Design and control of a vibration isolator using a biased magnetorheological elastomer

Saul Opie
University of Nevada, Las Vegas

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DESIGN AND CONTROL OF A VIBRATION ISOLATOR USING A BIASED MAGNETORHEOLOGICAL ELASTOMER

by

Saul Opie

Bachelor of Science in Mechanical Engineering
California State Polytechnic University, Pomona
December 2004

A thesis submitted in partial fulfillment of the requirements for the

Master of Science Degree in Engineering
Department of Mechanical Engineering
Howard R. Hughes College of Engineering

Graduate College
University of Nevada, Las Vegas
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Saul Opie

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Examination Committee Chair

Dean of the Graduate College

Examination Committee Member

Examination Committee Member

Graduate College Faculty Representative
ABSTRACT

Design and Control of a Vibration Isolator Using a Biased Magnetorheological Elastomer

by

Saul Opie

Dr. Woosoon Yim, Examination Committee Chair
Professor of Mechanical Engineering
University of Nevada, Las Vegas

The objective of this work is to explore the capability of a Semi-Active (SA) elastomer and control techniques in the area of shock and vibration isolation. Typical passive isolation methods have shortcomings in meeting competing objectives. A specific problem is isolating electronic packages mounted to military vehicle walls from shock. Often passive elastomer based isolators are used. The ideal solution for shock isolation is a soft lightly damped isolator. However a soft lightly damped isolator will cause excessive sway during normal driving conditions. Further, vehicle dynamics during normal driving conditions are typically in the range of a few hertz, presenting the possibility of a lightly damped soft system experiencing severe resonance. As a result most elastomer based isolators have significant damping, which decreases their ability to isolate shock. Active systems are able to theoretically reach an optimal compromise between shock isolation and sway, however for several reasons active systems are not practical. SA systems combine the benefits of passive systems, primarily cost and low
actuator power input, with the capability of varying system parameters in real-time with performance indexes nearing that of active systems.

This work investigates an interesting SA elastomer, a magnetorheological elastomer (MRE), that is able to change its properties with the application of an external magnetic field. Methods of controlling the field to achieve a desired response is discussed. Finally experimental data is presented of a MRE based device using a SA control scheme to isolate a payload from shock and vibration.
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CHAPTER 1

INTRODUCTION

Semi-active (SA) shock and vibration isolation has seen considerable interest in the last several decades. SA technology, with respect to shock and vibration isolation, changes physical properties or parameters of passive elements (i.e. springs, dampers, etc.) in an attempt to improve a performance index such as transmitted acceleration, absolute/relative displacement, or possibly both. A properly executed SA system will outperform a passive system, and in some respects an active system.

Typical active systems (e.g. voice coils, linear actuators) require a physically large envelope, power requirement, and cost. Additionally active systems have an inherent bandwidth limitation due to mechanical inertia (i.e. mass of voice coil, mass of actuator rod) and power electronic constraints. This bandwidth problem and weak control algorithms leads to active systems becoming unstable in the sense that under wide operation conditions a passive system will often significantly outperform the active system. Often the instability of an active system can lead to catastrophic results.

Magnetorheological (MR) technology has produced a promising field of SA devices such as MR dampers and MR Elastomers (MREs) that have many of the benefits of fully active systems while circumventing active drawbacks. MR technology relies on introducing ferrous particles within a fluid or solid. The properties of the fluid or solid are rapidly changed with the application of an external magnetic field which induces an
anisotropic dipole interaction energy among the ferrous particles. Ultimately this interaction energy leads to an apparent increase in fluid viscosity and/or modulus. As a result MR technology does not have the actuator mechanical inertia problems of typical active systems. Also the required power for MR technology devices is typically magnitudes less than active systems, so that power electronic constraints are not as significant in MR devices. Finally since the energy requirement of MR devices is much less than active systems the catastrophic instability issues observed in active systems, even if a weak SA controller is implemented, will not be present in isolation systems utilizing MR devices.

The objective of this research will be to explore the potential of an MRE device for shock and vibration isolation. Several SA controllers will be discussed for use with the device. This work will also provide valuable insight on how to improve the performance of MR dampers, since the operating principles of the developed MRE device are very similar to that of MR dampers.

1.1 Literature Review

Nearly every object is sensitive to some level of shock and/or sustained levels of vibration. The subcommittees (SC) of ISO technical committee (TC) 108 provide guidelines and testing procedures to quantify critical levels of shock and vibration [1]. For example, SC 4 provides ISO document 2631-1 “Evaluation of human exposure to whole-body vibration,” and advises that the whole-body human exposure rms vibration value for the 4-8hz band should be below $0.315m/s^2$ for an 8 hour period in order to avoid discomfort. Other SCs provide guidelines for categories such as buildings (ISO 4866),
machinery (ISO 7919), and cars (ISO 8002). Other organizations also provide guidelines or specifications for vibration, such as US Military Standards (e.g. MIL-STD-202G, electronic part specifications and test procedures).

Vibration and shock transmission is typically mitigated with passive elements. Passive elements are inexpensive, compact, and are available in nearly an infinite amount of configurations. Passive elements (e.g. springs and dampers) however have fixed properties, so that some passive systems – a combination of springs and dampers to mitigate transmitted vibration to some mass – outperform other passive systems depending on the spectral content of the vibration/shock source. Therefore in application passive elements are chosen such that a performance index (PI) is optimized [2] based on some knowledge of the vibration/shock source. Too further complicate matters the PI usually contains competing goals. A relevant example is isolating large packages mounted to military vehicle walls. To isolate the package from shock the isolating mounts should be as soft as possible. However the resonant frequency of the soft isolator system will be near the dominant frequencies of the vehicle dynamics, causing the package to resonate and sway excessively. To remedy the solution a stiffer isolator will likely be chosen, but at the expense of decreased shock isolation.

To improve particular PIs various active systems have been developed [list some active system articles or ref, TMC tech brochure]. The different control schemes, instrumentation, and equipment in active systems is immense. However a basic system consists of at least one sensor, a controller (analog or digital), and a device capable of producing a force to instantaneously improve the PI [3-8]. Active systems are then intrinsically more complicated and expensive than passive systems, this limits active
systems to critical applications where the best performance is necessary. Active systems also have the ability to perform worse than passive systems if the controller and sensors are not robust and reliable, since large amounts of energy can be added to the system in a detrimental way.

Semi-active (SA) systems attempt to bridge the gap between passive and active systems. SA devices are typically described as vibration control elements that cannot inject energy into the system to be controlled [8]. However this definition is used loosely, since variable stiffness devices are commonly accepted as SA devices, yet many variable stiffness devices can inject energy into a system[9,10]. A better definition then might be that SA devices are devices who’s parameters (stiffness or damping) can be changed rapidly in real-time [11].

One of the earliest and most popular discussions describing the benefit of a modern SA device is Karnopp’s [12] “skyhook” damper. In his paper a hydraulic damper capable of changing damping rates via fluid control valves and solenoids is described. This SA damper then tries to emulate a passive damper (the skyhook damper) connected to an inertial frame and a vibrating body to be isolated. The control strategy showed that the SA damper had a performance capability near that of an active system, yet the stability, cost, and energy problems of the active system were not carried over. In addition to the skyhook controller, several other SA control strategies for vibration isolation have been developed such as ON-OFF and modified skyhook [13,14], clipped LQR [15,16], predictive optimal [17], and a lyapunov stability controller [18].

Over the last thirty years several other hydraulic dampers have been proposed [19], however commercial success has been absent because, like active systems, SA hydraulic
dampers tend to be complicated and costly. This led to the development of Electrorheological (ER) and Magnetorheological (MR) dampers. Both are much simpler in construction and less costly than SA hydraulic dampers, yet provide superior performance in the respect of bandwidth.

MR dampers operate similar to passive hydraulic dampers in the respect that a fluid is forced through orifices, dissipating kinetic energy as heat. In a passive damper the viscosity is usually fixed in a small range, leading to a fixed damping rate. However the apparent fluid viscosity in an MR damper can be changed rapidly by several magnitudes with the application of a magnetic field. The typical power consumed to produced the field is in the range of 10 to 50 watts, yet several kilowatts of system kinetic energy can be removed [8]. These properties have led to MR dampers having commercial success [20,21,22], fueling further interest in SA device research and development.

Magnetorheological Elastomers (MREs) are variable stiffness elements discovered as a result of MR damper research [23]. MREs consist of micron ferrous particles embedded in an elastomer matrix. Similar to MR dampers, an applied magnetic field causes the ferromagnetic particles to develop an anisotropic dipole interaction energy which results in the elastomer, or fluid in the case of MR dampers, developing an increase in complex shear modulus. The interest in MREs is that the stiffness of the rubber compound can be changed in milliseconds and the cost and complexity of the smart elastomer is in some cases less than MR dampers. To date however the application of MREs has not been as significant as MR dampers, yet several patents have been filed [24-27] and an interesting steering column crash system has been developed by Thyssen Krupp [28].
The use of MREs for vibration and shock isolation has been limited to a few cases. Davis at the Ford Motor Company developed a control arm bushing incorporating an MRE [29], and an electromagnetic coil to provide the needed field. However the advantages of the device over a traditional passive bushing were marginal, and the power requirement of the coil was well above 100 watts cold. Other researchers have developed MRE based tunable vibration absorbers (TVAs) [30,31]. TVAs are complete assemblies that are mounted to a vibrating structure in attempt to shift the resonances of the vibrating structure to less vulnerable frequencies.

1.2 Organization

The thesis is organized as follows. Chapter 2 describes what an MRE is and what unique properties they possess. A simple dipole model is reviewed to explain the field-dependent elastomer modulus. Chapter 3 presents the basic design of an isolation device that utilizes a MRE element. In addition to mechanical considerations, such as the isolator's natural frequency for a given load, the magnetostatic aspects of the device are considered. A simple magnetic circuit model is given to pick geometry variables. A more complicated optimization method involving a genetic algorithm and a Finite Element Model (FEM) is also discussed as an optional design methodology if non-standard components are available. Chapter 4 reviews SA control. The Skyhook damper concept is discussed in detail in this section, several forms of this controller are explained and compared through simulations. In consideration of the importance of controller bandwidth, Chapter 5 looks at improving the time response of the MRE based device. Chapter 6 applies the work of the previous chapters through several experiments. The
experimental results provide encouragement for utilizing a MRE element in a commercial device. Future work and final thoughts are presented in Chapter 7. For completeness, several Appendices are attached to clarify terminology and to explain fabrication methods.
CHAPTER 2

MAGNETORHEOLOGICAL ELASTOMER PROPERTIES

2.1 Magnetorheological Elastomer (MRE) Description

A rubber compound is a term used to refer to a raw polymer/elastomer (e.g. silicone, EPDM, nitrile, natural rubber) that has been mixed with other ingredients. The other ingredients are composed of typically four groups: curing agents, fillers, softening/processing aids, and miscellaneous ingredients [32]. The curing agent bonds the polymer chains together strengthening the raw polymer. A filler, amorphous silica when silicone polymer is used or carbon soot in other polymers, is added to further strengthen the raw polymer by inhibiting relative chain movement. A softening agent (typically some form of oil) is used in to reduce the strength of a compound. This is commonly done when an inexpensive soft compound needs to be made by using large amounts of filler and a small amount of raw polymer. Miscellaneous ingredients typically include age resistors (e.g. antioxidants) and coloring additives. An MRE is a special rubber compound that includes a large volume (30 percent or more) of micron sized iron particles in addition to the ingredients described above.
2.2 MRE Model

MREs are attractive for use in SA devices because when a magnetic field is applied to an MRE compound it’s complex shear modulus changes in the order of milliseconds. The effect is commonly explained through a dipole interaction energy between neighboring ferrous particles within the compound, as shown in Figure 2.1. Basically the particles become polarized (see the Appendix for a description of magnetic terms) by the magnetic field and pull at one another adding strain energy to the rubber. More precisely the interaction energy between two dipoles (magnetic moments, $\vec{m}$) of equal strength and oriented in the same direction, see Figure 2.1, in a particle chain is [23]:

$$E = \frac{\vec{m}^2}{4\pi\mu_o\mu_i} \left(1 - 3 \frac{r_o^2}{r_o^2 + x^2}\right) \left(r_o^2 + x^2\right)^{3/2}$$

Figure 2.1. Neighboring Iron Particles with Magnetic Moments (m).

Where equation (2.1) represents the energy between two neighboring multi domain particles in the material, and $\mu_i$ is the relative permeability of the medium between the particles (typically $\mu_i = 1$). Summing the interaction energy between all the particles in
the material and noting that the shear strain is \( \gamma = \frac{x}{r_o} \), the energy density, \( U \), from magnetic attraction becomes,

\[
U = \frac{3\phi(\gamma^2 - 2)|\bar{m}|^2}{2\pi^2 \mu_0 \mu_i d^3 r_o^2 (1 + \gamma^2)^{3/2}} \tag{2.2}
\]

where \( \phi \) is the volume fraction of ferromagnetic particles contained within the MRE. Since the shear stress in the material is the derivative of energy density with respect to shear strain, and stress divided by shear strain is defined as the shear modulus \( G \), then from (2.2) the change in shear modulus, \( \Delta G \), due to magnetic attraction becomes,

\[
\Delta G = \frac{\partial U}{\partial \gamma} \approx \frac{\phi J^2}{2\mu_0 \mu_i h^3}, \gamma < 0.1
\]

where \( J = \frac{\bar{m}}{V_i} \) \( (V_i \) is the particle volume) is the average polarization of the particles, and \( h = r_o / d \). When the particles become saturated \( J \rightarrow J_s \), then (2.3) represents the maximum change in shear modulus.

The relative change, \( \%G \), in the MRE modulus is dependent on the MRE modulus before a magnetic field is applied, \( G_{MRE} \), and after a field is applied, \( G \), and is simply expressed as:

\[
\%G = \frac{G - G_{MRE}}{G_{MRE}} \times 100 \tag{2.4}
\]

and

\[
G(H) = G_{MRE} + \Delta G(H) \tag{2.5}
\]

where \( H \) is the applied magnetic field. Note that the polarization, \( J \) in (2.3), is a nonlinear function of \( H \) as described in Appendix A.

Attempts at predicting \( G_{MRE} \) with the Guth-Smallwood equation,
\[ G_{MRE} = G_o (1 + 2.5\phi + 14.1\phi^2) \]  

(2.6)

where \( G_o \) is the shear modulus in MPa of the polymer before the iron particles are added, has had success with some MRE compounds [33,34]. Predicting (2.4) to be about,

\[ \% G \approx \frac{0.19}{G_o} \times 100 \]  

(2.7)

with a iron polarization value of \( J = 2.0T \) and a ‘optimal’ volume fill fraction of \( \phi = 0.30 \) (30% iron) suggested by [35]. However, using (2.6) is not generally accurate. The interactions of processing aids and fillers in a rubber compound such as an MRE is very complicated; these interactions and the processing of the MRE will all have a significant effect on the value of \( G_{MRE} \).

Unfortunately equation (2.3) is valid only for small strains. Experimental results show the field induced modulus decreases sharply once the strain reaches 2%. The point-dipole model (2.1-3) predicts that this drop off should not occur so soon. Explanations for the large discrepancy center around imperfect particle chain structures. Researchers believe that the dipole interactions are as strong as predicted, but that particle spacing throughout the chain is not as ideal as modeled and consequently the chains are weaker in a few areas [23], leading to a weak gross modulus. Suggestions have been made on how to delay the strain dependent drop off [36].

2.3 MRE Experimental Properties

To validate previous work and obtain material properties for design purposes the dynamic properties of a few MRE compounds were found with a Dynamic Mechanical Analysis (DMA). Three sample compounds were tested and are listed in Table 1. All
three samples were made with RTV silicone (Dow HS III) and silicone oil (Dow 2000 50ct), while two samples used iron particles (ISP-3700).

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RTV silicone is likely not an ideal material to use, however the processing and molding equipment needed to properly utilize rubber compounds with heat activated cure systems (such as sulphur or peroxide) is costly and task specific. The results to follow therefore could likely be improved significantly by a skilled rubber compounding worker with modern processing and molding equipment. RTV silicone is used in this study because the iron particles can be blended in fairly easily, and the mold used to cure the rubber can be fabricated with aluminum and relatively loose tolerances. In heat activated cure systems molds are made of steel with flatness tolerances below a thousandth of an inch. This is because these cure systems create gaseous byproducts, and to maintain rubber part tolerances and eliminate porosity, high molding pressure are used (4MPa or more). With RTV silicone molding conditions are not as stringent.

To characterize the shear modulus properties of the compounds listed in Table 1, double lap shear specimens of 20mm x 20mm x 5mm were used, see Figure 2.2. The double lap shear fixtures were made of aluminum and coated with a silicone bond promoter (Dow 1200 Prime Coat). The fixtures were then arranged in a mold and the rubber compound was poured into the voids. To investigate the effect an aligning field
has on final properties two samples were made containing iron and a third sample of just
RTV and plasticizer (silicone oil), see Table 1. It should be noted that equations (2.1-3,
2.7) assume that an aligning field is used during the curing process.

A custom laboratory electromagnet with pole diameters of 1.5 inches and 2000 turn
coils (2 coils at 2000 turns each) of 14 gauge magnet wire was constructed to
accommodate the dimensions of the DMA equipment (Bose ElectroForce LM1). This
electromagnet was also used as the aligning field source for sample 3 during curing. The
aligning field was slightly above 0.4T (average flux density of sample cross section) at a
coil current of 2 amps provided by a DC bench power supply (Sorenson LHP 100-10).
The experiment setup is shown in Figure 3.3.

Each sample was tested at shear strains of 1, 2, 3, 5, 10, and 20%. At each strain level
the samples were tested at frequencies ranging from 0.1 to 100Hz. All samples were
tested at an average cross section flux density of zero. In addition samples 2 and 3 were
tested at an average flux density of 0.6T, approximately the magnetic saturation level of
the two samples. The flux densities were measured by carefully wrapping a search coil of
20 turns around the samples and integrating the induced voltage with an integrating flux
meter (Walker MF-3D).
Figure 2.2. DMA Sample Dimensions (5mm depth).

Figure 2.3. Experiment Setup for Characterization of MRE Samples.
Figures 2.4 through 2.6 show the results from the DMA. It should be noted that eddy currents in the aluminum fixtures did not introduce error into the results. This is because the field created by the electromagnet has a field uniformity better than 90% (measured with a Hall probe device) and the fact that the aluminum fixture attached to the load cell was fixed. Before any data was recorded all samples were put through a pre-extension treatment of 20% strain at 2 Hz for 2 minutes to reduce the effect of strain history on test results. In all the plots $G^*$ represents the complex shear modulus (2.8):

$$G^* = G' + iG''$$  \hspace{1cm} (2.8)

where $G'$ is the storage modulus, $G''$ is the loss modulus, and Tan Delta is ratio of the loss modulus to the storage modulus.

![DMA_results](image.png)

Figure 2.4. DMA Results for Sample No. 1.
Figure 2.5. DMA Results for Sample No. 2.

Figure 2.6. DMA Results for Aligned Sample No. 3.
Figures 2.5 and 2.6 show the benefit in using an aligning field during the curing process. The aligned sample (Sample No. 3, Figure 2.6) showed an increase in $G^*$ ranging from a factor of 2 (for 20% strain deflection) to about 3.5 (for a 1% strain deflection) when a field was applied during the DMA test. These results agree reasonably well to Equation 2.3, when the fact that the magnetic effect is weakened with increasing strain. In contrast the isotropic sample (Sample No. 2, Figure 2.5) showed an increase in $G^*$ ranging from only 1.25 to 2. Comparing the $G^*$ values when a field is not applied (top left hand corner plot in figures) in Figures 2.5 and 2.6 to Figure 2.4 shows that the Guth-Smallwood Equation (2.6) does a reasonable job of predicting the iron filled elastomer properties. Equation 2.6 predicts a value of 0.24 MPa (with $G_o = 0.08$ MPa), while the actual values from Figure 2.5 and 2.6 range from 0.17 to 0.30 MPa.
CHAPTER 3

MRE DEVICE DESIGN

3.1 Introduction

MR fluid dampers have had much success for reasons such as: design simplicity, low cost (with the possible exception of the MR fluid), low power requirements (typically less than 50W), and the performance capability of increasing damping by a magnitude or greater [22]. MRE devices have not yet shared this success. The following sections will explain some difficulties in utilizing MREs and introduce a new MRE based device with these factors in mind. The basic design can be used either in translational or rotational modes, and can be utilized in shock and vibration isolation, or other engineering applications where a variable stiffness feature is needed. A unique aspect to the design is that a bias flux is provided via a permanent magnet so that the device is capable of decreasing as well as increasing its stiffness.

3.2 Basic Design

Most elastomer based isolators are compact and support large loads. A practical MRE device should share these properties also, but should also utilize the unique MRE variable modulus properties. Therefore an MRE device also needs to be able to create a variable magnetic field through the MRE element. The requirement of a variable magnetic field will usually lead to the addition of an electromagnetic coil and significantly more
ferromagnetic material (i.e. steel), both add weight and size to the device. Additionally, the creation of a magnetic field requires energy, and because real conductors have losses power is needed to sustain the field. This leads to two competing goals, 1) reduction of MRE device power requirement and 2) reduction of MRE device weight and size. With these factors in mind the basic design shown in Table 3.1 and Figure 3.1 was arrived at.

Figure 3.1. Basic Schematic of MRE Isolator.

In order to utilize the MRE variable shear modulus (discussed in previous sections) the MRE elements in Figure 3.1 are designed to primarily support a vertical load in a shear mode. Meanwhile attention is given to maintain a small device profile and footprint while providing a sufficient ferromagnetic cross section for the electromagnetic coil and flux paths. To pick the parameter values (i.e. R1,R2,R2, etc.) shown in Figure 3.1, two approaches were used: 1) static magnetic circuit analysis 2) a genetic algorithm coupled to a static magnetic finite element model. The first approach is useful because it ends in a simple equation which gives the designer an empirical relation between parameter values
and values of interest, such as flux density passing through the MRE for a given amount of available power. The second approach helps to refine and validate the first approach using a Finite Element Method (FEM) that discretizes Maxwell's equations.

3.3 Magnetic Circuit Representation

The flux paths for the device with no current, forward current, and reverse current are shown in Figures 3.2, 3.3, and 3.4 respectively. The red arrows represent magnetic flux, the thicker the arrow the larger the flux. From these flux paths a simplified magnetic circuit can be created as shown in Figure 3.5. Appendix A reviews magnetic terminology and the derivation of magnetic circuit equations. Also recall that an increase in flux density through the MRE causes an increase in stiffness as discussed previously. Therefore a forward current increases stiffness while a reverse current decreases stiffness.

Figure 3.2. No Current Applied to Device.
From Figure 3.5, and using KCL, a simple equation relating MRE flux density to the parameters shown in Figure 3.1 can be arrived at:
Expressions for the reluctances can be found in most introductory electrodynamics textbooks [37,38]. For completeness the reluctances are listed below (3.2). Typically in such a circuit analysis, reluctance in iron portions of the circuit can be neglected (i.e. \( R_p \) is ignored as long as the pole does not become saturated) if flux density levels are kept below saturation, since the permeability of iron is much larger than that of air or an MRE elastomer. In the design phase, caution should be taken to make sure the operating point of the magnet is not pushed below the knee of the demagnetization curve, leading to irreversible demagnetization, for the expected operating ranges of the control coil.

\[
B_{\text{MRE}} A_{\text{MRE}} = \phi_{\text{MRE}} = \phi = \frac{H_C L_M R_G R_L - NI (R_G R_L + R_M R_L + R_M R_G)}{R_G R_L R_{\text{MRE}} + R_M R_L R_{\text{MRE}} + R_M R_G R_{\text{MRE}} + R_M R_G R_L} \tag{3.1}
\]

In (3.2) Equations (3.1 and 3.2) point out an obvious practical problem with MRE based devices. The MRE reluctance, \( R_{\text{MRE}} \), appears only in the denominator, so that a large radial MRE length increases the MRE reluctance and therefore the amp-turns, \( NI \), needed to effectively operate the device between minimum and maximum stiffness or equally flux density. If the device is only used for short-term shock events this might not be a problem, but for most other applications where a larger coil duty cycle is needed to
achieve the necessary amp-turns a large coil current will be needed (leading to a lot of resistive losses) or a large coil volume will be needed allowing the current to be less but adding a significant amount of weight and size to the device. In contrast, one reason MR dampers have been so successful is that the optimum piston-wall gap is conveniently very small, while the rest of the circuit is all iron leading to a very low reluctance circuit, so that comparatively very little amp-turns are needed to provide the necessary field to saturate the MR fluid [8]. Nevertheless Equations 3.1 and 3.2 provide the relationships necessary to pick and iterate the variables until satisfactory MRE flux density is achieved while meeting other design goals to be discussed later.

3.4 GA Optimization with Magnetostatic FEM

The magnetic circuit equations given previously are useful for quick calculations but assuming that iron in the flux paths doesn’t saturate and that the leakage flux can easily be calculated can lead to erroneous results. In reality iron has significant reluctance as it approaches saturation and leakage flux paths can be very complicated. So a designer who wants to minimize weight/volume of a device has to be careful that the iron portions of the circuit have significant cross section so that they don’t approach saturation, but at the same time are not larger than they need to be. This is a difficult optimization problem because iron’s relatively permeability is non-linearly dependent on the amount of flux density passing through it, as illustrated in Figure 3.6, and leakage flux paths change as the iron approaches saturation not to mention complicated geometries are hard to simplify into a simple circuit relation.
Figure 3.6. Typical Relation Between Iron Reluctance and Iron Flux Density.

To improve the flux density calculations a static magnetic FEM model can be evaluated. Coupling the FEM program to a genetic algorithm (GA) will also provide a way of optimizing the weight/volume of the device while meeting magnetic, power, and physical constraints. The subsections to follow will outline the general optimization procedure. The optimization problem is also similar to what would be required in an MR damper optimization problem.

3.4.1 Mechanical and Non Mechanical Constraints

An important property of an isolator is its natural frequency for a given static load. The natural frequency can be expressed as shown in Equation (3.3).

\[
\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{\delta}}
\]  

(3.3)

Where \( \delta \) is the vertical static deflection of the mass when placed on the device shown in Figure 3.1. Equation (3.3) provides an upper limit for the static deflection for a given natural frequency lower limit. If the static deflection is equal to 100% shear strain, then
the maximum static deflection gives you the desired MRE radial thickness for the device. Further, to guarantee that the device operates in shear the radial thickness of the MRE element should be about one fourth the height of the MRE ($H_{MRE}$) in Figure 3.1, assuming the radial thickness of the MRE is small compared to the radius of the MRE ring (i.e. $(R_4 - R_3) << R_1$) [34]. Therefore by picking a natural frequency lower limit, and assuming small perturbations about the static deflection, you have equality constraints for the MRE radial thickness and height. Of course the exact load that causes the static deflection will also depend on the radius of the MRE ring radius $R_1$, but this can be left as a variable. For a vertical load the stiffness can easily be shown to be:

$$k = 2\pi G_{MRE} H_{MRE} \left( \frac{1}{\ln(R_4 / R_1)} + \frac{1}{\ln(R_3 / R_1)} \right)$$

(3.4)

The other constraints will be magnetic and electrical. All of the constraints are listed below:

1. $I^2 R \leq P_{\text{available}} \Rightarrow$ Resistive losses can’t exceed a given available power.

2. $B_{MRE} \geq 0.55T$ at zero input current (i.e. $I=0$). $\Rightarrow$ Bias flux density in MRE should be near MRE saturation point so that maximum stiffness is close to the operating point $I=0$.

3. $|B_{MRE}| \leq 0.1T$ at maximum current, $I = \left( \frac{P_{\text{available}}}{R} \right)^{1/2} \Rightarrow$ Flux density should near zero when maximum reverse current is applied.

4. $B_{\text{magnet}} \geq 0.15T \Rightarrow$ During no part of operation should the ceramic magnet approach the knee of its demagnetization curve.
5. \( R_2 - R_1 \approx R_4 - R_3 = \delta \Rightarrow \) Thickness of MRE should equal desired static deflection, as described above.

6. \( H_{MRE} = 4\delta \Rightarrow \) MRE height should be four times its thickness, as described above.

3.4.2 Problem Statement

![BMRE Schematic for Optimization](image)

Figure 3.7. BMRE Schematic for Optimization.

With the aid of Figure 3.8 the optimization problem is:

Variables:

\[ x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7] \]

Constraints:

1) \( I^2R \leq P_{\text{available}} \)

2) \( B_{MRE} \geq 0.55T \) at zero input current (i.e. \( I=0 \)).

3) \( |B_{MRE}| \leq 0.1T \) at maximum current, \( I = \sqrt{\frac{P_{\text{available}}}{R}} \).

4) \( B_{magnet} \geq 0.15T \)
Equality constraints of section 3.4.1 are used to eliminate variables from Figure 3.1. Even with equality constraints there are seven independent variables.

Objective Function:

Minimize $F(x): F(x) = \text{weight}(x) + P\text{(constraints)}$

Where a penalty function, $P$, is made from the constraints. Therefore the objective is to minimize the weight of the device, given available power and mechanical performance (desired natural frequency) requirements.

3.4.3 Optimization Solution Method

The problem in section 3.4.2 is solved with a Genetic Algorithm (GA), provided by Matlab's optimization toolbox. A flow diagram of the general solution procedure is shown in Figure 3.8.

There are several options for the stopping criteria. For this problem the stopping criteria was to keep track of the best fitness value of the population and end the optimization routine once a plateau or perceived minimum was reached. The minimum fitness value was monitored by looking at a real-time text file that Matlab created as the GA was running. It typically took about 15 generations for a minimum to be reached with a population of 500. A magnetostatic FEM program, FEMM, was used to evaluate whether or not a constraint was violated and if a penalty should be enforced in the fitness function evaluation [39]. See Appendix C for the Matlab Code.
3.5 Discussion

The GA optimization utilizing a magnetostatic FEM program is ideal, however the resulting best design is difficult to construct because it specifies a non standard permanent magnet size. For this reason, in this BMRE design, magnetic circuit equations were used since many of the dimensions became fixed by the use of standard permanent magnets available. After the basic dimensions of Figure 3.7 were arrived at, the design was checked with FEMM, and modifications were made. Table 3.1 lists the final dimensions of the BMRE device and some design values used. If non standard permanent
magnets were available the weight could have been reduced by about 30%. The weight could be reduced even further if one added more variables to the geometry to thin out sections of the design that were far from saturating, see Figure 3.9. Also note that the procedure described above along with much of the code in Appendix C could be used to optimize a MR fluid device.

Table 3.1. Slotted\textsuperscript{1} BMRE Final Dimensions and Design Values

<table>
<thead>
<tr>
<th>Dimension/Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_1)</td>
<td>15.2mm</td>
</tr>
<tr>
<td>(R_2)</td>
<td>17.4mm</td>
</tr>
<tr>
<td>(R_3)</td>
<td>18.16mm</td>
</tr>
<tr>
<td>(R_4)</td>
<td>20.3mm</td>
</tr>
<tr>
<td>(R_{im})</td>
<td>37.5mm</td>
</tr>
<tr>
<td>(R_{ig})</td>
<td>59.9mm</td>
</tr>
<tr>
<td>(R_{og})</td>
<td>68.6mm</td>
</tr>
<tr>
<td>(L_m=L_g)</td>
<td>12.0mm</td>
</tr>
<tr>
<td>(L_s=H_{mre})</td>
<td>9.53mm</td>
</tr>
<tr>
<td>(N)</td>
<td>500 turns of 24AWG</td>
</tr>
<tr>
<td>Resistance</td>
<td>8.2 ohms</td>
</tr>
<tr>
<td>(H_c) (Ceramic Magnet Coercivity)</td>
<td>291666 amp/meter</td>
</tr>
<tr>
<td>(\mu_{MRE})</td>
<td>2 (linear permeability)</td>
</tr>
<tr>
<td>(P) (Power Available)</td>
<td>(P &lt; 50W)</td>
</tr>
<tr>
<td>MRE B – Pole B (current)\textsuperscript{2}</td>
<td>0.6 T – 1.2 T (0 amps)</td>
</tr>
<tr>
<td>MRE B – Pole B (current)\textsuperscript{2}</td>
<td>0.0 T – 1.8 T (+1.25 amps)</td>
</tr>
<tr>
<td>MRE B – Pole B (current)\textsuperscript{2}</td>
<td>0.8 T – 0 T (-2.5 amps)</td>
</tr>
</tbody>
</table>

\textsuperscript{1} See Ch. 5 for a description of "Slotted" BMRE.
\textsuperscript{2} MRE Flux Density (B) is a estimated value based on actual pole flux density reading. See Ch. 6.
Figure 3.9. Optimum Design. Above design is optimum for the problem formulation. However if more geometry variables are added, the design could be improved, since areas near outside radius are far from the saturation value of iron (~2.0T).
4.1 Skyhook Damper

The skyhook damper concept is a term generally used when a passive, active, or semi-active element produces a force that is proportional to an absolute velocity. Considering the system shown in Figure 4.1, [12,19] found that to minimize the weighted mean square payload velocity and the mean square relative payload-base displacement a force, \( F_c \), involving the position and velocity states as shown in Equation (4.1) was optimal when the base moved with a white noise velocity. It's entirely possible, and hence the name "skyhook damper", that the passive system shown in Figure 4.2 can produce the optimum force, however in most environments an inertial reference frame is not available, and therefore some active actuator is often use.

\[
F_c = -b\ddot{x} - k(x - y)
\]

Figure 4.1. SDOF System. Objective is to minimize competing weighted objectives, relative displacement and payload acceleration, by optimizing \( F_c \).
The skyhook damping concept is also commonly used in other motion control objectives different from the one mentioned previously. As an example, to minimize the weighted mean square payload acceleration and the mean square relative payload-base displacement, a feedback force (using the relative position and absolute payload velocity states) of the form of (4.1) is again used. Although the values of $b$ and $k$ differ when compared to the previous problem. For instance, considering the performance indexes below in Equations (4.2) and (4.3), the optimum feedback force, $F_c$, can be found with a Linear Quadratic Regulator (LQR) for the system described by Equation (4.4) and Figure 4.1. Solving the Algebraic Riccati Equation (ARE) you arrive at the optimal force given in Equations (4.5) and (4.6) for a white noise disturbance. Notice the form of (4.5) and (4.6) is the same as (4.1) and could be produced by the passive system shown in Figure 4.2. Solving the LQR problem for a conventional passive system described by (4.3) and (4.7), arrives at the same solution as (4.6), except relative velocity is used as feedback instead of absolute payload velocity. Examination of (4.6) reveals that the optimum damping ratio is the often quoted $\xi = 0.707$. The transfer functions for the optimal
conventional passive system and optimal skyhook passive system, using (4.6), are shown in Equation (4.8) and (4.9) respectively, where \( \xi = 0.707 \) for both instances, and the choice of \( w_n \) depends on the ratio of \( c_2 \) to \( c_1 \). Transmissibility curves are shown in Figure 4.3.

\[
J_1 = \lim_{T \to \infty} \frac{1}{2T} \int_0^T \left[ c_1 \dot{x}^2 + c_2 (x - y)^2 + c_3 F_c^2 \right] dt \quad (4.2)
\]

\[
J_2 = \lim_{T \to \infty} \frac{1}{2T} \int_0^T \left[ c_1 \dot{x}^2 + c_2 (x - y)^2 \right] dt \quad (4.3)
\]

\[
\begin{bmatrix} \dot{x} - \dot{y} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x - y \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F_c + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \dot{y} \quad (4.4)
\]

\[
F_{c1} = -\left( \frac{c_2}{c_3} \right)^{1/2} (x - y) - \left( \frac{2m \sqrt{c_2 c_3}}{c_3} \right)^{1/2} \dot{x} \quad (4.5)
\]

\[
F_{c2} = -m \left( \frac{c_2}{c_1} \right)^{1/2} (x - y) - m \sqrt{2} \left( \frac{c_2}{c_1} \right)^{1/4} \dot{x} \quad (4.6)
\]

\[
\begin{bmatrix} \dot{x} - \dot{y} \\ \ddot{x} - \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x - y \\ \dot{x} - \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} F_c + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \ddot{y} \quad (4.7)
\]

\[
\frac{X}{Y} = \frac{2 \xi w_n s + w_n^2}{s^2 + 2 \xi w_n s + w_n^2} \quad \text{and} \quad \frac{X - Y}{Y} = \frac{s^2}{s^2 + 2 \xi w_n s + w_n^2} \quad (4.8)
\]

\[
\frac{X}{Y} = \frac{w_n^2}{s^2 + 2 \xi w_n s + w_n^2} \quad \text{and} \quad \frac{X - Y}{Y} = \frac{s^2 + 2 \xi w_n s}{s^2 + 2 \xi w_n s + w_n^2} \quad (4.9)
\]
Figure 4.3. Comparison of Optimal Transmissibility Curves. Skyhook system has superior high frequency characteristics.

4.1.1 Continuous MR Damper Skyhook Controller

Controllers for active and semi-active elements (such as real-time adjustable dampers) have been utilizing a skyhook damper like element for several decades now [19]. Figure 4.4 illustrates the skyhook damper concept for an MR damper. However since the MR damper is a passive device, in the sense it cannot add energy to the system, it is only able to closely match the skyhook damper force in certain situations, as shown in Equation (4.10). In contrast an active system could in theory always produce the desired optimal force by injecting energy into the system when necessary.

$$F_{MRdamper} = -C(u)(\dot{x} - \dot{y}) = \begin{cases} -b\dot{x}, & \dot{x}(\dot{x} - \dot{y}) > 0 \\ 0, & \dot{x}(\dot{x} - \dot{y}) < 0 \end{cases}$$

$$\Rightarrow C(u) = \begin{cases} -\frac{b\dot{x}}{\dot{x}(\dot{x} - \dot{y})}, & \dot{x}(\dot{x} - \dot{y}) > 0 \\ 0, & \dot{x}(\dot{x} - \dot{y}) < 0 \end{cases} \quad (4.10)$$
Payload Mass

K_{\text{passive}}

Controller chooses 'u' so that MR damper attempts to create skyhook damper force.

Figure 4.4. Schematic of Semi-Active MR Damper System.

An alternative MR Damper Skyhook Controller is shown in Figure 4.5 and described by (4.11). The controller is more difficult to implement however because relative position is now needed and the spring properties $K_1$ and $K_{\text{passive}}$ need to be known well, however better performance may result. This idea has been used in automotive suspensions systems, where softer springs than normal (i.e. $K_1 < K_{\text{passive}}$) are used because the adjustable properties of the MR Damper can compensate for the lack of stiffness when necessary, such as body roll in corners [40].

$$
C_{\text{test}} = \frac{(x - y)(K_{\text{passive}} - K_1) + \dot{x}b}{(\dot{x} - \dot{y})}
$$

$$
C(u) = \begin{cases} 
C_{\text{test}}, & C_{\text{test}} > 0 \\
0, & C_{\text{test}} < 0 
\end{cases} \tag{4.11}
$$
4.1.2 Continuous MRE Skyhook Damper Controller

Using a similar approach to the previously mentioned MR Damper Skyhook Controller, an MRE’s adjustable properties can be used to emulate the passive skyhook damper system. The idea is explained with the aid of Figure 4.6. As discussed in Ch. 3 the MRE physical properties can be described as a function of the flux density passing through the MRE as shown in Equation (4.12). If you estimate $h(B)$ and $w(B)$ in (4.12) as linear functions of MRE flux density, $B$, as in Equation (4.13), then a controller can vary the flux density to the MRE according to Equation (4.14) and (4.15). Where $\phi_{vol}$ is the volume fraction of the MRE that is iron, as discussed in Ch. 2.
Controller chooses 'B' so that MRE attempts to create skyhook damper system force.

\[ K = g(B) = K_{\text{rubber}} + h(B), \quad K \in [K_{\text{rubber}}, K_{\text{max}}] \]
\[ C = f(B) = C_{\text{rubber}} + w(B), \quad C \in [C_{\text{rubber}}, C_{\text{max}}] \]  

(4.12)

\[ h(B) = \rho_h B > 0 \quad \text{and} \quad w(B) = \rho_w B > 0, \quad B \in [0, B_{\text{sat}} \approx 2 \phi_{\text{vol}}] \]

where \( \rho_h = \frac{K_{\text{max}} - K_{\text{rubber}}}{B_{\text{sat}}} \), \( \rho_w = \frac{C_{\text{max}} - C_{\text{rubber}}}{B_{\text{sat}}} \)  

(4.13)

\[ B_{\text{test}} = \frac{-K_{\text{passive}}(x - y) - b\dot{x} + K_{\text{rubber}}(x - y) + C_{\text{rubber}}(\dot{x} - \dot{y})}{-\rho_h (x - y) - \rho_w (\dot{x} - \dot{y})} \]  

(4.14)

\[ B = \begin{cases} 
0, & B_{\text{test}} < 0 \\
B_{\text{test}}, & B_{\text{test}} \in [0, B_{\text{sat}} \approx 2 \phi_{\text{vol}}] \\
B_{\text{sat}}, & B_{\text{test}} > B_{\text{sat}}
\end{cases} \]  

(4.15)

4.1.3 On/Off MRE Skyhook Controller

Simulations of the MRE or MR skyhook controller will sometimes lead to the observation that the control input (i.e. flux density) is clipped at either zero or the saturation value, \( B_{\text{sat}} \), as outlined in (4.15). Therefore to simplify calculations and the
controller, an On/Off control scheme could be used [13]. From a different perspective, if you heavily weight the payload velocity (i.e. $c_i$ in (4.5)) then the skyhook damper force becomes much larger than the passive spring shown in Figure 4.2. This leads to the payload having a small absolute velocity at the expense of more relative displacement (notice this says nothing about the acceleration). If the skyhook damping term, $b$, is very large than (4.14) simplifies to Equation (4.16). If $b$ is large, then $B_{test}$ will almost always be less than zero, or greater than $B_{sat}$, causing (4.15) to become an On/Off type control. Therefore all we’re really concerned about in (4.16) is the sign, leading to the On/Off switching logic of (4.17) and (4.18). Equations (4.17) and (4.18) are simple enough that a simple analog circuit could be used for the switching logic. The only model dependent parameters are the maximum and minimum rubber properties. In an application a potentiometer could be used to adjust the $K_{max}-K_{rubber}$ to $C_{max}-C_{rubber}$ ratio until satisfactory performance is reached, similar to how PID gains might be adjusted by a technician.

$$B_{test} = \frac{b\dot{x}}{\rho_h(x-y) + \rho_w(\dot{x} - \dot{y})}$$ \hspace{1cm} (4.16)

$$A = \dot{x} \left[ \frac{\rho_h}{\rho_w} (x-y) + (\dot{x} - \dot{y}) \right] = \dot{x} \left[ \frac{(K_{max} - K_{rubber})}{(C_{max} - C_{rubber})} (x-y) + (\dot{x} - \dot{y}) \right]$$ \hspace{1cm} (4.17)

$$B = \begin{cases} 
0 & \text{(i.e. min K and C),} \quad A < 0 \\
B_{sat} & \text{(i.e. max K and C),} \quad A > 0
\end{cases}$$ \hspace{1cm} (4.18)

An alternative On/Off controller is possible by using (4.14) to calculate $A$ in (4.17) and then applying the switching logic of (4.18). However (4.14) is more difficult to calculate than (4.17) and strongly depends on material properties that could be nonlinear.
and time dependent. To minimize absolute acceleration (4.14) would likely be ideal, however for practical reasons (4.17) might be better.

4.2 Comparison of Continuous and On/Off Skyhook Controllers

The following sections compare the controllers previously mentioned with passive and active systems, when subjected to various base excitations.

4.2.1 Band Limited White Noise Input

A white noise band limited input from 1 to 30hz is used. The base acceleration input is shown in Figure 4.7. The systems used in the comparison are described in Table 4.1. Each system using a controller has a first order time constant of 1.5ms.

![Figure 4.7. Band Limited White Noise Input.](image-url)
Table 4.1. Systems Used in Figures 4.6 and 4.7. All semi-active systems have a first order
time lag of 1.5ms.

<table>
<thead>
<tr>
<th>System Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>Minimum stiffness and damping MRE. ( \xi = 0.213, f_n = 3\text{hz} ).</td>
</tr>
<tr>
<td>Stiff</td>
<td>Maximum stiffness and damping MRE. ( \xi = 0.707, f_n = 10\text{hz} ). This is equivalent to the optimal conventional passive system.</td>
</tr>
<tr>
<td>OPS</td>
<td>Optimal Passive Skyhook system. ( \xi = 0.707, f_n = 10\text{hz} ). This system could also be produced by an active system.</td>
</tr>
<tr>
<td>CSMRE</td>
<td>Continuous Skyhook MRE Controller. MRE is characterized by maximum values ( \xi = 0.707, f_n = 10\text{hz} ), and minimum values ( \xi = 0.213, f_n = 3\text{hz} )</td>
</tr>
<tr>
<td>OOSMRE1</td>
<td>On-Off Skyhook MRE Controller 1 using (4.14)</td>
</tr>
<tr>
<td>OOSMRE2</td>
<td>On-Off Skyhook MRE Controller 2 using (4.17)</td>
</tr>
<tr>
<td>SAD1</td>
<td>Semi-Active Skyhook Damper Controller 1. Passive spring gives ( f_n = 10\text{hz} ), and MR damper is able to adjust damping properties according to (4.10), except damping ratio, for a practical MR damper and system, is never allowed below ( \xi = 0.1 ), and the damping dynamic range ( (C_{\text{max}}/C_{\text{min}}) ) is 6.</td>
</tr>
<tr>
<td>SAD2</td>
<td>Semi-Active Skyhook Damper Controller 2. Passive spring gives ( f_n = 3\text{hz} ), and MR damper is able to adjust damping properties according to (4.11), except damping ratio, for a practical MR damper and system, is never allowed below ( \xi = 0.1 ), and the damping dynamic range ( (C_{\text{max}}/C_{\text{min}}) ) is 6. This range is representative of commercial MR dampers.</td>
</tr>
</tbody>
</table>

Figure 4.8. Passive and Active (OPS) Systems Response.
Figure 4.9. CSMRE System Response.

Figure 4.10. OOSMRE1 System Response.

Figure 4.11. OOSMRE2 System Response.
Figure 4.12. SAD1 System Response.

Figure 4.13. SAD2 System Response.

Figure 4.14. Comparison of SAD2 and OPS Responses.
Table 4.2. Maximum and RMS Values for Several Systems for a Band Limited (1-30hz) White Noise Input. Each system is ranked (1-8).

<table>
<thead>
<tr>
<th>System</th>
<th>RMS Acc (g)</th>
<th>Max Acc (g)</th>
<th>RMS Rel. Disp. (mm)</th>
<th>Max Rel. Disp. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>0.33 (4)</td>
<td>1.25 (3)</td>
<td>8.34 (8)</td>
<td>31.72 (8)</td>
</tr>
<tr>
<td>Stiff</td>
<td>0.45 (8)</td>
<td>1.75 (4)</td>
<td>0.71 (1)</td>
<td>2.41 (1)</td>
</tr>
<tr>
<td>CSMRE</td>
<td>0.33 (4)</td>
<td>1.80 (5)</td>
<td>2.69 (4)</td>
<td>11.13 (5)</td>
</tr>
<tr>
<td>OOSMRE1</td>
<td>0.32 (3)</td>
<td>1.92 (8)</td>
<td>3.88 (7)</td>
<td>17.39 (7)</td>
</tr>
<tr>
<td>OOSMRE2</td>
<td>0.35 (6)</td>
<td>1.77 (7)</td>
<td>2.52 (3)</td>
<td>10.76 (3)</td>
</tr>
<tr>
<td>SAD1</td>
<td>0.42 (7)</td>
<td>1.80 (5)</td>
<td>1.25 (2)</td>
<td>4.37 (2)</td>
</tr>
<tr>
<td>SAD2</td>
<td>0.28 (1)</td>
<td>0.98 (1)</td>
<td>3.05 (5)</td>
<td>11.15 (6)</td>
</tr>
<tr>
<td>OPS</td>
<td>0.29 (2)</td>
<td>0.97 (2)</td>
<td>3.11 (6)</td>
<td>11.05 (4)</td>
</tr>
</tbody>
</table>

4.2.2 Shock Input

In many cases a shock input can be the most destructive event for a payload. The shock input can impose maximum forces beyond the yield point of the payload materials causing irreversible damage. The typical approach is to use the softest isolator possible that still constrains the payload to a defined rattles space (i.e. a relative displacement constraint). The Table and Figures below show simulated results of a shock event for the systems mentioned previously. In the simulations a shock input is added to the white noise in the previous simulations.
Figure 4.15. Base Shock Input.

Figure 4.16. Passive and Active (OPS) Systems Response.

Figure 4.17. CSMRE System Response.
Figure 4.18. OOSMRE1 System Response.

Figure 4.19. OOSMRE2 System Response.

Figure 4.20. SADI System Response.
Figure 4.21. SAD2 System Response.

Figure 4.22. Comparison of SAD2 and OPS Responses.
Table 4.3. Maximum Values for Several Systems Subjected to a Shock Input.

<table>
<thead>
<tr>
<th>System</th>
<th>Max Acc (g)</th>
<th>Max Rel. Disp. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft</td>
<td>4.04 (2)</td>
<td>25.93 (8)</td>
</tr>
<tr>
<td>Stiff</td>
<td>42.47 (8)</td>
<td>5.86 (1)</td>
</tr>
<tr>
<td>CSMRE</td>
<td>7.25 (4)</td>
<td>16.21 (5)</td>
</tr>
<tr>
<td>OOSMRE1</td>
<td>25.75 (7)</td>
<td>19.49 (7)</td>
</tr>
<tr>
<td>OOSMRE2</td>
<td>9.84 (5)</td>
<td>14.04 (3)</td>
</tr>
<tr>
<td>SAD1</td>
<td>10.04 (6)</td>
<td>6.90 (2)</td>
</tr>
<tr>
<td>SAD2</td>
<td>5.58 (3)</td>
<td>15.88 (4)</td>
</tr>
<tr>
<td>OPS</td>
<td>2.36 (1)</td>
<td>16.23 (6)</td>
</tr>
</tbody>
</table>

4.2.3 Comparison Conclusions

From the Tables and Figures above it’s hard to pick an obvious optimum controller. What does stand out is that the Soft system has excessive sway (relative displacement) during normal driving conditions (band limited white noise input); and while it reduces transmitted shock acceleration, transients after the shock lead to large amounts of sway. Conversely the Stiff system does a good job of limiting sway, but tends to transmit a large amount of shock acceleration. The OPS system (or equally the optimum active system) is a good compromise to the passive systems. It provides superior shock acceleration isolation, even when compared to the Soft system, and does a good job of limiting sway during normal driving conditions, unlike the Soft system.

If shock was not an issue than the Stiff system would probably be the ideal solution when system complexity and cost is considered, but since shock is an issue the Stiff system is not ideal for electronic component survivability. The OPS is a fictional system, and the equivalent active system is too complex, costly, and heavy. The SA systems then
start to have some value. Tables 4.2 and 4.3 show that the CSMRE and SAD2 systems provide a good compromise when compared to the passive systems. In fact the SAD2 system closely approximates the performance of the active system for low frequencies near and at the vehicle dynamics, however because of bandwidth limitations; the SAD2 controller becomes less effective during a shock event but is still much better than the Stiff system. Another benefit of all the SA systems is that resonance is avoided. The Soft system, and to a lesser degree the Stiff system, has the potential of resonating. In vehicles this can be a serious problem because a vehicles suspension natural frequency tends to be below 4 hz, which is near the Soft systems resonant frequency [41]. Therefore the passive systems (particularly the soft system) could have significant sway if the white noise was filtered through the vehicle dynamics first.

4.3 Other Semi-Active Controllers

Several other Semi-Active controllers exist. Controllers based off of Lyapunov stability theory have been developed [18,50]. In order to meet the requirements of stability the Lyapunov controllers developed are usually no different than the skyhook controllers where the absolute payload velocity is heavily weighted. Meanwhile the development of the Lyapunov controller is more difficult. However the benefit of the Lyapunov controller is that stability is guaranteed.

Clipped Optimal Semi-Active controllers exist, which are closely related to the Skyhook Damper controllers. In optimal SA controllers the available feedback states are used in the LQR problem to find the optimal control law. Clipped Optimal Semi-Active control is often just an extension of the skyhook concept to higher order systems [19].
An instantaneous SA optimal controller was recently developed, an optimal predictive controller. Here the damping/stiffness values are changed in real time to minimize an objective function. The controller predicts the future value with a Taylor series expansion and changes the material properties accordingly to minimize the future value of the objective function. This controller is theoretically effective, but is very sensitive to modeling errors and controller saturation [17].
CHAPTER 5

TIME RESPONSE

5.1 Introduction

Controller and actuator bandwidth is a critical parameter in active as well as SA systems. In active systems insufficient bandwidth can lead to instability, and in SA systems it can lead to degraded performance as was seen in the shock simulations of Ch. 4. In MR systems the bandwidth is dependent on the ability to make rapid flux density changes in the MR fluid or elastomer. An MR device can therefore increase its bandwidth by decreasing the time it takes for the flux density to step from one flux density value to another, or in other words to decrease the response time, see Figure 5.1. To improve the response time of the BMRE device a flux feedback sensor is used and will be explained. The response time is further improved by cutting radial slots in the steel structures to reduce eddy currents.

Response Time = \(D_1 + D_2 + D_3 \approx D_1 + D_2\), (i.e. \(D_1, D_2 >> D_3\))

\[ \text{Response Time} = D_1 + D_2 + D_3 \]

\[ \text{Voltage} \rightarrow \text{Delay 1, D1} \rightarrow \text{Current} \rightarrow \text{D2} \rightarrow \text{Flux} \rightarrow \text{D3} \rightarrow \text{C and K Change} \]

Figure 5.1. Response Time Definition.
5.2 Flux Sensing and Feedback

Many devices using MR technology are controlled by an open loop. For instance, the Lord's MR Brake RD-2087-01 resistance is strongly related to the flux passing through the MR media [42]. During steady state operation the flux is dependent on the current passing through the electromagnet coil, if magnetic hysteresis is ignored. Therefore in most circumstances, MR devices are current controlled. A steady state relationship between coil current and MR Brake force is found and used in a controller. For small control bandwidths this is acceptable, but for higher bandwidths transient effects like eddy currents become significant and a simple relationship between coil current and flux does not exist. However the relationship between flux passing through the MR media and the MR properties still does exists for larger bandwidths. Therefore utilizing flux feedback will lead to better control of the MR properties [44]. An alternative feedback scheme is to feedback the MR force directly [8]. While this will lead to good results too, it is expensive and often cumbersome because of the need of a durable and accurate sensor. Flux sensors such as search coil loops or hall effect sensors are accurate, small, and inexpensive. Flux is typically measured in one of two ways 1) integrating the induced voltage in a search coil, from Faraday's Law of Induction, or 2) using a hall effect sensor.

5.2.1 BMRE Flux Sensor

The BMRE uses a search coil and a low drift analog integrator to sense the flux density in the pole. The flux in the pole is used as a measure of the flux in the MRE elements since nearly all the flux that passes through the MRE also passes through the pole, and it is impossible to wrap a search coil around the MRE elements, but it is easily done with the pole. Figure 5.2 shows a setup to verify that the pole flux density is an
accurate measure of the flux passing through the MRE elements (for more details about the experiment setup hardware see Ch. 6). In the experiment a fixed deflection is applied to the device causing the MRE elements to flex. Then a “reverse” current step input is applied to the control coil (see Ch. 3), causing the flux in the pole to decrease. The plot to the right in Figure 5.2 shows the transient response of the BMRE system to the current step. Notice that the pole flux lags the current step, but that the change in deflection force does not noticeably lag the flux change. This is expected since from Ch. 2 the magnetic field induced structural properties of the MRE are dependent on the flux passing through the MRE. The flux lags the coil current because of eddy currents.

![Diagram](image)

Figure 5.2. Response Time Experiment. A constant deflection is applied, while flux density through MRE element is varied.

5.2.2 BMRE Flux Feedback

Using the MRE flux, or a measure of it, as feedback to a controller is important because without knowing the non field-induced properties of the rubber (e.g. modulus, modulus vs. temperature, modulus vs. strain rate) you can accurately state that the gross MRE modulus monotonically increases with the absolute value of the flux passing
through the MRE, even if the exact MRE modulus value depends strongly on other factors. So if we have a controller that simply desires to switch between maximum and minimum stiffness, such as the OOSMRE controllers of Ch. 4, then flux feedback will be especially useful and not many details of the MRE properties are needed. However, if a detailed model of the MRE is developed and a more complicated controller is used, such as the CSMRE controllers from Ch. 4, then flux feedback will still be useful along with the other important MRE parameters (e.g. temperature, strain, strain rate, etc.), which will also need to be sensed. Note that all MR based devices have a similar relationship with flux, so that flux feedback can be applied to improve the performance of other MR devices also.

The pole flux is controlled with a simple PI or PID regulator. A schematic of the setup is shown in Figure 5.3. Figure 5.4 shows the improvement in response when using flux feedback for the same experiment setup shown in Figure 5.2. From the figure it was found that the response time – time needed to reach 95% of final value – was decreased by about a factor of 2. For convenience the PID regulator was implemented with digital hardware (Quanser Q8 DAQ board) running compiled C code obtained from a Simulink block diagram via Quanser’s WinCon software. A real-time OS (Ardence RTX) runs the C code. The PID regulator sends an analog command to a PWM current driver (Advanced Motion Controls 12A8). The 12A8 PWM, using a 65V bus voltage, had to be tuned by replacing a gain resistor on it’s on board analog PI current loop, since the model is designed for small DC motors with relatively small inductances.
5.3 Eddy Current Formation and Reduction

The fundamental reason for the flux lagging the control coil current is the generation of eddy currents. Maxwell’s equations govern the formation of eddy currents, and for materials with nonlinear permeabilities undergoing large changes of flux density, the relevant Maxwell equations are extremely nonlinear time-dependent PDEs. However, simply put, as you try to change the flux density in a solid conducting structure, large circular potentials are generated as a result of Faraday’s law of induction. These potentials generate circular currents which produce their own magnetic field opposing the applied field. The net result being that the change in flux density through the cross...
section is slowed down. The faster you try to change the flux density in the solid the stronger the eddy currents become in a nonlinear relationship. Flux feedback with the PID regulator results in the current driver (see Figure 5.3 and 5.4) applying large initial currents to the control coil in an attempt to compensate for the eddy current formation. However as the desired response becomes faster the current needed to compensate for the eddy current formation becomes exponentially large. The physical result is that an impractical large current driver is needed, and huge losses are created in the form of heat generated in the conducting structures of the MR device.

5.3.1 Reducing Eddy Currents with Radial Slots

Typically eddy currents are reduced with laminations, such as those seen on power transformers. The desired flux flow is in plane with the laminations, see Figure 5.5, so that the magnetic circuit reluctance does not increase, yet the non-conductive varnish between the laminations prevents eddy currents from forming. This allows transformers and electric motors to operate at high efficiencies in the presence of quickly changing magnetic fields. However laminations are difficult to apply to designs where the magnetic flux follows radial and axial paths. The reason is that the stacked disc laminations, like those in induction motors, allow flux to travel in radial directions easily, but inhibit axial paths. Ideally the laminations would be 'pie' shaped so that flux could travel easily in axial and radial direction. Pie shaped laminations however are not practical to manufacture. Other options to reduce eddy currents in the BMRE device would be to replace the steel components with ferrite. Ferrite is nonconductive and can carry large flux densities; however Ferrite has a much smaller permeability than steel so more magnetomotive force (i.e. amp-turns, coil current, permanent magnet) would be
needed to produce the same change in the MRE modulus. Also a ferrite device would require a high quality mold and press, which is beyond the budget for this work. Silicone steel could be used in the place of iron, which has more resistance than steel and thus smaller eddy currents, except that silicone steel has a lower magnetic saturation point than steel so that the size of the MRE device would have to be increased so that magnetic saturation is avoided.

Figure 5.5. Eddy Currents in a Transformer. Solid core allows eddy currents to form easily and forces flux to follow skin (outside) of core. Laminations reduce eddy currents, and while flux will still tend to follow the skin of the laminations, there are many more laminations and the penetration depth of the flux is close to the core of the lamination.

Another alternative to reduce eddy currents, and the one proposed in this work, is to cut thin radial slots in the steel components and fill the slots with a ferrite like material. If the slots are thin enough, such as can be done with a laser or water jet, the reluctance of the steel paths won’t change much. Also radial slots will not affect axial or radial flux paths; however the radial slots will prevent the formation of circular eddy currents. Figure 5.6 and 5.7 shows a 3D FEA harmonic simulation of the BMRE device with and without slots for a 50 Hz 1000 amp-turn control coil input. Notice that the slotted design, for the exact same control coil current input, better distributes the flux. While the solid
design pushes the flux to the outside edges as a result of eddy currents. A caveat to the simulations is that typically harmonic magnetic FEA assume/require that the materials have linear relative permeabilities [39]. This is an acceptable assumption if no part of the device in the simulation approaches saturation. However if a large part of the device does approach saturation (such as the outside surface of the pole in Figure 5.6) in the simulation, than the results will give flux densities that are larger than can actually be expected in operation. Therefore the MRE flux density estimated in Figure 5.6 is overestimated as a result of the pole surface flux density being greater than 2.0T which is the saturation upper limit of iron.

Figure 5.6. Solid BMRE Harmonic FEA Results. Notice how flux is pushed towards pole surface (red circle), causing the pole to saturate.
5.3.2 Slotted BMRE

Another BMRE demonstration device was built to the specifications of the previous unit, except that radial slots, via a water jet process, were cut into the device. The geometry of the slots is the same as the simulation in Figure 5.7 (i.e. 32 slots were cut on the disks and 8 slots on the pole). The pole slots were filled with an improvised ferrite mixture made from a two-part epoxy (Devcon 5 Minute Epoxy) mixed with iron particles (ISP-3700). This smoothes the poles surface so the MRE has something to bond to and it also helps improve the poles ability to carry flux despite the slots. The two disks were not filled with an epoxy since they, even with the slots, are nowhere near magnetic saturation. The non slotted and slotted BMRE devices are shown for comparison in Figure 5.8. The response time setup shown in Figure 5.2 was repeated for the slotted and non slotted BRME. Both tests used flux feedback. Figure 5.9 shows the results of the tests.
Figure 5.8. Comparison of BMRE Designs. Slotted design to right has 32 slots on bottom and top plate, and 8 slots on pole. Pole slots were later filled with an epoxy-iron mixture.

Figure 5.9. Comparison of Non Slotted Design to Slotted Design. Slotted design not only has a faster time response but wastes less energy since less eddy currents are formed.
CHAPTER 6

EXPERIMENTAL VIBRATION AND SHOCK ISOLATION RESULTS

6.1 BMRE Device Characteristics

Some preliminary tests had to be performed with each BMRE device to find specific operating points. Specifically, since the device has a bias field, the steady state control coil current needed to cancel the bias field had to be experimentally determined since the exact bias field value is not known and can't be measured directly. To find this current a few passive tests were made with the device excited horizontally by a shaker (APS Model 113). The tests consisted of a chirp input (sine sweep) from 2 to 100 Hz. A few tests were run with the coil current set at different levels. The test setup and a typical acceleration transmissibility plot are shown in Figures 6.1 and 6.2. From these tests you can tell at what coil current the MRE becomes its softest, or in other words at what coil current the MRE flux was reduced to zero. The change in pole flux density was recorded at this steady state current level. Depending on the particular unit the change in pole flux density needed was about 1.2 to 1.3T. The control coil current was around 1200 to 1400 amp-turns, up to this level the resonant frequency decreases. Naturally if the ~1300 amp-turn value is exceeded the resonant frequency starts to increase again as a result of the MRE flux passing through zero and flowing in the opposite direction of the bias field. The pole flux density change was measured with a commercial integrating flux meter (Walker MF-3D) for high accuracy.
Figure 6.1. Setup and Functional Schematic. The payload is 5lbs.

Figure 6.2. Typical BMRE Passive Acceleration Transmissibility Curves.

All of the results presented in this chapter are for the BMRE excited horizontally. The device was mounted horizontally because the APS shaker is only able to produce a 40 lb force up to 20 Hz at which point the force available drops off sharply due to limitations
of the shaker amplifier (APS Model 114-EP). To bring the BMRE device resonant frequency below 20 Hz a 50 lb payload would be required, which would exceed the recommended static vertical load of the shaker. For the horizontal tests, the payload was between 5 and 10 lbs, and the moment arm varied depending on the unit, but was typically around 3 or 4 cm.

6.2 BMRE Controller and Test Setup

A simple switching controller is used to gain some insight into the potential Semi-Active capabilities of the BMRE device and MR elastomers in general. A switching controller was decided on for simplicity, considering that a more complicated controller would have needed an accurate model of the elastomer. The passive tests showed that the MRE had a lot of non linear properties, more so than a typical elastomer because of the non linear magnetic interactions discussed in Ch. 2. For these reasons the OOSMRE1 controller was decided on since only one parameter is needed. This parameter can be tuned to gain satisfactory response, and a system model is not necessary. A schematic of the experimental setup is shown in Figure 6.3. A few comments about the setup:

1) The search coil is looped around the pole as described in Ch. 4, it consists of ten loops. A search coil is not an ideal choice for a feedback signal since DC drift is a significant problem, and can’t be filtered out since the DC value is important. However the tests were usually a minute or less so that drift was not a problem. In a practical application a cleverly placed hall element could replace the search coil sensor. The search coil induced voltage is integrated with a custom built low-drift analog integrator instead of using the Walker MF-3D unit. This is because the
current driver used to supply the control coil would couple a lot of noise on the search coil and this was carried through to the MF-3D output signal. The reason for this is unknown. The MF-3D is an analog integrator, which should have acted like a low pass filter and eliminated the PWM noise from the current driver. The custom analog unit did not have this problem.

2) For diagnostic reasons a current probe (PDI CA60) was used to monitor the control coil current.

3) The high pass (HP) filters were 1st order filters with a cutoff frequency of about 0.5 hz, the filtered signal was then digitally integrated to obtain velocity and position states.

4) For convenience the HP filters, OOSMREI controller, and the PID regulator were implemented with digital hardware (Quanser Q8 DAQ board) running compiled C code obtained from a Simulink block diagram via Quanser's WinCon software. A real-time OS (Ardence RTX) runs the C code.

5) The OOSMREI controller also includes an anti-chatter provision, where the absolute value of A from Equation 4.17 has to be greater than a chatter value (CV). If the absolute value of A is not greater than CV then the control coil current is set to zero amps.

6) The digital hardware and controller isn’t necessary but is used, as mentioned, for convenience. This is a benefit of the using flux feedback and a simple controller such that complicated calculations aren’t needed, as opposed to other attempts in this area [44]. It would not be difficult to design a complete analog controller, leading to a fast inexpensive control system.
7) The Simulink Block diagrams used to compile the run time C code can be seen in Figure 6.4 and 6.5.

Figure 6.3. Functional Block Diagram for Experimental Setup.

Figure 6.4. Simulink Block Diagram. Diagram is converted to executable C code by WinCon.
6.3 Controlled BMRE with Chirp and Harmonic Disturbances

The first base motion disturbance looked at was a chirp, or swept sine input, from about 2 to 25 Hz. Above 25 Hz the BMRE time response becomes a problem, which will be discussed in more detail later. A frequency response plot is shown in Figure 6.6.
A caveat with frequency response analysis is that it is intended primarily for linear systems. As has been previously mentioned the MRE is not linear, but setting that aside the OOSMRE1 controller is extremely nonlinear. The result is that for a harmonic disturbance the controlled system outputs higher order harmonics. For instance, excite the BMRE structure at the resonant frequency of the soft system (i.e. the MRE with no flux flowing through it), and take the Fourier transform. Figures 6.7 and 6.8 show the frequency response spectrum for the soft system and the controlled system. Figures 6.9 and 6.10 show the steady state time domain results for the soft and OOSMRE1 controlled systems. Notice from Figure 6.8 that while the controlled system does a good job of limiting the resonance, it also inserts a harmonic at twice the forcing disturbance. This is a result of the OOSMRE1 switching at a rate of twice the disturbance, which means for the controller to be effective it must have a bandwidth of twice the disturbance. Hence the reason the controlled BMRE performance tapers off at disturbances above 25 Hz - recall from Ch. 5 that the response time of the BMRE is about 5ms.

![Frequency Magnitude Spectrum of Passive Soft System](image)
Figure 6.8. Frequency Magnitude Spectrum of Controlled System.

Figure 6.9. Time Response Plot for Passive Soft System (no flux passing through MRE).

Figure 6.10. Time Response Plot for Controlled System.
6.4 Controlled BMRE with Band-Limited White Noise Disturbance

In this experiment a band limited disturbance was used, the PSD of the input is shown in Figure 6.11. Figure 6.12 shows the controlled BMRE response. Table 6.1 compares performance values of the passive systems to the controlled system. The results, of course, are not general and depending on the base input one system may perform better than the others.

![Figure 6.11. Base Input PSD for Band-Limited White Noise Excitation.](image1)

![Figure 6.12. Controlled Transient Response to White Noise Base Input.](image2)
Table 6.1. System Response Values for Base PSD Shown in Figure 6.11.

<table>
<thead>
<tr>
<th>System (current)</th>
<th>RMS Acc (g)</th>
<th>Max Acc (g)</th>
<th>RMS Rel. Disp (mm)</th>
<th>Max Rel. Disp (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft (-2.5 A)</td>
<td>0.29 (1)</td>
<td>1.36 (3)</td>
<td>5.34 (3)</td>
<td>16.5 (3)</td>
</tr>
<tr>
<td>Hard (+1.25 A)</td>
<td>0.36 (3)</td>
<td>1.25 (1)</td>
<td>4.85 (1)</td>
<td>13.8 (1)</td>
</tr>
<tr>
<td>Controlled</td>
<td>0.29 (1)</td>
<td>1.26 (2)</td>
<td>5.13 (2)</td>
<td>14.2 (2)</td>
</tr>
</tbody>
</table>

6.5 Controlled BMRE with Shock Disturbance

The last disturbance looked at is a shock input. The test rig used is shown in Figure 6.13. The plunger is pulled back to a repeatable position and released. A typical result from the test is shown in Figure 6.14. There were several problems with the shock tests. Primarily it was impossible to consistently produce a consistent shock input, even with the plunger pulled back to the same starting point. Also it was hard to tell a difference between the soft, hard, and controlled system performance as far as shock acceleration isolation is concerned, this may be due to the hardware limitations (i.e. sampling rate and sensor bandwidth) or the hot glue used to attach the accelerometers to the payload and base.

Some useful results were found nevertheless:

1) The SA controller behaved as expected, that is, decreasing the stiffness at the onset of the impact and switching the stiffness to control transient motions after the impact had passed, see Figure 6.14.

2) Tuning the controller such that it quickly responded to the shock impacts, but wasn’t so sensitive that it chattered excessively between maximum and minimum stiffness resulted in a lower chatter value (CV), and a slightly more aggressive high pass filter.
than the white noise base input tests (cutoff is 1hz for shock tests, and 0.5hz for white noise input).

Figure 6.13. Shock Test Rig.

Figure 6.14. Shock Test Results for SA Controlled BMRE. PI controller for flux feedback should have been tuned better.
6.6 Discussion

In the above experiments a simple controller (OOSMRE1) is used to control the MRE flux density that was optimal for payload velocity mitigation, but not necessarily for payload acceleration isolation. Regardless the results were interesting. The harmonic tests, Figures 6.7-6.10, showed that there is a significant improvement in resonance control by switching the MRE flux between low and high states. Also the white noise disturbance test, Figure 6.12 and Table 6.1, showed promise in using a MRE for broad band isolation. There is no doubt that MREs have potential in SA vibration isolation systems. The challenge is finding the right application where the design constraints of the MRE based device will not be an issue (i.e. limiting the thickness of the MRE so that large amp-turns are not need as discussed in Ch. 3) and developing an appropriate controller which may, but preferably not, involve developing an accurate dynamic model of the MR elastomer.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

This study explained the shortcomings of current linear passive technology for combined shock and vibration isolation. Specifically it was shown that competing design goals, limiting sway and high shock isolation, can’t be met with passive systems. An ideal active system (or equally a fictitious skyhook system) was shown to provide the best performance in light of the design goals.

Because of the practical problems of implementing active systems, SA systems were proposed as an alternative. A MRE based device was developed in response to the commercial success of MR dampers. The MRE device has the unique ability of varying it’s stiffness in response to an externally applied magnetic field. To exploit this property, SA control schemes were investigated to isolate a payload from an excited base. This work concluded by mounting a payload to a custom built MRE device and exciting the base of the system with various disturbances. The SA properties of the device were then utilized by applying a simple SA controller. The results were encouraging, showing that the SA controlled device significantly reduced resonance when compared to the same device in a passive mode.
7.2 Future Work

The simulations of Chapter 4 suggest that the SAD2 system has much potential if the MR damper can be controlled accurately and quickly. Current literature has shown that MR damper steady state properties can be modeled very accurately. The problem is extending this model to higher bandwidths. Currently most researchers develop a linear model, or use a look-up table, relating coil current to MR properties [8]. Others, realizing that a linear model isn’t appropriate for high bandwidths or precise control have explored complicated models to account for hysteresis and other nonlinear effects [44]. The models however are not very accurate.

Therefore extending flux feedback to MR devices should result in a noticeable improvement in not only MR response time but accurate control of the MR property (i.e. the yield stress of the fluid). Controlling the MR property accurately and quickly opens MR devices to other applications such as torque control [44] for robotics and improving other current applications such as a automotive MR dampers ability to handle potholes or other rapid inputs [43].
APPENDIX A– MAGNETIC TERMINOLOGY

A.1 Sources

The following was gathered primarily from [45,37,38]. [45] provides the clearest explanations and would definitely be a good starting point for someone beginning to study magnetic materials. [37] provides a very good overview of modern materials and fundamental equations necessary for design and numerical methods. [38] is a comprehensive electrodynamics textbook.

A.2 Magnetic Moment and Interaction Energy

The most basic unit of magnetism is the Magnetic Moment $\vec{m}$. Which can be defined in two equivalent ways:

$$\vec{m} = p\vec{l} = iA\hat{n} \quad (A.1)$$

where $p$ is the strength of the poles separated by a distance $l$ and in the second expression $i$ is a current in amps and $A \ [m^2]$ is the area of the current loop with an outward normal unit vector. The first expression was commonly used before computers because simple calculations could be performed. The second term is the basic description used today in electromagnetic calculations utilizing Maxwell’s equations.

Since opposite poles are attracted to each other, the first expression in (A.1) along with Coulombs laws of magnetism can be used to define the potential energy (or Interaction Energy) of two dipoles. The derivation is common in introductory books in electrodynamics [38].
A.3 Magnetization and Induction Curves

Ferromagnetic materials are composed of Magnetic Domains which are tiny volumes of $\sim 10^{12}$ atoms. Each atom has a magnetic moment associated with it and in a domain all of these moments have the same direction. Ferromagnetic materials can then be classified as either a Soft Material, when the domains are randomly oriented in the material producing a net moment of zero in zero applied field, or a Hard Material (permanent magnets), when the domains have a ordering that produces a net magnetic moment in zero applied field. As the name suggests pure iron is a soft ferromagnetic material while steel could be considered a mild hard ferromagnetic material. However alloying iron with rare earth elements (a discovery made in the early 70's and improved significantly in the 80's) produces an extremely hard ferromagnetic material. Another hard material is ferrite $\text{Fe}_3\text{O}_4$, which is commonly used in refrigerator magnets.

Terms common to both hard and soft magnetic materials can be explained with the aid of the Magnetization $\vec{M}$ and Induction (or commonly flux density) $\vec{B}$ curves shown in Figures A1 and A2. Basically, as a Magnetic Field $\vec{H}$ is applied to a ferromagnetic material the domains in a direction close to that of $\vec{H}$ grow (domain wall growth) at the expense of domains in non favorable directions, the vector sum of these domains is the materials magnetization. Once the field is removed in soft materials, the domains become randomly oriented again (because this is a state of minimum energy) and the net $\vec{M}$ becomes zero again. In hard materials the process is more complicated. Alloying materials are added that inhibit domain wall growth and also produce a crystalline structure that allows for the material to have domain directions in only one direction (magnetocrystalline anisotropy). This is the reason for the square appearance of the
magnetization curves for hard materials. Once the Intrinsic Coercive Force $H_c$ is reached the domains flip $180^\circ$ in direction. A good introduction to modern permanent magnetic material science can be found in [Campbell].

The Magnetic Saturation $\vec{M}_s$, is defined as the point where any further increase in the applied field $\vec{H}$ results in no increase to the materials magnetization and only a proportional increase to induction, where:

$$\vec{B}(\vec{H}) = \mu_0(\vec{H} + \vec{M}(\vec{H})) \ [T]$$  \hspace{1cm} (A.2)

then as $\vec{M} \rightarrow \vec{M}_s$ the induction becomes

$$\vec{B}(\vec{H}) = \mu_0\vec{H} + \mu_0\vec{M}_s$$ \hspace{1cm} (A.3)

Figure A1. Hard Magnetization Curve. Typical of a hard ferromagnetic material. To visualize $B$ over other quadrants use equation (A.2) and the figure above.
The induction above is expressed in SI units of Tesla [T], and $\mu_0$ is the Permeability of Free Space with a value of $1.25 \times 10^{-6} \frac{mkg}{s^2 A^2}$.

Material Polarization $\vec{J}$ then is simply the magnetization of the material multiplied by $\mu_0$.

$$\vec{J} = \mu_0 \vec{M} \quad \text{or as} \quad \vec{M} \to \vec{M}_s \quad \vec{J}_s = \mu_0 \vec{M}_s \quad (A.4)$$

Magnetization as shown in (A2) is a function of $\vec{H}$ until saturation is reached, where until saturation is reached:

$$\vec{M}(\vec{H}) = \chi \vec{H} \quad (A.5)$$
The simplicity of the above equation is misleading. As Figure A.2 shows, the Magnetic Susceptibility $\chi$ is a complicated function of $\vec{H}$, and for hard materials it is history dependent. The Relative Permeability $\mu_r$ and the Material Permeability $\mu$ are defined as follows:

$$\vec{B}(\vec{H}) = \mu_0(\vec{H} + \vec{M}(\vec{H})) = \mu_0(1 + \chi)\vec{H} = \mu_0\mu_r\vec{H} = \mu\vec{H} \quad (A.6)$$

The intrinsic coercive force $H_{ic}$ is the applied field needed to remove all magnetization (i.e. $\vec{M} = 0$). In soft materials $H_{ic} \approx 0$. A large value of $H_{ic}$ (where it is implied $H_{ic}$ is in the reverse direction of the current magnetization) is an important quality factor for permanent magnets. Alnico magnets (prevalent pre WWII) had a very low $H_{ic}$ and demagnetization was common if the magnets were not handled properly. Recent rare earth magnets have a very large intrinsic coercive force value so that they can made in flat shapes, like those used in computer hard drives, and can be placed in close contact to opposing fields without the risk of demagnetization. Remnant Magnetization $\vec{M}_R$ is the magnetization remaining after the applied field has been removed.

A.4 Magnetic Circuit Analysis

Magnetic circuit analysis is analogous to electric circuit analysis and Kirchoff’s voltage and current laws. Where voltage is replaced by Magnetomotive Force $mmf$ and current is replaced with Magnetic Flux $\phi$. Magnetic circuit equations are used to estimate static or slowly changing flux densities in devices. The magnetic circuit method can be derived from the Ampere Circuit Law and Gauss’s Law for magnetism:
\[ \int H \cdot dl = i \]  \hspace{1cm} (A.7)

\[ \phi_{\text{Total}} = \sum_{i=1}^{n} \phi_i \]  \hspace{1cm} (A.8)

where the \( n \) in (C.8) represents the number of paths the flux can take.

![Diagram of a magnetic device and equivalent magnetic circuit.](https://example.com/diagram.png)

**Figure A3. Magnetic Device and Equivalent Magnetic Circuit.**

As an example of the use of (A.7) and (A.8) consider Figure A3. Using equation (A.7) you arrive at:

\[ H_{\text{Steel}} L_{\text{Steel}} + H_{\text{Gap}} L_{\text{Gap}} = NI \]  \hspace{1cm} (A.9)

From (A.6) you have:

\[ \mu_{\text{Steel}} \mu_0 H_{\text{Steel}} = B_{\text{Steel}} = \frac{\phi_{\text{Steel}}}{A_{\text{Steel}}} \quad \text{and} \quad \mu_{\text{Gap}} \mu_0 H_{\text{Gap}} = B_{\text{Gap}} = \frac{\phi_{\text{Gap}}}{A_{\text{Gap}}} \]  \hspace{1cm} (A.10)

Also from (A.8) and Figure A3:

\[ \phi_{\text{Steel}} = \phi_{\text{Gap}} \]  \hspace{1cm} (A.11)

Solving for the gap flux with (A.9-11):
\[ \phi_{\text{Gap}} = \frac{NI}{R_{\text{Gap}} + R_{\text{Steel}}} \]  

(A.12)

Where the Reluctances R are given as:

\[ R_{\text{Gap}} = \frac{L_{\text{Gap}}}{\mu_{\text{Gap}} \mu_0 A_{\text{Gap}}} \quad \text{and} \quad R_{\text{Steel}} = \frac{L_{\text{Steel}}}{\mu_{\text{Steel}} \mu_0 A_{\text{Steel}}} \]  

(A.13)

Generally the reluctances can have a more complicated form, for instance if the flux traveled in radial directions.

A.5 Inductance and Nonlinear Time Delays

Inductance (L) is typically thought of as being constant in most electromechanical systems. However for certain high performance systems this is not a good assumption. Take for instance the simple magnetic circuit shown in Figure A3. The governing equation for the coil current is:

\[ V_{\text{bus}} = RI + \lambda \]  

(A.14)

where \( \lambda \) is the flux linkage for the coil given by:

\[ \lambda = N \phi \]  

(A.15)

where \( N \) is the number of coil loops and \( \phi \) is the flux passing through the coil. From (A.12) we know what the flux passing through the coil is such that:

\[ \lambda = N \phi = N \frac{NI}{R_{\text{total}}} \text{, where } R_{\text{total}} = R_{\text{Gap}} + R_{\text{Steel}} \]  

(A.16)

which leads to, in this example, the definition of the self inductance, L:

\[ \lambda = N \phi = \frac{N^2}{R_{\text{total}}} I = LI \]  

(A.17)
Now (A.17) says that if the total magnetic circuit reluctance is constant (i.e. the self inductance is constant) then coil current and magnetic flux will be in phase, and that (A.14) is a simple linear 1st order ODE. This is true in many devices like air core inductors, or laminated electric motors running under stated operating limits. For MR systems this has significant meaning, because if you have such a simple relationship between coil current and magnetic circuit flux, then you can accurately control the properties of the MR device (which are dependent on magnetic circuit flux) by simply controlling the coil current, similar to how the torque might be controlled in a DC motor. However from (A.17) and (A.13) it is easy to see that if the steel components in the flux path near saturation then the self inductance will change (as a result of the relative permeability approaching one) and there won’t be a simple linear relationship between current and flux. Therefore (A.14) won’t be a linear ODE. Usually in MR devices this is the case, since MR devices typically deal with large flux densities that push the materials towards saturation. This is also true of electric motors if the rated armature coil current is far exceeded.

Now if eddy currents are not present, then even with saturation, the coil current and magnetic flux will be in phase. However if significant eddy currents are generated, as a result of Faraday’s Law of Induction, then not only will (A.14) be non linear it will also become a PDE as a result of the flux linkage becoming time dependent. To make matters more complicated (A.12) will not hold, as a result of Faraday’s Law of Induction (A.18):

\[ \oint E \cdot d\ell = \dot{\phi} \]  

(A.18)

When eddy currents are significant and have to be considered, Maxwell’s equations have to be solved numerically [39]. Only in certain simplified cases can eddy currents be
treated analytically [46,47]. From (A.18) it is obvious that as the rate of flux change is increased, larger and larger circular potentials are generated, which will generate large eddy currents in solid conducting structures. The eddy currents then create their own field opposing the applied field. Some insight can be drawn from the simplified electrical schematic in Figure A4. Note that the inductance and eddy resistance are not constant, but instead our dependent on voltage frequency and the flux density in the magnetic core.

![Simplified Electrical Schematic of Figure A.3.](image)

Figure A4. Simplified Electrical Schematic of Figure A.3. Current and flux are not in phase as a result of $R_{\text{eddy}}$.

A.6 What does all of this mean?

It means that if flux values are important to know, to say control the stiffness of a MRE or the damping of a MR damper, then you can’t rely on open loop equations to calculate these flux values if:

1) The flux density values change rapidly and/or

2) The flux density values approach the materials saturation value.

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In high performance applications 1) and 2) will likely be true, so that to accurately control the MR properties you have no choice but to attempt to measure the flux values directly.
APPENDIX B – BMRE FABRICATION TIPS

B.1 Steel Components

All steel and aluminum components were modeled in SolidWorks and fabricated with a CNC machine or a manual lathe or mill.

B.2 Coil Components

Coil components were hand wound with the aid of a lathe turning at about 40 rpm. The coils are free formed, meaning that the finished coils do not have a bobbin. The general procedure is as follows:

1) A temporary two-piece bobbin is made, typically from aluminum.

2) The bobbin is then covered in foil so that the magnet wire does not directly contact the bobbin.

3) The exposed foil is lightly wiped with a cloth wetted with silicone oil.

4) A few loops of magnet wire are wrapped around the bobbin.

5) The lathe is turned on so that the magnet wire begins to wind around the bobbin. As the magnet wire is pulled to the bobbin it is passed through a piece of cardboard that has been saturated in epoxy (Devcon 5 Minute Epoxy). The epoxy allows the coil to harden and retain the form of the bobbin.

6) After sufficient turns have been wound the lathe is turned off and the epoxy is given time to harden.
7) After the epoxy has hardened the bobbin is disassembled and the freeform coil is removed.

8) To help the coil retain its shape in the event of severe thermal cycling a few wraps of fiberglass tape can be used.

B.3 MRE Components

The MRE components are the most difficult part to fabricate. The fabrication can be broken down into two parts: 1) formulation and mixing of the MRE rubber compound and 2) molding the MRE compound.

B.3.1 MRE Formulation and Mixing

All the MRE compounds used in this work were two-part room temperature vulcanizing (RTV) based compounds, particularly Dow’s HS line of silicones. RTV rubber is easy to work with, and doesn’t require expensive molds like many heat activated sulfur and peroxide based cures. The formulation process is relatively straightforward:

1) First a RTV compound is chosen, preferably one with a low shear modulus (below 0.1MPa) so that the magnetic dipolar effect of the iron particles will be large in comparison (see Chapter 2).

2) Then various fillers are chosen to alter the RTV compound properties. For instance adding silicone oil helps to decrease the compound modulus. Adding fillers, amorphous silica or iron particles, increases the modulus. In this work iron particles (ISP-3700) were added so that the volume of the cured rubber was about 30% iron. The amount of RTV and silicone oil is experimented with so that the final cured
rubber had a low shear modulus, about 0.1MPa. It helps to develop a spreadsheet containing information like desired volume by part, and the density of each ingredient. The volume quantities can then be converted to weight units which can be easily measured out with a scale.

3) Mixing the formulation is usually done by adding small portions of the iron particles to the silicone RTV. The RTV and iron are mixed until the iron particles are completely dispersed, then more iron is added. When it becomes difficult to disperse any more iron then add some of the measured quantity of silicone oil to the mixture. Once all the iron has been dispersed any remaining silicone coil is added and mixed. For mixing small quantities a mortar and pestle do a good job. The pestle breaks up iron conglomerates, helping to achieve good dispersion.

B.3.2 MRE Molding

Since an RTV compound does not require heat or pressure to cure properly a simple mold can be used. However, unlike most rubber compounds, a magnetic aligning field is required with MRE compounds so that the magnetic effect of the iron particles is maximized. This involves incorporating a electromagnet into the mold, such that the electromagnet produces flux lines that pass through the MRE in a desired path. Figure B1 shows a schematic of the mold used for the BMRE device.
For the BMRE device two molding steps were needed. First the inner ring was molded, while an outer aluminum ring held steel mount centered around the pole. After the inner ring was cured the outer aluminum ring was removed and a outer MRE ring was molded. During each molding process the electromagnet was energized so that the particles in the rubber were aligned in a radial matter, so that the iron magnetic effect was maximized. Also it should be noted that the steel surfaces, where the MRE was molded to, were treated with a bond promoter (Dow 1200 OS Primer). Without the bonding agent the cured MRE would peel off the steel surfaces relatively easily.

B.4 Custom Voltage Integrator

A custom integrator with low DC drift was built to integrate the induced voltage in the BMRE pole. Usually voltage integrators contain a high pass filter to avoid DC drift, but the DC component is often the value of interest in search coil measurements. Commercial
magnetic flux integrators obtain low drift by using chopper stabilized op-amps, or use fast sampling digital integration. However for short term integration periods of a few minutes or less high quality analog components can be used instead [48].

Figure B2. Low Drift Analog Integrator. Adjust potentiometer to eliminate DC drift. Voltage follower should be adjusted to eliminate voltage offset (for instance use a 741 op amp). Voltage supply rail should be regulated (for instance use a battery powered rail with 7812 and 7912 regulators).
APPENDIX C – MATLAB OPTIMIZATION CODE

C.1 Overview

Matlab’s optimization toolbox, in some versions, includes a genetic algorithm function, ga(arguments), which relieves the programmer from developing their own routine. The ga function can be accessed via a graphical window by typing ‘gatool’ or through a Matlab script which was used in this work. The options to the ga() function can then be included in the script as variables. Type ‘help ga’ in the Matlab command window for more information on the ga() function and to reverse engineer the scripts listed in the sections to follow.

C.2 Initialization Script

The following script calls the ga() function with the appropriate options and specifies a fitness function to use. The fitness function is the ‘meat’ of the program and includes calls to the magnetostatic FEM program.

```
% **** Initialize GA Optimization ****

%Range first population will be picked from
initRange=[.01 .1 .3 .4 .1 .1 .001 ;.1 .7 1.54 1.52 1]

%non default options for the ga function
options=gaoptimset('StallTimeLimit',inf,'PopInitRange',initRange,...
    'MutationFcn',@mutationuniform,'SelectionFcn',@selectionroulette,...
    'PopulationSize',500,'TimeLimit',inf,'Generations',100,...
    'CrossoverFcn',@crossoverintermediate)

%call the ga function
[x, fval, reason, output, population, scores]=ga(@fitnessFunction, 7, options);
```
C.3 Fitness Function

The following is the actual fitness function. In addition to calculating the fitness function value, it collects a lot of information from the magnetostatic FEM simulation and saves this data, along with the variables used to create the FEM model, in a text file. The input variables are determine by the ga() function routine of course.

```matlab
function z=fitnessFunction(x)
    %x=[x1 x2 x3 x4 x5 x6 x7]

    %***** writing input vector to text file ***************
    fid_open=fopen('C:\FVectorIN.txt','at');
    for i=1:1:6
        fprintf(fid_open, '%10.5f',x(i));
    end
    fprintf(fid_open, '%10.5fn',x(7));
    fclose(fid_open);

    % ************* design values ***********************
    BiasMRE=0.55; % 0.55T
    tolMRE=.1; % tolerance in T
    BmagMin=.15; % 0.15T, knee of demagnetization curve for Ceramic Grade 5

    %************** get values from FEM ***************
    noP=femmEngine2act([x,0]); %0watts
    power=femmEngine2act([x,50]); %50watts
    Bmre0=noP(1)*-1;
    Bmag0=noP(2);
    Bleak0=noP(3);
```
Bpole0=noP(4);
weight0=noP(5);
Bmre50=power(1)*-1;
Bmag50=power(2);
Bleak50=power(3);
Bpole50=power(4);
weight50=power(5);

% ***************** calculate fitness function *****************
if BiasMRE<Bmre0 && Bmre50<tolMRE && -1*tolMRE<Bmre50
    Pmre=0; % want MRE saturation (.55T) at 0 watts (maxK),
else % and want 0.0T at 50 watts (minK)
    Pmre=1;
end

if BmagMin<Bmag50 % don't want to demagnetize magnet
    Pmag=0;
else
    Pmag=1;
end

fitness=weight0+Pmre*10^9*((BiasMRE-Bmre0)^2 +(abs(tolMRE)-abs(Bmre50))^2) + ... 
Pmag*10^9*(BmagMin-Bmag50)^2;

% ******** writing input vector and results to text file ********
fid_open=fopen('C:\\FVectorOUT.txt','at');
for i=1:1:7
    fprintf(fid_open, '% 10.5f,x(i));
end
fprintf(fid_open,'% 10.4f,Bmre0);
fprintf(fid_open,'% 10.4f,Bmag0);
fprintf(fid_open,'% 10.4f,Bleak0);
fprintf(fid_open,'% 10.4f,Bpole0);
fprintf(fid_open,'% 10.1f,weight0);
fprintf(fid_open,'% 10.4f,Bmre50);
fprintf(fid_open,'% 10.4f,Bmag50);
fprintf(fid_open,'% 10.4f,Bpole50);
fprintf(fid_open,'% 10.4f,Bleak50);
fprintf(fid_open,'% 15.1f','fitness');
fclose(fid_open);

% ********************* output to GA *********************
z=fitness;
C.4 FEM Evaluation

The fitness function obtains information about the magnetic properties of the device from a magnetostatic FEM simulation. The open source program ‘FEMM’ is used to obtain this information via OctaveFEMM, which uses ActiveX controls to link Matlab to the FEMM program [49]. OctaveFEMM provides a Matlab toolbox so that FEMM features can be controlled via Matlab. The code below develops the 2D axisymmetric model, assigns material properties, solves the resulting FEM problem, and post processes the results. Finally relevant results are returned to the script calling the function.

```matlab
function z=fem2act(x)
% x is a vector of dimension 8.
% z=[Bmre Bmag Bleak Bpole weight]
% actual dimensions of built BMRE
%x=[.329 .325 .6 .781 .677 .885 0.0 50] = [x1 x2 x3 x4 x5 x6 x7 % power]
% 0.001<x7<1
%
% writing input vector to text file
fid_open=fopen('C:\Vector\IN.txt','at');
for i=1:1:7
    fprintf(fid_open, '%10.5f,x(i));
end
fprintf(fid_open, '%10.5f',x(8));
fclose(fid_open);%
%******************************************** CONSTANT AND GEOMETRY ****************************
intomil=25.4;
mu=4*pi*10^-7;
% design values
staticDeflection=0.030;
MREthickness=.066;
MREheight=.33;
mountthickness=.066;
mountheight=2*MREheight;
Althickness=.05;
power=x(8); % 50Watts available
% other constants
Rwire=25.7/1000*1/.3048; % resistance of 24awg copper wire [ohms/m]
diaWire = .0201/12*.3048; % diameter of 24awg wire [m]
currlimit=5; % 5amp limit
packEff = .8; % packing efficiency of winding
dcopper = 0.00896; % density of copper [g/mm^3]
dsteel = .00787; % density of steel [g/mm^3]
```

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mu_rel_MRE=2;

%geometry
r1=0;
z1=0;
r2=x(3)*intomil+x(1)*intomil+MREthick*intomil*2+mountthickness*intomil+....
   Althickness*intomil+x(5)*intomil+x(6)*intomil;
z2=x(2)*intomil;
r3=x(3)*intomil;
z3=z2+x(4)*intomil;
r4=r3+MREthick*intomil;
z4=z3-MREheight*intomil;
r5=r4+mountthickness*intomil;
z5=z4+mountheight*intomil;
r6=r5;
z6=z4;
r7=r6+MREthick*intomil;
z7=z3;
r8=r2;
z8=z6;
r9=r7;
z9=z6;
r10=r9+Althickness*intomil+x(5)*intomil;
z10=z2;
r11=r10+x(6)*intomil;
z11=z9;
r12=r1;
z12=z2;
r13=r9+Althickness*intomil;
z13=z9;
r14=r13;
z14=z10;
r15=r11;
z15=z2+(z8-z2)*x(7);

%calculate current
N1=round(packEff*(r10-r13)/(diaWire*1000));
N2=round(packEff*(z13-z10)/(diaWire*1000));
Lw=2*pi*(r13/1000*(N1+1)+(N1+N1^2)*diaWire/2)*N2;
icurr=1*(power/(Lw*Rwire))^0.5; %multiply by negative one to increase pole
if currlimit<icurr
   icurr=currlimit;
end
N0turns=N1*N2; %density

%max mesh size per component
maxmesh=[min(((z2-z1)/4,(r2-r1)/4)), ... %bottom plate
   min(((z3-z12)/4,(r3-r12)/4)), ... %pole
   min(((z3-z4)/4,(r4-r3)/4)), ... %inner MRE
   min(((z5-z4)/4,(r5-r4)/4)), ... %mount
   min(((z7-z6)/4,(r7-r6)/4)), ... %outer MRE
   min(((z7-z8)/4,(r8-r7)/4)), ... %top plate
   min(((z13-z10)/4,(r10-r13)/4)), ... %coil
   min(((z11-z10)/4,(r11-r10)/4)), ... %magnet
   ...];
\[
\begin{align*}
&\min\left(\frac{(z_9-z_{10})/4,(r_9-r_3)/4}{\text{inside air}}\right), \quad ... \%\text{inside air} \\
&\min\left(\frac{(z_9-z_{14})/4,(r_{14}-r_9)/4}{\text{outside air}}\right), \quad ... \%\text{bobbin} \\
&\min\left(\frac{(z_{15}-z_2)/4,(r_2-r_{15})/4}{\text{flux leakage path block}}\right) \\
\end{align*}
\]

% **************************************** SETUP ****************************************
openfemm;
main_restore
% need to create a new Magnetostatics document to work on.
newdocument(0);

% Define the problem type. Magnetostatic; Units of mm; Axisymmetric;
% Precision of 10^-6 for the linear solver; a placeholder of 0 for
% the depth dimension, and an angle constraint of 30 degrees
mi_probdef(0, 'millimeters', 'axi', 1.e-8, 0, 30);

% **************************************** MATERIALS ****************************************
% Add some materials properties
mi_addmaterial('Air', 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0);
mi_addmaterial('Coil', 1, 1, 0, 0, 58*0.65, 0, 0, 1, 0, 0, 0);
mi_addmaterial('MRE', mu_rel_MRE, mu_rel_MRE, 0, 0, 0, 0, 0, 1, 0, 0, 0);
mi_addmaterial('Magnet', 1, 1, 293000, 0, 0, 0, 0, 1, 0, 0, 0); % ceramic magnet grade 5
mi_addmaterial('Bobbin', 1, 1, 0, 0, 20, 0, 0, 1, 0, 0, 0);
mi_addmaterial('Ferrite', 21, 21, 0, 0, 0, 0, 0, 1, 0, 0, 0);

% A more interesting material to add is the iron with a nonlinear
% BH curve. First, we create a material in the same way as if we
% were creating a linear material, except the values used for
% permeability are merely placeholders.
mi_addmaterial('Iron', 2100, 2100, 0, 0, 9.9, 0, 0, 1, 0, 0, 0);

% laminated iron
laminationT=.015; %[in]
laminationFactor=.95;
parallelr=1;
parallelz=2;
mi_addmaterial('IronR', 2100, 2100, 0, 0, 9.9,laminationT, 0, laminationFactor, parallelr, 0, 0);
mi_addmaterial('IronZ', 2100, 2100, 0, 0, 9.9,laminationT, 0, laminationFactor, parallelz, 0, 0);

% A set of points defining the BH curve is then specified. IRON
bhcurveIRON = [0,0.3,0.8,1.12,1.32,1.46,1.54,1.62,1.74,1.87,1.99,2.046,2.08; 
0, 40, 80, 160, 318, 796, 1590, 3380, 7960, 15900, 31800, 55100, 79600];

% A set of points defining the BH curve is then specified. MRE
Hsat=.6/(4*pi*10^-7*mu_rel_MRE);
Hmid1=Hsat+.4/(4*pi*10^-7);
Hmid2=Hmid1+.4/(4*pi*10^-7);
Hmid3=Hmid2+.4/(4*pi*10^-7);
Htop=Hmid3+.4/(4*pi*10^-7);
bhcurveMRE = [0 .6 1 1.4 1.8 2.2 0 Hsat Hmid1 Hmid2 Hmid3 Htop];

% plot(bhcurve(:,2),bhcurve(:,1))
% Another command associates this BH curve with the iron and MRE material:
mi_addbhpoints('Iron', bhcurveIRON);
mi_addbhpoints('Iron', bhcurveIRON); % may change to IronR
mi_addbhpoints('Iron', bhcurveIRON); % may change to IronZ
mi_addbhpoints('MRE', bhcurveMRE);

% Add a "circuit property" so that we can calculate the properties of the
% coil as seen from the terminals.
mi_addcircprop('icoir', icurr, 1);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% COMPONENTS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Draw steel parts;
mi_drawrectangle([r1 z1;r2 z2]); %bottom plate
mi_drawrectangle([r12 z12;r3 z3]); %pole
mi_drawrectangle([r4 z4;r5 z5]); %mount
mi_drawrectangle([r7 z7;r8 z8]); %top plate
mi_drawrectangle([r15 z15;r2 z2]); %flux leakage block

mi_addblocklabel((r2+r1)/2,(z2+z1)/2);
mi_addblocklabel((r12+r3)/2,(z12+z3)/2);
mi_addblocklabel((r4+r5)/2,(z4+z5)/2);
mi_addblocklabel((r7+r8)/2,(z7+z8)/2);
mi_addblocklabel((r15+r2)/2,(z15+z2)/2);

% MRE
mi_drawrectangle([r3 z3;r4 z4]); %inner MRE
mi_drawrectangle([r6 z6;r7 z7]); %outer MRE

mi_addblocklabel((r3+r4)/2,(z3+z4)/2);
mi_addblocklabel((r6+r7)/2,(z6+z7)/2);

%coil
mi_drawrectangle([r13 z13;r10 z10]);

mi_addblocklabel((r13+r10)/2,(z13+z10)/2);

%inside air
mi_addblocklabel((r9+r3)/2,(z9+z10)/2);

%magnet
mi_drawrectangle([r10 z10;r11 z11]);

mi_addblocklabel((r10+r11)/2,(z10+z11)/2);

%bobbin
mi_drawrectangle([r9 z9;r14 z14]);

mi_addblocklabel((r9+r14)/2,(z9+z14)/2);

% Draw a half-circle to use as the outer boundary for the problem
mi_drawarc([0 -(r8^2+z3^2)^(.5)]*2; 0 ((r8^2+z3^2)^(.5))^2], 180, 2.5);
mi_addsegment([0 -(r8^2+z3^2)^(.5)]*2; 0 ((r8^2+z3^2)^(.5))^2]);
mLaddblocklabel(((r8^2+z3^2)^.5)*2-2,0):

%**************************************************************************
% BC'symology

% Define an "asymptotic boundary condition" property. This will mimic
% an "open" solution domain
muo = pi*4.e-7;
mi_addboundprop('Asymptotic', 0, 0, 0, 0, 0, 0, 1/(muo*0.2), 0, 2);

% Apply the "Asymptotic" boundary condition to the arc defining the
% boundary of the solution region
mi_selectarcsegment(2*r8,0);
mi_setarcsegmentprop(2.5, 'Asymptotic', 0, 0);

%**************************************************************************
% ASSIGN MATERIALS TO COMPONENTS
%**************************************************************************
% Apply the materials to the appropriate block labels
%coil
mi_selectlabel((r13+r10)/2,(z13+z10)/2);
mi_setblockprop('Coil', 0, maxmesh(7), 'icoil', 0, 0, Noturns);
mi_clearselected

%air
mi_setblockprop('Air', 0, maxmesh(10), '<None>', 0, 0, 0);
mi_clearselected

%iron
mi_selectlabel((r1+r2)/2,(z1+z2)/2);
mi_setblockprop('Iron', 0, maxmesh(1), '<None>', 0, 0, 0);
mi_clearselected

%MRE
mi_selectlabel((r3+r4)/2,(z3+z4)/2);
mi_setblockprop('MRE', 0, maxmesh(3), '<None>', 0, 0, 0);
mi_clearselected

%bottom plate

%pole

%mount

%top plate

%flux leakage block

%MRE
mi_selectlabel((r6+r7)/2,(z6+z7)/2);
mi_setblockprop('MRE', 0, maxmesh(5), '<None>', 0, 0, 0);
mi_clearselected

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%magnet
mi_selectlabel((r10+r11)/2,(z10+z11)/2);
mi_setblockprop('Magnet', 0, maxmesh(8), '<None>', 90, 0, 0);
mi_clearselected

%bobbin
mi_selectlabel((r9+r14)/2,(z9+z14)/2);
mi_setblockprop('Bobbin', 0, maxmesh(11), '<None>', 0, 0, 0);
mi_clearselected

% We have to give the geometry a name before we can analyze it.
projectName=['funcEval.fem'];
mi_saveas(projectName);
% Now, analyze the problem and load the solution when the analysis is finished
mi_analyze
mi_loadsolution

%*********************************************************************** POST PROCESS  ***********************************************************************
mo_seteditmode('contour');
mo_addcontour(r10,(z10+z11)/2);
mo_addcontour(r15,(z10+z11)/2);
Bmag=mo_lineintegral(0); %magnet B
mo_clearcontour;

mo_seteditmode('contour');
mo_addcontour((r7+r5)/2,z7);
mo_addcontour((r7+r5)/2,z9); %outer MRE B
Bmre=mo_lineintegral(0);
mo_clearcontour;

mo_seteditmode('contour');
mo_addcontour(r11,(z11+z15)/2);
mo_addcontour(r8),(z11+z15)/2);
Bleak=mo_lineintegral(0); %leakage gap B
mo_clearcontour;

mo_seteditmode('contour');
mo_addcontour(r1),(z12+z4)/2);
mo_addcontour(r3),(z12+z4)/2);
Bpole=mo_lineintegral(0); %pole B
mo_clearcontour;

%*********************************************************************** END  ***********************************************************************

%volume of pieces
v1=(z2-z1)*pi*(r2-r1)^2; %bottom plate
v2=(z3-z2)*pi*(r3-r12)^2; %pole
v3= ((z13-z10)*pi*(r10-r13)^2)*(pi/4); %coil 1-(1-pi/4)=pi/4 -> CopperVol=TotVol-AirVol
v4= (z11-z10)*pi*(r11-r10)^2; %magnet
v5=(z15-z2)*pi*(r2-r15)^2; %flux block
v6=(z7-z8)*pi*(r8-r7)^2; %top plate
weight=((v1+v2+v4+v5+v6)*dsteel+v3*dcopper); %weight of device in kg

%writing input vector and results to text file
fid_open=fopen('C:\VectorOUT.txt','at');
for i=1:1:8
    fprintf(fid_open, '%10.5f,x(i));
end
fprintf(fid_open,'%10.5f,Bmre(2));
fprintf(fid_open,'%10.5f,Bmag(2));
fprintf(fid_open,'%10.5f,Bleaf(2));
fprintf(fid_open,'%10.5f,Bpole(2));
fprintf(fid_open,'%15.1fn',weight);
fclose(fid_open);

%output
z=[Bmre(2) Bmag(2) Bleak(2) Bpole(2) weight];
BIBLIOGRAPHY


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VITA

Graduate College
University of Nevada, Las Vegas

Saul Allen Opie

Home Address:
7737 Langdon Ave
Hesperia, CA 92345

Degrees:
Bachelor of Science, Mechanical Engineering, 2004
California State Polytechnic University, Pomona

Thesis Title: Design and Control of a Vibration Isolator Using a Biased Magnetorheological Elastomer

Thesis Committee:
Chairperson, Dr. Woosoon Yim, Ph.D.
Committee Member, Dr. Brendan O'Toole, Ph.D.
Committee Member, Dr. Mohamed Trabia, Ph.D.
Graduate Faculty Representative, Dr. Sahjendra Singh, Ph.D.