1-1-2003

Using the concrete-representational-abstract teaching sequence to increase algebra problem-solving skills

Kyle B Konold
University of Nevada, Las Vegas

USING THE CONCRETE-REPRESENTATIONAL-ABSTRACT TEACHING SEQUENCE TO INCREASE ALGEBRA PROBLEM-SOLVING SKILLS

by

Kyle B. Konold

Bachelor of Science
University of Wisconsin, Eau Claire
1992

Master of Science
Southern Illinois University, Edwardsville
1995

Specialist of Education
Southern Illinois University, Edwardsville
1996

A dissertation submitted in partial fulfillment
of the requirements for the

Doctor of Philosophy Degree in Special Education
Department of Special Education
College of Education

Graduate College
University of Nevada Las Vegas
May 2004

UMI Number: 3143383

INFORMATION TO USERS

ProQuest Information and Learning Company
300 North Zeeb Road
P.O. Box 1346
Ann Arbor, MI 48106-1346

UNLV
UNIVERSITY OF NEVADA LAS VEGAS

Dissertation Approval

The Graduate College
University of Nevada, Las Vegas

April 13, 2004

The Dissertation prepared by

Kyle Konold

Entitled

Using the Concrete-Representational-Abstract Teaching Sequence to Increase Algebra Problem-Solving Skills

is approved in partial fulfillment of the requirements for the degree of

Ph.D. - Doctor of Philosophy Degree in Special Education

Examination Committee Chair

Dean of the Graduate College

Examination Committee Member

Examination Committee Member

Graduate College Faculty Representative

1017-52

ABSTRACT

Using the Concrete-Representational-Abstract Teaching Sequence to Increase Algebra Problem-Solving Skills

By

Kyle Brian Konold

Dr. Susan Miller, Examination Committee Chair
Professor of Special Education
University of Nevada, Las Vegas

Researchers have demonstrated the effectiveness of using direct instruction, learning strategy instruction and the Concrete-Representational-Abstract teaching sequence for teaching a variety of basic math skills, but little research has been conducted related to their effectiveness for teaching more complex skills such as algebra. This study investigated the effects of teaching secondary school students with and without mild disabilities a strategy for solving algebra equations and word problems using the concrete-representational-abstract (CRA) teaching sequence. There were 169 secondary students who participated in this study. Of the 169 participants, 79 were male and 90 were female, they ranged in age from 11 to 19, and 61 had mild disabilities (i.e., learning disabilities and emotional disturbances). Students in the treatment group participated

in 11 algebra lessons using the CRA teaching sequence. Students in the control group participated in 11 algebra lessons using traditional textbook-based instruction. Both groups of students received the same practice problems during their respective lessons. Student scores were compared across Teacher-Made Pretests, Posttests, and Maintenance tests. All students increased their ability to solve the algebra problems. The CRA approach and the traditional teaching method were equally effective. The results from this research show that both general education and special education students can learn to solve algebra problems.

TABLE OF CONTENTS

LIST OF TABLES

ACKNOWLEDGMENTS

The completing of this project never would have happened without the contribution of so many people. To each person involved in the many aspects of this project, I am very thankful. Additionally, I would like to thank each person who contributed to my making it to the dissertation stage—a stage I often thought I would never reach. I would like to use this section for special acknowledgements to those who contributed to the culmination of my doctoral degree.

I would like to thank my committee chair, Susan Miller. Without her reassurances, kind words when I needed them, and directness when I needed it, I never would have completed this project. When my life was upside-down, she gave the space I needed and when the time was right she gave me a friendly reminder that my program was not complete.

I give thanks to my committee members: John Filler, Sherri Strawser, and Paul Jones. Each member has his/her area of expertise, which I drew upon to finish this process. Whether it was to answer a specific research question, to provide words of wisdom and reassurance or to provide a

little comic relief. All of you made this very difficult process a little bit easier to bear.

To the students and parents who agreed to participate in the study, I give special thanks. Their willingness to engage in innovative teaching methods assists in improving the education of our youth. Without their assistance, none of this would have been possible.

To the teachers involved in the study, I owe more than can be written here. Your willingness, dedication and hard work exceeded all of my expectations. I want to thank Michael Calkins, Jeff Comer, Chris DeYonke, Stephanie Foresta, and Tiffany Seibel. I only hope that my children have as dedicated teachers in school as they were.

I would like to give special thanks to a handful of people that have been by my side throughout this process helping me in one way or another. To my parents, Pearce Konold and Marilyn Konold, you have always been a guiding light for me. You have been there to celebrate in my joys and successes as well as to comfort me in my times of need. You are the two people whom I admire most and I love you dearly. To my children, Emily Konold and Adam Konold, I have been in the doctoral program your entire lives and am grateful for your patience and understanding that daddy sometimes has to do school work. The two of you make me

realize what is important in life and you always manage to make me smile…and I promise, after this we will go to Chuck E. Cheese! My friends have been very supportive in this process. Throughout the dissertation stage, they asked me how my paper was going, which made me get back to work. Thank you: Lisa Bartes, Kristian Borla, Craig and Jan Butz, Loretta Donovan, Bill Eddins, Kelly Haas, Barb Hardy, Dan Kissinger, Jennifer Powers, and Keith Spresser.

CHAPTER 1

INTRODUCTION

Since October 4th, 1957, when Russia sent Sputnik I into orbit, improved mathematics instruction has been a priority in American schools. In the fifties and sixties, there was a marked increase in funding by the federal government to develop the field of mathematics. This increased funding was intended to produce more and better math teachers and to raise the math performance of the American youth. The "new math" movement also began with this federal funding.

The "new math" programs were developed to increase achievement by focusing on computational and problem solving skills. By the mid-1970's, mathematics achievement in America still lagged behind its foreign competitors. There was a public outcry to get "back to basics" in math instruction. This "back to basics" movement was interpreted by math professionals as the need to go back to the skill and drill approach to teaching math.

In the 1980's, national reports (A Nation at Risk, 1983; Making the Grade, 1989) were written to address teaching,

curriculum, and standards in the American educational system. These reports increased public awareness of the recurring poor math performance among students and the lack of research to validate current educational practices. Also in the 1980's, the National Council of Teachers of Mathematics published *An Agenda for Action* and the *Curriculum and Evaluation Standards for School Mathematics*. These publications resulted in broadening the mathematics curriculum and reducing the emphasis on basic computational skills.

In the new millennium, mathematics is still a priority in American schools. However, in cross-national studies, the United States continues to lag behind a number of its international competitors in mathematics achievement (Hong, 1995; Stedman, 1997; Tuss & Zimmer, 1995). Many researchers have argued that the discrepancy in math achievement between American and international students is due to differences in curricula, length of school year, and quality and quantity of exposure to math rather than in true math ability (Barrett, 1994; Stevenson, et al., 1990; Stevenson & Stigler, 1992). Although many researchers have focused on the comparison of American students to their international counterparts in the area of mathematics, others have simply focused on the poor achievement among American students.

Researchers have indicated that students with learning disabilities experience even greater difficulty in math than their non-disabled peers (Ackerman, Anhalt, & Dykman, 1986; Cawley, Parmar, Yan, & Miller, 1996). Cawley and Miller (1989) reported that students with learning disabilities progress approximately one year in math achievement for every two years of math instruction. They also reported that third and fourth grade students with learning disabilities performed at about a first grade level and twelfth grade students with learning disabilities performed at a fifth grade level.

According to the *National Assessment of Educational Progress* ("National Assessment," 1990), 83 percent of American seniors stated they had taken one algebra course in high school and 56 percent stated they had taken two algebra courses. Yet, less than half of the American seniors demonstrated an understanding of percents, fractions, and simple algebra and only 5 percent showed an understanding of higher-level algebra and geometry (Mullis, et al., 1991). Two things may be concluded from this information. Either the students never understood the concepts taught in their algebra class or they knew the concepts at one time, but failed to retain them.

Politicians and educators have been trying to find ways to increase mathematics achievement in America. Unfortunately, there has been a tremendous amount of disagreement among professionals regarding mathematics instruction. Secretary of Education, Richard Riley, stated in his 1998 address, *The State of Mathematics Education,* that educational professionals must stop their fighting over the best way to teach mathematics ("The State of," 1998). Riley believes students become the losers when paradigm arguments receive too much time and attention because poor student achievement often results. In addition to paradigm battles, several curricular factors have been identified as influencing math performance.

Pickreign (2000) noted that significant differences exist among material presented in mathematics textbooks, the math standards that are expected to be taught, and the math being assessed in school districts using state standardized assessments. This mismatch between curricular materials, instruction, and assessment undoubtedly hinders student understanding and subsequent performance in math.

Hollingsworth and Ybarra (2000) noted additional curricular problems that have negatively influenced math performance. In their study, they found that curricular content taught to students in kindergarten and first grade

is commensurate with what is expected in these grades, but this was not true in the second grade. In second grade, the teachers only covered 77% of the math curriculum required for that grade level. By the fifth grade, only 2% of the material presented in class was at the fifth grade level, according to state standards. Hollingsworth and Ybarra stated that schools need to determine which material should be taught in which grades and realign the curriculum with the state standards. Similarly, Peck and Jencks (1981) analyzed a basal math series and reported that the majority of the material presented to secondary students was review (76% of the material presented in the sixth grade, 80% in seventh grade, and 82% in eighth grade). Clearly, math curricula and related instructional practices need to be examined further.

Porter (1989) identified four factors that negatively affect student understanding of mathematical problem solving. The first is the significant amount of time spent on teaching computational skills. Porter notes that the time spent teaching these basic skills is taking away from time spent teaching higher-level problem-solving skills. The second factor is that 70% of material is taught at the exposure level (less than 30 minutes of instructional time spent on the topic). The third factor is the lack of

consistency related to the amount of time teachers actually spend teaching math. Some teachers devote more of their instructional time to math than others. The final factor that negatively influences math problem solving is the low-intensity curriculum. Porter states that some teachers choose a math curriculum that does not emphasize the higher-level problem solving skills.

Fortunately, over the past decade researchers and educators have advanced their knowledge regarding effective teaching methodologies in the area of mathematics. Specifically, three methodologies have emerged as being appropriate for students having difficulty with math; direct instruction, strategy instruction and the concrete-representational-abstract teaching sequence.

Direct Instruction

Direct instruction (or explicit instruction) is task-oriented and organized teacher-directed instruction where information is presented in a clear and focused manner to promote student understanding (Miller, 2002). The instruction typically is presented in a five-step sequence. The first step is to provide an advanced organizer. This organizer precedes each lesson and gives the students a "heads up" as to the material being covered in the upcoming lesson. This is done to gain student attention. The second

step is describe and demonstrate. The teacher pairs the verbal explanation of the lesson with a step-by-step demonstration of the problem the students are expected to solve. The third step of direct instruction is to provide guided practice. In this step the students have the opportunity to work through a problem with teacher support. The fourth step is to provide the students with independent practice. After the students have demonstrated success in solving the problem during the guided practice step, the students are given the opportunity to solve problems independently. The final step is to provide the students with a post-organizer. During this organizer, the teacher reviews the information discussed in the day's lesson and emphasizes its importance, provides feedback related to the students' performance, and previews upcoming lessons.

Strategy Instruction

Instruction in learning strategies, as described by Deschler, Ellis, and Lenz (1996), is based on a cognitive approach to teaching that provides instruction consistent with how a student thinks in the context of learning tasks. The goal is to teach the learner skills that facilitate learning (i.e., teach students how to learn). The teacher and the material used are only efficient when they provide experiences that enable the learner to construct and retain

new meanings. When using the cognitive approach to teaching, the instruction must be developed based on an understanding of the interaction between the individual and the learning environment, which includes the instructional process and settings where accurate performance is required (Deschler, Ellis, & Lenz, 1996). The teacher's role is to analyze the students' performance and formulate hypotheses about how a student identifies, interprets, organizes, and applies information. The teacher then tests those hypotheses through the use of specifically designed instruction that provides the student with strategies to use in guiding the student's learning. Deschler, et al. (1996) stated that "Instruction must either promote the development of more effective and efficient ways of learning, or it must compensate for a perceived mismatch between how the student processes information and how information is being presented by the teacher and the instructional materials"(p.12). Deshler's long-term research with colleagues at the University of Kansas Center for Research on Learning has resulted in the identification of a curricular and instructional framework that is effective for teaching students how to learn and perform when faced with complex academic challenges. Major components of the learning strategy instructional approach include the use of organizers, describing and modeling the

problem-solving procedures, and using guided and independent practice to ensure student mastery.

One of the most important components of learning strategy instruction is the use of acronym mnemonics. Acronym mnemonics are words formed from the initial letters of other words, which are used to enhance learning and memory (Miller & Mercer, 1993; Miller, Strawser, & Mercer, 1996). The sequential steps of a mnemonic device require students to be actively involved in the academic task and reduce passive learning behaviors.

Most research related to the effectiveness of using mnemonic devices for solving math problems has involved basic computational skills (Miller & Mercer, 1991-1994) and word problems, (Montague, Applegate, & Marquard, 1993; Montague, 1996; Snyder 1998; Watanabe, 1991; Case, Harris, & Graham, 1992). Unfortunately, little research has been conducted related to the use of mnemonic devices for solving complex algebraic word problems. To successfully solve these problems, students must follow a specific set of sequential steps, so the use of mnemonic devices may be particularly appropriate. Additional research is needed to make this determination.

Concrete-Representational-Abstract Teaching Sequence

The concrete-representational-abstract (CRA) teaching sequence has been found to facilitate math learning in a variety of basic skills including addition (Miller, Mercer, & Dillon, 1992), place value (Peterson, Mercer, & O'Shea, 1988), subtraction (Mercer & Miller, 1992), multiplication, (Miller, Harris, Strawser, Jones, & Mercer, 1998; Morin & Miller, 1998), division (Mercer & Miller, 1992; Miller, Mercer, & Dillon, 1992), and fractions (Butler, 1999). This method of instruction places an emphasis on teaching students to understand the concepts of math before memorizing facts, algorithms, and operations. Instruction begins at the concrete level where students use three-dimensional objects to solve math problems. Instruction progresses to the representational level during which students use drawings to solve math problems (e.g., tally marks). The abstract component of the CRA sequence requires students to solve the math problem without using objects or drawings. The student reads the problem, recalls the answer or thinks of a way to solve the problem, and writes the answer.

Researchers have demonstrated the effectiveness of using direct and learning strategy instructional models to implement the CRA mathematical sequence when teaching basic

math skills to students with and without disabilities. Most research related to the CRA teaching sequence has been conducted with elementary-aged students. Additional studies are needed to determine whether this teaching sequence also is effective for secondary students who are learning to solve complex math skills such as algebraic word problems.

Statement of the Problem

Despite the increased emphasis on math education over the past three decades and increased knowledge related to factors that influence math performance, students with and without disabilities continue to struggle with mathematics. The current mathematics reform movement has resulted in higher performance expectations and standards for all students. Included among these standards is the expectation that students will learn sophisticated problem solving techniques and increase the ability to use symbols in reasoning. To meet these standards, researchers and teachers must work together to identify effective practices for teaching complex math skills.

Purpose of Study and Related Research Questions

The present study is designed to investigate the effects of teaching high school students with and without mild disabilities a strategy for solving algebra equations and

word problems using the concrete-representational-abstract (CRA) sequence. Specifically, the following questions will be addressed:

1. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra equations?
2. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra word problems?
3. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra equations?
4. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra word problems?
5. Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra equations?

6. Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra word problems?
7. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students with mild disabilities?
8. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem skills among students with mild disabilities?
9. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students without disabilities?
10. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem skills among students without disabilities?
11. Is there a change in student attitudes toward mathematics after receiving algebra instruction using

the Concrete-Representational-Abstract teaching sequence?

Significance of the Study

The latest mathematics standards proposed by the National Council of Teachers of Mathematics (NCTM, 2000) suggest that students should: (a) build on prior knowledge and learn more varied and sophisticated problem-solving techniques; (b) increase their ability to visualize, analyze, and describe situations in mathematical terms; and (c) increase their ability to use symbols in reasoning. The NCTM standards also state that all students, regardless of future aspirations, should study math all four years they attend high school, and that this course of study should include instruction in algebra.

To meet increased standards for problem solving and higher algebraic performance, students will need intensive instruction that includes effective learning strategies for solving complex algebra problems. Seven critical components have been identified to successfully instruct secondary students with learning disabilities in the area of algebra (Maccini, 1999). These components are: (1) teach prerequisite skills, definitions, and strategies; (2) teach

conceptual knowledge; (3) provide direct instruction in self-monitoring procedures; (4) provide direct instruction in problem representation and problem solution; (5) provide effective instruction; (6) use organizers; and (7) incorporate manipulatives. Additionally, to increase overall math achievement as well as assist students who encounter specific difficulty in learning algebra, students' needs and misunderstandings, should be assessed. Identification of student misconceptions or errors assists educators in planning appropriate instruction.

Konold (2000) found, in a pilot study, that high school freshman and sophomores who received instruction on the concepts and processes for solving algebra word problems could not solve them one month following initial instruction. Although they exhibited strong calculation skills, a majority of the students chose the wrong calculation process to compute the answer. This suggests that if the students had learned and recalled a strategy to properly convert the word problem to an algebraic formula, they should have been able to complete the problem successfully. Several researchers have noted the importance of teaching students specific learning strategies to assist in their understanding of mathematical concepts and processes (Mercer & Miller, 1992; Miller & Mercer, 1993).

Definition of Terms

Abstract Instruction

Abstract instruction requires the student to solve problems using numbers only. It does not allow the learner to use manipulatives or visual stimuli to assist in the problem solving process (Underhill, Uprichard, & Heddens, 1980).

Acronym Mnemonic

An acronym mnemonic is a word formed from the initial letters of other words, which is used to enhance learning and memory (Miller & Mercer, 1993).

Concrete Instruction

This instruction involves the use of manipulative and computational processes, which allows the learner to focus on both the manipulated objects and the symbolic processes involved in solving the problem (Underhill, Uprichard, & Heddens, 1980).

Concrete-Representational-Abstract (CRA) Instructional Sequence

Instruction begins at the concrete level where students use three-dimensional objects to solve the math problems. Instruction progresses to the representational level. At this stage, students use drawings to solve the math problems (e.g., tally marks). The final stage of the CRA sequence is the abstract. In this stage, the student solves math

problems without using objects or drawings. The student reads the problem, thinks of a way to solve the problem, and writes the answer (Gagnon & Maccini, 2001).

Direct Instruction

Direct instruction involves explicit instruction, mastery learning, fading teacher support, examples and modeling, reviewing prior knowledge, and teacher-led instruction and correction (Maccini & Gagnon, 2000).

National Council of Teachers of Mathematics (NCTM)

The NCTM was founded in 1920 and is the largest mathematics education organization in the world. The mission of NCTM is to provide the vision and leadership necessary to ensure a mathematics education of the highest quality for all students (www.nctm.org/about/intr.htm).

Problem-solving

Problem-solving requires students to retrieve previously learned information and apply it to new or varying situations (Bley & Thornton, 2001).

Retention

The ability to remember information after time has passed (Friend & Bursuck, 2002)

Representational Learning

During this stage of learning, the student uses pictures or tallies to represent the numbers used in solving the problem (Underhill, Uprichard, & Heddens, 1980).

Strategy Instruction Model (SIM)

SIM involves an eight stage instructional sequence designed to promote the acquisition and generalization of the strategy being taught (Deshler, Ellis, & Lenz, 1996).

Limitations of the Study

This study will include students without disabilities and students with mild disabilities in grades 6 to 12. Thus, the findings should not be generalized to students with severe disabilities or to students in other grades. This study will address solving algebra word problems. Therefore, the findings should not be generalized to other math skills or other algebra skills. Finally, the study will be conducted in three schools within two school districts. Caution should be exercised in extrapolating results of the study to students who attend other schools in the districts. Caution also should be exercised when generalizing results to students who attend schools in other districts.

Summary

Mathematics achievement in America has been an emphasis in American politics and in academia. Over the years, many math movements have emerged and failed. Researchers have noted there are many contributing factors (e.g., curricular designs and instructional methods) to the continued poor math achievement among school-aged students. Researchers have demonstrated the effectiveness of using direct instruction, learning strategy instruction and the CRA sequence for teaching a variety of basic math skills, but little research has been conducted related to the effectiveness of the CRA sequence for teaching more complex skills such as algebra. This study is intended to provide new information related to teaching algebra problem-solving skills. Specifically, comparisons will be made between the concrete-representational-abstract sequence and the traditional abstract method of teaching these skills. Also, students with mild disabilities and without disabilities will be compared to determine if any differences exist in their ability to progress through the CRA teaching sequence.

Chapter 2

REVIEW OF RELATED LITERATURE

Literature Review Procedures

A systematic search through four computerized data-bases - Education Resources Information Center, Journal Storage (JSTOR), Mathscinet, and Elton B. Stephens Company (EBSCO) was conducted. The following descriptors were used: algebra, word problems, algebra and remediation, algebra and intervention, algebra and techniques, algebra and special education, mathematics and remediation, mathematics and intervention, mathematics and special education, and concrete-representational-abstract. An ancestral search through the references lists of the articles obtained in the computer search also was completed.

Selection Criteria

Studies were included in this review of literature if: (a) the procedures and data-based results were published between 1980 and 2003, (b) the subjects were elementary or secondary students without disabilities or with mild

disabilities, (c) the purpose of the study was to examine the effectiveness of an instructional method on students' problem-solving ability. Studies were excluded from this review if: (a) the subjects had a moderate or severe disability (e.g., mental retardation, autism) (b) the purpose of the study was to identify characteristics of students experiencing math difficulties, (c) the purpose of the study was to assess the problem-solving abilities of students without implementing an instructional intervention.

Problem Solving Using Cognitive, Metacognitive, or Self-Regulation Strategies

Maqsud (1998) examined the effects of metacognitive instruction on mathematics achievement and attitude toward math of low math achievers. Maqsud reviewed the files of 310 seventh grade students. Of these 310 students, 80 of these pupils were selected due to low math achievement scores. Maqsud then administered the Raven's Progressive Matrices to determine general ability level. Based on the results of the matrices test, the 80 students were divided into a low group and a high group. The low group then was randomly assigned to the experimental group and the control group.

Both experimental and control groups were given four tests: Raven's Standard Progressive Matrices; Swanson

Metacognitive Questionnaire; Aiken Scale of Attitude toward Mathematics; and a teacher-made achievement test. In the experimental group, the researcher interviewed each student to determine the process used in leading to errors on the students' class work. The students then were redirected to apply a strategy in solving the problem. In general, the researcher taught strategies to the students so they could find the correct solutions to the problems and avoid the earlier errors. In the control group, the class work was graded and returned to the students with no formal feedback.

The researcher used a repeated measures t-test to compare the means of the four variables between the control group and experimental group. The comparisons of pretest and posttest measures of general ability, metacognitive awareness, attitude toward math and math achievement revealed that the posttest scores of all four variables for the experimental group were significantly higher than those for the control group.

The author concluded that an individual remedial approach was an effective way of increasing math achievement among middle school students. Also, this individualization can bring about positive changes in the students' attitudes toward mathematics.

The weakness of this study lies within the activities of the control group. The author concludes the study shows individualizing remediation produces positive effects. The control group received no remediation. The researcher compared remediation to no remediation. Had the author provided the control group with a class-wide remediation technique and compared that with the individual remediation technique, then the author could have concluded that individual remediation provided a better result. However, as the study stands, the only conclusion the author can make is that remediation is better than no remediation.

Bottge and Hasselbring (1993) compared two groups of adolescents having difficulty in math on their ability to generate solutions to a contextualized problem after being taught problem-solving skills under two conditions. The first condition involved teacher-guided instruction in standard word problems, while the second condition involved teacher mediation of students' efforts to solve a contextualized problem presented on videodisc.

The subjects in this study were 36 ninth-grade students in two remedial math classes. Of the 36 students, 17 of them received special education services. Before the study began, the authors administered a researcher-made fractions-computation test. Test scores were ranked from lowest to

highest. Students having the two lowest scores were randomly assigned to either a contextualized problem (CP) group or a word problem (WP) group. Then, students with the next lowest pair of scores were randomly assigned to a group. This procedure continued until all 36 students were either assigned to the CP group or the WP group.

Students in the CP group were shown a video problem and asked to describe the challenge presented by the video. The teacher guided a class discussion regarding how to better define the problem. To end the first day's instruction, the teacher replayed portions of the video and the students completed a worksheet that reviewed the video's content. During the second day, the students corrected their worksheets and were given time to calculate solutions to the subproblems. On day three, the students were given a teacher-guided quiz to check their understanding of the relationship between the subproblems and the challenge problem. The students were encouraged to generate several ways to solve the challenge problem on the fourth day. Alternative methods to solve the problem were summarized on the blackboard and then reviewed using a worksheet. The focus of the last day's instruction focused on questions to help the students focus on the problem, yet invited the student to think about how solutions could be altered.

The WP group was led through a series of word problems by the teacher. The word problems paralleled the contextualized problems and required identical mathematical procedures to solve the subproblems. Each of the five-days of instruction followed the same format. First, a student read the problem aloud and then the teacher asked the students to identify all extraneous information. Once the students were able to explain how to solve the problem, they computed the answer. Following the last day of instruction, the students were combined into one group and administered the contextualized problem posttest and the word problem posttest. A 2 x 2 repeated measures ANOVA was used to analyze the data. Both groups improved their ability to solve word problems, but the CP group performed significantly better than the WP group on the contextualized problems posttest. The authors concluded that students with a history of difficulty in mathematics can be taught how to solve complex, meaningful math problems.

Weaknesses of this study include a fairly small sample size ($n = 36$) and the fact that the intervention was limited to the use of one video problem. Based on the previous statement, this study has limited generalization.

Allsopp (1997) compared the effectiveness of using classwide peer tutoring to using traditional independent

student practice to teach beginning algebra problem solving skills between both students at-risk for math failure and students not at-risk for math failure. The students in the study included 262 eighth grade students in 14 different general education math classes. Ninety-nine of those students were classified as being at-risk for math failure (stanine of three or less on the math portion of the California Test of Basic Skills and receiving a D or lower in math class). One hundred and sixty-three of the students were classified as not being at-risk for failure in mathematics (a stanine of 4 or higher on the math portion of the California Test of Basic Skills and a grade of C or better in math class). Two groups were created with an equal number of students at-risk for math failure and those not at-risk for math failure. The students were assessed using a researcher-made assessment tool. This tool was administered as the pretest measure, posttest measure, and maintenance measure.

The study was implemented in four phases. Phase one included teacher training. Teachers involved in the study were trained on a math curriculum used for problem-solving instruction. The curriculum included three learning strategies in the form of mnemonic devices. The curriculum begins with the use of concrete manipulative devices. The

curriculum progressed toward the more abstract problem-solving skills. The teachers also were instructed on the Classwide Peer Tutoring (CWPT) technique. Phase two involved teaching the students the CWPT technique. Phase three was the implementation phase. Treatment group A was instructed using the problem-solving curriculum and then student independent practice after the completion of the lesson. Treatment group B also received the problem-solving curriculum, but after the lesson the students engaged in CWPT to actively practice the skills.

Data were analyzed using a 2 x 2 x 2 ANOVA. Neither method was more effective than the other, but the at-risk group demonstrated greater performance gains than the students not at risk. The author concluded that the problem-solving curriculum was effective with both types of student practice (CWPT and Independent practice). However, neither of the practice types appear to be more effective than the other. The weakness of this study is that it appears as though there were actually two studies instead of one. One study was determining whether the problem-solving strategy was an effective method of teaching the particular algebra skill. The other was to determine if CWPT was more effective than independent practice. It seems that the study may have

been more powerful if it had been separated into two separate studies to prevent possible confounding variables.

Montague, Applegate, and Marquard (1993) investigated the effects of cognitive strategy instruction on math problem-solving performance of junior high school students with learning disabilities. The subjects in this study consisted of 72 junior high school students receiving special education services in the area of learning disabilities. In order to participate in the study, the students had to have an IQ of at least 85, knowledge of basic operations using whole numbers, poor performance on the math word problems (as judged by their math teacher) and a reading grade level of at least 3.5. A comparison group of 24 general education students also was used in this study. Three treatment conditions were investigated. Subjects in the first condition received direct instruction in cognitive strategies, subjects in the second condition received instruction in metacognitive activities for solving math word problems, and subjects in the third condition received a combination of cognitive and metacognitive strategy instruction. All conditions were taught by the investigator and two research assistants. Each of the groups were taken out of their general math class to be instructed using the three different models. In the first condition, the students

learned only the names of the processes and their descriptions. The teacher modeled the problem solving, but did not explain how to apply the processes. Subjects in the second condition were taught only the metacognitive activities associated with each cognitive process. The teacher modeled the application using word problems and the students practiced on their own. Students in the third condition were required to memorize the processes and paraphrase the metacognitive activities associated with the process. The teacher modeled the strategy and its application and gave the students corrective and positive feedback during guided practice. A repeated measures ANOVA was used to analyze the data. All subjects in the experimental groups improved in their mathematical problem solving performance, but no one condition was significantly better than the other. At the completion of the study, no significant difference existed between the experimental groups and the control group. The authors concluded that the effectiveness of the instructional routine for improving math word problem solving for students with learning disabilities was demonstrated. The subjects improved over time and achieved a level comparable to their non-disabled peers.

The weakness of the study is the fact that it did not take teacher effect into account. Each of the groups were taken out of their general math class and given specialized instruction by someone other than their math teacher. The improved performance could have been a function of something new occurring in the lives of the students.

Case, Harris, and Graham (1992) examined the effectiveness of a self-regulated strategy to improve word problem skills among students with learning disabilities. The seven participants in this study were fourth, fifth and sixth grade students who had been identified as having a learning disability. Two undergraduate students majoring in special education served as the students' instructors. The students were taught how to be an active collaborator, which included principles of interactional scaffolding and Socratic dialogue. The students did not move on to the next level of instruction until they had mastered the previous level. The strategy instruction was approximately 35 minutes in length and occurred two to three times per week. The authors used a multiple baseline across subjects design. The students were given a seven question researcher-made test periodically throughout the study. At the end of the strategy instruction, overall performance on mixed sets of word problems improved, but maintenance of skills was not

shown for the strategy. The authors concluded that the strategy was effective in teaching the students which of the two operations (addition and subtraction) should be used in solving the various word problems. Weaknesses of this study include poor generalizability due to the single-subject design. Also, the students were pulled out of class and taught by someone other than their teacher. This strategy may not have practical applications within the general school environment.

Montague and Bos (1986) investigated the effects of an eight-step cognitive strategy on verbal math problems solving performance of adolescents with learning disabilities. Six adolescents identified as having a learning disability were used for this study. All subjects had scaled scores on the arithmetic subtest of the WISC-R or WAIS of at least one standard deviation below the mean. Also, the subjects had at least a fourth grade reading level and at least a three and one-half years delay in mathematics as measured by the Woodcock-Johnson Psycho-educational Battery. The authors used a multiple baseline design with baseline, treatment, generalization, maintainance, and retraining. The authors developed 19, 10-item tests of two-step verbal math problems. Baseline data were recorded and continued until a stable baseline was established for the

first subject. During treatment, the students received strategy acquisition training, strategy application practice and testing. There were eight steps in the problem-solving strategy (1. Read the problem aloud; 2. Paraphrase the problem aloud; 3. Visualize; 4. State the problem; 5. Hypothesize; 6. Estimate; 7. Calculate; and 8. Self-check). The subjects were taught this strategy in a resource setting during regular school hours. The strategy trainer was one of the subjects' teachers. The authors designated 7 of 10 correct answers during the treatment phase as acceptable and 5 of 10 correct during the maintenance phase as acceptable. The results indicated six of the seven subjects reached the acceptable level during treatment and four of the seven reached the acceptable level in the maintenance phase. The authors concluded that the eight-step strategy appeared to be an effective intervention for students having difficulty in verbal math problem solving. Weaknesses of this study include the low acceptance level established for the maintenance phase (50% correct). In most schools, 50% is an "F." Another weakness is within the strategy itself. Steps one and two require the student to read the problem aloud and paraphrase the problem aloud. In a class of 30 students, reading aloud could make for a very noisy environment.

Montague (1992) investigated the effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. Out of 14 middle-school students placed into a special education program, she randomly selected six to use as subjects. The Mathematical Problem Solving Assessment-Short Form was administered to the subjects as a pretest and posttest measure. Montegue created 35, 10-problem tests from a pool of 400 math word problems taken from middle school textbooks. Each test contained 3 one-step, 4 two-step and 3 three-step problems requiring the use of all four basic operations. These tests were used for screening and experimental conditions.

Montegue used a multiple baseline across subjects design including a baseline, two levels of treatment, setting and temporal generalization and retraining. During treatment 1, the subjects received either cognitive strategy instruction (CSI) or metacognitive strategy instruction (CMSI). Treatment 2 consisted of instruction in the complementary component of the instructional program so that all subjects eventually received both cognitive and metacognitive strategy instruction. The study was conducted over a four-month period of time. Each subject received individual instruction and test sessions from the researcher in a

separate room during the regularly scheduled math time. Each session lasted for approximately 55 minutes. The treatment consisted of strategy acquisition training, strategy application practice for the CSMI only and testing sessions using the dependent measures.

Visual inspection of the data indicated that three days of CSI did not improve the math problem-solving ability, but the same amount of CMSI resulted in some improvement of the subjects' math ability. The author concluded that a combination of cognitive and metacognitive strategies may be more effective in teaching math problem-solving skills than either strategy alone.

The weakness of this study is in the ability to generalize the results to a classroom setting. The researcher worked with the subjects individually over 26 55-minute sessions. It does not seem practical to expect a special education teacher to work one-on-one with a child for 55 minutes a day for 26 days.

Hutchinson (1993) investigated the effects of a two-phase cognitive strategy on algebra problem solving of adolescents with learning disabilities. The treatment consisted of individual meetings with the researcher. The subjects met with the researcher for 40 minutes every-other day for around four months. Each session used the same procedures.

The procedures were: 1. Remind student of the purpose; 2. Give students five problems and a prompt card for self-questioning; 3. Ask the students to read the self-questions; 4. Have students read the problems silently; 5. Ask students to model the use of the strategy by thinking aloud for the first two problems; 6. Provide corrective feedback for problems three and four; 7. Provide corrective feedback after problem eight; 8. Fade out prompts; 9. Test student with an assessment sheet (to be completed independently); and 10. Plot student progress on a graph.

The study used a modified multiple baseline with 11 replications as well as a two-group design. Visual analysis of the single subject data showed the strategy to be an effective intervention for this sample of students. Statistical analysis of the two-group data showed that the instructed students had a significantly higher posttest score than the comparison group. The findings indicate the instructed students demonstrated improved performance on algebra word problems. Maintenance and transfer of the strategy were evident.

The weakness of this study lies in the administration of the procedure. The students are required to use a think-aloud procedure. Given the typical classroom consists of

more than one student, this strategy does not seem to be very practical.

Summary of Research Related to Problem Solving Using Cognitive, Metacognitive, or Self-Regulation Strategies

A total of eight studies were reviewed in the previous section. Six of the eight studies used subjects receiving special education services. Five of the studies used a form of single-subject design and three used a group design. All of the studies reported an improvement in ability to solve math problems after strategy instruction. Three of the studies assessed for maintenance of skills (Case, Harris, & Graham, 1992; Montegue & Bos, 1986; and Hutchinson, 1993) and two of the strategies were found to be effective over the maintenance period (Montegue & Bos, 1986; and Hutchinson, 1993). After reviewing these studies, it appears that providing strategy training is an effective way to improve student problem-solving ability.

Problem Solving Using Schema-Based Drawings

Jitendra, Hoff, and Beck (1999) investigated the generalization of the schema strategy from one-step addition and subtraction word problems to two-step addition and

subtraction word problems by middle school students with learning disabilities.

The subjects included four middle school students with learning disabilities. The study also included 21 normally achieving third grade students. The authors used third graders because the majority of instruction on how to solve addition and subtraction word problems occurs in the third grade. All subjects were given a 10-item word problem test at the beginning of the study to serve as the pretest.

The researchers used a multiple baseline design across subjects and across behaviors. The experimental phases included a baseline, two levels of instruction and postinstructional tests.

During the first phase of schema-based instruction, the students were taught to pick out the distinguishing features of the story. Diagrams were provided to allow students to map out the features of the story. Once the students were able to pick out the important information and diagram the information correctly, the students were taught which math operation was required to find the missing information. During the second level of instruction, the students were taught a backward chaining strategy to solve two-step word problems. Backward chaining utilized a top-down approach where the student identifies the primary problem to be

solved and then identifies the secondary problem, which must be solved before the primary problem can be solved. The results indicated that the schema strategy led to an increase in word problem solving performance for all students within the experimental group. Further, these results were maintained at a 2 and 4 week follow up. The performance on two-step word problems by the students receiving the schema-based strategy surpassed that of the typical third grade control group.

The authors concluded that the schema-based instruction improved the word problem solving ability of the four junior high school students in the study. The weaknesses of the study include a single-subject design, which limits generalizability and comparing the four subjects receiving strategy instruction to a group of third graders with whom no instruction on solving word problems was given.

Jitendra, Griffin, McGoey, Gardrill, Bhat, and Riley (1998) compared the effects between schema-based instruction and a traditional-based instruction on the acquisition, maintenance, and generalization of mathematical word problem solving for students at-risk for math failure or those with mild disabilities.

Students included in the study had to meet three criteria. First, the students' teachers had to identify them

as having adequate addition and subtraction skills, but poor word problem-solving skills. Second, the students had to successfully complete a measure of their addition and subtraction skills and the last criterion was the students had to perform at or below a 60% on a measure of word problem-solving skills. A total of 34 students in elementary school made up the sample. Twenty-five of those students had been identified as having a mild disability (learning disabilities, mild mental retardation and seriously emotionally disturbed). The remaining nine students were non-identified low achieving students experiencing difficulty in mathematics. The students were randomly assigned to either the schema group or the traditional group.

A 15-item problem-solving instrument was designed to be given as a pretest, posttest, and delayed posttest. Instruction was delivered in a small group setting (three to six students per group) in a quiet room in the school building. Each session lasted approximately 40 minutes and was delivered by four doctoral students and two master's students. In the schema-based instruction, the students were taught to find the important information in the text, develop a solution strategy or action schema, and then select and execute the appropriate arithmetic operation. For

the traditional group, the researchers used the students' textbook to teach word problem solving skills. The instruction used a five-step checklist procedure to solve word problems.

The authors used a 2 x 2 ANCOVA with repeated measures to test for treatment effects. Differences between the schema-based group and traditional-based group were significant favoring the schema group. The authors concluded that when elementary school students with learning problems were taught to use a schema strategy to solve word problems, their performance increased on measures of acquisition, maintenance, and generalization. Points of weakness in this study include all of the instruction took place in a small group setting, which is likely not the case in "real world" application. Also, researchers, not the teachers, taught and assessed the students participating in the study.

Summary of Research Related to Problem Solving Using Schema-Based Drawings

Two studies were reviewed in the previous section. Both studies investigated the use of concept mapping to solve math word problems. One study (Jitendra, Hoff, & Beck, 1999) utilized a single-subject design while the other (Jitendra,

et al., 1998) utilized a group design. The subjects in each study were identified as having learning disabilities, or being low-achievers in mathematics. One study (Jitendra, Hoff, & Beck, 1999) used middle school students as subjects. Jitendra, et al., (1998) used elementary school students. Increased ability to solve word problems was evident for both studies.

Problem Solving Using Manipulative Devices

Marsh and Cook (1996) examined the effects of using manipulative devices in teaching students with learning disabilities to identify the correct operation to use when solving math word problems. The study consisted of three, third-grade boys identified as having learning disabilities in the areas of reading, written language and mathematics. Psychological testing results indicated that all three of the subjects were below grade level and experiencing difficulty in word problem solving tasks.

The authors used a multiple baseline across students design. Sets of Cuisenaire rods were used during the manipulative treatment portion only. The examiner developed 10 word problem probes. The word problems were one-step problems. Each student received 20 minutes of instruction each day. The rods were placed in a tray until the students

needed to use them. The instructor gave the student a worksheet and asked the student to read the first problem aloud. The instructor asked the student to go back and re-read the first sentence. The subject was asked to identify any important information within the first sentence and use the rods to represent the numbers within the sentence. This continued until the entire problem was read and the rods were set up to answer the problem. In each case, there was immediate and sustained improvement in the manipulative condition. The researchers stated that one of the subjects moved beyond using manipulatives to solve the problems and began to solve the problems without any representations. This information was provided as an anecdotal observation.

The weakness of this study is in the strategy itself. The students never were explicitly taught to move beyond using manipulative devices. Although, it appears using manipulative devices is an effective way to teach students to solve problems, it does not seem to be the most efficient way to solve problems. Manipulative devices can become cumbersome when the numbers increase in size. Also, the student may not have access to manipulative devices during testing situations.

Cass, Cates, Smith, & Jackson (2003) investigated the use of manipulative instruction on the acquisition and retention

of solving perimeter and area problems. Two high school and one junior high school students with learning disabilities participated in this study. The researchers used a multiple baseline design across subjects and two behaviors (perimeter and area problem-solving skills). The teachers used modeling, prompting/guided practice and independent practice when teaching the problem solving with manipulative devices. The students were taught to solve problems involving perimeter first and once the students mastered solving for a perimeter, they were taught to solve for area. The teacher used geoboards and geobands to model perimeter. The geoboard consisted of a 9 x 9 array, which limited the problems to single digit addition (perimeter) and single digit multiplication (area). The teacher taught the students to count the markers on the geoboard to determine the perimeter of the design. The teacher then created five shapes on the geoboard and prompted the students to follow the same step as before to determine the perimeter. After the students completed that exercise, the teacher selected two perimeter problems from the math book and demonstrated how to solve the problems using the geoboards. Once the students completed problems from the book, the teacher asked the students to measure items in the classroom (e.g. tabletop, rug) and determine the perimeter. The teacher followed a

similar process for teaching the students to solve for the area.

The results indicate that all three students increased their ability to solve problems involving perimeter and area. An assessment of skill retention revealed that after two weeks all students still were proficient in solving these problems. The authors concluded that the study extends previous findings that use of manipulative devices results in long-term retention of skills learned. The authors also report that the semiconcrete or representational stage may not be a vital component of instruction. The weaknesses of this study are that all of the modeling and problems solved contained 90 degree angles. It is a certainty that the students will need to determine perimeters and areas of figures with angles other than 90 degree angles. Also, the findings cannot be generalized to a larger population due to the small sample size.

Summary of Research Related to Problem Solving Using Manipulative Devices

Two studies were reviewed in the previous section. Both studies investigated the use of manipulative devices in math problem-solving skills. Both studies utilized a single-subject design. All of the subjects involved in both of the

studies had been identified as having a learning disability. One study (Cass, Cates, Smith, & Jackson 2003) used middle school students as subjects. Marsh and Cook (1996) used elementary school students. In both studies, the use of manipulative devices increased the ability of the students to solve math problems.

Concrete-Representational-Abstract Studies

Harris, Miller and Mercer (1995) conducted a study to evaluate the effectiveness of teaching multiplication skills to elementary school students with learning disabilities within general education classrooms. The subjects consisted of 112 second grade students (13 students had mild disabilities). The students selected for this study had to meet two criteria. The first was a signed permission slip by the student's parent and the second was passing the Prerequisite Skills Test. The test required the students to write 30 digits 0-9 in one minute and fill in missing numbers up to 81.

The authors analyzed the effectiveness of the Concrete-Representation-Abstract teaching sequence on the ability of the students to complete multiplication computations and word problems. Six general education teachers implemented this strategy within their classrooms. The strategy consists of 21 lessons. The first 10 lessons focused on the concept

of multiplication and solving simple problems. The remaining lessons focused on solving word problems and increasing the rate of computations. The results were that students with disabilities performed just as well as their nondisabled peers on the computation portion, but not on the word problems.

The weakness of this study was the use of a multiple baseline across classes design. No control group was used to assess the effectiveness of the treatment as compared to what is typically done within the general education class. One cannot state that this 21-lesson strategy is any more effective than following the students' textbook instructions for 21 lessons.

In 1998, Morin and Miller studied the effectiveness of teaching multiplication facts and related word problems using the CRA teaching sequence. There were three seventh grade students used in the study. Each of the subjects was receiving special education services under the funding category of mental retardation. The criteria for including the subjects in the study were: the subjects had not mastered computation and problem-solving skills in multiplication; each subject was able to count to 81 and compute addition problems with sums to 18; and parent and student permission to participate in the study. The

researchers used a single-subject multiple baseline design across individuals. When subject one obtained the 80% accuracy criterion, the intervention was introduced to subject two and then with subject three.

The researchers used a scripted manual from the Strategic Math Series for the study (Multiplication Facts 0-81). The pretest and the posttest were taken from the manual. A special education teacher was trained on the materials and procedures of the manual. The special education teacher conducted all of the 35 minute instructional sessions. There were three sessions at the concrete level, three sessions at the representational level, one session instructing the use of a mnemonic, and three sessions at the abstract level. The results indicated an improvement for each subject in their ability to solve multiplication problems. The researchers concluded that students with mental retardation can learn to solve multiplication facts and word problems using the CRA teaching sequence. Also, the researchers concluded that use of mnemonic devises can be beneficial in cueing the specific cognitive functions required in solving multiplication problems. A weakness in the study falls within its limited generalizability. The students were taught this strategy individually. This intervention may not have classroom applicability.

Summary of Research Related to Concrete-Representational-Abstract Teaching Method

Two studies were reviewed in the previous section. Both studies investigated the use of the concrete-representational-abstract teaching sequence when instructing students in mathematics. One study utilized a single-subject design (Morin and Miller, 1998) involving secondary students with mental retardation. Harris, Miller, and Mercer (1995) utilized a group research design involving elementary school students with learning disabilities. In both studies, using the CRA teaching sequence increased the ability of the students participating in the study to solve mathematics computations and word problems.

Problem Solving and Algebra Instruction

Witzel, Mercer, and Miller (2003) investigated the use of the Concrete-Representational-Abstract teaching sequence to instruct students with math learning disabilities and/or students who were at-risk for algebra difficulty to solve inverse algebraic operations. The subjects consisted of 68 students matched according to pretest score, standardized math test scores, teacher, similar age, and same grade. Half

of the students participated in the 19-lesson curriculum using the CRA program while the other half received a 19-lesson curriculum using traditional instruction. Results indicate that all students increased their ability to solve algebra equations, but the students who received the CRA instruction scored higher than those who received the traditional method of instruction on post-test and follow-up tests.

The strength of this study is within the research procedure of matching the subjects on various test scores, grade and age.

Maccini and Hughes (2000) investigated the use of a strategy to improve solving word problems involving addition, subtraction, multiplication, and division of integers. The strategy utilized a concrete-semi-concrete-abstract teaching sequence. The subjects consisted of six students with learning disabilities that had targeted math goals on their Individual Education Program.

The strategy began with teaching the subjects to use manipulative devices to solve the problems. They were given a worksheet and were guided through the process of solving the problems. The second phase of the treatment was to teach the subjects to use a two-dimensional representation of the numbers to solve the problems. During the final stage of the

treatment, the subjects were given a worksheet and asked to solve the problems using numerical symbols and to review the solution to check for reasonableness.

The researchers used a single subject multiple baseline design to study the effectiveness of this treatment. The CRA was found to be an effective instructional method in teaching the six students with learning disabilities in the study to solve the word problems.

The limitation of this study is in its generalizability. With six students, it is difficult to generalize to other students with learning disabilities, students with other disabilities or to the general education population.

Maccini and Ruhl (2000) piloted an instructional strategy to teach secondary students with learning disabilities to solve word problems involving subtraction of integers. There were three subjects in the study. Each subject had a diagnosed learning disability and demonstrated a deficit in the ability to solve word problems involving subtraction of integers.

The treatment included three phases. The first phase was the concrete phase. In this phase, the subjects were taught to use algebra tiles to compute the problems and to self-regulate their thinking process through the use of questioning. Each subject needed to reach mastery (80%)

before moving onto the second phase. The second phase of the treatment was the semi-concrete phase. The subjects were taught to move from the three-dimensional representation to a two-dimensional representation. The subjects were taught to draw pictures to represent the problem instead of using the tiles to represent the problem. In the third phase, the abstract phase, the subjects solved the word problems using numeric representations.

The authors used a multiple probe across subjects design. All three subjects were given four baseline probes and once stability was achieved for the first subject, the instruction began. When the first subject showed improvement, the strategy was started for the second subject and again for the final subject.

The results indicated that all three of the subjects learned to solve word problems involving the subtraction of integers. The weakness of the study is within the generalizability. The study cannot be generalized to other students with learning disabilities, other students with different disabilities, or to general education students.

Summary

There has been a limited amount of research that has been completed related to problem-solving interventions for

algebra instruction. At this point in time, only three studies (Witzel, Mercer, and Miller, 2003; Maccini and Hughes, 2000; and Maccini and Ruhl, 2000) have been conducted to assess the effectiveness of the Concrete-Representational-Abstract teaching sequence for teaching algebra word problems. Two of the studies involved single-subject designs (Maccini and Hughes, 2000; and Maccini and Ruhl, 2000) and all included only students with learning disabilities. The studies were developed to examine the effectiveness of the CRA teaching sequence. The current dissertation adds to the literature in several ways. First, a group design is used to allow for comparison between the CRA teaching sequence and traditional instruction. Second, the study includes a larger number of students with and without disabilities than previous studies. Third, the study includes a comparison of performance between students with disabilities and student without disabilities. Finally, attitude toward mathematics is investigated before and after strategy instruction.

CHAPTER 3

METHOD

The purpose of this study is to investigate the effects of the concrete-representational-abstract teaching sequence on students' algebraic equation and problem-solving skills. Specifically, this study addresses the following questions:

1. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra equations?
2. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra word problems?
3. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra equations?
4. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level

instruction for teaching students without disabilities to solve algebra word problems?

5. Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra equations?
6. Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra word problems?
7. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students with mild disabilities?
8. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem skills among students with mild disabilities?
9. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students without disabilities?

10. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem skills among students without disabilities?
11. Is there a change in student attitudes toward mathematics after receiving algebra instruction using the Concrete-Representational-Abstract teaching sequence?

Methods and procedures used in this study are detailed in this chapter. The chapter is organized into four sections: description of subjects and setting, description of the research instrumentation, procedures, and treatment of the data.

Description of the Subjects and Setting

The participants in this study are five high school teachers, one middle school teacher and their students in two schools located in the southwest portion of the United States and one located in Alaska. Three of the teachers teach general education classes while the other three teachers teach math within a pull-out resource room model. All of the teachers teach multiple sections of the same math course.

The total number of signed consent forms returned was 194. The number of subjects with usable data was 169. The

data from twenty-five subjects were excluded from analysis due to the following reasons: 1) student demonstrated noncompliance with regard to completing the pretest or posttest; 2) student was absent during either pretest or posttest administration; and 3) student transferred to different school or class. Twenty-four of the subjects were identified as having a high incidence disability (e.g. learning disability or emotional disturbance). The remaining subjects were general education students. All subjects were selected for participation in this study using two criteria. First, parental consent for minors was required for reporting results. Consent forms were sent home with every student in all participating sections. The second criterion for subject selection was current enrollment in an Algebra 1A (similar to Pre-Algebra) class or in a resource room math class. The students ranged in age from 11 to 19 years. Demographic data collected on the students with disabilities are contained in Table 3.1. Demographic data collected on the students without disabilities are contained in Table 3.2.

Table 3.1

Demographic Information for Students with Mild Disabilities Participating in this Study

Characteristics	CRA (n=37)	Traditional(n=24)
Gender		
Male	18	11
Female	19	13
Grade Level		
6	2	1
7	4	3
8	8	5
9	6	4
10	9	4
11	6	6
12	2	1
Disability Category		
Learning Disability	29	20
Emotional Disturbance	6	3
Mild Mental Retardation	2	1
Mean Intelligence Quotient	91.73	94.79
Standardized Achievement		
TOMA - 2 (SS)	86.24	79.56

Table 3.2

Demographic Information for Students without Disabilities Participating in this Study

Characteristics	CRA (n=46)	Traditional (n=62)
Gender		
Male	19	31
Female	27	31
Grade Level		
9	16	21
10	18	25
11	8	14
12	4	2
Achievement Score		
TOMA - 2	87.16	88.34

Description of Research Instrumentation

Standardized Test

The calculation and attitude toward mathematics portions of The Test of Mathematical Abilities-2 (TOMA-2) was group administered to each student. The TOMA-2 is a group administered norm-referenced test measuring math computation, ability to solve story problems, student

attitude toward math, student understanding of the language of math, and their familiarity of math terms and concepts used in everyday life. This assessment tool has a mean of 100 and a standard deviation of 15. The authors of the TOMA-2 report the internal consistency coefficient of the attitude toward math portion to be .84. Test-retest reliability is reported to be .70.

Teacher-Made Test

The teacher-made test was used for the pretest measure (See Appendix A) and the posttest measure (See Appendix B). These two measures consist of 20 one-variable algebra equations and one-variable algebra word problems. There are six word problems and 14 equations. These teacher-made tests were constructed and field-tested by the author of the algebra strategy being used for this study (Allsopp, 2001).

Lesson Materials

The strategy lessons used for this study were taken from The Building Algebra Skills Series (Allsopp, 2001). Unit four within the series, Solving One-Variable Algebra Equations and One-Variable Algebra Word Problems, was implemented as the treatment in this study. Unit four consists of one pretest lesson, 11 scripted teaching lessons, and a posttest lesson.

The materials for the strategy include three strategy sheets, which explain the three different mnemonic devices used, one strategy rules sheet, eleven learning sheets (one for each of the lessons), a pre-test, a post-test, a learning contract the student signs, and a progress chart.

Procedures

There are four phases in this study. These phases are: (a) preparation and teacher training, (b) preassessment, (c) implementation of treatment, and (d) postassessment.

Preparation and Teacher Training: Phase 1

Obtaining Research Approvals

Permission for the study was obtained from the University of Nevada Las Vegas Social Behavioral Sciences Institutional Review Board, the University of Nevada Las Vegas College of Education Center for Research and Planning, and from the Clark County School District Office of Testing and Evaluation. Prior to starting the study, explanatory letters and consent forms were sent home with the students. Only data from students whose parents returned a signed consent form were included in the study.

Group Assignment

Intact classes were randomly assigned to either a treatment or control group. Each teacher taught multiple

sections of the same Algebra 1A course. Two of the sections from each teacher were randomly assigned as treatment, while the other two were assigned as control. Therefore, each of the teachers involved in this study taught two treatment group classes and two control group classes. In addition to having an equal distribution of treatment and control classes, this method of group assignment simultaneously controlled for teacher effect.

Teacher Training

The teachers participating in the study were given a four-hour training session on the strategy. The training focused on the CRA teaching sequence and the importance of following the scripted lessons. The training began with a description of the CRA teaching sequence and the lesson format (i.e. advanced organizer, describe and model, guided practice, independent practice, and corrective feedback). While describing the lesson format, the trainer provided the teachers with the rationale behind following the format. The trainer discussed literature related to the effectiveness of the CRA and the lesson format. During the next portion of the training, the teachers were shown the correct way to complete a lesson in each of the phases of the strategy. Once this modeling was completed, the teachers were asked to review a lesson and demonstrate it. Feedback was provided to

the teachers about their performance. This portion of the training was repeated until each of the teachers taught one lesson in each of the phases to a criteria of 100% on the Treatment Fidelity Checklist (See Appendix D).

Pre Assessment: Phase 2

The pre assessment was administered on the first two days of the study. The Test of Mathematical Abilities-2 (TOMA-2) was group administered and the teachers adhered to the administration guidelines for the TOMA-2. The students also were given the teacher-made test. A teacher script was used to introduce the pre-assessment. Specifically, the students were told, "Over the next few weeks we will be learning how to solve one-variable algebra equations. Today we are going to find out how well you can perform this task. To do this, you will be taking a short test. The results of the test will tell us what you already know and what you need to learn. The results of this test will not affect your grade for this grading period." After this explanation, the teacher distributed the test and pointed, to the first problem, and said, "Begin with this problem and try to answer every problem on these pages. If you are not able to do a problem, skip it and move on to the next. Don't be upset if you have difficulty answering the problems. When

you are finished, turn your paper over and I will collect it. Are there any questions?"

The pre-assessments were scored that evening. On the following day, the students were given a piece of paper with their percentage correct on the teacher-made test. The teacher provided feedback and then discussed the rationale for learning how to solve one-variable algebra equations. A teacher script was used for the discussion. The students were told, "Knowing how to solve one-variable algebra equations can benefit you in several ways. First it will help you understand the relationship between basic mathematics (such as addition, subtraction, multiplication, and division) and algebra, and its use of letters (such as x and y), which we know represents variables. Second, it will help you in school to earn higher grades in math and will provide you a better opportunity to earn a diploma by passing the math proficiency test. That diploma will assist you in obtaining a better job or it will give you the opportunity to go to college." The students were then asked to make a commitment to participate in the lessons and learn how to solve one-variable algebra equations. To facilitate the commitment process, the students and the teacher signed a learning contract. The students then were given a progress chart. The teacher explained that this chart would be used

to monitor their progress within the program. The students were told they would plot their scores on the chart and turn the charts back in to the teacher.

Implementation of Instruction: Phase 3

Treatment Group Lesson Sequence

There were 11 thirty-minute lessons that addressed solving one-variable algebra equations and one-variable algebra word problems. The lessons were scripted to minimize the possibility of teacher effects. Each lesson follows a similar teaching sequence including advanced organizer, describe and model, guided practice, independent practice, and feedback. At the beginning of each lesson, the teacher provided the students with an advanced organizer. This organizer involved telling the students what they would be doing in the upcoming lesson and the rationale for doing it. During this organizer, the teacher also reminded the students what was covered in the preceding lesson. The next portion of the lesson was describe and model. The teacher demonstrated how to solve problems for the lesson being taught. The teacher then conducted guided practice of solving the problems. During this portion of the lesson, the teacher and the students solved a problem together. After the guided practice, the students practiced their problem solving skills independently. The teacher then provided

corrective feedback. The students continued to practice until they mastered the lesson (completed the problems with 90% accuracy).

Treatment Group Lesson Content

In lesson one, students were taught the concrete method of solving one-variable algebraic equations. In lesson two, the students learned to solve one-variable algebraic word problems and one-variable algebra equations using the concrete method. In lesson three, students were taught the "DRAW" strategy (i.e. Discover the variable, Read the equation and combine like terms on each side of the equation, Answer the equation or draw and check, and Write the answer for the variable and check the equation). The "DRAW" strategy is used for solving one-variable algebraic equations at the representational level. In lesson four, students were introduced to the concept of solving one-variable algebra equations that require the combining of like terms that included variables. The students used the DRAW strategy to answer one-variable equations when they did not know the answer from memory. Lesson five was used to promote the relevance of one-variable algebra equations by solving word problems through the use of one-variable algebra equations. In lesson six, students were taught the FAST DRAW strategy (i.e. Find what you are solving for, Ask

yourself what information is given, Set up the equation, and Take the equation and solve it). The students used the "FASTDRAW" strategy to solve word problems during lesson seven. During lesson eight, the students were taught how to use the "FASTDRAW" strategy to solve more complex algebra word problems. In lesson nine, students were taught the CAP strategy (i.e. Combine like terms, Ask yourself how can I isolate the variable, and Put the value of the variable in the initial equation and check to see if the equation is balanced). In lessons ten and eleven, the students practiced solving one-variable algebraic equations and word problems at the abstract level of understanding.

Control Group Instruction

The control group received the same amount of instructional time to address solving one-variable algebra equations and one-variable algebra word problems. The teachers used the same lesson problems as the treatment group, but did not use the concrete or representational illustrations. The teachers followed instructions as specified by the teacher's manual of the class textbook. Problems were demonstrated on the board. Once the instruction was completed, the students were given the lesson problem worksheet to complete.

Fidelity of Treatment

Each teacher was observed by two people three times during the study (one time for each phase of the CRA instruction). The observers used the treatment fidelity checklist (See Appendix D) to ensure the sequence of instruction and instruction components were used consistently throughout the study. Inter-observer reliability was computed for the fidelity of treatment observations using the formula agreements divided by agreements and disagreements times 100.

Post Assessment: Phase 4

The post assessments were administered on the final two days of the study. The TOMA-2, the teacher-made test, and the math proficiency test were group administered. The teacher adhered to the administration guidelines set forth by the TOMA-2 manual. When administering the teacher-made test, the students were told what they would be doing and why. The teacher said, "Today we are going to find out what kind of progress you have made in learning to solve one-variable algebra equations. To do this, you'll be taking a short test. If you score 90% or better on this test, you will have reached mastery." Then the teacher passed out the tests and said, "Begin with problem one and try to answer each problem on the page. Take your time and do your best

work. If you need help solving a problem, think about the DRAW, FAST DRAW or CAP strategy and the rules that you've learned. However, don't look at your DRAW strategy sheet, the DRAW strategy rule sheet, the FAST DRAW strategy sheet, or the CAP strategy sheet. When you are finished turn your test over and I will pick it up. Any questions?" The posttests were scored that night and the students were provided with feedback the next day. Two weeks after the final lesson was taught, the teacher-made test was re-administered to measure student retention. Twenty percent of the tests were scored by two individuals to ensure inter-scorer reliability using the formula agreements divided by agreements and disagreements times 100.

Treatment of the Data

Data from the teacher-made test were analyzed to answer Research Question 1. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra equations? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 2. Is the Concrete-Representational-

Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra word problems? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 3. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra equations? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 4. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra word problems? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 5. Is the Concrete-Representational-

Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra equations? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 6. Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra word problems? An analysis of covariance (ANCOVA) with the pretest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 7. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students with mild disabilities? An analysis of covariance (ANCOVA) with the posttest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 8. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problems-solving skills among students with mild disabilities? An analysis of covariance (ANCOVA) with the posttest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 9. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students without disabilities? An analysis of covariance (ANCOVA) with the posttest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the teacher-made test were analyzed to answer Research Question 10. Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem-solving skills among students without disabilities? An analysis of covariance (ANCOVA) with the

posttest score as the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Data from the TOMA-2 were analyzed to answer Research Question 11. Is there a change in student attitudes toward mathematics after receiving algebra instruction using the Concrete-Representational-Abstract teaching sequence? An ANCOVA with pretest score being the covariate was used to analyze the data. A .05 confidence level was used to determine statistical significance.

Chapter 4

DATA ANALYSIS AND RESULTS

The purpose of this study was to investigate the effects of the concrete-representational-abstract teaching sequence on students' algebraic equation-solving and problem-solving skills. Data were collected to answer 11 research questions comparing students' ability to solve one-variable algebra equations and one-variable algebra word problems instructed in one of two conditions. The treatment condition involved the use of a concrete-representational-abstract teaching sequence and the control condition used the traditional (abstract only) instructional method. Following the results related to each research question, interscorer reliability for the various measures in this study is reported. The content in this chapter is organized according to the eleven research questions. Each question is restated. Then the results of the statistical analyses of data obtained in the study are provided.

Research Questions

Equation Solving with Students with Mild Disabilities

Question 1: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra equations?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra equations. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the pretest and posttest, which contained 14 similar one-variable algebra equations. All subjects were given the pre- and posttest by their special education teacher within the resource classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,61) = .003$, $p = .957$, indicating that there was no significant difference in ability to solve

algebra equations between the traditional group and the CRA group (see Table 4.1 for mean and standard deviation).

Table 4.1

ANCOVA for traditional v. CRA teaching sequence for students with disabilities on ability to solve algebra equations

(N = 61)		
Method	Pretest *M* (SD)	Posttest *M*(SD)
Traditional (n = 24)	2.88(4.11)	10.75(4.10)
CRA (n = 37)	3.14(3.03)	10.86(4.10)

*Significant at the p<0.05 level.

Word Problem Solving with Students with Mild Disabilities

Question 2: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra word problems?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra word problems. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the pretest

and posttest, which contained 6 similar one-variable algebra word problems. All subjects were given the pre- and posttest by their special education teacher within the resource classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,61) = .575$, $p = .451$, indicating that there was no significant difference in ability to solve algebra word problems between the traditional group and the CRA group (see Table 4.2 for mean and standard deviation).

Table 4.2

ANCOVA for traditional v. CRA teaching sequence for students with disabilities on ability to solve word problems

(N = 61)		
Method	Pretest *M* (SD)	Posttest *M* (SD)
Traditional (N = 24)	1.25(1.19)	2.83(2.10)
CRA (N = 37)	1.43(1.17)	3.32(2.01)

*Significant at the $p<0.05$ level.

Equation Solving with Students without Disabilities

Question 3: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra equations?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra equations. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the pretest and posttest, which contained 14 similar one-variable algebra equations. All subjects were given the pre- and posttest by their teacher within the classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,108) = .453$, $p = .502$, indicating that there was no significant difference in ability to solve algebra equations between the traditional group and the CRA group (see Table 4.3 for mean and standard deviation).

Table 4.3

ANCOVA for traditional v. CRA teaching sequence for students without disabilities on ability to solve algebra equations

	(N = 108)	
Method	Pretest *M* (SD)	Posttest *M* (SD)
Traditional (N = 62)	11.52(3.07)	12.71(2.03)
CRA (N = 46)	10.85(2.90)	12.46(2.04)

*Significant at the p<0.05 level.

Word Problem Solving with Students without Disabilities

Question 4: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra word problems?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra word problems. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the pretest and posttest, which contained 6 similar one-variable algebra word problems. All subjects were given the pre- and posttest by their teacher within the classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,108) = .168$, $p = .683$, indicating that there was no significant difference in ability to solve algebra word problems between the traditional group and the CRA group (see Table 4.4 for mean and standard deviation).

Table 4.4

ANCOVA for traditional v. CRA teaching sequence for students without disabilities on ability to solve word problems

(N = 108)		
Method	Pretest *M* (SD)	Posttest *M* (SD)
Traditional (N = 62)	2.84(1.55)	3.53(1.17)
CRA (N = 46)	2.91(1.70)	3.65(1.66)

*Significant at the $p<0.05$ level.

Comparison of Equation Solving Skills for Students With and Without Disabilities

Question 5: Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra equations?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra equations. All treatment group subjects (i.e., recipients of CRA teaching sequence) participated in the pretest and posttest, which contained 14 similar one-variable algebra equations. All

subjects were given the pre- and posttest by their teacher within the classroom.

To determine if there was a significant difference between the performance of students with disabilities and students without disabilities, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was disability or no disability; the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,83) = 1.226$, $p = .271$, indicating that there was no significant difference in ability to solve algebra equations between students with disabilities and students without disabilities (see Table 4.5 for mean and standard deviation).

Table 4.5

ANCOVA for students with disabilities v. students without disabilities on ability to solve algebra equations after being taught using the CRA teaching sequence

(N = 83)		
Group	Pretest *M* (SD)	Posttest *M* (SD)
Disability (N = 37)	3.14(3.03)	10.86(4.10)
No Disability (N = 46)	10.85(2.90)	12.46(2.04)

*Significant at the $p<0.05$ level.

Comparison of Word Problem Solving Skills for Students With and Without Disabilities

Question 6: Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra word problems?

The Teacher-Made Pretest and Posttest were used to assess the students' ability to solve algebra word problems. All treatment group subjects (i.e., recipients of CRA teaching sequence) participated in the pretest and posttest, which

contained 6 similar one-variable algebra equations. All subjects were given the pre- and posttest by their teacher within the classroom.

To determine if there was a significant difference between the performance of students with disabilities and students without disabilities, a univariate analysis of covariance (ANCOVA) was used with the pretest scores as the covariate. The independent variable used was disability or no disability; the dependent variable was the scores on the posttest. The covariate was the scores on the pretest. No significance was found $F(1,83) = .3.862$, $p = .053$, indicating that there was no significant difference in ability to solve algebra word problems between students with disabilities and students without disabilities (see Table 4.6 for mean and standard deviation).

Table 4.6

ANCOVA for students with disabilities v. students without disabilities on ability to solve algebra word problems after being taught using the CRA teaching sequence

(N = 83)		
Group	Pretest *M* (SD)	Posttest *M* (SD)
Disability (N = 37)	1.43(1.17)	3.32(2.01)
No Disability (N = 46)	2.91(1.70)	3.65(1.66)

*Significant at the $p<0.05$ level.

Retention of Skills to Solve Algebra Equations by Students With Mild Disabilities

Question 7: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students with mild disabilities?

The Teacher-Made Posttest and Maintenance Test (See Appendix C) were used to assess the students' retention

related to solving algebra equations. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the posttest and maintenance test, which contained 14 one-variable algebra equations. All subjects were given the posttest and maintenance test by their special education teacher within the resource classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the maintenance test. The covariate was the scores on the posttest. No significance was found $F(1,61) = .562$, $p = .347$, indicating that there was no significant difference between the traditional and CRA groups' retention related to solving algebra equations(see Table 4.7 for mean and standard deviation).

Table 4.7

ANCOVA for traditional v. CRA teaching sequence for students with disabilities on retention to solve algebra equations

(N = 61)		
Method	Posttest *M* (SD)	Maintenance *M*(SD)
Traditional (n = 24)	10.75(4.10)	8.79(3.24)
CRA (n = 37)	10.86(4.10)	9.21(2.71)

*Significant at the $p<0.05$ level.

Retention of Skills to Solve Algebra Word Problems by Students With Mild Disabilities

Question 8: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem-solving skills among students with mild disabilities?

The Teacher-Made Posttest and Maintenance Test (See Appendix C) were used to assess the students' retention

related to solving algebra word problems. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the posttest and maintenance test, which contained 6 one-variable algebra word problems. All subjects were given the posttest and maintenance test by their special education teacher within the resource classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the maintenance test. The covariate was the scores on the posttest. No significance was found $F(1,61) = .783$, $p = .623$, indicating that there was no significant difference between the traditional and CRA groups' retention related to solving algebra word problems(see Table 4.8 for mean and standard deviation).

Table 4.8

ANCOVA for traditional v. CRA teaching sequence for students with disabilities on retention to solve algebra word problems

(N = 61)		
Method	Posttest *M* (SD)	Maintenance *M*(SD)
Traditional (n = 24)	2.83(2.10)	1.76(1.63)
CRA (n = 37)	3.32(2.01)	2.05(1.84)

*Significant at the $p<0.05$ level.

Retention of Skills to Solve Algebra Equations by Students Without Disabilities

Question 9: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students without disabilities?

The Teacher-Made Posttest and Maintenance Test were used to assess the students' retention related to solving algebra equations. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the posttest and maintenance test, which contained 14 one-variable algebra equations. All subjects were given the posttest and maintenance test by their general education teacher within the classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the maintenance test. The covariate was the scores on the posttest. No significance was found $F(1,108) = 1.397$, $p = .171$, indicating that there was no significant difference between the traditional and CRA groups' retention related to solving algebra equations (see Table 4.9 for mean and standard deviation).

Table 4.9

ANCOVA for traditional v. CRA teaching sequence for students without disabilities on retention to solve algebra equations

(N = 108)		
Method	Posttest *M* (SD)	Maintenance *M* (SD)
Traditional (n = 62)	12.71(2.03)	12.16(2.14)
CRA (n = 46)	12.46(2.04)	11.67(2.36)

*Significant at the p<0.05 level.

Retention of Skills to Solve Algebra Word Problems by Students Without Disabilities

Question 10: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problems-solving skills among students without disabilities?

The Teacher-Made Posttest and Maintenance Test were used to assess the students' retention related to solving algebra word problems. All subjects for the CRA teaching sequence and traditional (abstract only) teaching method participated in the posttest and maintenance test, which contained 6 one-

variable algebra word problems. All subjects were given the posttest and maintenance test by their general education teacher within the classroom.

To determine if there was a significant difference between the performance of the treatment group and control group, a univariate analysis of covariance (ANCOVA) was used. The independent variable used was method of instruction (traditional v. CRA instruction); the dependent variable was the scores on the maintenance test. The covariate was the scores on the posttest. No significance was found $F(1,108) = .088$, $p = .767$, indicating that there was no significant difference between the traditional and CRA groups' retention related to solving algebra word problems (see Table 4.10 for mean and standard deviation).

Table 4.10

ANCOVA for traditional v. CRA teaching sequence for students without disabilities on retention to solve algebra word problems

(N = 108)		
Method	Posttest *M* (SD)	Maintenance *M*(SD)
Traditional (n = 62)	3.53(1.17)	3.06(1.10)
CRA (n = 46)	3.65(1.66)	3.20(1.47)

*Significant at the $p<0.05$ level.

Attitude Toward Mathematics

Question 11: Is there a change in student attitudes toward mathematics after receiving algebra instruction using the Concrete-Representational-Abstract teaching sequence?

Results from the TOMA-2 pretest and posttest were used to assess the students' attitude toward mathematics. All subjects instructed using the CRA teaching sequence participated in the pretest and posttest, which contained 15 questions regarding how the student felt about completing math problems. All of these subjects were given the pre- and posttest by their teacher within the classroom.

To determine if there was a significant difference in the attitudes of the students before being instructed using the CRA teaching sequence and after the instruction, a univariate analysis of variance (ANOVA) was used. No significance was found $F(1,58) = .153$, $p = .697$, indicating that there was no significant difference in attitude toward mathematics after being instructed using the CRA teaching sequence (see Table 4.11 for mean and standard deviation).

Table 4.11

ANOVA for change in attitude toward mathematics after CRA instruction

	(N = 58)	
Method	Pretest *M* (SD)	Posttest *M* (SD)
CRA (N = 58)	37.24(6.01)	38.17(5.97)

*Significant at the p<0.05 level.

Interscorer Reliability

The researcher and a research assistant independently scored 20% of the pre- and posttests to assess reliability of the scoring system. An agreement was obtained when both

scorers recorded the same score for items on each test. The percentage of agreement was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying 100. There were 500 agreements out of 500 opportunities. Interscorer reliability was 100% (see Table 4.12 for a summary of reliability measures).

Table 4.12

Interscorer Reliability

Measure	Interscorer Reliability
Pre/Postests	100%

Interobserver Reliability

Each teacher was observed by two people three times during the study (one time for each phase of the CRA instruction). The observers used the treatment fidelity checklist (See Appendix D) to ensure the sequence of instruction and instruction components were used consistently throughout the study. Interobserver reliability was computed for the fidelity of treatment observations using the formula agreements divided by agreements and disagreements times 100. Each of the five

teachers were observed three times. There were 73 agreements out of 75 opportunities. Interobserver reliability was 97% (See Table 4.13 for a summary of reliability measures).

Table 4.13

Interobserver Reliability

Observations	Interscorer Reliability
Treatment Sessions	97%

Chapter 5

DISCUSSION

Introduction

Researchers have demonstrated the effectiveness of using direct instruction, learning strategy instruction and the CRA sequence for teaching a variety of basic math skills, but little research has been conducted related to the effectiveness of the CRA sequence for teaching more complex skills such as algebra. This study compared the concrete-representational-abstract sequence to the traditional abstract method of teaching algebra equation solving and algebra word problem solving skills. Also, students with mild disabilities and without disabilities were compared to determine if any differences exist in their ability to progress through the CRA teaching sequence. Findings related to each research question in this study are discussed in the subsequent section of this chapter. Next, conclusions drawn from these findings are shared. Finally, practical implications of the study are described and recommendations for future research are provided.

Discussion of Findings

The first question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra equations?

The analysis of the data indicates that there was no significant difference between the CRA group and the control group in their ability to solve algebra equations. It is important to note that the students with disabilities increased the number of problems they were able to solve by 345 - 373%. The re-authorization of the Individuals with Disabilities Education Act requires that students with disabilities have access to the general education curricula. Many of the students and parents of the students reported, before the study began, that there was no way the students would be able to complete any algebra problems. By the conclusion of the study, they doubled and tripled their ability to perform algebra equations. The students were taught grade level and above grade level (for the sixth grade students) material.

A challenge related to the implementation of this study was that the intervention extended over winter break. The students were provided instruction beginning in November and ending in January. After the two weeks of winter break, the

teacher had to provide extensive review before proceeding onto the next lesson in the series. This represents one of the many typical challenges involved in conducting educational research in natural settings.

The second question discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students with mild disabilities to solve algebra word problems?

The results of the analysis indicate that there was no significant difference between the CRA and traditional teaching methods in instructing students with disabilities on algebra word problem-solving skills. There was, however, an increase in the number of word problems the students with disabilities were able to solve. The accuracy rate increased by approximately 230%. Previous research indicated that students with disabilities did increase their ability to solve algebra word problems with the use of the CRA teaching sequence (Witzel, Mercer, & Miller, 2003; Maccini & Hughes, 2000). It is interesting to compare the results of this study to those of Witzel et al and Maccini and Hughes. Witzel, Mercer, & Miller (2003) found a significant difference between the treatment group (CRA) and the control group (abstract only). The Witzel, Mercer, and Miller study

had 19 instructional sessions using the CRA method while the present study had 11 sessions. Additional sessions may be needed to solidify the math reasoning skills necessary to problem solve. Maccini and Hughes used a single-subject design with students with learning disabilities. The strategy was found to be effective, but there were no comparison groups and no students without disabilities participating in the study.

As mentioned in the discussion of question one, the strategy sessions extended over winter break. This may have hindered the learning process.

The third question to discuss is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra equations?

The data showed no significant difference in the ability to solve algebra equations with students without disabilities after being instructed in either the CRA or Traditional teaching method. It should be noted that in both conditions, accuracy on the pretest was around 80% (Traditional - 82%; CRA - 77%). Therefore, the students without disabilities were near mastery level before instruction began. There was not much room for improvement.

Both groups showed improvement (up to an approximate 93% accuracy rate).

The fourth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for teaching students without disabilities to solve algebra word problems?

Results indicate that there was no significant difference in ability to solve algebra word problems among students without disabilities after being instructed in either the CRA or Traditional teaching method. As observed with the students with disabilities, the students without disabilities did increase their ability to solve algebra word problems (Traditional - increase of 17 percentage points; CRA - increase of 19 percentage points). The increase for the students without disabilities was not as significant as the increase for the students with disabilities, but the students without disabilities started at a higher level of accuracy.

The fifth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra equations?

Analysis of the data indicated that the treatment was equally effective for both students with disabilities and students without disabilities. The students with disabilities had more room for improvement because their pretest percentage correct (i.e., ≈ 22%) was lower than the pretest percentage correct (i.e., ≈ 80%) for students without disabilities. The students with disabilities improved their percentage correct to a mean of 77% by the end of the study representing a 55 percentage point increase. This is still not quite as good as the posttest performance of students without disabilities, but is much closer.

The sixth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective for students with disabilities than for students without disabilities for teaching algebra word problems?

The CRA teaching sequence appears to be equally effective for teaching students with disabilities and for teaching students without disabilities. The percentage correct for the students with disabilities increased 22 percentage points (23% at pretest to 55% at posttest) over the course of the instructional lessons. The percentage correct for the students without disabilities increased 12 percentage points

(48% at pretest to 60% at posttest) over the course of the strategy.

The seventh question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students with mild disabilities?

The data indicate that the students' retention levels were the same regardless of teaching style. The mean percentage correct of algebra equations for students with disabilities decreased by 12 percentage points for the CRA group and 14 percentage points for the traditional group over a two-week period. This decrease in ability to solve the algebra equations suggests that students with disabilities require continued review to maintain previously learned skills.

The eighth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem-solving skills among students with mild disabilities?

An analysis of the data indicates that students with disabilities had approximately the same retention level for solving algebra word problems regardless of teaching method.

The percentage correct for the group who were taught using the CRA teaching method decreased 21 percentage points over the two-week time period. The percentage correct for the students in the traditional group decreased 18 percentage points. Konold (2000) noted high school students had difficulty finding the appropriate information, determining the needed operation, and setting up the equation. This difficulty was exhibited one month after instruction in the skills assessed. Students may need more instructional time in problem solving and may need periodic review of skills previously mastered.

The ninth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra equation-solving skills among students without disabilities?

The data indicate that the students' retention levels were the same regardless of teaching style. The mean percentage correct of algebra equations for students without disabilities decreased by 6 percentage points for the CRA group and 4 percentage points for the traditional group over a two-week period. The decrease in ability to solve algebra equations for students without disabilities was much lower than the decrease for students with disabilities.

The tenth question to be discussed is: Is the Concrete-Representational-Abstract teaching sequence more effective than traditional abstract level instruction for promoting retention of algebra word problem-solving skills among students without disabilities?

An analysis of the data indicates that students with disabilities had approximately the same retention level for solving algebra word problems regardless of teaching method. The percentage correct for both groups (CRA and traditional instruction) decreased 8 percentage points over the two-week time period. Again, it is noteworthy to point out that the decrease for students without disabilities was less than the decrease for students with disabilities.

The final question to be discussed is: Is there a change in student attitudes toward mathematics after receiving algebra instruction using the Concrete-Representational-Abstract teaching sequence?

No change in student attitude toward mathematics was noted among students who received CRA instruction. The attitudes before treatment and after treatment remained the same from the pretest to the posttest. It should be noted that the teachers reported some changes in classroom behaviors and attitude toward math. One teacher stated the students appeared to be more motivated to learn this

material than previously taught material. The TOMA-2 may not have captured those changes in attitude.

Conclusions

The following conclusions are based on quantitative data collected in this study.

1) Students with disabilities perform equally well on solving one-variable algebra equations regardless of whether they received instruction using the CRA teaching sequence or the traditional method.
2) Students with disabilities perform equally well on solving one-variable algebra word problems regardless of whether they received instruction using the CRA teaching sequence or the traditional method.
3) Students without disabilities perform equally well on solving one-variable algebra equations regardless of whether they received instruction using the CRA teaching sequence or the traditional method.
4) Students without disabilities perform equally well on solving one-variable algebra word problems regardless of whether they received instruction using the CRA teaching sequence or the traditional method.
5) The CRA teaching sequence has similar effects on students with disabilities and students without

disabilities with regard to algebra equation solving skills.

6) The CRA teaching sequence had similar effects on students with disabilities and students without disabilities with regard to algebra word problem solving skills.

7) The traditional teaching method has similar effects on students with disabilities and students without disabilities with regard to algebra equation solving skills.

8) The traditional teaching method has similar effects on students with disabilities and students without disabilities with regard to algebra word problem solving skills.

9) Students with and without disabilities have similar attitudes toward mathematics (generally positive) and these attitudes remained constant over the course of the study.

10) Students with disabilities can be successful in learning algebra skills when taught using the CRA teaching sequence or when taught using with traditional text-book based instruction.

11) The retention rate for algebra equation solving and algebra word problem solving can be expected to decrease without continuous review.

Practical Implications

There has been a great push to provide students with disabilities access to the general education curricula. Some educators, parents, and students believe that algebraic concepts are beyond the ability levels of the students with mild disabilities. Consequently, they are tracked into non-college bound math courses (e.g. consumer mathematics) with subsequent lowered expectations. This research indicates that students with disabilities can learn how to solve algebraic equations. According to anecdotal comments from their teachers, the students with disabilities who participated in this study were motivated to learn these concepts. They volunteered to work problems on the board more frequently than previously seen in class and made comments about how their older brother or sister (who did not have a disability) was working on the same type of problems at home. This gave the students a sense of accomplishment and pride. Too often, these feelings are not experienced within the classroom. Many secondary students with disabilities have extensive histories of academic

failure and know that the material they are working on is not the same as the material their peers without disabilities are completing. The effects of low expectations for students in general and students with disabilities in particular can be quite harmful.

Several important implications emerged from this study. First, teachers and parents should resist the temptation to assume that students with disabilities will be unsuccessful in higher level math skills (e.g. algebra). Second, students with disabilities should be given access to the general education curricula and attempts should be made to help these students recognize that, with appropriate supports, content with the general education curriculum is within their grasp. Third, in order for students with disabilities to maintain the skills previously learned, continued review and support needs to occur.

Suggestions for Further Research

The results of the study showed no significant difference between the CRA and traditional method of teaching, but students with and without disabilities increased their ability to solve the algebra problems.

Future research should be conducted to investigate the number of lessons required to acquire and retain the skill

of solving one-variable algebra equations and word problems. This information is needed to ensure skill mastery and to ensure instructional efficiency.

Future research should be conducted to investigate the effectiveness of CRA for teaching algebra equation and word problem solving skills to students within a smaller grade level range. This study was conducted with sixth through twelve graders. The strategy may be more effective with one age group (e.g., 6-8 graders) than another (e.g., 9-12 graders). Further research is needed to determine whether the use of CRA with middle school students differs from the use of CRA with high school students.

A longitudinal study should be conducted to determine if there is a relationship between the CRA instructional model and the learning of subsequent math skills. Mathematics is hierarchical in nature. Simpler skills are prerequisites for more complex math problems. Future research is needed to determine if instructing students using the CRA teaching sequence leads to a quicker and more comprehensive understanding of subsequent complex tasks.

APPENDIX A

TEACHER MADE PRE-TEST

1) $7a = 28$

2) $4C = 32$

3) $3y + 6y = 54$

4) $2r + 9r = 77$

5) $3d + 2 = 20$

6) $9x + 8 = 80$

7) $8s - 7 = 33$

8) $3t - 9 = 21$

9) $5m + 3m + 3 = 67$

10) $2p + 4p + 6 = 36$

11) $6g + 6g - 6 = 42$

12) $3i + 2i - 1 = 44$

13) $4b + 3b + 18 - 9 = 37$

14) $7f + 2f + 12 - 5 = 34$

15) Eric spent $6.00 playing 3 video games at the arcade. If each game cost the same amount, how much did Eric spend on each game?

16) On their camping trip, Mark, Andy, and Ross gathered firewood in the morning. In the afternoon, Paul and Bob gathered more firewood. In the evening, they counted the pieces of firewood and discovered that each boy had found the same amount of firewood. If they had 30 pieces of firewood altogether, how many pieces did each boy find?

17) Lori is 20 years old. She is 3 times plus two years older than her younger sister, Ellen. How old is Ellen?

18) In ceramics class, Angela and Denise each made the same number of animal figures for the science exhibit. One figure was dropped and broken on the way to setting up the exhibit. If 19 figures were in the display, how many animal figures did each girl make in ceramics class?

19) The 4 students in Mr. Gomez's first period math class each completed the assigned page of algebra problems. During second period, 6 completed the same page in their books. In addition, 1 student completed 4 geometry problems. If Mr. Gomez had a total of 74 problems to grade, how many algebra problems were completed by each student?

20) Penny likes to do word processing to earn extra money. She has a standard charge for business letters. She did 7 letters on Thursday and 4 letters on Friday. On Saturday, she had to spend $4.00 on paper. If she still had $106.00 after she bought the paper, what did she charge for each letter?

APPENDIX B

TEACHER-MADE TEST

POSTTEST

1) $6a = 24$

2) $5c = 30$

3) $2y + 5y = 49$

4) $3r + 6r = 72$

5) $3d + 8 = 38$

6) $7x + 8 = 43$

7) $9s - 6 = 48$

8) $4t - 7 = 21$

9) $6m + 2m + 9 = 57$

10) $2p + 3p + 5 = 40$

11) $5g + 5g - 8 = 52$

12) $2i + 4i - 2 = 16$

13) $3b + 5b + 14 - 7 = 63$

14) $6f + f + 11 - 7 = 67$

15) During their summer vacation, Sam's family bicycled 48 miles around Washington DC on a sightseeing tour. They rode 4 days and covered the same distance each day. How many miles did they ride each day?

16) At the school store, you can buy pencils with your name printed next to the name of the football team. Since Jim is always losing his pencils, he decided to buy 6 pencils. His best friend, Bobby, bought 3 pencils with his name on them Together they spent $0.81. What was the cost of each pencil?

17) Ms. Garcia, the biology teacher, had 9 notebooks full of science experiments to grade. There were 73 experiments including 10 that should have been turned in to the chemistry teacher. If each notebook contained the same number of biology experiments, how many biology experiments were in each notebook?

18) Nine students from Ms. Anderson's room each earned the same number of points in the school homework contest. Unfortunately, 5 points were lost by Ms. Anderson's room for a late paper. When the points were totaled, Ms. Anderson's room had 85 points. How many points did each of the 9 students earn before the penalty?

19) During the morning race, Matt and Phil ran the full length of the course. In the afternoon, Jerry, Juan, and Dwayne also ran the full length of the course. Tim hurt his ankle and only ran 5 miles. The combined number of miles for all runners was 40 miles. How long was the course?

20) Jerry has a paper route. Last week he collected payments on 3 afternoons. This week he collected payments on 4 afternoons. He collected the same amount each day. After he finished collecting, he had to send $30 to the newspaper company. He had $26 left to spend. How much did he collect each day?

APPENDIX C

TEACHER-MADE TEST

MAINTENANCE TEST

1) $6a = 24$

2) $5c = 30$

3) $2y + 5y = 49$

4) $3r + 6r = 72$

5) $3d + 8 = 38$

6) $7x + 8 = 43$

7) $9s - 6 = 48$

8) $4t - 7 = 21$

9) $6m + 2m + 9 = 57$

10) $2p + 3p + 5 = 40$

11) $5g + 5g - 8 = 52$

12) $2i + 4i - 2 = 16$

13) $3b + 5b + 14 - 7 = 63$

14) $6f + f + 11 - 7 = 67$

15) During their summer vacation, Sam's family bicycled 48 miles around Washington DC on a sightseeing tour. They rode 4 days and covered the same distance each day. How many miles did they ride each day?

16) At the school store, you can buy pencils with your name printed next to the name of the football team. Since Jim is always losing his pencils, he decided to buy 6 pencils. His best friend, Bobby, bought 3 pencils with his name on them Together they spent $0.81. What was the cost of each pencil?

17) Ms. Garcia, the biology teacher, had 9 notebooks full of science experiments to grade. There were 73 experiments including 10 that should have been turned in to the chemistry teacher. If each notebook contained the same number of biology experiments, how many biology experiments were in each notebook?

18) Nine students from Ms. Anderson's room each earned the same number of points in the school homework contest. Unfortuantely, 5 points were lost by Ms. Anderson's room for a late paper. When the points were totaled, Ms. Anderson's room had 85 points. How many points did each of the 9 students earn before the penalty?

19) During the morning race, Matt and Phil ran the full length of the course. In the afternoon, Jerry, Juan, and Dwayne also ran the full length of the course. Tim hurt his ankle and only ran 5 miles. The combined number of miles for all runners was 40 miles. How long was the course?

20) Jerry has a paper route. Last week he collected payments on 3 afternoons. This week he collected payments on 4 afternoons. He collected the same amount each day. After he finished collecting, he had to send $30 to the newspaper company. He had $26 left to spend. How much did he collect each day?

APPENDIX D

TREATMENT FIDELITY CHECKLIST

Teacher Checklist

Teacher Name: ______________________________

Date: ______________________________

Components of Instruction	**Concrete (0/1)**	**Representational (0/1)**	**Abstract (0/1)**	**Total (0/3)**
Advanced Organizer				
Describe and Model				
Guided Practice				
Independent Practice				
Feedback				

Percent of components completed correctly:

____ / 15 * 100 = ____%

REFERENCES

A nation at risk. (1983). Retrieved May 12, 2001, from http://www.ed-gov/pub/NatAtRisk/risk.html

Ackerman, P. T., Anhalt, J. M., & Dykman, R. A. (1986). Arithmetic automatization failure in children with attention and reading disorders: Associations and sequilae. *Journal of Learning Disabilities, 19*, 222-232.

Allsopp, D. H. (1997). Using classwide peer tutoring to teach beginning algebra problem-solving skills in heterogeneous classrooms. *Remedial and Special Education, 18*, 367-380.

Allsopp, D. H. (2001). The building algebra skills series: Unit 4 - solving one-variable algebra equations and one-variable algebra word problems. Unpublished strategy guide.

Bley, N. S., & Thornton, C. A. (2001). *Teaching mathematics to students with learning disabilities* (4th ed.) Austin, TX: Pro-ed.

Bottge, B. A., & Hasselbring, T. S. (1999). Teaching mathematics to adolescents with disabilities in a

multimedia environment. *Intervention in School and Clinic, 35,* 113-117.

Butler, F. (1999). Fraction instruction for students with disabilities: Comparing two teaching sequences. Unpublished doctoral dissertation, University of Nevada, Las Vegas.

Case, L. P., Harris, K. R., & Graham, S. (1992). Improving the mathematical problem-solving skills of students with learning disabilities: Self-regulated strategy development. *The Journal of Special Education, 26,* 1-19.

Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice, 18*(2), 112-120.

Cawley, J. F., & Miller, J. H. (1989). Cross-sectional comparisons of the mathematical performance of children with learning disabilities: Are we on the right track toward comprehensive programming? *Journal of Learning Disabilities, 23,* 250-254.

Cawley, J. F., Parmar, R. S., Yan, W. F., & Miller, J. H. (1996). Arithmetic computation abilities of students with learning disabilities: Implications for instruction. *Learning Disabilities Research and Practice, 11,* 230-237.

Deshler, D. D., Ellis, E. S., & Lenz, B. K. (1996). *Teaching adolescents with learning disabilities: Strategies and methods* (2nd ed.). Denver: Love.

Friend, M., & Bursuck, W. D. (2002). *Including students with special needs: A practical guide for classroom teachers* (3rd ed.). Boston: Allyn and Bacon.

Gagnon, J. C., & Maccini, P. (2001, September/October). Preparing students with disabilities for algebra. *Teaching Exceptional Children,* 8-15.

Harris, C. A., Miller, S. P., & Mercer, C. D. (1995). Teaching initial multiplication skills to students with learning disabilities in general education classrooms. *Learning Disabilities Research and Practice, 10,* 180-195.

Hollingsworth, J. & Ybarra, S. (2000). Curriculum calibration. *Thrust for Educational Leadership, 29*(5), 22-23.

Hong, E. (1995). Mental models in word problem-solving: A comparison between American and Korean sixth-grade students. *Applied Cognitive Psychology, 9,* 123-142.

Hutchinson, N. L. (1993). Effects of cognitive strategy instruction on algebra problem solving of adolescents with learning disabilities. *Learning Disability Quarterly, 16,* 34-63.

Jitendra, A. K., Griffin, C. C., McGoey, K., Gardill, C.,

Bhat, P., & Riley, T. (1998). Effects of mathematical word problem solving by students at-risk or with mild disabilities. *Journal of Educational Research, 91,* 345-356.

Jitendra, A. K., Hoff, K., & Beck, M. M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial and Special Education, 20,* 50-65.

Konold, K. B. (2000). [Speak aloud protocol review for algebra problem solving]. Unpublished raw data.

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children, 32*(5), 1-22.

Maccini, P., & Hughes, C. A. (2000). Effects of problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research and Practice, 15,* 10-21.

Maccini, P., & Ruhl, K. L. (2000). Effects of a graduated instructional sequence on the algebraic subtraction of integers by secondary students with learning disabilities. *Education and Treatment of Children, 23*(4), 465-489.

Making the grade. (1989). Retrieved May 12, 2001, from http://nces.ed.gov/pubs89/teaching9394/references.html

Maqsud, M. (1998). Effects of metacognitive instruction on mathematics achievement and attitudes towards mathematics of low mathematics achievers. *Educational Research, 40,* 237-243.

Marsh, L. G., & Cooke, N. L. (1996). The effects of using manipulatives in teaching math problem solving to students with learning disabilities. *Learning Disabilities Research & Practice, 1,* 58-65.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13*(3), 19-35.

Mercer, C. D., & Miller, S. P. (1991-1994). Strategic Math Series. Lawrence, KS: Edge.

Mercer, C. D., & Miller, S. P. (1992). Teaching students with learning problems in math to acquire, understand, and apply basic math facts. *Remedial and Special Education, 13,* 19-35.

Miller, S. P., (2002). *Validated practices for teaching students with diverse needs and abilities.* Boston: Allyn and Bacon.

Miller, S. P., Harris, C. A., Strawser, S., Jones, W. P.,

& Mercer, C. D. (1998). Teaching multiplication to second graders in inclusive settings. *Focus on Learning Problems in Mathematics, 21*(4), 49-69.

Miller, S. P., & Mercer, C. D. (1993). Mnemonics: Enhancing the math performance of student with learning difficulties. *Intervention in School and Clinic, 29,* 78-82.

Miller, S. P., Mercer, C. D., & Dillon, A. S. (1992). CSA: Acquiring and retaining math skills. *Intervention in School and Clinic, 28,* 105-110.

Miller, S. P., Strawser, S., & Mercer, C. D. (1996). Promoting strategic math performance among students with learning disabilities. *Learning Disabilities Forum, 21*(2), 34-40.

Montague, M. (1992). The effects of cognitive and metacognitive strategy instruction on the mathematical problem solving of middle school students with learning disabilities. *Journal of Learning Disabilities, 25,* 230-248.

Montague, M., & Bos, C. S. (1986). The effect of cognitive strategy training on verbal math problem solving performance of learning disabled adolescents. *Journal of Learning Disabilities, 19,* 26-33.

Montague, M., Applegate, B. & Marquard, K. (1993).

Cognitive strategy instruction and mathematical problem-solving performance of students with learning disabilities. *Learning Disabilities Research & Practice, 8,* 223-232.

Morin, V. A., & Miller, S. P. (1998). Teaching multiplication to middle school students with mental retardation. *Education and Treatment of Children, 21,* 22-37.

Mullis, I. V. S., Owen, E. H., & Phillips, G. W. (1991). Accelerating academic learning: A summary of findings from 20 years of NAEP. *Curriculum Review, 30*(7), 24-26.

National assessment of educational progress. (1990). Retrieved May 12, 2001, from http://nces.ed.gov/nationsreportcard

National Council of Teachers of Mathematics (2000). *Math standards and expectations*. Retrieved May 12, 2001, from http://standards.nctm.org

Peck, D. M. & Jencks, S. M. (1981). Conceptual issues in the teaching and learning of fractions. *Journal for Research in Mathematics Education, 12*(5), 339-348.

Peterson, S. K., Mercer, C. D., & O'Shea, L. (1988). Teaching learning disabled students place value using the concrete to abstract sequence. *Learning Disabilities Research, 4,* 52-56.

Pickreign, J. (2000). Alignment of elementary geometry curriculum with current standards. *School Science & Mathematics, 100,* 243-252.

Porter, A. (1989). A curriculum out of balance: The case of elementary school mathematics. *Educational Researcher, 18*(5), 9-15.

Stevenson, H. W., Lee, S., Chen, C., Stigler, J. W., Hsu, C., & Kitamura, S. (1990). Contexts of achievement: A study of American, Chinese and Japanese children. *Monographs of the Society for Research in Child Development, 55,* 223-256.

The state of mathematics education. (1998). Retrieved May 12, 2001, from http://www.ed.gov/speeches/01-1998/980108.html

Tuss, P., & Zimmer, J. (1995). Causal attributions of underachieving fourth-grader students in China, Japan, and the United States. *Journal of Cross-Cultural Psychology, 26,* 408-426.

Underhill, R. G., Uprichard, A. E., & Heddens, J. W. (1980). *Diagnosing mathematical difficulties.* Coumbus, OH: Merrill.

Watanabe, A. K. (1991). *The effects of a mathematical word problem solving strategy on problem solving performance by middle school students with mild disabilities.*

Unpublished doctoral dissertation, University of Florida, Gainsville.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice, 18*(2), 121-131.

VITA

Graduate College
University of Nevada, Las Vegas

Kyle Konold

Home Address:
9912 Long Cattle Avenue
Las Vegas, NV 89117

Degrees:
Bachelor of Arts, Psychology, 1992
University of Wisconsin, Eau Claire

Master of Science, Psychology, 1995
Southern Illinois University, Edwardsville

Educational Specialist, School Psychology, 1996
Southern Illinois University, Edwardsville

Special Honors and Awards:
Awarded certificate of distinction for exceptional performance on the doctoral written comprehensive examination.

Publications:

Konold, K. E., Miller, S. P., & Konold, K. B. (Under review). Using teacher feedback to enhance student learning. *Teaching Exceptional Children.*

Konold, K. K. (2002). *Nevada systems change grant for community living* (Annual report, FY 2002). University of Nevada, Las Vegas, Department of Special Education.

Dissertation Title: Using the Concrete-Representational-Abstract Teaching Sequence to Increase Algebra Problem Solving Skills

Dissertation Examination Committee:

Chairperson, Dr. Susan Miller, Ph.D.

Committee Member, Dr. John Filler, Ph.D.

Committee Member, Dr. Sherri Strawser, Ph.D.

Graduate Faculty Representative, Dr. W. Paul Jones, Ph.D.