Self-stabilizing protocol for anonymous oriented bi-directional rings under unfair distributed schedulers with a leader

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SELF-STABILIZING PROTOCOL FOR ANONYMOUS ORIENTED
BI-DIRECTIONAL RINGS UNDER UNFAIR DISTRIBUTED
SCHEDULERS WITH A LEADER

By

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Bachelor of Engineering
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of the requirements for the

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School of Computer Science
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ABSTRACT

Self-Stabilizing Protocol For Anonymous Oriented Bi-directional Rings
Under Unfair Distributed Schedulers With A Leader

By

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School of Computer Science
University of Nevada, Las Vegas

We propose a self-stabilizing protocol for anonymous oriented bi-directional rings of any size under unfair distributed schedulers with a leader. The protocol is a randomized self-stabilizing, meaning that starting from an arbitrary configuration it converges (with probability 1) in finite time to a legitimate configuration (i.e. global system state) without the need for explicit exception handler of backward recovery. A fault may throw the system into an illegitimate configuration, but the system will autonomously resume a legitimate configuration, by regarding the current illegitimate configuration as an initial configuration, if the fault is transient. A self-stabilizing system thus tolerates any kind and any finite number of transient faults. The protocol can be used to implement an unfair distributed mutual exclusion in any ring topology network.

Keywords: self-stabilizing protocol, anonymous oriented bi-directional ring, unfair distributed schedulers. Ring topology network, non-uniform and anonymous network, self-stabilization, fault tolerance, legitimate configuration.
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CHAPTER 1

INTRODUCTION

In this thesis, we present a self-stabilizing protocol for anonymous oriented bi-directional rings of any size under unfair distributed schedulers. Self-stabilization is a well-known paradigm of non-masking fault tolerant distributed algorithms [21, 10, 9]. Self-stabilization introduced by Dijkstra, [1], provides an uniform approach to fault-tolerance, [8]. We are particularly interested in non-uniform (i.e. all processors don’t perform the same algorithm) and anonymous network (i.e. no processor has a distinct identifier). This protocol guarantees that, regardless of the initial state, the system will eventually converge to the intended behavior without the need for explicit exception handler of backward recovery.

1.1 Distributed Systems

A distributed system in its simplest form can be presented as a set of processors connected over a communication medium. The processors make local computations and exchange messages using the communication medium. Distribution systems can be classified as synchronous or asynchronous. Processors can be synchronous or asynchronous depending on how the local computations are made. The communication medium can be synchronous or asynchronous depending on how the communication between the processors is accomplished.
1.2 Self-Stabilization

Self-Stabilization is an important concept for distributed computing and communication networks. It describes a system’s ability to recover automatically from unexpected failure. It is also an important issue for multiagent systems, as they are distributed and communicative systems. Self-stabilization is a framework for dealing with channel or memory failures. After a failure the system is allowed to temporarily exhibit an incorrect behavior, but after a period of time as short as possible, it must behave correctly, without external intervention, [22]. The practical appeal of stabilizing protocols is that they are simpler (i.e., they avoid a slew of mechanisms to deal with a catalog of anticipated faults), and they are more robust (e.g., they can recover from transient faults such as memory corruption as well as common faults such as link and node crashes), [12].

1.3 Related Work

The first self-stabilizing algorithms was introduced by Dijkstra[1]. Schneider[21] presented a survey on early research on self-stabilization. Katz and Perry [3] showed how to compile an arbitrary asynchronous protocol into a stabilizing equivalent. Their general transformation is expensive; hence more efficient (and possibly less general) techniques are needed. Techniques that transform any locally checkable protocol into a stabilizing equivalent are given in [12, 13].

of prime size. In their model, there is a central daemon that picks an enabled processor each time to make an atomic move. The chosen processor can read the states of its two neighbors at the same time to determine its next state.


1.4 Contribution

Many of the previous works on the self-stabilizing mutual exclusion problem either assume a central daemon or assume unfair daemon for uniform unidirectional rings or assume unfair daemon for non-uniform bi-directional rings but use message passing model. We present an self-stabilizing protocol under unfair daemon for oriented bi-directional non-uniform ring without using message passing model. In [16], Kakugawa and Yamashita claimed that “there is no such system when the number $n$ of processes (i.e., ring size) is composite, even if a fair central-daemon (c-daemon) is assumed” and there was an open question to design a self stabilizing algorithm that solves the mutual
exclusion problem under an unfair distributed scheduler. We answer the open question of [16] and present an self-stabilizing algorithm for anonymous oriented bi-directional rings of any size under unfair distributed schedulers with a leader.

1.5 Outline of the Thesis

We give definitions of some topics involved in this research and an overview of dining philosopher problem including survey of self-stabilizing and non self-stabilizing dining philosopher problem in Chapter 2.

In Chapter 3, first we give the solution to the simplified version of dining philosophers problem. We consider chain topology instead of ring topology and present a solution by DPCHAIN algorithm. Then we consider ring topology instead of chain topology and present a solution by DPRING algorithm. It also includes the proof of correctness of both DPCHAIN and DPRING algorithm.

We finish with concluding remarks in Chapter 4.
CHAPTER 2

ORIGIN OF DINING PHILOSOPHERS PROBLEM

2.1 Definitions

Mutual Exclusion: Mutual Exclusion is a fundamental problem in the area of distributed computing. Concurrent processes come into conflict with each other when they are competing for the use of the same resource. They are not necessarily aware of each other, but the execution of one process may affect the behavior of competing processes. Mutual Exclusion is a collection of techniques for sharing resources so that different processes do not conflict and cause unwanted interactions. Examples of such resources are fine-grained flags, counters or queues, used to communicate between code that runs concurrently, such as an application and its interrupt handlers.

Consider a system of $n$ processors. Every processor, from time to time, may need to execute a critical section in which exactly one processor is allowed to use some shared resource. A distributed system solving the mutual exclusion problem must guarantee the following two properties [18]:

(i) Mutual Exclusion: Exactly one processor is allowed to execute its critical section at any time.

(ii) Fairness: Every processor must be able to execute its critical section infinitely often.

One of the most commonly used techniques for mutual exclusion is the semaphore.
**Starvation:** Starvation is a control problem due to the enforcement of mutual exclusion. Consider we have three processes, P1, P2, and P3, competing for a resource R. Suppose each of them require periodic access to R, which is not sharable, and P1 is first granted access to R. Then when P1 exits its critical section, either P2 or P3 may be allowed access to R. Assume that R is allocated to P3 and P1 requires access to R again. If the operating system alternately allocates R to P1 and P3, then P2 has to wait indefinitely and thus experience starvation, [24].

**Deadlock:** Deadlocks form one of the important error categories of concurrent computer systems, [32]. A set of processes, or threads, is resource deadlocked if each process in the set requests a resource, a lock, held by another process in the set, forming a cycle of lock requests. In communication deadlocks, messages are the resources for which processes wait.

Four conditions must hold for deadlock to occur:

1. **Exclusive use** – when a process accesses a resource, it is granted exclusive use of that resource.

2. **Hold and wait** – a process is allowed to hold onto some resources while it is waiting for other resources.

3. **No preemption** – a process cannot preempt or take away the resources held by another process.

4. **Cyclical wait** – there is a circular chain of waiting processes, each waiting for a resource held by the next process in the chain.

Deadlock can occur whenever two or more processes are competing for limited resources and the processes are allowed to acquire and hold a resource (obtain a lock)
thus preventing others from using the resource while the process waits for other resources.

**Scheduler:** All Components of (Processors and communication links) of distributed systems may not share the same speed assumptions (i.e. one processor may execute its code speedily, while many others are very slow.). The scheduler is a way to model such different behaviors. A scheduler chooses processors to execute their code at a given time. The scheduler (also known as daemon) is said to be *fair* if it selects every process infinitely many times; otherwise, it is *unfair*, [18].

### 2.2 Dining Philosophers Problem (DPP)

The problem of the dining philosophers, proposed by Dijkstra in [17], is a very popular example of control problem in distributed systems, and has become a typical benchmark for testing the expressiveness of concurrent languages and of resource allocation strategies. The dining philosophers problem is a simple case of general resource-allocation problem. The situation is modeled by a graph on the set of processors with an edge between two nodes if they share some resource (Each resource is thus represented by the edges of a complete graph connecting the processors that have access to it). Each processor handles a sequence of jobs; each job in the sequence of a processor has a resource requirement that is a subset of the resources accessible to that processor, [23]. For a job to be executed, all of the required resources must be available for exclusive use by its processor. This can be interpreted as saying that the processor must control the edges incident to it corresponding to the needed resources.
Traditionally, the problem is described in terms of the following informal scenario. There are \( n \) philosophers (users) seated around a table, usually thinking. Between each pair of philosophers is a single fork (resource). From time to time, any philosopher might become hungry and attempt to eat. In order to eat, the philosopher needs exclusive use of the two adjacent forks. After eating, the philosopher needs exclusive use of the two adjacent forks. After eating, the philosopher relinquishes the two forks (i.e., perform an exit protocol) and resume thinking.

![Figure 1.1: Dining Philosophers Problem (for n=5)](image)

2.3 Survey of Non Self-Stabilizing DPP

The dining philosophers problem was first introduced in a specialized setting of a ring of five philosophers by Dijkstra in [17]. The problem was later generalized to the current setting of arbitrary graphs by Lynch in [19]. In this generalization, processes and resources are modeled by a graph with each vertex representing a process, and each edge representing a resource shared by the end vertices. The first work to consider the response time explicitly was the seminal work by Lynch [33] who considered the
problem in the context of resource allocation. Lynch’s algorithm provides an upper bound on the response time of a job. The solution of dining philosophers problem proposed by M. Rabin and D. Lehmann [25] is fully distributed and does not involve any central memory or any process with which every philosopher can communicate. They exhibit a probabilistic solution for dining philosophers problem which guarantees, with probability one, that every hungry philosopher eventually gets to eat.

Styer and Peterson [38] extended and augmented Lynch’s idea [33] to give an algorithm that guarantees a bound on the waiting time of a job that is polynomial in the number of processors at some maximum distance from the processor to which job is assigned. B. Awerbuch and M. Saks,[23] presented a new deterministic algorithm for a general job scheduling problem (generalizing the drinking (and dining) philosophers problem) that guarantees a response time that is not much more than the square of the lower bound. The unique feature of their algorithm is that resources are not explicitly collected; rather a job at the front of the queue simply executes its job, and the properties of the queue ensure that no conflicting job will execute at the same time.

A few non-stabilizing solution to the diners problem with optimal failure locality are also known [34, 42,49]. Choy and Singh [42] investigated the fault-tolerance of distributed algorithms in asynchronous message passing systems with undetectable process failures. They considered two specific synchronization problems the dining philosophers problem and the binary committee coordination problem. The abstraction of a bounded doorway is introduced as a general mechanism for achieving individual progress and good failure locality. Using it as a building block, optimal fault-tolerant algorithms are constructed for the two problems. Sivilotti, Pike and Sridhar [34]
presented a new algorithm for the dining philosophers problem that has optimal failure locality. As a refinement, the algorithm can be easily parameterized by a simple failure model to achieve super-optimal failure locality in the average case. Tsay and Bargodia [49] presented an algorithm that combines the idea of a dynamic priority scheme with the use of a preemptive fork collecting strategy. Its response time is $O(n)$, where $n$ is the total number of processes, if no failures actually occur or $O(n^2)$ in the presence of failures.

2.4 Survey of Self-Stabilizing DPP

Besides the non-stabilizing solution to the diners problem, a number of stabilizing solutions are published as well [35, 36, 45, 48]. Antonoiu and Srimani [35] proposed a new protocol that is id-based and does not use any shared variable as opposed to the self-stabilizing traditional mutual exclusion algorithm, which is anonymous and does use shared link registers. It is also based on read/write atomicity [26] of operations and operates under a distributed demon.

Beauquier, Datta, Gradinariu and Magniette [36] presented a self-stabilizing solution to the local mutual exclusion problem that is the extension of dining philosophers problem to any arbitrary network. They proposed a transformation technique that to transform self-stabilizing algorithms under weaker daemons into algorithms, which maintain the self-stabilization property, and also work under any arbitrary distributed daemon. Arora and Nesterenko [51] combined the stabilization and crash fault tolerance to present an efficient and inexpensive solution to the dining philosophers problem for a rich class of faults-malicious crashes.
Hoover and Poole [46] presented self-stabilizing dining philosophers algorithm that was inspired by the self-stabilizing dining philosophers algorithm presented by Gouda [52]. In Gouda’s solution, one of the philosophers is required to behave differently than the others in order to introduce asymmetry. Datta, Gradinariu and Raynal [27] presented a self-stabilizing solution to the mobile philosophers problem (for asynchronous model) that is a new version of the dining philosophers problem. They assume that the resources form a logical ring (as in dining philosophers problem) and the philosophers can move around a logical ring formed out of a dynamic network.
CHAPTER 3

DINING PHILOSOPHERS

3.1 The Dining Philosophers Problem

In this section, we give a self-stabilizing asynchronous distributed algorithm for the Dining Philosophers Problem, in the composite model of computation. We first describe the problem formally, guided by the presentation given by Lynch [16].

Each philosopher Pi is represented by two processes, the user Ui, and the agent , which we also call Pi. The user decides when to request and return the resources, and the agent actually executes the algorithm. We are also given resources f1, . . . fn. In order for a request by Ui to be satisfied, the Pi must have use of both fi-1 and fi (except that P1 uses fn and f1), and no two philosophers may simultaneously have use of the same resource. We refer to fi-1 and fi as the left and right resources of Pi, and to Pi-1 and Pi+1 as the left and right neighbors of Pi.

Figure 3.1, which is similar to Figure 11.2 of [16], shows the network of processes and resources in the case n = 5.
• $U_i$ has only two states, request and sat.
• $P_i$ can either lock or release either of its resources.
• $P_i$ can use $f_j$ if $P_i$ is holding $f_j$ and no other process is holding $f_j$.
• If $U_i$ is in state request and $P_i$ can use both neighboring resources, $P_i$ will begin executing. Eventually the request will be satisfied, after which $U_i$ will change its state to sat.

A solution to the Dining Philosophers Problem consists of a protocol (program) for each agent process, such that every request by any user process is eventually satisfied. A configuration is said to be illegitimate if two processes are simultaneously holding the same resource, or if a processor is attempting to execute without holding both resources. We call the first situation contention, and we call the second situation premature execution. A configuration is legitimate otherwise.
3.1.1 Livelock and Deadlock

Note that, if two processes are holding the same resource, neither can be executing. It could happen that two processes simultaneously lock the same resource. One of the processes must release the resource if this occurs. But if both release the resource simultaneously, they could both lock it again simultaneously. This cycle could continue indefinitely, a situation known as livelock.

On the other hand, a configuration could occur where each user Ui is in state request, and each agent process Pi holds fi, and refuses to release it until it can lock fi+1 also. In this situation, called deadlock, nothing can happen, and the requests are never satisfied.

3.2 The Chain Version of the Dining Philosophers Problem

We first give a solution to a simplified version of the Dining Philosophers Problem, where the topology is that of a chain rather than a ring. We still have philosophers P1,..., Pn, with user processes U1,...,Un, but we have n + 1 resources, namely f0, f1,..., fn, as shown in Figure 3.2. Our solution is a distributed algorithm in the composite model of computation. Each process has shared variables that can be read by its neighbor processes.
We write:

User(Pi) = Ui
Left(Pi) = Pi-1
Right(Pi) = Pi+1

Variables of DPCHAIN:

P.flag ∈ {A,B}. This variable can be read by P's neighbors.

P.state ∈ {waiting, executing, idle}. This variable can be read by User (P), but not by P's neighbor agents.

Functions of DPCHAIN:

Define reverse(A) = B and reverse(B) = A.

Holds_Left(P) = P is holding its left resource.
Holds_Right(P) = P is holding its right resource.
Left_Free(P) = no process is holding P's left resource.
Right_Free(P) = no process is holding P's right resource.

Left_Nbr_Flag(P) =

\[
\begin{cases} 
\text{reverse(P.flag)} & \text{if } P = P1 \\
\text{Left(P).flag} & \text{otherwise}
\end{cases}
\]
Right_Nbr_Flag(P) = \begin{cases} 
P.\text{flag} & \text{if } P = P_0 \\
Right(P).\text{flag} & \text{otherwise}
\end{cases}

Left_Enabld(P) \equiv Left_Nbr_Flag(P) \neq P.\text{flag}

Right_Enabld(P) \equiv Right_Nbr_Flag(P) = P.\text{flag}

Has_Tokens(P) \equiv Left_Enabld(P) \land Right_Enabld(P)

Error(P) \equiv (Holds_Left(P) \land \neg Left_Enabld(P)) \lor
(Holds_Right(P) \land \neg Right_Enabld(P)) \lor
((P.\text{state} = \text{executing}) \land (\neg Holds_Right(P) \lor \neg Holds_Left(P))) \lor
((P.\text{state} = \text{idle}) \land (Holds_Right(P) \lor Holds_Left(P)))

Macros of DPCHAIN:

Lock_Right(P): P locks its right resource.

Lock_Left(P): P locks its left resource.

Release_Right(P): P releases its right resource.

Release_Left(P): P releases its left resource.

Release_Tokens(P): P.\text{flag} \leftarrow \text{reverse}(P.\text{flag}).

The Flags A and B, and virtual tokens. Since livelock is caused by different processes simultaneously locking the same resource, we can avoid that problem by a scheme which enables only one process to lock a resource at any given time. We use the concept of a token, where possession of a token enables a process to lock resources.

In DPCHAIN tokens are virtual. There is no variable called “token” in our code. Instead, we implement tokens by the use of a shared variable P.\text{flag} for each process P. The value of P.\text{flag} is always either A or B, and a process P “has its right token” if P’s flag is the same as that of its right neighbor (if it has a right neighbor) and P “has its left
token" if P's flag is different from that of its left neighbor (if it has a left neighbor). By default, P1 always "has its left token" and Pn always "has its right token." By this simple method, using only one bit per process, we guarantee that no two adjacent processes have both both of its tokens, while simultaneously guaranteeing that at least one process in the chain has both of its tokens.

**Clauses and Priorities:** The third column of each action given in Table 3.1 consists of a list of clauses, each of which is a Boolean expression over the variables and functions which are computable by a process P. All of those clauses must be true for the action to be enabled. Priorities are also assigned in Table 3.1. The guard of each action contains the unwritten clause that no action whose priority number is lower is enabled. For example, if Error (P) holds, then no action other than Action A1 is enabled.

**The Program:** The algorithm DPCHAIN is almost anonymous, i.e., all processes have the same program, except for the two end processes of the chain, whose programs are very slightly different, due to the slightly different definition of Left_Enabld for P1 and Right_Enabld for Pn.
<table>
<thead>
<tr>
<th>Priority</th>
<th>Action</th>
<th>Description</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Detect</td>
<td>Error</td>
<td>→ Release_Left(P) Release_Right(P) P.state ← idle</td>
</tr>
<tr>
<td>2</td>
<td>Request</td>
<td>P.state = idle User(P).state = request</td>
<td>→ P.state ← waiting</td>
</tr>
<tr>
<td>2</td>
<td>Read</td>
<td>Satisfaction P.state ≠ idle User(P).state = sat</td>
<td>→ P.state ← idle</td>
</tr>
<tr>
<td>3</td>
<td>Release Tokens P.state = idle Has_Tokens(P)</td>
<td>→ Release_Tokens(P)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Release Left ¬Has_Tokens(P) P.state ≠ executing Holds_Left(P)</td>
<td>→ Release_Left(P)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Release Right ¬Has_Tokens(P) P.state ≠ executing Holds_Right(P)</td>
<td>→ Release_Right(P)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Lock     Left P.state = waiting Has_Tokens(P)</td>
<td>→ Lock_Left(P)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Lock     Right P.state = waiting Has_Tokens(P)</td>
<td>→ Lock_Right(P)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Start Execution P.state = waiting Has_Tokens(P)</td>
<td>→ P.state ← executing</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Holds_Left(P) Holds_Right(P)</td>
<td>Release_Tokens(P)</td>
</tr>
</tbody>
</table>
3.3 Proof of Correctness of DPCHAIN

**Lemma 3.1:**

(a) From an arbitrary configuration, the network will reach a legitimate configuration within one round.

(b) From a legitimate configuration, the network will never reach an illegitimate configuration.

**Proof.** If the configuration is illegitimate by contention, i.e., two processes simultaneously hold a resource, within one round, at least one of these processes will notice the contention and release the process by executing Action A1. Thus, there will be no more contention after one round has elapsed. If the configuration is illegitimate by premature execution of some P, then P will execute Action A1, returning to the state idle. No action of DPCHAIN can cause a new contention or premature execution to occur, so the system will never enter an illegitimate configuration from a legitimate configuration. Henceforth, we will assume that the network is always in a legitimate configuration.

**Pseudo-Time.** We define an integral function \( \tau(P) \) for all processors \( P \) as follows:

\[
\tau(P_i) = \begin{cases} 
0 & \text{if } i = 1 \\
\tau(P_{i-1}) - 1 & \text{if } i > 1 \text{ and } \text{Left}_\text{Enabld}(P_i) \\
\tau(P_{i-1}) + 1 & \text{otherwise}
\end{cases}
\]

**Remark 3.1** For any \( 1 \leq i, j \leq n, |\tau(P_i) - \tau(P_j)| \leq |i - j| \)

Let Num(P) be the number of times that P has executed Release_Tokens since the network was initialized. Let \( \Delta i = \text{Num}(P_i) - \text{Num}(P_1) - \frac{1}{2} \tau(P_i) \).

**Lemma 3.2** For any \( 1 \leq i \leq n, \Delta i \) is constant.
Proof. Whenever Pi executes Release_Tokens, Num(Pi) increases by 1 and \( \tau (Pi) \) increases by 2. Whenever P1 executes Release_Tokens, Num(P1) increases by 1 and \( \tau (Pi) \) decreases by 2.

Let

\[
T = \frac{1}{4} n(n-1) + n \text{Num}(P1) + \frac{1}{2} \sum_{i=2}^{n} \tau (Pi) \tag{3.1}
\]

Remark 3.2 T is an integer, and \( n \text{Num}(P1) \leq T \leq n \text{Num}(P1) + \frac{1}{2} n(n-1) \)

Lemma 3.3 During any given step, T increases by the number of processes that execute Release_Tokens during that step.

Proof. Execution of Release_Tokens(Pi) causes \( \tau (Pi) \) to increase by 2 if \( i > 1 \), and hence causes T to increase by 1. Execution of Release_Tokens(P1) causes Num(P1) to increase by 1 and causes \( \tau (Pi) \) to decrease by 2 for all \( i > 1 \), and hence causes T to increase by 1.

Lemma 3.4 Starting from any configuration, T eventually increases.

Proof. Pick i such that \( \tau (Pi) \) is minimum. Then Has_Tokens(Pi), which implies that Pi will eventually executes Release_Tokens, by executing Action A4 or A9. By Lemma 3.3 we are done.

Lemma 3.5 Starting from any given configuration, for any \( 1 < i < n \), Pi eventually executes Release_Tokens.

Proof. By Lemma 3.4, T increases without bound. By (3.1), Num(P1) increases without bound, since the other two terms of the right side are bounded.

\[
\text{Num}(Pi) = \text{Num}(P1) + \frac{1}{2} \tau (Pi) + \Delta i \quad \text{by the definition of } \Delta i, \quad \tau (Pi) \geq -i, \quad \text{and } \Delta i \text{ is constant.}
\]

Thus Num(Pi) increases without bound.

Lemma 3.6 If User(P).state = request , P.state \neq executing, and Has_Tokens(P), then P will eventually execute Action A9.

Theorem 3.1 The algorithm DPCHAIN is correct.

Proof. If a process receives a request from its user, then, by Lemma 3.6, it must eventually execute Action A9, after which it must eventually complete that execution.

3.4 The Algorithm DPRING

We now adapt the algorithm DPRING to the ring topology, as described at the beginning of this chapter. If we use the same code as DPCHAIN, deadlock can occur.

Note that in DPCHAIN, both end processes, P1 and Pn, have programs that are slightly different from the middle processes, P2, . . . , Pn-1. We will do the same for DPRING. The difference is that in DPRING the end processes are neighbors, and so we must ensure that they do not execute simultaneously.

As in DPCHAIN, we let each process have two virtual tokens, one that it shares with its left neighbor, the other with its right. Each middle process has just one flag, and it uses the same rules as DPCHAIN to decide whether it holds none, one, or both of its tokens. Each end process has two flags, P1.left_flag and P1.right_flag. P1.right_flag and Pn.left_flag are their “normal” flags, which are used to determine whether they hold the resources they share with their middle neighbors. The other flag, P1.left_flag or Pn.right_flag, is used by the end process to decide whether it holds the “end token,” i.e., permission to use the end resource. If both end processes’ end flags are equal, Pn has the end token. If they are different, P1 has the token.
As long as neither end resource has a request, the end token shuttles endlessly back and forth between the end processes. Each time an end process has the end token, it checks to see whether it has its other token and also whether its status is "waiting." If both are true, it keeps the end token and waits, if necessary, until its middle neighbor (P2 or Pn−1) has finished executing, and then locks both resources and starts executing and releases both tokens by reversing both flags. In all other cases, it immediately releases the end token by releasing just its end flag.

An end process has more variables than a middle process, but it allows their neighbors to see its variables selectively in such a way that, to its middle neighbor, the end process appears to be just another process. Thus, P2 sees P1.right but not P1.left, while Pn−1 sees Pn.left but not Pn.right.

Each middle process runs exactly the same code as a middle process of DPCHAIN. In fact, there is no need for the process to even know that it is running DPRING instead of DPCHAIN. For that reason, we simply use Table 3.1 for the actions of a middle process.

**The Deadlock Problem.** There is a deep mathematical reason that it is difficult for an asynchronous algorithm on the ring to avoid deadlock. This has to do with the fact that the topology of the ring is not simply connected, i.e., it has a non-contractible cycle. (The same kind of problem arises some certain other distributed problems on any non-simply connected topology, such as construction of a synchronizer.)

If we attempt to use a strict analog of DPCHAIN on the ring, deadlock may result. The key to resolving this problem is to break the cycle in some way. We do this by
designating P1 and Pn to be end (or leader) processes, with codes that differ from that of the normal processes, P2, ..., Pn-1.

We do this by assigning colors to the tokens. The normal tokens that shuttle back and forth between the normal processes we assign the color 0. the one end token that shuttles back and forth between P1 and Pn, we assign the color 1.

The tokens have different priorities. If a process has a token of color 0, it holds it until it has the other token. But if a process (always an end process) needs tokens of both colors, and it has a token of color 1 but not the token of color 0, it releases the token of color 1 even if its state is waiting. This scheme prevents deadlock.

The scheme can be extended to other topologies by having more colors, although that is beyond the scope of this thesis.

3.4.1 Formal Definition of DPRING

We let P1, ..., Pn be the agent processors, and Ui = User(Pi) the corresponding user processors. We assume a ring topology, i.e., Pi and Pi+1 are adjacent for i < n, and Pn and P1 are adjacent. Each Pi has the same code, except for P1, which is the leader.

The code for P1 is given in Table 3.2. We have carefully designed DPRING so that, from the viewpoint of any process other than the leader, it is identical to DPCHAIN. Thus, the code for Pi, for i > 0 is given in Table 3.1

We write:

\[
\begin{align*}
\text{User}(Pi) & = Ui \\
\text{Left}(Pi) & = \begin{cases} 
  Pn & \text{if } i = 1 \\
  Pi-1 & \text{otherwise}
\end{cases}
\end{align*}
\]
\[
\text{Right}(\text{Pi}) = \begin{cases} 
\text{Pi} & \text{if } i = n \\
\text{Pi+1} & \text{otherwise}
\end{cases}
\]

Variables of DPRING:

\( P.\text{flag} \in \{A,B\} \) if \( P \in \{P_2, \ldots, P_{n-1}\} \). This variable can be read by \( P \)'s neighbors.

\( P.\text{left\_flag} \in \{A,B\} \) if \( P \in \{P_1, P_n\} \). This variable can be read by Left(P).

\( P.\text{right\_flag} \in \{A,B\} \) if \( P \in \{P_1, P_n\} \). This variable can be read by Right(P).

\( P.\text{state} \in \{\text{waiting, executing, idle}\} \) This variable can be read by User(P), but not by its neighbor agents.

Functions of DPRING:

Holds_Left(P) \equiv P \text{ is holding its left resource.}

Holds_Right(P) \equiv P \text{ is holding its right resource.}

Left_Free(P) \equiv \text{no process is holding } P \text{'s left resource.}

Right_Free(P) \equiv \text{no process is holding } P \text{'s right resource.}

Left_Nbr_Flag(P) = \begin{cases} 
P_n.\text{right\_flag} & \text{if } P = P_1 \\
P_1.\text{right\_flag} & \text{if } P = P_2 \\
\text{Left}(P).\text{flag} & \text{otherwise}
\end{cases}

Right_Nbr_Flag(P) = \begin{cases} 
P_1.\text{left\_flag} & \text{if } P = P_n \\
P_n.\text{left\_flag} & \text{if } P = P_{n-1} \\
\text{Right}(P).\text{flag} & \text{otherwise}
\end{cases}

Left_\text{Enabld}(P) \equiv \text{Left_Nbr_Flag}(P) \neq P.\text{flag}

Right_\text{Enabld}(P) \equiv \text{Right_Nbr_Flag}(P) = P.\text{flag}

Has_Tokens(P) \equiv \text{Left_Enabld}(P) \land \text{Right_Enabld}(P)
\[\neg \text{Holds}_\text{Right}(P) \land \neg \text{Holds}_\text{Left}(P) \land (\text{P.state} = \text{idle})\]

\[\text{Rest}(P) = \begin{cases} \text{\neg Left}_\text{Enabld}(P) \land \neg \text{Right}_\text{Enabld}(P) & \text{if } P \in \{P_1, P_n\} \\ \neg \text{Holds}_\text{Right}(P) \land \neg \text{Holds}_\text{Left}(P) \land (\text{P.state} = \text{idle}) & \text{otherwise} \end{cases}\]

\[\text{Error}(P) = (\text{Holds}_\text{Left}(P) \land \neg \text{Left}_\text{Enabld}(P)) \lor (\text{Holds}_\text{Right}(P) \land \neg \text{Right}_\text{Enabld}(P)) \lor ((\text{P.state} = \text{executing}) \land (\neg \text{Holds}_\text{Right}(P) \lor \neg \text{Holds}_\text{Left}(P))) \lor ((\text{P.state} = \text{idle}) \land (\text{Holds}_\text{Right}(P) \lor \text{Holds}_\text{Left}(P)))\]

Macros of DPRING:

- Lock\_Right(P): P locks its right resource.
- Lock\_Left(P): P locks its left resource.
- Release\_Right(P): P releases its right resource.
- Release\_Left(P): P releases its left resource.
- Release\_Tokens(P):
  
  if \(P = P_1\) then
  
  \(P\_\text{left\_flag} \leftarrow (P_n\_\text{right\_flag})\)
  
  \(P\_\text{right\_flag} \leftarrow \text{reverse}(P_2\_\text{flag})\)
  
  else if \(P = P_n\) then
  
  \(P\_\text{left\_flag} \leftarrow P_{n-1}\_\text{flag}\)
  
  \(P\_\text{right\_flag} \leftarrow \text{reverse}(P_1\_\text{left\_flag})\)
  
  else
  
  \(P\_\text{flag} \leftarrow \text{reverse}(P\_\text{flag})\)
  
  endif
<table>
<thead>
<tr>
<th>Priority</th>
<th>Action</th>
<th>Description</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correct Error</td>
<td>Error (P)</td>
<td>Release_Left(P) Release_Right(P) P.state ← idle</td>
</tr>
<tr>
<td>2</td>
<td>Read Request</td>
<td>P.state = idle User(P).state = request</td>
<td>P.state ← waiting</td>
</tr>
<tr>
<td>2</td>
<td>Satisfaction</td>
<td>P.state ≠ idle User(P).state = sat</td>
<td>P.state ← idle</td>
</tr>
<tr>
<td>3</td>
<td>Release Right Token</td>
<td>P = P1 P.state = idle Right_Enabld(P)</td>
<td>Release_Tokens(P)</td>
</tr>
<tr>
<td>3</td>
<td>Release Left Token</td>
<td>P = Pn P.state = idle Left_Enabld(P)</td>
<td>Release_Tokens(P)</td>
</tr>
<tr>
<td>3</td>
<td>Lock Left</td>
<td>P.state = waiting Has_Tokens(P) Left_Free(P)</td>
<td>Lock_Left(P)</td>
</tr>
<tr>
<td>3</td>
<td>Lock Right</td>
<td>P.state = waiting Has_Tokens(P) Right_Free(P)</td>
<td>Lock_Right(P)</td>
</tr>
<tr>
<td>3</td>
<td>Start Execution</td>
<td>P.state = waiting Has_Tokens(P) Holds_Left(P) Holds_Right(P)</td>
<td>P.state ← executing Release_Tokens(P)</td>
</tr>
<tr>
<td>4</td>
<td>Shuttle Left</td>
<td>P = P1 Left_Enabld(P)</td>
<td>P.left_flag ← Pn.right_flag</td>
</tr>
<tr>
<td>4</td>
<td>Shuttle Right</td>
<td>P = Pn Right_Enabld(P)</td>
<td>P.right_flag ← reverse(P1.left_flag)</td>
</tr>
</tbody>
</table>
3.5 Proof of Correctness of DPRING

Lemma 3.7

(a) From an arbitrary configuration, the network will reach a legitimate configuration within one round.

(b) From a legitimate configuration, the network will never reach an illegitimate configuration.

The proof is the same as that of Lemma 3.1. Henceforth, we will assume that the network is always in a legitimate configuration.

Pseudo-Time. We define an integral potential $\tau(P)$ for all processors $P$ as follows:

$$
\tau(P_i) = \begin{cases} 
1 & \text{if } i = 1 \\
\tau(P_{i-1}) - 1 & \text{if } i > 1 \text{ and } \text{Left}_{\text{Enabld}}(P_i) \\
\tau(P_{i-1}) + 1 & \text{otherwise}
\end{cases}
$$

Remark 3.3 For any $1 < i, j < n$, $|\tau(P_i) - \tau(P_j)| \leq |i - j|

Let $\text{Num}(P)$ be the number of times that $P$ has executed Release_Tokens since the network was initialized. Let $\Delta_i = \text{Num}(P_i) - \text{Num}(P_1) - \frac{1}{2} \tau(P_i)$.

Lemma 3.8 For any $1 < i < n$, $\Delta_i$ is constant.

The proof is the same as that of Lemma 3.2.

Let

$$
T = \frac{1}{4} n (n-1) + n \text{Num}(P_1) + \frac{1}{2} \sum_{i=2}^{n} \tau(P_i)
$$

Remark 3.4 $T$ is an integer, and $n \text{Num}(P_1) \leq T \leq n \text{Num}(P_1) + \frac{1}{2} n(n-1)$

Lemma 3.9 During any given step, $T$ increases by the number of processes that execute Release_Tokens during that step.

The proof is the same as that of Lemma 3.3.

Lemma 3.10 If the configuration is legitimate, then within four rounds, either $P_1$.left_flag...
or Pn.right_flag will change.

Proof.

Case I: Left_Enabld(P1). We have two subcases.

Subcase I.a: P1.state = waiting and Right_Enabld(P1). Within three rounds, P1 will execute Action B8. (P1 may have to execute one or both locking actions, B6 and B7, first)

Subcase I.b: P1.state ≠ waiting or ~Right_Enabld(P1). Then, P1 is enabled to execute Action B9. Within one round, either P1 will execute Action B9, or that action will be neutralized by Action B8 being enabled, reducing to subcase I.a.

Case II: Right_Enabld(Pn). Similar to Case I.

Lemma 3.11 If P is a minimum of the function τ, then P eventually executes Release_Tokens.

Proof. P = Pi for some 1 ≤ i ≤ n.

Case I: 1 < i < n.

Subcase I.a: P.state = waiting. Then P will execute Action A9 within three rounds.

Subcase I.b: P.state = idle. Then within one round, either P will execute Action A4, and we are done, or P will execute Action A2, reducing to Subcase I.a.


Case II: i = 1. Then Right_Enabld(P).

Subcase II.a: P1.state = waiting. By Lemma 3.10, Left_Enabld(P) will hold within four rounds, and within three more rounds, P1 will execute Action B8.
Subcase II.b: P1.state = idle. Within one round, either Action B4 will execute, in which case we are done, or P1.state ← waiting, reducing to Subcase II.a.

Subcase II.c: P1.state = executing. Eventually User(P).state = sat, after which P1 will execute Action B3 followed by Action B4, and we are done.

Case III: i = n. Similar to Case II.

**Lemma 3.12** Starting from any configuration, T eventually increases.

**Proof.** Let P be a process such that τ(P) is minimum. By Lemma 3.11, P eventually executes Release_Token. Then, by Lemma 3.9, we are done.

**Lemma 3.13** Starting from any given configuration, for any 1 ≤ i ≤ n, Pi eventually executes Release_Tokens.

The proof is the same as that of Lemma 3.5, except that we use Lemma 3.12 instead of Lemma 3.4.

**Lemma 3.14** If User(P).state = request, P.state ≠ executing, and Has_Tokens(P), then P will eventually execute Action A9 or B8.

**Proof.**


Case III: P = Pn. Similar to Case II.

**Theorem 3.2** The algorithm DPRING is correct.
**Proof.** If a process receives a request from its user, then, by Lemma 3.14, it must eventually execute Action A9 or Action B8, after which it must eventually complete that execution.
CHAPTER 4

CONCLUSION AND FUTURE RESEARCH

We propose a self-stabilizing solution for non-uniform bi-directional rings under unfair distributed daemon without using message passing model and token circulation method. We present solution for composite size of ring for which Burns and Pachl claim in [4] that there is no solution possible. First we adapt DPCHAIN algorithm for chain topology and then we adapt DPRING Algorithm for ring topology to make sure that both end processes don’t execute simultaneously. By this virtual token scheme we prevent livelock, starvation and deadlock in ring topology without using any real token. The scheme can be extended to other topologies by having more colors, although that is beyond the scope of this thesis.


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