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Design and spectral analysis of a six-axis shaker system

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DESIGN AND SPECTRAL ANALYSIS OF A SIX-AXIS SHAKER SYSTEM

by

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ABSTRACT

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Shaker table assemblies can exhibit resonances within their frequency range of operation. The spectral analysis of a six-axis shaker system permits the identification and prediction of resonance frequencies. In the project described here, a small six-axis shaker was tested experimentally. In parallel with the experiments, Finite Element (FE) models of the shaker were generated, and their dynamic performance was simulated. Time data from experimental test series were compared with the responses from FE analysis obtained for identical test conditions. The comparison of the response spectra between experiments and simulation permits the assessment and validation of FE analysis as a predictive tool for designing larger multi-axis shakers. The power density and coherence frequency response spectra for the entire 6x6 control matrix are computed in Matlab, creating a detailed performance assessment for all aspects of the 6x6 control matrix.A new FE model of a larger 6-axis shaker table has been created. The design seeks to

minimize the inertial table mass, while maintaining platform stiffness such that all eigenvalues are above the shaker table's operational frequency range.

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LIST OF ACRONYMS

C:	Damping coefficient
DFT:	Discrete Fourier Transform
DSP:	Digital Signal Processing
f:	Frequency
F:	Force
FEA:	Finite Element Analysis
FEM:	Finite Element Model
FFT:	Fast Fourier Transform
g:	Acceleration of gravity, 386.087 in./sec^2
G:	Power
Hz:	Hertz
ISO:	International Standards Organization
j:	Imaginary part
k:	Spring constant
m:	Mass
MDOF:	Multiple Degrees of Freedom
MIMO:	Multiple Input, Multiple Output
SISO:	Single Input, Single Output
t:	Time
x:	Linear displacement in X axis
y:	Linear displacement in Y axis
z:	Linear displacement in Z axis

LIST OF TERMINOLOGY

Acceleration	Vector quantity that specifies the time rate of velocity
Accelerometer	A transducer whose output is proportional to the acceleration input.
Amplitude	The maximum value of a sinusoidal quantity
Angular frequency	The angular frequency of a periodic quantity, in radians per unit time, is the frequency multiplied by 2Π.
Autocorrelation Function	The autocorrelation function of a signal is the average of the product of the value of the signal at time t with the value at time $t+\tau : R(\tau) = \overline{x(\tau)x(t+\tau)}$ For a stationary random signal of infinite duration, the power spectral density (except for a constant factor) is the cosine Fourier transform of the autocorrelation function.
Auto spectral Density (*power spectral density*)	The limiting mean-square value (e.g. of acceleration, velocity, displacement, stress, or other random variable) per unit bandwidth, i.e. the limit of the mean-square value in a given rectangular bandwidth divided by the bandwidth, as the bandwidth approaches zero.
Auxiliary Mass Damper (Damped Vibration Absorber)	A system consisting of a mass, spring, and damper which tend to reduce vibration by the dissipation of energy in the damper as a result of relative motion between the mass and the structure to which the damper is attached.
Center-of-Gravity	CG is the point through which passes the resultant of the weights of its component particles for all orientations of the body with respect to a gravitational field. If the gravitational field is uniform, the CG corresponds with the Center-of-Mass. Correlation Function: The correlation function of two variables is the average value of their product $(\overline{x_1(t)\,x_2(t)})$
Coupled Modes	Modes of vibration that are not independent but which influence one another because of energy transfer from one mode to the other.

Critical Damping	The minimum viscous damping that will allow a displaced system to return to its initial position without oscillation
Damper	A device used to reduce the magnitude of a shock or vibration by one or more energy dissipation methods.
Degrees-of-Freedom	The number of degrees-of-freedom of a mechanical system is equal to the minimum number of independent coordinates required to define completely the positions of all parts of the system at any instant of time. In general, it is equal to the number of independent displacements that are possible.
Deterministic Function	A deterministic function is one whose value at any time can be predicted from its value at any other time.
Displacement	A vector quantity that specifies the change of position of a body or particle and is usually measured from the mean position or position at rest.
Distortion	An undesirable change in waveform. Noise and certain desired changes in waveform, such as those resulting from modulation or detection, are not usually classed as distortion.
Dynamic Vibration Absorber (Tuned Damper)	An auxiliary mass-spring system which tends to neutralize vibration of a structure to which it is attached. The basic principle of operation is vibration out-of-phase with the vibration of such structure, thereby applying a counteracting force.
Effective Bandwidth	The effective bandwidth of a specified transmission system is the bandwidth of an ideal system which (1) has a uniform transmission in its pass band equal to the maximum transmission of the specified system and (2) transmits the same power as the specified system when the two systems are receiving equal input signals having a uniform distribution of energy at all frequencies.
Effective Mass	The complex ratio of force to acceleration during simple harmonic motion.
Equivalent System	An equivalent system is one that may be substituted for another system for the purpose of analysis.
Excitation	Excitation is an external force (or other input) applied to a system that causes the system to respond in some way.

Filter	A device for separating waves on the basis of their frequency. It introduces relatively small gain to waves in one or more frequency bands and relatively large gain to waves of their frequencies.
Forced Vibration	The oscillation of a system is forced if the response is imposed by the excitation. If the excitation is periodic and continuing, the oscillation is steady-state.
Foundation	Is a structure that supports the gravity load of a mechanical system. It may be fixed in space, or it may undergo a motion that provides excitation for the supported system.
Frequency	The frequency of a function periodic in time is the reciprocal of the period. The unit is the cycle per unit time and must be specified: the unit *cycle per second* is called *hertz* (Hz).
Induced Environments	Induced environments are those conditions generated as a result of the operation of a structure or equipment.
Isolation	A reduction in the capacity of a system to respond to an excitation, attained by the use of a resilient support.
Linear System	A system is linear if for every element in the system the response is proportional to the excitation. This definition implies that the dynamic properties of each element in the system can be represented by a set of linear differential equations with constant coefficients, and that for the system as a whole superposition holds.
Mechanical System	An aggregate of matter comprising a defined configuration of mass, stiffness, and damping.
Mode of Vibration	In a system undergoing vibration, a mode of vibration is a characteristic pattern assumed by the system in which the motion of every particle is simple harmonic with the same frequency. Two or more modes may exist concurrently in a multiple degree-of-freedom system.
Multiple Degrees-of-Freedom System	One for which two or more coordinates are required to define completely the position of the system at any instant.
Natural Frequency	The frequency of free vibration of a system. For a multiple degree-of-freedom system, the natural frequencies are the frequencies of the normal modes of vibration.

Oscillation	Oscillation is the variation, usually with time, of the magnitude of a quantity with respect to a specified reference when the magnitude is alternately greater and smaller than the reference
Random Vibration	The vibration whose instantaneous magnitude is not specified for any given instant of time. The instantaneous magnitude of a random vibration is specified only by probability distribution function giving the probable fraction of the total time the magnitude lies within a specified range
Resonance	Resonance of a system in forced vibration exists when any change, however small, in the frequency of excitation causes a decrease in the response of the system.
Stiffness	The ratio of change of force (or torque) to the corresponding change on translational (or rotational) deflection of an elastic element.
Time History	The magnitude of a quantity expressed as a function of time.
Transducer	A device which converts shock or vibratory motions into an optical, a mechanical, or most commonly to an electrical signal that is proportional to a parameter of the experienced motion.
Transfer Impedance	Transfer impedance between two points is the impedance involving the ratio of force to velocity when force is measured at one point and velocity at the other point. The term transfer impedance is also used to denote the ratio of force to velocity measured at the same point but in different directions.
Transmissibility	The nondimensional ratio of the response amplitude of a system in steady-state forced vibration to the excitation amplitude.
Uncorrelated	Two signals or variables $x_1(t)$ and $x_2(t)$ are said to be uncorrelated if the average value of their product is zero. If the correlation coefficient is unity, then the signals are said to be completely correlated.
Vibration	An oscillation wherein the quantity is a parameter that defines the motion of a mechanical system.
Vibration Machine	A device for subjecting a mechanical system to a controlled

and reproducible mechanical vibration.

Viscous Damping	The dissipation of energy that occurs when a particle in a vibrating system is resisted by a force that has a magnitude proportional to the magnitude of the velocity of the particle and direction opposite to the direction of the particle.

CHAPTER 1

INTRODUCTION

1.1 Problem Definition

The project objective is the design and spectral analysis of a six-axis shaker system. The shaker should meet the following specifications:

1. The Shaker system should be designed to perform six Degrees of Freedom vibration.
2. Accelerations in six directions simultaneously to 20 g pk-pk at maximum pay load.
3. Frequency range 10 Hz - 3000 Hz
4. Payload up to 25 pounds
5. We should develop a sufficiently complete Finite Element Model of the Team electrodynamic shaker so that experiments and FE results match over the range of experiments within a narrow error band.

Shakers, vibration and shock testing equipment are force generators or transducers that provide a vibration, shock or modal excitation source for testing and analysis. Shakers are used to determine product or component performance under vibration or shock loads, detect flaws through modal analysis, verify product designs, measure structural fatigue of a system or material or simulate the shock or vibration conditions found in aerospace, transportation or other areas.

Shakers can operate under various principles. Mechanical shakers use a motor with an eccentric on the shaft to generate vibration. Electrodynamic models use an electromagnet to create force and vibration. Hydraulic systems are useful when large forces and amplitudes are required, such as in testing large aerospace or marine structures or when the magnetic fields of electro dynamic generators cannot be tolerated. Pneumatic systems, known as air hammer tables; use pressure air to drive a table. Piezoelectric shakers work by applying an electrical charge and voltage to a sensitive piezoelectric crystal or ceramic element to generate deformation and motion.

Common features of shakers consist of an integral slip table and active suspension. An integral slip allows horizontal or both horizontal and vertical sample testing. The slip table is a large flat plate that rests on an oil film placed on a granite slab or other stable base. An active suspension system compensates for environmental or floating platform variations.

The most important specifications for shakers are peak sinusoidal force, frequency range, displacement, peak acceleration and peak velocity. Some of these specifications may be ratings without a load, as the manufacturers cannot always predict how the shakers will be used.

The three main test modes shakers can have are random vibration, sine wave vibration and shock or pulse mode. In a random vibration test mode, the force and velocity of the table and test sample will vary randomly over time. A sine wave test mode varies the force and velocity of the table and test sample sinusoidal over time. In a shock test mode, the test sample is exposed to high amplitude pulses of force.

1.2 Shaker Selection

The shaker selection begins with understanding what is suitable or needed for the test being conducted. The three types of shaker systems that are available include electro dynamic, hydraulic and mechanical, which are typically used for random vibration, sine vibration and many types of shock tests. The major specifications to consider are frequency range and waveform, displacement and velocity, the number of axis to be tested, payloads and vibration levels, and force rating. This information is got from Reference [5].

1.2a Frequency Range

Shakers can be become unstable or unpredictable at higher frequencies. Since project testing is conducted in a 2 KHz frequency range, we cannot use a hydraulic shaker, suitable for the frequency range of 500 Hz; instead we use an electro dynamic shaker. The electro dynamic shaker can provide reliable and accurate results at up to 3 KHz, making it an ideal choice.

1.2b Displacement and Velocity

Shakers have definite limits on the vibration levels they can produce. These limits are quantified in terms of acceleration, velocity and displacement. With sinusoidal motion, simple formulas define the relationship between displacement in inches, velocity in inches per seconds, acceleration and frequency in hertz.

1.2c Number of Axes Tested

The decision to test in one, two or three axes is application-specific. Many tests are performed in only one axis, yet others require two or even three axes of vibration. Multi-axis testing can be done sequentially or simultaneously, which can significantly affect the selection of the shaker system. The main advantage of this multi-axis shaker is its ability to reduce the test time [Ref 6].

1.2d Payloads and Vibration Levels

A certain amount of force is required to cause a particular payload and armature to vibrate at a given vibration level. This can be stated from Newton's law, $F = ma = wg$. The force, F, is required to accelerate the weight, w, at a certain vibration level, g. The project at hand will be conducted at a 20 g vibration level. If a hydraulic shaker were selected, we must account for the piston weight in the equation $F = wg$, since the piston of the hydraulic shaker also vibrates. This excessive force required may cause some surprises while running the simulation. Hence we can prefer electro dynamic shaker system.

Experimental values can be obtained from this electro dynamic shaker. However, it is not possible to conduct the entire experiment every time for the small modifications or the results expected. It is highly expensive, time consuming and involves excessive labor. The best way to correlate this work is Finite Element Analysis (FEA), which is highly reliable and predictable.

The name it self defines as dividing the model in to number of elements, i.e. finite elements. A study for each and individual element is performed to get most reliable output. Various software is used in FEA, but here for the project at hand MSC.Patran and MSC.Patran was used. We generate the same model in FEA and create same boundary conditions that the real modal has. By applying same forces as the real modal has driving forces, we will get the output values. Just like we measure acceleration values by accelerometer for real modal, we can get output acceleration values. Two test series have been completed so far, and we are trying to study the results with margin error. Once proper results are obtained, FE analysis can be applied for any additional modifications of the model.

CHAPTER 2

COMPONENTS OF THE VIBRATION SYSTEM

Vibratory motion is a key characteristic of many physical systems. The structure of physical systems can be modeled as mass and spring like elements that are connected together. Thus, vibrating system consist of (1) mass and (2) elasticity components. The energy of the system consists of the kinetic energy of the mass elements and drives the spring like elements that possess potential or stored energy. The spring-like elements are therefore the elastic part of the vibration system which determine the strength and position of the vibratory motion experienced. In constant or fluctuating vibration motion, the energy is the system consistently shifts between potential and kinetic energy. In the absence of any external force, a vibrating system can potentially experience constant vibration motion forever; nevertheless, although theoretically correct, everlasting vibration motion is very unlikely to occur.

The inclusion of a third key element, damping, dissipates energy in the form of heat. For example, a free vibration will eventually have all its internal energy dissipated due to the damping and bring the vibration to a halt [Ref 12].

2.1 Simple Harmonic Motion

Oscillation is the most common form of motion exhibited most by simple mechanical systems. Such motion is called simple harmonic motion. Simple harmonic motion consists of a particle that begins at rest or equilibrium position and achieves maximum displacement in the opposite direction and starts to switch position from each direction. The motion this particle experiences can be described mathematically as a sinusoidal function,

$$y(t) = Y_p \sin \omega t \tag{2.1}$$

where,

Y_p = maximum displacement amplitude (inches or meters)

ω = angular frequency of oscillation (radians/second)

t = time (seconds)

The angular frequency ω can be expressed,

$$\omega = 2\pi f \text{ or } f = \omega/2\pi \tag{2.2}$$

where,

π = 3.141593

f = frequency (cycles/second or Hz)

The frequency f represents the number of complete cycles of oscillations an object makes in one second.

$$T = 1/f \tag{2.3}$$

The period of oscillation represents the time it takes an object to complete one cycle of oscillation.

2.2 Relationship between Displacement, Velocity, and Acceleration Signals

It is important to understand the relation between the displacement, velocity and acceleration signals of the vibration of a simple mechanical system.

Displacement is the distance that a physical object moves with respect to its initial position. Static displacement refers to a shift in the physical position of an object, for example, a car that has moved from one city to another. Here, displacement is called distance.

Dynamic displacement is the magnitude of the vibration of an object or a portion of an object, for example, how much the wing tips of an aircraft move up and down during flight. Dynamic displacement can be measured directly using displacement transducers or by using accelerometers and double integration of the acceleration signal. Let the displacement $y(t)$ be described by a cosine function:

$$y(t) = Y \cos \omega t \tag{2.4}$$

The velocity of the system is the time rate of change of displacement,

$$v(t) = \frac{dy(t)}{dt} \tag{2.5}$$

Or ,

$$v(t) = -\omega Y \sin \omega t \tag{2.6}$$

The velocity can be expressed,

$$\dot{y}(t) = \omega Y \cos(\omega t + \frac{\pi}{2}) \tag{2.7}$$

Which indicates the velocity leads the displacement by a phase of 90° degrees. The acceleration is the time rate of change of velocity and is the second derivative of the displacement signal with respect to time,

$$a(t) = \frac{d^2 y(t)}{dt^2} \tag{2.8}$$

or,

$$\ddot{y}(t) = -\omega^2 Y \cos \omega t \tag{2.9}$$

This indicates the acceleration lead the velocity by a phase 90° and the displacement by a phase 180°.

2.3 Damping Elements

Vibration systems often contain damping elements. If the damping element is a viscous damping element, the force acting through a translational damper is,

$$f_d = Cv \text{ or } f_d = Cx \tag{2.10}$$

where,

f_d = damping force (lb_f or N)

C = viscous damping coefficient (lb_f-s/in) or N-s/m) of the damping element

x = displacement (inches or meters)

v = velocity (in/s or m/s)

2.4 Free Vibration with Coulomb Damping

Coulomb damping results from the friction force associated with two surfaces in contact moving relative to each other. The friction force is equal to the normal force on the sliding surface associated with the weight of the mass times the surface friction coefficient. When the mass is starting from a "stopped" position, the friction coefficient is the static friction coefficient, U_s. When the mass is moving, the friction coefficient is the dynamic friction coefficient, U. The friction force always acts in a direction opposite the velocity of the mass and is independent of the amplitude of the velocity and displacement of the mass.

2.5 Free Vibration with Structural Damping

All mechanical systems possess mechanisms for dissipating energy although some systems may not have damping elements. Many structural materials exhibit a stress-strain relationship characterized by a hysteresis loop when subjected to cyclic stresses below their elastic limits. The energy dissipated per cycle is associated with the internal friction, and is proportional to the area within the hysteresis loop. This type of damping is referred to as hysteresis or structural damping and the corresponding damping force is proportional to the elastic or spring force.

2.6 Deterministic and Random Vibration Signals

Deterministic and random vibration signals are two classes of signals that can excite vibration systems. Deterministic signals can be expressed by explicit mathematical functions and can be broken down into periodic and non-periodic signals.

Random signals are more complex and are described by statistical functions rather than explicit mathematical functions. Random signals can be divided into stationary and non-stationary signals.

Periodic signals repeat themselves over specified time intervals, T such that

$$f(t) = f(t+T) \tag{2.11}$$

Harmonic or sinusoidal and complex periodic or Fourier series signals are the two types of periodic signals that have been discussed in the preceding sections. The response of vibration systems to harmonic signals can be more easily described and understood from an analytical perspective. Furthermore, the response of a linear system to many types of complex deterministic signals and to some types of random signals can be described by using the principle of superposition to sum the individual responses of the system to the series of harmonic components that often make up a these signals.

Non-periodic signals can be divided into almost periodic and transient signals. Almost periodic signals are very similar to complex periodic signals. Whereas in a complex periodic signal all of the higher harmonics that make up the signal are integral multiples of the fundamental or lowest frequency component of the signal, in an almost periodic signal there will be at least one and possibly more higher frequency components in the signal that will not be an integral multiple of the lowest frequency component of the signal. Transient signals usually occur for a brief period of time, long or short, and then disappear. Transient signals can be described using Fourier and Laplace transforms.

Certain types of signals cannot be described by explicit mathematical functions. For example, if an identical experiment is repeated many times and the measured outputs always differ, the process is probably random. Random signals must be described in terms of probability statements and statistical functions. Random signals may be either stationary or non-stationary. If several sample lengths of a data record of a process, which form an ensemble of individual data records from the overall record, and if the results have the same statistical properties as other ensembles of data from the same data record, then the process is stationary. Otherwise, the process is non-stationary.

Four principal types of statistical functions are generally used to describe the basic properties of random signals. They are mean squared values and variances, probability and probability density functions, correlation functions, and spectral density functions. The mean squared values and variances furnish an elementary statistical description of the overall amplitude of a signal. The probability and probability density functions yield more specific information with respect to the statistical properties of a signal in the amplitude domain. The correlation and power spectral density functions furnish information concerning the statistical properties of a signal in the time and frequency domains, respectively.

CHAPTER 3

FINITE ELEMENT ANALYSIS

Finite elements are mainly defined by their shape and their properties. The two shapes and properties supported depend on the analysis program that uses in MSC.Patran, as defined in Analysis Preferences. This information is got from the basic concepts and definitions [Ref 13].

When creating a finite element mesh using the Finite Elements application form, elements are defined purely in terms of their topology. Other properties such as materials, thickness and behavior types are then defined for these elements in subsequent applications. The structural uses column describes typical usage conditions for the element shapes.

3.1 Mesh Generation Techniques

There are four basic mesh generation techniques available in MSC.Patran: IsoMesh, Paver Mesh, Auto TetMesh, and 2-1/2D Meshing. This section describes each meshing technique. Selecting the right technique for a particular model must be based on geometry, model topology, analysis objective, and engineering judgment, [Ref 13]. The model shown in Figure 1 is the meshed geometry, using the IsoMesh generation technique.

Figure 1: Meshed Geometry

The FE model that used for the simulation was defined with 16,000 elements and 30,000 degrees-of-freedom. The major difference between the two FE models is in the way the contacts have been defined.

The FE model also consists of 12 small rigid surfaces shown in figure 2. They don't have any mass. The contact between the deformable cruciform and these rigid surfaces is defined as sliding contacts with a friction coefficient of zero. Due to this type of contact definition, the FE model is non-linear and thus a complex system to analyze. We are using MSC.Marc along with MSC.Nastran to conduct the non-linear simulations.

The springs are modeled by beam elements. These springs are attached to the center of the rigid surfaces. The other ends of these beam elements are attached to another smaller beam element. One end of these smaller beam elements is completely fixed to the ground while the other end has 4 degrees of freedom: three rotational and one translational. The spring and the rigid plate are constrained in their respective X-Y plane.

The beam elements are made of spring-steel material. The length of the beam representing the actuators and the preload piston vary slightly so that the difference in their volume reflects the difference in their actual mass accurately.

A preload of 90 lbs is applied at the end of each spring element. The sinusoidal excitation force is applied at the center of the surface where the beam is connected.

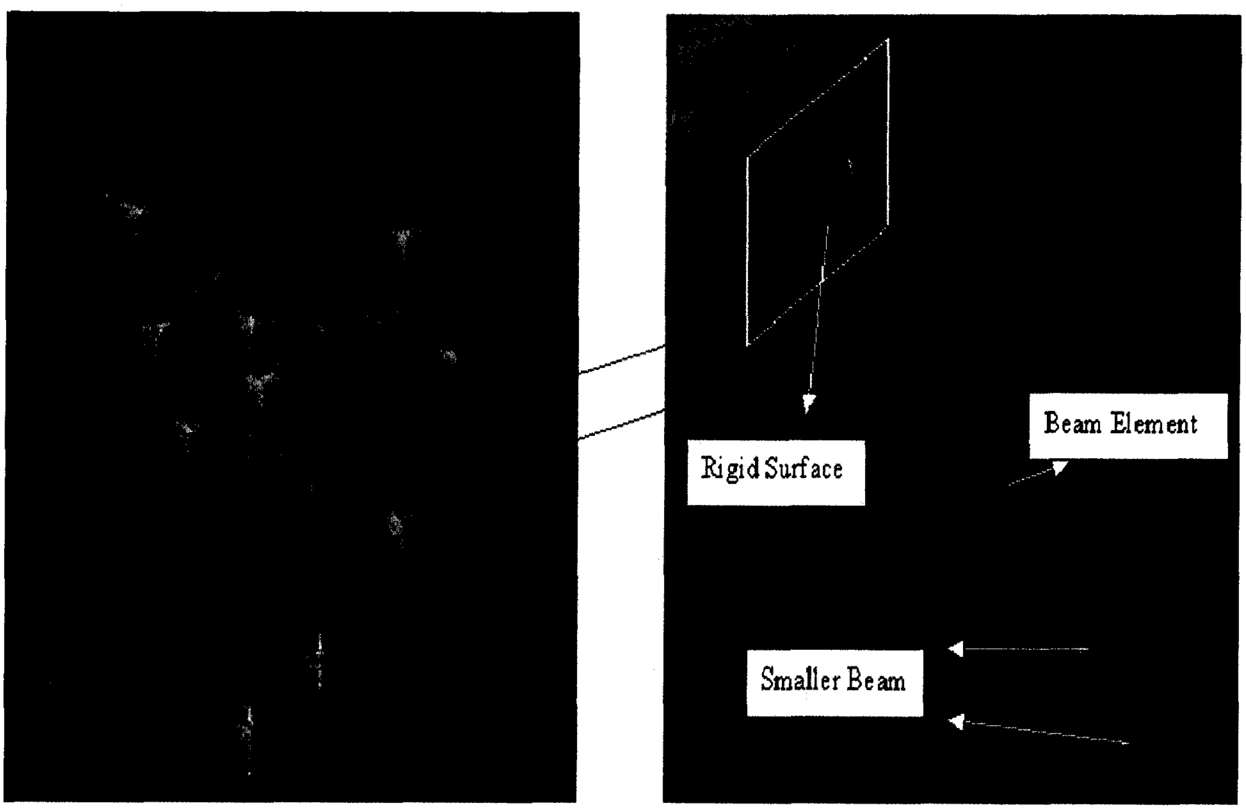

Figure 2: Model with Rigid Surfaces and Beam Elements

3.2 Transient Dynamic Analysis

Transient dynamic analysis solves for the dynamic response (displacements, velocities, accelerations, stresses and strains) of a structure subjected to the action of time dependent loads. The basic equation of the transient dynamic analysis of the dynamic equilibrium equation is,

$$[K]U(t)+[C]\dot{U}(t)+[M]\ddot{U}(t)=F(t) \quad (3.1)$$

where,

[K], [M] and [C]	the stiffness, mass and damping matrices respectively
$U(t)\ \dot{U}(t)$ *and* $\ddot{U}(t)$	the displacements, velocities and accelerations at time t
F(t)	the force vector acting at time t

It is implicitly assumed that the stiffness, mass and damping matrices remain constant (linear transient dynamic analysis) [Ref 13].

3.3 Modal Transient Dynamic

The step-by-step direct integration method of solving the dynamic equilibrium *equation 3.1* can be very costly for large models, especially if the integration has to be carried out for many time steps. In general, at every time step we have to solve the resulting n (n = number of model degrees-of-freedom) simultaneous equations. In the modal method, we use the modal superposition principle to transform the dynamic equilibrium *equation 3.1* in terms of the modal coordinates.

Let,

$$U(t)=[\phi]X(t) \qquad (3.2)$$

where,.

$[\phi]$ — The matrix of the m (mass-orthonormalized) eigenvectors.

X(t) — A vector of m components (generalized displacements).

m — The number of vibration modes (eigenvectors).

Substituting *equation 3.2* into equation *3.1* and pre-multiplying by $[\phi]^T$, we obtain,

$$[K_r]X(t)=[C_r]\dot{X}(t)+[M_r]\ddot{X}(t)=F_r(t) \quad (3.3)$$

where,

$$[K_r]=[\phi]^T[K][\phi] \quad (3.4)$$

$$[M_r]=[\phi]^T[M][\phi] \quad (3.5)$$

$$[C_r]=[\phi]^T[C][\phi] \quad (3.6)$$

$$[F_r]=[\phi]^T[F][\phi] \quad (3.7)$$

The objective of the transformation is to reduce the order of the stiffness, mass and damping matrices. Usually *m* is much smaller than *n* ($m \ll n$). A special advantage of using the normalized mode shapes $[\phi]$ as the transformation matrix is that the reduced matrices $[K_r]$, $[M_r]$ and $[C_r]$ are all diagonal. Thus, ignoring damping for the time being,

$$[M_r]=[I](m\times m) \quad (3.8)$$

and,

$$[K_r]=[\omega]^2(m\times m) \quad (3.9)$$

Consequently, the m equations in equation 3.3 are uncoupled. The resulting uncoupled equations can be solved by using the Newmark beta method, Wilson theta method, or any other integration scheme.

Having calculated the generalized displacements, velocities and accelerations $X(t)$, $\dot{X}(t)$ and $\ddot{X}(t)$, we can obtain the response of the structure $U(t)$, $\dot{U}(t)$ and $\ddot{U}(t)$ as,

$$U(t)=[\phi]X(t) \tag{3.10}$$

$$\dot{U}(t)=[\phi]\dot{X}(t) \tag{3.11}$$

$$\ddot{U}(t)=[\phi]\ddot{X}(t) \tag{3.12}$$

3.4 Solution Type

The main solution types that are available in MSC.Patran are described as follows,

Static:

This solution requests a linear static analysis.

Modal:

This solution requests a modal vibration analysis using one of three eigensolvers.

Bifurcation Buckling:

This solution requests a bifurcation buckling analysis. This can also be run as a restart from a linear, static analysis.

Steady-State Heat Transfer:

This solution requests a steady state heat transfer conduction and/or convection analysis for constant loads and properties.

Nonlinear State:

This solution requests a material and/or geometric nonlinear, static analysis.

Direct Linear Transient:

This solution requests a linear, transient dynamic structural analysis.

Modal Linear Transient:

This solution requests a linear, transient dynamic structural analysis as a restart from a Modal analysis.

Frequency Response:

This solution requests a linear frequency response analysis. This analysis type is always run as a restart to a modal analysis.

Shock Spectrum:

This solution requests a linear shock spectrum analysis. This analysis type is always run as a restart to a modal analysis.

Design Sensitivity:

This solution requests a design sensitivity structural analysis. This analysis type is always run as a restart to a static analysis.

3.5 Free Vibration Analysis

MSC.Patran FEA can perform free vibration analysis to compute the natural frequencies and associated mode shapes of linear elastic structures. The structure is assumed to be initially unstressed. A real eigenvalue analysis is performed, which assumes that there is no damping and that the structure is not spinning (i.e., no Coriolis force).

3.6 Eigen Solution

MSC.Patran FEA has three eigensolvers: subspace iteration, the Householder-QL method (which can be used with or without Irons-Guyan reduction), and the Lanczos method for solving problems in free vibration and bifurcation buckling [Ref13].

The Subspace Iteration Technique

The subspace iteration technique, which is a special form of the Rayleigh-Ritz method, is a very efficient eigensolver for cases when a small number of the system's lowest eigenpairs are sought. The essential steps of this technique for solving the generalized eigenvalue problem are as follows:

1. Choose some trial load vectors $\{P\}_i$
2. Compute a trial displacement vector $\{X\}_i$ for each trial "load" vector $\{P\}_i$ by solving

$$[K][X]_i = \{P\}_i \tag{3.13}$$

These displacement vectors will be used as trial eigenvectors.

3. Use the trial vectors $\{X\}_i$ to form the reduced eigenvalue problem,

$$\left(\left[K^*\right]-\omega^2\left[M^*\right]\right)\{\psi\}=0 \tag{3.14}$$

Where $[K^*]$ and $[M^*]$ are reduced system stiffness and mass matrices, respectively, defined as,

$$\left[K^*\right]=[X]^T[K][X] \tag{3.15}$$

$$\left[M^{*}\right]=[X]^{T}[M][X] \tag{3.16}$$

4. Solve this reduced eigenvalue problem. When the number of eigenpairs sought is much less than the number of degrees-of-freedom in the model, the computer time necessary to solve this reduced problem is significantly less than would be required to solve the much larger original problem. MSC.Patran FEA uses the QL method to solve the reduced eigenvalue problem.

5. If the eigenvalues have converged, then use the eigenvectors of the reduced system to calculate the eigenvectors of the original system:

$$\{\delta\}_i=\{X\}_i\{\psi\}_i \tag{3.17}$$

6. If further iteration is required, calculate new load vectors $\{P\}_i$ from the inertial forces generated by the eigenvectors $\{\psi\}$ of the reduced problem,

$$\{P\}_i=[M][X]\{\psi\}_i \tag{3.18}$$

steps 2 through 6 repeat until the eigenvalues have converged to the desired accuracy, or for a pre-assigned number of iterations. The convergence tolerance used by MSC.Patran FEA is the fractional change in eigenvalues in continuous iterations to the next.

CHAPTER 4

SIGNALS

4.1 Random Signals

Random signals occur when the data representing a physical phenomenon cannot be predicted or is random and when the future time history record cannot be generated or predicted without experimental error. The collection of all time history records can be described as $x_i(t)$, $i = 1, 2, 3 \ldots$, which is produced by the experiment is considered the ensemble that defines a random process $\{x(t)\}$ describing the phenomenon.

4.2 Statistical Sampling Considerations

The number of time history records that might be available for analysis by ensemble-averaging procedures, or the length of a given sample record available for analysis by time-averaging procedures, will always be finite. It follows that the average values of the data can only be estimated and never computed exactly.

4.3 Transient Inputs

Transient data is unique as it is the only type of non-stationary data that results from a short duration non-stationary phenomena with defined commencement and ending. Using similar techniques of that for stationary data, we can easily interpret and analyze transient data.

4.4 Frequency Response

Frequency response is the gain and phase response of a circuit or other unit under test at all frequencies of interest. In common usage though, only the magnitude or gain is considered important when referring to frequency response. The frequency response, $H(f)$, is defined as the inverse Fourier Transform of the Impulse Response, $h(\tau)$, of a system,

$$H(f)=\int_{-\infty}^{\infty}h(\tau)e^{-j2\pi f\tau}d\tau \tag{4.1}$$

Frequency response measurements require the excitation of the UUT with energy at all relevant frequencies. The fastest way to perform the measurement is to use a broadband excitation signal that excites all frequencies simultaneous, and use FFT techniques to measure at all of these frequencies at the same time. Noise and non-linearity is best minimized by using random noise excitation, but short impulses or rapid sweeps may also be used. When a resolution bandwidth of less than about 100 kHz is obtained, the fastest way to measure the frequency response functions is to use FFT based techniques.

Units: Comparison of Decibel (dB) and/or Velocity/Angular Velocity (V, Radians, or Degrees) to Hertz (Hz).

Approach 1: Sine Generator/Voltmeter

Apply a sine wave to the input of the system under test and measure the output voltage and repeat this process for each frequency. The gain of the system is the ratio of the output voltage to the input voltage.

Approach 2: Transient or Noise Excitation with Cross Spectral Techniques

You can use any signal that contains frequency components in the range of interest. The signals aren't required to have the same amplitude. However, all measurements using Cross Spectral Techniques require simultaneous measurement of both input and output signals, using simultaneously sampling A/D Converters. The frequency response, H_{xy}, can be computed as,

$$H_{xy} = G_{xy} / G_{xx} \tag{4.2}$$

where, G_{xy} is the cross spectrum and G_{xx} is the auto spectrum of the input.

This technique computes the correlation between the input and output signals (as a function of frequency) and hence, rejects noise and distortion. The more statistical samples that are included in the averaging yields greater noise and distortion rejection and hence, making the measurement more accurate. The resulting statistical function, called the *cross spectrum*, is then normalized for the actual amplitude of the signal at each frequency on the input (called the *auto spectrum*, or more commonly, the *averaged spectrum*). This gives the Frequency Response Function (FRF), which contains both magnitude and phase information. The magnitude is typically shown on a logarithmic *Y* axis (in dB), and the phase is often shown on a 0 to 360 degree scale. In systems with output noise, the most accurate evaluation of resonance peaks is made using the *H2* method of frequency response computation, whereas the *H1* technique gives the best response for anti-resonances. *H2* is also useful when inadequate resolution is used in the measurement of a resonance. Since the phase often shifts thousands of degrees, a technique called *phase unwrapping* is used to remove the discontinuities every time the phase jumps from 360 to 0 degrees.

This approach has the advantage of overcoming noise, distortion, and non correlated effects. It also corrects for any loading effects on the input to the system. In addition, the technique can be extremely rapid, because it measures all frequencies of interest simultaneously. Its only weakness is that its signal-to-noise ratio can be lower than the swept sine with tracking filter technique.

Approach 3: Naturally Occurring Excitation

Sometimes one cannot insert an excitation signal into the system to be tested. However, if one wants to measure the Frequency Response Function of a shock absorber in a car, one can use the naturally occurring "input signals" coming from bumps in the road as the excitation signals. Using cross spectral techniques, one can measure the input signal on the axle of the wheel and cross correlate it with the output signal picked up on the automotive chassis. Since the bumps are transients, they have relatively broad frequency components and make a broadband measurement possible. When making this measurement, one should take extreme care to account for triggering and windowing conditions, and also consider potential time delays between the input and output. Thus, this technique is only recommended for experienced professionals with a thorough understanding of digital signal processing techniques.

4.5 Gain

Gain is the factor in which a signal is amplified in terms of decibels, dB and is considered the magnitude of the frequency response function. Measure in dB or decibels gain can be written as mathematically function in the form of the logarithmic ratio of two signal amplitudes as shown below:

For signals in volts,

$$dB = 20\log\frac{V}{V_{ref}} \tag{4.3}$$

For power ratios,

$$dB = 10\log\frac{V^2}{V_{ref}^2} \tag{4.4}$$

Many quantities are expressed in decibels such as sound pressure level, sound power level, sound intensity level, transmission loss, and many more. Although they all have "units" of decibels, they are typically *not* interchangeable because they use different reference levels.

4.6 Phase

The shift of a periodic signal or phase is often measured with reference to its zero crossing, compared to a reference signal of the same frequency. One period of the signal is defined as having duration of 360 degree of two pi radians. On occasion a phase shift is often related to as a time delay; however, since phase shifts occur in both the positive and negative direction, great caution should be used in directly relating phase shift to a time shift. Many forms of apparent time shift of a waveform are actually due to a distortion of the waveform. Phase can be described using two units, either radians or degrees.

In circuit analysis, the slope of the phase curve, also known as the group delay, is used to characterize the delay of a circuit. Group delay can be measured using the phase portion of the frequency response function.

4.7 Frequency Spectrum

The representation of a signal in the frequency domain is called the frequency spectrum. A signal is broken into multiple periodic signals each consisting of a specific magnitude and phase. Recognizing repeated is a key aspect of the frequency spectrum of which the sine wave is the most ideal to use. Frequency spectra are broken into three main groups which appropriately define there application.

4.8 Types of Frequency Spectra

Frequency Spectrum also referred to as Instantaneous Spectra, Fourier Spectra, FFT Spectrum, or Complex Spectrum. This spectrum is measured in one black data with no averaging and consists of a number of periodic components (one vector per frequency) displayed as magnitude and phase information. The magnitude of the spectrum is measured in volts, V, or power, V2, and the entire spectrum is computed using an FFT, the frequency components will have linear spacing.

Averaged Complex Spectrum also referred to as Averaged Fourier Spectrum. The primary usage of the averaged complex spectra involves averaging the complex spectra, which already accounts for magnitude and phase, for repeated signals. In order to fully understand the signal, the averaging is done for both the real and imaginary components of the signal individually. This type of measurement picks out periodic frequency components and is very useful in removing noise that is not correlated to the repeated signal.

4.9 Power Spectrum

Power spectrum is also referred to as Auto Spectrum or Auto Power Spectrum.These terms refer to an average of the power of the individual frequency components over a number of instantaneous spectra. When this averaging occurs, the magnitude of each frequency component is squared, and added to the previous sum for that frequency. Hence, the phase information is discarded.

The averaging to compute a power spectrum does not reduce the unwanted noise in the system. However, it can be extremely useful in getting a reliable statistical estimate of the level of random signals being measured. For example, if one only displays the instantaneous spectrum of white noise, the spectrum will be extremely jagged. But after computing the power spectrum with adequate averaging, the shape of the spectrum will converge on a flat spectrum. Power spectra can be shown in units of power, or the square root of the averaged result can be taken to give a voltage value (which is the RMS value).

In addition, the power spectrum may be scaled in several different ways depending on the type of signal under analysis. They are Power Spectral Density (PSD) and Energy Spectral Density (ESD).

- PSD is used when measuring continuous broadband noise, and normalizes the power to an equivalent bandwidth of 1 Hz, irrespective of the actual bandwidth of the filter being used. For example, if a signal is measured at -93 dB in a 10 Hz bandwidth, then the spectral density would be -103 dB (in a 1 Hz bandwidth). This makes it possible to compare noise measurements made with different bandwidth settings of the spectrum analyzer.
- ESD is used to measure the energy of transient signals. Since transients also are spread out over a broad frequency range, they must be normalized to 1 Hz (as with noise). In addition, the duration of transients may vary significantly, so their

duration is also normalized to a standard equivalent duration of 1 second. This makes it possible to compare the spectra of different transients.

4.10 Fourier series and Transforms

The Fourier method is a mathematical technique used to analyze periodic signals. Consider any periodic record $x(t)$ of period T; then for any value of t,

$$X(t) = x(t \pm kT), \ where\ k = 1, 2, 3 \ldots \tag{4.5}$$

The fundamental frequency, f_1, satisfies, $f_1 = 1/T$.

Using the mathematical concept shown, we can expand such periodic data using the formula,

$$x(t) = \frac{a_o}{2} + \sum \{a(k) \times \cos(2\pi \times f(k)t) \times b(k) \times \sin(2\pi \times f(k)t)\} \tag{4.6}$$

where,

$$f(k) = k \times f_1 = k/T,\ k = 1, 2, 3 \ldots \tag{4.7}$$

Therefore, we can define $x(t)$ as a sinusoidal wave function at discrete frequencies spaced $df = f_1$ apart. The coefficients of the function, $a(k)$ and $b(k)$, can be obtained by calculating the following integrations over a defined period T, either $-T/2$ to $T/2$ or zero to T.

$$a(k) = 2/T \int_0^{\infty} \{x(t) \cos(2\pi \times f(k)t\} dt \quad k = 0, 1, 2 \ldots \tag{4.6}$$

$$b(k) = 2/T \int_0^{\infty} \{x(t) \sin(2\pi \times f(k)t\} dt \quad k = 0, 1, 2 \ldots \tag{4.7}$$

4.11 Spectral Analysis

Periodogram analysis can be a useful way of assessing whether there is a strong cyclic component in a time series. However, its most serious fault is that the sampling errors associated with estimates of sums of squares are quite large. Spectral analysis techniques were developed to reduce this problem of sampling error. A power spectrum is a slightly modified version of a periodogram. There are many different versions of spectral analysis that involve different ways of modifying the periodogram estimates to reduce their sampling error [Ref 4].

The term "smoothing" refers to a process in which each periodogram intensity is replaced by a weighted average that includes intensity estimates for a few neighboring frequencies. Smoothing procedures differ in two ways: First, the width of the "window", that is, the number of neighboring frequencies that are included in this weighted average, can vary. Second, the weights used for this weighted average can be of different forms; some smoothing windows give equal weight to all included frequencies, whereas others give more weight to frequencies near the centre of the window than to frequencies near the edges. When the weighting function is graphed, it may have various different shapes: for instance, a Daniell window looks like a rectangle, whereas many of the other popular smoothing windows have a bell shape. Thus, windows can vary in width and also in shape.

A power spectrum is a periodogram that has been smoothed, using one of many possible smoothing functions, or "windows", to reduce the sampling error. A periodogram partitions the variance of the overall time series into a discrete set of frequency components: the sum of squares associated with each frequency was called "intensity". In a spectrum these sums of squares are averaged together across neighboring frequencies to provide a smoother and more reliable estimate of the distribution of variance continuously across the entire range of frequencies from $1/N$ to ½. The term "power" is typically used to refer to the estimated amount of variance in the time series that is accounted for by a particular band of frequencies.

4.12 An Alternative Approach to Smoothing: The Bartlett Window

The other approach to the estimation of a spectrum is to calculate the lagged autocorrelation function (ACF) for a time series up to some lag M: then do an FFT or periodogram analysis on this lagged ACF. This method is called the Bartlett window. The set of Fourier frequencies that is fitted to the data is now based on *M*, the maximum lag in the ACF, instead of *N*, the number of observations in the original time series. A benefit of this approach is that the analyst can choose various values of *M* as a means of fitting different sets of periods to the data. This can be a way of avoiding the problem of leakage when N is not an integer multiple of the cycle length that the analyst is trying to detect, making it possible to vary the set of periods or Fourier frequencies that are fitted to the time series, while still making use of all the data.

This method of obtaining the spectrum (first computing the ACF to lag *M*, Then applying the FFT) is the procedure that Koopmans calls the Bartlett (1) window. For the resultant spectral estimates in this case the $edf = N/M$, where N is the number of observations in the original time series and M is the maximum lag in the ACF that is used as the basis for the FFT.

4.13 Fourier Transforms

The use of the Fourier Series was introduced for periodic signals and here will outline the use of Fourier Transforms to analyze non-periodic signals obtain from transient data, either deterministic or random, or for stationary random data. Consider the Fourier series as T approaches to infinity giving us the following Fourier integral,

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f(t)} dt \quad -\infty < f < \infty \tag{4.8}$$

The quantity $X(f)$ is called the Fourier transform or spectrum of $x(t)$. Alternatively, if $X(f)$ is known, the inverse Fourier transform will give you the $x(t)$ using the formula,

$$X(f) = \int_{-\infty}^{\infty} X(t) e^{j2\pi f(t)} df \quad -\infty < t < \infty \tag{4.9}$$

4.14 Correlation and Spectral Density Functions

4.14a Cross-Spectral Analysis

To perform a bi-variate or cross-spectral analysis for a pair of time series (denoted X and Y), it is first necessary to obtain the univariate spectrum for each of the individual time-series variables.

The cross-spectrum is essentially the cross-product of these two smoothed univariate spectra. (An alternative ways of estimating a cross-spectrum is to do a discrete Fourier transform of the lagged cross-correlation function (CCF) between X and Y time series.) The complex numbers that result are not directly interpretable but they are converted into an estimate of coherence, and an estimate of phase, for each of the N/2 frequencies in the spectrum. Coherence, like an R^2, indicates the percentage of shared variance between the two time series at a particular frequency. Phase, like a time lag, indicates the timing of peaks in the Y series relative to peaks in the X time series at a given frequency (however, phase is usually given in terms of fractions of a cycle, whereas time lag is usually given as the number of lagged observations).

A wide range of engineering applications of random data analysis centers on the determination of linear relationships between two or more sets of data. These linear relationships are generally extracted in terms of a correlation function or its Fourier transform, called a spectral density function. Correlation and spectral density functions provide basically the same information, but from a historical viewpoint, they evolved separately. Correlation functions were a product of mathematicians and statisticians, whereas spectral density functions were developed more directly as an engineering tool.

4.14b Correlation Functions

Often when analyzing a random signal, it is necessary to know the general dependence of the values of the signal at one instant in time to the values at another instant in time. For example, it may be necessary to identify periodic signals that are embedded in random noise. For a stationary random process, the autocorrelation function describes the general dependence of the values of the data at one time on the values on the other time, reference [12].

The autocorrelation function $R_x(T)$ associated with a time delay T between $x(t)$ and $x(t+T)$ is obtained by multiplying $x(t)$ by $x(t+T)$ and averaging over the sample time T, or,

$$R_x(t) = \lim(1/T)\int_0^T \{x(t) \times x(t+T)\}dt \qquad (4.10)$$

In terms of expected values, $R_x(T)$ is expressed,

$$R_x(t) = E[x(t) \times x(t+T)] \qquad (4.11)$$

The autocorrelation function is always a real-valued even function which has a maximum at t=0. In equation form,

$$R_x(T) = R_x(-T) \qquad (4.12)$$

The cross-correlation function of two sets of stationary random data describes the dependence of one set of random data upon the other. For stationary random process, the cross-correlation between two signals *x(t)* and *y(t)* can be determined by multiplying *x(t)* by *y(t + T)* and averaging the results over the sample time, *T*, or,

$$R_{xy}(t) = \lim(\tfrac{1}{T}) \int_0^T \{x(t) \times y(t+T)\} dt \qquad (4.12)$$

in terms of expected values, $R_{xy}(t)$ is expressed,

$$R_{xy}(t) = E[x(t) \times y(t+T)] \qquad (4.13)$$

The cross correlation function is always real-valued. However, $R_{xy}(T)$ may not always be maximum at T=0 and it is not an even function, as were both the cases for the autocorrelation function. Some useful relationships are,

$$R_{xy}(-T) = R_{yx}(T) \qquad (4.14)$$

4.15 Spectra via Correlation Function

The spectral density function between two time history records *x(t)* and *y(t)* representing stationary random processes *{x(t)}* and *{y(t)}* may be defined as the Fourier transform of the correlation function between those records as follows,

$$S_{xy}(f) = \int\{R_{xy}(\tau)e^{-j2\pi f(\tau)}\}d\tau \qquad (4.15)$$

For the general case where *x(t)* and *y(t)* represent different data, $S_{xy}(f)$ is called the cross-spectral density function, or more simply the cross-spectrum, between $x(t)$ and $y(t)$.

For the special case where *y(t) = x(t)*,

$$S_{xx}(f) = \int\{R_{xx}(\tau)e^{-j2\pi f(\tau)}\}d\tau \qquad (4.16)$$

$S_{xx}(f)$ *is* called the auto spectral density function or auto spectrum of $x(t)$, or sometimes the power spectral density function.

4.16 Coherence

Coherence is the frequency domain correlation between the input and output signals of a system. It is defined as the ratio of the correlated cross spectrum to the uncorrelated cross power, which includes all uncorrelated noise. A mathematical model of coherence is shown below.

$$\gamma_{xy}^2 = \frac{|G_{xy}|^2}{G_{xx}G_{yy}} \qquad (4.1)$$

A very useful tool, the coherence function can greatly assist in validating the quality of your frequency response measurement. To determine whether the system under test is being measured accurately, the coherence should slightly deviate above or below one. Furthermore, the coherence can also help to you determine measurement flaws and can act as a unique tool that keeps the system in check [Ref 12].

Examples of problems that the coherence will detect are:

UUT Problems

- Uncorrelated noise on the output
- Unstable system (Unit Under Test) characteristics

- Non-linear behavior of the Unit Under Test (distortion, inter-modulation)
- Loose cables and loose transducers on the output of the system.

Measurement Problems

- Inadequate frequency resolution
- Improper delay between channels.
- Incorrect frequency weighting windows
- Computational errors.
- Inadequate averaging time.

For the coherence to be meaningful; more than one measurement must be averaged. If only one average is made, the coherence, by definition is unity.

The random error in the gain of the frequency response is given by

$$\varepsilon = \frac{\left[1-\gamma_{xy}^{2}\right]^{\frac{1}{2}}}{|\gamma_{xy}|\sqrt{2\eta_{d}}}$$

From this it can be seen that with a high signal to noise ratio, the error is very low, and that averaging will reduce the error.

4.17 Propagation - Path Identification

To produce an accurate overall linear relationship, we must calculate the frequency response function within the input and output measurements. However, it will not in itself identify the contributions of individual paths and it will then be required to first distinguish clearly between frequency dispersive and non-dispersive propagations. In other words, we must determine whether or not the propagation speeds are a function of frequency.

4.18 Non-Stationary Data Analysis Techniques

A collection of time history records measured under statistically equivalent conditions are theoretically required to allow the time dependent properties of the data to be estimated at specific instants by ensemble-averaging procedures. There is a well-developed methodology for the analysis of non-stationary data using higher-order spectra.

Figure 3 illustrates the process. It explains each and every step of input signals and out put signals and from then, experimental and FEA values of out put values. From the time-histories from the experiments or FE simulations the auto-correlation, cross-correlation and the frequency response are computed, and finally the experimental and simulation results are compared.

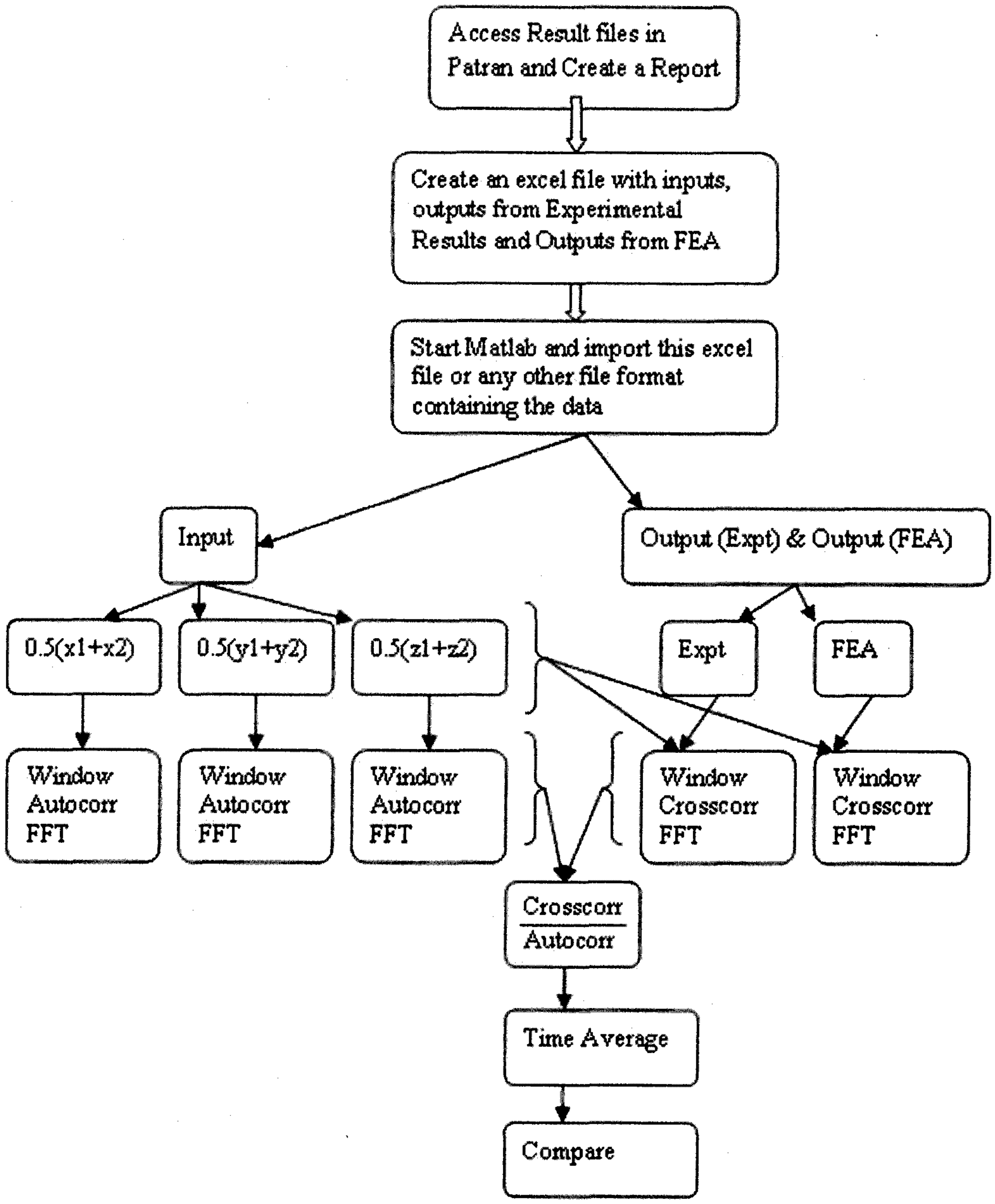

Figure 3: MATLAB Flowchart

CHAPTER 5

EXPERIMENTS AND ANALYSIS

5.1 Standard Analysis from 2005 Experiments

A linear finite element (FE) model was developed for the six-axis shaker system with the objective of comparing experimental results from Team Corp Tensor performance records with predictive modeling. Experiments were conducted under sinusoidal sweep and random multiple forcing function inputs.

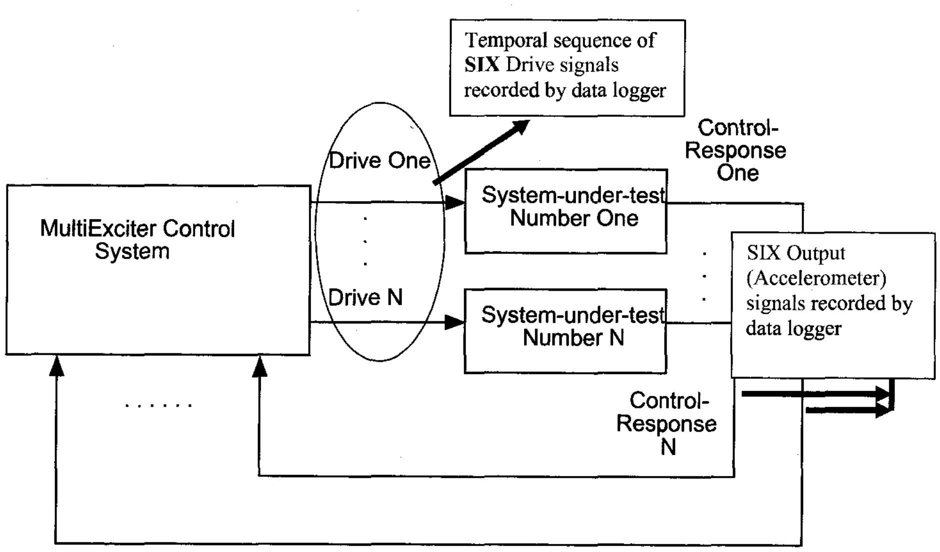

Figure 4: Shaker Experiment

The Figure 4 represents accelerometer time-domain responses to six drive inputs. TEAM Corp six-axis shaker. The controller maintains the signal amplitude within the user-specified limits by adjusting the six 'Drives' such that the output error (accelerometer Freq. Responses – trapezoidal reference spectra) is minimized.

5.1a Drive Forces on Finite Element Model

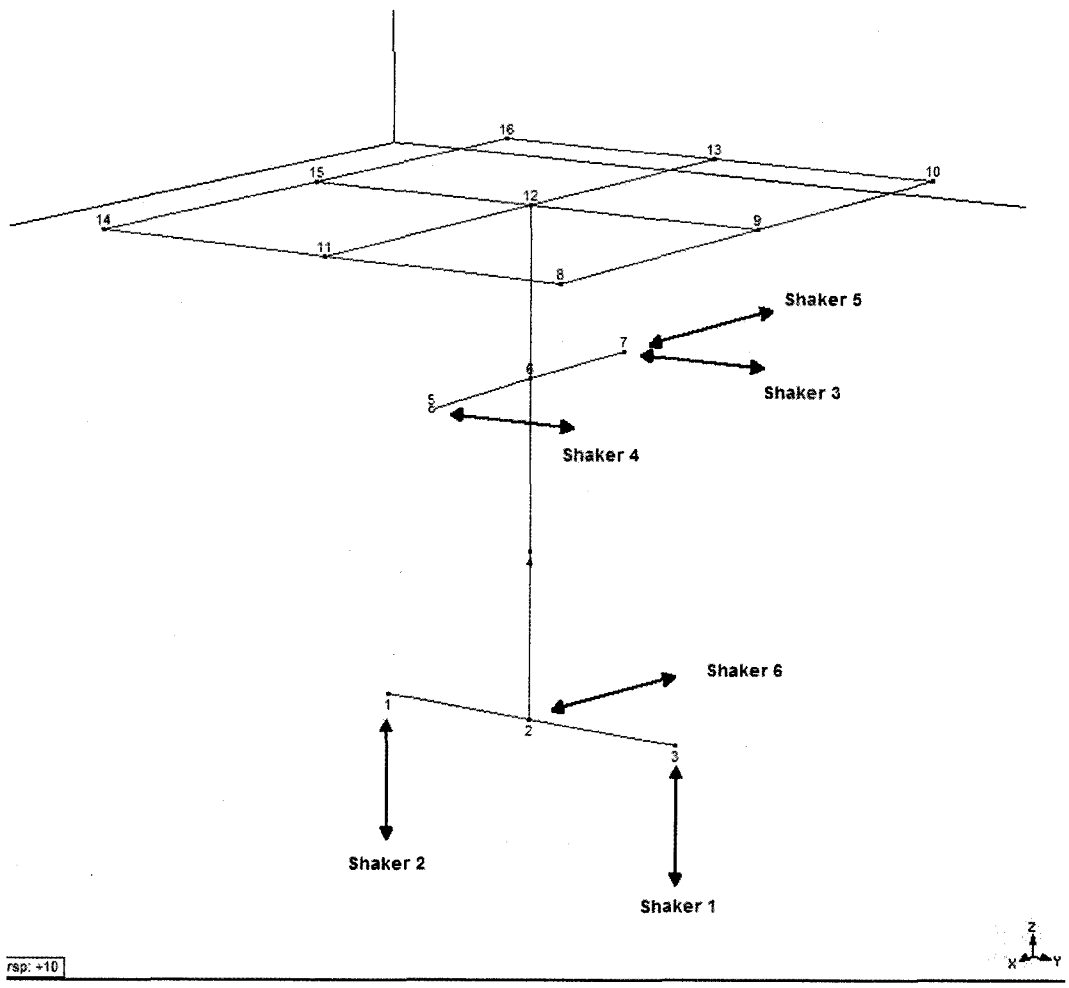

Figure 5: The Input Drive Forces on the Shaker System

There are six drive forces acting on the shaker system; two forces in each Cartesian direction. These drive forces are the input variables to the shaker system. The points where these drive forces are acting are shown in Figure 5.

The assignment of drive input forces to three Cartesian directions are shown in the transfer function matrix below.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \end{bmatrix} \tag{5.1}$$

The output variables of the shaker system are recorded accelerations at six distinct locations on the table top. The response accelerometers are 14z, 8z, 13z, 13y, 14y, and 13x these points are shown in figure 6.

Point 8	Table Top Corner
Point 12	Table Top Center
Point 13	Table Top rim midway between corners
Point 14	Table Top Corner

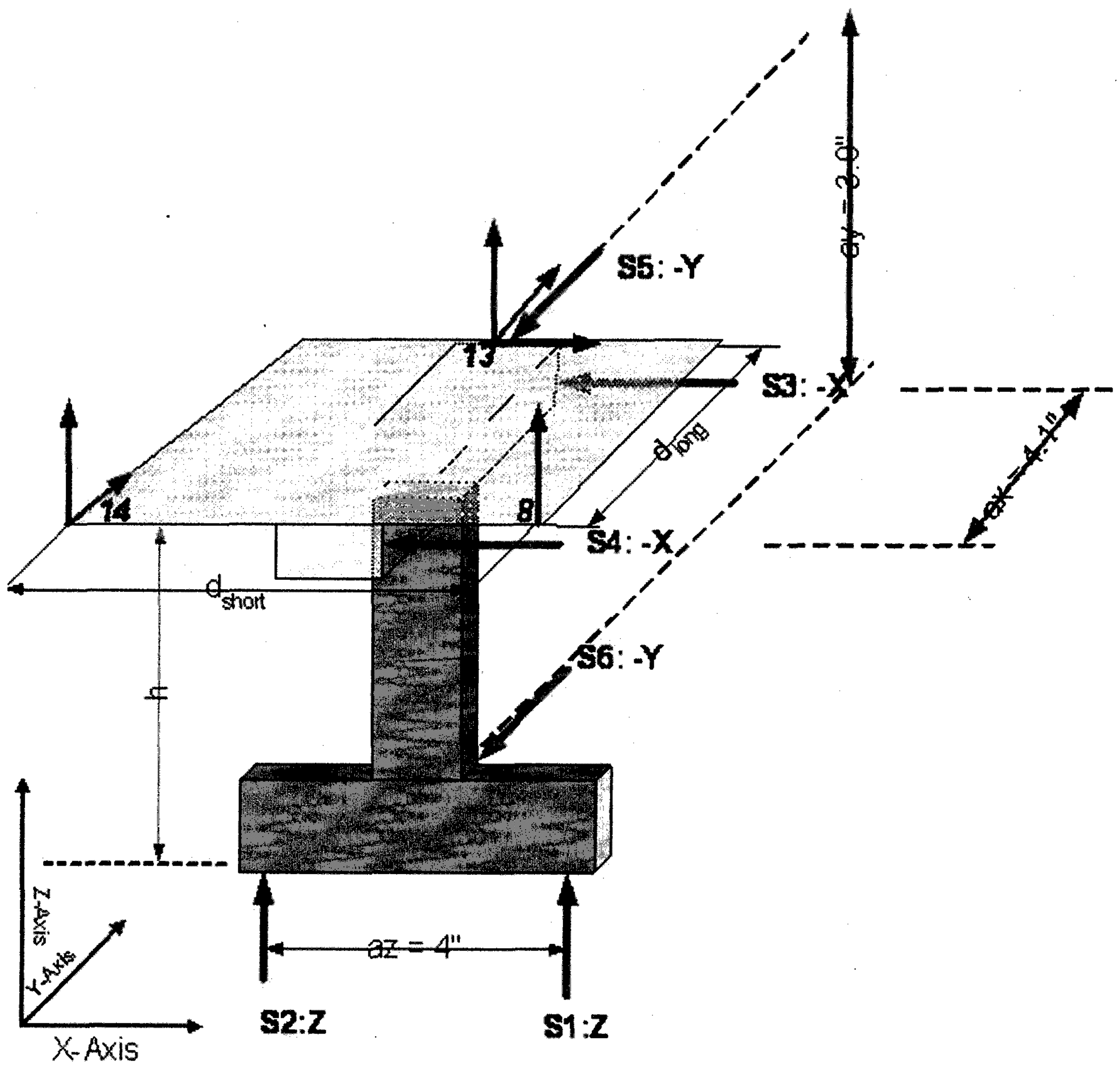

Figure 6: The Input Drive Forces and Output Accelerometers on the Shaker System

The model was excited in all six axes with random input signal without any coherence. The resultant input force and output acceleration signals from TEAM TENSOR time histories were recorded and stored on the Jaguar System's Throughput Disk (TPD) and this process is shown in figure 4 so clearly [Ref 9]. The recorded six-axis force-input time histories were applied to the finite element model.

5.1b Frequency Response Function Calculations

Consider a single input/single output system with extraneous noise at the output point. The noise represents many practical physical problems where the input measurement *x(t)* is essentially noise-free while the output measurement y*(t)* consists of the sum of the ideal linear output *v(t)* due to *x(t)*, plus all possible deviations *n(t)*. From measurements of *x(t)* and *y(t)*, the system frequency response function *H(f)* is calculated by,

$$H(f) = \frac{G_{xy}(f)}{G_{xx}(f)} \tag{5.2}$$

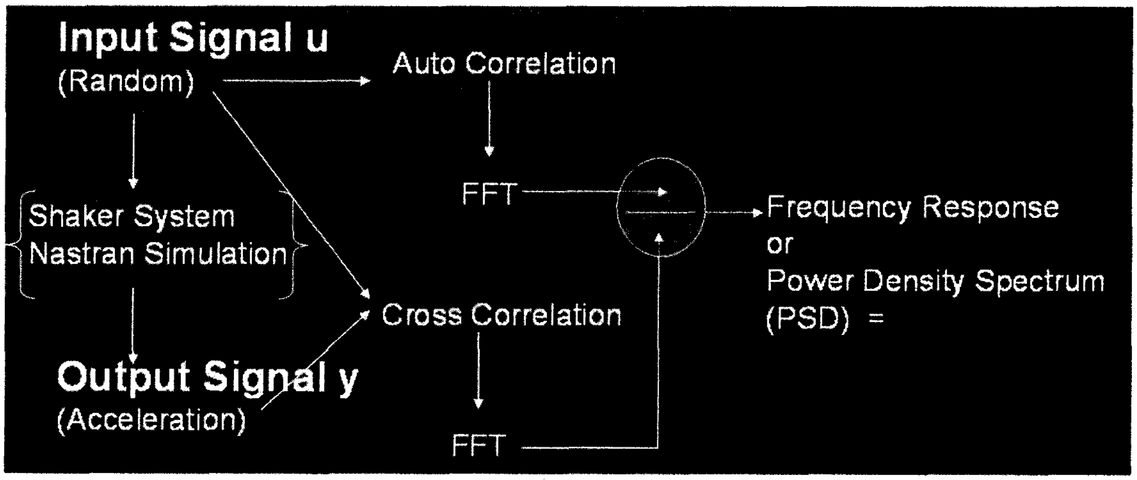

Figure 7: Mathematical Concepts for Frequency Response Analysis

The mathematical concepts for frequency response analysis follows the established methodology, as described for instance in Bendat and Piersol, 1980. The same methodology is implemented in the Jaguar controller's algorithms for real time analysis and shaker control. The auto and cross-spectra referenced in the above equation may be obtained either by using correlation methods or by using Discrete Fourier Transform techniques (DFT) [Ref 2].

In the following, the terminology of Fig. 7 will be used, which is consistent with that of the Jaguar Controller.

We define,

S_{uu} Auto spectral density function of input $u(t)$.

S_{uy} Cross spectral density function (input u, output y).

PSD or Power Density Spectrum $= \dfrac{S_{uu}}{S_{uy}}$

The following relationships that exist between recorded signals and the physical quantities represent:

Drive Signals: The sensitivity of the current sensor is $57.1 mV/\ lbf$. This conversion factor was used in all analyses to compute the drive force magnitudes at each actuator.

Accelerometers: The sensitivity of the Dytran triaxial accelerometers used in the experiments is $10 mV/\ g$ in each Cartesian axis direction. This conversion factor was used in all analyses to compute the acceleration magnitudes in each recorded direction and location.

In MIMO Random control, the reference is a matrix of spectral densities. The controller minimizes the error by modifying the impedance matrix, $Z(f)$ which is the inverse of the system-under-test's frequency response matrix.

The Output Power spectrum (PSD or Frequency Response) of a signal y due to an input u is found as the cross-correlation, S_{uy}, divided by the input autocorrelation, S_{uu},

$$PSD = \frac{S_{uu}}{S_{uy}}$$

The graphs below represent the frequency response values for the October 2005 and calculating the frequency response values for the output values at point 8 in Z-direction.

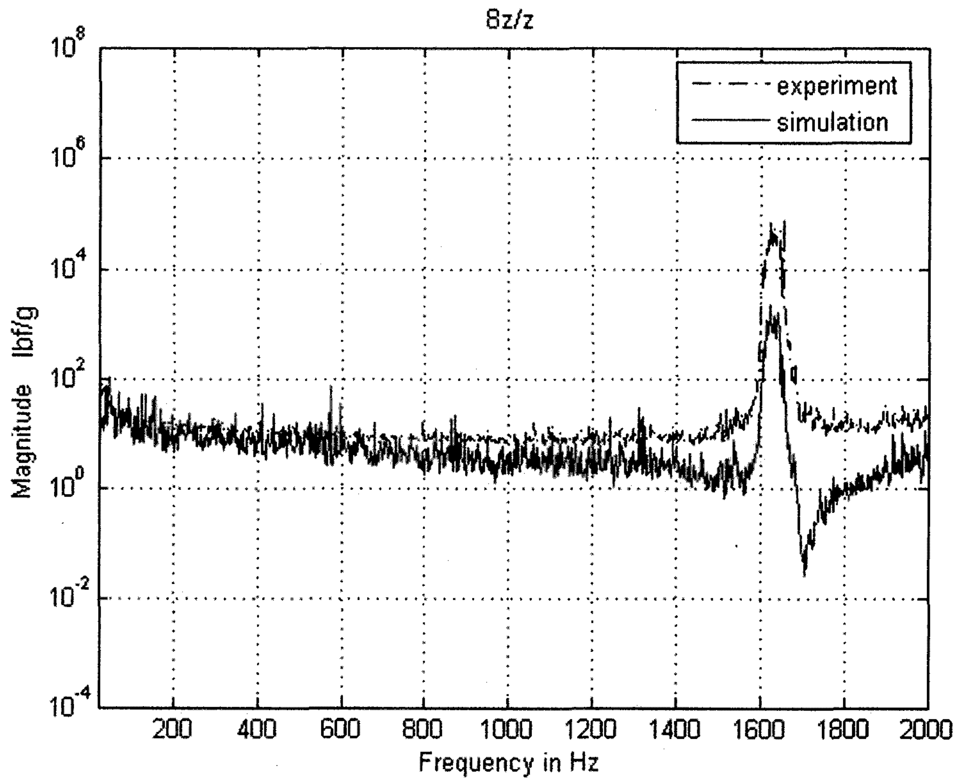

Figure 8: Frequency Response Acceleration/Force (Output =Acceleration/ Input=Vertical Force) of experimental and FEA simulation at point 8Z (no Coherence) unfiltered.

Physical Interpretation of the Frequency Response plot of Fig. 8: The ordinate, labeled 'Magnitude' and scaled logarithmically, represents the ratio of S_{uy} (*PSD* physical units in lbf^2/Hz) over S_{uu} (*PSD* physical units in g^2/Hz). Thus the 'Magnitude' ordinate of Fig. 8 has the dimension *lbf/g*. In the frequency band from approximately 200 Hz to 1,200 Hz, the experimental and FE magnitude in this frequency band is approximately10 *lbf/g*. A significant discrepancy between experiment and FE model amplitudes exists around the 1,600-Hz resonance in all records. Possible reasons for the discrepancies are:

1. Structural damping in the shaker center member is not accurately modeled in the FE model.
2. Numerical analysis issues: The frequency response spectra result from the ratio of S_{uy}/S_{uu}; therefore, the spectrum computed from the experimental involves a division of the S_{uy} spectral values by higher value of S_{uu} values than the FEA S_{uu}'s. Since S_{uy} value is much bigger than the FEA S_{uy}, a large increase in amplitude at the frequency range of 1,600 Hz is present. This has been explained in section 5.5 in more detail.

The following two graphs, Figure 9 and Figure 10 shows the frequency response at two other points, 13Z and 14Z in Z-direction. The frequency response patterns are consistent with those observed in Figure 8.

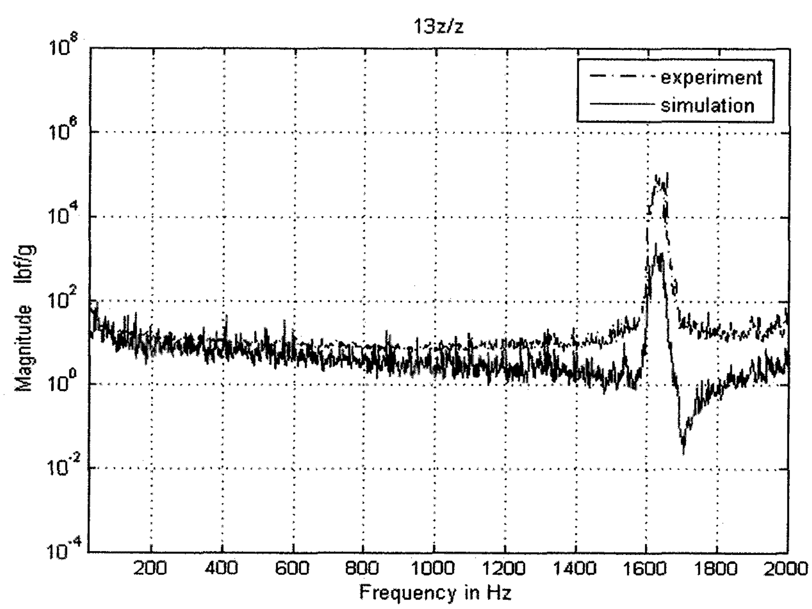

Figure 9: Frequency Response at point 13 in Z direction

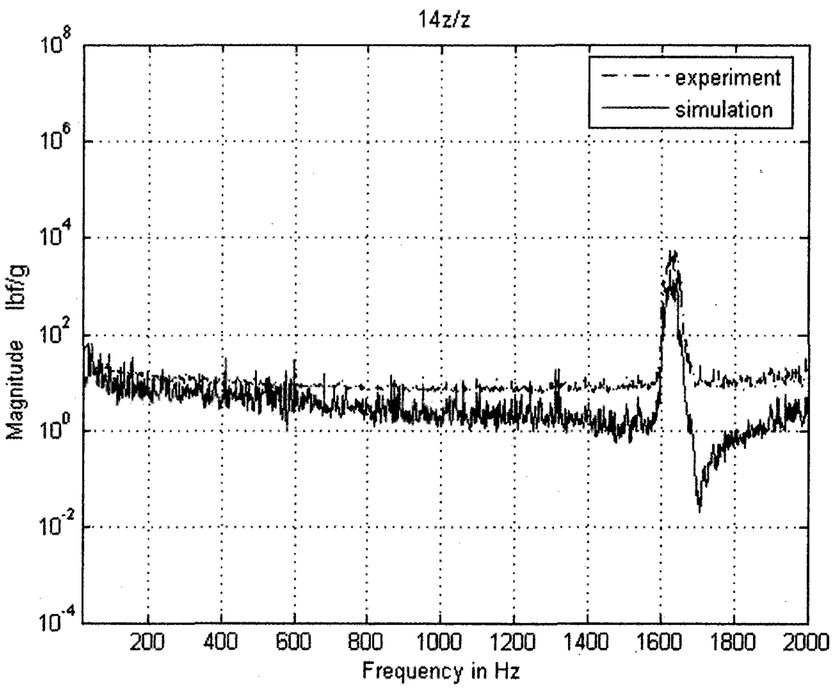

Figure 10: Frequency Response at point 14 in Z direction

5.2 Second Test Series

New sets of data were collected in order to accomplish the following objectives:

- To collect time history data that can be analyzed post test to validate the FE model of the system. The purpose was to make the FE model sufficiently accurate to make useful predictions of the capabilities of future shaker designs.
- Conduct a variety of tests that will illustrate the capability of the Jaguar system to conduct realistic 6 DOF shaker tests.

Test plan:

The response accelerometers are 14z, 8z, 13z, 13y, 14y, and 13x. The response of these accelerometers will be represented by a vector, r = [14z 8z 13z 13y 14y 13x]'. The positive sense of each accelerometer shall be in the positive axis direction. The rigid body degrees of freedom, c, will be the control variables. The response of these rigid body degrees of freedom will be represented by the vector, c = [x y z rz ry rz]', where x, y, z are the rigid body translations, and *rx*, *ry*, *rz* are the rigid body rotations about the x, y, z axes.

The transformation between the response accelerometers and the rigid body degrees of freedom is given by,

$$c = Gr \tag{5.3}$$

$$c = \begin{Bmatrix} x \\ y \\ z \\ rx \\ ry \\ rz \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1/4 & 1/4 & 1/2 & 0 & 0 & 0 \\ -1/4 & -1/4 & 1/2 & 0 & 0 & 0 \\ +1/2 & -1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{Bmatrix} 14z \\ 8z \\ 13z \\ 13y \\ 14y \\ 13x \end{Bmatrix} \tag{5.4}$$

The electrical drives to the actuators are represented by vector, d = *[d1 d2 d3 d4 d5 d6]'*.

System ID tests:

This is essential test to identify the system in the system ID phase. The only difference includes recorded data outputs of drive voltages, the drive currents, and all accelerometer response time histories, response channels used for control and auxiliary channels, used for later analysis. The system will be driven with all the six electrical drives having independent white noise over the control bandwidth and the time histories will be recorded as well. The system transfer functions as computed by the control system, which will also be saved for comparison with the later off line estimates using the recorded time histories. This data will be used to validate the FEM model. In all the tests the same reference spectrum will be used. The spectrum is shown in Figure 12.

Nominal Random Shape RMS-Acceleration = 5.78 g

RMS-Velocity = 1.11 inches/seconds

RMS-Displacement = 0.0069 inches

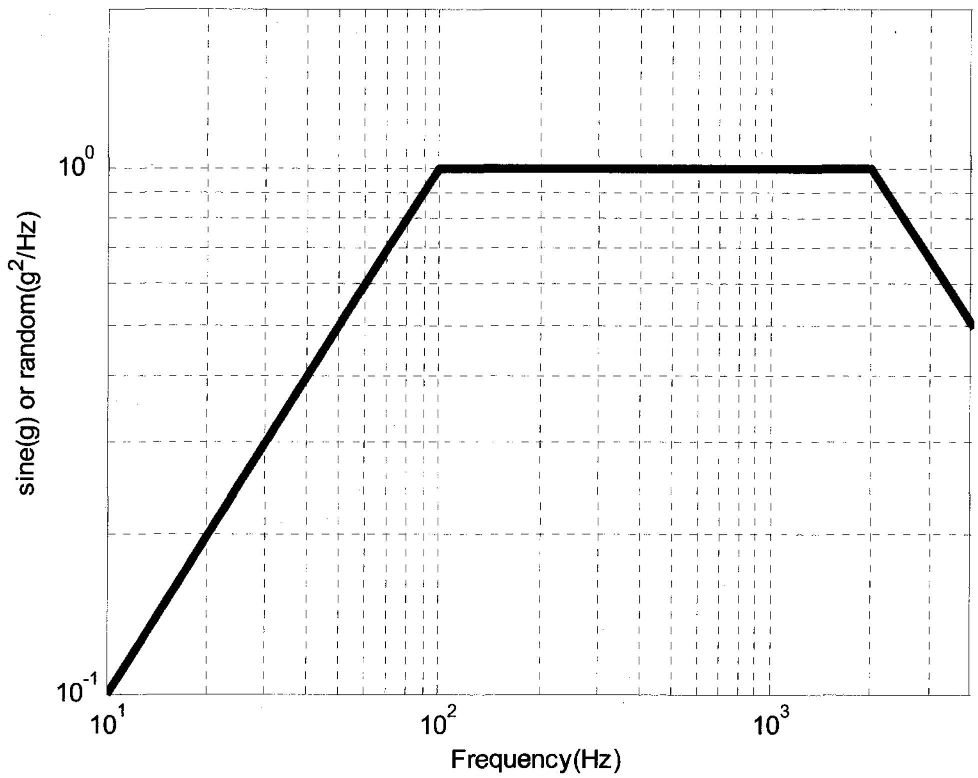

Figure 11: Random Multi-Axis Control: Reference Spectrum

5.3 Experiment Series 2

The frequency response of the shaker system is represented in the graphs below at various points represented by 8, 13, 14, all in z direction. The frequency response of the system is obtained by taking the ratio of the cross correlation and auto correlation of the vibrating system under a frequency range of 0 to 2,000 Hz.

$$\text{Frequency Response} = \frac{CrossCorrelation}{AutoCorrelation}$$

The following graphs are from the data of November 2006.

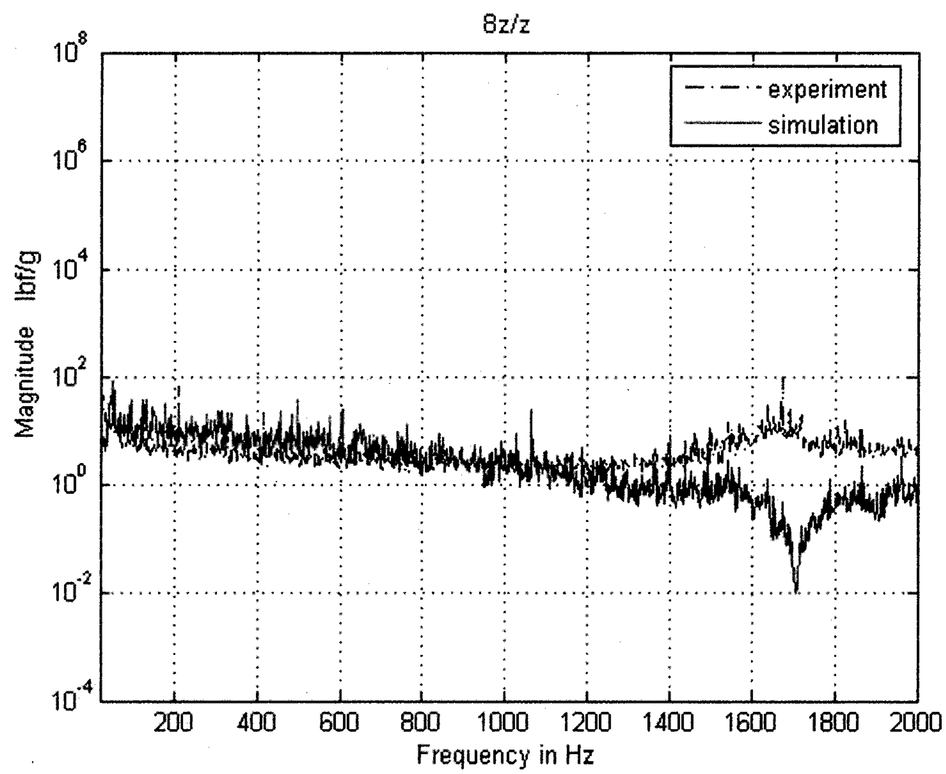

Figure 12: Frequency Response at point 8 in Z direction, (output = acceleration/ input = vertical force) of experimental and FEA simulation at point 14Z (Coherence = 0.96) unfiltered.

Interpretation of the Frequency Response plot of Figure 12:

The ordinate, labeled Magnitude and scaled logarithmically, represents the ratio of S_{uy}, PSD physical units in lb_f^2/Hz, over S_{uu}, PSD physical units in g^2/Hz. Thus the Magnitude ordinate of Figure 12 has the dimension lb_f/g. In the frequency band from approximately 200 Hz to 1,200 Hz, the experimental and FE magnitude in this frequency band is $10lb_f/g$.

Figure 13 and 14 display the frequency response at two other points, 13Z and 14Z in Z-direction. The frequency response patterns are consistent with those observed in Figure 12.

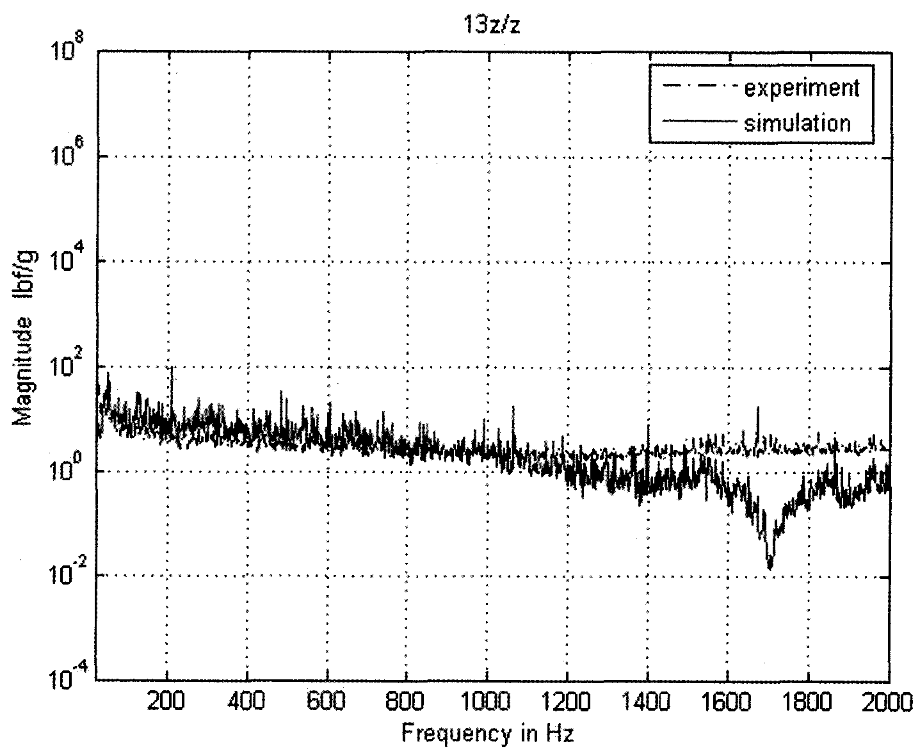

Figure 13: Frequency Response at point 13 in Z direction

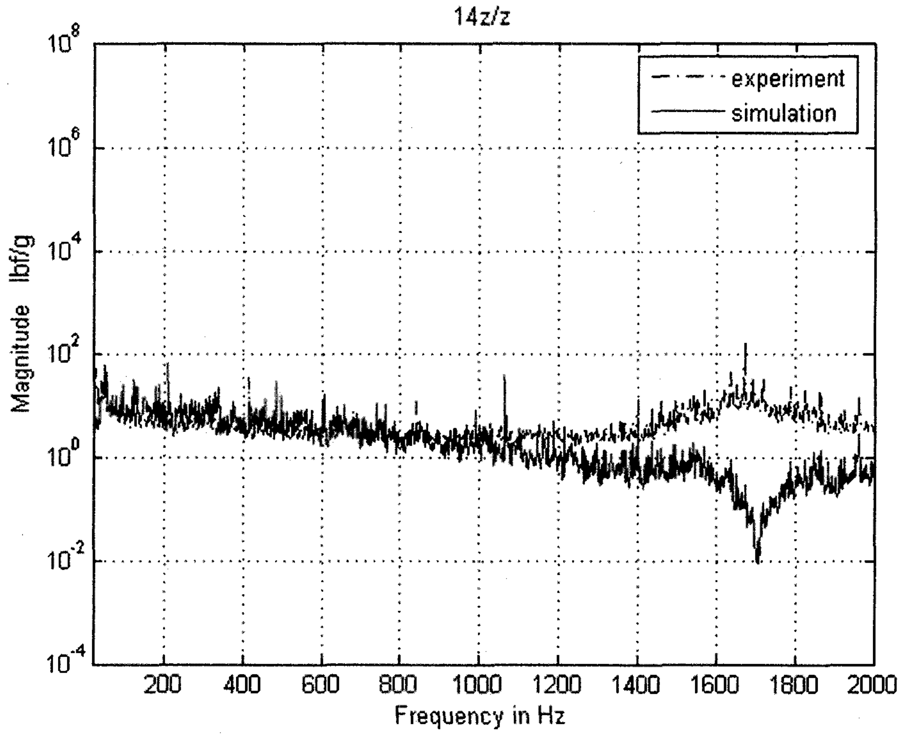

Figure 14: Frequency Response at point 14 in Z direction

From graphs 12 through 14, it can be determined that the experimental and simulation values are in reasonable agreement over the frequency range of 0 Hz to 1,000 Hz. Similar to the 2005 results, the FE model exhibits lower amplitudes than the experiment in the frequency band around the resonance. Again, possible reasons for the discrepancies could arise either from an inaccuracy in modeling structural damping within the center member, or from the numerical analytical process. Both hypotheses are investigated below in section 5.5.

5.4 Structural Damping of the FE Model

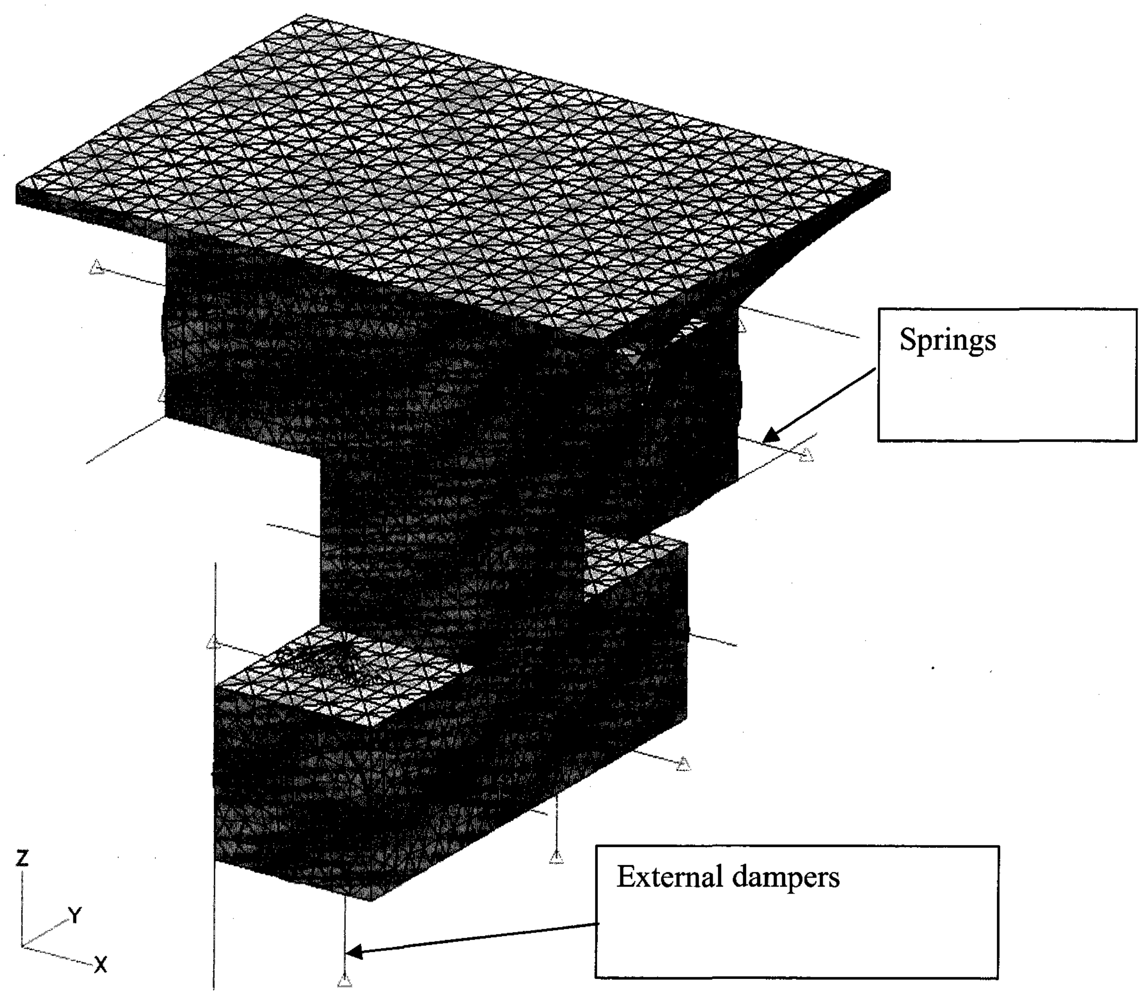

Figure 15: The Forces on the Shaker System through Rigid Threads, FE Model

1. **Viscous Damping** in the Rotational Joints Linking Actuators and the Center Member: The Finite Element model displayed above consists of actuators or drive forces are connected to the shaker model via rigid threads. Since the resulting forces act at a single point would produce excessive deformation, the load is distributed over an area as shown in Figure 16. To create a slight damping effect in the FEA model simulation, an oil film layer between the FE model and acting force was created.

However, the damping created was very small and did not significantly affect the output.

The simulations have been run for FE model for different conditions. The study mainly focused at 1,600 Hz frequency because the model is showing some damping at this point. We placed two dampers in each x, y, and z directions respectively. However, the final result has not showed much effect.

2. **External Dampers** between Actuators and Center Member: We sought to add structural dampers between the actuators and the center member. Since the model is three-dimensional, three dampers would have to be attached from the same point on the actuators to the three different points on the model, each one-hundred twenty degrees apart. This attempt created difficulties in the FE model which could not be resolved.
3. **Zero Damping**: Figure 15 shows 12 external dampers, 4 in each direction. Similarly, there are 12 spring elements through which the drive forces acts. These springs have an internal damping coefficient value. By selecting zero damping coefficient values for all external dampers and spring elements, the frequency response of the shaker changes. The results for the points 8, 13, and 14 in Z-direction are shown below.

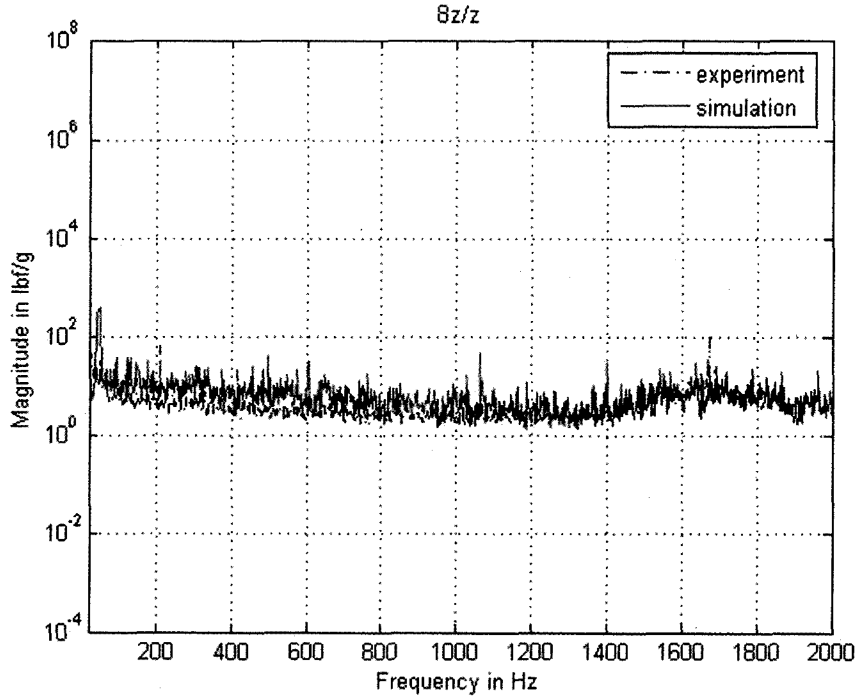

Figure 16: Frequency Response at point 8 in Z direction

Interpretation of the Frequency Response plot of Figure 16

The ordinate, labeled Magnitude and scaled logarithmically, represents the ratio of S_{uy}, PSD physical units in lb_f^2/Hz, over S_{uu}, PSD physical units in g^2/Hz. Thus the Magnitude ordinate of Figure 16 has the dimension lb_f/g. In the frequency band from approximately 200 Hz to 2,000 Hz, the experimental and FE magnitude in this frequency agree well. Therefore, internal damping of the experimental system appears to be negligible, and the FE model represents the experimental reality best when assuming no viscous damping in the structure.

Figure 17 and 18 display the frequency response at two other points, 13Z and 14Z in Z-direction. The frequency response patterns are consistent with those observed in Figure 16.

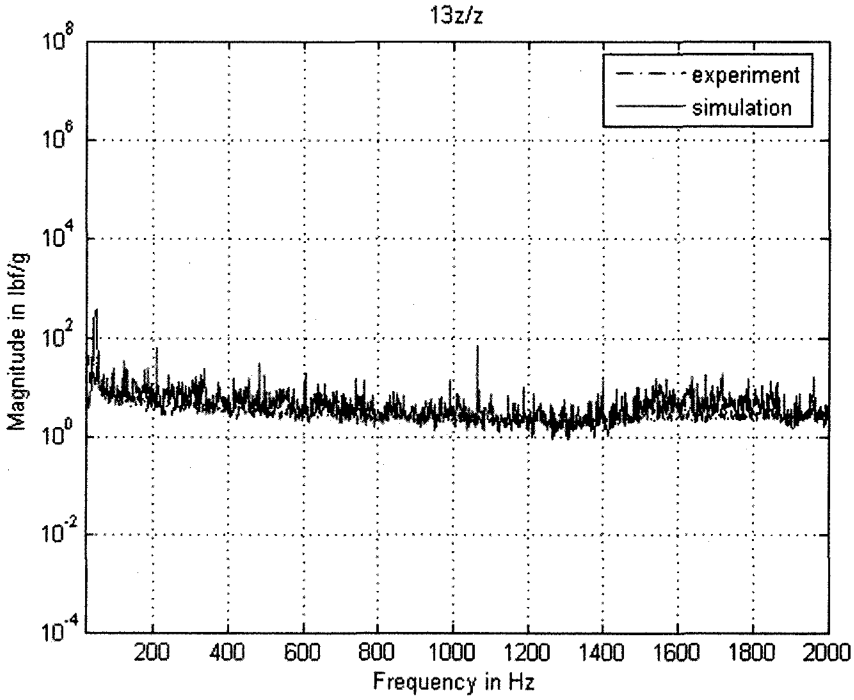

Figure 17: Frequency Response at point 13 in Z direction

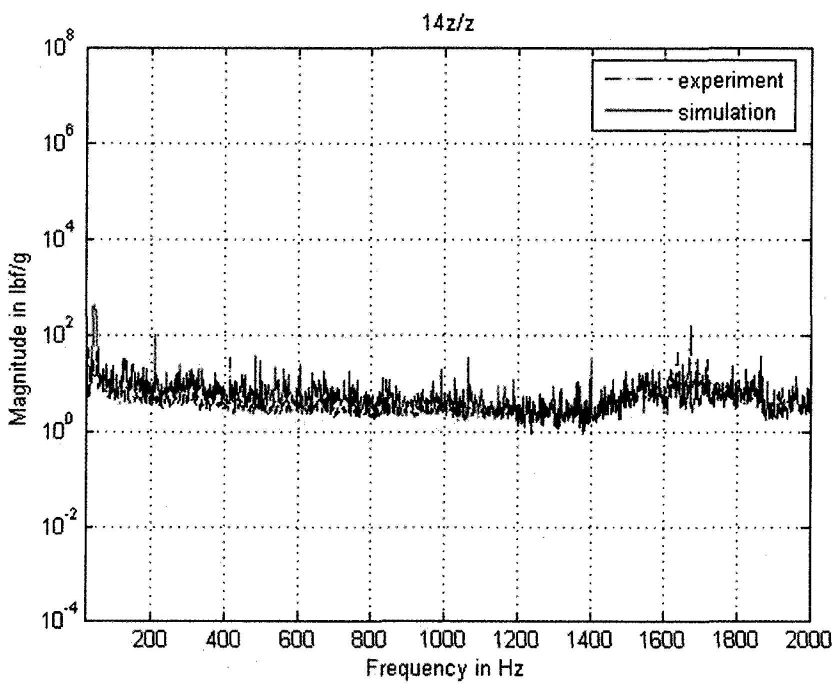

Figure 18: Frequency Response at point 14 in Z direction

4. Non-Linear Analysis:

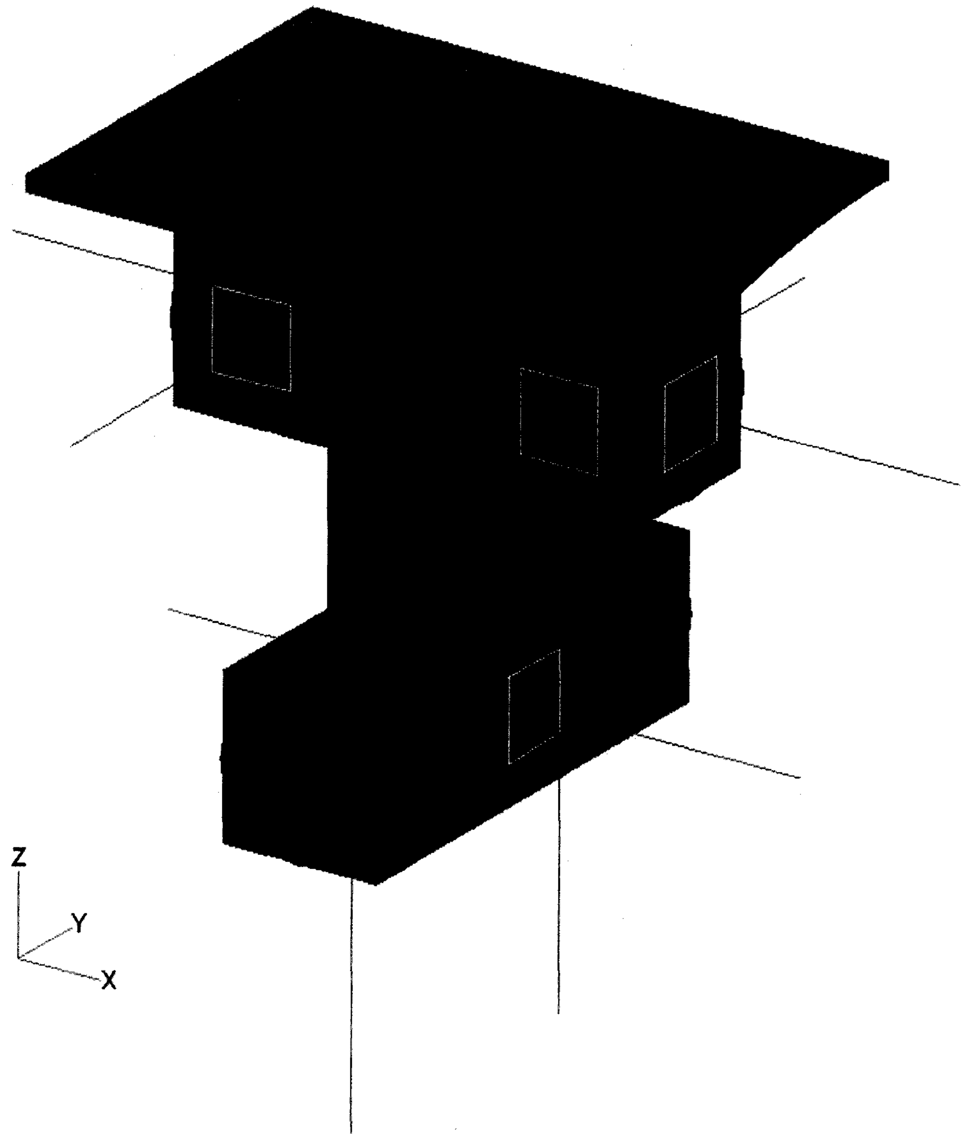

Figure 19: The actuators connected to the Shaker System on the contact surfaces

The Finite Element model above shows the point forces or drive forces acting on the shaker model. As compared to linear analysis, non-linear analysis allows for viscous sliding relative motion between two flat surfaces, i.e. the center member and the flat plate connecting to the actuator. The contact surfaces, represented by the yellow square boxes in Figure16, were used to incorporate viscous damping into vibrating model. The viscous damping consisted of an oil film between both surfaces. Some tests were performed, but they were not conclusive. Computing time increases dramatically with Non-Linear Analysis as compared to Linear Simulation; therefore, non-linear simulation was conducted for just 1,632 points which took forty eight hours. The frequency responses of the Non-Linear Simulation are shown in the graphs 20 through 22 below.

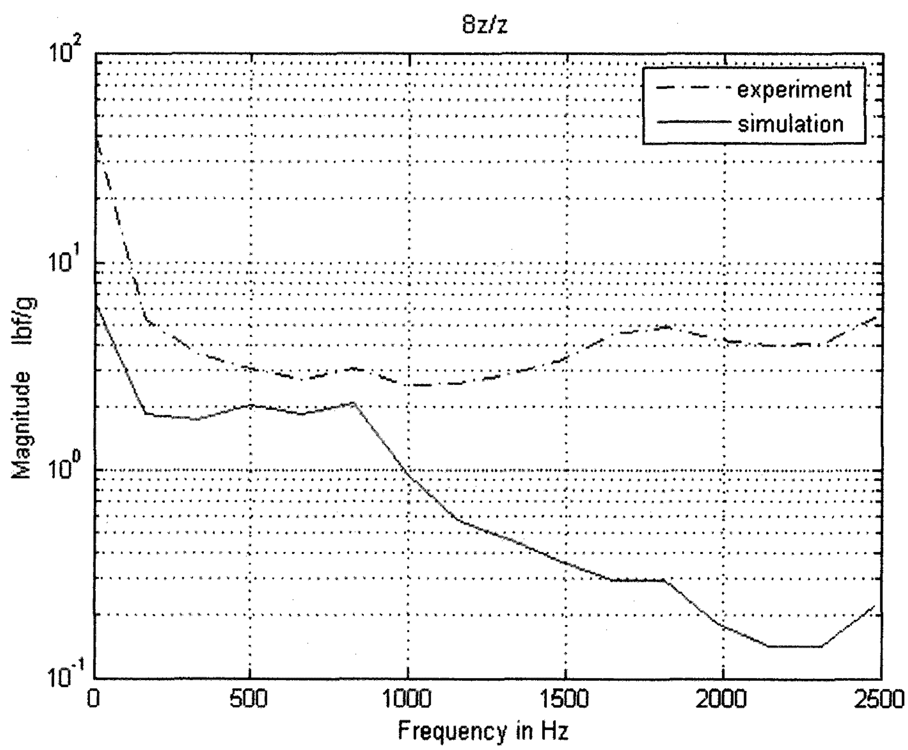

Figure 20: Frequency Response at point 8 in Z direction, (output = acceleration/ input = vertical force) of experimental and FEA simulation at point 14Z unfiltered.

Interpretation of the Frequency Response plot of Figure 20: The ordinate, labeled Magnitude and scaled logarithmically, represents the ratio of S_{uy}, PSD physical units in lb_f^2/Hz) over S_{uu}, PSD physical units in g^2/Hz. Thus the Magnitude ordinate of Figure 20 has the dimension lb_f/g. In the frequency band from approximately 200 Hz. to 800 Hz, the experimental and FE magnitude in this frequency band is approximately 2 lb_f/g. Clearly, the addition of viscous oil film damping in the nonlinear simulation does nothing to improve the agreement between experiments and FEA. Similar frequency response curves were recorded for coordinate points 13Z and 14Z see Figures 21 and 22.

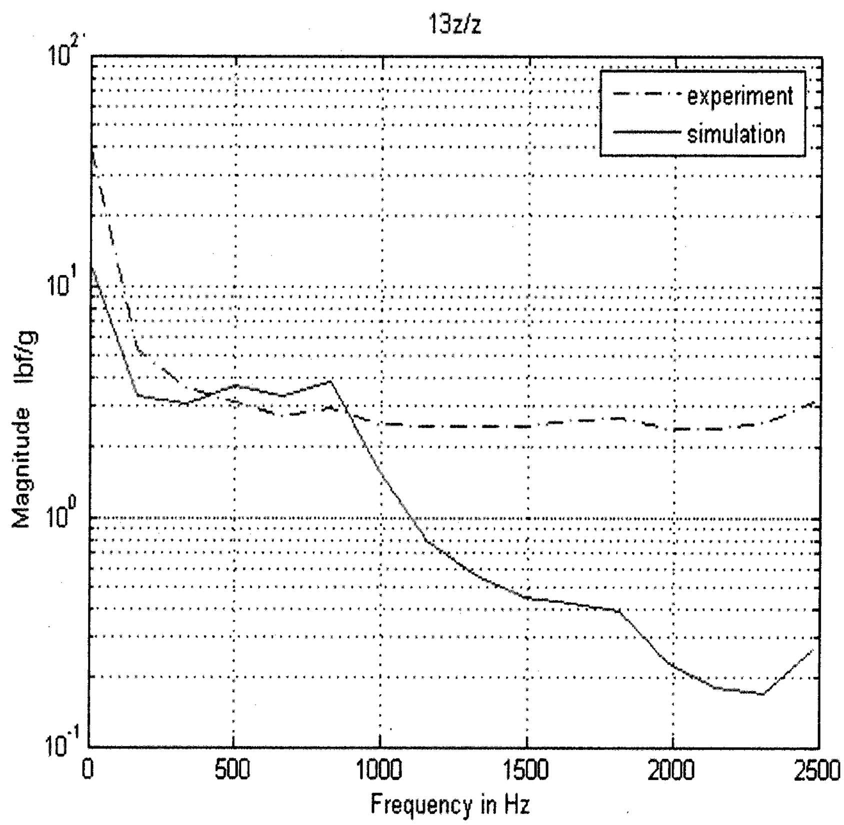

Figure 21: Frequency Response at point 13 in Z direction

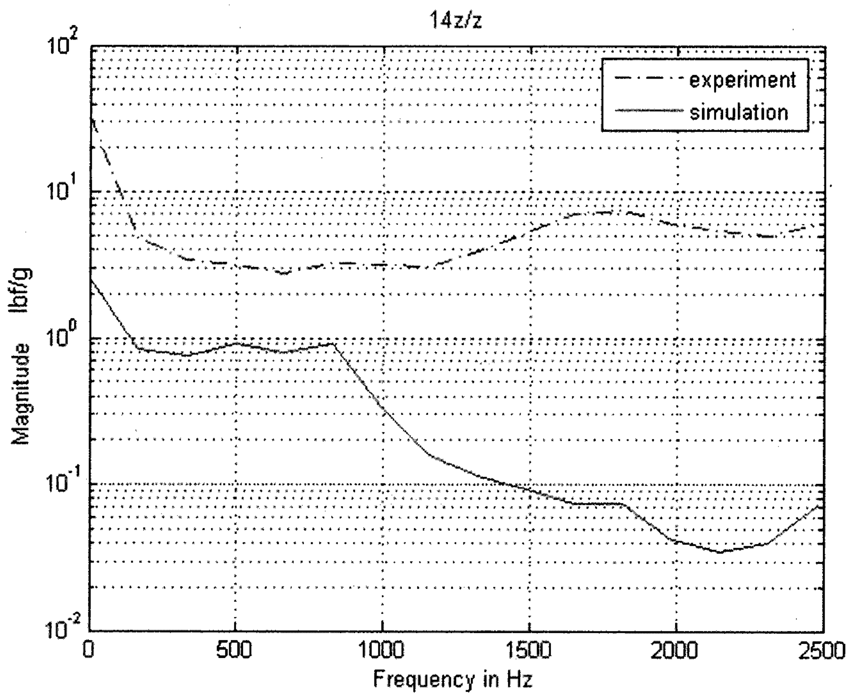

Figure 22: Frequency Response at point 14 in Z direction

5.4 Numerical Analysis of the Frequency Response Computation

Computation of a Sample PSD

Following the schematic of Figure 7, the autocorrelation of the input, and the cross correlation of output/input are computed.

The following describes in detail the steps of the analysis on the example of Accelerometer 14Z, excited by a rocking input in vertical direction. All computations were done in Mat lab.

Source: Data set recorded at Team Corp on November 9, 2006

Data set: Excitation (Drive) force (vertical) time series see Figure 23, and Accelerometer 14Z time series, see Fig. 25. Both time series are represented in their proper physical units, i.e. lb_f and g's, respectively.

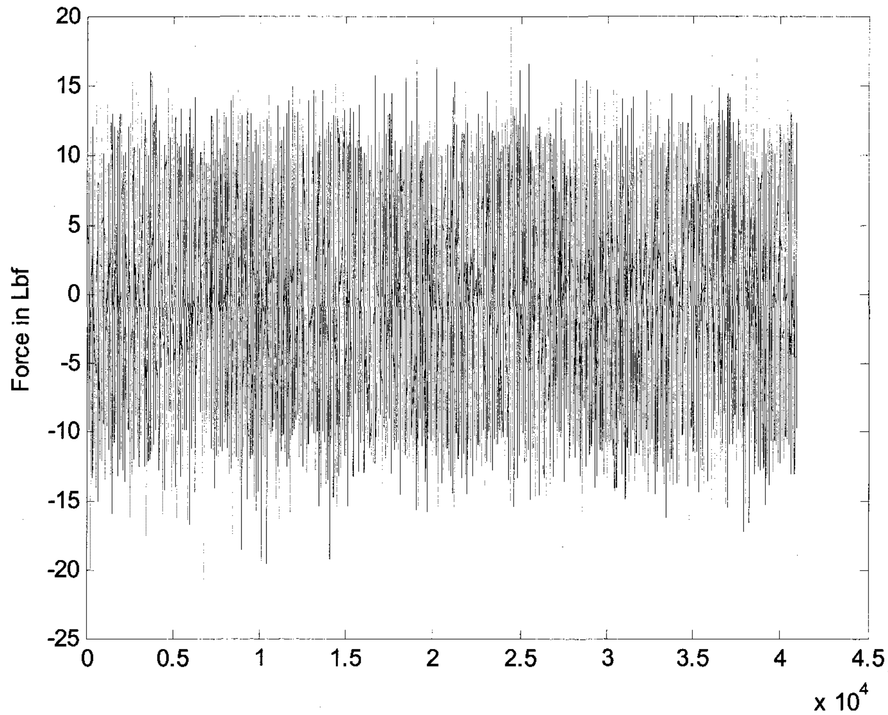

Figure 23: Excitation force time series (in lbf)

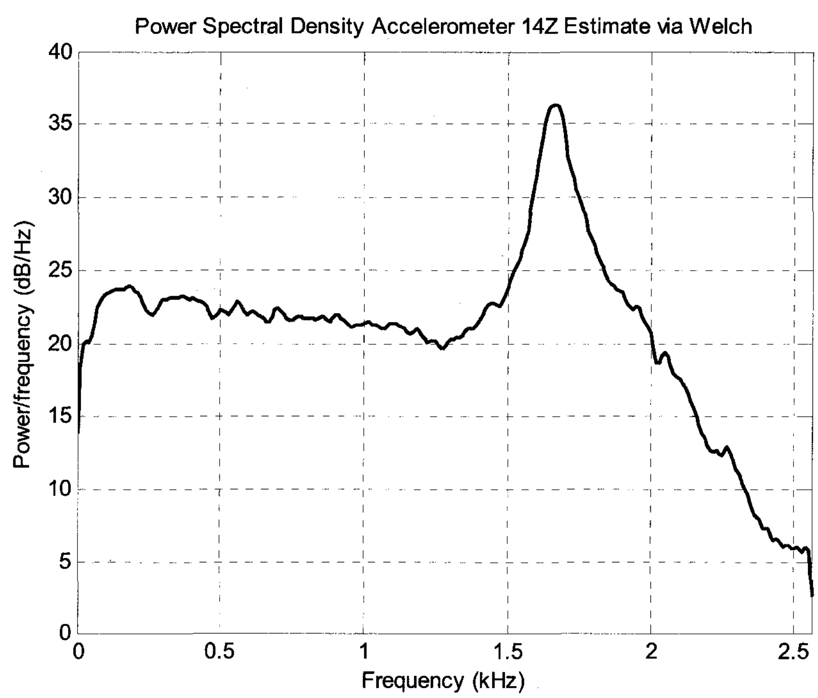

Figure 24: Power Density Spectrum (Autocorrelation) of Experimental Force Input signal of Fig. 20 (PSD physical units in lbf2/Hz), Welch Filtering.

Output Spectrum for point 14Z

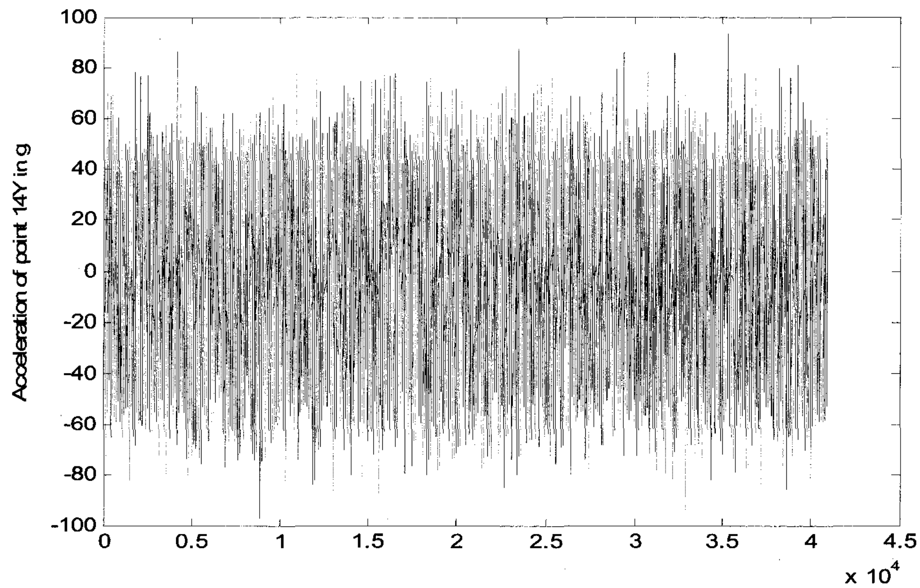

Figure 25: FEA Accelerometer 14Z time series (in g's).

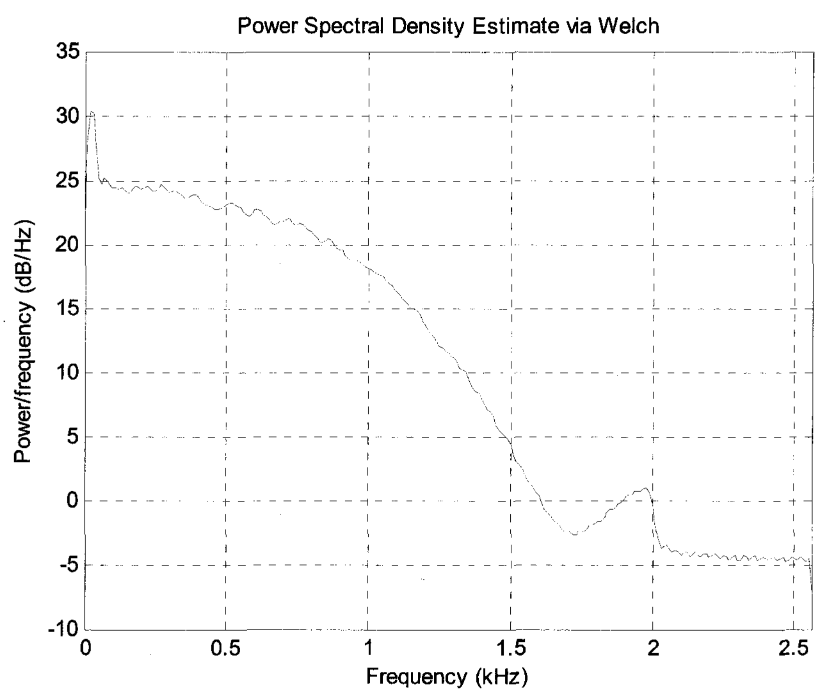

Figure 26: Power Density Spectrum (Autocorrelation) of FEA Acceleration signal of Fig. 7, Welch Filtering (PSD physical units in g2/Hz)

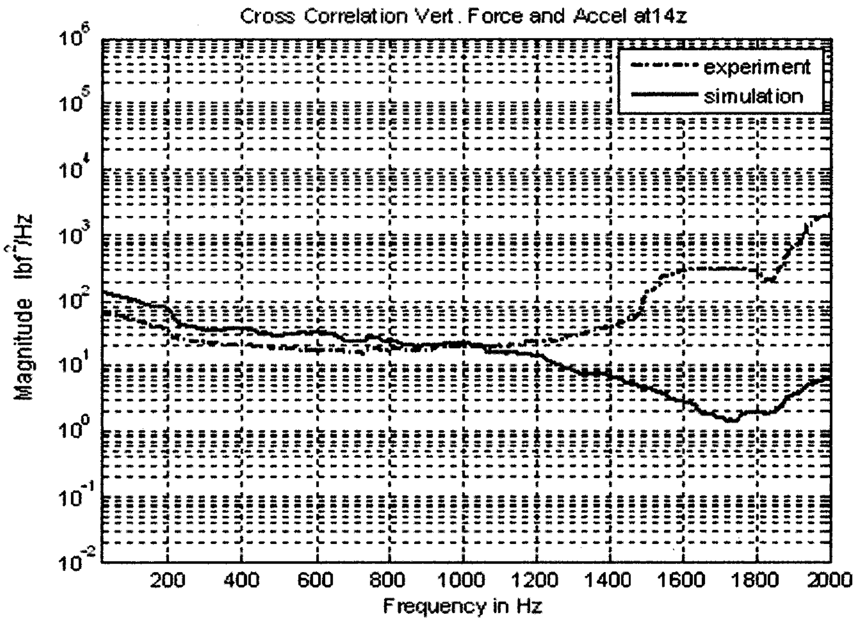

Figure 27: Power Density Spectrum (Cross Correlation) of Acceleration signal 14Z with vertical Force input.

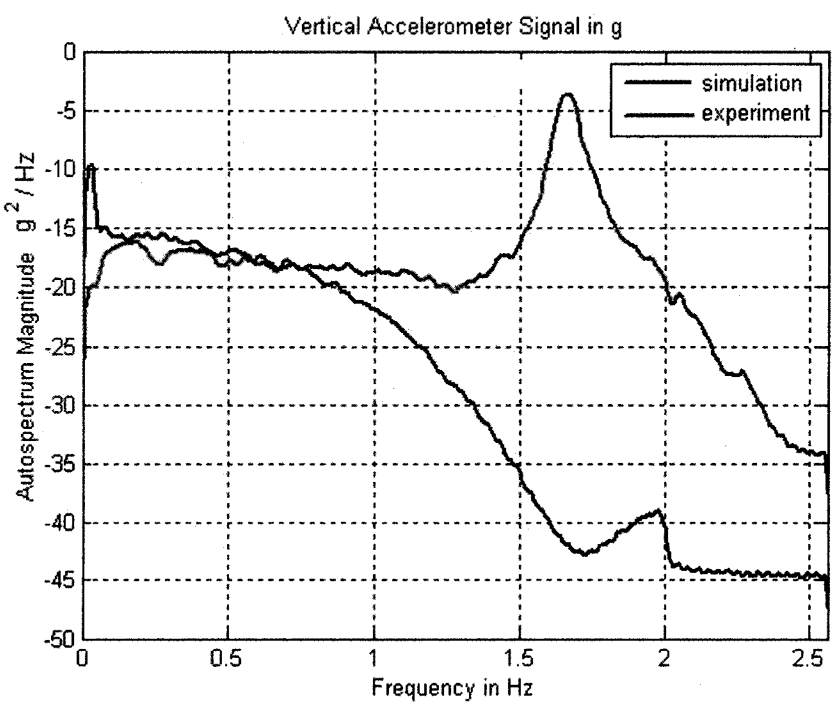

Figure 28: Power Density Spectra (Autocorrelation) of experimental and FEA acceleration signals, Welch Filtering

In Figure 28, the simulated acceleration PSD's from the FE analysis and those recorded experimentally differ substantially around the resonance region. While the 1,600 Hz resonance is clearly visible in the experiment, the controller has reduced the drive PSD (shown in Figure 28) to almost -5 in the region around 1,600 Hz. Consequently, the simulated accelerometer 14Z records about -42 amplitude around the 1,600 Hz resonance. The frequency response spectra result from the operation S_{uy}/S_{uu}. The spectra computed from the experimental and FE records involve a division of the S_{uy} spectral values shown in figure 24 by the respective S_{uu} values. The resulting input/output spectra are presented in Figure 14.

At 1600 Hz, the cross-correlation value of the experiment is much higher, 500, than the FEA S_{uy} value, which is close to zero. Dividing the larger experimental S_{uy} value by a low experimental S_{uu} value yields large frequency response amplitude. Similarly, when dividing the lowest FEA S_{uy} value, close to zero, by the large magnitude FEA auto spectrum, the frequency response magnitude becomes rather small. Clearly, the internal damping in the FE model is larger than the actual structural damping of the center member observed in the experiments. In the FE model, the damping parameters are those of Magnesium, which are defined internally by the software.

CHAPTER 6

SIX-AXIS 25 POUND SHAKER SYSTEM DESIGN

Design Constraints

Experiments with the TEAM Corp. small electro dynamic shaker have shown the feasibility of building a six-axis electro dynamic shaker with a frequency range to 2 KHz.

Design Issues

When scaling up to a larger center member, two major design issues will arise,

1. The size of the electro-dynamic actuators will increase considerably. If the actuators diameter is larger than ½ the width of the projected shaker table surface, it might become necessary to increase the size of the center member so as to accommodate the actuators.

2. The lowest eigen frequency of the center member is proportional to $\sqrt{\frac{k}{m}}$, where m is the center member mass, and k the (equivalent) spring elasticity. Thus, as the center member's mass increases, its lowest eigenfrequency will decrease. While designing the new center member, great care should be applied to maximize its stiffness in all coordinate directions. Yet one must expect that eigenfrequencies below 2 kHz will exist. The eigenmodes can be controlled by the electronic controller and to a limited extent by additional viscous damping of the center member.

3. Thermal Effects: Active Cooling will likely be required for the actuators and for the hydraulic fluid. Effective damping of center member resonances can result in thermal power dissipated to the hydraulic fluid on the order of kilowatts.
4. Compression Spring Preload: The compression springs are preloaded in order to maintain contact with the center member irrespective of shaker dynamics. The preload also contributes to the center member's viscous damping. Raising the spring preloads beyond the shaker dynamics structural requirements would be a means to increase viscous damping, If necessary.
5. Center Member Manufacture: Complex geometries that maximize stiffness while reducing weight are easily designed and analyzed. The choice and cost of a manufacturing method will determine the design choice.

Center Member Design Concepts

A. Option 1: Scale center member of existing electro dynamic shaker up to meet requirements for 25-lbs load shaker, see Fig. 29 (shown with single actuators, assuming passive springs on opposite sides)

Design needs to accommodate larger voice coil actuators and also by considering,

- We seek to avoid structural resonances if possible, i.e. first eigenfrequency is greater than 2 KHz.
- Need to minimize centre member mass while maintaining structural stiffness.

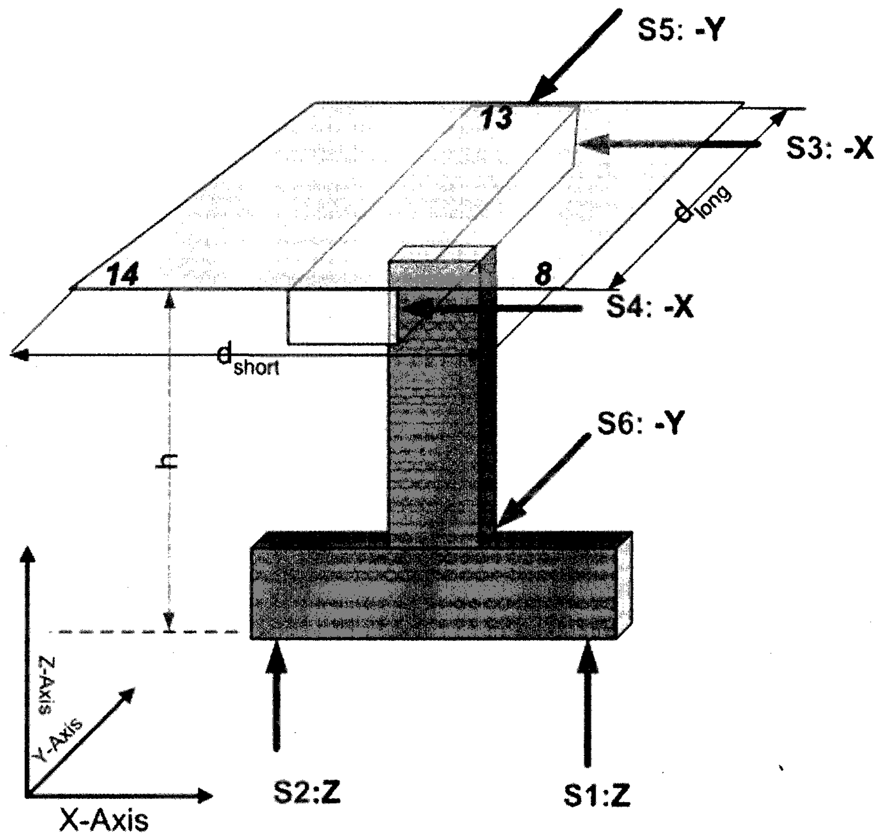

Figure 29: Current Electrodynamic Center Member Configuration, the six actuators are labeled S1 through S6

Experience exists with existing design. Shaker table is accessible from all sides. Placement of the vertical actuator underneath the platform might impose unacceptable height requirements on vertical beam of the center member, resulting in reduced center member stiffness.

B. Option 2: Cube-shaped Center Member: Vertical actuators attached to either sides, Figure 30, or at opposite diagonal endpoints, Figure 28.

- A hollow, internally stiffened cube structure would optimize the stiffness to weight ratio. Horizontal actuators are located on the sides of cube surface.

- Vertical actuators would restrain the accessibility on the platform, because two actuators would have to be placed on the side.

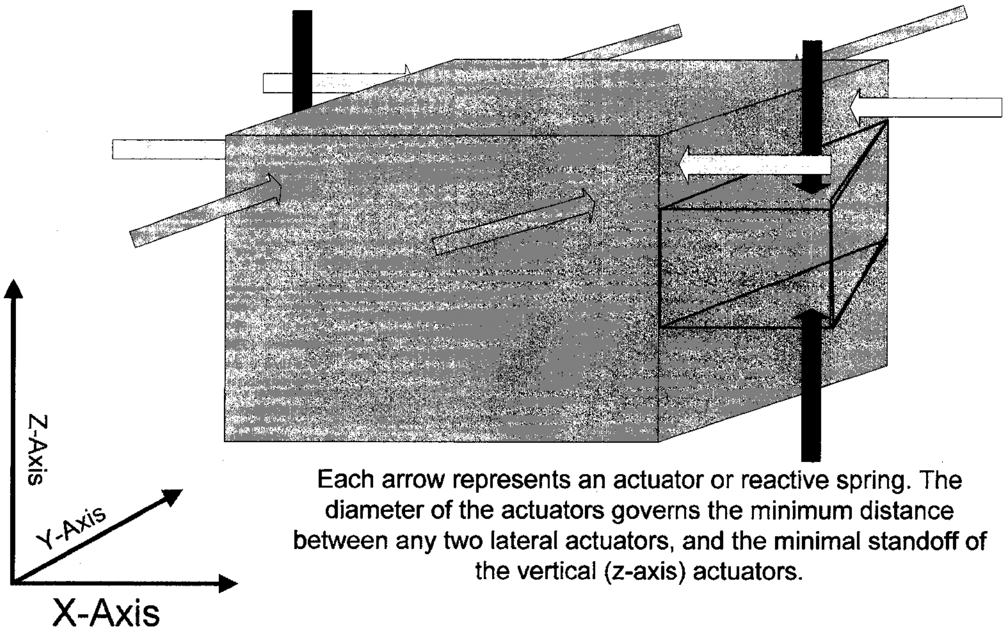

Figure 30: Cubic Center Member, Vertical actuators on sides

C. Cylindrical Center Member: Basically a variant of the cube.

- Better stiffness/weight ratio than cube shape. Plane contact surfaces must be created for all actuators on sides of cylinder surface.
- Vertical actuators would restrain the accessibility on the platform, because two actuators would have to be placed on the side.

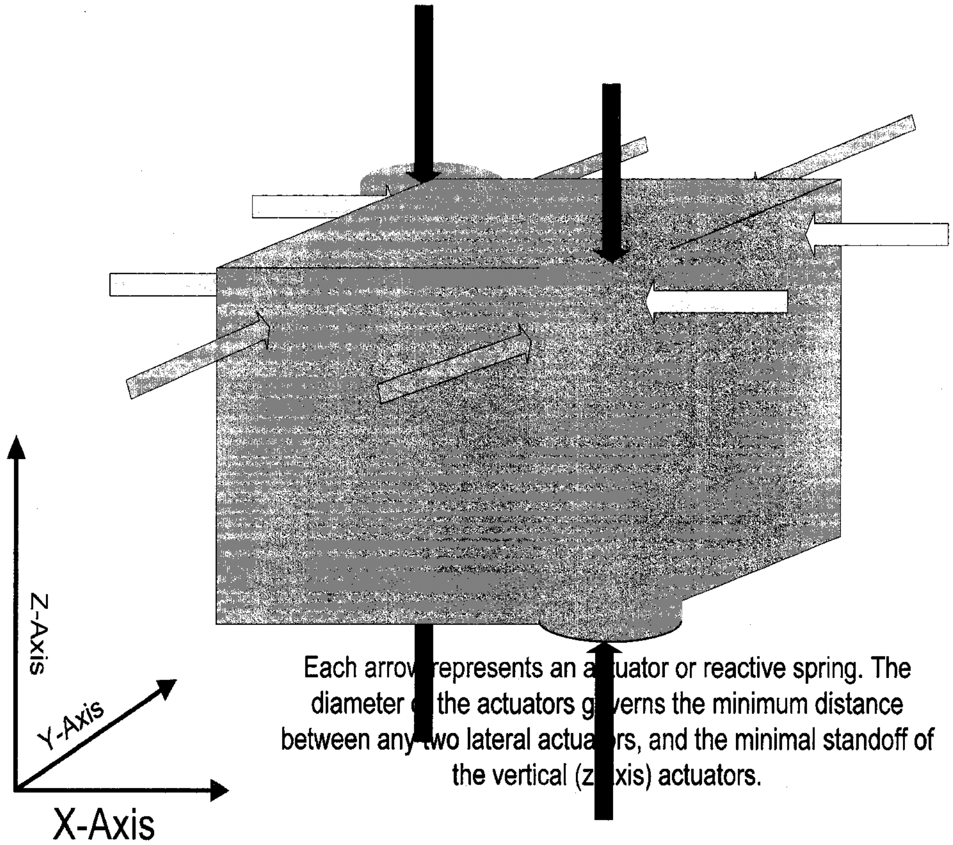

Figure 31: Cubic Center Member, Vertical actuators on diagonally opposite ends

D. Other Center Member Shapes:

Several variants of the basic shapes B and C can be devised. The FEA will show the optimal design configuration.Some modified center member designs were been constructed by using Solid Works, and the MSC-Nastran modal analysis module was used to compute the natural frequencies of the FE model. Testing was done for many designs, but among all of them the following center member design (see Figure 32) conformed quite well to the requirements.

The modal values of the model are

TABLE 6.1: The mode values of the modified center member (Figure 29)

Mode No.	Mode Values, Hz	[illegible]
	Magnesium	Magnesium
1	1,856	52.8
2	2,220	

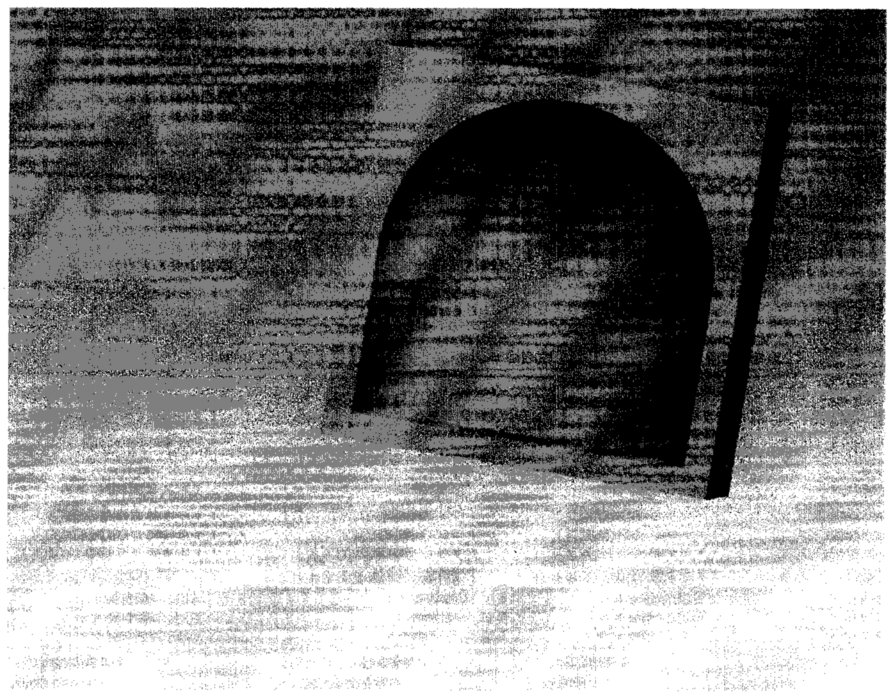

Figure 32: Hollowed-out cube design of the center member

The sides of the cube are 15 inches and there is a cylindrical hole with the radius of 6.5 inches and depth of 7.5 inches. There is also a semi sphere cap at that end for the cylinder with the radius of 6.5 inches.

CHAPTER 7

CONCLUSION

The objective of the research effort described here is the design and spectral analysis of a six-axis shaker system. The project focused on analyzing the results at the resonance frequency of 1,600 Hz. Finite Element Analysis. Spectral response analysis was used to compare FE model predictions and experimental results. The FEA and experimental spectra agree with discrepancies at or near the frequency of 1,600 Hz. The design concept was validated through two experimental test series. The internal damping assumed by the FE model is larger than the actual damping observed during the experiments, resulting in the observed larger resonance amplitudes in the experiments. A new center member design was developed for a 25-lbs payload shaker system. In spite of the large mass of the new center member, it is possible to optimize its stiffness such that its first eigenfrequency is larger than 2 kHz.

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Design and spectral Analysis of a Six-Axis Shaker System

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