Adaptive, Neural and Robust Control of Wing-Rock and Aeroelastic System

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**Prince Ghorawat**

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is approved in partial fulfillment of the requirements for the degree of

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Modern aircraft exhibit wing-rock phenomenon and aeroelastic instability. Wing-rock (roll single degree of freedom motion) and aeroelastic systems’ (two degrees of freedom) behavior are described by complex nonlinear differential equations. The nonlinearities in the dynamics of these systems give rise to limit cycle oscillations beyond critical speed of aircraft. The onset of wing-rock and aeroelastic instability limits the performance of aircraft and can even lead to catastrophic consequences. Therefore, control of wing-rock motion and stabilization of aeroelastic systems are important. In the past, several studies have been made and experimental and analytical results have been obtained to explain the wing-rock and aeroelastic phenomena in wind-tunnel tests, and also control systems have been derived.

Motivation for this research is the importance of flying aircraft in a large flight envelope in which complex uncertain aerodynamic nonlinearities appear, causing in-
stabilities and flutter in the aircraft wings. For the control of wing-rock motion and the stabilization of aeroelastic instabilities, new control systems are designed. Because modeling of nonlinear dynamics of wing-rock motion and aeroelastic systems are imprecise, the control algorithms must be insensitive to model uncertainties. Apparently control theory for deterministic systems is not applicable to uncertain systems.

For the stabilization of wing-rock, two non-certainity equivalent adaptive (NCEA) laws are designed. The first control system includes a finite form realization of a speed-gradient adaptation law, and the second controller is based on the Immersion and Invariance (I&I) theory. For the nonlinear multi-input multi-output (MIMO) aeroelastic systems, equipped with leading- and trailing-edge control surfaces, four distinct control systems are designed. First, a Chebyshev neural adaptive control law is derived for the suppression of limit cycle oscillations (LCOs) of the prototypical wing. For this derivation SDU decomposition of the high-frequency constant gain matrix is utilized for obtaining a singularity free controller. Then for a multi-input aeroelastic system with state dependent input matrix, a higher-order robust sliding mode control law for finite-time stabilization is derived. This is followed by the design of a suboptimal controller based on the state-dependent Riccati equation (SDRE) method. Finally, a suboptimal control law is designed for the control of the aeroelastic system, based differential game theory. In this approach, the wind gust is treated as an adversary which tries to destabilize system. These control algorithms are simulated using MATLAB and SIMULINK to verify their performance. Results show that the designed controllers are effective in suppressing the limit cycle oscillations.
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NOMENCLATURE

\(a\) = nondimensionalized distance from the midchord to the elastic axis

\(a_i, \hat{a}_i, b\) = parameters in the wing dynamics

\(A, B, C, D\) = system matrices

\(b\) = semichord of the wing

\(c_h, c_\alpha\) = plunge and pitch structural damping coefficient

\(f(x)\) = nonlinear function

\(h\) = plunge displacement

\(H\) = Hamiltonian

\(I_\alpha\) = moment of inertia of the wing about the elastic axis

\(J\) = performance index

\(k_h(h), k_\alpha\) = plunge and pitch structural spring nonlinearity

\(L\) = Feedback matrix

\(M, M_g, L, L_g\) = aerodynamic and disturbance moments and lift

\(m_t\) = mass of the plunge-pitch system

\(m_W\) = mass of the wing

\(P\) = Riccati equation matrix

\(R_u, r_w\) = weighting matrix, parameter
\( u \) = control input  
\( V_f \) = airspeed  
\( V_1, V_2 \) = Lyapunov functions  
\( V(x) \) = value function  
\( w_G(\tau) \) = disturbance velocity  
\( W(x) \) = Lyapunov function  
\( x_{1r}, \tilde{x}_1 \) = reference trajectory, tracking error  
\( x \) = state vector  
\( x_\alpha \) = nondimensionalized distance measured from the elastic axis to the center of mass  
\( \theta, \hat{\theta}, \tilde{\theta} \) = parameter vector, parameter estimate, parameter error  
\( \hat{\theta}_p, \hat{\theta}_I, \hat{\theta}_d \) = parameter estimates  
\( \chi_\nu, \chi \) = nonlinear vector functions  
\( \psi = 0 \) = target manifold  
\( \alpha \) = angle of attack  
\( s_p \) = span  
\( U, u = (\beta, \gamma)^T \) = free-stream velocity, flap deflection vector  
\( (\zeta_i, \omega_i) \) = reference generator parameters  
\( \lambda \) = manifold gain  
\( \lambda_\alpha, \lambda_h \) = Feedback parameters  
\( \Gamma \) = adaptation gain  
\( \gamma(\psi) \) = nonlinear function
\( \sigma \) = \( \sigma \)-modification parameter

\( \Psi_a(x,t), \mu_a(x,t) \) = algebraic vector functions

\( \Theta, \Theta_a, \Theta_b \) = unknown parameter vectors

\( \hat{\Theta}, \hat{\theta}_a \) = parameter estimates

\( \phi, \dot{\phi}, \phi_r \) = roll angle, roll rate, reference trajectory

\( \eta_i \) = Design parameters

\( \rho \) = density of air
CHAPTER 1

INTRODUCTION

Wing rock is described by complex nonlinear differential equations having one type of lateral-directional instability for aircraft flying at high speed. It is triggered by flow asymmetries, developed by negative roll damping, and sustained by nonlinear aerodynamic roll damping.

Aeroelasticity is termed for the interaction between the elastic, inertial and aerodynamic forces. It is broadly classified into two fields: static aeroelasticity and dynamic aeroelasticity. But as a part of thesis, our work is mainly contributed to dynamic aeroelasticity, which deals with vibration response and leads to Flutter. Hence, aeroelasticity is an extremely important phenomenon in the aircraft design. Modern aircraft exhibit wing-rock phenomenon and aeroelastic instability. Aeroelastic behavior are described by two degree of freedom. Among different aeroelastic instability such as wing flutter, buffeting, divergence, control-surface effectiveness, reversal and buzz, and gust load, flutter is the most dangerous phenomenon which occurs when wing mode oscillations extract energy from the airstream and leads to sudden catastrophic failure. For example United States third longest suspension bridge, the Tacoma Narrow bridge in Washington collapsed on Nov 7, 1940 because of strong winds. For nonlinear systems, flutter is usually interpreted as a limit cycle oscillation (LCO).
1.1 Literature review

Modern aircraft operating in nonlinear flight regimes often exhibit wing rock phenomenon (a limit cycle oscillation in the roll angle). The onset of wing rock adversely affects the handling qualities and maneuverability of aircraft and could even lead to catastrophic consequences. Therefore, in the past, considerable effort has been made to analyze wing rock phenomenon and design control systems for suppressing wing rock [4, 6, 16, 18, 25, 31]. Hsu and Lan [16] developed mathematical models to study wing rock characteristics of swept slender wings. Based on analytical models, roll divergence and existence of limit cycle oscillations have been examined [6, 25]. Brandon and Nguyen [4] and Suarez et al. [31] have shown that vortices emanating from the forebody of an aircraft are primarily responsible for wing rock at high angles of attack. Katz [18] studied wing-vortex interaction and wing rock phenomenon. The measurement of unsteady surface pressure on a slender wing undergoing self-induced oscillation has been done [2]. Guglieri and Quagliotti [14, 15] and Guglieri [13] performed experiments to examine wing rock phenomenon. An analysis of wing rock due to rolling-moment hysteresis with respect to the sideslip angle has been presented in Go and Lie [10]. Based on multiple time scales method, center manifold reduction principle, and bifurcation theory, Go and Ramnath [11, 12] have analyzed coupled roll and pitch wing rock dynamics as well as three-degree-of freedom (roll, pitch, and yaw) wing rock dynamics of advanced aircraft. An analysis of the lateral-directional aircraft dynamics with cubic sideslip-dependent nonlinearity has been also considered by using the multiple time scales method [9].
Authors have developed a variety of optimal and suboptimal control systems for wing rock control [22, 28, 36]. Based on the \( \theta-D \) technique, a nonlinear suboptimal control law for the wing rock suppression has been developed (Xin and Balakrishnan, [36]). Nusawardhana et al. [26] have designed a synergetic optimal controller and a sliding mode control system for the control of wing rock. A variable phase control system for the control of wing rock with hysteresis has been proposed [21]. For uncertain models, adaptive law and neural control system have been designed for wing rock suppression. Singh et al. [29], Sreenath et al. [30] and Joshi et al. [17] have designed wing rock control systems using neural networks and fuzzy logic. An adaptive fuzzy control system for the wing rock control has been proposed by Lin (2005). An \( L_1 \) adaptive control system has been proposed for the control of wing rock motion [5]. A discrete-time sliding mode controller and a variable structure model reference adaptive control system for wing rock control have been designed [1, 7]. The wing rock control using adaptive feedback linearization has also been considered [23].

The traditional adaptive systems [24] are based on integral type update laws. Based on immersion and invariance (I&I) theory [3] and attractive manifold design technique [27] for uncertain linearly parameterized systems, noncertainty-equivalent adaptive (NCEA) wing rock controllers have been designed [3, 19]. Recently, adaptation algorithms in finite form (integral-algebraic form) [8] for a large class of uncertain systems with nonconvex as well as nonlinear parameterizations have been developed [32-35].
For system with two degree of freedom, aeroelasticity, impose severe constraints on the performance of new generation of flexible and combat aircraft. In the past, researchers have made many contributions related to the stability analysis for aeroelastic systems [37, 38]. Also considerable effort has been made for developing active control systems for avoiding instability in aeroelastic systems. Readers may refer to [37] which provides many references related to control of aeroelastic systems. At the NASA Langley Research Center, a benchmark active control technology (BACT) wind-tunnel model was developed for the analysis of aeroelastic behavior and validating control algorithms. For the flutter control of the BACT wing, several flutter control algorithms have been designed [39-41]. Mukhopadhyay [40] has designed a transonic flutter suppression control law and performed wind-tunnel tests. A passivity-based control system for the stabilization of the BACT wing has been proposed [41].

At the Texas A&M University, a two-degree-of-freedom plunging and pitching laboratory model has been developed [42-44] to study aeroelastic instability and limit cycle oscillations of a nonlinear wing section with structural nonlinearity. Seta et al. [43] have provided the computational and experimental investigation of the LCOs for this model. Also for this aeroelastic model, a variety of linear and nonlinear (adaptive and nonadaptive) control systems for suppressing the limit cycle oscillations (LCOs) have been developed using a trailing-edge flap as well as trailing-and leading-edge flaps [42, 44-52, 54-56]. An output feedback adaptive system has been designed for the suppression of LCOs [47]. Based on immersion and invariance (I&I) theory, adaptive control law for a MIMO aeroelastic has been designed [52]. The multifidelity control
of aeroelastic systems using I&I approach has been considered [53]. An $L_1$ adaptive controller has been proposed for the stabilization of the LCOs [55]. Recently, a finite-time robust control law based on the higher-order sliding mode control technique has been developed [56].

For the multi-input multi-output (MIMO) aeroelastic system, neural control of the wing section using leading-and trailing-edge control surfaces has been considered [49, 54]. For the purpose of design, Gujjula et al. [49] have used Gaussian activation functions for the approximation of the pitch-axis structural nonlinearity. The adaptation law in [49] uses 42 weights in the neural network for control. But the control law of [49] is valid only as long as the estimated high-frequency gain matrix remains nonsingular. A three-layer neural network has been considered by Wang et al. [54] for the adaptive flutter control using leading-and trailing-edge flaps in the presence of external disturbance inputs. The authors of [54] have chosen sigmoid function as activation function. However, the adaptation law associated with this neural controller is of large dimension (eighty two); and therefore, this controller has complicated structure from the viewpoint of implementation. Furthermore, for the design of the neural controllers of [49, 54], the plunge-axis structural nonlinearity has been ignored. As such, it is of interest to develop neural controllers for aeroelastic systems which have simple structure and provide robustness with respect to external disturbances and unmodeled structural nonlinearities.

The differential game theory was originally developed by Isaacs [63] for obtaining optimal control strategies for dynamical systems in which two sets of players have
conflicting goals. For dynamical systems, one can formulate a game problem in which the control vector adopt its strategy to minimize certain objective functional, and the external disturbance input vector aims to maximize it. For the solution of any differential game problem, it is essential to solve the Hamilton-Jacobi-Isaacs equation, which is a partial differential equation with appropriate boundary conditions. For nonlinear systems, this is an extremely difficult task, and a closed-form solution is rarely possible. For infinite-horizon differential game problems with quadratic objective function, it is possible to design suboptimal control laws based on the state-dependent Riccati equation (SDRE) method [67-69]. In the SDRE approach, one solves a nonlinear algebraic Riccati equation, instead of the partial differential equation. In a recent paper, a differential game-based guidance law for missiles by solving Riccati equation using SDRE method has been designed [74]. Earlier SDRE method has been considered for designing controllers for nonlinear aeroelastic systems without external disturbance inputs [70-71]. The unknown gust load can be treated as an adversary that adopts an strategy to destabilize the aeroelastic system. As such, one can formulate a differential game problem to obtain stabilizing control laws for aeroelastic systems despite the best strategy of the gust load. For a linear aeroelastic model, a related $\mathcal{H}_\infty$ control system has been designed [72]. However, it appears from literature that the design of controllers based on the differential game theory for nonlinear aeroelastic systems in the presence of external disturbance inputs (gust load) has not been attempted. Therefore, it is of interest to derive a game theory-based control system for nonlinear aeroelastic systems to preserve stability, despite
the worst destabilizing effect of unknown wind gusts.

1.2 Thesis Outline

The contribution of this thesis lies in the design of two adaptation algorithms in finite form (integral-algebraic form) for the control of wing-rock (Chapter 2) motion of a highly swept wing aircraft. For this study, the wing rock dynamics developed by Guglieri [13] is considered (see Figure 2.1). It is assumed that the aerodynamic parameters in the model are unknown. The uncontrolled wing rock model exhibits limit cycle oscillations at various angles of attack. The objective is to suppress the undesirable roll motion by the application of a control signal. Two adaptive control laws in finite form are designed for the trajectory control of the roll angle. The first control system includes a finite form realization of a speed-gradient adaptation law, based on the design method of Tyukin (2003) and Tyukin at al. (2003, 2007) [32, 34]; and the second controller is based on the I&I theory of Astolfi et al. [3]. Unlike the certainty-equivalent control laws, these adaptation laws include an integral term as well as a judiciously chosen nonlinear algebraic vector function. The algebraic vector function in the update law provides stronger stability property in the closed-loop system. It is pointed out that the I&I-based adaptive law developed in this paper for the model of Guglieri [13] differs from that of Lee and Singh [19] in which filtered signals were used for synthesis. Simulation results are obtained to verify the capability of the designed controllers. These results show that each adaptive system suppresses the wing rock motion, despite parameter uncertainties at different angles.
of attack.

In chapter 3, the nonlinearities has two degree of freedom and the design of a Chebyshev neural network-based adaptive control system for the control of a nonlinear aeroelastic system using leading-and trailing-edge flaps is considered. The dynamics of this two-degree-of freedom model describe the plunge and pitch motion of a wing section. In this study the plunge and pitch axis structural nonlinearities are treated as unmodeled nonlinear functions. Furthermore, the model includes uncertain parameters and external disturbance input (wind gust). This uncontrolled aeroelastic model exhibits limit cycle oscillations when the free-stream velocity exceeds a critical value. The objective is to stabilize the plunge and pitch responses of the system. An adaptive control law is designed for suppressing the oscillatory state vector of the system. The control system includes Chebyshev neural networks for the representation of the unmodeled structural nonlinearities. For the derivation of a singularity-free adaptive law, the SDU decomposition of the high-frequency gain matrix as the product of a positive definite symmetric matrix, a diagonal matrix and an upper triangular matrix is considered. Unlike the aeroelastic dynamics of [49, 54], in this wing model both the pitch-and plunge-axis unmodeled nonlinearities are included. The dimension of the neural adaptation law of this paper is smaller compared to the neural adaptation law of [49, 54]. Therefore, this is attractive from the viewpoint of implementation. By the Lyapunov stability analysis, it is shown that the state vector of the aeroelastic system is uniformly ultimately bounded. For the evaluation of the controller performance, numerical results are presented. These results show that the controller suppresses the
oscillatory motion of the system, despite unmodeled structural nonlinearities, large parameter uncertainties and gust loads acting on the model.

A finite-type adaptive scheme is applied in the chapter 4, in which Sheta [43] model is used, which accurately predicts the limit-cycle oscillations (LCOs) of an aeroelastic system with combined structural and aerodynamic nonlinearities. The model includes parametric uncertainties as well as external disturbance force and moment. A robust control system is designed for the tracking of reference plunge and pitch angle trajectories. The control law includes a nominal finite-time stabilizing continuous control signal designed for the model without uncertainties and a discontinuous control signal for nullifying the effect of uncertain functions in the model. In the closed-loop system, finite-time control of the complete state vector of the aeroelastic model to the origin is accomplished.

This is followed by the design of a suboptimal controller based on the state-dependent Riccati equation (SDRE) method is derived in chapter 5. In chapter 6, based on the theory of differential games, a control law for the stabilization of a nonlinear multi-input aeroelastic system in the presence of gust load is presented. The two degree-of-freedom aeroelastic model is equipped with leading-and trailing-edge control surfaces for the purpose of control. The model includes structural nonlinearity as well as nonlinear function of the pitch rate. The uncontrolled system exhibits limit cycle oscillations beyond a critical freestream velocity. A nonlinear zero-sum game with quadratic cost is formulated by treating the gust load as an adversary. For the derivation of the control law, the nonlinear model of the aeroelastic system is
represented as linear system with state-dependent system matrices. Then a suboptimal control law is obtained for the stabilization of the aeroelastic system by solving a state-dependent Riccati equation derived from the Hamilton-Jacobi-Isaacs equation. It is shown that the gust free aeroelastic closed-loop system is asymptotically stable, and the system trajectories remain bounded if the gust load is of limited strength. Simulation results are presented which show that the control system suppresses the oscillatory responses of the system, despite triangular, exponential and sinusoidal gust loads.
CHAPTER 2

WING ROCK CONTROL BY FINITE-FORM ADAPTATION

2.1 Introduction

In this chapter, the design of two adaptation algorithms in finite form (integral-algebraic form) for the control of wing-rock motion of a highly swept wing aircraft. For this study, the wing rock dynamics developed by Guglieri [13] is considered (see Figure 2.1). It is assumed that the aerodynamic parameters in the model are unknown. The uncontrolled wing rock model exhibits limit cycle oscillations at various angles of attack. Two adaptive control laws in finite form are designed for the trajectory control of the roll angle. The first control system includes a finite form realization of a speed-gradient adaptation law, based on the design method of Tyukin [32] and Tyukin at al. [34, 35]; and the second controller is based on the I&I theory of Astolfi et al. [3]. Unlike the certainty-equivalent control laws, these adaptation laws include an integral term as well as a judiciously chosen nonlinear algebraic vector function. The algebraic vector function in the update law provides stronger stability property in the closed-loop system.
2.2 Wing rock dynamics and control

A variety of mathematical models governing the single-degree-of-freedom wing rock motion have been developed by researchers. Here for definiteness, the mathematical model, developed by Guglieri [13] from the experimental data obtained for 80° delta wings, is considered. The wing configurations for model A and model C of Guglieri [13] are shown in Figure 2.1. The wing rock model is described by

\[
\ddot{\phi}(t) = -\frac{\hat{a}_0}{t_s^2} \phi(t) - \frac{\hat{a}_1}{t_s} \dot{\phi}(t) - \hat{a}_2 |\dot{\phi}(t)| \dot{\phi}(t) - \frac{\hat{a}_3}{t_s} \phi^3(t) - \frac{\hat{a}_4}{t_s} \phi^2(t) \dot{\phi}(t) + \frac{\omega_u}{t_s^2} u(t) \\
\triangleq -a_0 \phi(t) - a_1 \dot{\phi}(t) - a_2 |\dot{\phi}(t)| \dot{\phi}(t) - a_3 \phi^3(t) - a_4 \phi^2(t) \dot{\phi}(t) + bu(t) \\
\triangleq f(\phi, \dot{\phi}) + bu
\]  

(2.1)
where $\phi$ is the roll angle, $\dot{\phi}$ is the roll rate, $\omega_u = 0.9$, $t_s = (b_s/2V_f)$, $b_s$ is the span, $V_f$ is the airspeed, and $u$ is the control input. The parameters $\hat{a}_i$ depend on the angle of attack and $b$ is the control effectiveness gain. The aerodynamic parameters of model A and C for various angles of attack ($\alpha$) identified by Guglieri [13] are given in the appendix. The nonlinear function $f(\phi, \dot{\phi})$ is defined in equation (2.1). For simplicity, it is assumed here that the actuator dynamics are ignorable.

Defining the state vector $x = (x_1, x_2)^T = (\phi, \dot{\phi})^T \in \mathbb{R}^2$, the wing rock dynamics can be written as

$$
\dot{x} = \begin{bmatrix}
    x_2 \\
    \theta_h^T \chi_h(x_1, x_2)
\end{bmatrix} + \begin{bmatrix}
    0 \\
    b
\end{bmatrix} u
$$

(2.2)

where $f(x, t) = \theta_h^T \chi_h(x_1, x_2)$, and the parameter vector $\theta_h$ and the nonlinear vector function $\chi_h(x)$ are given by

$$
\theta_h = [-a_0, -a_1, -a_2, -a_3, -a_4]^T \in \mathbb{R}^5
$$

$$
\chi_h(x) = [x_1, x_2, x_2|x_2|, x_1^3, x_1^2x_2]^T \in \mathbb{R}^5
$$

(2.3)

($T$ denotes matrix transposition.) It is noted that $\theta_h$ changes with the angle of attack. Guglieri [13] showed that the wing configurations A and C exhibit limit cycle oscillations. Here the open-loop responses of the model A with the initial condition $\phi(0) = 0.1$ [deg] and $\dot{\phi}(0) = 0$ for $\alpha = 32.5$ [deg] obtained by simulation are shown in Figure 2.2. It is seen that the roll-angle trajectory undergoes oscillations of growing amplitude and asymptotically converges to the stable limit cycle.
**Assumption 1:** It is assumed that the control effectiveness gain $b$ is not known but its sign is known. Furthermore, it is assumed that the parameter vector $\theta_h$ is unknown.

Suppose that a smooth reference roll angle trajectory, $x_{1r}(t) = \phi_r(t)$, converging to zero is given. The objective is to design state variable feedback adaptive control systems so that the tracking error $\tilde{x}_1 = x_1 - x_{1r}$ asymptotically converges to zero, despite uncertainties in the roll dynamics. For this purpose, consider a target manifold given by

$$\psi(x, t) = \tilde{x}_2 + \lambda \tilde{x}_1 = 0 \quad (2.4)$$

where $\tilde{x}_2 = x_2 - x_{2r}$, and $x_{2r} = \dot{x}_{1r}$. In view of equation (2.4), one observes that if the trajectory of the system evolves on the target manifold, then the tracking error satisfies $\tilde{x}_1(t) = e^{-\lambda t} \tilde{x}_1(0)$. Therefore, for the suppression of the wing rock motion, it suffices to regulate $\psi$ to zero.

### 2.3 Speed-Gradient-Based Finite Form Adaptive System

This section presents an adaptive wing rock control system based on a finite form (integral-algebraic form) realization of a speed-gradient adaptation algorithm. The derivation of control law is based on the design technique of Tyukin [32], and Tyukin et al. [34, 35]. The original design approach proposed in these references are applicable to a large class of systems with nonconvex as well as nonlinear parameterization with known control effectiveness gain. However, it is possible to extend their design method to the wing rock model with unknown control input gain $b$. 
Because the interest is to steer the roll angle trajectory to the target manifold, consider the dynamics of $\psi(x, t)$ along the solution of equation (2.1). Differentiating $\psi$ gives

$$
\dot{\psi} = \dot{x}_2 + \lambda \dot{x}_1
$$

$$
= \theta_h^T \chi_h(x) + bu - \ddot{x}_2 + \lambda \ddot{x}_1
$$

(2.5)

Consider a nonlinear function $\gamma(\psi)$ given by

$$
\gamma(\psi) = k_1 \psi + k_2 \psi^3
$$

(2.6)

where $k_1 > 0$ and $k_2 > 0$ are design parameters. Note that for this choice of $\gamma(\psi)$, the inequality $\psi \gamma(\psi) > 0$ holds for all $\psi \neq 0$. It is possible to select other nonlinear functions $\gamma(\psi)$ which lie in the first and third quadrant for the design. Adding and
subtracting $\gamma(\psi)$ and factoring $b$, one can write equation (2.5) as

$$\dot{\psi} = b[b^{-1}\{\theta^T_h \chi_h(x) - \ddot{x}_1 + \lambda(x_2 - \dot{x}_1 + \gamma(\psi)\} + u] - \gamma(\psi)$$

$$\triangleq b[\theta^T \chi(x,t) + u] - \gamma(\psi)$$

(2.7)

where the parameter vector $\theta \in \mathbb{R}^6$ and the regressor vector $\chi(x,t) \in \mathbb{R}^6$ are given by

$$\theta = b^{-1}[-a_0, -a_1, -a_2, -a_3, -a_4, 1]^T$$

$$\chi(x,t) = [x_1, x_2, |x_2| x_2, x_1^3, x_1^2 x_2, (-\ddot{x}_1 + \lambda \dot{x}_1 + \gamma(\psi))]^T$$

(2.8)

In equation (2.7), the parameter vector $\theta$ is not known.

Let $\hat{\theta} \in \mathbb{R}^6$ be an estimate of the parameter vector $\theta$. Now in view of equation (2.7), an adaptive feedback linearizing control signal is chosen as

$$u = -\hat{\theta}^T \chi(x,t)$$

(2.9)

Then substituting the control law equation (2.9) in equation (2.7) gives

$$\dot{\psi}(x,t,\hat{\theta}) = b[-\hat{\theta}^T \chi(x,t)] - \gamma(\psi)$$

(2.10)

where $\tilde{\theta} = \hat{\theta} - \theta$ is the parameter vector error. For the chosen function $\gamma(\psi)$, it is seen that if $\tilde{\theta}=0$ or $\tilde{\theta}^T \chi(x,t) = 0$, then the following equation holds:

$$\frac{d\psi^2}{dt} = -\psi \gamma(\psi) \leq 0$$

(2.11)
This implies asymptotic convergence of $\psi$ to zero; and therefore, subsequently $\tilde{x}_1$ will converge to zero.

For the derivation of the structure of the adaptation law, consider an objective function $Q(t)$ given by (Fradkov et al., [8])

$$Q = \frac{1}{2} \int_0^t [\psi(x(s), s, \hat{\theta}(s)) + \gamma(\psi(x(s), s))]^2 ds \quad (2.12)$$

The objective is to regulate $Q(t)$ to zero by the choice of the adjustable parameter vector $\hat{\theta}(t)$. The speed of change of $Q$ is given by

$$\dot{Q}(t) = \frac{1}{2} [\dot{\psi}(x(t), t, \hat{\theta}(t)) + \gamma(\psi(x(t), t))]^2 \quad (2.13)$$

Using (10) gives

$$\frac{\partial \dot{\psi}}{\partial \hat{\theta}} = -b \chi(x, t) \quad (2.14)$$

Therefore, the partial derivative of $\dot{Q}$ with respect to $\hat{\theta}$ takes the form

$$\frac{\partial \dot{Q}}{\partial \hat{\theta}} = [\dot{\psi} + \gamma(\psi)] \frac{\partial \dot{\psi}}{\partial \hat{\theta}} = -|b| \text{sign}(b) [\dot{\psi} + \gamma(\psi)] \chi(x, t) \quad (2.15)$$

(Often the arguments of functions are suppressed for simplicity.) Then according to Fradkov et al. [8], in view of equation (2.15), the speed-gradient (velocity-gradient) algorithm for the adaptation of $\hat{\theta}$ is selected as

$$\dot{\hat{\theta}} = \Gamma [\dot{\psi} + \gamma(\psi)] \chi(x, t) \text{sgn}(b) \quad (2.16)$$

where $\Gamma > 0$ is an adaptation gain. One observes that according to equation (2.16),
the change of $\hat{\theta}$ is selected to be proportional to the negative gradient of $\dot{Q}(t)$ (the speed of change in the objective function $Q(t)$). Note that $|b| > 0$ is not included in the update rule because it is not known.

The adaptation law equation (2.16) cannot be implemented because $\dot{\psi}$ given in equation (2.10) is a function of the unknown parameter vector $\tilde{\theta}$. To obtain a realizable adaptive law, consider the parameter estimate $\hat{\theta}$ of the finite form

$$\hat{\theta} = \Gamma(\hat{\theta}_p(x,t) + \hat{\theta}_I)\text{sgn}(b) \quad (2.17)$$

The chosen $\hat{\theta}$ in equation (2.17) is the sum of an integral term, $\theta_I$, as well as an algebraic vector function $\hat{\theta}_p(x,t)$. The component $\theta_I$ of $\hat{\theta}$ will be generated by a dynamic system of integral form. The choice of $\hat{\theta}_I$ and $\hat{\theta}_p(x,t)$ will be made such that the derivative of $\hat{\theta}$ satisfies the speed-gradient adaptation law equation (2.16). Furthermore, the algebraic vector function $\hat{\theta}_p(x,t)$ must be chosen so that equation (2.17) is free of unknown parameters and the derivative of state vector.

For the derivation of $\hat{\theta}_I$ and $\hat{\theta}_p$, consider the derivative of $\hat{\theta}$ in equation (2.17) which is given by

$$\dot{\hat{\theta}} = \Gamma(\dot{\hat{\theta}}_p + \dot{\theta}_I)\text{sgn}(b) \quad (2.18)$$

To this end, the algebraic vector function $\hat{\theta}_p(x,t)$ is chosen as

$$\hat{\theta}_p(x,t) = \psi(x,t)\chi(x,t) - \Psi(x,t) \quad (2.19)$$

where the nonlinear vector function $\Psi(x,t)$ is yet to be determined. Then differen-
Substituting \( \dot{x}_2 \) derived from equation (2.10) in equation (2.20) will cause appearance of unknown parameter vector \( \tilde{\theta} \). Naturally, one selects \( \Psi_a(x,t) \) to eliminate \( \dot{x}_2 \)-dependent functions in equation (2.20). This is achieved by setting

\[
\frac{\partial \Psi_a}{\partial x_2} = \psi \frac{\partial \chi}{\partial x_2} \quad (2.21)
\]

Note that although \( \dot{\psi} \) depends on unknown parameters, it is retained in view of equation (2.16). Then using equation (2.21) in equation (2.20) yields

\[
\dot{\hat{\theta}}_p = \dot{\psi} \chi + \psi \left\{ \frac{\partial \chi}{\partial t} + \frac{\partial \chi}{\partial x_1} x_2 + \frac{\partial \chi}{\partial x_2} \dot{x}_2 \right\} - \frac{\partial \Psi_a}{\partial t} - \frac{\partial \Psi_a}{\partial x_1} x_2 - \frac{\partial \Psi_a}{\partial x_2} \dot{x}_2 \quad (2.22)
\]

Substituting \( \dot{\hat{\theta}}_p \) from equation (2.22) in (2.18) gives

\[
\dot{\hat{\theta}} = \Gamma \left[ \dot{\psi} \chi + \psi \left\{ \frac{\partial \chi}{\partial t} + \frac{\partial \chi}{\partial x_1} x_2 \right\} - \frac{\partial \Psi_a}{\partial t} - \frac{\partial \Psi_a}{\partial x_1} x_2 + \dot{\hat{\theta}}_I \right] sgn(b) \quad (2.23)
\]

In view of equation (2.23), for cancelling the known functions, the integral update law is chosen as

\[
\dot{\hat{\theta}}_I = -\psi \left\{ \frac{\partial \chi}{\partial t} + \frac{\partial \chi}{\partial x_1} x_2 \right\} + \frac{\partial \Psi_a}{\partial t} + \frac{\partial \Psi_a}{\partial x_1} x_2 + \gamma(\psi) \chi \quad (2.24)
\]

Note that an additional function \( \gamma(\psi) \chi \) has been introduced in equation (2.24). Then
substituting $\dot{\theta}_I$ from equation (2.24) in (2.23) gives the desired speed gradient adaptation law; that is,

$$\dot{\theta} = \Gamma[\dot{\psi} + \gamma(\psi)]\chi \text{sgn}(b)$$  \hspace{1cm} (2.25)

It is pointed out that the update law equation (2.25) is only for the purpose of analysis.

The adaptation law in finite form is realized by using the algebraic vector function $\theta_p$ given in equation (2.19) and the integral adaptation equation (2.23) for generating $\theta_I$. Of course, $\Psi_a(x, t)$ is to be obtained by solving equation (2.21).

Now that the adaptation law of a finite form has been designed, one proceeds to to establish stability in the closed-loop system. Substituting $\dot{\psi}$ from equation (2.10) in equation (2.25) gives

$$\dot{\theta} = -|b|\Gamma(\tilde{\theta}^T \chi(x, t))\chi(x, t)$$  \hspace{1cm} (2.26)

Now consider a Lyapunov function

$$V = \frac{\psi^2}{2} + l_1 \frac{\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}}{2}|b|^{-1}$$  \hspace{1cm} (2.27)

where $l_1 > 0$. Differentiating $V$ along the solution of (10) and (26) gives

$$\dot{V} = \psi[-b\tilde{\theta}^T \chi - \gamma(\psi)] + l_1 \tilde{\theta}^T \Gamma^{-1} \{-\Gamma|b|(\tilde{\theta}^T \chi)\chi\}|b|^{-1}$$

$$\leq -\psi \gamma(\psi) + |b| \cdot |\psi \dot{\theta}^T \chi| - l_1 (\tilde{\theta}^T \chi)^2$$  \hspace{1cm} (2.28)

Using the Young’s inequality, for any $p_1 > 0$, one has

$$|\psi \dot{\theta}^T \chi| \leq p_1 \psi^2 + \frac{(\dot{\theta}^T \chi)^2}{4p_1}$$  \hspace{1cm} (2.29)
Substituting (2.29) in (2.28), and collecting terms gives

\[
\dot{V} \leq -[\psi \gamma(\psi) - |b|p_1 \psi^3] - (\tilde{\theta}^T \chi)^2 \left( l_1 - \frac{|b|}{4p_1} \right)
\]

(2.30)

Because \(l_1\) and \(p_1\) are arbitrary positive numbers, these can be chosen so that \(l_2\) and \(p_2\) (defined below) satisfy

\[
l_2 = k_1 - |b|p_1 > 0; \quad p_2 = l_1 - |b|(4p_1)^{-1} > 0
\]

(2.31)

In view of equation (2.6), for such a choice of \(l_1\) and \(p_1\), one has

\[
\dot{V} \leq -l_2 \psi^2 - k_2 \psi^4 - p_2 (\tilde{\theta}^T \chi)^2 \leq 0
\]

(2.32)

Because \(V\) is a positive definite function of \(\psi\) and \(\tilde{\theta}\), in view of equation (2.32), \(\psi\) and \(\tilde{\theta}\) are bounded. Using the definition of \(\psi\), one finds that \(\tilde{x}_1\) and \(\tilde{x}_2\) are bounded. This also implies that the control input \(u\) is bounded. Furthermore, integrating equation (2.32), one finds that \(\psi\) and \(\tilde{\theta}^T \chi\) are square integrable functions. Then using the Barbalat’s lemmas (Astolfi et al., 2008), one concludes convergence of \(\psi\) and \(\tilde{\theta}^T \chi\) to zero. Of course, convergence of \(\psi\) to zero implies that \(\phi - \phi_r\) tends to zero as \(t \to \infty\).

Because the chosen reference trajectory \(x_{1r}\) converges to zero, \((\phi, \dot{\phi})\) also tends to zero. It is interesting to note that eventually the trajectory of the system lies in the manifold defined by

\[
\Omega = \{(x, \tilde{\theta}) : \tilde{\theta}^T \chi(x, t) = 0\}
\]

(2.33)

Based on these arguments, the following theorem is stated.
**Theorem 1:** Consider the closed-loop system including the wing rock model (2.1), the control law equation (2.9) and the adaptation law equations (2.17), (2.19) and (2.24). Then for any initial condition \((x(0), \hat{\theta}_I(0)) \in \mathbb{R}^2 \times \mathbb{R}^6\), all the signals in the closed-loop system are bounded, and \(\phi, \dot{\phi}\) and \(\hat{\theta}^T \chi(x, t)\) asymptotically tend to zero.

Now the design is completed by solving for \(\Psi_a(x, t)\) using equation (2.21). Integrating equation (2.21) gives the algebraic vector function

\[
\Psi_a(x, t) = \int_0^{x_2} \psi(x_1, \xi, t) \frac{\partial \chi(x_1, \xi, t)}{\partial \xi} d\xi
\] (2.34)

For the computation of the derivative of \(\hat{\theta}_I\) in equation (2.24) and \(\hat{\theta}_p\) in equation (2.19), the expressions for \(\Psi_a\) and the partial derivatives of \(\Psi_a\) and \(\chi\) with respect to \(t\) and \(x_1\) are provided in the appendix.

### 2.4 I&I-Based Adaptive Law

In this section, based on the immersion and invariance theory [3], an adaptive law is designed. In view of equation (2.7), similar to equation (2.9), the control law is chosen as

\[
u = -\hat{\theta}^T \chi(x_1, x_2, t)
\] (2.35)

where \(\hat{\theta}\) is an estimate of \(\theta\). But now \(\hat{\theta}\) is assumed to be of the form

\[
\hat{\theta} = \mu_a(x_1, x_2, t) + \hat{\theta}_d
\] (2.36)
in which $\mu_a(x_1, x_2, t)$ is the algebraic part and $\hat{\theta}_d(t)$ is generated by a dynamic adaptation law. Therefore, in the closed-loop system, similar to equation (2.10), one has

$$\dot{\psi} = -b\tilde{\theta}^T\chi(x, t) - \gamma(\psi)$$  \hspace{1cm} (2.37)

where $\tilde{\theta} = \hat{\theta}_d + \mu_a(x, t) - \theta$ is the parameter vector error.

Now the derivation of the adaptation law is considered. Differentiating $\tilde{\theta}$ gives

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}}_d + \frac{\partial \mu_a}{\partial t} \frac{\partial}{\partial t} + \frac{\partial \mu_a}{\partial x_1} x_2 + \frac{\partial \mu_a}{\partial x_2} \dot{x}_2$$  \hspace{1cm} (2.38)

Using (5), one solves for $\dot{x}_2$ to obtain

$$\dot{x}_2 = \dot{\psi} + \ddot{x}_1 r - \lambda(x_2 - \dot{x}_1 r)$$  \hspace{1cm} (2.39)

Substituting for $\dot{\psi}$ from (2.37) in (2.39) gives

$$\dot{x}_2 = -b\tilde{\theta}^T\chi - \gamma(\psi) + \ddot{x}_1 r - \lambda(x_2 - \dot{x}_1 r)$$  \hspace{1cm} (2.40)

In view of equation (2.40), the derivative of $\tilde{\theta}$ takes the form

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}}_d + \frac{\partial \mu_a}{\partial t} + \frac{\partial \mu_a}{\partial x_1} x_2 + \frac{\partial \mu_a}{\partial x_2} [-b\tilde{\theta}^T\chi - \gamma(\psi) + \ddot{x}_1 r - \lambda(x_2 - \dot{x}_1 r)]$$  \hspace{1cm} (2.41)

The parameter error dynamics includes known functions as well as an unknown $\tilde{\theta}$-dependent function. To this end, the dynamic adaptation law is chosen to cancel all the known terms in equation (2.41) by setting
\[ \dot{\theta}_d = -\frac{\partial \mu_a}{\partial t} - \frac{\partial \mu_a}{\partial x_1} x_2 + \frac{\partial \mu_a}{\partial x_2} [\gamma(\psi) - \ddot{x}_1 + \lambda(x_2 - \dot{x}_1)] \] (2.42)

Then using equation (2.42) in (2.41), one obtained

\[ \dot{\tilde{\theta}} = -b \frac{\partial \mu_a}{\partial x_2} \tilde{\theta}^T \chi(x, t) \] (2.43)

For examining the stability of the parameter error dynamics, consider a Lyapunov function

\[ W = |b|^{-1} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \] (2.44)

Its derivative along the solution of (43) is

\[ W = -|b|^{-1} \tilde{\theta}^T \Gamma^{-1} \left( b \frac{\partial \mu_a}{\partial x_2} \tilde{\theta}^T \chi \right) \] (2.45)

For the stability of the equilibrium point \( \tilde{\theta} = 0 \) of equation (2.43), a selection of the algebraic vector function \( \mu_a(x, t) \) is made such that

\[ \frac{\partial \mu_a}{\partial x_2} = \text{sgn}(b) \Gamma \chi(x_1, x_2, t) \] (2.46)

Solving equation (2.46), one obtains the algebraic vector function of the form

\[ \mu_a(x, t) = \text{sgn}(b) \Gamma \int_0^{x_2} \chi(x_1, \xi, t) d\xi \] (2.47)

Substituting (2.46) in (2.45) yields

\[ \dot{W} = -\left( \tilde{\theta}^T \chi(x, t) \right)^2 \leq 0 \] (2.48)
Because $W$ is a positive definite function of $\tilde{\theta}$ and and $\dot{W}$ is negative semidefinite, one concludes that $\tilde{\theta} = 0$ is globally stable, and $\tilde{\theta}$ remains bounded for all time. Now stability in the closed-loop system can be established using a quadratic Lyapunov function $V(\psi, \tilde{\theta})$ given in equation (2.27). Because the proof is similar to the proof of the previous section, it is not repeated here. The computation of the expressions for $\dot{\hat{\theta}}_d$ and $\mu_a$ can be completed as done for the speed-gradient algorithm in the appendix. (This is not included here in order to save space.)

It is noted that the two adaptation algorithms developed here are not identical. This is evident if one compares the algebraic vector function $\hat{\theta}_p(x, t)$ of the first adaptation algorithm in equation (2.34) and the algebraic function $\mu_a(x, t)$ in equation (2.47). Because the algebraic functions differ, the integral adaptation rules for $\hat{\theta}_I(t)$ and $\hat{\theta}_d(t)$ are also not identical. In the next section, simulation results are presented and effectiveness of the two algorithms is examined.

In this study, for the derivation of the speed-gradient adaptation law and I&I-based update law, unmodeled dynamics and external disturbance moment have been assumed to be ignorable. But the linearly parameterized function $f(\phi, \dot{\phi})$ in Eq. (2.1) computed using test results can represent the actual aerodynamic nonlinearities only in an approximate way. However, the proposed adaptive laws can be extended to wing rock models, including unmodeled nonlinearities and external disturbance input. In the literature, several modifications of update laws, such as parameter projection, $\sigma$-modification, $e_1$-modification, etc., have been proposed for ensuring stability in the closed-loop system (Narendra and Annaswamy, 1989). Of course, it is possible to use
neural networks to model aerodynamic nonlinearities with a high degree of accuracy so that the unmodeled functions have insignificant effect. This will simply require adaptation laws of larger dimensions in the speed-gradient adaptive scheme and the I&I-based adaptive system.

2.5 Simulation Results

This section presents the simulation results for wing rock model A and C of Guglieri [13]. The aerodynamic parameters given in Table 2.1 and Table 2.2 of the appendix are used for simulation.

The performance of the controllers for various angles of attack and initial conditions ($\phi_0 = \phi(0), \dot{\phi}_0 = \dot{\phi}(0)$) is evaluated. The initial values of the estimates $\hat{\theta}_p(0) \in R^n$ and $\hat{\theta}_d(0) \in R^n$ are arbitrarily set to zero. This is rather not a good
choice, but it is made here to examine the robustness of the adaptive law. The parameter in the manifold equation is \( \lambda = 10 \) and the adaptation gain is chosen to be \( \Gamma = 10 \). For the nonlinear function \( \gamma(\psi) \) the gains chosen are \( k_1 = 10 \), and \( k_2 = 10 \).

A fourth-order command generator

\[
(s^2 + 2\zeta_1 \omega_1 s + \omega_1^2)(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)\phi_r(t) = 0 \tag{2.49}
\]

is used for generating reference trajectories for tracking. (Here \( s \) denotes the differential operator \( d/dt \).) The command generator provides flexibility in shaping the roll angle trajectory in the closed-loop system. Its parameters are \( \zeta_i = 1 \) and \( \omega_i = 10 \). The initial conditions are set as \( \phi_r(0) = \phi(0) \) and \( (d^j/dt^j)\phi_r(0) = 0, j = 1, 2, 3 \). In the figures, instead of control signal \( u \), the applied acceleration denoted as \( u_{acc} \) (in rad/s\(^2\)) for control is plotted, where

\[
u_{acc} = \frac{\omega_u u}{l_s^2} = 1.1480 \times 10^5 u
\]

In the previous sections, it has been shown that stability in the closed-loop system is preserved for a range of the design parameters. However, similar to any adaptive system for obtaining satisfactory transient responses, here the controller and adaptation parameters have been selected after observing simulated responses. First, simulation results are presented using the speed-gradient adaptation algorithm.

A. Control of model A by speed-gradient adaptation: \( \alpha = 32.5 \text{ [deg]} \)

The complete closed-loop system including equation (2.1) for the wing model
A, the control law equation (2.9) with the adaptation law equations (2.17), (2.19) and (2.24) is simulated. Note that the initial estimate $\hat{\theta}_I(0)$ has been assumed to be zero. The responses for $\alpha = 32.5$ [deg] with the initial condition $(\phi(0), \dot{\phi}(0)) = (10$ [deg], 0 [deg/s]) are shown in Figure 2.3. Figure shows smooth convergence of the roll to zero in about one second. The tracking error is very small. The norm of $\hat{\theta}_p$ converges to zero. This could have been predicted in view of equation (2.19). But $||\hat{\theta}(t)||$ converges to a small nonzero value. It is interesting to note that $||\hat{\theta}_p||$ is considerably large compared to $||\hat{\theta}||$ in the transient period. In the steady-state, the applied acceleration $u_{acc}$ converging to zero can be observed in the figure.
B. Control of model A by speed-gradient adaptation at several angles of attack

The model parameter vector $\theta$ widely varies with the angle of attack as seen in Table 2.1 and Table 2.2. To examine the performance of the control system, the closed-loop system for model A is simulated for several values of angles of attack ($\alpha = 25, 30, 35, \text{ and } 40 \text{ [deg]}$). But initial parameter estimate $\hat{\theta}_I(0)$ is assumed to be zero for each angle of attack. The initial conditions are $\phi_0 = 10 \text{ [deg]}$ and $\dot{\phi}_0 = 0 \text{ [deg/s]}$. The controller of Case A is retained. The selected responses for model A are shown in Figure 2.4. It is observed that the adaptive law succeeds in stabilizing the roll motion in about one second for each angle of attack. It is interesting to observe that although the angles of attack are different, the roll angle responses are almost overlapping. The norm $||\hat{\theta}||$ converges to a constant value. It is observed that peak values of control magnitude, tracking error, and $||\hat{\theta}||$ monotonically increase with the angle of attack.

C. Control of model A by speed-gradient adaptation for several initial conditions

The model A is simulated for a set of values of initial roll angles ($\phi_0=-45, -30, -15, 15, 30, \text{ and } 45 \text{ [deg]}$) with $\dot{\phi}_0 = 0$. The angle of attack is $\alpha =32.5 \text{ [deg]}$. It is observed in Figure 2.5 that the roll angle converges to zero for each initial condition. The response time is of the order of one second. Again the norm of the estimated parameters $||\hat{\theta}||$ tends to a constant value. It is observed that for the suppression of the wing rock motion, larger peak values of the roll rate, control signal $u_{acc}$ and $||\hat{\theta}||$
D. Control of model C by speed-gradient adaptation

Now the performance of the speed gradient adaptive law in finite form is examined for the wing model C at a angle of attack ($\alpha = 32.5$ [deg]). The parameters of model C differ from those of model A. But the controller designed for model A is retained. The initial condition is assumed to be $(\phi_0, \dot{\phi}_0) = (10$ [deg], $0$ [deg/s]) and one has $\hat{\theta}_I(0) = 0$. The uncontrolled and controlled responses are shown in Figure 2.6 (a) and Figure 2.6 (b),(c), and (d), respectively. It is seen that the finite form adaptive law suppresses the oscillation of the roll angle in about one second (Figure 2.6 (d)).
Figure 2.5: Control of model A by speed-gradient adaptation for several initial conditions of $\phi_0 = -45, -30, -15, 15, 30, 45$ [deg]: $\alpha = 32.5$ [deg], $\dot{\phi}_0 = 0$ [deg/s]. (a) Roll angle [deg], (b) Roll rate [deg/s], (c) $u_{acc}$ [rad/s$^2$], (d) Norm of parameter estimate $\hat{\theta}$.

2.6(b)). The norm of the parameter estimate vector converges to a constant value.

**E. I&I-based adaptive control of wing model A**

The closed-loop system including the roll dynamics equation (2.1) for wing model A and the control law (2.35) with the I&I-based adaptation law equations (2.36), (2.42) and (2.47) is simulated. The initial condition is $(\phi_0, \dot{\phi}_0) = (10$ [deg], 0 [deg/s]), and the angle of attack is 32.5 [deg].

It is pointed out that the controller parameters $\lambda$, $k_1$, $k_2$, and $\Gamma$ of the speed gradient adaptive system of Case A are retained for this I&I-based adaptive system. Similar to Case A, it is observed that the control law with I&I-based adaptation
Figure 2.6: Control of model C by speed-gradient adaptation: $\alpha = 32.5$ [deg], $\phi_0=10$ [deg], $\dot{\phi}_0=10$ [deg/s]. (a) Open-loop roll angle [deg], (b) Closed-loop roll angle [deg], roll rate [deg/s], (c) $u_{acc}$ [rad/s^2], (d) Norm of parameter estimates $\hat{\theta}$ and $\hat{\theta}_p$.

suppresses the oscillations in the roll angle in about one second (Figure 2.7(b)). The responses are somewhat similar to those in Figure 2.3.

**F. I&I-based adaptive control of wing model C**

To examine the sensitivity of the I&I-based adaptive controller, simulation is done for the wing model C at the angle of attack ($\alpha = 32.5$ [deg]) with the initial state $(\phi_0, \dot{\phi}_0) = (10$ [deg], 0 [deg/s]). Selected responses are shown in Figure 2.8. Similar to the speed gradient adaptation law (Case D), this adaptive law suppresses oscillations in the roll angle in about one second. The responses of Case D and Case F are somewhat similar. The norm of the parameter estimate vector $\hat{\theta}$ (not shown...
Figure 2.7: Control of model A based on I&I adaptive law: \( \alpha = 32.5 \text{ [deg]} \), \( \phi_0 = 10 \text{ [deg]} \), \( \dot{\phi}_0 = 0 \text{ [deg/s]} \).(a) Closed-loop roll angle [deg], roll rate [deg/s].(b) \( u_{acc} \) [rad/s\(^2\)].(c) Norm of parameter estimate \( \hat{\theta} \). (d) Tracking error [deg].

here) converges to a constant value.

**G. Control of perturbed model C by speed-gradient adaptation**

The aerodynamic parameters given the appendix of Guglieri (2012) are derived from test results. Furthermore, these parameter values vary with the wing configuration. Therefore, it is of interest to investigate the performance of the controller by introducing perturbations in the parameters given in the tables. For the purpose of illustration, here the model C with perturbed parameters is considered. The computed (nominal) parameters of model C at \( \alpha = 32.5 \text{ [deg]} \) (Table 2.2) are

\[
\{\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4\} = \{0.00729, -0.01260, 0.33063, -0.00506, -0.00378\}
\]
These parameters of the nominal model are arbitrarily perturbed to obtain three set of parameters $M_1$, $M_2$, and $M_3$ using different multiplying factors for simulation. These three sets of perturbed parameters are:

(i) $M_1 = \{-0.5\hat{a}_0, 1.2\hat{a}_1, 1.2\hat{a}_2, -1.2\hat{a}_3, -1.2\hat{a}_4\} = \{-0.0036, -0.0126, 0.3968, 0.0061, 0.0045\}$

(ii) $M_2 = \{-0.8\hat{a}_0, 0.8\hat{a}_1, 1.2\hat{a}_2, -1.2\hat{a}_3, 1.2\hat{a}_4\} = \{-0.0058, -0.0101, 0.3968, 0.0061, -0.0045\}$

(iii) $M_3 = \{-1.2\hat{a}_0, 1.2\hat{a}_1, 1.2\hat{a}_2, -1.2\hat{a}_3, 1.2\hat{a}_4\} = \{-0.0087, -0.0151, 0.3968, 0.0061, -0.0045\}$

The closed-loop system including the perturbed models $M_i$ and the speed gradient adaptive system of Case A (without any change in feedback gains) is simulated. The initial state is assumed to be $(\phi_0, \dot{\phi}_0) = (45 \text{ [deg]}, 0 \text{ [deg/s]})$. Note that the initial value of $\phi(0)$ is large compared to the value in Case D. The oscillatory open-loop responses of the nominal model C as well as the perturbed models $M_1$, $M_2$, and $M_3$
Figure 2.9: Control of perturbed model C by speed-gradient adaptation: $\alpha = 32.5$ [deg], $\phi_0 = 45$ [deg], $\dot{\phi}_0 = 0$ [deg/s]. (a), (b), (c), (d) Periodic oscillations of open-loop nominal and models with perturbed parameters $M_1$, $M_2$, $M_3$, respectively. (e) Roll angle responses of nominal and perturbed models. (f) Roll rate responses of nominal and perturbed models. (g) Control signals $u_{acc}$ of nominal and perturbed models. (h) Norms of parameter estimate $\hat{\theta}$.

are shown in Figure 2.9 (a), (b), (c) and (d), respectively. One observes distinct limit cycles with significant biases (offsets) for the perturbed models. The responses of the closed-loop nominal and the three perturbed systems are also shown in Figure 2.9 (e)-(h). It is interesting to observe that despite different types of perturbations in the nominal model, and large initial deviation $\phi(0)$, the waveforms of the roll angle, roll rate and $||\hat{\theta}||$ for the nominal and the perturbed models ($M_1$, $M_2$, and $M_3$) are almost overlapping. The oscillatory responses converge to zero for each model in about one second. The control input magnitudes differ for the perturbed models, as one would have expected.
Table 2.3: Performance for model A (Fig. 2.3, 2.7) and model C (Fig. 2.6, 2.8)

<table>
<thead>
<tr>
<th>Speed-gradient-based law</th>
<th>I&amp;I-based law</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A (Fig. 2.3)</td>
</tr>
<tr>
<td>Convergence time [sec]</td>
<td>1.0046</td>
</tr>
<tr>
<td>Max control $u_{acc}$ [rad/sec$^2$]</td>
<td>156.9749</td>
</tr>
<tr>
<td>Max tracking error [deg]</td>
<td>0.0117</td>
</tr>
</tbody>
</table>

Table 2.4: Performance for model A and model C with parameters ($M_1$, $M_2$, $M_3$) of Case G. $\alpha = 32.5$ [deg]

<table>
<thead>
<tr>
<th>$\phi_0$ or parameters $M_i$</th>
<th>$45^\circ$:A</th>
<th>$30^\circ$:A</th>
<th>$15^\circ$:A</th>
<th>$M_4$:C</th>
<th>$M_5$:C</th>
<th>$M_6$:C</th>
</tr>
</thead>
<tbody>
<tr>
<td>I&amp;I</td>
<td>CT</td>
<td>1.2130</td>
<td>1.1552</td>
<td>1.0616</td>
<td>1.2129</td>
<td>1.2130</td>
</tr>
<tr>
<td></td>
<td>$u_{acc}$</td>
<td>387.5224</td>
<td>370.4039</td>
<td>227.5413</td>
<td>131.2516</td>
<td>273.2750</td>
</tr>
<tr>
<td></td>
<td>TE</td>
<td>0.0345</td>
<td>0.0223</td>
<td>0.0068</td>
<td>0.0348</td>
<td>0.0348</td>
</tr>
<tr>
<td>Speed-gradient</td>
<td>CT</td>
<td>1.2130</td>
<td>1.1528</td>
<td>1.0622</td>
<td>1.2128</td>
<td>1.2130</td>
</tr>
<tr>
<td></td>
<td>$u_{acc}$</td>
<td>387.5589</td>
<td>370.4039</td>
<td>227.5413</td>
<td>131.2038</td>
<td>273.2664</td>
</tr>
<tr>
<td></td>
<td>TE</td>
<td>0.0385</td>
<td>0.0088</td>
<td>0.0068</td>
<td>0.0388</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

To this end, a comparison of the performance of the two adaptive systems designed in this chapter for the control of linearly parameterized wing rock model Eq. (2.1) is provided. Table 2.3 summarizes the convergence time (within 2% of the final value), maximum control magnitude and the maximum tracking error obtained for the speed-gradient adaptation algorithm in Figure 2.3 and 2.6, and for the I&I-based adaptive law in Figure 2.7 and 2.8. It is seen that these performance measures for model A (denoted as A in Table 2.3), with the speed-gradient law in Figure 2.3, and with the I&I-based law in Figure 2.7, are almost identical. This similarity in performance of the two adaptive systems is also observed in Figure 2.4 and 2.5 and Table 2.3 for model C (denoted as C). Also, for the model A with different initial conditions ($\phi_0 = 45^\circ$, $30^\circ$, $15^\circ$) and for the model C with perturbed parameters $M_1$, $M_2$, $M_3$ listed in Case G, similarity in the performance measures for the two adaptive systems can be noticed in Table 2.4. In the Table 2.4, CT, $u_{acc}$, and TE denote convergence time, maximum control input and maximum tracking error, respectively. However, it is important to point out that unlike the I&I-based adaptive system, the speed-gradient adaptive law is applicable to a wider class of nonlinear systems with nonconvex and nonlinear parameterization [33].
Now the performance of the two adaptive systems designed in this chapter, the $L_1$ adaptive controller of Capello et al. [5], and the neural controller of Lee and Singh (2014) for the control of the wing rock model of Guglieri [13] is examined. First of all, the I&I-based adaptation law of this chapter differs from the I&I-based neural adaptive system of Lee and Singh [19], which uses filtered signals for design. The adaptive system of Lee and Singh [19] is not simple because additional state variables associated with the filtered signals are essential for the implementation of the control law. The simulated responses of this section confirm that the speed-gradient and I&I-based adaptation algorithms accomplish wing rock control, despite uncertainties in the parameters and large initial deviations $\phi_0$ at various angles of attack. It is interesting to point out that for similar initial conditions ($\phi_0$, $\dot{\phi}_0$) and angles of attack, the neural controller of Lee and Singh [19] requires significantly large control magnitudes compared to magnitudes observed here. For example, for the model A with ($\phi_0 = 10^\circ$, $\dot{\phi}_0 = 0$, $\alpha = 32.5^\circ$), the neural controller (see Figure 2.4 (b) of Lee and Singh (2014)) requires $u_{acc}$ more than 275 [rad/s$^2$]; but the peak value of $u_{acc}$ by using the speed-gradient and I&I-based adaptation rules is less than 162 [rad/s$^2$] (see Figure 2.3 (b) and 2.7 (b)). However, roll angle response characteristics observed here are somewhat similar to those obtained by the use of the neural controller in Lee and Singh [19]. Capello et al. [5] have noted that the $L_1$ adaptive controller, designed in their paper, is not able to suppress the wing rock of model A for starting roll angles larger than 32 deg, when $\dot{\phi}_0$ is zero, at the angle of attack 32.5 deg. But adaptive systems designed here succeed in suppressing the wing rock motion of model A and C even for large initial roll angle 45 degrees at the angle of attack 32.5 deg, as observed in Figure 2.5 and Figure 2.9. However, for smaller initial conditions, the waveforms and convergence time of the roll angle obtained by Capello et al. [5] are somewhat similar to those obtained by using the controllers derived here. Moreover, unlike the adaptive systems using only traditional integral adaptation law for wing rock control [5, 29], once the estimated parameters coincide with the true values at certain instant $t_e$ in this chapter, they remain frozen for all $t \geq t_e$. This happens because $\tilde{\theta} = 0$ is the
stable equilibrium point of the parameter error dynamics Eq. (2.26) (for the speed-gradient scheme) and Eq. (2.43) (for the I&I-based adaptive system). In adaptive systems including traditional integral adaptation law, parameter estimates keep on drifting even if parameter estimate error is zero at \( t = 0 \).

2.6 Conclusions

In this chapter, two adaptive systems in finite form for wing rock control were developed. The first algorithm was obtained by a finite form realization of an adaptation law derived by the application of the speed-gradient method; and the second adaptation scheme was based on the immersion and invariance approach. Both the adaptation laws included an integral update rule and an algebraic state-dependent vector function. Based on the Lyapunov analysis, stability in the closed-loop system, and the convergence of the roll angle tracking error to zero were established. It was shown that unlike traditional adaptive systems, the stability property of the closed-loop systems was enhanced due to the inclusion of the nonlinear algebraic term in the parameter estimate. Due to the use of the algebraic function, the derivative of the Lyapunov function included additional nonnegative function. Simulation results showed that both the adaptive systems are capable in suppressing the wing rock motion, despite uncertainties in the model parameters at various angles of attack.
CHAPTER 3

ADAPTIVE CHEBYSHEV NEURAL CONTROL OF A MULTI-INPUT AEROELASTIC SYSTEM

3.1 Introduction

This chapter presents a Chebyshev neural network-based adaptive control system for the stabilization of a multi-input multi-output prototypical aeroelastic wing section. The two degree-of-freedom aeroelastic model is equipped with a trailing-edge and a leading-edge control surface. This aeroelastic system describes the plunge and pitch motion of a wing section. The model includes unmodeled structural plunge and pitch axis nonlinearities, parameter uncertainties and gust loads. The uncontrolled aeroelastic model exhibits limit cycle oscillations beyond a critical free-stream velocity. A nonlinear adaptive control law is designed for the stabilization of the oscillatory state trajectories. For the derivation of the control law, Chebyshev neural networks are used to represent the unmodelled structural plunge and pitch axis nonlinearities, and SDU decomposition of the high-frequency gain matrix is considered for avoiding singularity in the control law.

Unlike the aeroelastic dynamics of [49, 54], in this wing model both the pitch- and plunge-axis unmodeled nonlinearities are included. The dimension of the neural adaptation law of this paper is smaller compared to the neural adaptation law of [49, 54]. Therefore, this is attractive from the viewpoint of implementation. By the Lyapunov stability analysis, it is shown that the complete state vector is uniformly ultimately bounded. Simulation results are presented which show that the control system suppresses the oscillatory responses of the system, despite large parameter uncertainties, unmodeled structural nonlinearities and gust loads.
3.2 Aeroelastic Model and Control Problem

The dynamical model of the aeroelastic system considered here has been developed in [42, 45] (Fig. 3.1).

The second-order differential equations governing the evolution of the pitch angle ($\alpha$) and the plunge displacement ($\hat{h}$) are given by

\[
\begin{bmatrix}
I_\alpha & m_W x_\alpha b

m_W x_\alpha b & m_t
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha} \\
\dot{\hat{h}}
\end{bmatrix}
+ \begin{bmatrix}
c_\alpha & 0 \\
0 & c_h
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\hat{h}
\end{bmatrix}
+ \begin{bmatrix}
k_\alpha(\alpha) & 0 \\
0 & k_h(h)
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\hat{h}
\end{bmatrix}
= \begin{bmatrix}
M + M_g \\
-L - Lg
\end{bmatrix}
\]

(3.1)
where $m_W$ is the mass of the wing section; $m_t$ is the total mass; $b$ is the semichord of the wing; $I_\alpha$ is the moment of inertia; $x_\alpha$ is the nondimensionalized distance of the center of mass from the elastic axis; $c_\alpha$ and $c_h$ are the pitch and plunge damping coefficients, respectively; $L$ and $M$ are the aerodynamic lift and moment; and $L_g$ and $M_g$ are the lift and moment due to wind gust. In this study, for simplicity a quasi-steady form of the aerodynamic force and moment given by

\[
L = \rho U^2 b C_{l\alpha} s_p \left[ \alpha + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U) \right] + \rho U^2 b C_{l\beta} s_p \beta + \rho U^2 b C_{l\gamma} s_p \gamma
\]

\[
M = \rho U^2 b^2 C_{m_{\alpha-eff}} s_p \left[ \alpha + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U) \right]
\]

\[
+ \rho U^2 b^2 C_{m_{\beta-eff}} s_p \beta + \rho U^2 b^2 C_{m_{\gamma-eff}} s_p \gamma
\]

(3.2)
is considered, where $a$ is the nondimensionalized distance from the midchord to the elastic axis, $s_p$ is the span, and $\beta$ and $\gamma$ are the trailing-edge and leading-edge flap deflections, respectively. The lift and moment derivatives due to $\alpha$ and control surface deflections are $C_{l\alpha}$, $C_{l\beta}$, $C_{l\gamma}$, and $C_{m_{\alpha-eff}}$, $C_{m_{\beta-eff}}$, $C_{m_{\gamma-eff}}$, respectively, where

\[
C_{m_{\alpha-eff}} = \left( \frac{1}{2} + a \right) C_{l\alpha} + 2C_{m_{\alpha}}
\]

\[
C_{m_{\beta-eff}} = \left( \frac{1}{2} + a \right) C_{l\beta} + 2C_{m_{\beta}}
\]

\[
C_{m_{\gamma-eff}} = \left( \frac{1}{2} + a \right) C_{l\gamma} + 2C_{m_{\gamma}}
\]

and $C_{m_{\alpha}} = 0$ for a symmetric airfoil. Similar to [54], the lift and moment caused by wind gust are assumed to be of the form

\[
L_g = \rho U^2 b s_p C_{l\alpha} w_G(\tau)/U = \rho U b s_p C_{l\alpha} w_G(\tau)
\]

\[
M_g = (0.5 - a)b L_g
\]
where \( w_G(\tau) \) denotes the disturbance velocity and \( \tau \) is a dimensionless time variable defined as \( \tau = Ut/b \). In this study, \( \alpha k_\alpha(\alpha) \) and \( h k_h(h) \) are assumed to be unstructured (unmodeled) functions.

Define the state vector as \( x = (x_1, ..., x_4)^T = (\alpha, h, \dot{\alpha}, \dot{h})^T \in \mathbb{R}^4 \). Solving Eqs. (3.1) and (3.2) for \( \ddot{\alpha} \) and \( \ddot{h} \), one obtains a state variable representation of the form

\[
\dot{x} = \begin{bmatrix} x_3 \\ x_4 \\ a_{11}\dot{\alpha} + a_{12}\dot{h} + f_1(\alpha, h) \\ a_{21}\dot{\alpha} + a_{22}\dot{h} + f_2(\alpha, h) \end{bmatrix} + + \begin{bmatrix} 0_{2 \times 2} \\ B_g \\ M_g(t) \\ L_g(t) \end{bmatrix} \begin{bmatrix} \beta \end{bmatrix} + \begin{bmatrix} 0_{2 \times 2} \\ B \end{bmatrix} \begin{bmatrix} \gamma \end{bmatrix} \tag{3.3}
\]

where \( B \in \mathbb{R}^{2 \times 2} \) and \( B_g \in \mathbb{R}^{2 \times 2} \) are appropriate constant matrices, and \( a_{ij} \) are constant parameters. The nonlinear functions \( f_1 \) and \( f_2 \) include the unmodeled structural nonlinearities, as well as linear aerodynamic functions. Note that \((\dot{\alpha}, \dot{h})\) dependent linear functions are written separately in Eq. (3.3).

The open-loop system without the wind gust exhibits limit cycle oscillations (LCOs) beyond a critical free-stream velocity. In this study, it is assumed that \( a_{ij}, B_g, B \) and the disturbance inputs \( M_g \) and \( L_g \) are unknown. Furthermore, \( f_1 \) and \( f_2 \) are unstructured (unmodeled) nonlinear functions. The objective is to design an adaptive control system for the suppression of the oscillatory plunge and pitch angle responses in the presence of uncertainties and gust load in the model. It is assumed that the complete state vector is available for feedback.

### 3.3 Neural Adaptive Control System

In this section, for the nonlinear time-varying system Eq. (3.3), an adaptive control system is designed. The control law includes Chebyshev neural networks for obtaining approximations of the unmodeled functions \( f_1 \) and \( f_2 \).

The Chebyshev polynomials are orthogonal functions which have played important role in numerical analysis. These polynomials have good approximation proper-
ties. There are several kinds of Chebyshev polynomials, but in this study Chebyshev polynomials $T_n(z)$ ($z \in \{\alpha, h\}$) of the first kind are used [57]. The range of argument of these polynomials is in the interval $[-1, 1]$. The recurrence relation

$$T_n(z) = 2zT_{n-1}(z) - T_{n-2}(z); n = 2, 3, ...$$  

(3.4)

together with the initial conditions

$$T_0(z) = 1, T_1(z) = z$$  

(3.5)

recursively generates all the polynomials $T_n(z)$.

Because the range of $\alpha$ and $h$ differ from $[-1, 1]$, shifted Chebyshev polynomials are constructed. Suppose $\alpha$ and $h$ lie in the range $[\alpha_m, \alpha_M]$ and $[h_m, h_M]$, respectively. Then making these ranges correspond to the range $[-1, 1]$, new variables $\xi_\alpha$
and $\xi_h$ are formed by linear transformation given by

$$
\begin{align*}
\xi_\alpha &= \frac{2\alpha - (\alpha_m + \alpha_M)}{\alpha_M - \alpha_m} \\
\xi_h &= \frac{2h - (h_m + h_M)}{h_M - h_m}
\end{align*}
$$

(3.6)

The shifted Chebyshev polynomials of the first kind are now $T_n(\xi_\alpha)$ and $T_n(\xi_h)$.

It is pointed out that for the aeroelastic model, $f_i(\alpha, h)$ in Eq. (3.3) can be written as

$$
f_i = f_{i1}(\alpha) + f_{i2}(h), i = 1, 2
$$

(3.7)

because the nonlinear functions are separable. Now each nonlinear function $f_{ij}$ is approximated by shifted Chebyshev polynomials. By the choice of sufficient numbers of the Chebyshev polynomials, one can approximate these nonlinear functions as ($i = 1, 2$),

$$
f_i = \sum_{k=0}^{p} \theta_{i1k} T_k(\xi_\alpha(\alpha)) + \sum_{k=0}^{p} \theta_{i2k} T_k(\xi_h(\xi)) + e_i$$

where $\Phi_{ji} \in \mathbb{R}^{p+1}$, $\Theta_i \in \mathbb{R}^{2p+2}$ are constant unknown parameter vectors, and $e_i$ are approximation errors. In the following analysis the approximation error are ignored for notational simplicity because these errors can be lumped with the external disturbance inputs.

For the purpose of design, define a vector $s \in \mathbb{R}^2$ as

$$
s = \begin{bmatrix} \dot{\alpha} + \lambda_\alpha \alpha \\ \dot{h} + \lambda_h h \end{bmatrix}
$$

(3.9)

where $\lambda_\alpha, \lambda_h > 0$, are the design parameters. Note that if $s=0$, then according to Eq. (3.9), $(\alpha, h)$ converges to zero. Using Eqs. (3.3) and (3.8), the derivative of $s$ can be
written as

\[
\dot{s} = \begin{bmatrix}
\dot{\alpha} + \lambda \dot{\alpha} \\
\dot{h} + \lambda \dot{h}
\end{bmatrix} = \begin{bmatrix}
a_{11} \dot{\alpha} + a_{12} \dot{h} + (\Phi_{11}(\alpha), \Phi_{12}(h)) \Theta_1 + g_1(t) \\
a_{21} \dot{\alpha} + a_{22} \dot{h} + (\Phi_{21}(\alpha), \Phi_{22}(h)) \Theta_2 + g_2(t)
\end{bmatrix} + Bu_c \tag{3.10}
\]

where \(u_c = (\beta, \gamma)^T \in \mathbb{R}^2\), \(g = ([g_1, g_2])^T = B_g([M_g(t), L_g(t)])^T \in \mathbb{R}^2\), \(\bar{a}_{11} = a_{11} + \lambda \alpha\), and \(\bar{a}_{22} = a_{22} + \lambda h\).

For the design of a singularity-free adaptation law, the \(SDU_s\) decomposition of the high-frequency gain matrix \(B\) is obtained. The following assumption is made

**Assumption 1:**

The leading principal minors \(\Delta_i (i=1,2)\) of \(B\) are nonzero and sign of each \(\Delta_i\) is known.

For the aeroelastic model, \(\Delta_i\) is nonzero.

It can be verified that under Assumption 1, \(B\) can be factorized as

\[
B = SDU_s \tag{3.11}
\]

where

\[
S = \begin{bmatrix}
\eta_1^{-1} |b_{11}| & \eta_1^{-1} \text{sgn}(b_{11}) b_{21} \\
\eta_1^{-1} \text{sgn}(b_{11}) b_{21} & \eta_2^{-1} \text{sgn}(b_{21}) |b_{11}|^{-1} + |\Delta_2| \eta_2^{-1} |b_{11}|^{-1}
\end{bmatrix}
\]

\[
D = \text{diag} \left\{ \text{sgn}(\Delta_1) \eta_1, \text{sgn} \left( \frac{\Delta_2}{\Delta_1} \right) \eta_2 \right\} \tag{3.12}
\]

\[
U_s = \begin{bmatrix} 1 & \theta_s \\ 0 & 1 \end{bmatrix}
\]

\[
\theta_s = (b_{12} - \eta_2 \eta_1^{-1} b_{21} \text{sgn}(\Delta_2)) b_{11}^{-1}
\]

and the design parameters \(\eta_1\) and \(\eta_2\) are positive real numbers. These parameters are useful in shaping closed-loop responses. The matrix \(S\) is a positive definite symmetric matrix, the diagonal matrix \(D\) has elements \(\pm \eta_i\), and the matrix \(U_s\) is an upper triangular matrix. The matrix \(S\) and the parameter \(\theta_s\) of \(U_s\) are not known, but the
diagonal matrix $D$ is known in view of Assumption 1.

Define a regressor matrix $(\Phi_a^T) \in \mathbb{R}^{2 \times 4(p+2)}$

$$\Phi_a^T =
\begin{bmatrix}
\dot{\alpha} & \dot{h} & \Phi_{11}^T(\alpha) & \Phi_{12}^T(h) & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \dot{\alpha} & \dot{h} & \Phi_{21}^T(\alpha) & \Phi_{22}^T(h)
\end{bmatrix}$$

(3.13)

and

$$\Theta_a = [\bar{a}_{11} \ a_{12} \ \Theta_1^T \ a_{21} \ \bar{a}_{22} \ \Theta_2^T]^T \in \mathbb{R}^{4(p+2)}$$

Then using Eqs. (3.13) and (3.11), Eq. (3.10) can be written as

$$\dot{s} = \Phi_a^T \Theta_a + g(t) + SDU_s u_c$$

(3.14)

Note that

$$DU_s u_c = Du_c + [d_{11} \theta_s \gamma, 0]^T$$

(3.15)

Substituting Eq. (3.15) in Eq. (3.14) gives

$$\dot{s} = S[S^{-1}(\Phi_a^T \Theta_a + g(t)) + DU_s u_c]$$

(3.16)

where for an appropriate vector $\Theta_b$ and regressor matrix $\Phi_b^T$

$$\Phi_b^T \Theta_b = S^{-1} \Phi_a^T \Theta_a + [d_{11} \theta_s \gamma, 0]^T$$

(3.17)

It can be seen that

$$\Phi_b^T = [\Phi_a^T, (d_{11} \gamma, 0)^T] \in \mathbb{R}^{2 \times (4p+9)}$$

$$\Theta_b = [\Theta_a^T, \theta_s]^T \in \mathbb{R}^{4p+9}$$

(3.18)
For the derivation of the control law, consider a Lyapunov function

$$V_1(s) = \frac{s^T S^{-1} s}{2}$$  \hspace{1cm} (3.19)

Its derivative along the solution of Eq.(3.16) gives

$$\dot{V}_1(s) = s^T [\Phi_b^T \Theta_b + D u_c] \leq s^T [\Phi_b^T \Theta_b + G(s) g_m + D u_c] + \zeta$$  \hspace{1cm} (3.20)

where $S^{-1} g = (\bar{g}_1, \bar{g}_2)^T$ and $|\bar{g}_i| \leq g_{mi}$, a constant, $i=1,2$

According to [58], for any $\epsilon_i > 0$, $|s_i|$ satisfies the inequality

$$|s_i| \leq k_p \epsilon_i + s_i \tanh\left(\frac{s_i}{\epsilon_i}\right), i = 1, 2$$  \hspace{1cm} (3.21)

where $k_p = 0.2758$. Therefore,

$$\sum_{i=1}^{2} |s_i| g_{mi} \leq \sum_{i=1}^{2} [k_p \epsilon_i + s_i \tanh\left(\frac{s_i}{\epsilon_i}\right)] g_{mi}$$

$$\leq s^T G(s) g_m + \zeta$$  \hspace{1cm} (3.22)

where $\zeta = k_p [\epsilon_1 g_{m1} + \epsilon_2 g_{m2}]$, $g_m = [g_{m1}, g_{m2}]^T$ and the matrix $G(s)$ is

$$G(s) = \begin{bmatrix}
\tanh\left(\frac{s_1}{\epsilon_1}\right) & 0 \\
0 & \tanh\left(\frac{s_2}{\epsilon_2}\right)
\end{bmatrix}$$  \hspace{1cm} (3.23)

Substituting Eq. (3.22) in Eq. (3.20) gives

$$\dot{V}_1 \leq s^T [\Phi_b^T \Theta_b + G(s) g_m + D u_c] + \zeta$$

$$\leq s^T [\Psi^T \hat{\Theta} + D u_c] + \zeta$$  \hspace{1cm} (3.24)
where $\Psi^T = [\Phi^T_b, G(s)]$, $\Theta = [\Theta^T_b, g^T_m]$ and $\hat{\Theta}$ is the estimate of $\Theta$. In view of Eq. (3.24), one selects a control law of the form

$$u_c = D^{-1}[-\Psi^T \hat{\Theta} - Ls] \quad (3.25)$$

where $L = \text{diag}[l_{11}, l_{22}]$. Note that Eq. (3.25) is an implicit function of $\gamma$. However, because $U_s$ is an upper triangular matrix, Eq. (3.25) is solvable, and there is no algebraic loop. Substituting the control law Eq. (3.25) in Eq. (3.24) gives

$$\dot{V}_1 \leq s^T [\Psi^T \tilde{\Theta} - Ls] + \zeta \quad (3.26)$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$, the parameter error.

For the derivation of the adaptation law, consider a Lyapunov function

$$V_2 = V_1 + \frac{\tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta}}{2} \quad (3.27)$$

where the adaptation gain $\Gamma > 0$. Its derivative can be expressed as

$$\dot{V}_2 \leq -s^T Ls + s^T \Psi^T \tilde{\Theta} + \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta} + \zeta \quad (3.28)$$

In view of Eq. (3.28), one selects the adaptation law as

$$\dot{\hat{\Theta}} = \Gamma \Psi s - \sigma \Gamma \hat{\Theta} \quad (3.29)$$

where $\sigma > 0$. Here in order to avoid parameter divergence $\sigma$-modification has been introduced. Substitution of Eq. (3.29) in Eq. (3.28) yields

$$\dot{V}_2 \leq -s^T Ls + \sigma \tilde{\Theta}^T \tilde{\Theta} + \zeta \quad (3.30)$$

$$\dot{V}_2 \leq -s^T Ls + \sigma \tilde{\Theta}^T (\Theta - \hat{\Theta}) + \zeta$$

$$\dot{V}_2 \leq -s^T Ls - \sigma ||\tilde{\Theta}||^2 + \sigma \tilde{\Theta}^T \Theta + \zeta$$
Using Young’s inequality gives
\[
\tilde{\Theta}^T\Theta \leq \frac{||\tilde{\Theta}||^2}{2} + \frac{||\Theta||^2}{2}
\] (3.31)

Using inequality Eq.(3.31) in Eq.(3.30) gives
\[
\dot{V}_2 \leq -s^T L s - \frac{\sigma||\tilde{\Theta}||^2}{2} + \frac{\sigma||\Theta||^2}{2} + \zeta
\]
\[
\dot{V}_2 \leq -\lambda_{\text{min}}(L)||s||^2 - \frac{\sigma||\tilde{\Theta}||^2}{2} + \mu^*
\] (3.32)

where \(\lambda_{\text{min}}(L)\) is the minimum eigenvalue of \(L\) and \(\mu^* = \zeta + \frac{\sigma||\theta||^2}{2}\)

Define
\[
M = \text{diag}(\lambda_{\text{min}}(L), \frac{\sigma}{2})
\] (3.33)

Using Eq.(3.33), one can write Eq.(3.32) as
\[
\dot{V}_2 \leq -||s||, ||\tilde{\Theta}|| M[||s||, ||\tilde{\Theta}||]^T + \mu^*
\]

Because \(M > 0\), it follows that \(||s||\) and \(||\tilde{\Theta}||\) are uniformly ultimately bounded (UUB) [59]. This implies that \(\alpha\) and \(h\) are also uniformly ultimately bounded. Although, here only UUB of the state trajectory has been established, the simulation results of the next section confirm the convergence of \(\alpha\) and \(h\) to zero.

### 3.4 Simulation Results

This section presents the results of digital simulation. The model parameters taken from [45, 54] are collected in the appendix. Similar to [54], the velocity distributions of \(w_G(\tau)\) for simulation are assumed to be (i) triangular gust of finite duration.
and (ii) sinusoidal gust. For the triangular disturbance input, one has

\[ w_G(\tau) = 2w_0 \frac{\tau}{\tau_G} \left( H(\tau) - H\left( \tau - \frac{\tau_G}{2} \right) \right) + 2w_0 \left( \frac{\tau}{\tau_G} - 1 \right) \left( H(\tau - \tau_G) - H\left( \tau - \frac{\tau_G}{2} \right) \right) \]

(3.34)

where \( H(.) \) denotes the unit step function, \( \tau_G = Ut_G/b, t_G = 0.5 \) (s). The sinusoidal \( w_G(\tau) \) is

\[ w_G = w_0 \sin(\pi b\tau/U)H(\tau) \]

(3.35)

with \( w_0 = 0.7 \). Note that unlike the triangular \( w_G \), the sinusoidal velocity distribution is nonzero for all \( t > 0 \).

For simulation, two free-stream velocities \( U = 13.28 \) (m/s) and \( U = 15 \) (m/s) are considered. The value of the input matrix \( B \) for \( U = 13.28 \) (m/s) is

\[
B = \begin{pmatrix}
-82.0645 & -13.2214 \\
-6.9714 & 0.8979
\end{pmatrix}
\]

(3.36)

The design parameters \( \lambda_\alpha \) and \( \lambda_h \) in the definition of \( s \) in Eq. (3.9) are 8 and 20, respectively. For the approximation of the nonlinearities, two Chebyshev neural networks are constructed. The nonlinear functions of \( \alpha \) and \( h \) involve third degree polynomials of \( \alpha \) and \( h \), respectively [44, 45] (see the appendix). However, it is pointed out that the Chebyshev neural network can be used for the representation of other types of structural and aerodynamic nonlinearities. Therefore, the Chebyshev polynomials of the first kind given by \( T_j(\xi_\alpha) \) and \( T_j(\xi_h), j = 0, 1, 2, 3 \), are used for the neural networks. The scaling parameters are \( \alpha_m = -16, \alpha_M = 16 \) (deg), \( h_m = -0.07 \) and \( h_M = 0.07 \) (m). The feedback matrix in the control law Eq. (3.25) for damping is \( L = diag[3,1] \). The adaptation gain is selected as \( \Gamma = 50 \) and the parameters \( \epsilon_1 \) and \( \epsilon_2 \) are set to 0.1. The initial estimate of the unknown parameter vector is assumed to be \( \hat{\Theta}(0) = 0 \). This is not a good choice of the parameter estimates, but is made to examine the robustness of the controller. The parameter for \( \sigma \)-modification is \( \sigma = 0.0001 \).
The poles of the linearized uncontrolled system for $U = 13.28$ m/s and $a = -0.6719$ are $(1.3071 \pm 12.9398i, -2.8512 \pm 12.4120i)$. Therefore, $x = 0$ is unstable for $U = 13.28$ (m/s). The initial condition is set to $h(0) = 0$ (m), $\alpha(0) = 5.729$ (deg), and $\dot{\alpha}(0) = \dot{h}(0) = 0$. The responses of the open-loop disturbance-free system ($M_g = 0, L_g = 0$) are shown in Fig. 3.2. It is observed that for $U = 13.28$ (m/s), the aeroelastic model undergoes limit cycle oscillations. Now the closed-loop responses for the model equation (3.1) including the control law equation (3.25), adaptation law equation (3.29) and gust load are obtained. For a realistic simulation, similar to Wang and Behal [54], control surface deflection of each flap is allowed to saturate at 15 or 20 (deg).

**Case A. Adaptive control:** $U = 13.28$ m/s, $a = -0.6719$, $L_g = M_g = 0$,

For examining the performance of the controller, the closed-loop system for $U = 13.28$ m/s, and $a = -0.6719$ is simulated. It assumed that the external disturbance inputs ($L_g, M_g$) are zero. For the disturbance-free system, one has $\hat{\Theta} \in \mathbb{R}^{21}$ (that is, the last two update parameters of $\hat{\Theta}$ are set to zero). The flap deflection limits for $\beta$ and $\gamma$ are assumed to be 20 (deg). Selected responses are shown in Fig. 3.3.
Figure 3.4: Adaptive control; \( U = 15 \text{ m/s}, \ a = -0.6719, \ L_g = 0, \ M_g = 0 \).

It is observed that the plunge and pitch angle trajectories converge to zero in about two seconds. The control inputs saturates in the transient period and estimated parameters converge to certain constant values. Figure shows the plot of \( ||\hat{\Theta}_a|| \) and separately the estimated parameter \( \hat{\theta}_s \) of \( U_s \).

**B. Adaptive control:** \( U = 15 \text{ m/s}, \ a = -0.6719, \ L_g = M_g = 0 \).

For examining the robustness of the system, the closed-loop responses for a different value \( U = 15 \text{ (m/s)} \) are obtained. It is observed that the pitch angle and the plunge displacement converge to zero (Fig. 3.4). The response-time is of the order of 2 seconds. The control input saturates in the initial phase. The norm of the estimated parameter vector \( \hat{\Theta}_a \) and \( \hat{\theta}_s \) remain bounded and converge to certain constant values.

**C. Adaptive control:** Triangular gust, \( U = 13.28 \text{ m/s}, \ a = -0.6719 \)

Now the effect of a triangular gust load is examined. It is assumed that the velocity distribution in Eq. (3.34) has \( w_0 = 0.7 \). Because \( L_g \) and \( M_g \) are nonzero, update law for \( \hat{\Theta} \in R^{23} \) is considered. Also it is assumed that \( \beta \) and \( \gamma \) saturate at 15 [deg]. The responses are shown in Fig. 3.5. We observe the suppression of
the oscillations in $\alpha$ and $h$ in less than 2 seconds. The estimated parameters remain bounded and converge to constant values. Again the control input saturates in the transient period.

**D. Adaptive control: Sinusoidal gust, $U = 13.28 \text{ m/s}$, $a = -0.6719$ m/s**

For simulation, sinusoidal wind gust ($w_0 = 0.7$) given in Eq. (3.35) is used, where $U = 13.28 \text{ m/s}$ and $a = -0.6719$ is simulated. The flap saturation level of Case C is retained. Selected responses are shown in Fig. 3.6. Again the oscillations in the system are suppressed. The convergence time for the plunge and pitch angle trajectories is of the order of 2 seconds. In the steady-state, oscillatory flap deflections are observed. These nonzero oscillatory control surface deflections are essential for nullifying the effect of the persistent disturbance inputs.

**E. Adaptive control: $U = 13.28 \text{ m/s}$, $a = -0.6719$, $L_g = M_g = 0$**

The performance of the adaptive law (with the parameter vector $\hat{\Theta} \in \mathbb{R}^{23}$) for the control of disturbance-free model is examined. Note that for the disturbance-free case, actually $\hat{\Theta}$ of dimension 21 suffices, as used for Case A and B. But here $\hat{\Theta}$ of...
Figure 3.6: Adaptive control, sinusoidal gust: $U=13.8$ m/s, $a=-0.6719$, $w_0=0.7$.

dimension 23 is computed in order to examine the robustness of the adaptive law. The saturation level of 15 (deg) for both the flaps similar to Case D is assumed. It is seen that despite larger dimension of $\hat{\Theta}$, the controller accomplishes regulation of the pitch angle and plunge displacement to zero in less than 2 seconds (Fig. 3.7). The estimated parameters converge to certain constant values. Only the plot of $\theta_s$ is shown in the figure.

It is noted that similar to the neural controllers published in literature [49, 54] for this aeroelastic model, the suppression of the LCOs by the Chebyshev neural controller is accomplished. It may be pointed out that the Chebyshev neural adaptive controller of this paper has relatively simple structure compared to the multi-layer neural control system [54] and the Gaussian neural network of [49], and the dimension of the update law for the neural network weights is smaller. Moreover, unlike [49, 53], the model includes the plunge-and pitch-axis structural nonlinearities in this study. In the model of [49, 54], only the pitch-axis nonlinearity is assumed.
Figure 3.7: Adaptive control ($\hat{\Theta} \in R^{23}$): $U=13.8$ m/s, $a = -0.6719$, $L_g = 0$, $M_g = 0$.

3.5 Conclusions

In this chapter, a Chebyshev neural network based adaptive control system was designed for the control of the plunge and pitch angle trajectories of a two-dimensional aeroelastic system using trailing-and leading-edge flaps. The model included external perturbing force and moment due to wind gust and the system parameters were assumed to be unknown. The wing model included unmodeled plunge-axis and pitch-axis nonlinearities. The Chebyshev polynomials were used for the two neural networks for the representation of $\alpha$- and $h$-dependent nonlinearities for the adaptive law design. Based on the SDU decomposition of the high-frequency gain matrix, a singularity-free adaptive law was derived. In the closed-loop system, uniform ultimate boundedness of the trajectories was established using the Lyapunov stability theory. Simulated responses confirmed suppression of the LCOs in the closed-loop system, despite parameter uncertainties, unmodeled nonlinearities, and external triangular and sinusoidal disturbance inputs. It is seen that unlike the neural network-based adaptive laws published in literature, the controller designed here has update law of smaller dimension.
CHAPTER 4

HIGHER-ORDER SLIDING-MODE FINITE-TIME CONTROL OF AEROELASTIC SYSTEMS

4.1 Introduction

In this chapter, the model used is described in fig. 3.1. However, the mathematical expression is different. The model equations are referred from Sheta et al. [43], which has nonlinearity in the state matrix. A robust control system is designed for the tracking of reference plunge and pitch angle trajectory. The control law includes a nominal finite-time stabilizing continuous control signal designed for the model without uncertainties and a discontinuous control signal for nullifying the effect of uncertain functions in the model.

4.2 Nonlinear Aeroelastic Model

The dynamical model of the aeroelastic system considered here has been developed in [43, 45] (Fig. 3.1). The second-order differential equations governing the evolution of the pitch angle ($\alpha$) and the plunge displacement ($h$) are given by

\[ I_{EA} \ddot{\alpha} + [m_w x_\alpha b \cos(\alpha) - m_c r_c b \sin(\alpha)] \ddot{h} + c_\alpha \dot{\alpha} + k_\alpha (\alpha) \alpha = M(t) \]

\[ m_t \ddot{h} + [m_w x_\alpha b \cos(\alpha) - m_c r_c b \sin(\alpha)] \ddot{\alpha} + c_\alpha \dot{h} + \]

\[ [-m_w x_\alpha b \sin(\alpha) - m_c r_c b \cos(\alpha)] \dot{\alpha}^2 + k_h (h) h = -L(t) \]  

(4.1)

where $m_w$ is the mass of the wing section; $m_t$ is the total mass; $b$ is the semichord of the wing; $I_{\alpha}$ is the moment of inertia; $x_\alpha$ is the nondimensionalized distance of the center of mass from the elastic axis; $c_\alpha$ and $c_h$ are the pitch and plunge damping
coefficients, respectively; \( L \) and \( M \) are the aerodynamic lift and moment. In this study, for simplicity a quasi-steady form of the aerodynamic force and moment given by

\[
L = \rho U^2 b C_{l_a} s_p \left[ \alpha + \left( \frac{1}{2} - a \right) b(\dot{\alpha} / U) \right] + \rho U^2 b C_{l_3} s_p \beta + \rho U^2 b C_{l_\gamma} s_p \gamma
\]

\[
M = \rho U^2 b^2 C_{m_{a-\text{eff}}} s_p \left[ \alpha + \left( \frac{1}{2} - a \right) b(\dot{\alpha} / U) \right]
+ \rho U^2 b^2 C_{m_{3-\text{eff}}} s_p \beta + \rho U^2 b^2 C_{m_{\gamma-\text{eff}}} s_p \gamma \tag{4.2}
\]

is considered, where \( a \) is the nondimensionalized distance from the midchord to the elastic axis, \( s_p \) is the span, and \( \beta \) and \( \gamma \) are the trailing-edge and leading-edge flap deflections, respectively. The lift and moment derivatives due to \( \alpha \) and control surface deflections are \( C_{l_a}, C_{l_3}, C_{l_\gamma}, \) and \( C_{m_{a-\text{eff}}}, C_{m_{3-\text{eff}}}, C_{m_{\gamma-\text{eff}}} \), respectively, where

\[
C_{m_{a-\text{eff}}} = \left( \frac{1}{2} + a \right) C_{l_a} + 2 C_{m_a}
\]

\[
C_{m_{3-\text{eff}}} = \left( \frac{1}{2} + a \right) C_{l_3} + 2 C_{m_3}
\]

\[
C_{m_{\gamma-\text{eff}}} = \left( \frac{1}{2} + a \right) C_{l_\gamma} + 2 C_{m_\gamma}
\]

and \( C_{m_a} = 0 \) for a symmetric airfoil. For the purpose of illustration, the functions \( k_\alpha(\alpha) \) and \( k_h(h) \) are chosen as

\[
k_\alpha(\alpha) = 6.861422(1+1.1437925\alpha+96.669627\alpha^2+9.513399\alpha^3-727.664120\alpha^4)(N.m/rad)
\]

\[
k_h(h) = 2844.4(N/m) \tag{4.3}
\]
Now, from the equation (4.1), defining the system such that
\[
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{h}
\end{bmatrix} =
\begin{bmatrix}
F_1(\alpha, h, \dot{\alpha}, \dot{h}) \\
F_2(\alpha, h, \dot{\alpha}, \dot{h})
\end{bmatrix} + [B_1]u
\] (4.4)

Suppose that \((\alpha_r, h_r)\) is a bounded reference trajectory converging to zero. Define
\[
z_1 = (z_{11}, z_{21})^T = (\hat{\alpha}, \dot{\hat{\alpha}})^T \in \mathbb{R}^2; z_2 = (z_{12}, z_{22})^T = (\hat{h}, \dot{\hat{h}})^T \in \mathbb{R}^2
\] (4.5)
where \(\hat{\alpha} = \alpha - \alpha_r\) and \(\hat{h} = h - h_r\) are the pitch and plunge trajectory error, respectively. Also define \(z = (z_1^T, z_2^T)^T \in \mathbb{R}^4\). Using Eq. (4.4) gives
\[
\begin{bmatrix}
\dot{z}_{11} \\
\dot{z}_{12}
\end{bmatrix} = \begin{bmatrix}
z_{21} \\
z_{22}
\end{bmatrix} = F + Bu
\] (4.6)
where \(u = [\beta, \gamma]^T\).

4.3 Finite-Time Stabilizing Nominal Control Law

The matrices and nonlinear functions in the aeroelastic model are not precisely known. For the purpose of design, the unknown matrices and the function in Eq. (4.6) are decomposed into a known and an unknown parts as follows:
\[
F = F^* + \Delta F
\]
\[
B = B^* + \Delta B
\] (4.7)
where the quantities with the superscript \(*\) denote nominal values; and \(\Delta F\) and \(\Delta B\) denote uncertain portions of \(F\) and \(B\), respectively. The nominal matrix \(B^*\) is chosen...
such that it is nonsingular. Then the system dynamics Eq. (4.5) can be expressed as

\[ \dot{z}_{11} = z_{21}; \dot{z}_{12} = z_{22} \]

\[ [\dot{z}_{21}, \dot{z}_{22}]^T = F^* + \Delta F + (B^* + \Delta B)u \] (4.8)

In view of Eq. (4.7), one selects a control law of the form

\[ u = (B^*)^{-1}[-F^* + u_n + u_d] \] (4.9)

where \( u_n \) is a nominal control signal and \( u_d \) is a discontinuous signal, \( (u_n \text{ and } u_d \text{ are yet to be determined}) \).

The nominal control signal \( u_n \) is chosen such that the system without uncertainty becomes a homogeneous system. For a system described by a chain of \( n \) integrators, a control input which yields a homogeneous system has been derived by Bhat and Bernstein [75]. For the aeroelastic system, the continuous control signal \( u_n = (u_{n1}, u_{n2})^T \) takes the form

\[ u_{n1}(z_1) = -p_{11}\lambda_{11}^{\nu_{11}} \text{sgn}(z_{11}) - p_{21}\lambda_{21}^{\nu_{21}} \text{sgn}(z_{21}) \]

\[ u_{n2}(z_2) = -p_{12}\lambda_{12}^{\nu_{12}} \text{sgn}(z_{12}) - p_{22}\lambda_{22}^{\nu_{22}} \text{sgn}(z_{22}) \] (4.10)

where \( p_{ij} \) are selected so that

\[ \lambda^2 + p_{2i}\lambda + p_{1i} = 0 \] (4.11)

is a stable polynomial, \( (i = 1, 2) \), and \( \nu_{ij} \) are chosen to satisfy \( \nu_{1i} = \nu_{2i}/(2 - \nu_{2i}) \) with \( \nu_{2i} = \nu_i \in (1 - \epsilon_i, 1) \) and \( \epsilon_i \in (0, 1) \). For the system without uncertainties and with control input \( u_d = 0 \), the closed-loop system yields two decoupled systems and the \( \alpha \) dynamics \( (i = 1) \) and \( h \) dynamics \( (i = 2) \) are described by

\[ \dot{z}_{1i} = z_{2i} \]
\[
\dot{z}_{2i} = u_{ni} = -p_{1i} |z_{1i}|^{\nu_{i1}} \text{sgn}(z_{1i}) - p_{2i} |z_{2i}|^{\nu_{21}} \text{sgn}(z_{2i}) \quad (4.12)
\]

It is easily verified that the \(i\)th-subsystem Eq. (4.12) is homogeneous of negative degree \(\mu_{i} = (\nu_{i} - 1)/\nu_{i}\), with dilation \((\nu_{i1}^{-1}, \nu_{2i}^{-1})\). Based on a positive definite radially unbounded Lyapunov function, it has been proven in [75] that there exists \(\epsilon_{i} \in (0, 1)\) such that, for every \(\nu_{i} \in (1 - \epsilon, 1)\), the origin \(z_{i} = 0\) of Eq. (4.12) is globally finite-time stable.

Of course, in the presence of uncertainties, this nominal control law \(u_{n}\) cannot guarantee stability. Now to eliminate the effect of uncertainties, a discontinuous control signal \(u_{d}\) is designed.

### 4.4 Discontinuous Control Signal \(u_{d}\)

The design of the discontinuous control signal is based on a higher-order sliding mode control scheme of [76]. For this purpose, it is essential to make certain assumptions.

**Assumption 1:** There exist a function \(\gamma_{1}(z, t)\) and a \(\gamma_{0} \in [0, 1)\) such that the following inequalities hold:

\[
||\Delta Az + \Delta g(z, t) + \Delta B(B^*)^{-1}(u_{n} - A^*z - g^*(z, t))||_{\infty} \leq \gamma_{1}(z, t) \quad (4.13)
\]

\[
||\Delta B(B^*)^{-1}||_{\infty} \leq \gamma_{0} < 1 \quad (4.14)
\]

Although the first inequality does not pose any restriction on the uncertain functions, Eq. (4.14) limits the uncertainty in the input matrix \(B\). The inequality Eq. (4.14) is essential so that the control signal \(u_{d}\) dominates the uncertain vector function \(\Delta B(B^*)^{-1}u_{d}\).

For the design, similar to [76], a sliding vector function \(s(z, z_{a}) \in R^{2}\) is chosen as

\[
s(z) = [z_{21}, z_{22}]^T - z_{a} \quad (4.15)
\]
where \( z_a \in R^2 \) satisfies
\[
\dot{z}_a = u_n
\]  
(4.16)

It is noted that \( z_a \) is the integral of the nominal input \( u_n \). The derivative of \( s \) along the solution of (4.9) is given by
\[
\dot{s} = [I_{2\times2} + \Delta B(B^*)^{-1}]u_d + \Delta Az + \Delta g(z, t) + \Delta B(B^*)^{-1}(u_n - A^*z - g^*(z, t))
\]  
(4.17)

For the derivation of the signal \( u_d \), consider a Lyapunov function
\[
W(s) = (s^T s)/2
\]  
(4.18)

Differentiating \( W \) and using Eq. (4.17) gives
\[
\dot{W} = s^T[(I_{2\times2} + \Delta B(B^*)^{-1})u_d + \Delta Az + \Delta g(z, t) + \Delta B(B^*)^{-1}(u_n - A^*z - g^*(z, t))]
\]  
(4.19)

Using inequality Eq. (4.13) in Eq. (4.19) gives
\[
\dot{W} \leq s^T(I_{2\times2} + \Delta B(B^*)^{-1})u_d + ||s||_1||\Delta Az + \Delta g + \Delta B(B^*)^{-1}(u_n - A^*z - g^*(z, t))||_\infty
\]
\[
\leq s^T(I_{2\times2} + \Delta B(B^*)^{-1})u_d + \gamma_1(z, t)||s||_1
\]  
(4.20)

where \( ||.||_1 \) and \( ||.||_\infty \) denote 1 norm and \( \infty \) norm of a vector. For making the derivative of \( W \) negative, one selects \( u_d \) as
\[
u_d = -G(z, t)\text{sign}(s)
\]  
(4.21)

where the gain \( G(z, t) > 0 \).

4.5 Simulation Results

This section presents the results of simulation. Although stability in the closed-loop systems is ensured for any uncertainty satisfying the first assumption, here, for
Figure 4.1: Limit cycle in the open-loop system: $U=13.8 \text{ m/s, } a=-0.4$.

Figure 4.2: Limit cycle in the open-loop system: $U=18 \text{ m/s, } a=-0.4$. 
the purpose of illustrations, it is assumed that the nominal values are $F^*=0$, and $B^*=B$. The parameters of the control law $u_n$ are $p_{11} = 12$, $p_{21} = 35$, $p_{12} = 12$, $p_{22} = 35$, $\nu_{11} = 1/2$, $\nu_{21} = 2/3$, $\nu_{12} = 1/2$, $\nu_{22} = 2/3$. The value of $G$ in the discontinuous control law is $G = 0.05$. For simulation, the sign function in the discontinuous control law is replaced by the saturation function (sat(.)), where sat($p$) = $p/\epsilon$ if $|p| < \epsilon$, and sat($p$) = sign($p$), if $|p| \geq \epsilon$.

The initial condition is set to $h(0) = 0.01$ [m], $\alpha(0) = 5.729$ [deg], and $\dot{\alpha}(0) = \dot{h}(0) = 0$. The open-loop responses are shown in Fig. 4.2 and Fig. 4.3. It is observed that the open-loop disturbance-free system exhibits limit cycle oscillation. Apparently, it is necessary to suppress these undesirable oscillations in the plunge and pitch motion.

First the responses of the closed-loop system including Eqs. (4.1), (4.8), (4.10) and (4.21) for $U = 13.28$ [m/s] and $a = -0.4$ is obtained. The nominal values ($F^*$, $B^*$) are used in the control law Eq. (4.8), but model Eq. (4.5) with actual parameters is used for simulation. For a realistic simulation, similar to Wang and Behal [54],
control surface deflections are allowed to saturate at 15 [deg]. It is observed in Fig. 4.4 that the pitch angle and the plunge displacement converge to zero. The response-time is of the order of less than one second. The control input saturates in the initial transient phase.

Now, the same simulation is done, but for higher value of freestream velocity, $U = 18 \text{ m/s}$. It is seen that the system converges to zero in less than a second. This simulation is presented in figure 4.5.

4.6 Conclusion

In this chapter, control of a multi-input multi-output aeroelastic system for the finite-time stabilization in the presence of parametric uncertainties was considered. The control system included a primary feedback loop designed for the trajectory control for the nominal model. This nonlinear control law yielded a nominal homogeneous system which stabilizes in a finite time. Then a discontinuous control law
was developed based on second-order sliding mode control scheme for eliminating the
effect of uncertainties in the model. In the complete closed-loop system stabilization
of the state vector to the origin was accomplished. Simulation results were presented
which validated finite-time robust suppression of the limit cycle oscillations despite
uncertainties.
CHAPTER 5

SDRE BASED SUBOPTIMAL CONTROL SYSTEM

In this chapter, state dependent Riccati equation (SDRE) approach is introduced for the suboptimal controller design. The aeroelastic model is same as that of figure 3.1. This method is not applicable for gust load, however, gust load is applied in chapter 6 and differential game technique being used.

5.1 Mathematical Derivation

The dynamical model of the aeroelastic system considered here has been developed in [43, 45] (Fig. 3.1). The second-order differential equations governing the evolution of the pitch angle ($\alpha$) and the plunge displacement ($h$) are given by

\[
I_{\alpha} \ddot{\alpha} + \left[ m_w x_{\alpha} b \cos(\alpha) - m_c r_c b \sin(\alpha) \right] \ddot{h} + c_{\alpha} \dot{\alpha} + k_{\alpha}(\alpha) \alpha = M(t)
\]

\[
m_t \ddot{h} + \left[ m_w x_{\alpha} b \cos(\alpha) - m_c r_c b \sin(\alpha) \right] \ddot{\alpha} + c_h \dot{h} + \left[ -m_w x_{\alpha} b \sin(\alpha) - m_c r_c b \cos(\alpha) \right] \dot{\alpha}^2 + k_h(h) h = -L(t) \tag{5.1}
\]

where $m_w$ is the mass of the wing section; $m_t$ is the total mass; $b$ is the semichord of the wing; $I_{\alpha}$ is the moment of inertia; $x_{\alpha}$ is the nondimensionalized distance of the center of mass from the elastic axis; $c_{\alpha}$ and $c_h$ are the pitch and plunge damping coefficients, respectively; $L$ and $M$ are the aerodynamic lift and moment. In this study, for simplicity a quasi-steady form of the aerodynamic force and moment given by

\[
L = \rho U^2 b C_{l_s} s_p \left[ \alpha + \left( \frac{1}{2} - a \right) b (\dot{\alpha} / U) \right] + \rho U^2 b C_{l_s} s_p \beta + \rho U^2 b C_{l_s} s_p \gamma
\]

66
\[ M = \rho U^2 b^2 C_{ma-eff} s_p \left[ \alpha + (\dot{h}/U) + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U) \right] \]
\[ + \rho U^2 b^2 C_{m\beta-eff} s_p \beta + \rho U^2 b^2 C_{m\gamma-eff} s_p \gamma \]  
\hspace{1cm} (5.2)

is considered, where \( \alpha \) is the nondimensionalized distance from the midchord to the elastic axis, \( s_p \) is the span, and \( \beta \) and \( \gamma \) are the trailing-edge and leading-edge flap deflections, respectively. The lift and moment derivatives due to \( \alpha \) and control surface deflections are \( C_{l\alpha} \), \( C_{l\beta} \), \( C_{l\gamma} \), and \( C_{m\alpha-eff} \), \( C_{m\beta-eff} \), \( C_{m\gamma-eff} \), respectively, where

\[ C_{m\alpha-eff} = \left( \frac{1}{2} + a \right) C_{l\alpha} + 2 C_{m\alpha} \]

\[ C_{m\beta-eff} = \left( \frac{1}{2} + a \right) C_{l\beta} + 2 C_{m\beta} \]

\[ C_{m\gamma-eff} = \left( \frac{1}{2} + a \right) C_{l\gamma} + 2 C_{m\gamma} \]

and \( C_{m\alpha} = 0 \) for a symmetric airfoil. Similar to [55], the lift and moment caused by wind gust are assumed to be of the form In this study, \( \alpha k_\alpha(\alpha) \) and \( h k_h(h) \) are assumed to be unstructured (unmodeled) functions.

Using Eqs. (5.1) and (5.2) gives

\[ [S(\alpha)] \begin{bmatrix} \ddot{\alpha} \\ \dot{h} \end{bmatrix} = [A_1(\alpha, \dot{\alpha})] \begin{bmatrix} \alpha \\ h \\ \dot{h} \end{bmatrix} + [B_1]u \]  
\hspace{1cm} (5.3)

where \( A_1 = [A_{11}, A_{12}] \),

\[ [S(\alpha)] = \begin{bmatrix} I_{EA} & m_w x_\alpha b \cos(\alpha) - m_c r_c b \sin(\alpha) \\ m_w x_\alpha b \cos(\alpha) - m_c r_c b \sin(\alpha) & m_t \end{bmatrix} \]
\[ A_{11}(\alpha, \dot{\alpha}) = \begin{bmatrix} -k_\alpha(\alpha) + \rho U^2 b^2 C_{m\alpha - eff} s_p & 0 \\ -\rho U^2 b C_{\alpha s_p} & -k_h \end{bmatrix} \]

\[ A_{12}(\alpha, \dot{\alpha}) = \begin{bmatrix} -C_\alpha + \rho U b^2 C_{\alpha - eff} s_p (0.5 - a) b \\ (m_w x_b \sin(\alpha) + m_c r_c b \cos(\alpha)) \dot{\alpha} - \rho U b^2 C_{\alpha s_p} (0.5 - a) \end{bmatrix} \begin{bmatrix} \rho U b^2 C_{\alpha - eff} s_p \\ -\rho U b C_{\alpha s_p} \end{bmatrix} \]

\[ [B_1] = \begin{bmatrix} \rho U^2 b^2 C_{m\beta - eff} s_p & \rho U^2 b^2 C_{m\gamma - eff} s_p \\ -\rho U^2 b C_{\beta s_p} & \rho U^2 b C_{\gamma s_p} \end{bmatrix} \]

Define the state vector as \( x = [\alpha, h, \dot{\alpha}, \dot{h}]^T \). Then solving Eq. (3), one obtains

\[ \begin{bmatrix} \ddot{\alpha} \\ \ddot{h} \end{bmatrix} = [S(\alpha)]^{-1} [A_1(\alpha, \dot{\alpha})] x + [S(\alpha)]^{-1} [B_1] u \]

\[ \ddot{\alpha} = A_2(\alpha, \dot{\alpha}) x + B_2(\alpha) u \] (5.4)

A state variable representation of Eq. (4) is given by

\[ \dot{x} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ A_2(\alpha, \dot{\alpha}) \end{bmatrix} x + \begin{bmatrix} 0_{2 \times 2} \\ B_2(\alpha) \end{bmatrix} u \]

\[ \ddot{x} = A(\alpha, \dot{x}) x + B(\alpha) u \] (5.5)

We shall be interested in evolution of the trajectories of the system in the region \( \Omega_x \subseteq R^4 \).

The open-loop system without the wind gust exhibits limit cycle oscillations (LCOs) beyond a critical free-stream velocity, described in Figure 4.2 and 4.3. The objective is to design a control system for the suppression of the oscillatory plunge and pitch angle responses in the model. It is assumed that the complete state vector is available for feedback.
5.2 Control Law

For resolution of the problem, here a SDRE methodology is adopted.

Consider a quadratic performance criterion

\[
J(u) = \frac{1}{2} \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt
\]

where the weighting matrix \(Q\) is positive semi-definite symmetric (denotes as \(Q \geq 0\)) and \(R\) is positive definite symmetric \((R > 0)\). Although the matrices \(Q\) and \(R\), may be chosen as functions of \(x\), for simplicity only constant weighting matrices are considered. It of interest to obtain control input \(u \in U\), where \(U\) denotes admissible sets for \(u\).

For the following state-dependent Riccati equation (SDRE):

\[
A^T(x_1,x_3)P(x) + P(x)A(x_1,x_3) - P^T(x)B(x_1)R^{-1}B^T(x_1)P(x) + Q = 0
\]

The control law \(u\) now can be written as

\[
u_o(x) = -R^{-1}B^T(x_1)P(x)x
\]

5.3 Simulation Results

Simulation results are presented in this section. The mathematical model developed of Sheta et al. [43] is considered for numerical computation. Unlike the plunge-pitch aeroelastic model considered design in [42, 44, 45, 47-53, 55-59], this system in addition includes nonlinear function of the pitch rate \((\dot{\alpha}^2)\). For simulation, two free-stream velocities \(U = 13.8\) (m/s) and \(U = 18\) (m/s) are chosen.

The weighting matrices in the quadratic objective function are selected as \(R = 0.0001\) \(dig\{1,1\}\) and the matrix \(Q = 0.5\) \(dig\{1,1,1\}\).

These values have been selected by observing the simulated responses in several trials. The initial condition is set to \(h(0) = 0.01\) (m), \(\alpha(0) = 5.729\) (deg), and \(\dot{\alpha}(0) = \dot{h}(0) = 0\). For practical reasons, the flap deflection is limited to 30 (deg) for
First the open-loop system \((u = 0)\) without the gust load for \(a = -0.4\) is simulated. The stability property of the system depends on the free-stream velocity. For lower speed, the linearized model has asymptotically stable equilibrium point \((x = 0)\). The responses of the system for \(U = 13.28\) (m/s) and \(U = 18\) (m/s) are shown in Fig. 4.1 and Fig. 4.2, respectively. We observe limit cycle oscillations for both free-stream velocities. As expected, the limit cycle for \(U = 18\) (m/s) has larger amplitude compared to \(U = 13.28\) (m/s). Thus for suppressing the LCOs, it is essential to introduce stabilizing control signals by using the flaps. Now the closed-loop responses for the model Eqs. (5.1)-(5.4) including the gust load.

For examining the performance of the controller, the closed-loop system for \(U = 13.28\) m/s, and \(a = -0.4\) is simulated. It is assumed that the gust load is zero. The flap deflection is limited to 30 (deg). Selected responses are shown in Fig. 5.1. It is observed that the pitch angle exponentially converges to zero in about half second, and the plunge displacement tends to zero in 2 seconds. In figure 5.2, with the same data as that of figure 5.1 but the controller is turned on at \(t=3\) sec.

In similar fashion, for \(U = 13.28\) m/s, and \(a = -0.4\) is simulated and plot is described in figure 5.3 and in figure 5.4, the controller is switched on at \(t=3\) sec for
Figure 5.2: Controller is switched on at $t=3$ sec. (Closed-loop responses; $U=13.8$ m/s, $a=-0.4$.)

the same data as that of figure 5.3.

5.4 Conclusion

In this chapter, state dependent Riccati equation (SDRE) based suboptimal controller is implemented to suppress the fluttering effect. Through simulations, it is shown that the controller is turned on after few seconds and still the time to converge is same.
Figure 5.3: Closed-loop responses; U=18 m/s, \( \alpha = -0.4 \).

Figure 5.4: Controller is switched on at \( t=3 \) sec. (Closed-loop responses; U=18 m/s, \( \alpha = -0.4 \).)
CHAPTER 6

DIFFERENTIAL GAME-BASED CONTROL LAW FOR
STABILIZATION OF AEROELASTIC SYSTEM

6.1 Introduction

This chapter is based on the theory of differential games, a control law for the
stabilization of a nonlinear multi-input aeroelastic system in the presence of gust load
is presented. The two degree-of-freedom aeroelastic model is equipped with leading-
and trailing-edge control surfaces for the purpose of control. The model is same as
that of in chapter 4. But in this chapter we simulated the system having gust load
as an adversary. The uncontrolled system exhibits limit cycle oscillations beyond a
critical freestream velocity and is shown in figure 4.2 and 4.3.

For the derivation of the control law, the nonlinear model of the aeroelastic
system is represented as linear system with state-dependent system matrices. Then
a suboptimal control law is obtained for the stabilization of the aeroelastic system by
solving a state-dependent Riccati equation derived from the Hamilton-Jacobi-Isaacs
equation. It is shown that the gust free aeroelastic closed-loop system is asymptoti-
cally stable, and the system trajectories remain bounded if the gust load is of limited
strength.

6.2 Aeroelastic Model and Control Problem

The dynamical model of the aeroelastic system considered here has been de-
developed in [45, 54] (Fig. 3.1). The second-order differential equations governing the
evolution of the pitch angle ($\alpha$) and the plunge displacement ($h$) are given by

$$I_{EA} \ddot{\alpha} + [m_w x_{\alpha} b \cos(\alpha) - m_c r_{c} b \sin(\alpha)] \ddot{h} + c_{\alpha} \dot{\alpha} + k_{\alpha}(\alpha) \alpha = M(t) + M_g$$
\[
m_t \ddot{h} + [m_w x_\alpha b \cos(\alpha) - m_c r_c b \sin(\alpha)] \ddot{\alpha} + c_h \dot{h} + [-m_w x_\alpha b \sin(\alpha) - m_c r_c b \cos(\alpha)] \dot{\alpha}^2 + k_h(h) h = -L(t) - L_g \tag{6.1}
\]

where \(m_W\) is the mass of the wing section; \(m_t\) is the total mass; \(b\) is the semichord of the wing; \(I_\alpha\) is the moment of inertia; \(x_\alpha\) is the nondimensionalized distance of the center of mass from the elastic axis; \(c_\alpha\) and \(c_h\) are the pitch and plunge damping coefficients, respectively; \(L\) and \(M\) are the aerodynamic lift and moment; and \(L_g\) and \(M_g\) are the lift and moment due to wind gust. In this study, for simplicity a quasi-steady form of the aerodynamic force and moment given by

\[
L = \rho U^2 b C_{l_{\alpha}} s_p \left[ \alpha + (\dot{h}/U) + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U) \right] + \rho U^2 b C_{l_{\beta}} s_p \beta + \rho U^2 b C_{l_{\gamma}} s_p \gamma
\]

\[
M = \rho U^2 b^2 C_{m_{\alpha-eff}} s_p \left[ \alpha + (\dot{h}/U) + \left( \frac{1}{2} - a \right) b(\dot{\alpha}/U) \right] + \rho U^2 b^2 C_{m_{\beta-eff}} s_p \beta + \rho U^2 b^2 C_{m_{\gamma-eff}} s_p \gamma \tag{6.2}
\]

is considered, where \(a\) is the nondimensionalized distance from the midchord to the elastic axis, \(s_p\) is the span, and \(\beta\) and \(\gamma\) are the trailing-edge and leading-edge flap deflections, respectively. The lift and moment derivatives due to \(\alpha\) and control surface deflections are \(C_{l_{\alpha}}, C_{l_{\beta}}, C_{l_{\gamma}},\) and \(C_{m_{\alpha-eff}}, C_{m_{\beta-eff}}, C_{m_{\gamma-eff}},\) respectively, where

\[
C_{m_{\alpha-eff}} = \left( \frac{1}{2} + a \right) C_{l_{\alpha}} + 2C_{m_{\alpha}}
\]

\[
C_{m_{\beta-eff}} = \left( \frac{1}{2} + a \right) C_{l_{\beta}} + 2C_{m_{\beta}}
\]

\[
C_{m_{\gamma-eff}} = \left( \frac{1}{2} + a \right) C_{l_{\gamma}} + 2C_{m_{\gamma}}
\]

and \(C_{m_{\alpha}} = 0\) for a symmetric airfoil. Similar to [55], the lift and moment caused by wind gust are assumed to be of the form
\[ L_g = \rho U^2 b s_p C_{la} w_G(\tau)/U = \rho U b s_p C_{la} w_G(\tau) \]

\[ M_g = (0.5 - a)b L_g \quad (6.3) \]

where \( w_G(\tau) \) denotes the disturbance velocity and \( \tau \) is a dimensionless time variable defined as \( \tau = Ut/b \). In this study, \( \alpha k_\alpha(\alpha) \) and \( hk_h(h) \) are assumed to be unstructured (unmodeled) functions.

Using Eqs. (6.1) and (6.2) gives

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{h}
\end{bmatrix} = \begin{bmatrix}
\alpha \\
h
\end{bmatrix} + [B_1] u + [D_1] w_G(\tau) \quad (6.4)
\]

where \( A_1 = [A_{11}, A_{12}] \),

\[
[S(\alpha)] = \begin{bmatrix}
I_{EA} & m_w x_w b \cos(\alpha) - m_c r_c b \sin(\alpha) \\
m_w x_w b \cos(\alpha) - m_c r_c b \sin(\alpha) & m_t
\end{bmatrix}
\]

\[
A_{11}(\alpha, \dot{\alpha}) = \begin{bmatrix}
-k_\alpha(\alpha) + \rho U^2 b^2 C_{ma-eff s_p}(0.5 - a) & 0 \\
-\rho U^2 b C_{la s_p} & -k_h
\end{bmatrix}
\]

\[
A_{12}(\alpha, \dot{\alpha}) = \begin{bmatrix}
-C_\alpha + \rho U b^2 C_{ma-eff s_p}(0.5 - a) & \rho U b^2 C_{ma-eff s_p} \\
(m_w x_w b \sin(\alpha) + m_c r_c b \cos(\alpha)) \dot{\alpha} - \rho U b^2 C_{la s_p}(0.5 - a) & -\rho U b C_{la s_p}
\end{bmatrix}
\]

\[
[B_1] = \begin{bmatrix}
\rho U^2 b^2 C_{m\beta-eff s_p} & \rho U^2 b^2 C_{m\gamma-eff s_p} \\
-\rho U^2 b C_{l\beta s_p} & \rho U^2 b C_{l\gamma s_p}
\end{bmatrix}
\]
\[
[D_1] = \begin{bmatrix}
\rho U (0.5 - a) b^2 s_p C_{la} \\
-\rho U B s_p C_{la}
\end{bmatrix}
\]

Define the state vector as \( x = [\alpha, h, \dot{\alpha}, \dot{h}]^T \). Then solving Eq. (6.3), one obtains

\[
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{h}
\end{bmatrix} = [S(\alpha)]^{-1} [A_1(\alpha, \dot{\alpha})] x + [S(\alpha)]^{-1} [B_1] u + [S(\alpha)]^{-1} [D_1] w_G(\tau)
\]

\[
\dot{x} = [A_2(\alpha, \dot{\alpha})] x + [B_2(\alpha)] u + [D_2(\alpha)] w_G(\tau)
\] (6.5)

A state variable representation of Eq. (4) is given by

\[
\dot{x} = \begin{bmatrix}
0_{2 \times 2} & I_{2 \times 2} \\
A_2(\alpha, \dot{\alpha}) & B_2(\alpha)
\end{bmatrix} x + \begin{bmatrix}
0_{2 \times 2} \\
B_2(\alpha)
\end{bmatrix} u + \begin{bmatrix}
0_{2 \times 1} \\
D_2(\alpha)
\end{bmatrix} w_G(\tau)
\]

\[
\dot{x} = A(\alpha, \dot{\alpha}) x + B(\alpha) u + D(\alpha) w_G
\] (6.6)

We shall be interested in evolution of the trajectories of the system in the region \( \Omega_x \subset R^4 \).

The open-loop system without the wind gust exhibits limit cycle oscillations (LCOs) beyond a critical free-stream velocity. The objective is to design a control system for the suppression of the oscillatory plunge and pitch angle responses in the presence gust load in the model. It is assumed that the complete state vector is available for feedback.

### 6.3 Control Law

It is assumed that external disturbance input \( w_G(t) \) is completely unknown, but its energy \( E_w \) satisfies

\[
E_w = \int_0^\infty w_G^2 dt < \infty
\]
The objective in this paper is to design a control law for the suppression of the limit cycle oscillations by treating the disturbance input as an adversary. Thus it is assumed that \( w_G(t) \) acts to destabilize the system despite the best choice of stabilizing strategy of the control vector \( u \). For resolution of the problem, here a differential game theoretic methodology is adopted.

Consider a quadratic performance criterion

\[
J(u, w_G) = \frac{1}{2} \int_0^\infty [x^T(t)Qx(t) + u^T(t)R_u u(t) - r_w w_G^2(t)]dt
\]  

(6.7)

where the weighting matrix \( Q \) is positive semi-definite symmetric (denotes as \( Q \geq 0 \)), \( R_u \) is positive definite symmetric \( (R_u > 0) \), and \( r_w > 0 \). Although the matrices \( Q \) and \( R_u \), and \( r_w \) may be chosen as functions of \( x \), for simplicity only constant weighting matrices are considered. It of interest to obtain control input \( u \in U \) to minimize the cost function \( J(u, w_G) \) despite a maximizing counter action of the antagonist \( w_G \in W \), where \( U \) and \( W \) denote admissible sets for \( u \) and \( w_G \), respectively.

The problem posed has a solution on the set \( \Omega_x \subset R^4 \) if there exists a continuously differentiable positive define function \( V(x) : \Omega_x \rightarrow R^+ \) with respect to \( x \) on \( \Omega_x \), defined as

\[
V(x) = \inf_{u \in U} \sup_{w_G \in W} J(u, w_G)
\]  

(6.8)

The function \( V(x) \) satisfies the Hamilton-Jacobi-Bellman-Isaacs (HJBI) equation

\[
H(x, u^*(x), w_G^*(x)) = 0
\]  

(6.9)

where \( u^* \) and \( w_G^* \) denote the optimal strategies for \( u \) and \( w_G \), respectively; and \( H \) is the Hamiltonian

\[
H = \inf_u \sup_{w_G} \left\{ \left[ \frac{\partial V(x)}{\partial x} \right]^T \left[ A(x_1, x_3)x + B(x_1)u + D(x_1)w_G \right] + \frac{1}{2} [x^T Q x + u^T R_u u - r_w w_G^2 ] \right\}
\]
The function $V(x)$ satisfies $V(0) = 0$. Using Eq. (6.9), it is easily follows that the optimal controls satisfy

$$
\frac{\partial H}{\partial u} = R_u u^* + B^T(x_1) \frac{\partial V(x)}{\partial x} = 0; \frac{\partial^2 H}{\partial u^2} = R_u > 0
$$

$$
\frac{\partial H}{\partial w_G} = r_w w^*_G + D^T(x_1) \frac{\partial V(x)}{\partial x} = 0; \frac{\partial^2 H}{\partial w_G^2} = -r_w < 0
$$

(6.11)

According to Eq. (6.10), the optimal strategies are

$$
u^*(x) = -R_u^{-1} B^T_1(x_1) \frac{\partial V(x)}{x}
$$

$$
w^*_G(x) = r_w^{-1} D^T(x_1) \frac{\partial V(x)}{\partial x}
$$

(6.12)

Substituting $u^*$ and $w^*_G$ from Eq. (6.11) in (6.9) gives

$$
\left[ \frac{\partial V(x)}{\partial x} \right]^T A(x_1, x_3) x - \frac{1}{2} \left[ \frac{\partial V(x)}{\partial x} \right]^T [B(x_1) R_u^{-1} B^T(x_1) - D(x_1) r_w^{-1} D^T(x_1)] \frac{\partial V(x)}{\partial x}
$$

$$
+ \frac{1}{2} x^T Q x = 0
$$

(6.13)

Although one would like to obtain an analytical closed-form solution for $V(x)$ of the HJBI partial differential equation (6.12), it is not an easy task. To overcome this difficulty, the state-dependent Riccati equation (SDRE) approach will be used. For this linear-like state variable representation of the aeroelastic model is useful. For the derivation of the control law, certain assumptions are made.

Assumption 1: Let $Q$ be factorized as $Q = C^T C \geq 0$. For $x \in \Omega_x$, the following controllability and observability conditions are satisfied:

(i) rank $[B(x_1), A(x_1, x_3) B(x_1), A^2(x_1, x_3) B(x_1), A^3(x_1, x_3) B(x_1)] = 4$

(ii) rank $[D(x_1), A(x_1, x_3) D(x_1), A^2(x_1, x_3) D(x_1), A^3(x_1, x_3) D(x_1)] = 4$

(iii) rank $[C^T, (CA)^T(x_1, x_3), (CA^2)^T(x_1, x_3), (CA^3)^T(x_1, x_3)]^T = 4$
This assumption implies that (A,B) and (A,D) are point wise controllable pairs and (C,A) is an observable pair.

Assumption 2: Define

$$\Pi(x) = [B(x_1)R_u^{-1}B^T(x_1) - D_1(x_1)r_w^{-1}D^T_x]$$ (6.14)

The symmetric matrix $\Pi(x_1)$ is positive semi-definite for $x \in \Omega_x$.

It is noted that the weighting matrix $R_u$ and $r_w$ can be selected to satisfy Assumption 2. For the aeroelastic model, later it will be seen that along the trajectories of the system, the Assumption 1 holds.

According to the SDRE approach where $P(x)$ is an $n \times n$ matrix, one sets the partial derivative of $V(x)$ with respect to $x$ as

$$\frac{\partial V(x)}{\partial x} = P(x)x$$ (6.15)

Then substituting Eq. (14) in (12) gives

$$\frac{1}{2}[x^T(P^T(x)A(x_1, x_3) + A^T(x_1, x_3)P(x))x - \frac{1}{2}x^TP^T(x)[B(x_1)R_u^{-1}B^T(x_1) -

D(x_1)r_w^{-1}D^T(x_1)]P(x)x + \frac{1}{2}x^TQx = 0$$ (6.16)

Because $Q$ and $R_u$ are symmetric, this equation implies that $P(x)$ is symmetric. The Eq. (6.15) holds for $x \in \Omega_x$ if $P(x)$ satisfies the following state-dependent Riccati equation (SDRE):

$$A^T(x_1, x_3)P(x) + P(x)A(x_1, x_3) - P^T(x)[B(x_1)R_u^{-1}B^T(x_1) - D(x_1)r_w^{-1}D^T(x_1)]P(x) + Q = 0$$ (6.17)

In view of Eqs. (6.11) and (6.16), the control law $u$ now can be written as

$$u^o(x) = -R_u^{-1}B^T(x_1)P(x)x$$ (6.18)
and the maximizing disturbance input is

$$w^o_G(x) = r_w^{-1}D^T(x_1)P(x)x$$  \hspace{1cm} (6.19)$$

Theorem 1: Suppose that Assumption 1 and Assumption 2 hold, and a unique positive definite solution of the SDRE in Eq. (6.16) exists for \(x \in \Omega_x\), then in the closed-loop system including the control law Eq. (6.17), \(x = 0\) is locally asymptotically stable when (i) \(w_G = 0\), and (ii) \(w_G = w^0_G(x)\) given in Eq. (6.18). (The proof can be complete following [66, 68, 77].)

According to [66], \([A(x_1, x_3) - B(x_1)R_u^{-1}B^T(x_1)P(x)]\) is point-wise Hurwitz in \(\Omega_x\). This implies that the origin of the system given by

$$\dot{x} = [A(x_1, x_3) - B(x_1)R_u^{-1}B^T(x_1)P(x)]x + D(x_1)w_G(t)$$

$$\dot{x} = A_c(x_1, x_3)x + D(x_1)w_G(t)$$  \hspace{1cm} (6.20)$$

is exponentially stable if \(w_G(t)\) is zero. It is noted that the elements of the matrix \(D\) are periodic functions of \(x_1\); and therefore, \(D\) remains bounded. As such for any bounded gust

$$||D(x_1)w_G(t)|| \leq \mu_d$$  \hspace{1cm} (6.21)$$

where \(\mu_d\) is some positive real number.

Now the stability property of the closed-loop system in the presence of wind gust is analyzed. Because \(x = 0\) is exponentially stable for \(w_G = 0\), according to the converse Lyapunov theorem, there exists a function \(W(x)\) which satisfies [59]

$$c_1||x||^2 \leq W(x) \leq c_2||x||^2$$  \hspace{1cm} (6.22)$$

$$\left[\frac{\partial W(x)}{\partial x}\right]^TA_c(x_1, x_3)x \leq -c_3||x||^2$$  \hspace{1cm} (6.23)$$

$$||\frac{\partial W(x)}{\partial x}|| \leq c_4||x||$$  \hspace{1cm} (6.24)$$
for all $x \in B_r \subset \Omega_x$, where $c_1, ..., c_4$ are some positive constants, and

$$B_r = \{x \in \mathbb{R}^4 : ||x|| \leq r, r > 0\}$$

In the presence of gust load, differentiating $W(x)$ along the solution of Eq. (6.20) gives

$$\dot{W} = \left[ \frac{\partial W(x)}{\partial x} \right]^T [A_c(x_1,x_3)x + D(x_1)w_G(t)]$$

$$\leq -c_3||x||^2 + c_4\mu_d||x||$$

Then according to a results of [59], it follows that

$$\dot{W}(x) \leq -(1 - \theta)c_3||x||^2$$

(6.25)

for all $||x|| > \delta c_4/(\theta c_3)$, with $\theta < 1$, and

$$\delta < \frac{c_3}{c_4} \sqrt{\frac{c_1}{c_2^3} \theta r}$$

This implies that the trajectories of the system including the wind gust are uniformly ultimately bounded if $||x(0)|| < \sqrt{c_1/c_2^3} r$. Note that such a $\delta$ always exists provided that the maximum magnitude of $w_G(t)$ is sufficiently small. (The proof can be completed using results of [59].) In the next section, it will be seen that indeed the designed control law suppresses the LCOs in the presence of gust loads.

### 6.4 Simulation Results

This section presents the results of simulation. The mathematical model developed of Sheta et al. [43] is considered for numerical computation. Unlike the plunge-pitch aeroelastic model considered design in [42, 44,45, 47-53, 55-59], this system in addition includes nonlinear function of the pitch rate ($\dot{\alpha}^2$). Its parameters are listed in the appendix for convenience. However, gust load similar to [55] is included in the model. For simulation, the velocity distributions of $w_G(\tau)$ associated with
the gust load are assumed to have (i) triangular, (ii) sinusoidal, and (iii) exponential waveforms. For the triangular disturbance input, one has

\[
w_G(\tau) = 2w_0 \frac{\tau}{\tau_G} \left( H(\tau) - H\left( \tau - \frac{\tau_G}{2} \right) \right) + 2w_0 \left( \frac{\tau}{\tau_G} - 1 \right) (H(\tau - \tau_G) - H(\tau - \frac{\tau_G}{2}))
\]  

(6.26)

where \( H(.) \) denotes the unit step function, \( \tau_G = Ut_G/b \), \( t_G = 0.5 \) (s), and \( w_0 = 0.7 \). The sinusoidal \( w_G(\tau) \) is

\[
w_G = w_0 \sin(\pi b\tau/U)H(\tau)
\]

(6.27)

with \( w_0 = 0.15 \). The exponential velocity distribution is given by

\[
w_G = w_0 (1 - \exp(-4t))
\]

(6.28)

where \( w_0 = 0.3 \) or \( 0.7 \) For simulation, three free-stream velocities \( U = 13.2 \) (m/s), \( U = 15 \) (m/s) and \( U = 18 \) (m/s) are chosen.

The weighting matrices in the quadratic objective function are selected as \( R_u = 0.0001 \{1, 1\} \) and the matrix \( Q \) is

\[
Q = \begin{bmatrix}
10000 & 0 & 0 & 0 \\
0 & 100 & 0 & 0 \\
0 & 0 & 100 & 0 \\
0 & 0 & 0 & 10
\end{bmatrix}
\]

The scalar weighting parameter associated with the gust load is selected to be \( r_w = 10 \). These values have been selected by observing the simulated responses in several trials. The initial condition is set to \( h(0) = 0.01 \) (m), \( \alpha(0) = 5.729 \) (deg), and \( \dot{\alpha}(0) = \dot{h}(0) = 0 \). For practical reasons, the flap deflection is limited to 30 (deg) for simulation.

First the open-loop system \( (u = 0) \) without the gust load for \( a = -0.4 \) is
simulated. The stability property of the system depends on the free-stream velocity. For lower speed, the linearized model has asymptotically stable equilibrium point \((x = 0)\). The responses of the system for \(U = 13.28\) (m/s) and \(U = 18\) (m/s) are shown in Fig. 4.1 and Fig. 4.2, respectively. We observe limit cycle oscillations for both free-stream velocities. As expected, the limit cycle for \(U = 18\) (m/s) has larger amplitude compared to \(U = 13.28\) (m/s). Thus for suppressing the LCOs, it is essential to introduce stabilizing control signals by using the flaps. Now the closed-loop responses for the model Eqs. (6.1)-(6.4) including the gust load and the suboptimal state feedback control law Eq. (6.17) are obtained.

**Case A. Closed-loop responses (gust load zero):** \(U = 13.8\) m/s, \(a = -0.4\)

For examining the performance of the controller, the closed-loop system for

\[
U = 13.28\text{ m/s, and } a = -0.4
\]

is simulated. It is assumed that the gust load is zero. The flap deflection is limited to 30 (deg). Selected responses are shown in Fig. 6.1. It
is observed that the pitch angle exponentially converges to zero in about half second, and the plunge displacement tends to zero in less than a second. It is seen that there is no overshoot in the exponential pitch angle trajectory, but there is small overshoot in the $h$-response. The control input saturates in the transient period briefly.

**Case B. Adaptive control: without gust, $U = 18 \text{ m/s, } a=-0.4$**

The closed-loop responses for higher value $U = 18 \text{ (m/s)}$ and $a = -0.4$ are shown in Fig. 6.2. For this chosen speed, the plunge displacement convergence time has slightly increased (approximately to 1.25 seconds), but the pitch angle attains zero value in little over half second. It is observed that unlike Case A for lower speed $U$, the leading-edge surface deflection $\gamma$ takes only positive values. The control input saturates in the initial phase for smaller interval of time compared to Case A. This is attributed to increase in the effectiveness of the flaps at higher free-stream velocity.

**Case C. Closed-loop responses (gust load zero): $U = 15 \text{ m/s, a=-0.6719}$**

In this case, similar to Case A and B, simulation is done for the model without
wind gust input. But the different values for $a = -0.6719$ and the free-stream velocity $U = 15 \text{ m/s}$ are selected. The responses are shown in Fig. 6.3. We observed that for this set of values of $(U, a)$, the pitch angle converges to zero in about half second, and plunge displacement tends to zero in about one second. It is observed that controller $\beta$ takes only negative values, but $\gamma$ takes negative and positive values. It is seen that $\gamma$ saturates for longer period (about 0.25 s) compared to Case A and B.

**Case D. Closed-loop responses (triangular gust): $U = 15 \text{ m/s}, a = -0.4$**

Now the effect of a triangular gust load is examined. It is assumed that the velocity distribution in Eq. (6.26) has $w_0 = 0.7$. It is assumed that $U$ is 15 (m/s) and $a$ is -0.4. The responses of the closed-loop system and the gust load are shown in Fig. 6.4. We observe that, despite the presence of the wind gust, the oscillations in $\alpha$ and $h$ are suppressed in less than half second and one second, respectively. Also is plotted the determinant of the controllability matrix $C(x)$ (det. $C$) along the trajectory of the system. Its large magnitude indicates that the system is strongly controllable.
Figure 6.4: Closed-loop responses (triangular gust): $U=15 \text{ m/s}$, $a=-0.4$, $w_0 = 0.7$.

Figure 6.5: Closed-loop responses (exponential gust): $U=15 \text{ m/s}$, $a=-0.4$, $w_0 = 0.3$. 
Figure 6.6: Eigenvalues $\lambda_i(P(x))$, (exponential gust): $U=15$ m/s, $a=-0.4$, $w_0 = 0.3$.

Case E. Closed-loop responses (exponential gust load): $U = 15$ m/s, $a=-0.4$, $w_0 = 0.3$ or 0.7 Now simulation is done in the presence of exponential disturbance input (equation 6.29) for the model with $(U, a) =(15 \text{ (m/s), } -0.4)$ is examined. Responses are obtained for two values of $w_0$ ($w_0 = 0.3$ and $w_0 = 0.7$). First simulation is done for the gust load with $w_0 = 0.3$. Selected responses are shown in Fig. 6.5 and 6.6 for $w_0 = 0.3$. It is seen that the pitch angle and plunge displacement converge to zero in about half second for $w_0 = 0.3$ (see Fig. 6.5). Fig. 6.6 shows the four eigenvalues, $\lambda_i(P(x))$ of the matrix $P(x)$, $i = 1, 2, 3, 4$. It is observed all the eigenvalues have positive values as required for the existence of suboptimal control law. The determinant of the controllability matrix has large values along the trajectory of the system (Fig. 6.5).

Now the effect of the stronger exponential gust on the closed-loop responses is examine. For this purpose, simulation is done using a larger value $w_0 = 0.7$. The responses are shown in Fig. 6.7. One observes that the pitch angle tends to zero in half second. Although the oscillations in the plunge displacement trajectory are also
suppressed in little over two seconds, it is seen that the plunge displacement settles to a nonzero value in the steady-state. This has been caused by the gust load of higher strength in the steady-state.

**Case F. Closed-loop responses (sinusoidal wind gust):** $U = 15$ m/s, $a = -0.4$  

For simulation, sinusoidal wind gust ($w_0 = 0.15$) given in Eq. (6.28) is used. The model for $U = 15$ (m/s) and $a = -0.4$ is simulated. Selected responses are shown in Fig. 6.8. Again the oscillations in the system are suppressed. The convergence time for the pitch angle is about half second, but for $h$ is of the order of one second. In the steady-state, oscillatory flap deflections are observed. These nonzero oscillatory control surface deflections are essential for nullifying the effect of the persistent sinusoidal disturbance input.

To this end, a comparison of the performance of the differential game-based controller and the SDRE-based published works [33, 34] for this aeroelastic model is considered. Note that similar to [70, 71], model parameters are assumed to be known for the derivation of the controller. But unlike this paper, the aeroelastic
systems of [70, 71] have not included gust load for design. However the model of [70] includes unsteady aerodynamics. Similar to [70, 71], the LCOs are suppressed using the controller designed here. Simulation results presented here exhibit the suppression of the LCOs despite the adverse effect of the gust load. It is observed here that even in the presence of gust load, the pitch angle converges exponentially to zero without any overshoot, and the plunge trajectory overshoots only once in some cases. However, for the case of stronger exponential gust, the plunge trajectory settles to a nonzero value after undergoing few oscillations in the transient phase (see Fig. 6.10). It is also pointed out that the adaptive and nonadaptive control systems published in the literature for the plunge-pitch model have been designed assuming that the input coefficient matrix is constant. However, such an assumption is not necessary for designing game theory-based control system. It is seen here that the input-dependent matrix $B_2(\alpha)$ is indeed a nonlinear function of the pitch angle $\alpha$. Of course, the weighting matrices in the objective functional $J$ play a critical role in shaping the responses in the closed-loop system.
6.5 Conclusions

In this paper, based on the differential game theory, a control system was designed for the control of the plunge and pitch angle trajectories of a two-dimensional aeroelastic system in the presence of gust load, using trailing-and leading-edge flaps. For the purpose of design, an infinite-horizon optimal control problem with quadratic performance index was chosen. It was assumed that the two control surfaces played the role of minimizing player, and the gust load was maximizing adversary. The aeroelastic model was represented as a linear system with state-dependent system matrices. A suboptimal control law was computed based on a Riccati equation. In the gust free closed-loop system, the control system achieved local asymptotic stability. Simulation results were presented which showed that the designed controller is capable of suppressing limit cycle oscillations in the closed-loop system, despite the presence of exponential, sinusoidal, and triangular gust loads. It was found that appropriate selection of the weighting matrices in the objective function is important to obtain satisfactory performance. It was observed that for the larger magnitude of the exponential gust load, although the oscillations in the trajectories were suppressed, the plunge displacement converged to certain nonzero value. It is pointed out that unlike the published works for the plunge-pitch aeroelastic model, the differential game-based design is applicable to a larger class of aeroelastic systems including state-dependent input coefficient matrix.
CHAPTER 7

CONCLUSION

In this research work, among different aeroelastic instability such as wing flutter, buffeting, divergence, control-surface effectiveness, reversal and buzz, main motive was to suppress the effect of flutter, which is considered as most dangerous phenomenon for aircraft flying at high speed. Total five different techniques were used to control nonlinearities in the system. Wing-rock has roll single degree of freedom motion. The aeroelastic model chosen has two-degree of freedom in plunge and pitch and uses two control surface for flutter control. First, for each model, design of open-loop system, which shows limit cycle oscillations (LCOs) at critical freestream velocity. In chapter 2, two adaptive systems in finite form for wing rock control were developed. The first algorithm was obtained by a finite form realization of an adaptation law derived by the application of the speed-gradient method; and the second adaptation scheme was based on the immersion and invariance approach. Based on the Lyapunov analysis, stability in the closed-loop system, and the convergence of the roll angle tracking error to zero were established. In chapter 3, a Chebyshev neural network based adaptive control system was designed. The Chebyshev polynomials were used for the two neural networks for the representation of $\alpha$ and $h$-dependent nonlinearities for the adaptive law design. Based on the SDU decomposition of the high-frequency gain matrix, a singularity-free adaptive law was derived. In the closed-loop system, uniform ultimate boundedness of the trajectories was established using the Lyapunov stability theory. In chapter 4, the control system included a primary feedback loop designed for the trajectory control for the nominal model. This nonlinear control law yielded a nominal homogeneous system which stabilizes in a finite time. Then a discontinuous control law was developed based on second-order sliding
mode control scheme for eliminating the effect of uncertainties in the model. How-
ever, in chapter 5, using the same model equation as that of in chapter 4, a state
dependent Riccati equation (SDRE) approach was derived. In chapter 6, based on
the differential game theory, for the purpose of design, an in finite-horizon optimal
control problem with quadratic performance index was chosen. It was assumed that
the two control surfaces played the role of minimizing player, and the gust load was
maximizing adversary. A suboptimal control law was computed based on a Riccati
equation. It was observed that for the larger magnitude of the exponential gust load,
although the oscillations in the trajectories were suppressed, the plunge displacement
converged to certain nonzero value. It is pointed out that unlike the published works
for the plunge-pitch aeroelastic model, the differential game-based design is applica-
tible to a larger class of aeroelastic systems including state-dependent input coefficient
matrix.
APPENDIX

Define
\[ \psi = x_2 + g(x_1, t) \]
where \( g(x_1, t) = -x_{2r} + \lambda x_1 - \lambda x_1r \). Differentiating equation (2.3) with respect to \( x_2 \) gives
\[ \frac{\partial \chi}{\partial x_2} = \left[ 0, 1, 2x_2 \text{sgn}(x_2), 0, x_1^2, \lambda + \frac{\partial \gamma(\psi)}{\partial x_2} \right]^T \]
where
\[ \frac{\partial \gamma(\psi)}{\partial x_2} = k_1 + 3k_2(x_2 + g(x_1, t))^2 \]
Therefore, one has
\[ \psi \frac{\partial \chi}{\partial x_2} = \begin{bmatrix} 0 \\ x_2 + g(x_1, t) \\ 2\{x_2^2 + g(x_1, t)x_2\} \text{sgn}(x_2) \\ 0 \\ (x_2 + g(x_1, t))x_1^2 \\ \psi \left( \lambda + \frac{\partial \gamma(\psi)}{\partial x_2} \right) \end{bmatrix} \]
Note that
\[ \psi \left( \lambda + \frac{\partial \gamma(\psi)}{\partial x_2} \right) = (x_2 + g(x_1, t))[3k_2x_2^2 + 6k_2g(x_1, t)x_2 + (3k_2g^2(x_1, t) + k_1 + \lambda)] \]
Now using the relation
\[ \Psi_a = \int_0^{x_2} \psi(x_1, \xi, t) \frac{\partial \chi}{\partial \xi}(x_1, \xi, t)d\xi \]
one obtains

\[ \Psi_a(x, t) = \begin{bmatrix} 0 \\ \frac{x_2^3}{2} + g(x_1, t)x_2 \\ 2 \left( \frac{x_2^3}{3} + g(x_1, t)\frac{x_2^2}{2} \right) sgn(x_2) \\
0 \\
\left( \frac{x_2^3}{2} + g(x_1, t)x_2 \right) x_1^2 \\
q_3(x, t) \end{bmatrix} \]

where \( q_3 = k_2 \frac{x_1^4}{4} + 9k_2g(x_1, t)\frac{x_2^2}{2} + q_1(x, t)\frac{x_2^2}{2} + q_2(x, t)x_2 \), \( q_1(x, t) = 9g^2(x_1, t)k_2 + k_1 + \lambda \) and \( q_2(x, t) = g(x, t)[3k_2g^2(x_1, t) + k_1 + \lambda] \).

For the computation of the integral adaptation law, the partial derivatives of \( \chi \) and \( \Psi_a \) with respect to \( t \) and \( x_1 \) are required. The computation of these derivatives are given in the sequel. The partial derivatives of \( \chi \) obtained from equation are given by

\[
\frac{\partial \chi}{\partial t} = \begin{bmatrix} 0_{1 \times 5}, -\ddot{x}_{1r} - \lambda \dot{x}_{1r} + \frac{\partial \gamma(\psi)}{\partial t} \end{bmatrix}^T
\]

\[
\frac{\partial \chi}{\partial x_1} = [1, 0, 0, 3x_1^2, 2x_1x_2, (k_1 + 3k_2\psi^2)\lambda]^T
\]

where

\[
\frac{\partial \gamma(\psi)}{\partial t} = -(k_1 + 3k_2\psi^2)(\dot{x}_{2r} + \lambda \dot{x}_{1r})
\]

The partial derivatives of \( \Psi_a \) obtained from equation (A-4) are of the form

\[
\frac{\partial \Psi_a}{\partial t} = \begin{bmatrix} 0 \\ x_2 \frac{\partial q}{\partial t} \\ x_2^2 sgn(x_2) \frac{\partial q}{\partial t} \\
0 \\
x_1^2x_2 \frac{\partial q}{\partial t} \\
\frac{\partial q_3}{\partial t} \end{bmatrix}
\]
\[
\frac{\partial \Psi_a}{\partial x_1} = \begin{bmatrix}
0 \\
x_2 \frac{\partial g}{\partial x_1} \\
x_2^2 \text{sgn}(x_2) \frac{\partial g}{\partial x_1} \\
0 \\
x_1 x_2 \frac{\partial g}{\partial x_1} + 2gx_1 x_2 \\
\frac{\partial q_3}{\partial x_1}
\end{bmatrix}
\]

These partial derivatives are substituted to obtain \( \dot{\theta}_I \).

Chapter 3 : System Parameters

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = -0.6719 )</td>
<td>( b = 0.1905 , [\text{m}] )</td>
</tr>
<tr>
<td>( s_p = 0.5945 , [\text{m}] )</td>
<td>( \rho_a = 1.225 , [\text{kg/m}^3] )</td>
</tr>
<tr>
<td>( r_{cg} = -b(0.0998 + a) , [\text{m}] )</td>
<td>( x_\alpha = r_{cg}/b )</td>
</tr>
<tr>
<td>( c_h = 27.43 , [\text{kg/s}] )</td>
<td>( c_\alpha = 0.0360 , [\text{N} \cdot \text{s}] )</td>
</tr>
<tr>
<td>( k_{h0} = 2844 , [\text{N/m}] )</td>
<td>( k_{h1} = 0.09k_{h0} , [\text{N/m}^3] )</td>
</tr>
<tr>
<td>( m_{wing} = 4.340 , [\text{kg}] )</td>
<td>( m_w = 5.23 , [\text{kg}] )</td>
</tr>
<tr>
<td>( m_t = 15.57 , [\text{kg}] )</td>
<td>( I_{cgw} = 0.04342 , [\text{kg} \cdot \text{m}^2] )</td>
</tr>
<tr>
<td>( I_{cam} = 0.04697 , [\text{kg} \cdot \text{m}^2] )</td>
<td>( c_{t_\alpha} = 6.757 , [\text{rad}^{-1}] )</td>
</tr>
<tr>
<td>( c_{t_\beta} = 3.774 , [\text{rad}^{-1}] )</td>
<td>( c_{t_\gamma} = -0.1566 , [\text{rad}^{-1}] )</td>
</tr>
<tr>
<td>( c_{m_\alpha} = 0 , [\text{rad}^{-1}] )</td>
<td>( c_{m_\beta} = -0.6719 , [\text{rad}^{-1}] )</td>
</tr>
<tr>
<td>( c_{m_\gamma} = -0.1005 , [\text{rad}^{-1}] )</td>
<td>( k_{h}(h) = k_{h0} + k_{h1}h^2 )</td>
</tr>
<tr>
<td>( k_\alpha = 12.77 + 53.47a + 1003a^2 , [\text{N} \cdot \text{m}] )</td>
<td>( k_\alpha = 12.77 + 53.47a + 1003a^2 , [\text{N} \cdot \text{m}] )</td>
</tr>
<tr>
<td>( I_\alpha = I_{cam} + I_{cgw} + m_{wing} \cdot r_{cg}^2 , [\text{kg} \cdot \text{m}^2] )</td>
<td></td>
</tr>
</tbody>
</table>
## Chapter 6: System Parameters

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = -0.4</td>
<td>b = 0.1064 [m]</td>
</tr>
<tr>
<td>( s_p = 0.6 )</td>
<td>( \rho_a = 1.225 ) \ [kg/m(^3)]</td>
</tr>
<tr>
<td>( r_{cg} = (0.82 \times b - b - a \times b) ) \ [m]</td>
<td>( x_\alpha = r_{cg}/b )</td>
</tr>
<tr>
<td>( c_h = 27.43 ) \ [kg/s]</td>
<td>( c_\alpha = 0.0360 ) \ [N⋅s]</td>
</tr>
<tr>
<td>( k_h = 2844 ) \ [N/m]</td>
<td>( r_c = 1.1936 )</td>
</tr>
<tr>
<td>( m_c = 0.718 ) \ [kg]</td>
<td>( m_w = 1.662 ) \ [kg]</td>
</tr>
<tr>
<td>( m_t = 12.0 ) \ [kg]</td>
<td>( I_{EA} = 0.04325 + m_{wing} \cdot r_{cg}^2 ) \ [kg⋅m(^2)]</td>
</tr>
<tr>
<td>( c_{m_\gamma} = 0 ) \ [rad(^{-1})]</td>
<td>( c_{t_\alpha} = 6.757 ) \ [rad(^{-1})]</td>
</tr>
<tr>
<td>( c_{l_\beta} = 3.774 ) \ [rad(^{-1})]</td>
<td>( c_{r_\gamma} = -0.1566 ) \ [rad(^{-1})]</td>
</tr>
<tr>
<td>( c_{m_\alpha} = 0 ) \ [rad(^{-1})]</td>
<td>( c_{m_\beta} = -0.6719 ) \ [rad(^{-1})]</td>
</tr>
<tr>
<td>( k_\alpha = 6.861422(1 + 1.1437925\alpha + 96.669627\alpha^2 + 9.513399\alpha^3 - 727.664120\alpha^4) ) \ [N⋅m/rad]</td>
<td></td>
</tr>
</tbody>
</table>
BIBLIOGRAPHY


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[63] Isaacs, R., ” Differential Games,” Wiely, NY, 1966


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