Numerical and experimental investigation of deformation and strength properties of lithophysae-rich tuff and analog materials

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NUMERICAL AND EXPERIMENTAL INVESTIGATION OF DEFORMATION AND STRENGTH PROPERTIES OF LITHOPHYSAE-RICH TUFF AND ANALOG MATERIALS

by

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A dissertation submitted in partial fulfillment of the requirements for the Doctor of Philosophy Degree
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December 2002

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Numerical and Experimental Investigation of Deformation and Strength Properties of Lithophysae-Rich Tuff and Analog Materials

is approved in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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ABSTRACT

Numerical and Experimental Investigation of Deformation and Strength Properties of Lithophysae-Rich Tuff and Analog Materials

by

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Portions of the high-level nuclear waste repository in Yucca Mountain will be located in lithophysae-rich tuff formations. Understanding the mechanical properties of the lithophysae-rich tuff, including deformation modulus, deformation ratio and compressive strength, is an important issue for design and the performance of the repository tunnels. These properties are expected to be significantly affected by lithophysal porosity.

Two different research directions are implemented in this dissertation. First, uniaxial compression testing is simulated using finite difference technique on models containing circular holes in order to investigate the effect of porosity on deformation parameters. Numerical results are compared with biaxial test results of urethane specimens containing circular tubes to verify the numerical analysis results.
Second, an experimental program that consists of uniaxial compression tests on analog models and tuff is conducted. Two different configurations are implemented to model porosity using gypsum plaster as an analog material. In the first configuration analog models containing uniformly and randomly distributed open ended cylindrical tubes are produced. In the second configuration spherical cavities are introduced into the analog models Both models are tested under uniaxial compression and their deformation moduli and compressive strength are compared with lithophysae-rich tuff specimens that are obtained from outcrops of lithophysal tuff units.

Numerical modeling and testing are combined to assess that the deformation modulus of tuff where the porosity has a vital effect on mechanical behavior of the rock. Both numerical analysis and uniaxial testing on analog materials show that in deformation modulus exponentially decrease with increasing porosity. The deformation moduli and compressive strength of gypsum plaster specimens containing open ended cylindrical tubes are slightly lower than those containing spherical cavities due to confinement effects.

The deformation moduli and compressive strengths of the tuff specimens fall between the values determined for the plaster specimens with two different porosity configuration. Distribution of data for both analog and tuff specimens is very similar at low porosities. At higher porosities, a greater decrease in deformation modulus is observed in tuff due to larger and nonspherical cavities indicating that shape of the cavities is a factor affecting the modulus.
# TABLE OF CONTENTS

ABSTRACT ................................................................................................................. iii

LIST OF FIGURES ................................................................................................... viii

LIST OF TABLES ....................................................................................................... xi

ACKNOWLEDGEMENTS ........................................................................................ xi

CHAPTER 1  INTRODUCTION ................................................................................ 1
  1.1 Introduction ................................................................................................... 1
  1.2 Objective ...................................................................................................... 2
  1.3 Organization of the Dissertation ................................................................. 3

CHAPTER 2 LITERATURE REVIEW ..................................................................... 5
  2.1 Introduction ................................................................................................... 5
  2.2 Effective Medium Theories .......................................................................... 6
    2.2.1 Self-Consistent Scheme (SCS) ..................................................... 7
    2.2.2 Differential Scheme (DS) .............................................................. 9
    2.2.3 Mori-Tanaka Scheme (MTS) ....................................................... 10
    2.2.4 Comparison of Effective Medium Theories ................................ 11
    2.2.5 Effective Poisson's Ratio ............................................................ 13
  2.3 Numerical Studies on Solids Containing Holes ....................................... 13
  2.4 Experimental Studies of Solids Containing Holes .................................... 14
  2.5 Recent Studies on Mechanical Properties of Tuff ................................... 17
  2.6 Discussion .................................................................................................. 19
  2.7 Conclusion .................................................................................................. 20

CHAPTER 3 GENERAL DESCRIPTION OF LITHOPHYSAL TUFF ................. 22
  3.1 Introduction................................................................................................. 22
  3.2 Geology of the Repository Host Horizon ................................................. 23
  3.3 Description of Repository Lithophysal Tuff Units .................................... 24
  3.4 Conclusion.................................................................................................. 26

CHAPTER 4 NUMERICAL MODELING OF UNIAXIAL COMPRESSION TESTING ON SOLIDS CONTAINING CIRCULAR HOLES .......................................................... 27
  4.1 Introduction ................................................................................................. 27
  4.2 Using FLAC 2D as a Numerical Modeling Tool ........................................ 28
LIST OF FIGURES

Figure 2.1 Comparison of normalized elastic (Young's) moduli determined by different schemes for randomly distributed circular holes. ...12
Figure 3.1 Specimen of Topopah Spring Tuff, upper lithophysal zone. .....24
Figure 4.1 FLAC finite difference grid containing 24 randomly distributed circular holes with a total porosity of 11.9 %. .........................33
Figure 4.2 Configuration of uniformly distributed circular holes (a) 1 Hole, (b) 9 Holes and (c) 36 Holes ........................................................34
Figure 4.3 An example of randomly sized and distributed circular holes ...36
Figure 4.4 FLAC model boundary conditions ........................................41
Figure 4.5 Normalized deformation modulus versus percent porosity (a) for \( \nu_o = 0.1 \) and (b) for \( \nu_o = 0.2 \) ........................................48
Figure 4.5 Normalized deformation modulus versus percent porosity (c) for \( \nu_o = 0.3 \) and (d) for \( \nu_o = 0.4 \) ........................................49
Figure 4.5 Normalized deformation modulus versus percent porosity (e) for \( \nu_o = 0.45 \) and (f) for \( \nu_o = 0.49 \) ........................................50
Figure 4.6 Normalized modulus envelopes for models with uniformly distributed circular holes ........................................................................51
Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity (a) for \( \nu_o = 0.1 \) and (b) for \( \nu_o = 0.2 \) .................53
Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity (c) for \( \nu_o = 0.3 \) and (d) for \( \nu_o = 0.4 \) .................54
Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity (e) for \( \nu_o = 0.45 \) and (f) for \( \nu_o = 0.49 \) .................55
Figure 4.8 Normalized deformation modulus versus percent porosity with different deformation ratios ..................................................57
Figure 4.9 Normalized deformation modulus in both loading directions for (a) \( \nu_o = 0.1 \) (b) \( \nu_o = 0.2 \) ........................................58
Figure 4.9 Normalized deformation modulus in both loading directions for (c) \( \nu_o = 0.3 \) (d) \( \nu_o = 0.4 \) ........................................59
Figure 4.10 Normalized deformation ratios for \( \nu_o = 0.1 \) (circle), \( \nu_o = 0.2 \) (square) \( \nu_o = 0.3 \) (triangle) \( \nu_o = 0.4 \) (diamond). Solid line represents the 45 degree line. All models are loaded in y-direction ................................................61
Figure 4.11 Normalized deformation modulus in y-direction for (a) $\nu_0 = 0.1$
(b) $\nu_0 = 0.2$. Empty circles represent randomly distributed holes while solid circles, squares and diamonds represent models with 1, 9 and 36 holes, respectively.

Figure 4.11 Normalized deformation modulus in y-direction for (c) $\nu_0 = 0.3$
(d) $\nu_0 = 0.4$ for free sides. Empty circles represent randomly distributed holes while solid circles, squares and diamonds represent models with 1, 9 and 36 holes, respectively.

Figure 5.1 Stress versus strain curve for urethane. Circles are the data points and solid line is the best-fit curve.

Figure 5.2 Urethane specimens of different porosities and number of holes.

Figure 5.3 Proving ring assembly.

Figure 5.4 Proving ring assembly and LVDTs.

Figure 5.5 Axial and lateral strains versus axial stress for solid urethane specimen.

Figure 5.6 Normalized deformation modulus versus porosity for (a) test data and (b) data for both testing and corresponding numerical models. Solid and dotted lines are the best-fit curves for experimental and numerical data, respectively.

Figure 6.1 Stress versus strain curve for gypsum plaster. Circles are the data points and solid line is the best-fit curve.

Figure 6.2 Method for calculating deformation modulus from axial stress versus strain curve.

Figure 6.3 Compressive strength (a) and deformation modulus (b) with porosity for plaster specimens. Solid line and dotted line show the best fit curve for randomly and uniformly distributed tubes, respectively.

Figure 6.4 Porosity versus normalized deformation modulus for plaster specimens containing cylindrical tubes. Solid line and dotted line show the best fit curve for randomly and uniformly distributed tubes, respectively.

Figure 6.5 Porosity versus normalized deformation modulus for plaster specimens containing tubes determined through numerical analysis and testing. Solid line and dotted line show the best fit curve for numerically computed values and experimental values, respectively.

Figure 7.1 MTS uniaxial compression testing system.

Figure 7.2 Compressive strength (a) and deformation modulus (b) with porosity for plaster specimens. All porosities are bulk porosities.

Figure 7.3 Compressive strength (a) and deformation modulus (b) with porosity for lithophysal Tuff specimens. All porosities are bulk porosities.
Figure 7.4  Porosity versus normalized deformation modulus for plaster and Tuff specimens. Solid line and dotted line show the best fit curve for plaster and Tuff specimens, respectively. ..............................109

Figure 8.1  Deformation modulus versus porosity for gypsum plaster specimens containing open ended cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (dashed line is the best fit line). .................................................................113

Figure 8.2  Normalized deformation modulus versus porosity for gypsum plaster specimens containing open ended cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best fit line) and lithophysae-rich Tuff specimens (long dashed line is the best fit line). ......................114

Figure 8.3  Normalized deformation modulus versus porosity for gypsum plaster specimens containing open ended cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best fit line), lithophysae-rich Tuff specimens (long dashed line is the best fit line) and numerical values for \( \nu_s = 0.3 \). ........................................................................115

Figure 8.4  Compressive strength versus porosity for gypsum plaster specimens containing open ended cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (dashed line is the best fit line). ..............................................................................117

Figure 8.5  Normalized compressive strength versus porosity for gypsum plaster specimens containing open ended cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best fit line) and lithophysae-rich Tuff specimens (long dashed line is the best fit line). ......................118

Figure 8.6  Normalized deformation modulus versus porosity for numerical and experimental specimens including Tuff. The curves represent the relationships calculated using approximate methods. ........................................................................121
## LIST OF TABLES

| Table 3.1 | Phase Percentages Encountered within Topopah Spring Upper and Lower Lithophysal Zones .................................................................25 |
| Table 4.1 | Uniformly Distributed Circular Hole Geometries .............................................35 |
| Table 4.2 | Elastic material properties used in modeling ...........................................38 |
| Table 4.3 | Percent Errors for Solid Models ................................................................46 |
| Table 5.1 | Geometry of Holes in Urethane Specimens ................................................68 |
| Table 5.2 | Description of LVDTs .............................................................................71 |
| Table 5.3 | Actual and Normalized Deformation Modulus Values of Urethane Specimens .................................................................73 |
| Table 6.1 | Gypsum Plaster Specimens Containing Cylindrical Tubes........................80 |
| Table 6.2 | Compressive Strength, $\sigma_c$, and Deformation Modulus, $E$, of Gypsum Plaster Specimens Containing Open ended Cylindrical Tubes ........................................................................83 |
| Table 7.1 | Gypsum Plaster Specimens with Styrofoam Inclusions .........................93 |
| Table 7.2 | Coordinates of Structured Inclusions. The Center of the Base Plate Has Coordinates of (0,0,0) .................................................................94 |
| Table 7.3 | Gypsum Plaster Specimens with Styrofoam Inclusions .................................................................95 |
| Table 7.4 | Calculated Porosities of Gypsum Plaster Specimens Containing Styrofoam Inclusions .................................................................100 |
| Table 7.5 | Calculated Porosities for Tuff Specimens ................................................101 |
| Table 7.6 | Deformation Modulus, $E$, and Compressive Strength, $\sigma_c$, of Gypsum Plaster Specimens .................................................................103 |
| Table 7.7 | Deformation Modulus, $E$, and Compressive Strength, $\sigma_c$, of Tuff Specimens .................................................................106 |
ACKNOWLEDGEMENTS

I would like to thank Dr. Moses Karakouzian for his encouragement in the completion of this study.

I am grateful to Dr. Nick W. Hudyma for a detailed and critical review of my dissertation and his help during the tests and analysis.

I also thank Dr. Gerald Frederick, Dr. Samaan Ladkany and Dr. Vernon F. Hodge who served my graduate committee.

I also thank Mr. Venkatrao Thummala and Mr. C. D. Herrington of Bechtel Nevada Material Testing Laboratory for their help during the testing at the Nevada Test Site, Mr. Steven E. McCullough of Terracon, Inc., who kindly permitted me using the compression testing equipment, Mr. Mehmet Okuyan, and Mr. Barrett Applegate who helped me in data collection.

Many thanks to my parents for their moral support and believe in me. It was very difficult for them seeing me only couple of weeks every year.
CHAPTER 1

INTRODUCTION

1.1 Introduction

Rocks are structurally very complex materials. They consist of various crystals, grains, cementing materials and discontinuities such as joints, fractures, pores and cavities in different shapes and dimensions. These elements affect the performance of engineering structures in rock, such as excavations and tunnels.

Portions of the high-level nuclear waste repository in Yucca Mountain will be located in lithophysae-rich (or lithophysal) Tuff formations. Lithophysae are cavities that were formed by trapped air within the falling volcanic ash that formed the Tuff units. The porosity caused by lithophysae is called lithophysal porosity. The host rock surrounding the repository is expected to isolate the radionuclide migration for thousands of years. Understanding the mechanical properties of the lithophysae-rich Tuff, including deformation modulus, deformation ratio and compressive strength, is an important issue for design and the performance of the repository tunnels. These properties are expected to be affected by the amount of lithophysal porosity. To date there have been no in depth studies addressing the deformation and strength properties, and failure patterns of lithophysal Tuff with porosity.

Deformation and strength properties of porous materials, which are often called two-phase materials in material sciences and rock physics, where one of the phases has zero deformation modulus, have been analytically, or semi-analytically, studied by researchers in different areas (for instance, Walsh, 1980; Christensen, 1990; Kachanov et al., 1994; Roberts and Garboczi, 2000).
Although these studies demonstrate that deformation moduli, often elastic and/or bulk moduli, are affected by porosity; approximate solutions based on the assumptions that two-phase solids are effectively homogeneous and that pores are randomly distributed. Effective homogeneity means that the macroscale properties of a heterogeneous material can be averaged and calculated for the two-phase solid, and therefore the material can be considered as isotropic.

Rocks can be characterized as porous materials if the porosity between grains is considered (Mavko et al., 1998). However, this is in the scope of rock physics. In rock mechanics this porosity is mostly ignored because the porosity does not vary throughout the rock mass. Therefore, rocks without significant discontinuities can be classified as intact rock. Lithophysal Tuff, instead, contains wide ranges of voids in dimensions and shapes, and it might not support the assumptions of effective homogeneity.

Since porosity changes the mechanical properties of Tuff, the effect of lithophysal porosity on the deformation and strength properties of Tuff requires further investigation. This is the general scope of this dissertation.

The deformation parameters being investigated here are the deformation modulus and the deformation ratio. Elastic (Young's) modulus and Poisson's ratio are strictly appropriate only for intact rock. In this dissertation the slope of stress-strain curve (elastic modulus) and the ratio of vertical strain to the horizontal strain (Poisson's ratio) are called deformation modulus and deformation ratio, respectively, for specimens containing cavities.

1.2 Objective

The objective of this dissertation is to investigate the influence of varying porosity on deformation and strength properties of lithophysae-rich Tuff. Two different research directions are implemented. First, numerical compression tests are conducted on finite difference models containing circular holes in order to investigate the effect of porosity on deformation parameters. Second, an experimental program is developed. The experimental program includes two
approaches. First, an analog material is used to model porosity, which resembles lithophysal porosity, by creating cavities in the analog specimens and testing them under uniaxial compression to investigate the correlation between porosity and deformation and strength properties. Second, lithophysae-rich Tuff specimens obtained from Yucca Mountain are tested under uniaxial compression. Numerical modeling and an experimental program are combined to assess the deformation and strength properties of Tuff where the porosity has a vital effect on mechanical behavior of the rock.

The following tasks summarizes the dissertation outline:

1. Investigation of analytical solutions in the literature
2. Simulation of numerical experiments in two dimensions on models containing circular holes
3. Verification of numerical results by biaxial testing on urethane specimens containing circular holes
4. Uniaxial compression testing of gypsum plaster specimens containing open ended cylindrical tubes
5. Uniaxial compression testing of gypsum plaster specimens containing Styrofoam inclusions and lithophysae-rich Tuff specimens
6. Uniaxial compression testing of lithophysae-rich Tuff specimens
7. Correlations between deformation properties and porosity to assess the effect of porosity on deformation and strength properties of lithophysae-rich Tuff

1.3 Organization of the Dissertation

This dissertation is organized as follows.

Chapter 2 summarizes the recent analytical and experimental studies on materials containing holes or cavities that investigate effect of porosity on deformation and strength properties.
Chapter 3 describes the Topopah Spring lithophysae-rich Tuff and the porosity ranges encountered in the portions of proposed high-level nuclear waste repository. Shapes and orientations of lithophysae are evaluated to propose a numerical and experimental research plan in following chapters.

In Chapter 4, solids containing holes in two dimensions are modeled to compute the deformation and strength properties by numerical simulating uniaxial compression testing.

Chapter 5 presents biaxial compression testing on urethane specimens containing cylindrical tubes to compare and verify the results of numerical compression testing that are explained in detail in Chapter 4.

In Chapter 6 uniaxial compression testing which is conducted on gypsum plaster specimens containing open ended cylindrical tubes; a simple way of creating porosity, is explained. Deformation moduli determined from testing are compared with those computed numerically.

Chapter 7 presents uniaxial compression testing program and results (deformation modulus and compressive strength) from gypsum plaster specimens containing Styrofoam inclusions for attempts to model a material similar to Tuff in terms of its macro porous structure. Lithophysae-rich Tuff specimens taken from outcrops surrounding Yucca Mountain, Nevada are also tested under uniaxial compression.

Chapter 8 summarizes the results of this dissertation by comparing the numerical, analog (gypsum plaster) models testing and Tuff testing. Comparisons are made between the deformation parameters from numerical analysis, analog material testing and Tuff testing. Normalized compressive strength for Tuff and plaster specimens are also compared with each other. Recommendations for future research are also given.
LITERATURE REVIEW

2.1 Introduction

The problem of determining the elastic moduli of solids containing holes, cavities or inclusions has been studied in different engineering disciplines such as material sciences, rock mechanics and geophysics. Embedded inclusions or holes in a continuous solid material change the mechanical and physical properties of the material. In the case of embedded holes, the porosity is the most important factor influencing the overall properties of material. However, the definition of porosity takes different meanings in different disciplines. Recent analytical, numerical and experimental studies generally deal with microscopically heterogeneous materials. The term porosity is usually defined as the relative amount of pore space between minerals or individual grains. Highly porous materials such as sandstones and ceramics contain this type of porosity. The porosity in these materials is microscopic, that is, the pores cannot be seen with the naked eye. Although the analytical and semi-analytical studies are not typically conducted to investigate the effect of porosity due to large cavities on deformation and strength properties of materials, it is appropriate to mention that
these studies here to obtain a sense on how modulus changes with porosity. The analytical and semi-analytical can be grouped under effective medium theories. There are also a limited number of experimental studies on lithophysae-rich Tuff, however, their purpose is not the same as this dissertation.

2.2 Effective Medium Theories

The exact solutions to the elasticity problem of determining mechanical properties of solids containing many holes are very difficult to obtain (Zimmerman, 1991). However, there are semi-analytical or approximate solutions that can be used to determine the elastic properties of porous materials and materials containing inclusions and holes.

Approximate analytical solutions adapt constitutive laws and use continuum mechanics assuming that the matrix is continuous. The most common method is effective medium theories, which cover a wide range of materials such as cracked solids, porous media and multi-phase composites. Effective medium theories model the inclusions or cavities by replacing them into some kind of effective environment. This effective environment is either effective matrix (as in self-consistent scheme and differential scheme) or effective stress (for instance Mori-Tanaka scheme). The approach is to solve a one-hole problem and then use an averaging process to generate a formula that predicts effective elastic properties for a particular porosity (Garboczi and Day, 1995). The solutions provide equations in which the effective elastic modulus (bulk, shear or Young's modulus) is a function of matrix elastic modulus, porosity and sometimes a shape
factor for the holes. The elastic properties of the two-phase materials are described as "effective" properties. The term effective describes the average elastic properties by considering the properties of all phases of the heterogeneous media and their interaction. The mathematical background to determine the effective elastic properties by implementing effective medium theories is not within the scope of this dissertation. The final product of these studies, which provides relationships between effective elastic properties and properties of each phase, are investigated. The effective medium theories are in the linear elastic range and nonlinear effects due to cavity closures under compression are not investigated. A detailed description of these theories can be found in Christensen (1991).

2.2.1 Self-Consistent Scheme (SCS)

The self-consistent scheme has been widely used even though it has some limitations. The approximation is based upon the assumption that a macroscopic volume that contains holes can be replaced by an equivalent homogeneous material without changing the elastic behavior of the solid (Mackenzie, 1950). This is because the mean stresses and displacements at the boundary of the volume containing holes are equal to those at the boundary of the same volume in the equivalent elastic continuum. These conditions for consistency enable the effective elastic constants to be calculated. The interaction of holes is approximated by replacing the matrix material with the as-yet-unknown effective medium. The self-consistent approximation yields a set of
nonlinear simultaneous equations with the unknowns of elastic modulus such as bulk and shear modulus. These equations are then solved by simultaneous iterations.

The stiffness properties of solids containing spherical holes were studied by Mackenzie (1950) who used a self-consistent scheme. After his study, other researchers applied the same theory to different orientations and configurations of holes. Mackenzie's assumptions that were used to determine the elastic modulus of an effective medium are widely used in other studies. He assumed that (1) holes are randomly distributed and isolated, (2) concentration of holes is small, (3) the effective medium is isotropic and linear elastic, (4) the shapes of holes are idealized as spherical or ellipsoidal. The first assumption, isolated holes, assumes that there is a sufficient distance between each hole so that the interaction between stress fields is small enough to be ignored. This is true if the concentration of holes is small or diluted.

Korringa et al. (1979) used a self-consistent model to calculate the effective elastic moduli of dry rock as an isotropic, heterogeneous and porous medium. Porosity was represented by ellipsoidal and spherical pores of various sizes and shapes. They concluded that the different sizes of pores have the least importance in determining effective modulus prediction.

Walsh (1980) applied self-consistent scheme to predict the effective bulk modulus assuming that Poisson's ratio does not depend on porosity. He found that the SCS is satisfactory for porosities less than 25%, but this scheme predicts that bulk modulus should be zero at porosities higher than 50%.
Thorpe and Sen (1985) used two different self-consistent methods for a composite medium containing randomly positioned and oriented elliptical holes or rigid inclusions. Zhao and Weng (1990) considered tubular elliptical holes in plane strain for random and parallel orientation distributions.

Kachanov et al. (1994) determined an equation for normalized elastic modulus using SCS, which includes a shape factor. The equation is:

\[ \frac{E}{E_0} = 1 - (3p + q) \]  

(2.1)

where \( E \) is the matrix elastic modulus, \( E_0 \) is the effective elastic modulus, \( p \) is the overall porosity due to holes of all types (expressed as a fraction) and \( q \) is the shape factor. They emphasized that the moduli cannot be expressed as a function of porosity alone and a shape factor must be included in the equation. Otherwise, the effective modulus may be overestimated. For circular holes, \( q \) becomes zero.

2.2.2 Differential Scheme (DS)

The differential effective medium theory models composites containing holes by incrementally adding porosity to the matrix material (Norris, 1985; Zimmerman, 1991). This is different from the SCS scheme that introduces the holes in one step. In the differential scheme, the effective moduli depend not only on the final porosity but also on the order in which the incremental additions are
performed. Kachanov et al. (1994) provide the normalized elastic modulus calculated using DS for elliptical holes as:

\[
\frac{E}{E_o} = e^{(-3p-q)}
\]  

(2.2)

where \(E\) is the matrix elastic modulus, \(E_o\) is the effective elastic modulus, \(p\) is the overall porosity due to holes of all types and \(q\) is the shape factor.

2.2.3 Mori-Tanaka Scheme (MTS)

The Mori-Tanaka scheme (Mori and Tanaka, 1973) is often used in the study of the mechanics of composite materials (for instance in Christensen, 1990). The MTS places a representative hole into the average stress field in the matrix and obtains a solution. The scheme is applicable for both interactive and non-interactive holes. The approximation of non-interactive holes is a simpler approach to the problem than the approximation of interacting holes. However, in a solid with holes, the stress field that a particular hole is subjected to is influenced by the presence nearby holes. This interaction effect changes the volumetric strain of the solid under compression and thus increases the compressibility and decreases the effective elastic modulus. It is important to understand that the term interaction does not refer to physical interaction of holes or voids during the inelastic deformation, but to the interactions of the stress fields surrounding the cavities.
Zhao et al. (1989) and Kachanov et al. (1994) used MTS to address the problem of randomly distributed circular holes. Kachanov et al. (1994) also studied holes with irregular shapes by introducing a shape factor into the analysis. They stated that MTS is a reasonable approximation if the holes are randomly distributed. For randomly oriented elliptical holes, the solution of Kachanov et al. (1994) provides

\[
\frac{E}{E_0} = \frac{1}{1 + (3p + q)(1 - p)^{-1}}
\]  

(2.3)

where \((1-p)^{-1}\) accounts for interactions. If the influence of interactions is not taken into account the equation becomes

\[
\frac{E}{E_0} = \frac{1}{1 + (3p + q)}
\]  

(2.4)

2.2.4 Comparison of Effective Medium Theories

The comparison of the normalized elastic modulus for different schemes for circular holes is shown in Figure 2.1. The bottom horizontal axis is \(3p+q\), which includes both porosity and the shape factor. However, shape factor \(q\) is zero for circular holes. All schemes are the same for porosities up to approximately 20%. The normalized modulus curve for both interactive and non-
interactive holes are determined using Kachanov’s MTS. Non-interactive provides highest values. SCS does not appear to be valid for porosities above approximately 30%. Only the MTS is able to correctly predict the ratio of $E/E_o$, which is zero at 100 percent porosity. SCS reduces the normalized modulus to zero around 30% porosity. Assumption of non-interactive holes overestimates the effective modulus with respect to the other schemes. MTS is accurate in both small and high porosities like foam structure (Kachanov et al., 1994).

![Figure 2.1 Comparison of normalized elastic (Young’s) modulus determined by different schemes for randomly distributed circular holes.](image-url)
2.2.5 Effective Poisson's Ratio

Walsh (1980) stated that Poisson's ratio does not vary much with porosity and the error due to assuming constant Poisson's ratio in the solutions is reasonable. Jasiuk et al. (1994) demonstrated that the effective elastic modulus of a two-dimensional material containing holes is independent of the Poisson's ratio of the matrix and two-dimensional effective Poisson's ratio flows to a constant value as the percolation threshold is reached. Day et al. (1992) stated that Young's modulus is independent of the matrix Poisson's ratio. This result is exact for two-dimensional holes and can be proven analytically for low concentration of holes.

The DS of Zimmerman (1991) showed that increasing porosity decreases the elastic modulus in such a way as to cause the effective Poisson's ratio to approach 0.2 at a porosity of 100%. The trend that is followed is approximately linear regardless of matrix Poisson's ratio. The self-consistent equations of Hill (1965) and Budiansky (1965) also predicted similar trend. However, the effective Poisson's ratio reaches 0.2 at the porosity of 50% in their analyses.

2.3 Numerical Studies on Solids Containing Holes

Numerical applications to determine the elastic properties of heterogeneous materials have not been widely investigated due to difficulties of modeling the inclusions of holes. A few studies approach the problem of two-phase materials by comparing the numerical results with analytical solutions. Day et al. (1992) used an algorithm combining digital-image and spring network
technique to study the effective moduli of two-dimensional random isotropic composite sheets containing circular holes. They represented a continuum model by pixels and used the pixel lattice and material properties to define a spring network assigning elastic properties to linear Hooke springs to model the original continuum material. Effective elastic modulus was calculated using this discretized spring scheme with the help of a finite element algorithm. They studied circular holes of equal size in three different distributions including random distribution of holes and computed the normalized Young's modulus. They compared the numerical results with the effective medium theory for circular holes in a sheet (Thorpe and Sen, 1985). The numerically determined normalized modulus provided a good correlation with SCS at porosities lower than 20%, where SCS gives exact solution. For randomly distributed holes, the analysis was restricted to a maximum 50% porosity. The normalized modulus is lower for materials containing randomly distributed holes than those containing rectangular array of circular holes.

2.4 Experimental Studies of Solids Containing Holes

Although extensive studies were conducted on theoretical models for porous or cracked media, there is limited experimental data available with which to establish a relationship between theoretical models and experimental results. Experimental studies to predict the effective stiffness properties of solids containing holes have been limited to materials containing microporosity. Typically, tests are performed on specimens with variable porosities and results
are presented as a modulus versus porosity curve. An empirical correlation between modulus and porosity is then estimated by regression analysis from the experimental data. Experimental studies provide a reasonable means of describing and comparing different data sets and extrapolating results.

Rock mechanics literature contains only a limited number of studies where test results are compared with approximate analytical solutions. One reason for this is the difficulty of comparing data obtained from test materials containing holes with unknown shapes of the holes to correlations obtained from analytical methods using idealized hole shapes.

Experimental work on porous media, which contain either holes or cracks, is usually conducted on plates containing holes or cracks under static loading. The experimental work includes static testing (for instance uniaxial tension or unconfined compression tests) and dynamic testing (for instance ultrasonic velocity tests).

Experimental studies on porous materials usually investigate effective dynamic modulus by applying ultrasonic methods to determine effective elastic properties. The static elastic modulus is then calculated from the dynamic measurements. Numerous studies (for example Van Heerden, 1987; Eissa and Kazi, 1988) have shown there is a difference between the static and dynamic effective moduli. In engineering design, the statically determined properties are preferred over those obtained by dynamic methods because they better represent the actual high strain loading conditions.
Approximate analytical solutions have been compared to experimental data for materials other than rock to demonstrate how theories and experiments agree with each other. Most of these tests were conducted on ceramics, which have porosity between 30% and 50%, and materials such as porous glass and gypsum where porosities can be as high as 70%. The porosities in these specimens are microscopic porosities.

Comparison of analytical solutions and experimental data from ceramic and metals can be found in Rice (1977). Several analytical solutions (e.g. MacKenzie, 1950; Hashin and Shtrikman, 1963) give good but not excellent agreement with experimental data for Young's, shear and bulk moduli. Walsh et al. (1965) conducted bulk modulus measurements on a porous glass containing nearly spherical pores and Zimmerman (1991) compared their test results with some available analytical solutions. The Walsh et al. (1965) data set does not contain porosities greater than 50%, therefore it does not allow for discrimination between all analytical correlations. The data set shows good correlation with Norris (1985) and Kuster and Toksöz (1974) and SCS for low porosities. However, all methods give good correlation with experimental data for porosities lower than 20%.

Roberts and Garboczi (2000) compared the experimental data for the porous glass of Walsh et al. (1965) with FEM data for overlapping spherical pores and various effective medium theories. Agreement with experimental data is good for porosities lower than 30%. At higher porosities, their FEM results
underestimated the experimental data because the pores are not interconnected in glass.

Hilbert et al. (1994) compared the effective medium theories, assuming the pore shape is either spherical or tubular, with experimental data set of Berea sandstone. They calculated static and dynamic elastic modulus from static and dynamic testing. They found that SCS using tubular pores more closely approximates the effects of porous structure of Berea sandstone than using spherical pores.

Carvalho and Labuz (1996) conducted uniaxial tension test on aluminum plates containing randomly distributed circular holes. The results obtained from the plates indicate that the effective elastic modulus follows the predictions for the case of interacting holes from MTS and differential scheme.

Leite and Ferland (2001) tested the artificial rock consisting of a mixture of plaster, sand, water and polystyrene spheres using indentation tests indicated that both Young's modulus and compressive strength decrease with increasing porosity, which is created by polystyrene spheres.

2.5 Recent Studies on Mechanical Properties of Tuff

There are several studies to determine the mechanical properties and estimate the mechanical behavior of porous volcanic Tuff. However, experimental work on lithophysal Tuff, in which porosity is different from the porosity in sedimentary rocks, is limited. Schultz and Li (1995) conducted a detailed investigation of the strength properties of Calico Hills Tuff found in Yucca
Mountain. They tested cylindrical Tuff specimens that have total porosities between 24% and 39%. They did not record any large voids and cavities in the specimens. They documented the porosity dependence of elastic modulus and compressive strength of Tuff. However, elastic moduli of the specimens containing approximately 24% porosity exhibit a wide range of values between 1233 to 1668 ksi (8.5 to 11.5 GPa). Very few specimens outside of the porosity range were tested, therefore a clear decreasing trend of elastic modulus with increasing porosity is difficult to observe.

Wang and Kemeny (1993) presented a micromechanical model based on fracture mechanics. They verified their model by compression testing of samples from Topopah Spring Tuff under different confining pressures including zero confining pressure. Their specimens did not contain larger pores and inclusions. Their micro model predicted the nonlinear stress-strain behavior of Tuff and their experimental results indicated extensive cracking through pores.

Fuenkajorn and Daemen (1992) tested Apache Leap Tuff specimens to develop an empirical failure criterion. They observed large variations of the compressive strength and Young's modulus due to nonuniform distribution of pores, mineralogy, inclusions, welding and grain bonding. Again, Tuff they tested does not include large pores.

More recently, Price et al. (1994) conducted the uniaxial and triaxial compression tests on cylindrical lithophysal Tuff specimens and determined their compressive strengths. They presented correlations between statically and dynamically determined elastic properties of porous Tuff specimens recovered
from Yucca Mountain. Some of the specimens contained lithophysae, however due to small sizes of specimens, 4 inches (101.6 mm) in length and 2 inches (50.8 mm) in diameter, lithophysae has small dimensions. They did not test specimens containing large cavities. They found a significant reduction in static Young's modulus with increasing porosity.

2.6 Discussion

The approximate solutions have mainly focused on porous media. Porosity in porous media is microporosity and the media can be assumed as effectively homogeneous. The experimental studies performed to corroborate the results of approximate solutions have been conducted on materials containing micropores like porous ceramic, porous glass and sandstone. Furthermore, the analytical formulations were generated for microporosity but not for porosity due to large cavities such as lithophysae observed in Topopah Spring Tuff. Experimental applications have not been applied to materials containing larger voids. It is uncertain whether the effective homogeneity and isotropy can be pronounced the way it is for microscopically porous media.

One drawback of comparing experimental data with analytical correlations is that structure of two-phase material including shape of the cavities, sizes and their orientations corresponding to a particular scheme is not exactly known. Determining which scheme is more suitable is difficult since there is limited experimental verification of approximate schemes for elastic modulus. The schemes are approximate and use randomly distributed idealized shapes to
predict the elastic properties. It is difficult to include every actual detail in material structure in analytical correlations. Roberts and Garboczi (2000) demonstrated that the orientation of micropores, whether they are overlapped or not, is an important factor affecting the elastic properties while comparing the experimental and numerical data. Experimental data may or may not confirm a particular scheme. An agreement between analytical, numerical and experimental data is still valuable to predict elastic properties of two-phase material in preliminary engineering design and to understand the behavior of these materials with various porosity ranges.

Analytical and semi-analytical schemes and available experimental data show that the cavity or hole shapes are important for a reliable prediction of elastic properties of two-phase materials. The circular shape is the stiffest among various hole shapes (Zimmerman, 1986). The more elongated holes have higher compressibility, which is the reciprocal of bulk modulus, thus lower bulk modulus. Therefore, it is expected that elastic modulus also has a similar decreasing trend, but lower effective modulus with introduction of elongated holes, such as elliptical holes.

2.7 Conclusion

Literature review, including analytical, numerical and experimental studies on porosity-elastic modulus relationships has been summarized. The results show that normalized elastic modulus decreases with increasing porosity. Poisson's ratio does not vary much with porosity for Poisson's ratios between 0.1
and 0.2. The findings of semi-analytical and experimental studies are valid for microporosity and to date they have not been tested for porosity caused by lithophysae.
CHAPTER 3

GENERAL DESCRIPTION OF LITHOPHYSAL TUFF

3.1 Introduction

Characterization of the Tuff units at Yucca Mountain have been performed since the U.S. government enacted the Nuclear Waste Policy Act of 1982 and Yucca Mountain was chosen as one of the five potential sites for geologic disposal of high-level nuclear waste. In 1987, Yucca Mountain was chosen to be the only potential site for a high-level nuclear waste repository.

Information about the lithology, structure and geotechnical properties of the rock units within a 86.6 feet (26.4 meter) length of the cross drift at Yucca Mountain have collected by U.S. Bureau of Reclamation and U.S. Geology Survey (Mongano et al., 1999). The cross drift is a 1.68 miles (2.7 km) long and 16.4 feet (5.0 meters) in diameter tunnel designed to extend underground access to stratigraphic units within the proposed repository block. It is entirely excavated within the Topopah Spring Tuff formation of the Paintbrush group formed by pyroclastic flow and pyroclastic fall materials. The paintbrush group consists of four formations, the Tiva Canyon, Yucca Mountain, Pah Canyon and Topopah.
Spring Tuffs. The tunnel begins in the Topopah Spring crystal-poor upper lithophysae zone and passes through crystal-poor middle non-lithophysae zone and crystal-poor lower lithophysae zone.

The presence of the lithophysae has raised questions on the suitability of the rock mass for a geologic repository. Several studies have touched on the effect of porosity and lithophysal cavity content on the mechanical properties of Tuff.

3.2 Geology of the Repository Host Horizon

The proposed repository host horizon will be placed in Topopah Spring Tuff (Tpt). The Topopah Spring Tuff, where the repository would be located, was erupted about 12.8 million years ago (Sawyer et al. 1994) and has a maximum thickness of about 1150 feet (350 meters) in the vicinity of Yucca Mountain (Fox et al., 1990). Petrographically, Tpt is zoned from basal crystal poor high silica rhyolite, with silica content of approximately 75 percent to a capping crystal rich quartz latite with silica content of approximately 69 percent (Schuraytz et al., 1989). The actual repository will be approximately located in the middle to lower portion of the Topopah Spring Tuff. This section is densely welded, with variable fracture density and lithophysal content (BSC 2001). The repository will be located within two lithophysal zones, upper lithophysal zone (Tptpul) and lower lithophysal zone (Tptpll) (Mongano et al., 1999)

Of particular interest for this study are the upper and lower lithophysal zones of the Topopah Spring Tuff. Trapped pockets of gas within the volcanic
ash formed the macro-pores, called lithophysae or lithophysal cavities. Figure 3.1 is a photograph showing an outcrop rock specimen from the upper lithophysal zone from Topopah Spring Tuff.

Figure 3.1 Specimen of Topopah Spring Tuff, upper lithophysal zone.

3.3 Description of Repository Lithophysal Tuff Units

Excavations in the ECRB cross drift provides useful information regarding lithophysal cavities within the Tuff. Tuff lithologies encountered during the mapping are shown in Table 3.1. The table includes the percentage of different phases encountered in two lithophysae-rich zones during mapping. Both
lithophysal zones are moderately to densely welded, devitrified and vapor-phase altered.

In the upper lithophysal zone, lithophysal cavities generally comprise 25 to 40 percent of the rock, but as much as 60 percent locally. Aspect ratios are typically 1:1 to 5:4 with a few individual cavities up to 3:1 locally.

Table 3.1 Phase Percentages Encountered within Topopah Spring Upper and Lower Lithophysal Zones (After Mongano et al., 1999, 1999)

<table>
<thead>
<tr>
<th>Description</th>
<th>Phases Percentage (%)</th>
<th>Pumice</th>
<th>Phenocrysts</th>
<th>Lithic Fragments</th>
<th>Lithophysae</th>
<th>Matrix</th>
<th>Vapor-Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Lithophysal Zone</td>
<td></td>
<td>0 - 15</td>
<td>1 - 3</td>
<td>0 - 5</td>
<td>25 - 60</td>
<td>40 - 90</td>
<td>10 - 40</td>
</tr>
<tr>
<td>Lower Lithophysal Zone</td>
<td></td>
<td>3 - 7</td>
<td>1 - 2</td>
<td>12 - 25</td>
<td>5 - 30</td>
<td>56 - 90</td>
<td>3 - 12</td>
</tr>
</tbody>
</table>

Many of the larger cavities have irregular boundaries and appear to have formed from a number of coalesced lithophysal cavities. The lithophysae have pale red purple alteration margins from 0.04 to 0.2 inch (1 to 5 mm) wide. Vapor phase minerals coat the interior surfaces of lithophysal cavities (Mongano et al., 1999).

In the lower lithophysal zone, there are 5 to 30 percent lithophysae (locally 1 to 5 percent), however the size and shape of the lithophysal cavities vary widely depending on location within the drift. Shapes range from circular to gash-like and sizes range from 0.4 inch to 3.3 feet (1 to 100 cm) cavities (Mongano et al., 1999).
3.4 Conclusion

The proposed repository will be located in Topopah Spring Tuff formations of which parts consists of lithophysae. The lithophysal porosity controls the mechanical properties of Tuff that will be used in design. The shape of lithophysal cavities ranges from circular to gash-like form.
CHAPTER 4

NUMERICAL MODELING OF UNIAXIAL COMPRESSION TESTING ON
SOLIDS CONTAINING CIRCULAR HOLES

4.1 Introduction

Numerical techniques such as the finite element and finite difference are
widely applied to problems in solid mechanics, including rock and soil mechanics,
to determine the behavior of a system under loads and provide a designer insight
into physical mechanisms occurring within the system (Starfield and Cundall,
1988).

Here, numerical analysis is used as a tool to model and simulate
compression testing on solid models with randomly and uniformly distributed
holes. The analysis is done in two-dimensions enforcing condition of plane strain.
Although the actual media is three-dimensional, the advantage of two-
dimensional analysis over three-dimensional analysis is that the modeling effort
is relatively easier. The purpose of numerical analysis is to investigate the effect
of macro-porosity or lithophysal porosity on the deformation properties of solid
models containing circular holes without actually testing them. Macro-pores are
large, typically non-interconnected pores that have developed through a geological process.

The deformation parameters investigated for numerical models are deformation modulus and deformation ratio. These terms will be used in this chapter instead of elastic (Young's) modulus and Poisson's ratio that are appropriate for solid samples without voids or cracks.

4.2 Using FLAC 2D as a Numerical Modeling Tool

Several commercially available finite element or finite difference software packages can be used in numerical analysis. In this study, a two-dimensional finite difference code FLAC is used. FLAC was originally developed for applications in geotechnical and mining engineering, however now it can be used in variety of fields including mechanical engineering (Itasca, 1999). The versions used in this study were 3.5 and 4.1, which were developed for IBM-compatible microcomputers. FLAC version 4.1 is newer and has a better graphical user interface.

4.2.1 Finite Difference Method

The finite difference method is one of the basic discretization methods used to solve sets of differential equations. These differential equations are then replaced by an approximating, finite system of algebraic equations. In the finite differences method, every derivative in the sets of governing equations is
replaced by an algebraic expression written in terms of the field variables, such as stresses or displacements at a finite point set in space. These field variables are undefined within the elements. The finite difference equations are updated at each calculation timestep so that there is no need to create element matrices and store them into a global stiffness matrix. The grid generally used in finite differences is not restricted to rectangular shapes. Wilkins (1964) presents a method of deriving equations for elements of any shape, like the elements in finite element method.

4.2.2 FLAC 2D

FLAC is a two-dimensional explicit finite difference program. A finite difference mesh in FLAC is composed of quadrilateral elements. It is based on a Lagrangian calculation scheme in which the incremental displacements are added to the grid coordinates so that grid moves and deforms with the material it represents. This contrasts to Eulerian calculation scheme in which the material itself moves and deforms relative to a fixed grid. The Lagrangian formulation has an advantage for problems involving large distortion in the grid and material collapse. Although the constitutive formulation at each calculation step is a small-strain one, after many steps it is equivalent to a large strain formulation.

FLAC solves problems (static or dynamic) using a sequence of locally determined dynamic equilibrium states rather than a series of globally determined static equilibrium states (Last and Harkness, 1991). The reason for doing this is to provide numerical stability when the physical system is unstable.
In this calculation cycle, equations of motion are created to derive new velocities and displacements from forces and stresses. Then, strain rates are calculated from velocities, and are used to calculate new stresses using a built-in or user-defined constitutive law. One loop of the cycle occupies one timestep. All grid variables are updated from known values that remain unchanged, frozen, in that timestep. In other words, newly calculated stresses do not affect the velocities in one cycle. Timesteps should be very small so that the information cannot physically transfer from one element to the other in that time interval. The calculation speed should always keep ahead of the physical wave speed in finite difference grid so that the equations always use known values which are unchanged for the calculation step. This calculation scheme is called "explicit" and there is no iteration needed to compute stresses from strains in an element. Since no stiffness matrices are formed and updated memory requirements are at a minimum level. The disadvantage of the explicit method is that it needs very small timestep, which elevate the total computational duration. On the other hand, explicit methods are suitable to efficiently solve nonlinear, large-strain and physically instable system. They are not very efficient to solve for linear, small-strain models (Itasca, 1999).

Unlike the conventional finite element programs that produce a "solution" at the end of calculation, the explicit solution procedure is only conditionally stable. FLAC yields a solution when a mechanical equilibrium state is reached for a static analysis. There are two features at FLAC to help user determine whether the equilibrium is reached or not. These are unbalanced force and equilibrium
ratio. A gridpoint is surrounded by a maximum of four zones that contribute forces to that gridpoint. The algebraic sum of these forces, which is called unbalanced force, is almost zero at equilibrium. The model is considered to be in equilibrium when the maximum unbalanced force is small compared to the applied forces in the problem. Another and easier check can be made using equilibrium ratio which is a ratio of maximum unbalanced force to the representative internal force. Like the unbalanced force, the equilibrium ratio never decreases to zero, however a value of 0.01 or 0.001 can be accepted for equilibrium of the system. Both unbalanced force and equilibrium ratio are computed and displayed on computer screen during timestepping.

4.3 Numerical Model Setup

The one method to define a porous material is to consider it as a solid containing voids. Then the porous material can be modeled in two dimensions by introducing circular holes within a finite difference grid. In nature, like with lithophysae-rich Tuff, the shapes of the voids vary throughout the material and do not have a simple geometry. However, complex geometrical shapes are not feasible to model neither numerically nor experimentally. More difficulties arise when one considers the shapes and dimensions of voids are not the same throughout any representative physical body. To numerically model a porous material by reducing these difficulties, an idealized porous material is implemented. Since model geometry in numerical analysis will be used in
producing analog specimens, the geometry should be simple enough to produce analogs.

The simplest void geometry that can be generated in FLAC is a circle. Even though the lithophysae in Tuff are not perfectly spherical or ellipsoidal, it can be assumed that the lithophysae cross section can be idealized as circles and ellipses. However, elliptically shaped holes are not easy to produce for experimental specimens. In this dissertation, only circular holes are modeled and analyzed numerically. All holes are unpressurized, that is, there is no internal pressure applied to the internal boundaries of the holes.

Two different setups are used to represent solids with holes. In the first setup, uniformly distributed holes are chosen because of its simplicity in modeling. However, the lithophysae is not uniformly distributed, but randomly distributed. In second setup, this random nature is modeled using randomly distributed holes. FLAC finite difference grid containing 24 holes is shown in Figure 4.1.

Another important issue is the determination of porosity range of numerical models that will be analyzed. Since the main goal is to investigate the effect of lithophysal porosity on deformation properties of Tuff, assumed porosity for model should be within the range of existing porosity in lithophysal units of Topopah Spring Tuff. In this study, porosity is taken from 5 to 40%, which is comparable to the range of 5 to 30% by volume of lithophysae found in the Topopah lithophysae-rich Tuff units. Porosity in numerical models is created in
two ways by (1) placing uniformly distributed circular holes, (2) distributing circular holes randomly.

![Diagram of FLAC finite difference grid](image)

Figure 4.1 FLAC finite difference grid containing 24 randomly distributed circular holes with a total porosity of 11.9 %.

4.3.1 Uniformly Distributed Circular Holes

Porosity is introduced by uniformly distributed circles throughout the grid. The models are created by introducing 1, 9 and 36 equal size holes in 6 inch by 6 inch finite difference grid. The configuration and distribution of circular holes is presented in Figure 4.2. The purpose of using increasing number of holes for each setup is that these models will create a uniform distribution of porosity. The
different porosities are introduced by using different configurations of circular holes by increasing the diameter of each circle. In order to create isotropy, an equal center-to-center distance is established between circular holes in both directions in each configuration except the one with 1-hole where the center of the only circular hole is placed at the center of the grid. Porosity is defined as the total area of holes divided by the gross cross sectional area. The size of circular holes is determined by setting the porosity equal to 5%, 10%, 20%, 30% and 40%. The total porosity was divided by the total number of holes in each configuration to calculate the radius of each hole.

Matrix Poisson's ratio is chosen as 0.1, 0.2, 0.3, 0.4, 0.45 and 0.49 as an extreme case. The diameters of the circular holes and corresponding porosities are provided in Table 4.1.
Table 4.1 Uniformly Distributed Circular Hole Geometries

<table>
<thead>
<tr>
<th>Porosity</th>
<th>Number of Circular Holes</th>
<th>Radius (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>1</td>
<td>0.757</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.252</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>0.126</td>
</tr>
<tr>
<td>10%</td>
<td>1</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.357</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>0.1785</td>
</tr>
<tr>
<td>20%</td>
<td>1</td>
<td>1.515</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.5045</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>0.2525</td>
</tr>
<tr>
<td>30%</td>
<td>1</td>
<td>1.855</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.618</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>0.309</td>
</tr>
<tr>
<td>40%</td>
<td>1</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.714</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>0.357</td>
</tr>
</tbody>
</table>

4.3.2 Randomly Distributed Circular Holes

Numerical models containing randomly distributed holes are generated using PFC^{2D}, a discrete element software of ITASCA. PFC^{2D} generates a given number of circular solid particles, their radius and uniformly distributes them into a 6 inch by 6 inch square. The identification numbers, radii and locations of particles are written in an output file and then inserted to finite difference grid in FLAC. Overlapping circles are avoided. Particles touching each other are separated leaving a minimum of 0.12 inch distance between the particles. The same distance is also provided between any hole and the outside boundaries so that none of the holes has contacts with the boundaries of the finite difference grid.
grid. An example for models containing circular holes generated according to the procedure given above is seen in Figure 4.3. Some of the models containing randomly distributed circular holes are shown in Appendix I.

![Figure 4.3 An example for the model containing randomly sized and distributed circular holes.](image)

For randomly distributed models, 24 circular holes (except for two models) are placed in the finite difference grid. The porosity of models is between 4 and 38 percent. A minimum of three specimens containing randomly distributed holes is generated for every 5 percent porosity increment. After the computer models are generated, the radius of each hole is checked with those of available cylindrical rods, which are used to produce analog specimens for testing. If necessary, the radii of circular holes are approximated to the radii of cylindrical rods to be able to make a comparison between numerical and experimental results. The range of radii used for the holes in the models is between 0.12 and 0.62 in inches. Poisson's ratio for the matrix is chosen as 0.1, 0.2, 0.3 or 0.4.
4.3.3 Material Model

Elastic material model is assigned to the finite difference zones that create the solid part using built-in linear elastic model in FLAC whereas a null material model is assigned to the circular holes. Elastic behavior is described by two parameters, bulk modulus, $K$ and shear modulus, $G$. The density is required for each solid zones material. Bulk and shear moduli values can be calculated from elastic (Young's) modulus, $E$ and Poisson's ratio, $\nu$, as seen below.

$$K = \frac{E}{3(1-2\nu)} \quad (4.1)$$

$$G = \frac{E}{2(1+\nu)} \quad (4.2)$$

It is recommended to use bulk modulus and shear modulus rather than elastic modulus and Poisson's ratio because the first pair express the material behavior better than the second, especially for certain admissible materials (Itasca, 1999). The matrix elastic modulus is selected as an arbitrary value to initialize the model. Therefore, it is advantageous to use the second pair to calculate the parameters necessary for modeling elastic behavior in a parametric study.

In order to generate circular holes in the grid, a null material model is assigned to the zones that represent circles. A null material model represents
material that is removed from the grid. The stresses are zero within null zones and there are no body forces acting on them.

The required properties for the elastic model used in all models are shown in Table 4.2. The properties are selected arbitrarily because the material model is linearly elastic and there is no failure. An arbitrary value of 10,000 psi for matrix elastic modulus and a range of values for the Poisson's Ratio are chosen for each run. The effect of different values of density is negligible in the analysis.

Table 4.2 Elastic Material Properties Used in Models

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus, psi</td>
<td>10,000</td>
</tr>
<tr>
<td>Density, pci</td>
<td>0.08</td>
</tr>
</tbody>
</table>

4.3.4 Timestep Determination

In section 4.2.2, it was mentioned that the solution procedure in FLAC is not unconditionally stable. Any local disturbance of equilibrium is propagated at a stiffness dependent rate consistent with Newton's law of motion (Last and Harkness, 1991). Thus, to enable a stable solution, the speed of calculation must be greater than the maximum speed at which information propagates (Itasca, 1999). This condition can be satisfied using a timestep that is smaller than a critical timestep. A timestep that will satisfy the stability condition for an elastic solid is given as
\[ \Delta t < \frac{\Delta x}{c} \]  

(4.3)

where \( \Delta x \) is the size of the element and \( c \) is maximum speed, typically the P-wave velocity, at which the information can propagate. The P-wave velocity can be computed using the following relationship

\[ c = \sqrt{\frac{K + 4G/3}{\rho}} \]  

(4.4)

where \( \rho \) is the material density. The primary parameters, which will determine the timestep, are the element length, material density, bulk and shear moduli. The stability condition above shows that timestep decreases with increasing stiffness of the model.

Timestep is calculated separately for models that include circular holes and the ones that have elliptical holes because different element sizes are used in both models. Critical time increases with increasing Poisson's ratio. The critical timestep is determined to be \( 1 \times 10^{-4} \) second using Poisson's ratio of 0.45. Then, the timestep as chosen as \( 5 \times 10^{-5} \) and used in each run at which Poisson's ratio is smaller than 0.45. For the extreme case of Poisson's ratio of 0.49 for models with uniformly distributed holes, the timestep computed is still associated with a stable solution.
4.3.5 Boundary Conditions and Loading

The boundary conditions consist of the field variables, for instance stresses and displacements that are defined at the boundary of a numerical grid. Here prescribed-displacements are applied along the real boundaries that exist in the physical media being modeled, that is, the four sides of the square grid. In FLAC, the displacements cannot be directly controlled and in fact, they are not used in calculation process (Itasca, 1999). Instead, it is necessary to prescribe the velocities in order to apply a given displacement to a boundary. A zero velocity applied in any direction at a gridpoint fixes the gridpoint at that direction.

For simulation of models with uniformly distributed holes, bottom and top gridpoints are restrained by assigning a fixed-boundary condition and the boundaries at the vertical sides are set free to move horizontally as seen in Figure 4.4. In order to simulate rigid movement of an upper platen on a specimen zero velocities applied to the gridpoints at the top and bottom in horizontal directions.

The models containing random holes are uniaxially loaded under compression in (a) vertical direction (b) horizontal direction to investigate the effect of anisotropy due to the random sizes and distribution of circular holes. In both analyses, the lateral displacements along the gridpoints where the velocity loads are being applied are restrained.

The load is applied as a displacement load to very top and bottom gridpoints of the grid. The displacement can be explained as total velocity that
would occur over a particular number of steps. Using the presumption of linear elasticity, the amount of vertical displacement is arbitrarily chosen.

![Figure 4.4 FLAC model boundary conditions.](image)

The velocity value at one step should be kept very small with a large number of timesteps to minimize the shocks to the system being modeled. In section 4.3.4, the timestep is chosen as $5 \times 10^{-5}$. In order to satisfy timestep criteria, an arbitrary 0.5 inch displacement is applied as a velocity of $5 \times 10^{-5}$ inch at each timestep. In this way at the end of total 10,000 steps the value 0.5 inches displacement corresponds to a strain of 8.33% are being applied. The model generated numerically has a very small mass. This eliminates the significance of
stresses due to the weight of the material. Therefore, gravitational acceleration, which causes body forces to act on all gridpoints, is not introduced the analysis.

4.4 Analysis

Theory of linear elasticity is usually applicable to the cases where deformation of material is small and elastic. The linear relation between the stresses and strains in a spring is known as Hooke's law (for the uniaxial case)

\[ \varepsilon = \frac{\sigma}{E} \]  

(4.5)

where \( \sigma \) is the normal stress and \( \varepsilon \) is the normal strain and \( E \) is Young's modulus. However, Young's modulus is not enough to define the relationship between stresses and strains when models do not deform (extend or contract) only in one direction. The ratio between extension and contraction in orthogonal directions is defined as Poisson's ratio, \( \nu \). The strains in \( x \)-, \( y \)- and \( z \)-coordinates can be written as (Hooke's law for triaxial stress state)

\[ \varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \]
\[ \varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)] \]  

(4.6)
\[ \varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] \]
The linear elastic isotropic material model in FLAC implements the Hooke's law in plane strain where out-of-plane strains are zero. Since $\varepsilon_z = 0$ the last equation of equation (4.6) becomes

$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad (4.7)$$

where z-direction is the out-of-plane direction. Thus, the plane strain assumption reduces the determination of $\sigma_x$ and $\sigma_y$ as functions of $x$ and $y$ only (Timoshenko and Goodier, 1987). By substituting $\sigma_z$ given in the equation (4.7) the first two equations in equation (4.6) can be rewritten as

$$\varepsilon_x = \frac{1}{E} \left[ (1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y \right] \quad (4.8)$$
$$\varepsilon_y = \frac{1}{E} \left[ (1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x \right]$$

Rearranging the equations and solving for $E$ and $\nu$, yields

$$E = \frac{\varepsilon_y (\sigma_y)^2 - 2\varepsilon_x (\sigma_y)^2 + \sigma_z \varepsilon_z \sigma_y + \sigma_y \varepsilon_x \sigma_x - 2\varepsilon_y (\sigma_x)^2 + \varepsilon_x (\sigma_x)^2}{(\varepsilon_y - \varepsilon_x)(\varepsilon_y \sigma_x + \varepsilon_x \sigma_y - \varepsilon_x \sigma_x - \varepsilon_x \sigma_y)} \quad (4.9)$$

and
4.4.1 Calculation of Average Stresses and Strains

After the computation is executed, FLAC yields the stresses, that is $\sigma_x$, $\sigma_y$, $\sigma_z$, in each quadrilateral zone and zero stresses at the zones within the circular holes. In order to calculate the average stress that represents the average stress state in the model in any particular direction, the stress values of all zones are summed and then divided by the total number of zones.

Similarly, the average strain along the horizontal direction for models is determined from the average horizontal displacements. The average horizontal displacement are computed adding the horizontal displacements on the gridpoints at both boundaries then dividing the total number of gridpoints along the horizontal boundary. The average horizontal strain is calculated using $\varepsilon = \Delta L / L$ where $\Delta L$ and $L$ are the average horizontal displacement and the width of finite difference grid, respectively. The vertical strain is already known as 0.833 because the displacement load is calculated and applied from this predetermined strain level.

4.4.2 Calculation of Deformation Modulus and Deformation Ratio

The average values of $\sigma_x$, $\sigma_y$, $\sigma_z$, $\varepsilon_x$ and $\varepsilon_y$ determined for each matrix Poisson's ratio and porosity are substituted into the equation (4.9) to calculate
the deformation modulus and deformation ratio. Calculated deformation modulus and ratio are classified according to the porosity and number of holes and normalized with respect to matrix elastic modulus and matrix Poisson's ratio, respectively.

4.4.3 Verification of Formulations on Finite Elastic Medium

The implementation of plane strain Hooke's law described above is verified using a finite elastic solid medium. The purpose is to backcalculate the matrix elastic modulus and Poisson's ratio from the average stresses and strains computed by FLAC. The approximation depending on average stresses and strains assumptions would be assumed as correct if the difference between assigned and backcalculated values of matrix elastic modulus and Poisson's ratio are very small.

Table 4.3 shows the results of verification for solid model. The percent error between assigned and backcalculated elastic modulus varies between 0.4% and 0.5%. The error range between assigned and backcalculated Poisson's ratios are within 0.2% range. The percent error for elastic modulus and Poisson's ratio is very low.

Verification study proves that the elastic material properties, mainly $E$ and $\nu$, entered as input into the model are backcalculated within a very small range of error using the approximation of average stresses and strains. Thus, the same method of analysis can be used for models with circular holes.
Table 4.3 Percent Errors for Solid Models

<table>
<thead>
<tr>
<th>$E_0$ (psi)</th>
<th>$\nu_0$</th>
<th>$E_{calculated}$ (psi)</th>
<th>$\nu_{calculated}$</th>
<th>Error for $E$ (%)</th>
<th>Error for $\nu$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>0.1</td>
<td>10039.5379</td>
<td>0.09980</td>
<td>0.3954</td>
<td>0.198</td>
</tr>
<tr>
<td>10000</td>
<td>0.2</td>
<td>10040.9258</td>
<td>0.19963</td>
<td>0.4093</td>
<td>0.186</td>
</tr>
<tr>
<td>10000</td>
<td>0.3</td>
<td>10043.0084</td>
<td>0.29951</td>
<td>0.4301</td>
<td>0.164</td>
</tr>
<tr>
<td>10000</td>
<td>0.4</td>
<td>10046.3605</td>
<td>0.39943</td>
<td>0.4636</td>
<td>0.142</td>
</tr>
<tr>
<td>10000</td>
<td>0.45</td>
<td>10048.1880</td>
<td>0.44943</td>
<td>0.4819</td>
<td>0.126</td>
</tr>
<tr>
<td>10000</td>
<td>0.49</td>
<td>10048.6380</td>
<td>0.48947</td>
<td>0.4864</td>
<td>0.108</td>
</tr>
</tbody>
</table>

4.5 Results and Discussions

The results are presented as normalized deformation modulus and deformation ratio curves. The normalized modulus is established by taking the ratio of deformation modulus of the model over matrix elastic modulus, which is equal to 10,000 psi. The curves are extended to zero percent porosity where moduli and Poisson's ratios represent matrix properties.

4.5.1 Models with Uniformly Distributed Circular Holes

The normalized deformation moduli of models with circular holes for both boundary conditions, $E/E_0$, are plotted versus percent porosity for each matrix Poisson's ratio and can be seen in Figure 4.5 (a) through (b). The following discussion is based on the curves presented in Figure 4.5.
• The model with one hole where the concentration of zero stiffness at the center gives the lowest normalized modulus for varying matrix Poisson's ratios.

• The normalized modulus curves for \( \nu_o = 0.1 \), seen in Figure 4.5 (a), fall into a wider envelope because of higher compressibility of this particular model. The envelope becomes narrower with increasing matrix Poisson's ratio. The difference between normalized modulus curves for models 9 and 36 holes is small.

• The models with Poisson's ratio of 0.2 and higher produce almost same normalized deformation modulus distribution with porosity. Therefore, the Poisson's ratio of matrix material is not a significant factor contributing the deformation modulus of the model.

• Figure 4.6 shows an envelope of normalized deformation modulus computed for different Poisson's ratios and number of holes. Normalized deformation moduli of models containing only one hole are excluded from the graph because they are not representative for homogenous distribution of holes in a solid. An envelope is plotted between minimum and maximum modulus values at a given porosity without differentiating according to Poisson's ratios and the distribution of holes. The envelope gets wider with increasing porosity. The maximum percent difference that the envelope represents is 8% for models free sides.
Figure 4.5 Normalized deformation modulus versus percent porosity (a) for $\nu_0 = 0.1$ and (b) for $\nu_0 = 0.2$. 

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Figure 4.5 Normalized deformation modulus versus percent porosity (c) for \( \nu_o = 0.3 \) and (d) for \( \nu_o = 0.4 \).
Figure 4.5 Normalized deformation modulus versus percent porosity (e) for $\nu_o = 0.45$ and (f) for $\nu_o = 0.49$. 

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As seen in Figure 4.6, the bulging section at approximately 5% porosity is due to high deformation modulus computed for models with 36 holes.

![Normalized modulus envelopes for models with uniformly distributed circular holes.](image)

Figure 4.6 Normalized modulus envelopes for models with uniformly distributed circular holes.

Deformation ratios are calculated and plotted as a function of porosity at Figure 4.7. The Poisson's ratio for 0% porosity is taken as matrix Poisson's ratio.

Discussion of the results is the following:

- The models containing a single circular hole, at all Poisson's ratios, behave differently than the other models by showing decreases in compressibility with increasing porosity for and increases in compressibility for $\nu_o$ greater than 0.3.
• Other models show a similar trend without depending upon number of holes introduced into the model. As shown in Figures 4.7 compressibility decreases for porosities less than 20%. At higher porosities, compressibility slightly increases.

• Models having $\nu_o$ higher than 0.1 show a similar trend and compressibility increases slightly without depending on the configuration of uniformly distributed holes.

• The normalized deformation ratio curves for $\nu_o = 0.1$, as seen in Figure 4.7 (a), show a different trend comparing with others. The deformation ratios first increase then decrease.

• For a material of which matrix Poisson's ratio is between 0.2 and 0.3, for instance Topopah Spring Tuff, the dependency of deformation modulus ratio on porosity is small.

4.5.2 Models with Randomly Distributed Circular Holes

In order to investigate the effect of anisotropy due to random distribution of holes, the models are loaded in horizontal and vertical directions separately. The equations in section 4.4 are used with an adjustment in plane strain direction in two different axes.
Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity for (a) $v_o = 0.1$ and (b) $v_o = 0.2$. 

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Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity for (c) \( \nu_o = 0.3 \) and (d) \( \nu_o = 0.4 \).
Figure 4.7 Deformation ratios for uniformly distributed holes versus percent porosity for (e) \( \nu_0 = 0.45 \) and (f) \( \nu_0 = 0.49 \).
The normalized deformation moduli of models with circular holes in x- and y-directions, $E_x/E_0$ and $E_y/E_0$, are plotted versus percent porosity for each matrix Poisson's ratio and can be seen in Figure 4.8 for models loaded in (a) y- and (b) x-directions. The conclusions are:

- The normalized deformation modulus decreases with increasing porosity as seen in Figure 4.8. Two polynomial regression curves are generated for normalized modulus of matrix Poisson's ratios of 0.1 and 0.4. The regression curve belonging to $\nu_0 = 0.4$ demonstrates a somewhat higher deformation modulus as $\nu_0 = 0.4$ represents lower compressibility than $\nu_0 = 0.1$. At higher porosities, the differences between the two curves increase slightly. Thus, from Figure 4.8 it can be said the matrix Poisson's ratio does not significantly affect deformation moduli for models with randomly distributed circular holes.

- Figure 4.9 (a) through (d) shows the calculated deformation modulus in both loading directions. The solid line represents the 45-degree line. The normalized modulus values of different matrix Poisson's ratios and the 45-degree line overlay. Thus, models analyzed here are approximately elastically isotropic due to larger sizes of circular holes randomly introduced in the model.

Normalized deformation ratios are plotted as a function of porosity at Figure 4.10 for matrix Poisson's ratios of 0.1, 0.2, 0.3 and 0.4. Polynomial regression curves for each data set are also added into the plot. Discussion of the results as follows:
Figure 4.8 Normalized deformation modulus versus percent porosity with different deformation ratios. Models containing randomly distributed holes are loaded in (a) y-direction and (b) x-direction.
Figure 4.9 Normalized deformation modulus in both loading directions for (a) $\nu_o = 0.1$ (b) $\nu_o = 0.2$. Solid line represents the 45-degree line.
Figure 4.9 Normalized deformation modulus in both loading directions for (c) $\nu = 0.3$ (d) $\nu = 0.4$. Solid line represents the 45-degree line.
• Deformation ratios decrease slightly showing an increase in compressibility for models which have \( \nu_o \) greater than 0.2.

• Deformation ratios almost increase linearly for \( \nu_o = 0.1 \) showing a decrease in compressibility. Similar trend is also seen for models with \( \nu_o = 0.2 \) although not as sharp as those of 0.1.

• The deformation ratios for models with \( \nu_o = 0.3 \) and \( \nu_o = 0.4 \) are almost identical to the matrix Poisson’s ratios with a small decrease in compressibility. Although, models with \( \nu_o = 0.4 \) have higher compressibility.

4.6 Comparison of Deformation Modulus for Both Models

Deformation moduli for both models with uniformly and randomly distributed holes are plotted together in Figure 4.11 (a) through (d) for matrix Poisson’s ratios of 0.1, 0.2, 0.3 and 0.4. All models are loaded in y-direction. The normalized deformation moduli for models containing uniformly distributed holes are higher than the ones containing holes with random sizes and locations showing that models containing uniformly distributed holes have higher deformation moduli. This difference can be explained by the columns between uniformly distributed holes that carry the most of the load. The difference between the values for both models is more pronounced over 10\% porosity because increasing diameters of holes soften the model for randomly distributed holes.
The columns between the uniformly distributed holes in vertical direction still increase the stiffness of the model despite their reduced thickness due to increases in hole diameters.

As observed for models containing randomly distributed circular holes, dependency of deformation ratio on porosity is small for the values of matrix Poisson's ratio between 0.2 and 0.3.
Figure 4.11 Normalized deformation modulus in y-direction for (a) \( \nu_o = 0.1 \) (b) \( \nu_o = 0.2 \). Empty circles represent randomly distributed holes while solid circles, squares and diamonds represent models with 1, 9 and 36 holes, respectively.
Figure 4.11 Normalized deformation modulus in y-direction for (c) $\nu_o = 0.3$ (d) $\nu_o = 0.4$. Empty circles represent randomly distributed holes while solid circles, squares and diamonds represent models with 1, 9 and 36 holes, respectively.
4.7 Conclusion

Numerically conducted uniaxial compression testing on models in two dimensions containing both uniformly distributed and randomly sized and distributed holes are generated to investigate the effects of porosity on deformation modulus and deformation ratio. The deformation modulus and deformation ratios are then calculated using average stresses and strains developed in the model through Hooke’s law for plane strain assumptions. The deformation modulus decreases with porosity regardless of matrix Poisson’s ratio. Deformation ratios decrease with increasing porosity due to an increase in compressibility, especially at high matrix Poisson’s ratios. For matrix Poisson’s ratios between 0.2 and 0.3, the dependency of deformation ratio of porous media on porosity is small and can be neglected.
CHAPTER 5

VERIFICATION OF NUMERICAL ANALYSIS BY BIAXIAL COMPRESSION TESTING OF URETHANE SPECIMENS

5.1 Introduction

In order to verify the deformation modulus computed through numerically conducted compression testing under plane strain conditions, biaxial compression testing is conducted on analog urethane specimens in which the porosity is introduced by cylindrical holes that extend through the specimen. Urethane was chosen for testing material because it is an isotropic linear elastic material. The cubic urethane specimens containing open cylinder shape holes represent circular holes under plane strain conditions provided in biaxial compression testing.

5.2 Specimen Preparation

Urethane is a rubber-type material and is produced under a controlled environment. Uniaxial compression testing of a cylindrical urethane specimen, with length to diameter ratio of 2, was conducted to ensure that material behavior
is linear elastic under compression and does not show any hysteresis during unloading. As seen in Figure 5.1, the urethane specimen exhibits linear elastic behavior without any hysteresis.

![Stress versus strain curve for urethane](image.png)

Figure 5.1 Stress versus strain curve for urethane. Circles are the data points and solid line is the best-fit curve.

Urethane specimens were prepared using a 6 inch aluminum cubic mold and finely machined cylindrical rods. The mold consists of three parts: base plate, rods and side plates. The side plates were connected together and to the base plate to form a box with an open top. The cylindrical rods were used to produce holes into the specimen. The rods were fixed to the base plate by with setscrews. The holes are spaced in a rectangular pattern similar to those tested numerically, as seen in Figure 4.2 in Chapter 4. The location of each rod on the platen and its diameter are fixed within a specimen of a particular porosity. The
side plates were connected together and to the base plate to form a box with an open top. Using this procedure, thirteen urethane specimens, one solid and twelve with holes, were produced by VIP Rubber, Inc., located in LaHabra, California.

The Poisson’s ratio was not measured, but was estimated to be between 0.45 and 0.49. Figure 5.2 shows some of the specimens produced for testing.

![Urethane specimens of different porosities and number of holes.](image)

**Figure 5.2** Urethane specimens of different porosities and number of holes.

The number of holes, hole diameters and final porosities of urethane specimens are given in Table 5. Three different sets of holes (1, 9 and 36) are used to investigate the effect of different number of holes.
Table 5.1 Geometry of Holes in Urethane Specimens

<table>
<thead>
<tr>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Cylindrical Holes</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>10%</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>20%</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>30%</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>36</td>
</tr>
<tr>
<td>40%</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>36</td>
</tr>
</tbody>
</table>

5.3 Biaxial Compression Testing

Urethane specimens were tested under plane strain conditions. A special biaxial compression test assembly was manufactured for the analog cube specimens. The assembly consists of a base plate, two out-of-plane plates and Linear Variable Displacement Transducer (LVDT) clamps to secure the LVDTs in place. Figure 5.3 is a photograph of the plane strain compression test assembly. The specimens were placed in plane strain compression test assembly such that the cylindrical holes were facing the out-of-plane plates. The nuts were tightened so that the out-of-plane plates were snug against the specimen, resulting in a plane strain condition. During the testing, the axial load, axial strain, lateral strains and out-of-plane strains were monitored and recorded.
Securing rods for vertical plates

Out-of-plane plate

LVDT clamp for out-of-plane displacements

Base plate

Lateral strain LVDT clamp

Figure 5.3 Proving ring assembly.

5.3.1 Instrumentation and Data Collection

A small load frame was used to displace the specimen into the associated proving ring, shown in Figure 5.4. The load frame was a Soiltest Vera Loader with a calibrated 10,000 lb capacity proving ring. It was set at a displacement rate of 0.02 inches per minute. The maximum axial deformation for each test was 0.2 inches.

The data acquisition system consists of a Daytronic System 10 mainframe, eight LVDTs and the UtiliPAC410 software from Daytronic Corporation. The LVDTs were used to measure all of the displacements. They were calibrated to predetermined limits using a calibrated micrometer.
Table 5.1 contains a description for all the LVDTs, which are used to measure the horizontal and vertical strains and the axial force, and their associated channels of the data acquisition system. The LVDTs were calibrated to read displacements in inches. Electrical signals were sent from the LVDTs to the load frame, which converts the voltage signal to inches. The load frame interacted with a computer via the UtiliPAC410 software. The UtiliPAC410 software allowed the computer to store the readings, in inches, from the LVDTs via the load frame. The software was set to record readings from each of the eight channels every 0.5 seconds, that is a sampling rate of 2 Hz.
Table 5.2 Description of LVDTs

<table>
<thead>
<tr>
<th>Channel Number</th>
<th>Purpose</th>
<th>Limits (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Axial displacement</td>
<td>± 0.2</td>
</tr>
<tr>
<td>2</td>
<td>Proving ring displacement</td>
<td>± 0.1</td>
</tr>
<tr>
<td>3</td>
<td>Front side: out-of-plane displacement</td>
<td>± 0.02</td>
</tr>
<tr>
<td>4</td>
<td>Back side: out-of-plane displacement</td>
<td>± 0.02</td>
</tr>
<tr>
<td>5</td>
<td>Lateral displacement: upper left side</td>
<td>± 0.1</td>
</tr>
<tr>
<td>6</td>
<td>Lateral displacement: middle left side</td>
<td>± 0.1</td>
</tr>
<tr>
<td>7</td>
<td>Lateral displacement: lower right side</td>
<td>± 0.1</td>
</tr>
<tr>
<td>8</td>
<td>Lateral displacement: middle right side</td>
<td>± 0.1</td>
</tr>
</tbody>
</table>

A proving ring was used in conjunction with the LVDT on Channel 2. The Channel 2 LVDT measures the amount of deformation in the proving ring and the deformation was converted into force. The conversion from deformation to force is made using the data from the calibration of the proving ring by a qualified supplier.

5.3.2 Data Processing and Analysis

The axial and lateral strains were calculated from axial and lateral deformation readings respectively and divided by the original undeformed length of the specimens. Graphical representation of axial stress versus lateral and axial strain for the solid urethane specimen is shown in Figure 5.5. Deformation modulus is calculated by taking the slope of linear portion of axial stress-strain curve, and the deformation ratio is calculated by dividing deformation modulus by the slope of lateral stress-strain curve according to ASTM D3148 (2002). The
deformation moduli, which are determined by biaxial compression testing, are plane strain moduli.

5.4 Results

Labuz et al. (1996) state that in order to achieve 90% plane strain condition, out-of-plane strain ($\varepsilon_z$) would be 2% of the axial strain ($\varepsilon_y$) for a material with $\nu_0 = 0.2$ and zero confining pressure. The nuts are tightened to ensure a good contact between urethane and the plates, which prevents any out-of-plane displacements. However, some confining pressure may be applied to the specimen, but this confining pressure should be small comparing to the axial stress. The calculated out-of-plane strains are approximately 3% for porosities less than 10% and 0.1% for those higher than 10%. The Poisson's ratio of urethane is also higher than 0.2. Therefore, plane strain conditions are achieved within sufficient limits. The elastic modulus of the solid specimen was determined to be 1293 psi. The deformation modulus of each specimen was then normalized with respect to the elastic modulus of solid specimens. The actual and normalized deformation modulus of the specimens is provided in Table 5.3. Deformation modulus decreases with increasing porosity and normalized modulus values are close to each other showing that of different number of holes introduced into the specimens do not affect the decreasing trend in stiffness. The difference between numerical and testing data shows that testing underestimates the deformation modulus slightly. This is probably due to not providing a good confinement in out-of-plane direction.
Figure 5.5 Axial and lateral strains versus axial stress for solid urethane specimen.

In addition, the contact faces of specimens are not perfectly even because during manufacturing capillary action caused the urethane to rise along the rods, creating an uneven surface.

Table 5.3 Actual and Normalized Deformation Modulus Values of Urethane Specimens

<table>
<thead>
<tr>
<th>Porosity (%)</th>
<th>Deformation Modulus (psi)</th>
<th>Normalized Deformation Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 Hole</td>
<td>9 Holes</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>962</td>
<td>1005</td>
</tr>
<tr>
<td>20</td>
<td>850</td>
<td>836</td>
</tr>
<tr>
<td>30</td>
<td>661</td>
<td>715</td>
</tr>
<tr>
<td>40</td>
<td>502</td>
<td>548</td>
</tr>
</tbody>
</table>
Normalized deformation moduli for each of the porosities for different number of holes are plotted in Figure 5.5 (a). The decrease in modulus can be represented best by an exponential equation:

$$\frac{E}{E_0} = e^{-0.023p}$$  \hspace{5cm} (5.1)

where $p$ is the porosity. The $R^2$ is 0.9664. The value of $R^2$, which is between 0 and 1, is a measure of how data fit to the regression line. A value of $R^2$ closer to 1 indicates that data has a statistically good trend. Normalized moduli of the same models determined through numerical analysis are plotted together with experimental data as seen in Figure 5.5 (b) showing a good correlation between the results of numerical and actual testing. The best fit line for numerical data is also exponential and given as

$$\frac{E}{E_0} = e^{-0.0215p}$$  \hspace{5cm} (5.2)

The regression equations are nearly identical, which also demonstrates the good correlation between numerical and experimental results.
Figure 5.6 Normalized deformation modulus versus porosity for (a) test data and (b) data for both testing and corresponding numerical models. Solid and dotted lines are the best-fit curves for experimental and numerical data, respectively.
5.5 Conclusion

Normalized deformation moduli calculated using a finite difference method are compared with those determined through biaxial compression testing of urethane specimens. Numerically evaluated deformation modulus values slightly overestimate the modulus of urethane specimens. The difference between two best-fit curves for numerical and testing results is approximately 4%. Good correlation between numerically and experimentally determined deformation modulus for models with uniformly distributed holes proves that deformation moduli computed through numerical analysis are also relevant for solids containing randomly distributed holes.
CHAPTER 6

UNIAXIAL COMPRESSION TESTING OF GYPSUM PLASTER SPECIMENS CONTAINING CYLINDRICAL TUBES

6.1 Introduction

As a simple approach, a single pore can be represented by a cylindrical tube. This setup has been used by other researchers to study compressibility of porous media. An analytical model with cylindrical tubes of a circular or elliptical cross-section was used to correlate mechanical properties of rocks to their pore structures was used by Biot (1956) to study the attenuation of elastic waves in saturated porous media and by Scheidegger (1974) to study the effect of porosity and grain diameter on permeability. Also, Walsh et al. (1965) found the compressibility of a tubular pore of elliptical cross section under plane stress conditions for attempts to simulate porosity in sandstone.

Although cylindrical tubes do not truly represent the pore structure of lithophysae-rich Tuff, it is an appropriate starting point to attempt to correlate deformation modulus with porosity. This model is the two dimensional counterpart of the three dimensional composite spheres models that will be
introduced in next chapter. Furthermore, it is easy to produce analog specimens containing circular tubes of varying porosities by adjusting the radii of the cross sections of tubes.

The uniaxial compression test is performed for cubic gypsum plaster specimens of containing (1) uniformly and (2) randomly distributed open-ended cylindrical tubes in order to simulate porosity. Plaster specimens have same number of holes and distribution that were modeled numerically in Chapter 4. Deformation modulus and compressive strength are calculated, and deformation modulus computed for both test specimens and their numerical doubles are compared.

6.2 Test Specimens

Uniaxial compression testing on several cylindrical gypsum plaster specimens having a length to diameter ratio of 2 was conducted to assure that material behaves linearly elastic under compression and does not show any hysteresis during unloading. A plaster water mix was produced using 1:2 water to plaster ratio that recommended by the manufacturer. The same proportions were used for all batches. As seen in Figure 6.1, a plaster specimen shows linear elastic behavior without any hysteresis. The Poisson's ratio was measured as 0.31.

Gypsum plaster specimens containing open-ended cylindrical tubes were produced for testing using a 6 inch aluminum cubic mold. Specimens containing uniformly distributed cylindrical tubes were assembled as explained in section 5.2
in Chapter 5. Randomly distributed cylindrical tubes were produced using the locations and diameters of holes as they were created for input in FLAC. The rods were fixed to the base plate by using glue. They were removed after the plaster hardened and they were used to produce other specimens. After pouring the gypsum plaster into the mold, the top surface was leveled using a straight edge and the specimen was allowed to dry over night. The next day, the mold was removed and the specimen was weighed daily until it reached a constant weight. Then, loading surfaces of plaster specimens are ground flat to provide leveled surfaces because the uniformity of stress distribution on the loading surfaces controls the accuracy of strength (Demiris, 1974).

Figure 6.1 Stress versus strain curve for gypsum plaster. Circles are the data points and solid line is the best-fit curve.
The porosity and distribution of tubes for gypsum plaster specimens are shown in Table 6.1. All models contain 24 holes, except c24_64 (23 holes) and c24_66 (22 holes).

Table 6.1 Gypsum Plaster Specimens Containing Cylindrical Tubes

<table>
<thead>
<tr>
<th>Distribution of Cylindrical Tubes</th>
<th>Sample Number</th>
<th>Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>H36-P11</td>
<td>11.0</td>
</tr>
<tr>
<td></td>
<td>H36-P30.7</td>
<td>30.7</td>
</tr>
<tr>
<td></td>
<td>H36-P4.9</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>H36-P44.2</td>
<td>44.2</td>
</tr>
<tr>
<td></td>
<td>H9-P19.6</td>
<td>19.6</td>
</tr>
<tr>
<td></td>
<td>H9-P30.7</td>
<td>30.7</td>
</tr>
<tr>
<td>Random</td>
<td>c20-11</td>
<td>32.4</td>
</tr>
<tr>
<td></td>
<td>c24-17</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>c24-26</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>c24-28</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>c24-31</td>
<td>6.0</td>
</tr>
<tr>
<td></td>
<td>c24-34</td>
<td>25.9</td>
</tr>
<tr>
<td></td>
<td>c24-35</td>
<td>37.6</td>
</tr>
<tr>
<td></td>
<td>c24-36</td>
<td>30.7</td>
</tr>
<tr>
<td></td>
<td>c24-37</td>
<td>34.9</td>
</tr>
<tr>
<td></td>
<td>c24-38</td>
<td>35.5</td>
</tr>
<tr>
<td></td>
<td>c24-47</td>
<td>23.9</td>
</tr>
<tr>
<td></td>
<td>c24-60</td>
<td>18.2</td>
</tr>
<tr>
<td></td>
<td>c24-61</td>
<td>17.2</td>
</tr>
<tr>
<td></td>
<td>c24-63</td>
<td>9.1</td>
</tr>
<tr>
<td></td>
<td>c24-64</td>
<td>15.3</td>
</tr>
<tr>
<td></td>
<td>c24-65</td>
<td>19.6</td>
</tr>
<tr>
<td></td>
<td>c24-66</td>
<td>22.1</td>
</tr>
</tbody>
</table>
Six gypsum plaster specimens with uniformly distributed and seventeen with randomly distributed cylindrical tubes were produced for testing. The cross section of all specimens containing randomly distributed cylindrical tubes are shown in Appendix I. Final porosity is calculated as the ratio of total surface area of circles to total surface area of cubes where the circles are located.

6.3 Experimental Setup

Gypsum plaster specimens containing cylindrical tubes are tested under uniaxial compression. Uniaxial compression testing was conducted at Terracon, Inc, in Las Vegas, NV, using a 200,000 pound load frame with an accuracy of ± 250 lb. The strain rate applied during testing is nominal 10^-4 per second. Axial force and axial displacement were recorded manually during the testing. A dial gage was used to measure the axial displacements and axial load was read from an LCD panel attached to the machine.

6.4 Test Results

The uniaxial compressive strength is calculated according to the International Society of Rock Mechanics by dividing the maximum load carried by the specimen by the original cross-sectional area (ISRM, 1979). Deformation modulus, which is defined as the ratio of the axial stress change to axial strain produced by this stress change, is computed from the average slope of the more-
or-less straight line portion of the axial stress-axial strain curve, as seen in Figure 6.2. This method is also recommended by ISRM (1979).

Since the specimens contain open-ended cylindrical tubes, localized failures occurred during the testing. However, localized failures only caused a small decrease in axial load, specimens continued to carry more load exhibiting linear elastic behavior prior to failure.

![Figure 6.2 Method for calculating deformation modulus from axial stress versus strain curve (after ISRM, 1979).](image)

Table 6.2 shows the calculated deformation modulus and compressive strength values for gypsum plaster specimens containing open-ended cylinder tubes.

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Table 6.2 Compressive Strength, $\sigma_c$, and Deformation Modulus, $E$, of Gypsum Plaster Specimens Containing Cylindrical Tubes

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Porosity (%)</th>
<th>$\sigma_c$ (psi)</th>
<th>$E$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H36-P11</td>
<td>11.0</td>
<td>1055</td>
<td>402</td>
</tr>
<tr>
<td>H36-P30.7</td>
<td>30.7</td>
<td>528</td>
<td>152</td>
</tr>
<tr>
<td>H36-P4.9</td>
<td>4.9</td>
<td>1248</td>
<td>451</td>
</tr>
<tr>
<td>H36-P44.2</td>
<td>44.2</td>
<td>416</td>
<td>144</td>
</tr>
<tr>
<td>H9-P19.6</td>
<td>19.6</td>
<td>649</td>
<td>244</td>
</tr>
<tr>
<td>H9-P30.7</td>
<td>30.7</td>
<td>499</td>
<td>172</td>
</tr>
<tr>
<td>c20-11</td>
<td>32.4</td>
<td>168</td>
<td>85</td>
</tr>
<tr>
<td>c24-17</td>
<td>11.6</td>
<td>693</td>
<td>304</td>
</tr>
<tr>
<td>c24-26</td>
<td>12.4</td>
<td>607</td>
<td>275</td>
</tr>
<tr>
<td>c24-28</td>
<td>8.1</td>
<td>1086</td>
<td>293</td>
</tr>
<tr>
<td>c24-31</td>
<td>6.0</td>
<td>1516</td>
<td>407</td>
</tr>
<tr>
<td>c24-34</td>
<td>25.9</td>
<td>286</td>
<td>83</td>
</tr>
<tr>
<td>c24-35</td>
<td>37.6</td>
<td>143</td>
<td>68</td>
</tr>
<tr>
<td>c24-36</td>
<td>30.7</td>
<td>359</td>
<td>130</td>
</tr>
<tr>
<td>c24-37</td>
<td>34.9</td>
<td>193</td>
<td>102</td>
</tr>
<tr>
<td>c24-38</td>
<td>35.5</td>
<td>174</td>
<td>69</td>
</tr>
<tr>
<td>c24-47</td>
<td>23.9</td>
<td>372</td>
<td>121</td>
</tr>
<tr>
<td>c24-60</td>
<td>18.2</td>
<td>523</td>
<td>193</td>
</tr>
<tr>
<td>c24-61</td>
<td>17.2</td>
<td>500</td>
<td>150</td>
</tr>
<tr>
<td>c24-63</td>
<td>9.1</td>
<td>1026</td>
<td>265</td>
</tr>
<tr>
<td>c24-64</td>
<td>15.3</td>
<td>585</td>
<td>234</td>
</tr>
<tr>
<td>c24-65</td>
<td>19.6</td>
<td>621</td>
<td>125</td>
</tr>
<tr>
<td>c24-66</td>
<td>22.1</td>
<td>413</td>
<td>184</td>
</tr>
</tbody>
</table>

In Figure 6.3, compressive strength (a) and deformation modulus (b) are plotted versus porosity. The decrease in strength and deformation modulus can be represented by an exponential best-fit curve. A single best-fit curve is
calculated and plotted for all data for compressive strength. For the deformation modulus, two best-fit curves are developed, one for specimens containing uniformly distributed (shown as dotted line) and one for randomly distributed cylindrical tubes (shown as solid line). The coefficient of determination, $R^2$, is slightly higher for deformation moduli of specimens with uniformly distributed tubes than those with randomly distributed tubes. However, this is probably because a smaller number of uniformly distributed tubes specimens were tested.

Both compressive strength and deformation modulus values for specimens containing of uniformly distributed tubes have higher values than those containing randomly distributed tubes because the plaster columns between the uniformly distributed holes increase the stiffness. The columns continue to carry load even after cracks propagating from hole to hole.

The randomly distributed tubes better represent porous rock in nature due to their random sizes and locations. The deformation moduli for these specimens are rather dispersed yet it shows a good decreasing trend with increasing porosity.

In order to determine the normalized deformation modulus, the deformation modulus for zero porosity is taken as the value that best fit curve intersects vertical axis. Then, moduli are normalized with respect to the one for zero percent porosity value. Figure 6.4 shows the normalized deformation modulus for specimens containing both uniformly distributed and randomly distributed holes and the best-fit curves.
Figure 6.3 Compressive strength (a) and deformation modulus (b) with porosity for plaster specimens. Solid line and dotted line show the best fit curve for randomly and uniformly distributed cylindrical tubes, respectively.

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The normalized modulus can be given for specimens containing uniformly distributed tubes as

\[ \frac{E}{E_0} = e^{-0.03294 \rho} \quad \text{and} \quad R^2 = 0.9397 \]  

and for specimens containing randomly distributed tubes as

\[ \frac{E}{E_0} = e^{-0.05161 \rho} \quad \text{and} \quad R^2 = 0.8994 \]  

Figure 6.4 Porosity versus normalized deformation modulus for plaster specimens containing cylindrical tubes. Solid line and dotted line show the best-fit curve for randomly and uniformly distributed cylindrical tubes, respectively.
The same loading direction (vertical direction) was used in both test specimens and their equivalent numerical models. Only twenty-three plaster specimens, seventeen of them are containing randomly distributed holes, were produced and tested while thirty-six numerical models were analyzed. In order to compare two sets of deformation modulus (one from testing, the other from numerical model), plane strain deformation modulus computed through FLAC is transformed into three-dimensional deformation modulus using the relationship given by Chen and Thorpe (1994) as

\[ E' = E (1 - \nu'^2) \]  \hspace{1cm} (6.3)

for Poisson's ratio,

\[ \nu' = \left( \frac{\nu}{1 + 2\nu} \right) \]  \hspace{1cm} (6.4)

where primed notation is for three dimensional elastic constants and unprimed for two dimensions. Therefore, the plane strain deformation modulus is converted to three-dimensional modulus using equation (6.3) after equation (6.4) is substituted into equation (6.3). The difference in two and three dimensional elastic constants is because the Poisson's ratio is bounded by \(-1 < \nu < 1\) for two dimensional elasticity in contrast to the bounds \(-1 < \nu < 1/2\) for the three dimensional Poisson's ratio (Jasiuk et al., 1992). Both data sets are shown in Figure 6.5.
Figure 6.5 Porosity versus normalized deformation modulus for plaster specimens containing cylindrical tubes determined through numerical analysis and testing. Solid line and dotted line show the best fit curve for numerically computed values and experimental values, respectively.

As seen in Figure 6.5, numerically calculated values overestimated the deformation modulus while porosity increases. The main reason for the difference between two deformation modulus sets can be explained by the difference between modeling a three-dimensional medium in two dimensions. Furthermore, friction between the steel platen and specimens are not entered into numerical simulation since the effect of using steel platen on modulus is likely to be small.
6.5 Conclusion

Uniaxial compression testing was conducted on both cubic gypsum plaster specimens containing open-ended cylindrical tubes, which created porosity. Compressive strength and deformation modulus were computed and compared with those calculated through finite difference method using FLAC. Both testing and numerical models show decreasing compressive strength and deformation modulus with increasing porosity. Numerically calculated values overestimated the deformation modulus in an increasing way while porosity increased.
CHAPTER 7

UNIAXIAL COMPRESSION TESTING OF GYPSUM PLASTER SPECIMENS
CONTAINING STYROFOAM INCLUSIONS AND TUSS SPECIMENS

7.1 Introduction

Uniaxial compression testing is a common experimental procedure to determine the compressive strength and moduli of materials in which cylindrical or prismatic specimens are loaded axially to failure. Deformation moduli and deformation ratios of the specimens can be calculated using the linear elastic portion of the stress-strain curve. Furthermore, the strength of material is computed using the maximum stress value carried by the specimen.

Since the lithophysal cavities in rock mass make coring cylindrical specimens very difficult, if not impossible, prismatic specimens are preferred for testing of analog and lithophysal Tuff specimens. The uniaxial compression testing on cubic specimen is not common for rock and soil and not mentioned in ASTM standards. British standards documented compressive testing of concrete cubes (BS, 2002).
In this chapter, the results from the uniaxial compression tests performed on cubic specimens of gypsum plaster and Tuff are used to investigate the relationships between deformation modulus/compression strength and various porosities.

7.2 Test Specimens

Gypsum plaster specimens are produced to model the lithophysal porosity by using Styrofoam spheres. Styrofoam inclusions do not create exact porosity type that Tuff has because they do not create cavities in the specimens. However, Styrofoam is a highly compressible material and elastic modulus is very low compared with the plaster. Therefore, this two-phase (plaster and Styrofoam) material can be regarded as a solid with empty cavities.

7.2.1 Gypsum Plaster Specimens with Styrofoam Inclusions

Fourteen gypsum plaster specimens containing Styrofoam inclusions are produced for testing using a 6 inch aluminum cubic mold which is the same mold described in the Chapter 6. After pouring the gypsum plaster into the mold, the top surface is leveled using a straight edge and the specimen is allowed to dry over night. The next day, the mold is removed and the specimen is weighed daily until it reaches a constant weight. Then, loading surfaces of plaster specimens are ground flat to provide level surfaces because the uniformity of stress distribution on the loading surfaces controls the accuracy of strength.
The porosity of a specimen is calculated as the ratio of total volume of Styrofoam inclusions to total volume of specimen. For the fourteen specimens, the porosity varies between 9 to 36%. Two different distributions of Styrofoam inclusions in the specimen are considered and shown in Table 7.1.

For the specimens containing structured inclusions, Styrofoam spheres are placed in a structured manner using different sizes of Styrofoam spheres. The purpose of using such a configuration is not related to determination of deformation modulus and compressive strength, but ultrasonic and Acoustic Emission (AE) tests which are not covered in this dissertation. However, these specimens can be included in the modulus and compressive strength versus porosity comparison regardless of nonrandomness of the distribution of inclusions. Styrofoam spheres are attached to the mold using vinyl strings to maintain their positions. The locations of inclusions in structured specimens are shown in Table 7.2.

Nine specimens with randomly distributed Styrofoam inclusions are produced using either small or large, and small and large Styrofoam spheres. To produce the specimens containing random inclusions, a sufficient volume of Styrofoam spheres are mixed together with the wet plaster and poured into the mold.
Table 7.1 Gypsum Plaster Specimens Containing Styrofoam Inclusions

<table>
<thead>
<tr>
<th>Type of Inclusions</th>
<th>Specimen Number</th>
<th>Porosity (%)</th>
<th>Number of Inclusions</th>
<th>Diameter of Inclusions (inch)</th>
<th>Locations of Inclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structured</td>
<td>1652</td>
<td>8.5</td>
<td>1</td>
<td>3</td>
<td>Central</td>
</tr>
<tr>
<td></td>
<td>1653</td>
<td>22.1</td>
<td>1</td>
<td>4</td>
<td>Central</td>
</tr>
<tr>
<td></td>
<td>1661</td>
<td>16.7</td>
<td>4</td>
<td>2.5</td>
<td>Stacked</td>
</tr>
<tr>
<td></td>
<td>1663</td>
<td>9.7</td>
<td>4</td>
<td>2</td>
<td>Stacked</td>
</tr>
<tr>
<td></td>
<td>1666</td>
<td>30.3</td>
<td>8</td>
<td>2.5</td>
<td>Stacked</td>
</tr>
<tr>
<td>Random</td>
<td>1654</td>
<td>40.0</td>
<td>5</td>
<td>2.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1655</td>
<td>40.1</td>
<td>30</td>
<td>1.5</td>
<td>Random</td>
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<tr>
<td></td>
<td>1656</td>
<td>40.1</td>
<td>7</td>
<td>2.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1657</td>
<td>30.2</td>
<td>4</td>
<td>2.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1658</td>
<td>10.3</td>
<td>9</td>
<td>1.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1659</td>
<td>20.4</td>
<td>19</td>
<td>1.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1660</td>
<td>19.9</td>
<td>20</td>
<td>1.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1664</td>
<td>30.6</td>
<td>5</td>
<td>2.5</td>
<td>Random</td>
</tr>
<tr>
<td></td>
<td>1665</td>
<td>21.1</td>
<td>3</td>
<td>2.5</td>
<td>Random</td>
</tr>
</tbody>
</table>
Table 7.2 Coordinates of Structured Inclusions. The Center of the Base Plate Has Coordinates of (0,0,0)

<table>
<thead>
<tr>
<th>Type of Structured Inclusions</th>
<th>Nominal Porosity (%)</th>
<th>Inclusion Type</th>
<th>Coordinates (inches)</th>
<th>Inclusion Diameter (inch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central</td>
<td>10 and 20</td>
<td>Central</td>
<td>0 0 3</td>
<td>3 1 and 5 1 and 5 1 and 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Surrounding</td>
<td>-2 -2 -2</td>
<td>1 1 1 3 5 1 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 -2</td>
<td>1 1 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2 2</td>
<td>1 1</td>
</tr>
<tr>
<td>Stacked</td>
<td>10</td>
<td>Stacked</td>
<td>-1.5 -1.5</td>
<td>1 3 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.5 1.5</td>
<td>1 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5 -1.5</td>
<td>1 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5 1.5</td>
<td>1</td>
</tr>
<tr>
<td>Uniform</td>
<td>20</td>
<td>Stacked</td>
<td>-1.5 -1.5</td>
<td>0.875 3 5.125 1.5 2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.5 1.5</td>
<td>2 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5 -1.5</td>
<td>1 1</td>
</tr>
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<td></td>
<td>1.5 1.5</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.5 -1.5</td>
<td>1.5 2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.5 1.5</td>
<td>2 5</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5 1.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

7.2.2 Lithophysae-Rich Tuff Specimens

Lithophysal Tuff specimens were cut from blocks recovered from outcrops on Busted Butte, Fran Ridge and Sandia Quarry near Yucca Mountain on the Nevada Test Site. These specimens represent the Tuff from upper and lower lithophysal strata. Cubic specimens were cut in the Sample Management Facilities located in the Nevada Test Site (NTS) in Mercury, Nevada. Ten Tuff specimens were tested under uniaxial compression. All of the specimens contain
lithophysal cavities. The top and bottom surfaces of the specimens were ground flat at the Material Testing Laboratory (MTL) in NTS in order to achieve uniformly distributed load on the loading surfaces and not to lead to premature failure especially at edges. However, due to the presence of cavities located close to loading surfaces for some specimens, the bottom and top loading surfaces were not ground exactly parallel to each other to prevent any damage on the specimens during the grinding process. Pells (1993) states that when the ends are not parallel premature failure occurs but this has only minor effects on strength and modulus. Average dimensions and locations where the specimens are recovered are shown in Table 7.3

Table 7.3 Tuff Specimen Dimensions (Length/Width/Height) and the Nevada Test Site Locations Where the Samples Are Taken

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Average Dimensions (inch)</th>
<th>Nevada Test Site Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1667</td>
<td>5.9/5.9/5.8</td>
<td>Topopah Spring, Upper Lithophysal Zone in Sandia Quarry</td>
</tr>
<tr>
<td>1668</td>
<td>6.3/6.2/6.2</td>
<td>Topopah Spring Upper Lithophysal Zone in Sandia Quarry</td>
</tr>
<tr>
<td>1669</td>
<td>6.1/6.2/6.2</td>
<td>Topopah Spring Upper Lithophysal Zone in Sandia Quarry</td>
</tr>
<tr>
<td>1670</td>
<td>6.4/6.3/6.3</td>
<td>Crystal Poor Upper Lithophysal Zone in Busted Butte</td>
</tr>
<tr>
<td>1671</td>
<td>6.1/6.2/6.0</td>
<td>Topopah Spring, Upper Lithophysal Zone in Fran Ridge</td>
</tr>
<tr>
<td>1672</td>
<td>6.6/6.5/6.6</td>
<td>Topopah Spring, Lower Lithophysal Zone in Fran Ridge</td>
</tr>
<tr>
<td>1673</td>
<td>5.0/5.1/5.0</td>
<td>Topopah Spring Lower Lithophysal Zone in Fran Ridge</td>
</tr>
<tr>
<td>1674</td>
<td>5.0/5.2/5.1</td>
<td>Crystal Poor Upper Lithophysal Zone in Busted Butte</td>
</tr>
<tr>
<td>1675</td>
<td>6.0/5.9/6.2</td>
<td>Topopah Spring Upper Lithophysal Zone in Sandia Quarry</td>
</tr>
<tr>
<td>1676</td>
<td>5.4/5.3/5.4</td>
<td>Crystal Poor Upper Lithophysal Zone in Busted Butte</td>
</tr>
</tbody>
</table>
7.3 Experimental Setup

Uniaxial compression testing was conducted at MTL in NTS by using 1-million pound MTS stiff loading frame as shown in Figure 7.1.

All specimens were tested dry and at room temperature. The axial force and axial displacement were recorded during the testing using the parametric output of the AE data accusation system. The uniaxial compression testing is
conducted according to ASTM D2938 by loading gradually in a displacement controlled way. The strain rate during the testing of plaster specimens is $5 \times 10^{-4}$ per second. The strain rates for Tuff specimens are varied from $5 \times 10^{-4}$ per second to $8 \times 10^{-4}$ per second. The strain rate applied during testing on Tuff specimens is nominal $6 \times 10^{-4}$.

The difference in strain rates is due to the differences in thickness of Tuff specimens and input displacement values in the MTS testing machine. The strain rate affects the strength of brittle rocks more than their deformation properties. Martin et al. (1993) performed a series of uniaxial compression tests on cylinders of Topopah Spring Member welded Tuff using different strain rates and investigating the effect of strain rate. They found that strength decreases with decreasing strain rate. However, change in strength between the strain rates of $5 \times 10^{-4}$ and $8 \times 10^{-4}$ per second is small. Stavrogin and Tarasov (2001) collected strain rate versus compressive strength and elastic modulus data from different rocks including marble, sandstone and limestone. Their database shows that the strain rate dependence of compressive strength and elastic modulus is insignificant between the strain rates of $10^{-4}$ and $10^{-6}$ per second. Thus, the compressive strength and deformation properties of Tuff specimens tested can be compared with each other without considering a significant strain rate effect.

One important aspect in compression testing is that of applying uniformly distributed compressive load over the faces of prismatic specimen and eliminating the influence of frictional restraint between the loading platen and specimen (Brown, 1974). It is well known that prismatic specimens subjected to
uniaxial compressive loading may be confined along their loaded surfaces due to friction between the specimen and the loading platens so that this may result in an increase in the apparent strength of the specimens (Föppl, 1900). There are several techniques which are recommended to decrease the friction and maintain uniformly distributed loading on testing specimen such as using lubricants, cardboard sheet, epoxy or brush platens instead of solid ones (Brown, 1974). The effects of these techniques can be arguable. However, Gonano and Brown (1973) used brush and solid platens for uniaxial compression tests on cylindrical specimens of marble and gypsum plaster. The results showed that different platens affected the shape of stress-strain curve after the peak stress was achieved but not the linear portion of the curve.

In lieu of these studies, solid steel platens are used in uniaxial compression testing of plaster and Tuff specimens because the purpose is to understand and observe the change in modulus and compressive strength due to variable porosities.

7.4 Determination of Porosity

Plaster specimens are very porous. In this study, specimens without any Styrofoam inclusions are used as a reference solid material, that means they are assumed to have zero percent porosity even though they contain a large amount of microporosity. The term "porosity" refers to the totality of cavities caused by Styrofoam inclusions here. Since during the grinding process of top and bottom surfaces of specimens to create smooth surfaces, some of the Styrofoam
inclusions located very closely on these surfaces were ground so that the actual
Styrofoam volume decreased. This process also slightly changed the volume of
the cubes. Therefore, porosity needs to be recalculated rather than using the
inclusion volume that is introduced into the plaster during the production stage.

There are two solid specimens produced to calculate the average bulk
density of plaster. Assuming the contribution of Styrofoam inclusions is very
small, measured weight is the weight of solid phase. Therefore, the volume of
solid phase inside a particular cube is the weight of that cube divided by the bulk
density. Volume of Styrofoam inclusions is then volume of cube minus volume of
solid. The porosity is the ratio of volume of void space to total volume of the
specimen. Table 7.4 shows the calculated bulk porosities of gypsum plaster
specimens. For most of the specimens, actual Styrofoam volume is slightly lower
than the pre-determined ones during the production of specimens.

The porosity of Tuff specimens is calculated using specific gravities
determined in accordance to ASTM D854. The specific gravity of Tuff is
computed as the ratio of weight of a particular volume of pulverized Tuff to the
weight of an equal volume of distilled water. Specific gravity tests were
conducted at MTL. Dry unit weight of Tuff is determined by using total volume of
a specimen and its weight. Then the porosity of specimens are calculated using
Equations 7.1 and 7.2

\[ e = 1 - \frac{\gamma_{\text{dry}}}{G_s \gamma_w} \]  

(7.1)
where \( e \) is the void ratio, \( \gamma_{\text{dry}} \) is dry unit weight and \( G_s \) is specific gravity. Average \( G_s \) values are used for all specimens. Then porosity, \( p \), is

\[
p = \frac{e}{e+1} \times 100
\]  

(7.2)

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Description</th>
<th>Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1652</td>
<td>Central 10%</td>
<td>9.7</td>
</tr>
<tr>
<td>1653</td>
<td>Central 20%</td>
<td>18.5</td>
</tr>
<tr>
<td>1654</td>
<td>R-S &amp; L 40%</td>
<td>32.7</td>
</tr>
<tr>
<td>1655</td>
<td>R-Small 40%</td>
<td>34.7</td>
</tr>
<tr>
<td>1656</td>
<td>R-Large 40%</td>
<td>33.2</td>
</tr>
<tr>
<td>1657</td>
<td>R-S &amp; L 30%</td>
<td>20.9</td>
</tr>
<tr>
<td>1658</td>
<td>R-Small 10%</td>
<td>4.5</td>
</tr>
<tr>
<td>1659</td>
<td>R-Small 20%</td>
<td>17.4</td>
</tr>
<tr>
<td>1660</td>
<td>R-Small 30%</td>
<td>19.9</td>
</tr>
<tr>
<td>1661</td>
<td>Stacked 20%</td>
<td>16.7</td>
</tr>
<tr>
<td>1663</td>
<td>Stacked 10%</td>
<td>5.6</td>
</tr>
<tr>
<td>1664</td>
<td>R-Large 30%</td>
<td>28.6</td>
</tr>
<tr>
<td>1665</td>
<td>R-Large 20%</td>
<td>19.4</td>
</tr>
<tr>
<td>1666</td>
<td>Stacked 30%</td>
<td>22.4</td>
</tr>
</tbody>
</table>

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Porosity calculated in this way is bulk porosity that includes both microporosity and lithophysal porosity. Calculated porosities for Tuff specimens are given in Table 7.5.

Table 7.5 Calculated Bulk Porosities for Tuff Specimens

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>Porosity (%)</th>
<th>Lithophysal Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>1667</td>
<td>31.6</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1671</td>
<td>28.6</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1674</td>
<td>28.3</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1669</td>
<td>32.9</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1668</td>
<td>30.6</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1673</td>
<td>12.2</td>
<td>Lower Lithophysal Zone</td>
</tr>
<tr>
<td>1676</td>
<td>12.5</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1675</td>
<td>25.9</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1670</td>
<td>19.3</td>
<td>Upper Lithophysal Zone</td>
</tr>
<tr>
<td>1672</td>
<td>17.1</td>
<td>Lower Lithophysal Zone</td>
</tr>
</tbody>
</table>

7.5 Test Results

Axial force and axial displacements for both plaster and Tuff specimens were collected by AE data acquisition system with a sampling rate of 1 Hz. The deformation modulus and uniaxial compressive strength of the specimen are calculated according to the International Society of Rock Mechanics (ISRM, 1979). Since the specimens have cavities and introduced inclusions, which have
almost zero elastic modulus, local failures occurred during the testing. These local failures created small spikes on load versus displacement curves, but specimens continued to carry more loads exhibiting linear elastic behavior.

7.5.1 Gypsum Plaster Specimens

Stress-strain curves are virtually linear before specimens start showing yield point prior to the failure. The deformation moduli and compressive strengths of plaster specimens as a function of calculated bulk porosity are given in the Table 7.6.

In Figure 7.2, compressive strength and deformation modulus versus porosity is shown. Decrease in deformation modulus can be represented best by an exponential equation for plaster specimens:

\[ E = 444.1619 \times e^{-0.032p} \] (ksi) \hspace{1cm} (7.3)

Decrease in modulus and compressive strength are dispersed in a relatively wide range. This trend is probably due to the very porous nature of gypsum plaster used as an analog. Fuenkajorn and Daemen (1992) mentioned the effect of nonuniformly distributed pores on strength and modulus in porous sandstone and Tuff. The same behavior is observed for plaster. Furthermore, since the plaster-water gel was mixed by hand there are probably some zones left in the plaster cubes where the plaster is not properly mixed with water. These zones may cause the variations in strength. Another reason of scattered data
points is that the loading surfaces of the specimens are not perfectly parallel and caused premature failures at the edges although its effect on modulus is insignificant (Pells, 1993). Still, both compressive strength and deformation modulus data shows a good correlation between porosity and strength and modulus with a high coefficient of determination, \( R^2 \), 0.85 and 0.82 for compressive strength and deformation modulus, respectively.

Table 7.6 Deformation Modulus, \( E \), and Compressive Strength, \( \sigma_c \), of Gypsum Plaster Specimens

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Porosity (%)</th>
<th>( \sigma_c ) (psi)</th>
<th>E (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1652</td>
<td>9.7</td>
<td>1081</td>
<td>353</td>
</tr>
<tr>
<td>1653</td>
<td>18.5</td>
<td>998</td>
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<td>1654</td>
<td>32.7</td>
<td>499</td>
<td>164</td>
</tr>
<tr>
<td>1655</td>
<td>34.7</td>
<td>500</td>
<td>170</td>
</tr>
<tr>
<td>1656</td>
<td>33.2</td>
<td>418</td>
<td>159</td>
</tr>
<tr>
<td>1657</td>
<td>20.9</td>
<td>715</td>
<td>235</td>
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<td>1658</td>
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<td>1659</td>
<td>17.4</td>
<td>783</td>
<td>255</td>
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<td>16.7</td>
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<td>445</td>
<td>179</td>
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<td>1665</td>
<td>19.4</td>
<td>714</td>
<td>181</td>
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<tr>
<td>1666</td>
<td>22.4</td>
<td>1012</td>
<td>274</td>
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</table>

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Figure 7.2 Compressive strength (a) and deformation modulus (b) with porosity for plaster specimens. All porosities are bulk porosities.
These zones may cause the variations in strength. Another reason of scattered data points is that the loading surfaces of the specimens are not perfectly parallel and caused premature failures at the edges although its effect on modulus is insignificant (Pells, 1993). Still, both compressive strength and deformation modulus data shows a good correlation between porosity and strength and modulus with a high coefficient of determination, $R^2$, 0.85 and 0.82 for compressive strength and deformation modulus, respectively.

7.5.2 Lithophysal Tuff Specimens

The deformation modulus and compressive strength of lithophysal Tuff specimens as a function of calculated bulk porosity are given in the Table 7.7.

In Figure 7.3, compressive strength (a) and deformation modulus (b) versus porosity is shown. Stress versus strain curves from the uniaxial compression test on Tuff specimens show different behavior depending on the size and locations of lithophysal cavities. Due to local failures in proximity of cavities, stress drops and then recovers while rock continues carrying load until the failure. Testing was stopped when a continuous stress decreased is recorded. Catastrophic failure is not observed in any specimens. The compressive strengths and deformation moduli from Tuff specimens decrease with increasing porosity showing variations within approximately the range of 12 to 32 percent porosity. There are no specimens with zero lithophysal porosity found among the tuff specimens collected.
Table 7.7 Deformation Modulus, $E$, and Compressive Strength, $\sigma_c$, of Tuff Specimens

<table>
<thead>
<tr>
<th>Specimen Number</th>
<th>Porosity (%)</th>
<th>$\sigma_c$ (psi)</th>
<th>$E$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1667</td>
<td>31.6</td>
<td>2247</td>
<td>93</td>
</tr>
<tr>
<td>1668</td>
<td>30.6</td>
<td>5729</td>
<td>660</td>
</tr>
<tr>
<td>1669</td>
<td>32.9</td>
<td>2098</td>
<td>337</td>
</tr>
<tr>
<td>1670</td>
<td>19.3</td>
<td>7606</td>
<td>945</td>
</tr>
<tr>
<td>1671</td>
<td>28.6</td>
<td>889</td>
<td>138</td>
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<td>17.1</td>
<td>6508</td>
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<td>1673</td>
<td>12.2</td>
<td>9582</td>
<td>1090</td>
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<td>1674</td>
<td>28.3</td>
<td>3982</td>
<td>577</td>
</tr>
<tr>
<td>1675</td>
<td>25.9</td>
<td>2068</td>
<td>315</td>
</tr>
<tr>
<td>1676</td>
<td>12.5</td>
<td>10485</td>
<td>1151</td>
</tr>
</tbody>
</table>

For Tuff specimens the relationship between porosity and deformation modulus can be given as a linear equation:

$$E = -38.61p + 1604 \text{ (ksi)}$$  \hspace{1cm} (7.4)

Both compressive strength and deformation modulus data give a good correlation with porosity having $R^2$ values, 0.7616 and 0.7215 for compressive strength and deformation modulus, respectively. The variations in data seems depending on nonuniform distribution of lithophysal cavities and probably undetected microcracks and fractures, but not on large variations in physical and mineralogical characteristics of Tuff as mentioned by Zimmerman et al. (1985) and Price et al. (1984).
Figure 7.3 Compressive strength (a) and deformation modulus (b) with porosity for lithophysal Tuff specimens. All porosities are bulk porosities.
This is probably because of large cavities, and therefore higher porosity, governs the modulus and strength of lithophysal Tuff rather than the mineralogical variations, grain sizes, grain bonding and cementing in matrix material. The deformation modulus for zero porosity is taken as the value where best-fit curve intersects vertical axis. Deformation moduli for plaster and Tuff are then normalized with respect to the one for zero percent porosity value computed using curve fitting. The normalized modulus can be given for plaster specimens as

\[
\frac{E}{E_0} = e^{-0.0289 \, p} \quad \text{and} \quad R^2 = 0.8556
\]  

(7.5)

for Tuff specimens

\[
\frac{E}{E_0} = -0.0241 \, p + 1.0 \quad \text{and} \quad R^2 = 0.8321
\]  

(7.6)

Figure 7.4 shows the normalized deformation modulus for plaster and Tuff and the best-fit curves. The best-fit curve for Tuff is linear whereas that for plaster is exponential. The effective of elastic modulus drops off nearly linearly with porosity due to nonspherical shapes of larger cavities at high porosities (Zimmerman, 2002). The normalized deformation moduli of plaster usually overlap the one for Tuff except three very low values of Tuff, which normalized modulus is less than 0.2 within the range of 25-30 % porosity, so that normalized...
moduli of plaster are giving higher values. The decrease in modulus around 30% porosity is more than half of the modulus of same material with zero porosity. Distribution of both data with porosity proves that gypsum plaster specimens tested under uniaxial compression can be successfully used to study the effect of lithophysal porosity even if the physical behavior of two materials is different (Stimpson, 1970).

Figure 7.4 Porosity versus normalized deformation modulus for plaster and Tuff specimens. Solid line and dotted line show the best-fit curve for plaster and Tuff specimens, respectively.
7.6 Conclusion

The uniaxial compression testing was conducted on both gypsum plaster specimens containing spherical Styrofoam inclusions and lithophysal Tuff specimens. Porosity was created by introducing spherical Styrofoam inclusions into plaster. Compressive strength and deformation modulus were computed for both specimens and plotted as a function of porosity. Both testing shows decreasing compressive strength and deformation modulus with increasing porosity. Calculated normalized deformation moduli demonstrate similar decreasing trend with porosity for both specimens.
CHAPTER 8

COMPARISON OF NUMERICAL AND TEST RESULTS

8.1 Introduction

The dependence of the mechanical properties of lithophysal Tuff on porosity is an important issue for the design and performance of the repository tunnels. In the previous chapters, the effect of porosity on deformation modulus and compressive strength are investigated through numerical analysis and uniaxial compression testing on analog specimens (gypsum plaster) using different size and distribution of holes and cavities. This work provides a baseline to which the uniaxial compression testing of Tuff specimens will be compared.

In this chapter, normalized values of deformation modulus and compressive strength values are compared to each other as a function of porosity and the findings are summarized. It is important to remember that this dissertation does not recommend any mechanical and deformation property as an input for numerical modeling of repository tunnels but rather explains the changes in the properties with varying porosities. The relation between laboratory
rock properties and the in-situ rock mass properties in numerical modeling in design stage is out of the scope of this dissertation.

8.2 Deformation Modulus

The deformation moduli of gypsum plaster specimens containing open-ended cylindrical tubes and spherical Styrofoam inclusions are plotted in Figure 8.1 with their best-fit curves. For both cases, the best-fit regression curve is an exponential curve. Both data sets show a decreasing trend of deformation modulus with increasing porosity, however data points are dispersed in a relatively wide range. The moduli of specimens containing spherical Styrofoam inclusions show higher values than those containing open-ended cylinder tubes. This behavior is being expected because the tubes cross the cubes from one side to the other whereas spherical Styrofoam inclusions are localized inside the cubes, which leave stiff solid zones to carry the load.

Kachanov et al. (1994) used analytical methods to study the compressibility of different inclusions and cavities placed in a solid body. They showed that the compressibility of a cavity is a function of its shape. Although their work did not focus on solids containing large cavities, their findings on compressibility of different shapes can be used to explain the differences in the deformation modulus of specimens tested here. Their calculations show spherical shaped cavities are stiffer and cavities that have prolate shapes have higher compressibility, i.e. lower stiffness. The deformation moduli of specimens
containing long cylindrical tubes exhibit somewhat higher compressibility, thus lower deformation modulus than those containing spherical cavities.

![Graph showing deformation modulus versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit) and Styrofoam inclusions (dashed line is the best-fit line).](image)

Figure 8.1 Deformation modulus versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit) and Styrofoam inclusions (dashed line is the best-fit line).

The deformation moduli of plaster and Tuff specimens are normalized using the elastic modulus for zero percent porosity, which is calculated using best-fit curves for all data sets, as shown in Figure 8.2. Although there is a limited number of data for Tuff, the data is interspersed with those belonging to plaster specimens. The shape of the cavities observed in Tuff specimens are neither spherical nor tubular but between these two. This shows that the shape of the
cavities is an important factor affecting the distribution of data. The best-fit regression line used for Tuff data is linear unlike the exponential regression lines used for the plaster specimens. This may be because of the limited number of Tuff specimens tested. Additionally, the complex nature of Tuff may cause this linear trend.

Figure 8.2 Normalized deformation modulus versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best-fit line) and lithophysae-rich Tuff specimens (long dashed line is the best-fit line).

Normalized deformation moduli calculated through the finite difference analysis are plotted using randomly distributed circular holes after the values are converted to three dimensional constants as explained in Chapter 6. Since the
The effect of matrix Poisson's ratio on modulus is not significant and Poisson's ratio of rock and rock-type materials is between 0.2 and 0.3, modulus values computed for the matrix Poisson's ratio of 0.3 are used for comparison. Figure 8.3 shows the normalized deformation modulus determined through numerical analysis and testing. Best-fit curve is not plotted for numerical data set.

Figure 8.3 Normalized deformation modulus versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best-fit line), lithophysae-rich Tuff specimens (long dashed line is the best-fit line) and numerical values for ($\nu_o = 0.3$).

Normalized modulus values given by numerical modeling fall over the best-fit line for plaster specimens containing spherical Styrofoam inclusions since...
spherical cavities in three dimension and circular holes in two dimension are the stiffest among various shapes (Zimmerman, 1986). Distribution of data for both analog and Tuff specimens is very similar at low porosities. At higher porosities, a greater decrease in deformation modulus is observed in Tuff due to larger and nonspherical cavities (Zimmerman, 2002).

8.3 Compressive Strength

A similar trend to that seen with the relationship between deformation modulus and porosity is observed for compressive strength values of two different system of cavities inserted in gypsum plaster specimens, as shown in Figure 8.4. The best fit regression line for the gypsum plaster specimens is once again an exponential curve. The compressive strength of gypsum plaster specimens containing spherical Styrofoam inclusions has a slightly higher compressive strength than the specimens containing the open-ended tubes. This shows the shape of the cavities also has an effect on strength.

In order to plot compressive strength of Tuff with those for plaster specimens, compressive strength values of all three data sets is normalized with respect to the strength at zero porosity because strength values of Tuff is much higher than those of the plaster. Normalized compressive strength for plaster and Tuff versus porosity is shown in Figure 8.5. The normalized compressive strength of the Tuff specimens falls between the plaster specimens with two different systems of cavities. The best-fit regression line for compressive strength data is linear.
Figure 8.4 Compressive strength versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (dashed line is the best-fit line).

This shows that the change in compressive strength of Tuff cannot simply explained by the existence of cavities because the Tuff specimens also contain microcracks and fractures. As seen in Figure 8.5, porosity should be a dominant factor reducing the compressive strength.

8.4 Comparison of Results with Analytical Methods

In Chapter 2, effective medium theories, which explain the variance of elastic modulus with porosity and provide simple equations relating modulus and porosity, are summarized. Here, normalized deformation modulus is plotted with
these relationships. Only effective matrix schemes (SCS and DS) and an effective field scheme (MTS with and without interaction) are shown.

Figure 8.5 Normalized compressive strength versus porosity for gypsum plaster specimens containing cylindrical tubes (solid line is the best-fit line) and Styrofoam inclusions (short dashed line is the best-fit line) and lithophysae-rich Tuff specimens (long dashed line is the best-fit line).

All relationships are valid for two dimensions. It is important to remember that these approximations are given for an effective media where the cavities or holes are uniformly distributed so that they create some kind of effective homogeneity. Similar effective homogeneity may not be true for plaster specimens and Tuff. However, these approximate relationships give normalized
modulus as a function of porosity only since they are modified so that shape factor is zero for circular shapes.

Figure 8.6 shows the distribution of normalized modulus data plotted on relationships explained in detail in Chapter 2. The upper bound of the curves is given by relationship of non-interactive holes whereas the lower bound is given by discrete scheme (DS). The experimental and numerical data produced in this study falls between the bounds except the specimens containing cylindrical tubes.

Normalized modulus of plaster specimens are usually between upper and lower bounds. Those calculated through numerical modeling overlap with self-consistent scheme (SCS).

The normalized modulus values for Tuff overlap with the Mori-Tanaka scheme (MTS) curve for porosities less than 20%. For porosities higher than 20%, the normalized Tuff modulus falls between SCS and DS.

8.5 Recommendations for Future Research

This study is limited with the circular holes in numerical analysis and cylindrical tubes that do not contact each other or the outside boundaries of the specimen and spherical cavities that are not in contact with each other. However, as mentioned earlier lithophysae-rich Tuff has cavities that do not resemble any of these regular shapes like spheres and cylinders and are not always embedded in rock matrix. Further investigation on the deformation and strength properties of
materials containing cavities and voids of which shapes cannot be simply modeled using regular shapes should include following items:

- Numerical modeling of randomly distributed circular holes that contact each other and, therefore, create noncircular shapes should be studied. It is expected that due to the transformation of hole shapes from circular to those which are larger and no longer spherical, will increase the compressibility and decrease the stiffness more than the ones studied in here.

- Although numerical modeling the cavities in three dimensions using finite difference or finite element techniques is time consuming in both modeling and execution steps, for carefully selected models uniaxial compression testing can be numerically conducted to calculate deformation moduli as a function of porosity and nonspherical shapes which are similar to lithophysal porosity.

- Analog specimens using gypsum plaster and containing nonspherical cavities with varying porosities should tested under uniaxial compression. These analog models should include cavities similar to those observed in Tuff.

- Analog specimens mentioned above can also used to investigate the failure patterns on solids containing cavities under uniaxial compression. This type of research will help one to better understand
the failure of lithophysae-rich Tuff under loads without actually testing Tuff.

Figure 8.6 Normalized deformation modulus versus porosity for numerical and experimental specimens including Tuff. The curves represent the relationships calculated using approximate methods.
• Field testing of lithophysal Tuff should be conducted to determine the in situ behavior of the rock mass. Laboratory testing and numerical analysis do not represent the behavior of rock mass.

8.6 Conclusion

The deformation modulus as a function of porosity determined from testing of plaster and Tuff specimens and numerical results are compared to each other. They are also compared with semi analytical relationships. The correlation between tests, numerical and analytical data is very good by proving that analog (gypsum plaster) testing and numerical analysis are successful to simulate lithophysae-rich Tuff and explain the decrease in modulus with increasing porosity. A similar trend is also observed in compressive strength of plaster and Tuff specimens.
APPENDIX I

FIGURES OF MODELS CONTAINING RANDOMLY DISTRIBUTED
CIRCULAR HOLES
Figure A1.1 Models containing randomly distributed circular holes. Same configurations are used for analog models containing open ended cylindrical tubes.
Figure A1.2 Models containing randomly distributed circular holes. Same configurations are used for analog models containing open ended cylindrical tubes.
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UNLV Summer Graduate Assistantship Award (1998)
NSF EPsCoR Graduate Assistantship Award (1998)

Publications:


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Dissertation Title: Numerical and Experimental Investigation of Deformation and Strength Properties of Lithophysae-Rich Tuff and Analog Materials

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