Distributed importance-based fuzzy logic controllers for flexible link manipulators

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DISTRIBUTED IMPORTANCE-BASED FUZZY LOGIC CONTROLLERS
FOR FLEXIBLE LINK MANIPULATORS

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ABSTRACT

Distributed Importance-based Fuzzy Logic Controllers for Flexible Link Manipulators

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This research studies the design and tuning of the distributed importance-based fuzzy logic controllers (FLCs) for two dynamic systems: a single-link flexible manipulator and a two-link rigid-flexible manipulator. The importance analysis algorithm is introduced in the structure design of a FLC. The fuzzy rules for the former system are written based on observing the system behaviors. The fuzzy rules for the latter are selected to mimic the performance of the comparable linear controllers. A Modified Nelder and Mead Simplex Algorithm is used to tune the parameters of the membership functions in the distributed importance-based FLC. The tuned distributed importance-based FLC for the single-link flexible manipulator is compared with a linear quadratic regulator and the tuned distributed PD-like FLC. Similarly, the tuned distributed importance-based FLC for the two-link rigid-flexible manipulator is compared with the tuned importance-based linear controller and the tuned distributed PD-like FLC. The robustness of each tuned controller is tested under different conditions.
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CHAPTER 1

INTRODUCTION

Background of Research

Fuzzy logic is a model-free and rule-based reasoning approach that has been applied to the control of many dynamic systems. It starts by observing of the system, and articulates a corresponding system by fuzzy IF-THEN rules. In the early stage of fuzzy inference systems, the fuzzy logic was used to translate the expert's linguistic, mostly heuristic, control scenarios into IF-THEN rules as shown in the work of Zadeh (1973) and Mamdani (1977). As stated in Sayyarrodsari and Homaifar (1997), the promising results of these early experiments inspired widespread research activities in which the application domain included problems where mathematical models for the system, although generally imprecise and highly nonlinear, were available. Fuzzy logic controller (FLC) has the strength to deal with uncertainty and imprecision. The advantages of FLC over conventional controllers include the increased robustness and the ability to handle nonlinearities. Although FLC has already been implemented successfully in many applications, several questions remain however unanswered:

(1) How to determine the structure of a FLC for a dynamic system that has potentially a large number of input and output variables?

(2) How to derive fuzzy rules for a strongly coupled dynamic system?
(3) How to efficiently tune the structures / parameters of a FLC to achieve a better performance?

The above questions are all related to two important issues: the design and tuning of a FLC for a dynamic system. The following is a survey of the recent research to address these questions. Research papers are divided by topic in this survey.

**Design FLC for Complex Systems**

Design a FLC for complex systems with a large number of inputs and outputs is a challenging process. In many cases, it may not be practical to use all inputs to construct a single FLC for each output as the total number of fuzzy rules increases exponentially with the number of inputs. For example, consider a system with $n$ inputs and $m$ outputs. Choose $l$ as the number of membership functions for each variable. Then there will be totally $m \times l^n$ fuzzy rules if all the inputs are used to construct a single FLC structure for each output, as shown in Figure 1. Avoiding this dimensionality problem is critical to the success of the FLC. Many researchers attempted to address this problem through different ways. Lin and Lee (1994) used a reinforcement learning algorithm to delete useless fuzzy rules. Jang (1993) ignored some unimportant inputs to simplify the FLC structure. Sayyarrodsari and Homaifar (1997) proposed a hierarchical FLC to simulate an existing hierarchy in the human decision process. Chung and Duan (2000) pointed out that the dimensionality problem could be fundamentally addressed by adopting a multi-stage structure, that is, the output of one FLC can be the input of a FLC in the next stage. The total number of fuzzy rules will then be a linear function of the inputs. For the incremental structure proposed in Chung and Duan (2000), as shown in Figure 2, the total number of fuzzy rules is $m \times (n-1) \times l^2$.
Similar to the multi-stage FLC structure, a distributed FLC structure, as shown in Figure 3, is often adopted in the control applications, such as in Moudgal et al. (1994 and 1995), Trabia (1998), Shi and Trabia (2000), Trabia and Shi (2001). Most dynamic systems can be expressed by a set of second order differential equations, $\ddot{x} = f(x, \dot{x}, u, \tau)$

where $[x \; \dot{x}] = [x_1 \; \ldots \; x_N \; \dot{x}_1 \; \ldots \; \dot{x}_N]$, $n=2N$ is the state variable vector,
$u = [u_1 \ u_2 \ \cdots \ u_m]$ is the controller output vector. In the structure design of a FLC for these dynamic systems, it is intuitive to select $e_x, e_y$ as the two inputs of a FLC, and define $u_{ji}$ as the output of the FLC, where $i = 1, \cdots, N$ and $j = 1, \cdots, m$. In this arrangement, each FLC has a two-input one-output structure, and they are parallel with one another. The outputs of those FLCs are grouped together for each $u_j$, $j = 1, \cdots, m$. It is called a distributed FLC structure in literature. Choose $l$ as the number of membership functions for each variable, then the total number of fuzzy rules is $m \times N \times l^2 = m \times N \times l^2$ if all the inputs are used to control each output. This number will be reduced further if some inputs are deleted for a specific output based on the expert knowledge or an advanced algorithm. As a consequence, $e_x, e_y$ may not be both kept in the FLC structure. The following problems appear:

1. Difficult to determine which inputs to keep and which inputs to delete in the FLC structure for a specific output.

2. The input arrangement may become difficult when the coupling effect is strong for a nonlinear complex system.

3. Different experts may propose different distributed structures for each output.

To address these problems, Taylor Series Expansion is used in this study to analyze the importance degrees of inputs with respect to each output. This method has been effectively used in the areas of the system modeling and identification where the input-output data sets are easily obtained. The application of this method to the control area where the direct controller input and output data sets are not available is the motivation of this study.
Tuning Strategies of FLCs

Early FLCs did not have learning ability, which led many researchers to carry out comprehensive studies on integrating one of the following methodologies to the FLC structure:

1. Adaptive Fuzzy Control: In the case of large uncertainties or unknown variations in the plant parameters and structures, the adaptive fuzzy control can be used. The early adaptive controllers included the fuzzy model reference learning control (FMRLC) and the self-organizing fuzzy logic control. The former utilized the learning mechanics to make the closed-loop system perform according to the specifications given by the reference model. The latter had a learning algorithm and was capable of generating and modifying the control rules based on the evaluation of the system performance. FMRLC was applied to a two-link flexible robot in Moudgal et al. (1995), and to an antiskid break system in Layne and Passino (1996). An adaptive multivariable FLC was proposed to control a Puma
560 system and a two-inverted pendulum system in Yeh (1997). In order to guarantee the stability of the adaptive fuzzy controller, the direct and indirect adaptive fuzzy controllers were proposed in literature. Stable direct and indirect adaptive controllers for the automated highway system using Takagi-Sugeno fuzzy systems were presented in Spooner and Passino (1996). The design of the indirect adaptive fuzzy controller for the inverted pendulum tracking system was discussed in Wang (1997). A direct adaptive fuzzy control design method was developed for the general higher order nonlinear continuous system in Tsay et al. (1999). A combined indirect/direct adaptive fuzzy controller for a two-link planar manipulator was discussed in Yoo and Ham (2000).

(2) Fuzzy Neural Networks: The integration of Neural Networks and FLC brings the low-level learning and computational power of Neural Networks into FLCs and provides the high-level human-like thinking and reasoning of FLCs into Neural Networks. The supervised learning was used efficiently in the system modeling as shown in Chung and Duan (2000), and the system identification as shown in Buckley and Hayashi (1994), and Leu et al. (1999), where the input-output training data was available. The reinforcement learning in Lin and Lee (1994), Chiang et al. (1997), and Lin and Jou (2000), was more appropriate in many control applications where input-output training data was not readily available.

(3) Tuning FLC Using Genetic Algorithms: Genetic Algorithms is a global search method that does not use the local information about the promising search direction. Genetic Algorithms was used to tune the fuzzy rules for classical cart-pole benchmark, boat steering, and aircraft landing systems in Cooper (1995). A
genetic reinforcement FLC was proposed to learn the fuzzy rules for a cart-pole balancing problem in Chiang et al. (1997). Genetic Algorithms-based reinforcement learning method was applied to the control of a real magnetic bearing system in Lin and Jou (2000). Genetic Algorithms was used to tune the parameters of the membership functions of the FLC for a flexible-link manipulator in Shi and Trabia (2000).

(4) Tuning FLC Using Nonlinear Programming Techniques: Nonlinear programming techniques have been widely used in many engineering applications, as shown in Rekalitis et al. (1983). One of the powerful methods, Simplex Algorithm uses only the function evaluations to determine its search direction, which is especially useful when the training data of a controller are not available. It is a local search technique that uses the evaluation of the current data set to determine the promising search direction. The advantages of a local search technique include simplicity and computational efficiency. Simplex Algorithm, as stated in Rekalitis et al. (1983), starts by generating a simplex with n+1 vertices. The algorithm evaluates the function values at these points, and replaces the point of the highest function value with its reflection along a vector passing through the center of the remaining points. The Nelder and Mead Simplex Algorithm in Nelder and Mead (1965) was used to tune the parameters of the membership functions of the FLC in Trabia (1998), Shi and Trabia (2000), Trabia and Shi (2001).
Flexible Manipulators

To address the problems of the design and tuning of a FLC in control applications, flexible manipulators are chosen as the controlled plants due to their complex nonlinear dynamics, strongly coupled and non-minimum phase nature that might make accurate and robust control difficult.

A light-weight robotic manipulator provides faster response, lesser material, and lower energy consumption, when compared to the average industrial manipulators. These manipulators however exhibit flexible deformations, which can cause some deviations from the desired trajectories. Flexible manipulators cannot perform their tasks before dampening their vibrations, especially in high-speed applications.

The major difference between the flexible link manipulators and the rigid robots for the control purposes is that the number of inputs is far less than the number of degrees of freedom. It is called Reduced Control Effectiveness in Lewis et al. (1999). Therefore, many control strategies, which work well for the rigid-link robots, may not be directly applied to the flexible manipulators due to the flexibility effects on the control system performance. The situation is even worse, for it turns out that by selecting the control inputs to achieve a practical tracking performance of the rigid variable, one may actually excite the flexible modes. This is due to the non-minimum phase nature of the zero dynamics of flexible-link robot arms, as stated in Madhavan and Singh (1991), Wang and Vidyasagar (1991) and Martins et al. (2002).

Modeling Methodologies for Flexible Manipulators

The flexible manipulators can not be treated as collections of rigid bodies only. Many researchers have studied the dynamics of this kind of system extensively in the last
twenty years. The dynamic equations of the flexible manipulators, as stated in Yazdizadeh et al. (2000), are infinite dimensional and may be described by a set of nonlinear partial differential equations. There are two main modeling methods used in literature to reduce the complexities involved and also to readily implement the control algorithms. The first approach, the assumed modes method, Meirovitch (1967) and Book (1984), represents the deflection of a beam using series of separable functions. The other method, the finite element method, Kwon and Bang (1997), describes the beam as a sequence of elements. The governing equations of motion of the flexible manipulators can be obtained using Hamilton's principle, in Ge et al. (1996) or Lagrangian method in Book (1984). A single-link flexible manipulator moving in a horizontal plane was studied in Ge et al. (1996), Trabia (1998), Kubica and Wang (1999), Lewis et al. (1999), Rokui and Khorasani (2000), Shi and Trabia (2000), Trabia and Shi (2001), Su and Khorasani (2001), Mohamed and Tokhi (2002), Shaheed and Tokhi (2002). A two-link flexible manipulator moving in a horizontal plane was discussed in Asada et al. (1990), Moudgal et al. (1994 and 1995), Lee and Lee (2001), in a space surrounding in Cetinkunt and Book (1990), Gawronski et al. (1995), in a vertical plane in Nathan and Singh (1991), Madhavan and Singh (1991), Xi and Fenton (1994), Yazdizadeh et al. (2000) and Li et al. (2000). The general derivation for the multi-link flexible manipulator was given in Wang and Vidyasagar (1991), Cetinkunt and Book (1990), Asada et al. (1990).

Control Strategies for Flexible Manipulators

The controller’s objective for flexible manipulators is to make the joints of the flexible manipulators tracking the desired trajectories or moving from point to point during the active motion period with no higher-order vibrations excited at the final target
position. The control strategies in literature of flexible manipulators can be divided into five categories:

(1) Input Command Shaping: This type of open-loop method assumes that the system inputs can be shaped to inject a minimal energy into the flexible modes of the system, Singhose (1997). The input command shaping method for a two-link flexible robot was discussed in Hillsley and Yurkovich (1993), Magee and Book (1993), Romano et al. (2002) and for a single-link flexible manipulator in Mohamed and Tokhi (2002).

(2) Model-based Algorithms: This type of control methods assumes that the derived mathematical model of the flexible manipulators is fairly accurate. The inverse dynamics was used with a feed-forward compensation in Asada et al. (1990), Gawronski et al. (1995), with a linear stabilization in Madhavan and Singh (1991). To extend the control effectiveness, a singular perturbation approach of a flexible-link manipulator was derived in Siciliano and Book (1988) and Vandegrift et al. (1994). Using the singular perturbation technique, the flexible manipulator system was divided into a slow subsystem and a fast subsystem with different time-scales. A sliding mode control and an elastic mode stabilization for a flexible link manipulator were designed in Nathan and Singh (1991), and Qian (1992), where the discontinuous joint angle control law was designed to accomplish an asymptotic joint angle trajectory tracking. The major assumption of this type of control scheme is prefect modeling.

(3) Adaptive Control Methods: This type of control method is to improve the performance of the model-based algorithms by considering the unmodeled effect
in the flexible manipulator systems. An adaptive controller with a linearized continuous model was investigated in Feliu et al. (1990). An indirect adaptive control based on a discrete-time nonlinear model was proposed in Rokui and Khorasani (2000). Even through the adaptive control method has an on-line tuning capacity, the mathematical model with respect to the known parameters/structures must be accurate in order to achieve a good performance. Some types of adaptive control methods are integrated with the intelligent algorithms as shown in the following categories.

(4) Neural Networks and Genetic Algorithms: This type of control method is mainly used as a feed-forward controller in literature of flexible manipulators, such as a Neural-Network-based controller using the inverse dynamic approach in Su and Khorasani (2001) and an open-loop Genetic Algorithms in Shaheed et al. (2001). The learning feature of Neural Networks and Genetic Algorithms was combined with other types of feedback controllers by many researchers. A decimal genetic algorithms was used to tune and optimize the performance of a Lyapunov-base robust controller for a single-link flexible robot in Ge et al. (1996). An adaptive time delay neural networks for a two-link flexible manipulator was proposed in Yazdizadeh et al. (2000), where a neuro-dynamic structure was used to identify the system. Observer-based adaptive controller design for the flexible manipulators using the time-delay neuro-fuzzy networks was proposed in Deng et al. (2002).

(5) Fuzzy Logic Control: This type of control method has been widely used to control the flexible manipulators in the last ten years. A control law that consisted
of a FLC plus a nonlinear effects negotiator was derived in Lin and Lee (1993). A Neural-Network-like FLC for a flexible link manipulator was implemented by Arciniegas et al. (1993). A fuzzy model reference-learning controller for a flexible link manipulator was developed in Moudgal et al. (1994). A distributed FLC with an automatic parameter tuning procedure for a single-link flexible manipulator was proposed by Trabia (1998) and later expanded by Trabia and Shi (2001). A fuzzy control strategy to control the rigid body and the first flexural mode of vibration separately for a single-link robotic arm was described in Kubica and Wang (1999). A linear quadratic gaussian method was proposed to control a two-link flexible manipulator tracking a two-dimensional square trajectory in Green and Sasiadek (2001). A FLC with gravity compensation was applied for the point-to-point control of a two-link flexible manipulator in Oke and Istepanopoulos (2001). A neurofuzzy controller was used as a nonlinear compensator for a four-link flexible manipulator in Caswara and Unbehauen (2002).

Objective of Research and Methodologies

In view of the FLC literature and the control strategies for the flexible manipulators, the objective of this research is to study the design and tuning of a distributed FLC for the flexible manipulators. The controller's objective is to make the joints of the flexible manipulators tracking the desired trajectories during the tracking period with no higher-order vibrations excited at the final target position. Several issues affect the design of a distributed FLC:
(1) Identifying the variables of the controller (which variables should be included?).

(2) Designing the structure of the controller (how variables should be grouped?).

(3) Choosing the form and the number of the membership functions for each variable.

(4) Constructing fuzzy inference rules.

(5) Determining the parameter values of each membership function.

(6) Evaluating the performance of the controller to determine if any of the above elements, or even one of the FLCs, should be modified or deleted.

This study addresses the relation between these issues and proposes a distributed importance-based FLC structure for the flexible manipulator systems.

An importance analysis algorithm is proposed in this study based on Taylor Series Expansion. The results are applied to design a distributed FLC for the flexible manipulators. This analysis needs to have the controller's input-output training data sets, where the controller's inputs are the errors of some or all the state variables, and the controller's outputs are the system inputs (mostly torque) of the dynamic systems. The exact input-output data sets are not available for a feedback controller.

The purpose of the importance analysis is to come up with some information about the mapping relations between the input-output of the controller. The direct dynamics method is used, where the torque is generated randomly in the working range, and the system dynamic equations are solved for the state variables. The data sets of the state variables and the torque are used in the importance analysis.

The results of the importance analysis can be used to distribute the controller input variables to each output. The most important input variables will be used to construct the
distributed FLC for each output, and the remaining input variables may be used to add a minor modification to the output or deleted to reduce the dimensionality problem.

Constructing the fuzzy rules and determining the parameter values for each membership function are the two most important steps in the design of a FLC. In this study, the fuzzy rules for a simple dynamic system are written based on the observation of the system behaviors. The fuzzy rules for a more complicated coupling system are selected to mimic the performance of the comparable linear controllers. However, selecting the parameter values for each membership function can be challenging. A tuning algorithm will be needed to choose those values.

Reviewing the learning algorithms for a FLC, the Neural-Network-based FLC is not feasible for the control applications due to lack of exact training data. The GA-based FLC suffers the computational difficulty due to its slow convergence rate, and its inability to determine the best way to reach a minimum. On the contrary, a local search method, Nelder and Mead Simplex Algorithm, yields a satisfactory result. It will be modified in this study to achieve a faster convergence rate.

Organization of the Dissertation

This dissertation contains five chapters. In chapter 2, the design and tuning of the distributed importance-based FLC for a single-link flexible manipulator are studied. The importance analysis algorithm is proposed and the results are applied to design a distributed FLC for the single-link flexible manipulator. The fuzzy rules are written based on the observation of the system behaviors. The parameters of the membership functions are tuned using the Modified Nelder and Mead Simplex Algorithm. The performance of
the distributed importance-based FLC is further compared with a Linear Quadratic
Regulator and a distributed PD-like FLC. The robustness of the three controllers are
tested and compared under various conditions. The spillover effect of the distributed
importance-based FLC is discussed and the performance of the distributed importance-
based FLC using the Modified Nelder and Mead Simplex Algorithm is compared with
that using Genetic Algorithms.

In chapter 3 and 4, the design and tuning of the importance-based FLC for a two-link
rigid-flexible manipulator are studied. The purpose of the importance analysis is to
consider the coupling effect among the two joints and the payload. The importance
degrees of the tip deflection variables and the joint variables on the other link are studied
for the torque applied on one link. One FLC with the two most important input variables
is included to control each torque together with the FLC with the two joint variables on
that link. To address the difficulty in writing the fuzzy rules and determining the
parameter values of the membership functions, an importance-based linear controller that
has the same input-output structure as that of the importance-based FLC is constructed.
Fuzzy rules of the FLC are constructed to mimic the performance of the corresponding
linear controller. The parameters of the membership functions are tuned in Chapter 3
using the Modified Nelder and Mead Simplex Algorithm to match the corresponding
linear controller which is obtained in the process of constructing fuzzy rules. As a result,
the two importance-based controllers have similar responses under the same joint angle
trajectory. The gains of the linear controller and the parameters of the FLC are further
tuned using the same tuning technique to get better performances. The two importance-
based controllers are simulated and compared in Chapter 3. Robustness of each tuned
importance-based controller is tested by varying the joint angle trajectories in the working space.

The comparison of the distributed importance-based FLC with the distributed PD-like FLC for the two-link rigid-flexible manipulator is studied in Chapter 4. Both controllers use the corresponding linear controllers as a guide to write the fuzzy rules. The initial parameter values of the two FLCs are selected based on the working range of the linear controller and kept the same for the corresponding variables. The two distributed FLCs are simulated and compared in Chapter 4. The robustness of each distributed FLC is tested under various conditions.

Finally, the conclusions of this study and recommendations for future developments are discussed in Chapter 5.
CHAPTER 2

DESIGN AND TUNING OF DISTRIBUTED IMPORTANCE-BASED FLC FOR SINGLE-LINK FLEXIBLE MANIPULATOR

A single-link flexible manipulator system is chosen in this study to demonstrate the design and tuning of the distributed importance-based FLC on a multi-input single-output dynamic system. The first section presents the dynamic model of the single-link flexible manipulator. The second section introduces the importance analysis algorithm for a multi-input single-output dynamic system. The third section lists the results of the importance analysis for the single-link flexible manipulator. The fourth and fifth sections propose the design and tuning of the distributed importance-based FLC for the single-link flexible manipulator respectively. The sixth introduces two commonly used controllers: a Linear Quadratic Regulator and a distributed PD-like FLC for the single-link flexible manipulator. The seventh section tests the robustness of three controllers by decreasing and increasing the payload. The eighth section tests the spillover effect of the distributed importance-based FLC on the single-link flexible manipulator. The ninth section compares the performance of the distributed importance-based FLC using the Modified Nelder and Mead Simplex Algorithm with that of using Genetic Algorithms. The last section contains a summary of this chapter.
Dynamic Model

Many researchers have studied the dynamics and control of the single-link flexible manipulator. A finite element approach is used in this study to describe the dynamics of the flexible link. The link is considered as composed of finite elements satisfying Euler-Bernoulli’s theorem. The displacement of any point on the link is described in terms of modal displacements. Energy approach is used to formulate the equations of motion. The modeling steps, which are described briefly in this section, are based on Kwon and Bang (1997) and Logan (1997).

The beam is divided into \( n \) elements. The displacement of any point in element \( i \), Figure 4, is described using the nodal displacement and slope of nodes \( i \) and \( i+1 \) as follows:

\[
v = [N] \{d_i\} = [N_1 \quad N_2 \quad N_3 \quad N_4] \begin{bmatrix} \phi_i \\ \phi_{i+1} \\ v_i \\ v_{i+1} \end{bmatrix}
\]

(1)

\( N \)'s are called the shape functions, as shown in the following equations:

\[
N_1 = \frac{1}{L_i} \left( 2x_i^3 - 3x_i^2 L_i + L_i^3 \right)
N_2 = \frac{1}{L_i} \left( x_i^3 L_i - 2x_i^2 L_i^2 + x_i L_i^3 \right)
N_3 = -\frac{1}{L_i} \left( 2x_i^3 + 3x_i^2 L_i \right)
N_4 = \frac{1}{L_i} \left( x_i^3 L_i - x_i^2 L_i^2 \right)
\]

(2)
Figure 5 shows a single-link flexible manipulator of length $L$. Cantilever end boundary conditions are assumed in this model. The position vector of a point $P$ on this link, measured in the frame of the link is

$$P = \begin{bmatrix} x \\ v \end{bmatrix}^T$$ \hspace{1cm} (3)$$

The velocity of this point is

$$\dot{P} = \begin{bmatrix} 0 \\ x \dot{\theta} + \dot{v} \end{bmatrix} = \begin{bmatrix} 0 \\ x \dot{\theta} + [N]\dot{d_i} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ N\dot{\theta} \end{bmatrix}$$ \hspace{1cm} (4)$$

The kinetic and potential energies of an element are

$$KE_i = \frac{1}{2} \rho_i \frac{dx_i}{dt} = \frac{1}{2} \dot{\theta}^T M_i \dot{\theta}$$ \hspace{1cm} (5)$$

$$PE_i = \frac{EI_i}{2} \int x \left( \frac{\partial^2 v}{\partial x_i^2} \right)^T \left( \frac{\partial^2 v}{\partial x_i^2} \right) dx_i = \frac{1}{2} d_i^T K_i d_i$$ \hspace{1cm} (6)$$

where

$$M_i = \int_0^{L_i} \rho_i \begin{bmatrix} x \\ N \end{bmatrix} [x \\ N] dx_i$$ \hspace{1cm} (7)$$

$$K_i = \int_0^{L_i} EI_i \left( \frac{\partial^2 N}{\partial x_i^2} \right)^T \left( \frac{\partial^2 N}{\partial x_i^2} \right) dx_i$$ \hspace{1cm} (8)$$

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where $M_i$ is the mass per unit length of element $i$. $K_i$ is the product of Young’s modulus of elasticity by the cross-sectional moment of inertia for element $i$. Similarly, the kinetic energies of the payload, $m_i$, the mass moment of inertia, $J_r$, and the hub, $J_m$ are

$$KE_{tp} = \frac{1}{2} \sum_{i=1}^{n} m_i \dot{v}_i + \frac{1}{2} \sum_{i=1}^{n} \phi_i J_i \dot{\phi}_i \tag{9}$$

$$KE_{hub} = \frac{1}{2} \dot{\Theta} J_m \dot{\Theta} \tag{10}$$

Coefficients of $M_i$ and $K_i$ matrices can be expanded to $M_{xi}$ and $K_{xi}$ in terms of the global coordinate vector $q$:

$$q = [\theta \ v_2 \ \phi_2 \ \cdots \ v_{n+1} \ \phi_{n+1}] \tag{11}$$

The global mass and stiffness matrices are

$$M = \sum_{i=1}^{n} M_{xi} + \begin{bmatrix} J_m & [0]_{2 \times 2n} \end{bmatrix} + \begin{bmatrix} [0]_{(2n-1) \times (2n-1)} & [0]_{(2n-1) \times 2} \end{bmatrix} \begin{bmatrix} m_i & 0 \\ 0 & J_r \end{bmatrix} \tag{12}$$

$$K = \sum_{i=1}^{n} K_{xi} \tag{13}$$

Using the principles of the Lagrangian dynamics, the equations of motion are

$$[M][\ddot{q}] + [K][\dot{q}] = \{F\} \tag{14}$$

where $F$ is the force vector

$$F = [T \ 0]^T \tag{15}$$

$T$ is the torque applied at the hub.
Importance Analysis Algorithm

This section introduces the importance analysis algorithm for a multi-input single-output dynamic system. The process starts by normalizing the system to the form of,
\[ y = f(u_1, u_2, \cdots, u_n) \] such that \([u_1, u_2, \cdots, u_n] \in [0,1]^n\), where \(u\) is the input vector and \(y\) is the output. For a dynamic system, like the single-link flexible manipulator, \(u\) includes the system state variables, and \(y\) is the system input. A set of \(p+1\) sample data in the form of,
\[ [u_{j,1}, u_{j,2}, \cdots, u_{j,n}, y_j] \quad \forall j = 1, \cdots p+1 \], can be collected by solving the system dynamic equations using randomly generated system input signals under an initial condition, \(u_0\).

For a dynamic system, the value of \(u_{j+1}\) depends not only on the current value of \(y_j\), but also on the previous value of \(u_j\) unlike the system identification problem studied by Chung and Duan (2000). As a result, the importance analysis algorithm for a dynamic system cannot be done by mixing data sets. Two conjunct output values, \(y_j\) and \(y_{j+1}\), may be approximated using the following Taylor Series Expansion on a fixed point \([\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_n]^T\):

\[
y_j = f(\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_n) + \sum_{i=1}^{n} \frac{\partial f}{\partial u_i} igg|_{u_i = \mathcal{X}_i} (u_{j,i} - \mathcal{X}_i) + r_j \tag{16}
\]

\[
y_{j+1} = f(\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_n) + \sum_{i=1}^{n} \frac{\partial f}{\partial u_i} igg|_{u_i = \mathcal{X}_i} (u_{j+1,i} - \mathcal{X}_i) + r_{j+1} \tag{17}
\]
where $r$ is the higher order term. The following equation is derived by subtracting Equation (16) from Equation (17):

$$y_{j+1} - y_j = \sum_{i=1}^{n} b_i (u_{j+1,i} - u_{j,i})$$

where $b_i = \frac{\partial f}{\partial u_i}_{x_j=x_i}$. Equation (18) shows that each $b_i$ represents the ratio of the variance of an input variable $u_i$ with the variance of the output variable $y$, which in turn represents how important the input is to that output. Repeating the simulation $m$ times, and rewriting Equation (18) in a matrix form:

$$\{\Delta Y\} = [\Delta U] \{B\}$$

(19)

where the dimensions of $\{\Delta Y\}$, $[\Delta U]$, and $\{B\}$ are $(p \times m) \times 1$, $(p \times m) \times n$ and $n \times 1$, respectively. $\{B\}$ is an unknown vector whose element is parameter $b_i$. This problem can be usually solved using the pseudo-inverse formula:

$$\{B\}^* = \left[ [\Delta U]^T [\Delta U] \right]^{-1} [\Delta U]^T \{\Delta Y\}$$

(20)

If $[\Delta U]^T [\Delta U]$ is a singular matrix, let the $i^{th}$ row vector of matrix $[\Delta U]$ be $\Delta u_i$ and the $i^{th}$ element of $\{\Delta Y\}$ be $\Delta y_i$. $\{B\}$ can be calculated using the following sequential formulations as shown in Jang (1996):

$$\{B_{i+1}\} = \{B_i\} + [H_{i+1}] \Delta u_{i+1} (\Delta y_{i+1} - \Delta u_{i+1} \{B_i\})$$

(21)

$$[H_{i+1}] = [H_i] - \left[ \frac{[H_i] \Delta u_{i+1} \Delta u_{i+1}^T [H_i]}{1 + \Delta u_{i+1} [H_i] \Delta u_{i+1}^T} \right] i = 0, \cdots, (p \times m) - 1$$

(22)

where $[H_i]$ is called the covariance matrix in Miller (1990) and Seber and Wild (1989). The initial conditions to start solving Equation (21) are $B_0 = [0, 0, \cdots, 0]^T$ and $[H_0] = \gamma [I]$, where $\gamma$ is a positive large number and $[I]$ is the identity matrix of
dimension \((p \times m) \times (p \times m)\). Each \(b_i\) represents the ratio of the variance of an input variable \(u_i\) with the variance of the output \(y\) over the complete given data set. Therefore, \(b_i\) implies the importance degree of \(u_i\) with respect to \(y\) in a sense of statistics. Note that \(b_i\) can be positive or negative, the term \(IMP(u_i)\) is used to represent the importance degree of \(u_i\) and \(IMP(u_i) = |b_i|/\sum_{j=1}^{n} |b_j|\) to make \(\sum_{i=1}^{n} IMP(u_i) = 1\).

If the input vector \([u_1, u_2, \ldots, u_n]^T\) in the given data set exceeds the interval \([0,1]^n\), Taylor Series Expansion cannot be adopted for approximation. The above analysis algorithm can be applied by using the normalized data pairs: \([u'_{j,i}, u'_{j,2}, \ldots, u'_{j,n}, y'_j]^T\)

\[
\begin{align*}
u'_{j,i} &= \frac{(u_{j,i} - \sigma_i)}{\sigma_i} \quad (i = 1, \ldots, n; j = 1, \ldots, p \times m) \\
y'_j &= \frac{(y_j - \sigma_i)}{\sigma_i} \quad (j = 1, \ldots, p \times m)
\end{align*}
\]

where \(\sigma_i\) and \(\sigma_i\) are the minimum and range of a variable respectively over the corresponding column in the data set.

**Importance Analysis Results**

The above importance analysis algorithm is applied to a single-link flexible manipulator whose physical parameters are listed in Table 1. The first five natural frequencies of this system are listed in Table 2. As stated in the first section of this chapter, the flexible link is described by \(n\) elements. Eight elements were initially used to model the link. It was later found that the step response using four elements overlaps to a large degree with that using eight elements. The differences of those two cases under the step input are less than 1E-5 radians in the joint angle response as shown in Figure 6 and
less than 1E-5 meters in the tip deflection response as shown in Figure 7. They may be caused by computational errors. Therefore, four elements of equal length are used to describe the flexible link in this study. The degrees of freedom of the single-link flexible manipulator are eighteen.

Table 1  Physical Parameters of Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link length, $L$</td>
<td>Meter</td>
<td>1.0</td>
</tr>
<tr>
<td>Linear density, $\rho$</td>
<td>Kg/m</td>
<td>0.1</td>
</tr>
<tr>
<td>Bending stiffness, $EI$</td>
<td>Nm$^2$</td>
<td>2.0</td>
</tr>
<tr>
<td>Moment of inertia of the hub, $J_m$</td>
<td>Kg$m^2$</td>
<td>0.05</td>
</tr>
<tr>
<td>Radius of the hub, $L_0$</td>
<td>Meter</td>
<td>0.01</td>
</tr>
<tr>
<td>Payload, $m_i$</td>
<td>Kg</td>
<td>1.0</td>
</tr>
<tr>
<td>Tip mass moment of inertia, $J_t$</td>
<td>Kg$m^2$</td>
<td>10$^{-5}$</td>
</tr>
</tbody>
</table>

Table 2  First Five Natural Frequencies of Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Number of Natural Frequency</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Natural Frequency (Hz)</td>
<td>11</td>
<td>71</td>
<td>225</td>
<td>472</td>
<td>863</td>
</tr>
</tbody>
</table>
Figure 6  Difference of Joint Angle Response under Step Input of Using Four Elements and Eight Elements on Single-link Flexible Manipulator

Figure 7  Difference of Tip Deflection Response under Step Input of Using Four Elements and Eight Elements on Single-link Flexible Manipulator
The goals of the controller for the single-link flexible manipulator are:

(1) Make the joint angle tracking the desired trajectory.

(2) Reduce the tip displacement.

(3) Eliminate the potential higher-order vibrations at the final target position.

Based on those objectives, four system state variables are selected in the controller design: the joint angle, $\theta$, the joint angular velocity, $\dot{\theta}$, the tip displacement, $v(L)$, and the tip velocity, $\dot{v}(L)$. The motor torque $T$ is the controller output. The desired values of these variables are $\theta_d$, $\dot{\theta}_d$, 0, 0 respectively. The joint angle and its velocity can be measured using joint encoder and tachometer respectively. The tip displacement may be measured by attaching a laser source to the tip and a corresponding sensor at the manipulator base. Using series of strain gages along the manipulator link can be also used. The tip velocity may be calculated by differentiating the tip displacement signal.

The first step of the importance analysis is to generate sufficient random torque signals in the working space. These random signals should produce a reasonable range of the tip displacement (for example, less than one third of the length of the flexible link). After several attempts, the random torque range is chosen as $\pm 3$ Nm. The system equations of motion are solved under a zero initial condition. The time duration of each simulation is one second with one hundred samples. For the system under study, the torque signals result in $\theta$ motion between [-0.50, 0.46] radians, $v(L)$ motion between [-0.42, 0.45] meters, $\dot{\theta}$ motion between [-5.10, 4.77] radians/second, and $\dot{v}(L)$ motion between [-5.07, 5.38] meters/second. It should be noted that the tip displacement exceeds 0.33 meters (one third of the length of the flexible link) in few instances only. The simulation is repeated by 250 times to obtain 50000 data points.

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The second step is to analyze the importance degrees of the four state variables, $\theta$, $\dot{\theta}$, $v(L)$, and $\dot{v}(L)$ with respect to the torque $T$. The importance analysis algorithm in the previous section is applied to the data set $T = f(\theta, \dot{\theta}, v(L), \dot{v}(L))$. The results of $IMP$, as listed in Table 3, show that the two velocity variables, $\dot{v}(L)$ and $\dot{\theta}$, have higher importance degrees than the two displacement variables, $\theta$ and $v(L)$, which have significantly low importance degrees.

<table>
<thead>
<tr>
<th>Importance Degree</th>
<th>$IMP(\theta)$</th>
<th>$IMP(v(L))$</th>
<th>$IMP(\dot{\theta})$</th>
<th>$IMP(\dot{v}(L))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>11.47%</td>
<td>9.39%</td>
<td>38.43%</td>
<td>40.71%</td>
</tr>
</tbody>
</table>

**Design of Importance-based FLC**

Based on the results of Table 3, the controller is distributed between two FLCs: the Velocity FLC and the Displacement FLC as shown in Figure 8. Both FLCs use the error, which is defined as the difference between the desired value of a variable and its actual one, as the input. The output of each FLC is torque. The Velocity FLC has two inputs: the joint angular velocity error, $e_{d\theta}$, the tip velocity error, $e_{d\dot{v}}$, and one output: the torque needed to correct these errors: $T_v$. Similarly, the Displacement FLC has two inputs: the joint angle error, $e_{\theta}$, the tip displacement error, $e_{d\dot{v}}$, and one output: the torque needed to correct these errors: $T_d$. The sum of $T_v$ and $T_d$ is used to drive the joint motor.
The next step in the design of FLC is to choose the form and the number of membership functions that can best describe a fuzzy variable. Gaussian curve membership function, Equation (24), represents an attractive answer when attempting to tune a FLC, since it is described using two variables only.

\[ \mu(z, \sigma, c) = e^{-\frac{(z-c)^2}{2\sigma^2}} \]  

(24)

The curve is defined using the particular value of the fuzzy variable, \( z \), and two parameters: the center of the function, \( c \), and the shape factor, \( \sigma \).

The number of membership functions that can reasonably describe each fuzzy variable will determine the number of fuzzy rules, which in turn determine the smoothness of the control surface of a FLC. The more membership functions are selected, the smoother the FLC surface will be. At the same time, a large number of the membership functions will cause difficulties in choosing the initial parameter values in Equation (24). Furthermore, the process of tuning those parameters will be computationally intensive. In this chapter, we will attempt to use a minimal number.
(three) of membership functions per variable. Therefore, three membership functions are used: negative big (NB), zero (Z), and positive big (PB).

For the single-link flexible manipulator, the fuzzy rules of the Velocity FLC and the Displacement FLC, Table 4 and Table 5, are based on the observation of the system behaviors. For example, if $e_{d\theta}$ is NB, the joint is faster than expected. If $e_{d\phi}$ at the same instant is also NB, the tip velocity is pushing it away from the zero position. Both errors can be corrected by commanding the joint motor to produce a NB (clockwise) torque. Similarly, if $e_{\phi}$ is NB, the joint is beyond the expected position. If $e_{tip}$ is PB, the tip is below its zero position. Since these two errors tend to cancel each other, a Z torque should be supplied.

The degree of the membership function of a controller’s output may be related to those of the controller’s inputs by the following relationship:

$$\mu(y_j) = \min(\mu_x(x_1), \mu_x(x_2))$$

(25)

<table>
<thead>
<tr>
<th>$e_{d\phi}$</th>
<th>$e_{d\phi}$</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>PB</td>
<td>PB</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$e_{d\phi}$</th>
<th>$e_{d\phi}$</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PB</td>
<td>PB</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>PB</td>
<td>Z</td>
<td>NB</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>NB</td>
<td>NB</td>
<td></td>
</tr>
</tbody>
</table>
Since the results are fuzzy, they should be transformed into the real numbers through a process of defuzzification. Several defuzzification algorithms were proposed in Driankov (1993). The centroid method is used in this study.

The final step in the design of a FLC is to determine the parameter values of each membership function. Those values may not be as intuitive as determining the fuzzy rules of the FLC. Details of the proposed algorithm for selecting and tuning the parameters for an optimal performance are shown in the next section.

Tuning Parameters of Importance-based FLC

The performance of a FLC depends on the parameter values of its membership functions. In some cases, a good estimate of these values may be available through experience while in others such estimates may be unavailable or can be only obtained by operating the system extensively. This section proposes an automated method to tune a FLC by varying the parameter values using nonlinear programming. A controller can be tuned to minimize its performance index for the system. The proposed performance index for the single-link flexible manipulator is,

\[
PI = \sum_{i=1}^{nt} \left( e_{i}^2 + e_{i,p}^2 \right) dt + \sum_{i=1}^{nt} \left( e_{i,d}^2 + e_{i,dp}^2 \right) dt \\
+ \sum_{i=2}^{nt} \left( \left( e_{i,d} - e_{i-1,d} \right)^2 + \left( e_{i,dp} - e_{i-1,dp} \right)^2 \right) dt
\]

(26)

where \( nt \) is the total number of simulation samples. The first term in the above equation represents a measure of the displacement errors while the remaining two terms represent a measure of the velocity and accelerator errors respectively.
The tuning of a FLC may face the dimensionality problem due to the large number of parameters. In most control applications, it is reasonable to assume symmetry membership functions for a fuzzy variable, that is, the center value of $PB$ is the same as the absolute value of $NB$ of the same fuzzy variable. It requires $m$ parameters to describe $m$ membership functions (assuming the center of $Z$ is at the zero value). Therefore, each fuzzy variable is described by three parameters: $c_B$, $\sigma_B$, and $\sigma_Z$ as shown in Table 6. The total number of the optimization parameters is therefore eighteen for the distributed importance-based FLC. The Modified Nelder and Mead Simplex Algorithm, Appendix I, is used as the tuning method in this study. The termination criterion of the tuning algorithm for the single-link flexible manipulator is $2.5\times10^{-9}$.

In the simulation study for the single-link flexible manipulator, the initial joint angle is zero. The desired final joint angle is one radian. The desired joint angle motion is a bang-bang acceleration profile. The sampling frequency is one hundred samples per second. The desired active motion time is one second. The total simulation time is ten seconds.

<table>
<thead>
<tr>
<th>Parameter of Membership Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership Function</td>
</tr>
<tr>
<td>$NB$</td>
</tr>
<tr>
<td>$Z$</td>
</tr>
<tr>
<td>$PB$</td>
</tr>
<tr>
<td>Center</td>
</tr>
<tr>
<td>$c_B$</td>
</tr>
<tr>
<td>$-c_B$</td>
</tr>
<tr>
<td>$0$</td>
</tr>
<tr>
<td>$c_B$</td>
</tr>
<tr>
<td>Shape</td>
</tr>
<tr>
<td>$\sigma_B$</td>
</tr>
<tr>
<td>$\sigma_B$</td>
</tr>
<tr>
<td>$\sigma_Z$</td>
</tr>
</tbody>
</table>

Table 6 Parameters Describing Each Membership Function of Fuzzy Variable

There is no standard method for determining the initial parameter values of the membership functions for a FLC. The following arrangements are proposed in this
chapter in choosing the initial parameter values in the importance-based FLC for the single-link flexible manipulator:

(1) The values of \( c_B \) for each input variable as well as \( T_v \) are chosen based on what seems to be a sensible range of each variable, as shown in Table 7.

(2) The value of \( c_B \) for \( T_d \) is 4\% of that for \( T_v \) to reflect the reduction of the importance degrees of the two inputs in that FLC.

(3) The value of \( \sigma_B \) is chosen to be 30\% of \( c_B \) and the value of \( \sigma_Z \) is half of \( \sigma_B \) for each variable.

The response of the distributed importance-based FLC using the values in Table 7 is stable as shown in Figure 9 and Figure 10, but has a large maximum joint angle error (0.68 radians) and a long settling time (34.85 seconds).

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Initial Parameter Values of Importance-based FLC for Single-link Flexible Manipulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>( c_B )</td>
</tr>
<tr>
<td>( e_d \theta )</td>
<td>5.0</td>
</tr>
<tr>
<td>( e_d \text{tip} )</td>
<td>5.0</td>
</tr>
<tr>
<td>( T_v )</td>
<td>10.0</td>
</tr>
<tr>
<td>( e_{\theta} )</td>
<td>0.8</td>
</tr>
<tr>
<td>( e_{\text{tip}} )</td>
<td>0.3</td>
</tr>
<tr>
<td>( T_d )</td>
<td>0.4</td>
</tr>
</tbody>
</table>

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Figure 9  Initial Joint Angle Response of Importance-based FLC on Single-link Flexible Manipulator

Figure 10  Initial Tip Displacement Response of Importance-based FLC on Single-link Flexible Manipulator
Using the values in Table 7 as initial, the parameters of the importance-based FLC are tuned using the Modified Nelder and Mead Simplex Algorithm, Appendix I. The performance of the tuning algorithm is shown in Figure 11. The initial performance index value is 4.15. The tuning algorithm reaches a performance index value of 1.48 at the 3008\textsuperscript{th} iteration. The tuned parameter values of the importance-based FLC are listed in Table 8. A few parameters remain close to the initial values. Two-third parameters increase significantly in value, especially $\sigma_z$ of $e_{\text{tip}}$. One-third parameters, such as $c_B$ of $e_{\text{tip}}$, $\sigma_z$ and $c_B$ of $T_r$, $\sigma_B$ and $c_B$ of $e_{\text{tip}}$, experience some reductions in value.

![Figure 11](image)

**Figure 11** Tuning Progression of Importance-based FLC on Single-link Flexible Manipulator
Table 8  Tuned Parameter Values of Importance-based FLC for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$c_B$</th>
<th>$\sigma_B$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{d\theta}$</td>
<td>6.49</td>
<td>3.21</td>
<td>5.84</td>
</tr>
<tr>
<td>$e_{tip}$</td>
<td>2.46</td>
<td>7.37</td>
<td>4.56</td>
</tr>
<tr>
<td>$T_y$</td>
<td>8.41</td>
<td>4.70</td>
<td>1.32</td>
</tr>
<tr>
<td>$e_{\theta}$</td>
<td>2.18</td>
<td>0.48</td>
<td>0.12</td>
</tr>
<tr>
<td>$e_{tip}$</td>
<td>0.09</td>
<td>0.03</td>
<td>1.02</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.90</td>
<td>0.11</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The tuned response of the importance-based FLC is shown in Figure 12 and Figure 13. Two criteria are used in this chapter to compare the tracking and stabilizing performances of a controller based on the controller goals stated in the previous section:

1. The tip deviation of the flexible link with respect to the corresponding rigid manipulator during the tracking period. It is defined as:

\[
E_{tip1} = \sqrt{(x_d - x)^2 + (y_d - y)^2}\]  

where,

\[
x_d = L \cos(\theta_d)
\]
\[
y_d = L \sin(\theta_d)
\]
\[
x = L \cos(\theta) - v(L) \sin(\theta)
\]
\[
y = L \sin(\theta) + v(L) \cos(\theta)
\]

2. The settling time of the tip point. It is defined as the time after which the absolute differences of $E_{tip1}$, at a consecutive time duration (two seconds is chosen in this chapter) are always smaller than a specific value ($1E-4$ is chosen in this chapter).
Figure 12  Tuned Joint Angle Response of Importance-based FLC on Single-link Flexible Manipulator

Figure 13  Tuned Tip Displacement Response of Importance-based FLC on Single-link Flexible Manipulator
Comparing with the initial response, the tuned response of the distributed importance-based FLC has a smaller maximum tip deviation (0.74 meters vs. 0.82 meters), and a much shorter settling time (4.46 seconds vs. 34.85 seconds). The tuned torque of the distributed importance-based FLC is given in Figure 14. The torque magnitude applied to the joint varies from $-1 \text{ Nm}$ to $1.5 \text{ Nm}$. It is interesting to note that the total torque applied on the joint is mainly coming from the Velocity FLC (over 65%).

![Figure 14 Tuned Torque of Importance-based FLC on Single-link Flexible Manipulator](image)

Comparison with Two Other Controllers

To evaluate the effectiveness of the distributed importance-based FLC on the single-link flexible manipulator, two other controllers are compared in this section: a Linear Quadratic Regulator and a distributed PD-like FLC.
Linear Quadratic Regulator (LQR)

LQR is a widely used technique in the control field. It provides an optimal control for the system. LQR method can be defined as finding the appropriate state feedback controller that minimizes the following cost function:

\[
CF = \int_{t_0}^{t_f} \left( [e^T Q]e + u^T [R]u + 2e^T [NL] u \right) dt
\]

where \( e \) and \( u \) are the error and the control input matrices respectively. The above equation is subject to the state dynamic constraint,

\[
\dot{e} = [A]e + [B]u
\]

The optimal control is obtained through feedback with a control law defined as,

\[
u = [-K]e
\]

In the simulation study, both \( Q \) and \( R \) matrices are chosen to be identity matrices, while \( NL \) matrix is null. Observation shows that varying \( Q \) and \( R \) matrices in large ranges does not affect the response of LQR significantly. To properly compare LQR with the tuned distributed importance-based FLC, the feedback gain \( K \) is updated at every time step.

The response of LQR on the single-link flexible manipulator is shown in Figure 15 and Figure 16. Comparing with that of the tuned importance-based FLC using the two criteria in this chapter, LQR has a larger maximum tip deviation (0.76 meters vs. 0.74 meters), and a longer settling time (7.02 seconds vs. 4.46 seconds). The torque using LQR, as shown in Figure 17, is not as smooth as that using the importance-based FLC in Figure 14. Additionally, LQR is a full-state feedback controller. The error information of all the eighteen variables is needed to produce the feedback, which limits the possibilities.

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of implementing this controller. Using the gains corresponding to $e_\theta$, $e_{\dot{\theta}}$, $e_{\theta_p}$, and $e_{\dot{\theta}_p}$ only results in an unstable response.

Figure 15    Joint Angle Response of LQR on Single-link Flexible Manipulator
Figure 16  Tip Displacement Response of LQR on Single-link Flexible Manipulator

Figure 17  Torque of LQR on Single-link Flexible Manipulator
Distributed PD-like FLC

A common way to design a distributed FLC is to group a displacement variable and its time derivative variable together in one FLC, and sum up the outputs of all the FLCs as the final output. This type of arrangement was used by several researchers, such as Trabia (1998), Kubica and Wang (1999), and Trabia and Shi (2001). The controller structure is labeled as PD-like FLC in this study. Based on that rational, a PD-like FLC for the single-link flexible manipulator is distributed between two FLCs: the Joint Angle FLC and the Tip FLC, as shown in Figure 18. The Joint Angle FLC has two inputs: \( e_\theta \) and \( e_d\theta \), and one output: \( T_\theta \). Similarly, the Tip FLC has two inputs: \( e_{\text{tip}} \) and \( e_d\text{tip} \), and one output: \( T_{\text{tip}} \). The sum of the outputs of these two controllers is used to drive the joint motor.

![Figure 18 Distributed PD-like FLC for Single-link Flexible Manipulator](image)

Similar to the design procedure of the importance-based FLC, the fuzzy rules of the PD-like FLC are also based on the observation of the system behaviors. The goal of the Joint Angle FLC is to make the manipulator tracking a desired trajectory. The fuzzy rules...
of that FLC, as shown in Table 9, are selected to produce an output similar to that of a conventional PD controller, that is, to avoid the overshoot or lagging with respect to the desired joint trajectory. On the other hand, the fuzzy rules of the Tip FLC, Table 10, are based on observing the first mode behavior of the link, that is, to use the strain energy of the link to dampen the vibration of the arm. The FLC produces a torque when the tip is moving away from the desired target position.

Table 9  Fuzzy Rules of Joint Angle FLC for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>( e_{\theta} )</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>NB</td>
<td>Z</td>
<td>PB</td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>PB</td>
<td>PB</td>
</tr>
</tbody>
</table>

Table 10  Fuzzy Rules of Tip FLC for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>( e_{\text{tip}} )</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PB</td>
<td>PB</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>PB</td>
<td>Z</td>
<td>NB</td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>NB</td>
<td>NB</td>
</tr>
</tbody>
</table>

The parameter values used to initially describe the membership functions of the PD-like FLC are listed in Table 11. Note that the initial parameter values of the input variables are kept the same as the corresponding ones in Table 7. The parameter values of the two output variables, \( T_{\theta} \) and \( T_{\text{tip}} \) are kept the same as \( T_{\nu} \) in Table 7. No scale factor is applied to the parameter values of the output variables in Table 11 since the importance analysis is not considered in this structure. The initial response of the PD-like FLC excites higher vibration frequencies as shown in Figure 19 and Figure 20 comparing with
that of the importance-based FLC. It can be concluded that the importance information can be used as a guide in choosing the initial parameter values of the output variables.

Table 11 Initial Parameter Values of PD-like FLC for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$c_B$</th>
<th>$c_R$</th>
<th>$c_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_\theta$</td>
<td>0.8</td>
<td>0.24</td>
<td>0.12</td>
</tr>
<tr>
<td>$e_d\theta$</td>
<td>5.0</td>
<td>1.5</td>
<td>0.75</td>
</tr>
<tr>
<td>$T\theta$</td>
<td>10.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$e_{\text{tip}}$</td>
<td>0.3</td>
<td>0.09</td>
<td>0.045</td>
</tr>
<tr>
<td>$e_{d\text{tip}}$</td>
<td>5.0</td>
<td>1.5</td>
<td>0.75</td>
</tr>
<tr>
<td>$T_{\text{tip}}$</td>
<td>10.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 19 Initial Joint Angle Response of PD-like FLC on Single-link Flexible Manipulator
The initial response of the PD-like FLC is not acceptable. The parameters are tuned using the tuning method in Appendix I and the performance index in Equation (26). The performance of the tuning algorithm is shown in Figure 21. The initial performance index value is 14.18, which is about three times of that of the importance-based FLC. The tuning algorithm reaches a performance index value of 1.43 at the 2333\textsuperscript{th} iteration. The tuned parameter values of the PD-like FLC structure are listed in Table 12, which are in general different from those of the importance-based FLC in Table 8. Three-fourth parameters increase in value. The biggest increase occurs at $\sigma_z$ of $e_\beta$. The tuned response of the distributed PD-like FLC is shown in Figure 22 and Figure 23, which shows a remarkable improvement over the initial one. Comparing with the response of the tuned importance-based FLC using the two criteria in this chapter, the tuned PD-like FLC has a
larger maximum tip deviation (0.76 meters vs. 0.74 meters), and a longer settling time (5.3 seconds vs. 4.46 seconds). The tuned torque of the distributed PD-like FLC is shown in Figure 24. Note that the torque signs from the Joint Angle FLC and the Tip FLC are opposite, and the magnitude from the Joint Angle FLC is bigger than that from the Tip FLC. The total torque of the tuned PD-like FLC is of the same order as that of the importance-based FLC.

![Figure 21](image.png)

**Figure 21** Tuning Progression of PD-like FLC on Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$c_B$</th>
<th>$\sigma_B$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\theta}$</td>
<td>1.11</td>
<td>0.33</td>
<td>1.06</td>
</tr>
<tr>
<td>$e_{d\theta}$</td>
<td>3.10</td>
<td>2.10</td>
<td>2.48</td>
</tr>
<tr>
<td>$T_{\theta}$</td>
<td>9.38</td>
<td>2.92</td>
<td>1.56</td>
</tr>
<tr>
<td>$e_{\text{tip}}$</td>
<td>0.86</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>$e_{d\text{tip}}$</td>
<td>6.49</td>
<td>1.92</td>
<td>0.71</td>
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<tr>
<td>$T_{\text{tip}}$</td>
<td>11.18</td>
<td>3.81</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 12 Tuned Parameter Values of PD-like FLC for Single-link Flexible Manipulator

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Figure 22  Tuned Joint Angle Response of PD-like FLC on Single-link Flexible Manipulator

Figure 23  Tuned Tip Displacement Response of PD-like FLC on Single-link Flexible Manipulator

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Robustness Study under Different Payloads

The performance of the three controllers, the tuned distributed importance-based FLC, LQR, and the tuned distributed PD-like FLC, is acceptable as shown in the previous two sections. This section presents an evaluation of their robustness by decreasing / increasing the payload of the manipulator by 50%. The parameter values of the two tuned FLCs, Table 8 and Table 12, respectively, are used in this section. However, the gains of LQR in the previous section can not produce a stable result and they have to be recalculated at every time step during the robustness test.

The responses of the three controllers when the payload is decreased / increased by 50% are shown in Figure 25 through Figure 27, and Figure 28 through Figure 30 respectively. All controllers succeed in tracking the joint trajectory and stabilizing at the final target position within the testing period in the robustness tests.
The maximum tip deviation and the settling time of the three controllers are compared in Table 13 for the three payload cases: the payload decreased by 50%, the original payload and the payload increased by 50%. The tuned importance-based FLC has the best performance in all cases. Comparing with the response of LQR, the tuned PD-like FLC has shorter settling times in all cases, and a smaller maximum tip deviation in the decreasing payload case.

![Figure 25](image_url)

Figure 25 Joint Angle Response of Three Controllers on Single-link Flexible Manipulator after Decreasing Payload
Figure 26  Tip Displacement Response of Three Controllers on Single-link Flexible Manipulator after Decreasing Payload

Figure 27  Torque of Three Controllers on Single-link Flexible Manipulator after Decreasing Payload
Figure 28  Joint Angle Response of Three Controllers on Single-link Flexible Manipulator after Increasing Payload

Figure 29  Tip Displacement Response of Three Controllers on Single-link Flexible Manipulator after Increasing Payload
Figure 30 Torque of Three Controllers on Single-link Flexible Manipulator after Increasing Payload

Table 13 Criterion Values of Three Controllers for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Payload Value</th>
<th>Controller Structure</th>
<th>Maximum Tip Deviation (meter)</th>
<th>Settling Time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decreasing payload by</td>
<td>Importance-based FLC</td>
<td>0.61</td>
<td>4.09</td>
</tr>
<tr>
<td>50%</td>
<td>LQR</td>
<td>0.68</td>
<td>5.50</td>
</tr>
<tr>
<td></td>
<td>PD-like FLC</td>
<td>0.65</td>
<td>5.11</td>
</tr>
<tr>
<td>Original payload</td>
<td>Importance-based FLC</td>
<td>0.74</td>
<td>4.46</td>
</tr>
<tr>
<td></td>
<td>LQR</td>
<td>0.76</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td>PD-like FLC</td>
<td>0.76</td>
<td>5.30</td>
</tr>
<tr>
<td>Increasing the</td>
<td>Importance-based FLC</td>
<td>0.80</td>
<td>4.90</td>
</tr>
<tr>
<td>payload by 50%</td>
<td>LQR</td>
<td>0.80</td>
<td>16.42</td>
</tr>
<tr>
<td></td>
<td>PD-like FLC</td>
<td>0.82</td>
<td>5.02</td>
</tr>
</tbody>
</table>

Robustness Study on Spillover Effect

The performance of the distributed importance-based FLC is acceptable using four elements to describe the flexible link. Theoretically speaking, the flexible link should be
described by infinite number of elements. The truncation of elements in the model can cause spillover problem, as stated in Dadfarnia (2003). This section tests the robustness of the distributed importance-based FLC using larger numbers of elements to describe the flexible link in the model. Eight elements are initially used in this study. The difference in the tuned response of the distributed importance-based FLC is mainly from the computation errors (the difference is less than 1E-6 radians in the joint angle response, and 2E-6 meters in the tip deflection response). Sixteen elements are then used in the dynamic model. The difference of the tuned distributed importance-based FLC between using sixteen elements and using four elements is shown in Figure 31 and Figure 32. Note that the difference is less than 8E-4 radians in the joint angle response and less than 1.5E-4 meters in the tip deflection response. It can be concluded that the distributed importance-based FLC is very robust in the spillover test.
Figure 31 Difference of Joint Angle Response of Importance-based FLC of Using Sixteen Elements and Four Elements on Single-link Flexible Manipulator

Figure 32 Difference of Tip Deflection Response of Importance-based FLC of Using Sixteen Elements and Four Elements on Single-link Flexible Manipulator

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Comparison of Tuning Techniques

The Modified Nelder and Mead Simplex Algorithm is used in this study to tune the parameters of the membership functions in a FLC. To compare the performance of this tuning technique, Genetic Algorithms, Appendix III, is used to tune the parameters of the membership functions in the importance-based FLC structure. The initial parameter values in the first generation are randomly generated in the ranges of Table 14, which are twice of the initial parameter values using the Modified Nelder and Mead Simplex Algorithm.

The population number is chosen as six times of the parameter number (18×6=108). The algorithm terminates after 200 generations. The performance index in Equation (26) is used in the tuning process. The performance of the tuning algorithm is shown in Figure 33. The initial performance index value is 9.96 at this run of Genetic Algorithms. The tuning algorithm reaches a performance index value of 1.53 at the 200th generation. The tuned parameter values of the importance-based FLC using Genetic Algorithms are listed in Table 15, which are in general different from those using the Modified Nelder and Mead Simplex Algorithm in Table 8. The tuned response of the importance-base FLC using Genetic Algorithms is shown in Figure 34 and Figure 35. Comparing with the tuned response using the Modified Nelder and Mead Simplex Algorithm as, the tuned response using Genetic Algorithms has smaller maximum tip deviation (0.74 meters vs. 0.76 meters), but a longer settling time (6.14 seconds vs. 5.3 seconds). Note that the number of the function evaluations using Genetic Algorithms is much larger than that using the Modified Nelder and Mead Simplex Algorithm (14834 vs. 3008). As a result,
Genetic Algorithms may not be a good tuning technique for a complex dynamic system when solving the differential equations are relatively time consuming.

Table 14  Ranges of First Generation Using Genetic Algorithms for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$c_B$</th>
<th>$\sigma_B$</th>
<th>$\sigma_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_d\theta$</td>
<td>10.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$e_d\psi$</td>
<td>10.0</td>
<td>3.0</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_v$</td>
<td>20.0</td>
<td>6.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$e_\theta$</td>
<td>1.6</td>
<td>0.48</td>
<td>0.24</td>
</tr>
<tr>
<td>$e_{up}$</td>
<td>0.6</td>
<td>0.18</td>
<td>0.09</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.8</td>
<td>0.24</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Figure 33  Tuning Progression of Importance-based FLC using Genetic Algorithms on Single-link Flexible Manipulator
Table 15  Tuned Parameter Values of Importance-based FLC using Genetic Algorithms for Single-link Flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$c_B$</th>
<th>$\sigma_B$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{d\theta}$</td>
<td>0.996</td>
<td>0.187</td>
<td>0.21</td>
</tr>
<tr>
<td>$e_{dip}$</td>
<td>3.203</td>
<td>1.362</td>
<td>0.983</td>
</tr>
<tr>
<td>$T_v$</td>
<td>16.457</td>
<td>2.213</td>
<td>1.443</td>
</tr>
<tr>
<td>$e_{\theta}$</td>
<td>0.366</td>
<td>0.145</td>
<td>0.055</td>
</tr>
<tr>
<td>$e_{dip}$</td>
<td>3.874</td>
<td>1.972</td>
<td>0.449</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.877</td>
<td>1.500</td>
<td>3.312</td>
</tr>
</tbody>
</table>

Figure 34  Tuned Joint Angle Response of Importance-based FLC Using Genetic Algorithms on Single-link Flexible Manipulator
Chapter Summary

It is usually easy to design a FLC for a system when the designer is familiar with it. If the experience with a system is limited, developing a FLC for it becomes more difficult. This chapter proposes a new technique to design a distributed FLC based on studying the system responses of a single-link flexible manipulator under the random torque signals and analyzing the importance degrees of the selected four state variables with respect to the torque: the joint angle and its velocity, the tip displacement and its velocity. The importance analysis leads to the conclusion that the joint angular velocity and the tip velocity are significantly more important than the joint angle and the tip displacement for the torque of the single-link flexible manipulator. The controller inputs are distributed into two FLCs accordingly. The inputs to the Velocity FLC are the errors of the joint
angular velocity and the tip velocity while the inputs to the Displacement FLC are the errors of the joint angle and the tip displacement.

The fuzzy rules of the distributed importance-based FLC on the single-link flexible manipulator are written based on observing the system behaviors. Each fuzzy variable is described using three Gaussian membership functions. These membership functions are represented using three parameters based on symmetry. The results of the importance analysis are also helpful in selecting the initial parameter values of the FLC, and the initial response is stable. The parameters of the importance-based FLC are further tuned using the Modified Nelder and Mead Simplex Algorithm and a remarkable better performance is obtained.

To evaluate the effectiveness of the distributed importance-based FLC, it is compared with two other controllers: LQR and the distributed PD-like FLC. The gains of LQR are continually updated throughout the simulation while the parameters of the distributed PD-like FLC are tuned using the same tuning method as that of the distributed importance-based FLC. The robustness of each of the three controllers is tested by decreasing and increasing the payload by 50% respectively. Comparisons using the maximum tip deviation and the settling time show that the distributed importance-based FLC has the best overall tracking and stabilizing performances.

To test the spillover effect on the distributed importance-based FLC, sixteen elements are chosen to describe the flexible link in the dynamic model. The results show that the distributed importance-based FLC is very robust when the number of elements increases.

To compare the tuned response using the Modified Nelder and Mead Simplex Algorithm, Genetic Algorithms is chosen as an alternative tuning technique for the
distributed importance-based FLC structure. Results show that the tuned response using Genetic Algorithms is comparable to that using the Modified Nelder and Mead Simplex Algorithm, but the number of the function evaluation using the former method is much larger than the later.

This chapter emphasizes only the design and tuning of the importance-based FLC on a multi-input single-output dynamic system. The fuzzy rules can be written by the expert knowledge. The next two chapters deal with the design and tuning of the importance-based FLC for a multi-input multi-output coupling system where the coupling effects are strong and fuzzy rules can not be easily written.
CHAPTER 3

COMPARISON OF IMPORTANCE-BASED LINEAR CONTROLLER AND IMPORTANCE-BASED FLC FOR TWO-LINK RIGID-Flexible MANIPULATOR

This chapter extends the importance-based ideas of Chapter 2 to a multi-input multi-output dynamic system. A two-link rigid-flexible manipulator is chosen to be the controlled plant. The following is a brief summary of this chapter. The first section presents the dynamic model of the two-link rigid-flexible manipulator. The second section lists the results of the importance analysis for the two-link rigid-flexible manipulator. The third section proposes the structures of the distributed importance-based FLC and the corresponding importance-based linear controller. The fourth section proposes an algorithm to obtain the initial parameter values of the two importance-based controllers. The fifth section presents a procedure for tuning the parameters of the two importance-based controllers using the Modified Nelder and Mead Simplex Algorithm. The sixth section tests the robustness of the two importance-based controllers by varying the joint angle trajectories in the working space. The last section contains the summary of this chapter.

Dynamic Model

Figure 36 shows a schematic of the two-link rigid-flexible manipulator. The first link is rigid, and the second link is flexible. The two links move in a vertical plane where the
gravity field is active and pointing along the negative y-axis of the fixed frame. The Lagrangian approach and finite element approach are used to formulate the equations of motion. As stated in Madhavan and Singh (1991), the dynamic model of the two-link flexible manipulators is significantly more complex than that of a single-link flexible arm. Modeling steps of the two-link rigid-flexible manipulator are described briefly in this section.

The second flexible link is divided into \( n \) elements, as shown in Figure 4. The displacement of any point in element \( i \) is described using the nodal displacement and slope of nodes \( i \) and \( i+1 \) as shown in Equation (1) and Equation (2).

The position and velocity vectors of a point on the rigid link, the flexible link, and the payload, can be represented in the local frames of each link as shown below:

\[
P_i = \begin{bmatrix} x_i \\ 0 \end{bmatrix}, \quad \dot{P}_i = \begin{bmatrix} 0 \\ x_i \dot{\theta}_i \end{bmatrix}
\]  

(33)
\[ P_2 = \begin{bmatrix} x_2 \\ v \end{bmatrix} \]
\[ \dot{P}_2 = \begin{bmatrix} a_1 \dot{\theta}_1 s_2 \\ (x_2 + a_1 c_2) \dot{\theta}_1 + x_2 \dot{\theta}_2 + v \end{bmatrix} = [R][T]\dot{q}_l \]
\[ P_p = \begin{bmatrix} a_2 \\ r_{n+1} \end{bmatrix} \]
\[ \dot{P}_p = \begin{bmatrix} a_1 \dot{\theta}_1 s_2 \\ (a_2 + a_1 c_2) \dot{\theta}_1 + a_2 \dot{\theta}_2 + \dot{v}_{n+1} \end{bmatrix} = [R_p][T_p]\dot{q}_p \]

where \( c_i \) is \( \cos(\theta) \), \( s_i \) is \( \sin(\theta) \) and

\[
[R] = \begin{bmatrix}
1 & 0 & [0]_{6x4} \\
0 & 1 & N
\end{bmatrix} \\
[T] = \begin{bmatrix}
\begin{bmatrix} a_1 s_2 \\
x_2 + a_1 c_2 \\
[0]_{6x4}
\end{bmatrix} & 0 & [0]_{6x4}
\end{bmatrix} \\
[q]_i = \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
\]

\[
[R_p] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 1 & 1
\end{bmatrix} \\
[T_p] = \begin{bmatrix}
\begin{bmatrix} a_1 s_2 \\
0 \\
[0]_{4x4}
\end{bmatrix} & 0 & [0]_{4x4}
\end{bmatrix} \\
[q_p] = \begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
\]

The kinetic and potential energies of the rigid link can be expressed as:

\[
KE_1 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} \int_0^{a_1} \rho_1 \dot{\rho}_1 \dot{\theta}_1 dx = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} \rho_1 a_1^3 \dot{\theta}_1^2 \tag{36}
\]

\[
PE_1 = \int_0^{a_1} \rho_1 g x s_1 dx = \rho_1 g s_1 \frac{a_1^2}{2} = D_1 \tag{37}
\]

The first term in Equation (36) is the kinetic energy of the joint motor and the second term is the kinetic energy of the rigid link.

The modeling of the flexible link follows a procedure similar to that developed in Chapter 2. The kinetic and potential energies of the flexible link are,
\[ KE_2 = \frac{1}{2} J_2 \left( \dot{\theta}_1 + \dot{\theta}_2 \right)^2 + \sum_{i=1}^{n} \frac{1}{2} \rho_i \dot{q}_i \dot{q}_i \] (38)

\[ PE_2 = \sum_{i=1}^{n} \left[ \frac{EI_i}{2} \int_0^{l_i} \left( \frac{\partial^2 N}{\partial x_i^2} \right)^T \left( \frac{\partial^2 N}{\partial x_i^2} \right) d_d x_i \right] \]
\[ + \sum_{i=1}^{n} \rho_i g \int_0^{l_i} [a_i s_i + (X_i + x_i) s_{i2} + [N](d_i) c_{i2}] d_d x_i \] (39)

where \( X_i = L_i \times (i-1) \), \( c_{i2} \) is \( \cos(\theta_i+\theta_2) \), and \( s_{i2} \) is \( \sin(\theta_i+\theta_2) \).

The kinetic and potential energies of the payload are,

\[ KE_p = \frac{1}{2} J_p \dot{\phi}_{N+1}^2 + \frac{1}{2} m_p \dot{p}_p \dot{p}_p = \frac{1}{2} J_p \dot{\phi}_{N+1}^2 + \frac{1}{2} \dot{q}_p \dot{q}_p \] (40)

\[ PE_p = m_p g [a_1 s_1 + a_2 s_{12} + v_{n+1} c_{12}] = D_p \] (41)

Using Lagrangian dynamics, the equations of motion are,

\[ \frac{d}{dt} \left( \frac{\partial (KE)}{\partial q} \right) - \left( \frac{\partial (KE)}{\partial \dot{q}} \right) + \left( \frac{\partial (PE)}{\partial q} \right) = Q \] (42)

where,

\[ KE = KE_1 + KE_2 + KE_p \]
\[ PE = PE_1 + PE_2 + PE_p \]
\[ q = [\theta_1 \ \theta_2 \ \phi_2 \ \cdots \ \phi_{n+1}]^T \]
\[ Q = [T_1 \ \ T_2 \ \ 0 \ \ \cdots \ \ 0]^T \]

The equations of motion can be expressed in the following matrix form:
\[
\begin{align*}
\left[ M_{ex_i} + \sum_{i=1}^{n} M_{ex_i} + M_{ex_p} \right] \ddot{q} + \left[ \sum_{i=1}^{n} M_{ex_i} + M_{ex_p} \right] \dot{q} \\
- \left\{ \sum_{i=1}^{n} C_{ex_i} + C_{ex_p} \right\} + \left[ \sum_{i=1}^{n} K_{ex_i} \right] q_{\alpha} + \left[ D_{ex_i} + \sum_{i=1}^{n} D_{ex_i} + D_{ex_p} \right] = 0
\end{align*}
\]

(43)

The expressions of the coefficient matrices are given in Appendix II. The physical parameters of the two-link rigid-flexible manipulator under study are listed in Table 16. The first five natural frequencies of this system are listed in Table 17. Similar to the test in the third section of Chapter 2, eight elements were initially used to model the flexible link. It was later found that the step response using four elements overlaps to a large degree with that using eight elements. The differences of those two cases under the step inputs are less than 1E-4 radians for the two joint angle responses and less than 1E-5 meters for the tip deflection response. Therefore, four elements of equal length are used to describe the flexible link in this study. The degrees of freedom of the two-link rigid-flexible manipulator are twenty.

### Table 16 Physical Parameters of Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link length, ( a_1 )</td>
<td>Meter</td>
<td>0.43</td>
</tr>
<tr>
<td>Link length, ( a_2 )</td>
<td>Meter</td>
<td>0.43</td>
</tr>
<tr>
<td>Linear density, ( \rho_1 )</td>
<td>Kg/m</td>
<td>40.3</td>
</tr>
<tr>
<td>Linear density, ( \rho_2 )</td>
<td>Kg/m</td>
<td>11.12</td>
</tr>
<tr>
<td>Bending stiffness, ( EI )</td>
<td>Nm^2</td>
<td>20</td>
</tr>
<tr>
<td>Moment of inertia of the hub for the rigid link, ( J_1 )</td>
<td>Kg m^2</td>
<td>0.05</td>
</tr>
<tr>
<td>Moment of inertia of the hub for the flexible link, ( J_2 )</td>
<td>Kg m^2</td>
<td>0.05</td>
</tr>
<tr>
<td>Payload, ( m_p )</td>
<td>Kg</td>
<td>1.25</td>
</tr>
<tr>
<td>Payload moment of inertia, ( J_p )</td>
<td>Kg m^2</td>
<td>10^-5</td>
</tr>
</tbody>
</table>

### Table 17 First Five Natural Frequencies of Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Number of Natural Frequency</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Natural Frequency (Hz)</td>
<td>129</td>
<td>383</td>
<td>784</td>
<td>1448</td>
<td>2286</td>
</tr>
</tbody>
</table>
Importance Analysis Results

The goals of the controller for the two-link rigid-flexible manipulator are:

1. Make each joint angle tracking the desired trajectory.
2. Reduce the tip displacement.
3. Eliminate the potential higher-order vibrations at the final target position.

Based on those objectives, six state variables are selected in the design of the FLC: the joint angles, $\theta_1$ and $\theta_2$, the joint angular velocities, $\dot{\theta}_1$ and $\dot{\theta}_2$, the tip displacement, $v(a_2)$, and the velocity of tip point, $\dot{v}(a_2)$. The desired values of these variables are $\theta_{d1}$, $\theta_{d2}$, $\dot{\theta}_{d1}$, and $\dot{\theta}_{d2}$ for the joint variables, $v_{d}(a_2)$ for the static deflection of the flexible link due to the gravity effect, and zero for the tip velocity. The joint angles and the angular velocities can be measured using joint encoders and tachometers respectively. The tip displacement may be measured by attaching a laser source to the tip and a corresponding sensor at the manipulator base. Using series of strain gages along the flexible link can be also used. The tip velocity may be calculated by differentiating the tip displacement signal.

The first step of the importance analysis is to generate sufficient random motor torque signals in the working space. These random signals should however produce a reasonable range of the tip displacement (less than one third of the length of the flexible link). The random torque ranges of $\pm20$ Nm for the first joint and $\pm10$ Nm for the second joint are chosen. The equations of motion are solved using those random signals. The time duration of each simulation is one second with two hundred samples. Gravity effect is cancelled by a feedforward controller. For the system under study, the random torque signals result in $\theta_1$ motion between $[-0.88, 0.7]$ radians, $\theta_2$ motion between $[-2.12, 2.58]$
radians, \( v(a_2) \) motion between \([-0.17, 0.09]\) meters, \( \dot{\theta}_1 \) motion between \([-1.76, 1.72]\) radians/second, \( \dot{\theta}_2 \) motion between \([-13.04, 16.72]\) radians/second, and \( \dot{v}(a_2) \) motion between \([-7.17, 6.91]\) meters/second. It should be noted that the tip vibration exceeds 0.12 meters (one third length of the flexible link) in few instances only. The simulation is repeated by 120 times to obtain 23880 data points.

For the two-link rigid-flexible manipulator, the importance analysis is used to consider the coupling effect among the two joints and the payload. The motor torque \( T_1 \) and \( T_2 \) are the controller outputs. The multi-input multi-output system can be divided into two multi-input single-output systems. For the first joint, the joint variables of the rigid link, \( \theta_1 \) and \( \dot{\theta}_1 \) are used to construct the first FLC to generate the most torque to move the rigid link. The importance degrees of the remaining four variables, \( \theta_2, \dot{\theta}_2, v(a_2), \dot{v}(a_2) \), with respect to \( T_1 \) are analyzed. The two most important variables are used to construct the second FLC to consider the coupling effect of the flexible link and the payload on the first joint. Similarly, for the second joint, the joint variables of the flexible link, \( \theta_2 \) and \( \dot{\theta}_2 \) are used to construct the first FLC to generate the most torque to move the flexible link. The importance degrees of the remaining four variables \( \theta_1, \dot{\theta}_1, v(a_2), \dot{v}(a_2) \), with respect to \( T_2 \) are analyzed. The two most important variables are used to construct the second FLC to consider the coupling effect of the rigid link and the payload on the second joint.

The importance degrees of the four variables to each torque are therefore analyzed using the method in Chapter 2. The data sets are written in \( T_1 = f_1(\theta_2, \dot{\theta}_2, v(a_2), \dot{v}(a_2)) \) and \( T_2 = f_2(\theta_1, \dot{\theta}_1, v(a_2), \dot{v}(a_2)) \). The results of the importance analysis, as listed in
Table 18, show that $\dot{\varphi}(a_1)$ and $\dot{\theta}_2$ are the two most important variables for $T_1$ while the remaining two variables may be excluded from the controller design for $T_1$ due to their low importance degrees. Similarly, for $T_2$, $\dot{\varphi}(a_2)$ and $\dot{\theta}_1$ are the most important variables and the remaining two variables may be excluded from the controller design for $T_2$.

<table>
<thead>
<tr>
<th></th>
<th>$\text{IMP}(\theta_1)$</th>
<th>$\text{IMP}(\theta_2)$</th>
<th>$\text{IMP}(\dot{\varphi}(a_2))$</th>
<th>$\text{IMP}(\dot{\theta}_1)$</th>
<th>$\text{IMP}(\dot{\theta}_2)$</th>
<th>$\text{IMP}(\dot{\varphi}(a_2))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>------</td>
<td>21%</td>
<td>2%</td>
<td>------</td>
<td>39%</td>
<td>38%</td>
</tr>
<tr>
<td>$T_2$</td>
<td>24%</td>
<td>------</td>
<td>5%</td>
<td>33%</td>
<td>------</td>
<td>38%</td>
</tr>
</tbody>
</table>

Structures of Two Importance-based Controllers

Based on the importance analysis results in the previous section, a new controller is distributed in four controllers as shown in Figure 37. The first controller, $CT_{11}$, has two inputs: $e_{\theta 1}$ and $e_{d\theta 1}$, and one output: $T_{11}$. The second controller, $CT_{12}$, has two inputs: $e_{d\theta 2}$ and $e_{diap}$, and one output: $T_{12}$. The sum of $T_{11}$ and $T_{12}$ is used to drive the joint motor of the first link. The third controller, $CT_{21}$, has two inputs: $e_{\theta 2}$ and $e_{d\theta 2}$, and one output: $T_{21}$. The fourth controller, $CT_{22}$, has two inputs: $e_{d\theta 2}$ and $e_{diap}$, and one output: $T_{22}$. The sum of $T_{21}$ and $T_{22}$ is used to drive the joint motor of the second link. This arrangement maintains the coupling effects among the two joints and the payload in $CT_{11}$ for the first joint and $CT_{22}$ for the second joint.

The Gaussian membership function, Equation (24), is used to describe each membership function in the distributed importance-based FLC for the two-link rigid-flexible manipulator. As stated in Chapter 2, the number of membership functions will determine the number of fuzzy rules, which in turn determine the smoothness of the
control surface of a FLC. A large number of membership functions will make selecting the fuzzy rules difficult. For the two-link rigid-flexible manipulator, the following arrangement is proposed:

(1) Three membership functions are used to describe each input variable: negative big ($NB$), zero ($Z$), and positive big ($PB$).

(2) Five membership functions are used to describe each output variable: negative big ($NB$), negative small ($NS$), zero ($Z$), positive small ($PS$), and positive big ($PB$).

Figure 37 Importance-based Controllers for Two-link Rigid-flexible Manipulator (Gravity Feedforward Is Not Shown)
All membership functions are symmetric around $Z$ since gravity effect is canceled. Therefore, three parameters, $c_B$, $\sigma_B$, and $\sigma_Z$, are used to express each input variable and five parameters, $c_B$, $c_S$, $\sigma_B$, $\sigma_S$, and $\sigma_Z$, are used to express each output variable. The total number of the parameters is forty-four for the importance-based FLC structure.

For many systems, fuzzy rules can be constructed based on the observation of the system behaviors. This approach works successfully on the single-link flexible manipulator as shown in Chapter 2. For the two-link rigid-flexible manipulator, however, it may be hard to do so. To avoid the need to operate the system extensively in order to construct the proper fuzzy rules, a novel approach is proposed. A second structure, an importance-based linear controller, is introduced. This controller consists of the following four linear controllers, which have the same structure as that of the distributed FLC:

$$LT_{11} = W_{111} \cdot e_{\theta_1} + W_{112} \cdot e_{d\theta_1}$$  \hspace{1cm} (44)

$$LT_{12} = W_{121} \cdot e_{\theta_2} + W_{122} \cdot e_{d\theta_2}$$  \hspace{1cm} (45)

$$LT_{21} = W_{211} \cdot e_{\theta_2} + W_{212} \cdot e_{d\theta_2}$$  \hspace{1cm} (46)

$$LT_{22} = W_{221} \cdot e_{\theta_2} + W_{222} \cdot e_{d\theta_2}$$  \hspace{1cm} (47)

where $W_{jk}$, $k=1,2$, and $LT_{ij}$ are the linear gains and the torque output from $ij$th linear controller respectively.

Experience shows that it is relatively easy to observe the patterns of how the two gains in one linear controller affect the overall system performance. In the simulation study, the initial joint angle is zero for each joint. The desired final joint angle is $\pi$ radians. The desired joint angle motion is a bang-bang acceleration profile. The sampling frequency is two hundred samples per second. The desired active motion is one second, and the total simulation time is five seconds.
For each torque of the two-link rigid-flexible manipulator under study, choose large gain values for the first linear controller and small gain values for the second linear controller, for example, a set of gains using 10 for the gains of the first controller and 0.1 for the gains of the second controller, as shown in Table 19, can produce a stable response, as shown in Figure 38 through Figure 40. Observation of varying the two gains in $LT_{11}$ controller shows that a significant large gain change in $W_{111}$ (for example, varying it from 5 to 200) does not affect the overall system performance as much as a small gain change in $W_{112}$ (for example, varying it from 5 to 50). Same pattern is observed in $LT_{12}$ and $LT_{22}$ controllers. The fuzzy rules of the corresponding three FLCs, $CT_{11}$, $CT_{12}$ and $CT_{22}$ are constructed accordingly, as shown in Table 20 to reflect these observations. On the other hand, the gain change in either $W_{211}$ or $W_{212}$ (for example, varying either of them from 5 to 50) in $LT_{21}$ controller has an equal effect on the overall system performance. So the fuzzy rules of the corresponding FLC, $CT_{21}$ can be constructed as shown in Table 21.

<table>
<thead>
<tr>
<th>Gain</th>
<th>$W_{111}$</th>
<th>$W_{112}$</th>
<th>$W_{121}$</th>
<th>$W_{122}$</th>
<th>$W_{211}$</th>
<th>$W_{212}$</th>
<th>$W_{221}$</th>
<th>$W_{222}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>10</td>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
<td>10</td>
<td>10</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 19 Initial Gain Values of Importance-based Linear Controller for Two-link Rigid-flexible Manipulator
Figure 38 Initial Joint 1 Angle Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator

Figure 39 Initial Joint 2 Angle Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator
Figure 40  Initial Tip Displacement Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator

Table 20  Fuzzy Rules of $CT_{11}$, $CT_{12}$, and $CT_{22}$ FLCs for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>1st input</th>
<th>2nd input</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td></td>
<td>NB</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>PB</td>
<td></td>
<td>PS</td>
<td>PS</td>
<td>PB</td>
</tr>
</tbody>
</table>

Table 21  Fuzzy Rules of $CT_{21}$ FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>$e_{q2}$</th>
<th>$e_{dq2}$</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td></td>
<td>NB</td>
<td>NS</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td></td>
<td>NS</td>
<td>Z</td>
<td>PS</td>
</tr>
<tr>
<td>PB</td>
<td></td>
<td>Z</td>
<td>PS</td>
<td>PB</td>
</tr>
</tbody>
</table>

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The centroid defuzzification method is also used in this chapter. Details of the proposed procedure for selecting and tuning the parameter values of the two importance-based controllers for an optimal performance are discussed in the next two sections.

Obtaining Initial Parameter Values

As stated in Chapter 2, there is no standard method in determining the initial parameter values of the membership functions for a FLC in literature. On the contrary, an arbitrary set of linear gains in Table 19 can produce a stable response as shown in Figure 38 through Figure 40. Another set of linear gains, as listed in Table 22, is obtained by tuning the values manually in the process of constructing fuzzy rules of Table 20 and Table 21. The response of the importance-based linear controller using the gains in Table 22 yields a remarkable improvement as shown in Figure 41 through Figure 43. Comparing with the initial response in Figure 38 through Figure 40, the manually-tuned response of the importance-based linear controller has a smaller maximum tracking error in \( \theta_1 \) (0.4 radians vs. 1.4 radians), much shorter settling times (2.81 seconds vs. 5 seconds in \( \theta_1 \) and 2.09 seconds vs. 5 seconds in \( \theta_2 \)). The torque of the initial and the manually-tuned importance-based linear controller is shown in Figure 44.

<table>
<thead>
<tr>
<th>Table 22</th>
<th>Manually-Tuned Gain Values of Importance-based Linear Controller for Two-link Rigid-flexible Manipulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>( W_{111} )</td>
</tr>
<tr>
<td>Value</td>
<td>50</td>
</tr>
</tbody>
</table>

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Figure 41  Manually-tuned Joint 1 Angle Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator

Figure 42  Manually-tuned Joint 2 Angle Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator
Figure 43  Manually-tuned Tip Displacement Response of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator

Figure 44  Torque of Importance-based Linear Controller on Two-link Rigid-flexible Manipulator
The initial parameter values of the distributed importance-based FLC can be selected based on the following arrangements:

(1) The values of $c_B$ of the input variables, as shown in Table 23, are chosen based on what seems to be a sensible range of the variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$e_{01}$</th>
<th>$e_{d01}$</th>
<th>$e_{02}$</th>
<th>$e_{d02}$</th>
<th>$e_{tip}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_B$</td>
<td>0.4</td>
<td>2.5</td>
<td>1.2</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

(2) The value of $c_B$ of $T_{ij}$ is chosen to be 1.5 times of the output values $LT_{ij}$, as shown in Table 24. The output values $LT_{ij}$ can be calculated using the linear gains of Table 22 and the relations from Equation (44) to Equation (47).

<table>
<thead>
<tr>
<th>Variable</th>
<th>$T_{11}$</th>
<th>$T_{12}$</th>
<th>$T_{21}$</th>
<th>$T_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of $c_B$</td>
<td>226</td>
<td>150</td>
<td>90</td>
<td>12</td>
</tr>
</tbody>
</table>

(3) The remaining parameters, $c_s$, $\sigma_B$, $\sigma_s$ and $\sigma_z$ are chosen according to the following relations:

Input Variables: $\sigma_z = \sigma_B = 0.4 c_B$  \hspace{1cm} (48)

Output Variables: $c_s = 0.5 c_B$

$\sigma_B = \sigma_s = \sigma_z = 0.25 c_B$  \hspace{1cm} (49)
In order to compare the performances of the two importance-based controllers, the parameters of the importance-based FLC are tuned by matching the surfaces of the corresponding linear controllers.

The values of \( c_B \) in Table 23 and Table 24 are fixed in the matching procedure to ensure that the FLC surfaces remain within the reasonable limits. The remaining parameters, \( c_s, \sigma_B, \sigma_s \) and \( \sigma_z \) are tuned to match the control surfaces of the corresponding linear controllers. The performance index of each surface matching is,

\[
PIM_y = \left( \sum_{k=1}^{p} (LT_y(k) - T_y(k))^2 \right)
\]

where \( p \) is the total number of points in the matching procedure. Divide each input variable into \( m \) divisions, then \( p = m^2 \). The Modified Nelder and Mead Simplex Algorithm, Appendix I, is used as the tuning algorithm to minimize the value of Equation (50).

The parameter values in the importance-based FLC after surface-matching are shown in Table 25. The response of the importance-based FLC using those surface-matched values is shown in Figure 45 through Figure 47. Note that the two importance-based controllers produce very similar responses. Comparing with the response of the importance-based linear controller, the importance-based FLC produces a larger steady state error in \( \theta_i \) as shown in Figure 45 and some vibrations in \( v(a_2) \) as shown in Figure 47. The torque of the surface-matched importance-based FLC is shown in Figure 48. Comparing with the torque signal of the manually-tuned importance-based linear controller, the torque of the surface-matched FLC has the vibration after two seconds.
Table 25  Surface-matched Parameter Values of Importance-based FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>σZ</th>
<th>σB</th>
<th>cB</th>
<th>σS</th>
<th>cS</th>
</tr>
</thead>
<tbody>
<tr>
<td>e\₁₁</td>
<td>0.32</td>
<td>0.31</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_d\₁₁</td>
<td>1.46</td>
<td>1.82</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T\₁₁</td>
<td>12.18</td>
<td>23.42</td>
<td>226</td>
<td>7.42</td>
<td>208.66</td>
</tr>
<tr>
<td>e\₁₂</td>
<td>5.48</td>
<td>3.68</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_d\₁₂</td>
<td>8.20</td>
<td>10.60</td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T\₁₂</td>
<td>13.67</td>
<td>29.68</td>
<td>150</td>
<td>5.78</td>
<td>116.41</td>
</tr>
<tr>
<td>e\₂₂</td>
<td>0.46</td>
<td>0.57</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_d\₂₂</td>
<td>2.81</td>
<td>1.89</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T\₂₁</td>
<td>13.34</td>
<td>15.26</td>
<td>90.00</td>
<td>12.78</td>
<td>38.54</td>
</tr>
<tr>
<td>e\₂₁</td>
<td>3.89</td>
<td>2.80</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e_d\₂₁</td>
<td>6.44</td>
<td>10.42</td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T\₂₂</td>
<td>2.05</td>
<td>4.33</td>
<td>12.00</td>
<td>1.45</td>
<td>9.01</td>
</tr>
</tbody>
</table>

Figure 45  Surface-matched Joint 1 Angle Response of Importance-based FLC on Two-link Rigid-flexible Manipulator

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Figure 46  Surface-matched Joint 2 Angle Response of Importance-based FLC on Two-link Rigid-flexible Manipulator

Figure 47  Surface-matched Tip Deflection Response of Importance-based FLC on Two-link Rigid-flexible Manipulator
Tuning Parameters

The responses of the two importance-based controllers can be further improved by tuning the parameter/gain values using the Modified Nelder and Mead Simplex Algorithm, Appendix I. The proposed performance index for the two-link rigid-flexible manipulator is,

\[ PI = 1 = 1 \]

\[ r \]

\[ ôs \]

\[ ^ d & 2 i ^ dtip \]

\[ (51) \]

where \( n_t \) is the total number of samples, \( Q_s \) is the weighing factor that is set to one during the active motion period and to five afterwards to eliminate the potential higher-order vibrations. The first term in the above equation represents a measure of the displacement errors while the second term represents a measure of the velocity errors. \( G_s \) are the
weighting factors of the displacement and velocity errors that make each term of the same order, which are set to one for the three displacement errors, one-tenth for the joint angular velocity errors, and one-hundredth for the tip velocity error. The termination criterion of the tuning algorithm is 0.1, which is much higher than that for the single-link flexible manipulator due to the extensive simulation times required for the two-link rigid-flexible manipulator. The performances of the tuning algorithm on the parameters/gains of the two importance-based controllers are shown in Figure 49. The significant large number of parameters (forty-four) needed for the tuning of the importance-based FLC compared to the small number (eight) for the importance-based linear controller results in a larger number of the function evaluations (1821 vs. 212). The initial value of the performance index of the importance-based FLC is also larger than that of importance-based linear controller (472.89 vs. 423.77) due to the larger steady state error in \( \theta_i \) and the vibrations in \( v(a_2) \) in the initial response, as shown in Figure 45 through Figure 47. The final performance index value of the importance-based FLC is however smaller than that of the importance-based linear controller (192.7 vs. 268.5).

Table 26 lists the tuned gain values for the importance-based linear controller. The tuned values vary significantly comparing with the manually-tuned values in Table 22. Table 27 lists the tuned parameter values of the importance-based FLC. Twenty-seven tuned values vary slightly comparing with the surface-matched values in Table 25. The biggest variations appear in the parameters of \( e_{\theta_i} \).

<table>
<thead>
<tr>
<th>Gain</th>
<th>( W_{111} )</th>
<th>( W_{122} )</th>
<th>( W_{131} )</th>
<th>( W_{211} )</th>
<th>( W_{222} )</th>
<th>( W_{231} )</th>
<th>( W_{311} )</th>
<th>( W_{322} )</th>
<th>( W_{331} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>37.97</td>
<td>43.76</td>
<td>13.82</td>
<td>2.86</td>
<td>50.60</td>
<td>24.20</td>
<td>6.70</td>
<td>3.32</td>
<td></td>
</tr>
</tbody>
</table>

Table 26 Simplex-tuned Gain Values of Importance-based Linear Controller for Two-link Rigid-flexible Manipulator
Figure 49 Tuning Progression of Importance-based Controllers on Two-link Rigid-flexible Manipulator

Table 27 Simplex-tuned Parameter Values of Importance-based FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_Z$</th>
<th>$\sigma_B$</th>
<th>$e_B$</th>
<th>$\sigma_S$</th>
<th>$c_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\theta 1}$</td>
<td>0.46</td>
<td>0.78</td>
<td>0.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta 1}$</td>
<td>1.37</td>
<td>1.53</td>
<td>2.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{11}$</td>
<td>11.93</td>
<td>23.85</td>
<td>225.94</td>
<td>7.50</td>
<td>208.85</td>
</tr>
<tr>
<td>$e_{\theta 2}$</td>
<td>5.52</td>
<td>3.16</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta 2}$</td>
<td>9.42</td>
<td>10.30</td>
<td>15.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{12}$</td>
<td>13.95</td>
<td>29.84</td>
<td>150.18</td>
<td>5.53</td>
<td>116.21</td>
</tr>
<tr>
<td>$e_{\theta 2}$</td>
<td>0.004</td>
<td>0.49</td>
<td>1.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta 2}$</td>
<td>1.00</td>
<td>1.73</td>
<td>4.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>13.36</td>
<td>15.30</td>
<td>90.00</td>
<td>12.45</td>
<td>38.41</td>
</tr>
<tr>
<td>$e_{\theta 1}$</td>
<td>4.17</td>
<td>3.21</td>
<td>2.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta 1}$</td>
<td>6.46</td>
<td>10.71</td>
<td>15.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>2.18</td>
<td>4.25</td>
<td>11.94</td>
<td>1.46</td>
<td>8.85</td>
</tr>
</tbody>
</table>

The responses of the two tuned importance-based controllers are shown in Figure 50 through Figure 52. Comparing with those before tuning, both controllers achieve better
responses in $\theta_1$ and $\theta_2$, and a similar response in $v(a_2)$. Comparing with the response of the tuned importance-based linear controller, the tuned importance-based FLC has smaller maximum joint errors (0.32 radians vs. 0.40 radians in $\theta_1$, 0.41 radians vs. 0.64 radians in $\theta_2$), and shorter settling times (1.1 seconds vs. 2.47 seconds in $\theta_1$, 1.29 seconds vs. 2.32 seconds in $\theta_2$). Note that the vibration of the tip point in the surface-matched response of the importance-based FLC is eliminated after tuning. The torque of the two tuned importance-based controllers is shown in Figure 53.

![Figure 50](image_url)

**Figure 50** Simplex-tuned Joint 1 Angle Response of Importance-based Controllers on Two-link Rigid-flexible Manipulator
Figure 51  Simplex-tuned Joint 2 Angle Response of Importance-based Controllers on Two-link Rigid-flexible Manipulator

Figure 52  Simplex-tuned Tip Displacement Response of Importance-based Controllers on Two-link Rigid-flexible Manipulator
Robustness Study under Different Joint Angle Trajectories

The tuning results in the previous section are acceptable for the case when the two joints of the two-link rigid-flexible manipulator move from 0 to $\pi$ radians in one second. The robustness of the two tuned controllers may be evaluated by varying the joint angle trajectories in the working space. Table 28 shows the twenty-five cases of different initial and final joint angles. The active moving time is kept at one second, which results in varying the angular velocities for the two joints.

Similar to the procedure for the single-link flexible manipulator, the following two criteria are used to compare the tracking and stabilizing performances of a controller under the robustness test:

1. The tip deviation of the manipulator during the tracking period. It is defined as:
\[
E_{tip} = \sqrt{(\bar{x}_p - \bar{x})^2 + (\bar{y}_p - \bar{y})^2}
\]

where,
\[
\begin{align*}
\bar{x}_p &= a_1 c_{id} + a_2 c_{12d} - v_d(a_2) s_{12d} \\
\bar{y}_p &= a_1 s_{id} + a_2 s_{12d} + v_d(a_2) c_{12d} \\
\bar{x} &= a_1 c_1 + a_2 c_{12} - v(a_2) s_{12} \\
\bar{y} &= a_1 s_1 + a_2 s_{12} + v(a_2) c_{12}
\end{align*}
\] (53) (54)

(2) The settling time of the tip point. It is defined as the time after which the absolute differences of \(E_{tip}\) at a consecutive time duration (one second is chosen in this chapter) are consistently smaller than a specific value (\(1E^{-4}\) is chosen in this chapter).

<table>
<thead>
<tr>
<th>(\theta_1)</th>
<th>(0 \rightarrow \pi/2)</th>
<th>(0 \rightarrow \pi)</th>
<th>(\pi/2 \rightarrow \pi)</th>
<th>(\pi/2 \rightarrow 3\pi/2)</th>
<th>(\pi \rightarrow 3\pi/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\theta_2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0 \rightarrow \pi/2)</td>
<td>Case1</td>
<td>Case2</td>
<td>Case3</td>
<td>Case4</td>
<td>Case5</td>
</tr>
<tr>
<td>(0 \rightarrow \pi)</td>
<td>Case6</td>
<td>Case7</td>
<td>Case8</td>
<td>Case9</td>
<td>Case10</td>
</tr>
<tr>
<td>(\pi/2 \rightarrow \pi)</td>
<td>Case11</td>
<td>Case12</td>
<td>Case13</td>
<td>Case14</td>
<td>Case15</td>
</tr>
<tr>
<td>(\pi/2 \rightarrow 3\pi/2)</td>
<td>Case16</td>
<td>Case17</td>
<td>Case18</td>
<td>Case19</td>
<td>Case20</td>
</tr>
<tr>
<td>(\pi \rightarrow 3\pi/2)</td>
<td>Case21</td>
<td>Case22</td>
<td>Case23</td>
<td>Case24</td>
<td>Case25</td>
</tr>
</tbody>
</table>

The maximum tip deviations and the settling time of the two-link rigid-flexible manipulator using the two tuned importance-based controllers under the twenty-five cases are shown in Figure 54 and Figure 55 respectively. The maximum tip deviations in nineteen cases using the importance-based FLC are smaller than those using the importance-based linear controller, as shown in Figure 54. Similarly, the settling times in eighteen cases using the importance-based FLC are shorter than those using the
importance-based linear controller. It is also note that the variations of the settling times using the importance-based FLC are relatively smaller than those using the importance-based linear controller. So it can be concluded that the tracking and stabilizing performances using the importance-based FLC are better than those using the importance-based linear controller under various joint angle trajectories in the working space.

Figure 54 Maximum Tip Deviation of Importance-based controllers on Two-link Rigid-flexible Manipulator under Different Joint Angle Trajectories
Chapter Summary

This chapter presents two importance-based structures for a two-link rigid-flexible manipulator operating in a gravity field. The structures of both controllers are based on evaluating the importance degrees of the selected state variables with respect to the torque. The importance analysis algorithm identifies the most important variables beside the joint angle and angular velocity that affect the torque of each joint. The resulting importance-based structure has four controllers, with two for each joint. The first joint controller uses the errors of the joint angle and the angular velocity on that joint as inputs while the second one uses the errors of the two most important variables with respect to that joint as inputs.
The importance-based linear controller is used to deduce the fuzzy rules. Fuzzy rules of each FLC are constructed to mimic the overall performance of the corresponding linear controller. Using the Modified Nelder and Mead Simplex Algorithm, the parameters of each FLC are tuned to match the corresponding surface of the manually-tuned linear controller.

The parameters of both controllers are then further tuned to improve their performances. Results show that both tuned controllers have better responses over their initial ones. The tuned importance-based FLC has better tracking and stabilizing performances than the tuned importance-based linear controller. The advantage of the importance-based FLC is clearer when the joint angle trajectories are varied, which shows that the importance-based FLC has stronger robustness than that of the importance-based linear controller.

This chapter discusses the detailed tuning procedure of the importance-based FLC. It is interesting to note that the importance-based linear controller can be used not only to deduce the fuzzy rules, but also to select the initial parameter values for the output variables of FLCs. The responses using the initial parameter values in Table 23 and Table 24, as well as using the relations in Equation (48) and Equation (49) will be discussed in Chapter 4. The performance of the importance-based FLC will be also compared with that of the PD-like FLC for the two-link rigid-flexible manipulator.
CHAPTER 4

COMPARISON OF DISTRIBUTED PD-LIKE AND IMPORTANCE-BASED FLCS
FOR TWO-LINK RIGID-FLEXIBLE MANIPULATOR

Like the comparison made for the single-link flexible manipulator, the performance of the distributed importance-based FLC is further compared with the distributed PD-like FLC for the two-link rigid-flexible manipulator in this chapter. The first section proposes the distributed PD-like FLC structure that controls each joint separately. The second section proposes an algorithm to obtain the initial parameter values of the two FLCs. The third section presents a procedure of tuning the parameters of the two FLCs using the Modified Nelder and Mead Simplex Algorithm. The fourth section tests the robustness of the two FLCs by varying the joint angle trajectories in the working space and moving the tip along with a circle at a constant speed. The last section contains the summary of this chapter.

Structure of Distributed PD-like FLC

Similar to the design of the distributed PD-like FLC for the single-link flexible manipulator discussed in Chapter 2, this section presents a design of a distributed PD-like FLC for the two-link rigid-flexible manipulator. The coupling effect between the two joints is not explicitly considered in this controller. The two-link rigid-flexible manipulator is divided into two sub-systems: a rigid link and a flexible link.
For the rigid link subsystem, there are two state variables: $\theta_i$ and $\dot{\theta}_i$. One FLC, $PDFLC_i$, is constructed using the errors of those two variables: $e_{\theta i}$ and $e_{d\theta i}$, as shown in Figure 56. The output of this controller is the torque needed to correct these errors: $T_i$, which is used to drive the joint motor of the first link.

For the flexible link subsystem, there are four state variables selected for the controller design: $\theta_2$, $\dot{\theta}_2$, $v(a_2)$, and $\dot{v}(a_2)$. Similar to the design procedure in Chapter 2, the control action for the flexible link subsystem is accordingly distributed between two FLCs: $PDFLC_{21}$ and $PDFLC_{22}$, as shown in Figure 56. $PDFLC_{21}$ has two inputs: $e_{\theta 2}$ and $e_{d\theta 2}$, and one output: $T_{21}$. Similarly, $PDFLC_{22}$ has two inputs: $e_{ip}$ and $e_{dip}$, and one output: $T_{22}$. The sum of the outputs of these two controllers is used to drive the joint motor of the second link.

Gaussian curve membership function, Equation (24), is chosen to represent each fuzzy variable. Like the arrangement in the importance-based FLC in Chapter 3, three membership functions, $NB, Z, PB$, are used to describe each input variable and five membership functions, $NB, NS, Z, PS, PB$, are used to describe each output variable.

Similar to the procedure of constructing the fuzzy rules for the importance-based FLC, a conventional PD controller that has the same structure as that of the PD-like FLC is constructed to help construct the fuzzy rules for the PD-like FLC. The three PD controllers are defined as:

1. Rigid joint PD controller, $PDT_i = W'_{11}e_{\theta 1} + W'_{12}e_{d\theta 1}$
2. Flexible joint PD controller, $PDT_{21} = W'_{211}e_{\theta 2} + W'_{212}e_{d\theta 2}$
3. Tip PD controller, $PDT_{22} = W'_{221}e_{ip} + W'_{222}e_{dip}$
where $W'_{i(j)1}$ and $W'_{i(j)2}$ are the proportional and derivative gains respectively for the $i(j)$th PD controller.

![Distributed PD-like FLC for Two-link Rigid-flexible Manipulator](image)

Figure 56 Distributed PD-like FLC for Two-link Rigid-flexible Manipulator (Gravity Feedforward Is Not Shown)

Similar to the experience on the importance-based linear controller, it is relatively easy to observe the patterns of how the two gains of a PD controller affect the overall system performance. The observation on varying the two gains in $PDT_i$ controller shows that a significant large gain change in $W'_{i1}$ does not affect the overall system performance as much as a small gain change in $W'_{i2}$. Same pattern is observed in $PDT_{22}$ controller. The fuzzy rules of the corresponding two FLCs, $PDFLCT_1$ and $PDFLCT_{22}$ are constructed accordingly, as shown in Table 29. On the other hand, the gain change in either $W'_{11}$ or $W'_{12}$ in the $PDT_{21}$ controller has an equal effect on the overall system performance.
performance. So the fuzzy rules of the corresponding FLC, $PDFLCT_{21}$, can be written as shown in Table 30.

Table 29 Fuzzy Rules of $PDFLC_{1}$, $PDFLC_{22}$ for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>1st Input</th>
<th>2nd Input</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>PS</td>
<td>PS</td>
<td>PB</td>
<td></td>
</tr>
</tbody>
</table>

Table 30 Fuzzy Rules of $PDFLCT_{21}$ for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>$e_{\phi2}$</th>
<th>$e_{\phi2}$</th>
<th>NB</th>
<th>Z</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB</td>
<td>NS</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>NS</td>
<td>Z</td>
<td>PS</td>
<td></td>
</tr>
<tr>
<td>PB</td>
<td>Z</td>
<td>PS</td>
<td>PB</td>
<td></td>
</tr>
</tbody>
</table>

The same defuzzification process as shown in Chapter 2 is used in this chapter. Details of the proposed procedure for selecting and tuning the parameter values of the distributed PD-like FLC structure for an optimal performance are shown in the next two sections.

Obtaining Initial Parameter Values

This section presents an algorithm to select initial parameter values for the two FLCs. The physical parameters of the two-link rigid-flexible manipulator in Chapter 3 are used in this chapter.
In the distributed PD-like FLC for the two-link rigid-flexible manipulator, three parameters, \( c_B, \sigma_B, \) and \( \sigma_Z \), are used to express each input variable and five parameters, \( c_B, c_S, \sigma_B, \sigma_S, \) and \( \sigma_Z \), are used to express each output variable. The total number of parameters is thirty-three.

As stated in the previous two chapters, there is no standard method in determining the initial parameter values of the membership functions for a FLC in literature. Expert knowledge is used in Chapter 2 in selecting the initial parameter values for the two FLCs for the single-link flexible manipulator. Importance-based linear controller is used in chapter 3 to choose the initial parameter values of the importance-based FLC for the two-link rigid-flexible manipulator, and the parameters are later tuned by matching the control surface of the importance-based linear controller. The goal of this section is to compare the response of the distributed PD-like FLC with that of the distributed importance-based FLC on the two-link rigid-flexible manipulator. The initial parameter values in the importance-based FLC before the surface matching in Chapter 3 are shown in Table 31. The corresponding variables in the distributed PD-like FLC are kept the same as shown in Table 32. The initial responses of the two FLCs are shown in Figure 57 to Figure 59. Note that the two FLCs produce very similar responses. The tracking and stabilizing performances of the two FLCs are compared in Table 33. Observation shows that the importance-based FLC has smaller maximum tracking errors and shorter settling times in \( \theta_2 \) and \( v(a_2) \), and a less steady state error in \( v(a_2) \), while the PD-like FLC has a smaller maximum tracking error and a shorter settling time in \( \theta_1 \). Both FLCs have the same steady state errors in \( \theta_1 \) and \( \theta_2 \). The torque of the two FLCs is shown in Figure 61.
Table 31  Initial Parameter Values of Importance-based FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_Z$</th>
<th>$\sigma_B$</th>
<th>$c_B$</th>
<th>$\sigma_S$</th>
<th>$c_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\theta_l}$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_l}$</td>
<td>1.00</td>
<td>1.00</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{11}$</td>
<td>56.50</td>
<td>56.50</td>
<td>226.00</td>
<td>56.50</td>
<td>113.00</td>
</tr>
<tr>
<td>$e_{\theta_g}$</td>
<td>2.00</td>
<td>2.00</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_g}$</td>
<td>6.00</td>
<td>6.00</td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{12}$</td>
<td>37.50</td>
<td>37.50</td>
<td>150.00</td>
<td>37.50</td>
<td>75.00</td>
</tr>
<tr>
<td>$e_{\theta_g}$</td>
<td>0.48</td>
<td>0.48</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_g}$</td>
<td>2.00</td>
<td>2.00</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>22.50</td>
<td>22.50</td>
<td>90.00</td>
<td>22.50</td>
<td>45.00</td>
</tr>
<tr>
<td>$e_{\theta_l}$</td>
<td>1.00</td>
<td>1.00</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_l}$</td>
<td>6.00</td>
<td>6.00</td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>3.00</td>
<td>3.00</td>
<td>12.00</td>
<td>3.00</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Table 32  Initial Parameter Values of PD-like FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma_Z$</th>
<th>$\sigma_B$</th>
<th>$c_B$</th>
<th>$\sigma_S$</th>
<th>$c_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\theta_l}$</td>
<td>0.16</td>
<td>0.16</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_l}$</td>
<td>1.00</td>
<td>1.00</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{1}$</td>
<td>56.50</td>
<td>56.50</td>
<td>226.00</td>
<td>56.50</td>
<td>113.00</td>
</tr>
<tr>
<td>$e_{\theta_g}$</td>
<td>0.48</td>
<td>0.48</td>
<td>1.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_g}$</td>
<td>2.00</td>
<td>2.00</td>
<td>5.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>22.50</td>
<td>22.50</td>
<td>90.00</td>
<td>22.50</td>
<td>45.00</td>
</tr>
<tr>
<td>$e_{\theta_l}$</td>
<td>1.00</td>
<td>1.00</td>
<td>2.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_l}$</td>
<td>6.00</td>
<td>6.00</td>
<td>15.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>3.00</td>
<td>3.00</td>
<td>12.00</td>
<td>3.00</td>
<td>6.00</td>
</tr>
</tbody>
</table>
Figure 57  Initial Joint 1 Angle Response of Two FLCs on Two-link Rigid-flexible Manipulator

Figure 58  Initial Joint 2 Angle Response of Two FLCs on Two-link Rigid-flexible Manipulator
Figure 59  Initial Tip Deflection Response of Two FLCs on Two-link Rigid-flexible Manipulator

Figure 60  Initial Torque of Two FLCs on Two-link Rigid-flexible Manipulator
Table 33 Comparisons of Initial Responses of Two FLCs

<table>
<thead>
<tr>
<th>Performance</th>
<th>Variable</th>
<th>Importance-based FLC</th>
<th>PD-like FLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Tracking</td>
<td>$\theta_1$</td>
<td>0.64</td>
<td>0.61</td>
</tr>
<tr>
<td>Error</td>
<td>$\theta_2$</td>
<td>1.04</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>$v(a_2)$</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>Settling Time</td>
<td>$\theta_1$</td>
<td>1.81</td>
<td>1.16</td>
</tr>
<tr>
<td>State Error</td>
<td>$\theta_2$</td>
<td>2.27</td>
<td>2.28</td>
</tr>
<tr>
<td></td>
<td>$v(a_2)$</td>
<td>1.08</td>
<td>1.12</td>
</tr>
<tr>
<td>Steady State Error</td>
<td>$\theta_1$</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>$v(a_2)$</td>
<td>-0.000319</td>
<td>-0.000362</td>
</tr>
</tbody>
</table>

Tuning Parameters

The parameters of the two FLCs are further tuned to get better performances. The performance index expression, as shown in Equation (51) is used in the tuning process. The performances of the tuning algorithm of the two FLCs are shown in Figure 61.

A larger number of parameters (forty-four) needed for the tuning of the importance-based FLC compared to that (thirty-three) for the PD-like FLC results in a larger number of the function evaluations (1849 vs. 1100). The initial value of the performance index of the PD-like FLC is slightly higher than that of the importance-based FLC (497.61 vs. 464.02). The final tuned performance index value of the PD-like FLC is also higher than that of the importance-based FLC (235.46 vs. 210.29).

Table 34 lists the tuned parameter values of the distributed PD-like FLC. It is noted that the tuned values of the torque parameters are about the same as the initial ones. The biggest variations appear in the parameters of $e_{\theta_1}$ and $e_{\theta_2}$, especially in $\sigma_2$. Table 35 lists the tuned parameter values of the distributed importance-based FLC. Similar to the pattern in Table 34, the tuned values of the torque parameters are about the same as the initial ones. The biggest variations appear in the parameters of $e_{\theta_2}$, especially in $\sigma_2$.
The tuned response of the two FLCs is shown in Figure 62 through Figure 64. Note that the two tuned FLCs produce very similar responses, which are general better than the initial ones. The tracking and stabilizing performances of the two tuned FLCs are
compared in Table 36. Observation shows that the tuned importance-based FLC has smaller maximum tracking errors in $\theta_1$ and $\theta_2$, shorter setting times in $\theta_1$ and $v(a_2)$ and a smaller steady state error in $v(a_2)$, where the tuned PD-like FLC has a shorter settling time in $\theta_1$, and smaller steady state errors in $\theta_1$ and $\theta_2$. Both controllers has the same maximum tracking error in $v(a_2)$.

Table 35 Tuned Parameter Values of Importance-based FLC for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Variable in Importance-based FLC</th>
<th>$\sigma_Z$</th>
<th>$\sigma_B$</th>
<th>$c_B$</th>
<th>$\sigma_S$</th>
<th>$c_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\theta_1}$</td>
<td>0.13</td>
<td>0.08</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_1}$</td>
<td>0.63</td>
<td>1.01</td>
<td>2.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{11}$</td>
<td>57.02</td>
<td>56.44</td>
<td>225.86</td>
<td>56.35</td>
<td>112.84</td>
</tr>
<tr>
<td>$e_{\theta_2}$</td>
<td>1.99</td>
<td>1.94</td>
<td>4.49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_2}$</td>
<td>5.96</td>
<td>6.69</td>
<td>15.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{12}$</td>
<td>37.44</td>
<td>37.36</td>
<td>150.55</td>
<td>36.94</td>
<td>74.42</td>
</tr>
<tr>
<td>$e_{\theta_1}$</td>
<td>0.04</td>
<td>0.34</td>
<td>0.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_2}$</td>
<td>1.04</td>
<td>2.11</td>
<td>5.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{21}$</td>
<td>22.58</td>
<td>22.50</td>
<td>91.83</td>
<td>22.49</td>
<td>44.66</td>
</tr>
<tr>
<td>$e_{d\theta_1}$</td>
<td>0.82</td>
<td>0.83</td>
<td>2.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e_{d\theta_2}$</td>
<td>6.20</td>
<td>5.98</td>
<td>14.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{22}$</td>
<td>3.22</td>
<td>2.84</td>
<td>12.10</td>
<td>2.32</td>
<td>6.04</td>
</tr>
</tbody>
</table>
Figure 62 Tuned Joint 1 Angle Response of Two FLCs on Two-link Rigid-flexible Manipulator

Figure 63 Tuned Joint 2 Angle Response of Two FLCs on Two-link Rigid-flexible Manipulator
Figure 64  Tuned Tip Displacement Response of Two FLCs on Two-link Rigid-flexible Manipulator

Figure 65  Tuned Torque of Two FLCs on Two-link Rigid-flexible Manipulator
Table 36  Comparisons of Tuned Responses of Two FLCs

<table>
<thead>
<tr>
<th>Performance</th>
<th>Variable</th>
<th>Importance-based FLC</th>
<th>PD-like FLC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Tracking Error</td>
<td>$\theta_1$ (Radian)</td>
<td>0.44</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$ (Radian)</td>
<td>-0.36</td>
<td>-0.52</td>
</tr>
<tr>
<td></td>
<td>$\nu(a_2)$ (Meter)</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Settling Time (Second)</td>
<td>$\theta_1$</td>
<td>1.31</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$</td>
<td>1.58</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>$\nu(a_2)$</td>
<td>1.05</td>
<td>1.10</td>
</tr>
<tr>
<td>Steady State Error</td>
<td>$\theta_1$ (Radian)</td>
<td>-0.025</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>$\theta_2$ (Radian)</td>
<td>0.0017</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td>$\nu(a_2)$ (Meter)</td>
<td>-0.000059</td>
<td>0.00017</td>
</tr>
</tbody>
</table>

Robustness Study under Different Joint Angle Trajectories

The performance of the two tuned FLCs on the two-link rigid-flexible manipulator is acceptable as shown in the previous section. This section presents an evaluation of their robustness by varying the joint angle trajectories as shown in Table 37.

The maximum tip deviations and the settling time of the two-link rigid-flexible manipulator using the two tuned distributed FLCs under the twenty-five cases in Table 37 are shown in Figure 66 and Figure 67 respectively. The maximum tip deviations in nineteen cases using the importance-based FLC are smaller than those using the PD-like FLC, as shown in Figure 66. Similarly, The settling times in twenty-one cases using the importance-based FLC are shorter than those using the PD-like FLC, as shown in Figure 67.
Table 37 Joint Angle Trajectories for Two-link Rigid-flexible Manipulator

<table>
<thead>
<tr>
<th>Joint Angle Trajectory</th>
<th>Case1</th>
<th>Case2</th>
<th>Case3</th>
<th>Case4</th>
<th>Case5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1 \rightarrow \theta_2$</td>
<td>$0 \rightarrow \pi/2$</td>
<td>$0 \rightarrow \pi$</td>
<td>$\pi/2 \rightarrow \pi$</td>
<td>$\pi/2 \rightarrow 3\pi/2$</td>
<td>$\pi \rightarrow 3\pi/2$</td>
</tr>
<tr>
<td>$\theta_1 \rightarrow \theta_2$</td>
<td>$0 \rightarrow \pi/2$</td>
<td>Case6</td>
<td>Case7</td>
<td>Case8</td>
<td>Case9</td>
</tr>
<tr>
<td>$\theta_2 \rightarrow \pi/2$</td>
<td>$\pi/2 \rightarrow \pi$</td>
<td>Case11</td>
<td>Case12</td>
<td>Case13</td>
<td>Case14</td>
</tr>
<tr>
<td>$\pi/2 \rightarrow 3\pi/2$</td>
<td>Case16</td>
<td>Case17</td>
<td>Case18</td>
<td>Case19</td>
<td>Case20</td>
</tr>
<tr>
<td>$\pi \rightarrow 3\pi/2$</td>
<td>Case21</td>
<td>Case22</td>
<td>Case23</td>
<td>Case24</td>
<td>Case25</td>
</tr>
</tbody>
</table>

Figure 66 Maximum Tip Deviation of Two FLCs on Two-link Rigid-flexible Manipulator under Different Joint Angle Trajectories
Robustness Study under Constant Circular Movement at Tip Point

The objectives of the controllers in the above discussions are mainly concentrated on tracking the desired joint angle trajectories, reducing the tip displacement and eliminating the potential higher-order vibrations at the final target position. This section will test the robustness of the controllers under a different objective: Move the tip along with a circle at a constant speed.

Robot positioning requires that the joint angle positions be calculated as a function of tip position. This mapping is called inverse kinematics of a robot. The inverse kinematics problem is very nonlinear and cannot be solved in closed form, as stated in Rouvinen and Handroos (1997). The detailed discussion of the inverse kinematics of the flexible
manipulator is not the scope of this study. A simple case is included in the robustness study of the two tuned FLCs on the two-link rigid-flexible manipulator.

Assume that the tip of the two-link rigid-flexible manipulator moves along with a circle at a constant speed. Define the center of the circle as \((x_c, y_c) = (0.58, 0)\) meters, and the radius as \(R = 0.1\) meters. Define the angle of the tip position with the horizontal line at the center of the circle as \(\alpha\), and its moving speed as \(\dot{\alpha} = \pi/3\). The tip position is a function of the joint angles and the static tip deflection:

\[
\bar{x}_p = x_c + R\cos\alpha = a_1 c_{1d} + a_2 c_{12d} - v_d(a_2) s_{12d} \tag{55}
\]

\[
\bar{y}_p = x_c + R\sin\alpha = a_1 s_{1d} + a_2 s_{12d} + v_d(a_2) c_{12d} \tag{56}
\]

where \(c_\alpha\) is \(\cos(\alpha)\), and \(s_\alpha\) is \(\sin(\alpha)\).

Note that the static deflection \(v_d(a_2)\) is a linear function of \(c_{12d}\), which can be written as

\[
v_d(a_2) = \beta c_{12d} \tag{57}
\]

where \(\beta\) equals to \(-0.00401\) in the system under study. Substitute Equation (57) to Equation (55) and (56), and move the terms of \(\theta_{1d} + \theta_{2d}\) to the left side of the equations:

\[
x_c + R c_\alpha - a_2 c_{12d} + \beta c_{12d} s_{12d} = a_1 c_{1d} \tag{58}
\]

\[
x_c + R s_\alpha - a_2 s_{12d} - \beta c_{12d}^2 = a_1 s_{1d} \tag{59}
\]

Square both sides of Equation (58) and (59) and add them together. Define the nonlinear function of \(\theta_{1d} + \theta_{2d}\) as:

\[
f = \left( x_c + R c_\alpha - a_2 c_{12d} + \beta c_{12d} s_{12d} \right)^2 + \left( x_c + R s_\alpha - a_2 s_{12d} - \beta c_{12d}^2 \right)^2 - a_1^2 = 0 \tag{60}
\]
At an instant time \( t \), the angle \( \alpha = \dot{\alpha} \cdot t = \pi/3 \cdot t \). The corresponding joint angle \( \theta_{id} + \theta_{2d} \) at this instant can be calculated by solving Equation (60) using LSQNONLIN function in Matlab, where LSQNONLIN solves non-linear least squares problems in the form:

\[
\min \left[ \left( f(\theta_{id} + \theta_{2d}) \right)^2 \right] 
\]

Equation (61)

\( \theta_{id} \) can be solved using Equation (58) after \( \theta_{id} + \theta_{2d} \) is solved by LSQNONLIN:

\[
\theta_{id} = a \cos \left[ (x_c + R \cos \alpha - a_z c_{12d} + \beta c_{12d} s_{12d}) / a_i \right] 
\]

Equation (62)

In the robustness study, the simulation time is 17 seconds. The initial \( \alpha \) is set to 0. The tip starts moving from a static state and reaches the constant speed at 5 seconds. The tip of the manipulator moves two circles in the next 12 seconds. The moving path of the rigid link and the flexible link in the working space is shown in Figure 68.

The joint angular velocities \( \dot{\theta}_{id} \) and \( \dot{\theta}_{2d} \) can be solved by differencing Equation (58) and (59). The expressions are

\[
\begin{bmatrix}
\dot{\theta}_{id} \\
\dot{\theta}_{2d}
\end{bmatrix} =
\begin{bmatrix}
\dot{a}_1 s_{1d} + a_z s_{12d} + \beta (c_{12d} - s_{12d}) & a_z s_{12d} + \beta (c_{12d} - s_{12d}) \\
\dot{a}_1 c_{1d} + a_z c_{12d} - 2 \beta c_{12d} s_{12d} & a_z c_{12d} - 2 \beta c_{12d} s_{12d}
\end{bmatrix}^{-1}
\begin{bmatrix}
R \dot{s}_\alpha \\
R \dot{c}_\alpha
\end{bmatrix}
\]

Equation (63)

The performance of the tuned importance-based FLC and PD-like FLC is tested under the joint angles and joint angular velocities. The tip trajectory is shown in Figure 69, the joint angles of the rigid link and the flexible link are shown in Figure 70 and Figure 71 respectively, and the tip deflection is shown in Figure 72. From these figures, it is shown that both tuned FLCs have stable responses when the tip moving along with a circle at a constant speed. The total tracking errors is defined as

\[
TE = \sum_{t=0}^{T} \sqrt{\left( \bar{x}_p(t) - \bar{x}(t) \right)^2 + \left( \bar{y}_p(t) - \bar{y}(t) \right)^2}
\]

Equation (64)
where \( t_0 \) is the time when the tip reaches the constant speed, and \( t_f \) is the total simulation time. The total tracking errors using the tuned PD-like FLC are smaller than those of using the tuned importance-based FLC (10.41 vs. 17.34). The torque of the two FLCs is shown in Figure 73. Note that the torque of the PD-like FLC is less smooth than that of the importance-based FLC.

Figure 68 Moving Path of Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed
Figure 69  Tip Position of Two FLCs on Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed

Figure 70  Joint Angle 1 Response of Two FLCs on Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed
Figure 71  Joint Angle 2 Response of Two FLCs on Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed

Figure 72  Tip Deflection Response of Two FLCs on Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed
Chapter Summary

This chapter compares the performances of the two FLC structures for a two-link rigid-flexible manipulator operating in a gravity field. The first controller, the distributed PD-like FLC, has three FLCs: the Joint Angle FLCs for the two joints and the Tip FLC. The inputs to the first two controllers are the errors of the joint angle and its angular velocity on that joint while the inputs to the Tip FLC are the errors of the displacement and its velocity of the tip point on the flexible link. The other structure, the distributed importance-based FLC, as stated in Chapter 3, is based on evaluating the importance degrees of the selected state variables with respect to the torque. The algorithm finds the important variables beside the joint angle and the angular velocity that affect the torque.

Figure 73 Torque of Two FLCs on Two-link Rigid-flexible Manipulator when Tip Moving along with a Circle at Constant Speed
of each joint. As a result of the importance analysis, each joint is controlled by two FLCs as shown in Figure 37.

Similar to the procedure of constructing the fuzzy rules of the importance-based FLC, a conventional PD controller which has the same input-output structure as that of the PD-like FLC is constructed.

The parameters of the two FLCs are tuned using the Modified Nelder and Mead Simplex Algorithm starting from similar initial responses. Each of the tuned FLCs has better performance in terms of the tracking errors, the settling times, and the steady state errors in the joint angles and the tip deflection comparing with the initial responses. The performances of the two tuned FLCs are also compared. Results show that while the two tuned controllers exhibit comparable performance, the importance-based FLC has smaller maximum tracking errors throughout the motion. It however experiences a slightly larger steady state error at the final target position.

The robustness of the two tuned FLCs is tested by varying the joint angle trajectories in the working space and moving the tip of the two-link rigid-flexible manipulator along with a circle at a constant speed. Simulation results show that both tuned FLCs are able to generate stable responses under the two tests. In general, the distributed importance-based FLC has better tracking and stabilizing performances under different joint angle trajectories, while the distributed PD-like FLC has less total tracking error when the tip moves along with a circle at a constant speed.
CONCLUSIONS AND FUTURE WORK

This study introduces the importance analysis algorithm in the structure design of FLCs for two dynamic systems: a single-link flexible manipulator and a two-link rigid-flexible manipulator. To address the dimensionality difficulties in the design of single FLC, the number of variables per FLC is limited to two. The structure of the importance-based FLC is distributed based on the importance degrees of the selected state variables with respect to each torque.

For the single-link flexible manipulator, which is a multi-input single-output dynamic system, the importance analysis algorithm identifies the importance degrees of the selected four state variables, the joint angle and its angular velocity, the tip displacement and its velocity, with respect to the torque. Based on the importance analysis results, the controller is divided into two FLCs. The errors of the two state variables with higher importance degrees, $e_{dip}$ and $e_{d\theta}$, are grouped as the inputs for the first FLC, while the errors of the remaining two state variables with lower importance degrees, $e_{tip}$ and $e_{\theta}$, are grouped as the inputs for the second FLC. The sum of the outputs of the two FLCs is used to drive the joint motor. Fuzzy rules of the two controllers are selected based on observing the system behaviors. The importance information is further used as a guide in selecting the initial parameter values of the importance-based structure, that is, the initial torque range of the second FLC can be significantly less than that of the first FLC.
reflect the reduction of the importance degrees of the two inputs in the second FLC. The response of the distributed importance-based FLC is stable with a long settling time and large joint angle errors. These parameters are therefore tuned using the Modified Nelder and Mead Simplex Algorithm to achieve better performance. Simulation study shows that the output of the first FLC generates most of the torque during the tracking period, while the output of the second FLC produces some minor modifications near the final target position. This observation confirms the basic idea behind the distributed importance-based FLC.

This study evaluates the effectiveness of the distributed importance-based FLC by comparing it with a linear quadratic regulator (LQR). LQR method can be defined as finding the appropriate state feedback controller to minimize a cost function. The gains of the LQR are continually updated throughout the simulation to properly compare it with the tuned importance-based FLC. Simulation results show that the tuned distributed importance-based FLC has better tracking and stabilizing performances than those of the LQR.

The performance of the distributed importance-based FLC is also compared with the distributed PD-like FLC. There are also two FLCs in the distributed PD-like FLC. The errors of the joint angle and its angular velocity, \( e_\beta \) and \( e_{d\beta} \), are grouped as the inputs for the first FLC, while the errors of the tip deflection and its velocity, \( e_{\text{tip}} \) and \( e_{d\text{tip}} \), are grouped as the inputs for the second FLC. The sum of the outputs of the two FLCs is used to drive the joint motor. Fuzzy rules of the two controllers are selected based on observing the system behaviors. The initial parameter values of each input variable in the distributed PD-like FLC are selected the same as those of the comparable variable in the
distributed importance-based FLC, and the initial parameter values of the two output variables in the distributed PD-like FLC are selected the same as those of the first output variable in the distributed importance-based FLC (a scale factor is introduced to those of the second output variable in the importance-based FLC).

The robustness of the three controllers is tested by decreasing and increasing the payload by 50% respectively. Results show that the distributed importance-based FLC has the best overall performance in the robustness test.

Large numbers of elements are used to describe the flexible link in the dynamic model of the single-link flexible manipulator to test the robustness of the distributed importance-based FLC. Results show that the distributed importance-based FLC is very robust in this spillover test.

The importance analysis algorithm is also applied to design a distributed FLC for the two-link rigid-flexible manipulator, which is a coupled multi-input multi-output dynamic system. The multi-input multi-output system can be divided into two multi-input single-output systems. The importance analysis is used to consider the coupling effect of the two joints. The joint angle and its angular velocity on one joint are used to construct the first FLC to generate the most torque on that joint. The importance analysis algorithm identifies the importance degrees of the selected four state variables on one joint: the joint angle and its angular velocity on the other joint, the tip displacement and its velocity. The two most important variables are used to construct the second FLC to consider the coupling effect among the two joints and the payload. Based on the results of the importance analysis, the resulting importance-based structure has four controllers, with two FLCs for each joint. For the first joint, the errors of its joint angle and angular
velocity, \( e_{\theta_1} \) and \( e_{\omega_1} \) are the two inputs of the first FLC, while the errors of the two state variables with the higher importance degrees with respect to this joint, \( e_{d\theta_1} \) and \( e_{d\omega_1} \), are the two inputs of the second FLC. The remaining two state variables with lower importance degrees, \( e_{\theta_1} \) and \( e_{\omega_1} \), are not included in the controller design for this joint. Similarly, for the second joint, the errors of its joint angle and angular velocity, \( e_{\theta_2} \) and \( e_{\omega_2} \), are the two inputs of the first FLC, while the errors of the two state variables with higher importance degrees with respect to this joint, \( e_{d\theta_2} \) and \( e_{d\omega_2} \), are the two inputs of the second FLC. The remaining two state variables with lower importance degrees, \( e_{\theta_2} \) and \( e_{\omega_2} \), are not included in the controller design for this joint.

To avoid the need of operating the system extensively, the fuzzy rules of each FLC are constructed to mimic the overall performance of an equivalent linear controller. Linear gains of these controllers are selected to produce a stable response. This approach makes the procedure of constructing the fuzzy rules of the distributed importance-based FLC easy.

The distributed importance-based FLC is first compared with the importance-based linear controller in Chapter 3. In order to properly compare the response and robustness of the two importance-based controllers, the parameters of each FLC are tuned to match the control surface of the corresponding linear controller using the Modified Nelder and Mead Simplex Algorithm. The membership functions of the importance-based FLC and the gains of the importance-based linear controllers are subsequently tuned separately to improve performances. The robustness of the two importance-based controllers is tested by varying the joint angle trajectories in the working space. Results show that the distributed importance-based FLC has better overall performances in the robustness test.
The distributed importance-based FLC is further compared with the distributed PD-like FLC in Chapter 4. The distributed PD-like FLC of the two-link rigid-flexible manipulator has three FLCs. The inputs to the first FLC are the errors of the joint angle and its angular velocity on the first joint, $e_{\theta_1}$ and $e_{\dot{\theta}_1}$. The output of this FLC is used to drive the motor on the first joint. The inputs to the second FLC are the errors of the joint angle and its angular velocity on the second joint, $e_{\theta_2}$ and $e_{\dot{\theta}_2}$. The inputs to the third FLC are the errors of the tip displacement and its velocity on the flexible link, $e_{tip}$ and $e_{\dot{tip}}$. The sum of the outputs from the second and third FLCs is used to drive the second joint. Similar to the procedure of constructing the fuzzy rules of the distributed importance-based FLC, the fuzzy rules of the distributed PD-like FLC are also constructed to mimic the overall performance of the equivalent linear controller. The parameters of the two FLCs are tuned starting from the similar initial performances. The robustness of the two FLCs is further tested by varying the joint angle trajectories in the working space and moving the tip along with a circle at a constant speed. In general, the distributed importance-based FLC has better tracking and stabilizing performances in the first robustness test and the distributed PD-like FLC has less total tracking error in the second robustness test.

The importance analysis algorithm proposed in this study can be further applied to other dynamic systems in the future study. The idea of deducting the fuzzy rules and selecting the initial parameter values of the FLC using the corresponding linear controller may also be feasible in many applications.

The Modified Nelder and Mead Simplex Algorithm, like other local search techniques, suffer a slow converging rate, and the difficulty to reach a global minimum,
especially when the number of tuning parameters getting larger. Finding a better tuning method for flexible manipulators is one of the directions in the future study.

Another interesting topic is to derive FLC whose fuzzy rules and membership functions are a function of the manipulator parameters, so it does not need to tune the parameters.
APPENDIX I

MODIFIED NELDER AND MEAD SIMPLEX ALGORITHM

Simplex Algorithm is a local search technique that uses the evaluation of the current data set to determine the promising search direction. Simplex Algorithm starts by generating a simplex with n+1 vertices. The algorithm evaluates the function values at these points, and replaces the point of the highest function value with its reflection along a vector passing through the center of the remaining points. The following is a brief description of the Modified Nelder and Mead Simplex Algorithm for an n-dimensional problem.

(1) Start at an initial point $I$.

(2) Generate $n$ equally-spaced points at a distance $\alpha$ from point $I$ according to the equation

$$X_i = X_0 + \delta_i U_i + \sum_{j=A,j \neq i}^{n} \delta_j U_j$$

(I-1)

where

$$\delta_1 = \frac{\sqrt{n+1} + n - 1}{n\sqrt{2}} \alpha$$

(I-2)

$$\delta_2 = \frac{\sqrt{n+1} - 1}{n\sqrt{2}} \alpha$$

(3) Identify the point with the highest function value, $X_h$ and the point with the lowest function value, $X_l$. 

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(4) Calculate the coordinates of point $X_c$, which is the centroid of all the simplex points except $X_h$.

(5) Reflect the highest point into a new point, $X_{new}$, in the direction of the opposite of $X_h$, such that,

$$X_{new} = X_h + 2 \times (X_c - X_h)$$  \hfill (I-3)

(6) Compare the function value at the new point as follows:

- If $f(X_{new}) < f(X_i)$, expand: $X_{new1} = X_c + 3 \times (X_c - X_h)$
- If $f(X_i) \leq f(X_{new}) < f(X_h)$, replace $X_h$ by $X_{new}$
- If $f(X_{new}) \geq f(X_h)$, contract: $X_{new2} = X_h + 0.5 \times (X_c - X_h)$
- If $f(X_{new1}) < f(X_i)$, Replace $X_h$ with $f(X_{new1})$, otherwise replace $X_h$ with $f(X_{new})$. If $f(X_{new2}) < f(X_h)$, Replace $X_h$ with $f(X_{new2})$, otherwise, generate a smaller simplex around $X_i$ as follows:

$$X_i = \frac{X_{i-1} + X_i}{2} \quad i = 1, \cdots, n+1$$  \hfill (I-4)

$$\sum_{i=0}^{n} \left( f(x_i) - \left( \frac{\sum_{i=0}^{n} f(x_i)/(n+1)}{n+1} \right) \right)^2 < \varepsilon$$

(7) If $n+1 < \varepsilon$, terminate the search. Else, go to (3).

The distance factor, $\alpha$ is set to 0.5 and the termination value, $\varepsilon$ is different to each tuning case. Note that in the original Nelder and Mead Simplex Algorithm, Nelder and Mead (1965), $f(X_{new1})$ or $f(X_{new2})$ is compared with $f(X_{new})$. If $f(X_{new1})$ or $f(X_{new2})$ is lower than $f(X_{new})$, replace $X_h$ with the point with the lowest function value. If otherwise, generate a smaller simplex around $X_i$. Also, the expand expression in the original Nelder and Mead
Simplex Algorithm is $X_{\text{new}} = X_k + 3(X_c - X_p)$. The tuning of the parameters of FLCs in this study using the Modified Nelder and Mead Simplex Algorithm gives better results comparing with the original Nelder and Mead Simplex Algorithm.
APPENDIX II

COEFFICIENT MATRICES IN EQUATIONS OF MOTION OF TWO-LINK RIGID-FLEXIBLE MANIPULATOR

The coefficient matrices in Equation (38) to Equation (41) can be expanded in terms of the global coordinate vector \( q = [\theta_1, \theta_2, v_1, \phi_1, \ldots, v_{n+1}, \phi_{n+1}]^T \)

\[
\begin{align*}
\text{Mex}_i &= \\
&= \begin{bmatrix}
M_{\theta i} & [0]_{2x2(n-1)} & M_{\phi i} & [0]_{2x2(n-i)} \\
[0]_{2(i-1)x2} & [0]_{2(i-1)x2(i-1)} & [0]_{2(i-1)x4} & [0]_{2(i-1)x2(2(n-i))} \\
M_{\theta i}^T & [0]_{4x2(i-1)} & M_{\phi f} & [0]_{4x2(n-i)} \\
[0]_{2(n-i)x2} & [0]_{2(n-i)x2(i-1)} & [0]_{2(n-i)x4} & [0]_{2(n-i)x2(2(n-i))}
\end{bmatrix}
\end{align*}
\]

(II-1)

\[
\begin{align*}
\text{Kex}_i &= \\
&= \begin{bmatrix}
[0]_{2x2} & [0]_{2x2(n-1)} & [0]_{2x4} & [0]_{2x2(2(n-i))} \\
[0]_{2(i-1)x2} & [0]_{2(i-1)x2(i-1)} & [0]_{2(i-1)x4} & [0]_{2(i-1)x2(2(n-i))} \\
[0]_{4x2} & [0]_{4x2(i-1)} & K_i & [0]_{4x2(n-i)} \\
[0]_{2(n-i)x2} & [0]_{2(n-i)x2(i-1)} & [0]_{2(n-i)x4} & [0]_{2(n-i)x2(2(n-i))}
\end{bmatrix}
\end{align*}
\]

(II-2)

\[
\begin{align*}
\text{Dex}_i &= \\
&= \begin{bmatrix}
[0]_{2x4} \\
[0]_{2(n-1)x4} \\
D_i \\
[0]_{2(n-i)x4}
\end{bmatrix}
\end{align*}
\]

(II-3)

\[
\begin{align*}
\text{Mex}_p &= \\
&= \begin{bmatrix}
M_{\theta p} & [0]_{2x2n} & M_{\phi p} & [0]_{2x4} \\
[0]_{2x2n} & [0]_{2x2n} & [0]_{2x2} & [0]_{2x2} \\
M_{\theta p}^T & [0]_{4x2n} & M_{\phi p} & 0 \\
0 & [0]_{2x2n} & 0 & 0
\end{bmatrix}
\end{align*}
\]

(II-4)
\[
D_{ex_p} = \begin{bmatrix}
\{0\}_{2 \times 1} \\
\{0\}_{2 \times 1}
\end{bmatrix}
\]

\[
C_{ex_i} = \frac{\partial}{\partial q} \left( \frac{1}{2} \dot{q}^T M_{ex_i} \dot{q} \right)
\]  

\[
C_{ex_p} = \frac{\partial}{\partial q} \left( \frac{1}{2} \dot{q}^T M_{ex_p} \dot{q} \right)
\]

The first node has zero boundary condition (cantilever end). Therefore, it has to be eliminated from the system equations by deleting the 3\textsuperscript{rd} and 4\textsuperscript{th} rows and columns in the Equations (II-1) to (II-7).

The reduced variable vector is \( q = [\theta_1 \; \theta_2 \; \nu_2 \; \phi_2 \; \cdots \; \nu_{n+1} \; \phi_{n+1}]^T \).

The expressions of matrices \( M_{ex_i} \) and \( D_{ex_i} \) in Equation (43) with respect to the reduced variable vector are

\[
M_{ex_i} = \begin{bmatrix}
J_1 + J_2 + \rho_1 \frac{a_i^3}{3} & 0 & \{0\}_{1 \times (2n-1)} & 0 \\
0 & J_2 & \{0\}_{1 \times (2n-1)} & 0 \\
\{0\}_{(2n-1) \times 1} & \{0\}_{(2n-1) \times 1} & \{0\}_{1 \times (2n-1)} & \{0\}_{1 \times (2n-1)} \\
0 & 0 & \{0\}_{1 \times (2n-1)} & J_p
\end{bmatrix}
\]

\[
D_{ex_i} = \begin{bmatrix}
\rho_1 g s_i \frac{a_i^2}{2} \\
\{0\}_{1 \times (2n+1)}
\end{bmatrix}
\]  

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Appendix III

GENETIC ALGORITHMS

The initial population of $m$ chromosomes is randomly generated. The algorithm selects fifty percent of the population with the best fitness value as parents, as well as members of the next generation. The rest of the new population is generated by crossing over two randomly chosen parents using the weighted average operator in Michalewics (1994):

$$v'_i = a v_i + (1 - a) v_j$$  \hspace{1cm} (III-1)

$$v'_j = a v_j + (1 - a) v_i$$  \hspace{1cm} (III-2)

where $a$ is a randomly generated number from $[0, 1]$. A mutation rate of 0.01 is selected. At each generation, the number of mutated strings is equal to,

$$Mutate\_Number = Mutation\_rate \times Population\_size \times Number\_of\_strings$$  \hspace{1cm} (III-3)

The positions of the mutated strings are included in an array of random integer numbers that are selected from the array:

$$[1, 2, ..., Population\_size \times Number\_of\_strings]$$

The values of these strings are randomly generated. The process continues for a maximum number of 200 generations.
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