The influence of self-efficacy and working memory capacity on problem solving efficiency

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THE INFLUENCE OF SELF-EFFICACY AND WORKING MEMORY CAPACITY
ON PROBLEM SOLVING EFFICIENCY

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A dissertation submitted in partial fulfillment
of the requirements for the

Doctor of Philosophy Degree in Learning and Technology
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ABSTRACT

The Influence of Self-Efficacy and Working Memory Capacity on Problem Solving Efficiency

by

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The study investigated the influence of self-efficacy beliefs, working memory capacity and problem complexity on problem solving performance, response time, and problem solving efficiency. Previous research investigated these factors from an absolute performance perspective, but not from the perspective of efficiency, defined as the ratio of problem-solving accuracy over time.

Students completed an operational span working memory task, rated their self-efficacy for solving multiplication problems without the use of paper or calculation aids, and then solved computer-based cognitive multiplication problems, under conditions of varying complexity. Two competing hypotheses were proposed, which state that the efficiency of problem solving is either supported or inhibited as a function of individual beliefs and processing ability.

A within-subjects interaction between problem complexity and self-efficacy was found for both problem solving accuracy and efficiency, however interaction effects between complexity and working memory were not observed. Main effects indicated that individuals with increased self-efficacy, regardless of working memory capacity,
were more efficient problem solvers. Results suggested self-efficacy is a compensatory variable, which may influence problem solving efficiency. Conclusions indicated optimal problem solving efficiency is a function of self-efficacy beliefs, working memory and task complexity.
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CHAPTER 1

INTRODUCTION

The idealism of creating flawless instruction has preoccupied educational psychologists for almost one hundred years. Theorists have proposed holistic models of learning which account for the multitude of variables that influence the learning process (Bloom, 1976; Pintrich, 2000; Slavin, 1987). These systemic models describe the essential components of the learning process. Despite comprehensive effort, an analysis of effective knowledge acquisition that includes measuring the efficiency of learning is conspicuously absent.

Previous models of instruction have focused upon efficiency only in tangential ways. Carroll (1963) and Slavin (1987) included an element of time to learn, while Paas and Van Merriënboer (1993) and Van Gerven, Paas, Van Merriënboer and Schmidt (2002) described efficient learning from the context of invested mental effort compared to expected performance. Research concerning efficiency is limited; therefore, the current research focused upon both performance and efficiency using math problem solving as the domain of interest.

In the current study, efficient problem solving is defined as the ratio of problem solving accuracy to response time. This definition is adapted from a study conducted by Mory (1992) which indicated learning efficiency is the ratio of the amount of information learned to the amount of time needed to learn it (Mory, 1994). The domain of math problem solving has also addressed efficiency, as speed and the probability of
performance success (Campbell & Xue, 2001) or computational efficiency, being accurate in the context of more complicated problems (Kaye, deWinstanley, Chen & Bonnefil, 1989). The need for efficiency varies, as some learning or problem solving situations afford the luxury of unlimited time; while others are subject to rigid time constraints, such as post secondary classroom learning. Limitations in instructional time necessitate the need for greater efficiency. Instruction mismatched to a learner’s capacity or expectation prompts a disproportionate expenditure of mental energy or resources, and leads to inefficient learning (Mayer & Moreno, 2003). Thus, efficiency is deemed important and desirable in both organizational and scholarly settings.

One factor investigated in the current study, and a key determinant of problem solving performance, is problem complexity. Related to the learning context, complexity is determined by volume of information, type of task, and instructional format of the information to be learned (Sweller & Chandler, 1994). As problems become more complex, the accuracy of problem solving ability changes (Campbell & Xue, 2001; Hoffman, Schraw, McCrudden & Hartley, 2004; Kalyuga, Ayres, Chandler & Sweller; 2003; Sweller & Chandler, 1994). Complexity also affects learning efficiency (Hoffman et al., 2004).

A second factor investigated in the current study, and a probable determinant of learning efficiency, is the role of individual learner differences. Individual characteristics include diverse attributes such as background knowledge, cognitive preferences, beliefs, and processing capacity (Sternberg & Williams, 2002). These factors collectively or individually influence learning outcomes.

Two of these differences, working memory, a processing factor, and self-efficacy, a belief factor, are investigated in the current study. Self-efficacy is defined as an
individual's assessment of probabilistic success in a particular task or specific domain (Bandura, 1986). Specifically, when controlling for performance capabilities, efficacy beliefs have been shown to operate independently of underlying skills and mediate individual difference variables such as background knowledge (Pajares, 2003; Pajares & Miller, 1994), metacognitive awareness (McCombs & Marzano, 1990; Schunk & Ertmer, 2000), and overall ability (Bandura, 1986; Campbell & Hackett, 1986; Pajares & Kranzler, 1995; Pajares & Miller, 1994).

A second individual difference variable that affects learning is working memory capacity (WMC). Defined as the temporary storage and processing of information that has been read or heard, WMC influences learning (Baddeley, 1998; Baddeley & Logie, 1999; Bruning, Schraw, Norby & Ronning, 2004; Engle, Kane, & Tuholski, 1999). Learners use working memory to coordinate relationships between and among various pieces of information. The complexity of to-be-learned information, combined with working memory capacity, may also affect learning efficiency.

Previous research investigated the factors of information complexity, working memory, and self-efficacy from an absolute performance perspective, but not from the perspective of efficiency. The goal of this research was to determine the influence of self-efficacy and working memory upon problem solving efficiency, while controlling for item complexity when solving multiplication problems cognitively, without the use of paper or calculation aids.

It was predicted as the degree of self-efficacy increased; a greater degree of math problem solving efficiency would result. Additionally, it was expected greater working memory capacity enhanced math problem solving. Lastly, it was anticipated individuals with greater self-efficacy might compensate for lower levels of working memory.
ability, resulting in greater problem solving efficiency. Findings of this research will help determine the potential interaction effects between the roles of beliefs, specifically self-efficacy, and processing ability, i.e. working memory capacity. Each factor holds vast implications for the individualization of instructional approaches.

The paper begins with a brief research summary on the influence of self-efficacy and working memory upon math problem solving ability, followed by a description of the current study. Subsequent is a literature review that investigated studies addressing the role of self-efficacy in problem solving and learning, the influence of working memory in the learning process, and research related to efficiency. The relevance of the current research is discussed, and multiple competing hypotheses are proposed. Lastly, educational implications of the results are addressed.

**Research related to self-efficacy**

The influence of self-efficacy is difficult to dispute. Research and interest in the role of self-referent beliefs and efficacy is on the “verge of dominating the field of motivation” (Pajares, 2003, p. 140). Meta-analytic studies concluded moderate relationships (Cohen, 1988) exist among self-efficacy, academic performance, and studying (Multon, Brown & Lent, 1991), grade point average and university retention (Robbins, 2004), work related performance (Stajkovic & Luthans, 1998) and task performance (Stajkovic, 1997). The degree of confidence an individual possesses in their abilities, in many cases, is a better predictor of eventual performance than previous attainments (Bandura, 1986; Pajares, 2002).

Zimmerman, Bandura and Martinez-Pons (1992), developed a model of self-motivational variables and studied the relationship between perceived self-efficacy for academic achievement and setting of academic goals. Participants completed two
Likert- self-report scales indicating perceived efficacy for self-regulated learning and academic achievement as outcome measures. Results indicated efficacy and students’ self-regulation of learning accounted for 31% of the variance in students’ final grades in a social studies course. The researchers concluded self-regulatory factors not only mediate the influence of prior achievement, but also “contribute to academic attainment” (p. 672). The relationship between efficacy and performance was firmly established \( r = .37 \). The higher the student’s self-efficacy, the more challenging academic goals were set, which in turn led to greater resultant academic achievement.

Judgments of self-efficacy are task and domain specific (Pajares, 1996) therefore; it is prudent to examine the impact of efficacy from the perspective of one precise topic. Math problem solving was chosen in the current study, for several reasons. Foremost, math as a domain allows for the systematic control of problem complexity by manipulating the length of the problem solution (Campbell & Xue, 2001). Secondly, math efficacy has been found to be more predictive of performance than math background (Pajares & Kranzler, 1995), and math problem solving effectiveness is associated with the interaction of components of working memory (DeStefano & LeFevre, 2004). Lastly, many studies have demonstrated the positive effect of self-efficacy beliefs upon math achievement.

Hackett and Betz (1989) provided a series of questionnaires to college students and conducted a correlational analysis exploring the relationship between math performance and efficacy. An overall correlation of .44 was found between the two variables. Efficacy was concluded to influence attitudes towards mathematics, the perceived usefulness of math, and performance, as measured by a math inventory. A regression analysis partialing out the effects of math achievement variables found math self-
efficacy was the single most important predictor of college major choice, an indication of potential long-term success.

Pajares and Miller (1994) employed path analysis to determine the mediational influence of self-efficacy beliefs. Undergraduate participants completed the Mathematic Confidence Scale (MCS) and the Mathematic Problems Performance Scale (MPPS) both developed by Dowling (1978), along with other measures designed to assess math anxiety, math self-concept, and prior math experience.

Results suggested self-efficacy had both the greatest direct, and indirect effects, on math performance, more so than the other model variables. Students' beliefs about their performance capability proved more important than prior experience, self-worth, gender, and perceived usefulness of mathematical skill.

To compensate for the potential confounding of math ability with general intelligence (g), Pajares and Kranzler (1995), asked high school students to complete the Raven's Advanced Progressive Matrices (Raven, Court & Raven, 1983), in addition to measures of math efficacy, anxiety, math level, and math performance. Path analysis concluded self-efficacy had a dominant effect ($\beta = .324$), when controlling for ability. As general mental ability is "the single most largest component underlying individual differences" (p. 428), these findings demonstrate the powerful mediational role of efficacy beyond background knowledge.

Lent, Lopez, Brown and Gore, Jr. (1996) investigated sources of mathematics self-efficacy, using confirmatory factor analysis, to test a four-factor model consistent with Bandura's (1986) social cognitive perspective, which posits efficacy as resulting from personal accomplishment, vicarious learning, social persuasion, and physiological readiness. The pattern of relationship among the latent variables indicated strong
interrelations among self-efficacy and other source variables upon math performance.

Analysis of results inferred that social persuasion and emotional arousal are closely linked to academic performance, and may provide convergent information concerning performance. The model implies individuals' perceptions of domain success can be influenced by their own cognitive and physiological assessment, as well from external referent sources. In a follow-up study, (Lopez, Lent, Brown & Gore, Jr., 1997), tested a path model and determined perceived ability, in relation to perceived past performance, was the most salient determinant of self-efficacy perceptions. Efficacy was instrumental in determining outcome expectations and subject matter interest.

The efficacy research mentioned implies two main conclusions. First, efficacy is a powerful individual difference variable that has the potential to mediate performance outcomes. The degree of efficacy is strongly related to academic achievement (Pajares, 1996; Zimmerman et al., 1992), attitudes towards math related topics, such as career choice (Hackett & Betz, 1989) and can promote productive use of metacognitive strategies (Butler & Winne, 1995). The magnitude of self-referent beliefs can supersede other individual difference factors such as anxiety, physiological predisposition, and interest (Lent et al., 1996).

Second, efficacy can influence performance beyond basic capability. Even when controlling for general intelligence, (Pajares & Kranzler, 1995) or prior math experience (Pajares & Miller, 1994), judgments of efficacy predicted achievement outcomes. The beliefs individuals possess concerning anticipated success also determine what challenges individuals attempt (Pajares & Kranzler, 1994). In sum, these studies demonstrate a pervasive influence of self-efficacy on performance outcomes. It is
prudent to consider efficacy as an individual difference variable that potentially influences problem solving efficiency.

Research related to working memory

In the current study, participants solved multiplication problems cognitively, without the aid of paper, pencil, or any other computational aids. Cognitive arithmetic involves the mental representation of processes (Ashcraft, 1992), and the use of memory representations, such as stored associations and procedural processing (Campbell & Graham, 1985; DeStefano & LeFevre, 2004; Siegler, 1988). To solve an arithmetic problem, the solver must encode the presented information, perform the calculation, and then provide a response (DeStefano & LeFevre, 2004; Logie, Gilhooly & Wynn, 1994). The solving of cognitive arithmetic involves advanced cognitive processes beyond mere fact retrieval (Seitz & Schumann-Hengsteler, 2000). The combined solution procedure involves both the temporary storage and processing of information, a conventional definition of working memory (Hitch, 1978; Swanson & Beebe-Frankenberger, 2004; Mabbott & Bisanz, 2003).

Previous research indicated that individuals with higher levels of working memory capacity perform better on learning tasks (Daneman & Carpenter, 1980; Mousavi, S. Y., Low, R., & Sweller, J., 1995; Mayer, 2001). Additionally, working memory capacity is positively correlated with general fluid intelligence, (g), (Engle, Kane & Tuholski, 1999) and speed of processing (Bjorklund, 2005). Collectively, these factors are potentially instrumental in an individual’s ability to process information efficiently.

The ubiquitous influence of working memory capacity and the ability to solve math problems has been documented in many empirical studies (Adams & Hitch, 1997; DeStefano & LeFevre, 2004; Klein & Bisanz, 2000; Seitz & Schumann-Hengsteler, 2003).
2000; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). Although measurement of WMC varies, the current study and several others employed an operational or adding span task to measure working memory. Span tasks require participants to solve problems while concurrently remembering either the cumulative sums of a series of problems or a list of words or numbers that follow a sequence of problems.

Logie et al. (1994) studied the role of working memory in solving mental arithmetic problems. Using adding span techniques which involve addition of specific problems, while concurrently maintaining a cumulative running total, volunteer participants were required to solve either “single carry” or “multiple carry” (p. 399) mental arithmetic problems in both single and dual task conditions. Dual task conditions were designed to divert memory resources from the primary task of remembering problem solutions. Using articulatory suppression, irrelevant pictures or random generation of alphabet letters to disrupt memory ability, participants’ performance was vastly inhibited, regardless of interference method. Evidence of this nature supports the contention that working memory is impaired by disruption. Results for task disruption indicated errors were surprisingly close to correct answers, implying that participants have access to “a vocabulary of sums and totals that they can access relatively automatically” (p. 407). It is possible since automaticity evokes less working memory resources, available mental capacity can be dedicated towards problem solving. Thus, as a response evokes automaticity, processing can become more efficient.

Passolunghi and Siegel (2001) compared the role of working memory with children considered either poor or proficient math problem solvers, as measured by criterion-referenced tests. Using working memory measures of forward and reverse listening
span, as well as counting span completion tasks, children were required to answer topic
questions while simultaneously holding in memory certain structural problem
components. The results showed poor problem solvers demonstrated less recall of
relevant information, supporting the contention that working memory limitations have a
substantial influence upon problem solving. These findings found working memory
deficits of poor problem solvers were both general in nature, and specific to math
problem solving.

Swanson and Beebe-Frankenberger (2004) investigated the theoretical distinction
between working memory and short-term memory, posed math word-problems to
children considered at risk as well as those not at risk for math difficulties. Measures of
working memory included listening span, semantic association tasks, digit and sentence
span, and a visual matrix task. After statistically removing effects for phonological
processing associated with short-term memory, working memory was found to account
for “26% of variance for arithmetic calculation and 30% for problem solving” (p. 484).
No difference between predispositions towards math problem solving was evident,
leading to conclusions that general fluid intelligence and working memory combined
share a significant role in math problem solving.

A critical variable influencing problem solving ability and related to working
memory capacity is the nature of instructional material. Solving basic multiplication
problems, such as 3 x 4, involves association and the retrieval of a calculation algorithm
from long-term memory (Logie et al., 1994) and requires minimal working memory
resources. Solving problems of multiple digits involves greater complexity (Hitch,
and takes longer (Hitch, 1978; Hoffman et al., 2005; Royer, Tronsky, Chan, Jackson,
and Marchant III, 1999; Siegler, 1988) implying a greater demand on cognitive resources.

Campbell and Xue (2001) investigated the influence of problem size and the relationship with problem solution strategies. The objective of the study was to determine when, and under what conditions, participants used direct memory retrieval versus procedural strategies when solving problems of simple (product of operands < 25) and complex (product of operands > 25) cognitive arithmetic. Participants solved sets of problems while measures of efficiency were recorded that included accuracy and response latency. Upon completion of trials, participants indicated type of strategy used: transforming, counting, remembering, or unique. Results indicated regardless of solution strategy, larger problems were solved less efficiently. Retrieval was the most frequently used strategy for small problems, however, as complexity increased so did procedural strategy use, increasing response time and decreasing efficiency.

Kaye, deWinstanley, Chen & Bonnefil (1989) in a developmental study investigated the processing demands of memory retrieval in children and adults using a task of two-term addition with true-false verification. The methodology employed a dual task paradigm requiring problem solving and concurrent detection of auditory probes. Results concluded when participants were required to maintain constant sums in memory or attend to dual tasks response time increased. Dual tasks processing was related to efficient processing leading to the presumption that memory resources necessary in cognitive arithmetic are limited and a function of complexity and attentional demands. Thus, as the degree of complexity of materials is increased, greater demands are placed upon working memory and can potentially inhibit performance (Sweller, 1994; Pollack, Chandler & Sweller, 2002).
The research on WMC leads to two main conclusions. First, math problem solving ability is mediated by WMC (DeStefano & LeFevre, 2004; Logie et al, 1994; Passolunghi & Siegel, 2001; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004). The degree of WMC can influence how material is encoded, strategies used in problem solution, latency of response and overall performance accuracy, all factors potentially instrumental in the efficiency of problem solving. Tasks of mental calculation can be impaired by dual processing tasks (Kaye et al., 1989) or tasks of a complex nature (Ashcraft, 1992; Logie et al, 1994).

Secondly, problem complexity and problem length determine the efficiency of problem solving performance (Adams & Hitch, 1997; Campbell & Xue, 2001; Siegler, 1988). Problems requiring multiple stages of calculation take longer to solve, result in reduced accuracy and usurp precious working memory resources. Solving complex problems are likely less efficient according to current research on working memory.

These studies reveal a complex interdependent relationship likely exists among task complexity, self-efficacy, and working memory capacity. Results are inconsistent or unknown as to how the precise blend of complexity; assessment of confidence, and processing capability may influence performance. If tasks are easy and working memory capacity high, will efficacy influence problem solving? Are processing resources more critical for low efficacy learners, based upon degree of task complexity? Can varying degrees of efficacy compensate for processing limitations? How will the combination of these variables influence performance? These questions, variability in previous results, and the lack of emphasis upon the efficiency of learning, warranted further investigation.
The current study

The present study examined how the individual characteristics of working memory and perceived efficacy towards solving mental multiplication affect overall problem solving performance, problem solving time, and problem solving efficiency. Groups, described in more detail later, were created by results on a self-reported domain specific efficacy inventory, and performance on an operational span task of working memory. Problem complexity was manipulated using two different types of math problems. Each group solved 40 problems of mental arithmetic differing in complexity. The study design used (20) 2 digit x 1 digit problems with three digit solutions (49 x 9 = 441); and (20) 2 digit x 2 digit problems with three digit solutions (45 x 12 = 540).

The purpose of the study, then, was to determine how participants' self-efficacy and working memory-processing ability affect performance outcomes when differing degrees of problem complexity are controlled. Efficacy was controlled to determine the influence of beliefs upon problem solving performance and efficiency. Working memory was used to control for effects of processing capacity on problem solving and efficiency.

The association among the variables resulted in two main competing hypotheses in the current study. One assumption is that self-efficacy is the prevailing variable influencing problem solving performance and efficiency. Higher degrees of domain specific self-efficacy may create a compensatory effect, overcoming working memory constraints and resulting in more efficient problem solving. A competing view portends that higher levels of self-efficacy will not moderate learning efficiency or performance, and working memory alone should be the prevailing individual difference variable influencing performance and efficiency.
Previous research concluded the mediating effects of self-efficacy in many math achievement situations (Lent, Lopez, Brown & Gore, Jr., 1996; Pajares & Kranzler, 1995; Pajares & Miller, 1994). The application of the efficacy construct has been absent in situations of learning efficiency and therefore warranted investigation. Previous research indicates individuals with higher levels of WMC perform better on learning tasks (Daneman & Carpenter, 1980; Just & Carpenter, 1992; Mayer, 2001) and working memory contributes to math problem solving success (DeStefano & LeFevre, 2004; Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004). Prior research supports the contention that problem solving efficiency should be influenced by WMC.

The results of this research will help answer two important questions: Does self-efficacy and working memory, individually or collectively, have an impact upon problem solving efficiency? Although previous studies determined the impact of efficacy in absolute performance situations, research concerning the impact of efficacy, a belief factor, upon problem solving efficiency has not been investigated. Similarly the relationship between efficacy a belief factor and working memory capacity a processing factor has not been examined from the perspective of learning efficiency.

Secondly, does the influence of efficacy and working memory change as problem complexity increases? Some individuals may benefit from efficacy beliefs only when problems are easy; conversely, efficacy beliefs may have a lesser effect as task difficulty increases. As complexity increases, working memory may become a more significant factor, or perhaps, efficacy may provide a compensatory effect facilitating problem solving. Finally, as problem complexity increases, the performance of less efficacious learners should suffer (Hoffman et al., 2005; Stajkovic & Luthans, 1998).
Predictions

It was anticipated that math problem solving efficiency is a complex relationship mediated by the factors of self-efficacy, working memory capacity, and problem complexity. An interaction effect between self-efficacy and working memory capacity was anticipated. Main effects for problem complexity, self-efficacy, and working memory were also expected.

It was predicted as the degree of self-efficacy increased; a greater degree of math problem solving efficiency would result. Additionally, it was expected math problem solving could be enhanced by greater working memory capacity. Lastly, it was anticipated that greater working memory capacity results in greater problem solving efficiency, as problem difficulty increases.

These premises are consistent with the reciprocal nature of social cognitive theory, which postulates a mediational influence of factors contingent upon differential contributions (Bandura, 1986). As the relative necessity of requisite skills fluctuates based upon efficacy, a dynamic interrelation between problem difficulty and working memory should ensue.

Higher self-efficacy should increase problem solving performance, and also may decrease efficiency because individuals with a greater expectation of success will work harder to solve difficult materials. Social-cognitive theory indicates higher self-efficacy leads to trying harder and greater persistence, which should take relatively more time thereby decreasing efficiency. When working harder, individuals may invest more mental effort and take greater amounts of time to achieve superior performance, compared to their less efficacious peers. In tandem, higher working memory capacity should increase problem solving performance and efficiency because individuals can
process more information at any given point in time.

Findings of this research will help determine the potential interaction effects between the degree of self-efficacy and working memory capacity, which has vast implications for the individualization of instructional approach. Knowledge concerning the influence of individual difference variables contributing to problem solving efficiency will allow instructors to tailor instruction compatible with either objectives of performance accuracy or efficiency. Additional knowledge on the relationship between self-efficacy and working memory capacity under conditions of increasing problem complexity can help instructors instill cognitive equilibrium in students commensurate with perceived problem-solving success. It is unclear as to when self-referent beliefs such as efficacy are perceived irrelevant by students, and potentially may decrease learning efficiency. Some instructional situations require brevity, with a greater emphasis on efficiency, such as a typical university classroom. Other instructional conditions, such as web-based education, afford the luxury of limited contextual constraints, allowing the learner to proceed at their own pace. It is important to determine how, and under what conditions instructors should strive towards enhancing the efficacy of students. Identifying instructional variables, which contribute to the process of problem solving efficiency, may reap valuable rewards when constraints are present in the educational environment.
CHAPTER 2

LITERATURE REVIEW

The literature review is separated into three main parts. The first section describes research related to the influence of self-efficacy upon learning, from a social cognitive perspective. Studies related to math problem solving are emphasized. The second section provides an overview of the role of working memory capacity and problem complexity in relationship to math problem solving. Lastly, research related to learning efficiency is described.

Research on self-efficacy

Arguably, one of the most dominant individual differences related to learning is the role of self-referent beliefs. The perception of self-efficacy, defined as an individual’s assessment of probabilistic success in a particular task or specific domain, embodies the influence of beliefs. Self-efficacy is the foundation of motivational effort towards learning (Pajares, 2002), as an individual’s control over their thoughts, actions, and feelings in a proactive and self-regulating manner is vital to academic success (Pajares, 2003). Human agency, a catalyst for optimal academic functioning, is governed by individual assessment of capability and motivation, which leads to a learner’s representation of the learning context (Bandura, 1993).

Meta-analytic studies have concluded moderate relationships exist among self-efficacy, academic performance, and studying (Multon, Brown & Lent, 1991), grade point average and university retention (Robbins, 2004), work related performance
The degree of confidence an individual possesses in their abilities, in many cases, is a better predictor of eventual performance than previous attainments (Bandura, 1986; Pajares, 2002).

Multon, Brown and Lent (1991) conducted a meta-analysis to determine the relation of self-efficacy beliefs to academic outcomes and persistence towards academic goals. A total of 38 samples including 4,988 subjects were used to analyze performance, and 15 samples with 1,194 subjects were included for persistence. Academic outcomes were categorized as standardized achievement tests, course grades or GPA, and tests of basic skills. Unbiased effect sizes indicated a relationship between self-efficacy and performance accounted for 14% of total variance. Persistence measures included time spent on task, number of tasks or items completed, and academic terms completed. Self-efficacy was attributed to 12% of the variance between persistence and measured outcomes.

Similarly, Stajkovic and Luthans (1998) investigated the magnitude of the relationship between self-efficacy and performance. A meta-analysis of 114 different studies encompassing 21,616 participants was conducted. Overall, 11.4% of the variability in performance was accounted for by self-efficacy.

Specifically, when controlling for performance capabilities, efficacy beliefs have been shown to operate independently of underlying skills and mediate individual difference variables such as gender (Bandura, 1986), background knowledge (Pajares, 2003; Pajares & Miller, 1994), personality traits (Stajkovic & Luthans, 1998), metacognitive awareness (McCombs & Marzano, 1990; Schunk & Ertmer, 2000), and affective arousal (Litt, 1988; Meece, Wigfield & Eccles, 1990).

In the domain of math problem solving, the focus of the current study, beliefs
regarding self-efficacy play a powerful role concerning choice, persistence, effort, strategy and interest (Lopez, Lent, Brown & Gore, 1997; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Self-efficacy for math performance has been linked to college major choice (Hackett, 1985) and holds implications for mathematics career choice (Hackett & Betz, 1989).

Several studies investigated the relative contribution of efficacy to actual attainment of academic results. Pajares and Miller (1994) employed a path analysis model to determine if self-efficacy has greater impact on math problem solving than math self-concept, math anxiety, perceived usefulness of mathematics, prior experience, and gender. Undergraduate participants completed the Mathematic Confidence Scale (MCS) and the Mathematic Problems Performance Scale (MPPS), both developed by Dowling (1978), along with other measures designed to assess math anxiety, math self-concept, and prior math experience. Results suggested self-efficacy had the greatest direct effects on math performance, followed by math self-concept and high school grade level. Although significant mean differences between genders were found on performance and math self-concept, the strength of efficacy perceptions mediated these differences. Students’ beliefs about their performance capability proved more important than prior experience, self-worth, gender, and perceived usefulness of mathematical skill.

To compensate for the potential confounding of math ability with general intelligence (g), Pajares and Kranzler (1995) asked high school students to complete the Raven’s Advanced Progressive Matrices (Raven, Court & Raven, 1983), in addition to measures of math efficacy, anxiety, math level, and math performance. Path analysis, when controlling for general mental ability, concluded self-efficacy had a dominant effect ($\beta = .324$). As general mental ability was "the single most largest component
underlying individual differences” (p. 428), these findings demonstrate the powerful mediational role of efficacy beyond background knowledge.

Pajares and Graham (1999) reviewed task specific mathematics performance to determine the influence and development of various motivational variables and changes over the course of a year during the 6th grade. Math self-efficacy was hypothesized to mediate the influence of other determinants such as anxiety, self-regulation, engagement, bias (expectation of performance) and engagement (degree of effort) upon academic outcomes. Self-efficacy was found to be the lone significant motivational variable to predict math outcomes at both the beginning and end of the school year. Generally, decreasing value, engagement, effort and persistence in math influenced students' overall performance.

Relationships concerning difficulty of task, task interest, and strength of self-efficacy were reported by Campbell and Hackett (1986). Students completed a math number series task (i.e., 3, 12, 30, ?) and assessed efficacy with a nine-point rating scale after finishing each task. Participants were segregated into two groups based upon relative subjective difficulty of the number series. Self-efficacy was found to fluctuate as a function of task difficulty. Participants in the easy condition reported higher efficacy ratings than those in the difficult condition. Efficacy ratings diminished over trials in the difficult group while rising in the easy task group. Similarly, interest levels vacillated based upon the corresponding degree of efficacy, commensurate with task difficulty.

Self-efficacy influences the caliber and nature of learner chosen goals. The greater the degree of confidence a learner has in a particular domain, the greater the likelihood of setting challenging goals, the more effort is expended, and the greater probability of
resilience achieving the goals (Zimmerman, Bandura and Martinez-Pons, 1992; Pajares, 2002).

Schunk (1990) in a literature review observed the relationship between goal setting and efficacy is moderated by the degree of self-observation (personal monitoring), self-judgment (comparing progress to objectives), and self-reaction (evaluation of attained results). If the students believe they can meet goals, they feel better about their progress, set challenging goals, and create feelings of greater efficacy. Proximal goals, process goals, and goals with performance standards (Zimmerman & Kitsantas, 1999) were found to trigger feelings of greater self-efficacy. Specific goals also prompted greater feelings of efficacy. Realistic and obtainable goals were found to influence both self-efficacy for achievement and use of regulatory strategies (Schunk, 1990).

Self-efficacy is influential in determining what activities learners will engage in, and what strategies they will use (Schunk & Ertmer, 1999). Participants were given process goals, analogous to mastery goals, or performance-based product goals, while their adaptivity to computer-based learning was assessed. Students receiving process goals reported enhanced self-efficacy and exhibited more frequent and effective use of self-regulatory strategies. Although self-evaluation enhanced learning in both process and product goals, self-evaluation was more prevalent when focused upon process goals (Schunk & Ertmer, 1999). Additionally, providing opportunities for self-evaluation increased self-efficacy.

Attributions, or how learners account for success or failure, are influential in assessing goal progress (Ames & Archer, 1988). If a learner attributes success to a teacher, luck, specific content, or other factors unrelated to effort, the learner may risk suffering lower self-efficacy when performance falters. Conversely, if the learner
accounts for success by the notion of hard work, perseverance, and setting challenging goals, efficacy will be substantial. “Students’ self-perceptions of ability were found to vary considerably and mediate motivated cognitions” (p. 265). How the learner attributes results can confound feelings of efficacy, and ultimately influence motivation.

Self-efficacy is related to the tactics students will use when solving problems. Bandura (1986) described the connection between a learner’s ability to control the learning environment and self-regulation. As learners believe they have the capacity to create change, they seek to control their environment as opposed to being at the mercy of external forces. The sense of control, in turn, enhances the belief about their capabilities and potential to control their destiny (Pajares, 2002). The learner who believes s/he is capable of achieving academic results uses more strategies, works harder, and persists longer (Lodewyk & Winne, 2005).

The ability to confidently control the environment triggers the use of cognitive monitoring and subsequent strategies. The highly efficacious individual will frequently and effectively employ self-regulatory skills (Pajares, 2002), and evoke a greater use of cognitive strategy. “When students believe added effort will produce success, they persist longer and achieve at a higher level” (Schunk, 1990, p. 79).

Self-efficacy has an influence in the self-regulation process. Carver and Scheier (2000) stated, “…if expectations are for a successful outcome, the person returns to effort towards the goal. If doubts are strong enough, the result is an impetus to disengage from further effort and potentially from the goal itself” (p. 61). Lack of confidence, called “negative rumination” (p. 62), akin to low self-efficacy, results in lack of “self-focus” (p. 62) or cognitive withdrawal, which potentially causes performance deficits.
How accurately a learner assesses efficacy judgments can determine impact upon achievement. Chen (2002) postulated the predictability of self-efficacy beliefs from a calibration perspective. The issue of calibration, or an accurate assessment of prospective ability, in a student's self-efficacy judgments is pedagogically important because poor calibration may undermine the predictive power of self-efficacy judgments. In other words, if a student over estimates task specific capability, the influence of self-efficacy upon performance is diminished.

Middle school students completed seventh grade math items, confidence assessments of solutions based upon item difficulty, and post-question effort judgments. Significant linear trends for item difficulty were found across all dependant measures indicating as problem difficulty increased performance, efficacy, and calibration accuracy decreased. Significant correlations between strength of efficacy perceptions and calibration accuracy were not found. The most salient finding from the study indicated as perceived self-efficacy to solve problems increased, the effort expenditure of students decreased. As difficulty increased, effort judgments increased. Secondly, calibration accuracy improved predictions of math performance by 40%. These findings support relationships between underlying skill and accurate efficacy predictions, while also demonstrating the differential role of effort judgment.

Bandura (1986) found effective calibration of efficacy is a motivational force in achievement, whereby marginally inflated calibration increases effort and persistence. Pajares (1996a) cautions although accuracy of self-perceptions is helpful in problem solving, students who accurately predict lower levels of performance may lose optimism in face of the reality of lower achievement. Schraw, Potenza and Nebelsick-Gullet (1993), investigated the effect of incentives and feedback upon calibration
accuracy and found if a student is motivated to accurately calibrate performance, precision of evaluation is more likely. Undercalibration of performance was less in incentive conditions, leading to the conclusion that monitoring of performance is a “flexible, controllable attribute of the learner” (p. 461).

If calibration is accurate, feelings of self-efficacy are fostered (Stone, 2000). Learners achieving success as anticipated become more confident as results are realized. Increased efficacy can amplify the effectiveness of cognitive monitoring (Butler & Winne, 1995) as learners refine skills in achieving results. Inaccuracy can be detrimental. Pajares and Kranzler (1995) investigated calibration of self-efficacy for math problem solving. Eighty-six percent of the students overestimated their anticipated success compared to performance outcomes, implying that uncertain academic expectations may lead to maladaptive approaches, such as conceding when confronted with difficulty.

Conflicting evidence exists as to the sustainability of performance and efficacy over time. Valentine, DuBois and Cooper (2004) evoked a broader approach to meta-analytic review and determined the relationship between self-beliefs and academic achievement on a longitudinal basis. Self-concept (perceptions of self gained through experience), self-esteem (qualitative evaluations of self-concept), and self-efficacy were synthesized to determine overall effect size relationships with academic outcomes when controlling for initial achievement. Overall effects size for the influence of beliefs was nominal ($\beta = .08$), however, with respect to specific academic domains, self-beliefs were a more dominant predictor of performance. These results lend support to the potential of self-beliefs to effect learning over time.

Bong and Skaalvik (2003) however caution that self-concept and self-efficacy,
although predictively similar, are not the same. Self-efficacy is posited to act as a precursor to academic self-concept, but efficacy is more context specific, future oriented, and malleable.

Two studies, Vancouver, Thompson and Williams (2001) and Vancouver, Thompson, Tischner and Putka (2002) found although on the personal level efficacy does typically moderate performance, on a within-subject basis the role of efficacy may not be as dominant. Specifically, when participants with low performance expectations encounter a task, there may be a reduced allocation of resources directed towards that task. In these particular studies, efficacy was found to be unrelated to performance on an analytical gaming task, and had a partial negative influence on outcomes.

Bandura and Locke (2003) adamantly defended the role of reactive discrepancy reduction as moderated by efficacy, both over time, and within the same individual. Perceived high efficacy may inhibit effort, which can create higher discrepancies between exhibited performance and intention. In this case, high efficacy inspires individuals to set higher goals from the onset. In these scenarios, the discrepancy can be perceived as a motivating factor enhancing, not enervating, performance as suggested by Vancouver et al. (2001, 2002). In both scenarios, precise deployments of self-regulatory strategies are necessary to moderate performance between current states and desired results. The findings from Valentine et al. (2004), Vancouver et al. (2001, 2002) and Bandura and Locke (2003) illustrate the importance of the differential effects and precise calibration of efficacy in determining sustainability and prevalence of efficacy judgments.

The current study has an emphasis on the efficiency of problem solving performance. Research concerning the relationship between self-efficacy and efficiency
outcomes is severely limited. Three studies directly investigated efficacy perceptions when using efficiency as an outcome variable. Kulhavy, White, Topp, Chan and Adams (1985) asked students to read a passage about Naval operations, record reading times, and rate the perceived correctness of their responses to questions based upon the text. Four types of correctness feedback, additively more complex, were provided before students answered the same questions again. Efficiency was measured by the proportion of correct responses on a posttest and reading time during the program. A related ratio concerning time spent on feedback was recorded to determine feedback efficiency.

No significant differences related to response efficiency were found; however, as the complexity of feedback increased, the efficiency of feedback decreased. More importantly, as response confidence increased, feedback efficiency increased as well. These results, while methodologically questionable due to self-report and testing effects, lend support to the potential influence of confidence upon response efficiency.

Mory (1994) presented undergraduate students with computer-based verbal information tasks, or concept knowledge tasks, both in adaptive or non-adaptive conditions, to determine the effects of feedback upon performance, study time, and lesson efficiency. In the adaptive condition, learners indicated the degree of confidence in their responses. In the non-adaptive condition, ratings of response confidence were not requested.

Efficiency, or the ratio of total number of correct responses divided by study time, yielded inconsistent results. Feedback efficiency, a product of feedback study time, indicated feedback in the adaptive group was significantly more efficient than feedback presented in the non-adaptive group. Lesson efficiency, dividing correct responses by lesson time, yielded significantly different results in favor of the more efficient non-
adaptive group.

Hoffman et al. (2005) employed a mixed-model repeated measures design to investigate the effect of self-efficacy beliefs and working memory capacity on problem solving, problem solving time, and learning efficiency. Students rated their self-efficacy, then completed a working memory task and computer-based problem solving of mental multiplication, under progressively more complex conditions.

A significant within-subject effect was observed for problem solving time in relation to problem complexity and for the difficulty of math problems on learning efficiency. Main effects were found for self-efficacy, indicating a compensatory relationship in which working memory limitations were offset by self-efficacy beliefs. These results indicate efficacy is a mediating variable that influences learning and learning efficiency at all levels of working memory ability. Results supported the conclusion that the degree of efficacy may compensate for processing limitations. Collectively, these efficiency studies illustrate the differential effects of confidence level, which can influence performance outcomes.

The research presented on self-efficacy leads to three main conclusions. First, efficacy has a powerful and pervasive role in mediating math problem solving ability. Pajares and Miller (1994) found students’ beliefs about their performance capability proved more important than prior experience, self-worth, gender, and perceived usefulness of mathematical skill. Similarly, when controlling for the effects of general ability (Pajares & Kranzler, 1995) and multiple motivational variables (Pajares & Graham, 1999), efficacy judgments are the dominating factor. Beliefs regarding efficacy also influence choice, persistence, and interest in math (Lopez et al., 1997) lending support to the domain specific nature of the efficacy, which is not a
"decontextualized variable" (Bandura, 1997, p. 42).

Second, learner and task variables have a differential impact upon efficacy assessments. The degree of skill (Bandura, 1986; Bouffard-Bouchard, 1990; McCombs & Marzano, 1988), type of goals learners’ set (Ames & Archer, 1988; Schunk, 1990; Zimmerman et al., 1992) and accuracy of efficacy calculations (Chen, 2002; Schraw et al., 1993; Stone, 2000) influence the impact of efficacy assessments. Increasing task difficulty (Campbell & Hackett, 1986; Hoffman et al., 2004, 2005) lowers the impact of efficacy assessments. Two moderators reported from the Stajkovic and Luthans (1998) meta-analysis are especially relevant to the current study; first, greater positive effect size estimates due to self-efficacy were found for low achieving students, indicating the effects of self-efficacy may be proportionally greater for low ability performers. Secondly, strongest effect sizes were observed for measures of basic skills, while the impact of efficacy upon standardized achievement scores was least, potentially meaning that the emphasis of self-efficacy on classroom activities may be especially important.

Lastly, the degree of self-efficacy has strategic implications. Motivated cognition, strategic choice, and monitoring tactics are related to efficacy assessments (Bandura, 1986; Butler & Winne, 1995; Schunk & Ertmer, 1999). Stajkovic and Luthans (1998) concluded developing effective behavioral and cognitive strategies are necessary to cope with complex tasks that individuals encounter. “Low self-efficacy tends to cause people to become more self-focused and interferes with the optimal deployment of cognitive resources necessary to develop and test complex task strategies” (p. 254). Gauging performance accomplishment is a function of self-monitoring and assessment of progress towards goals. The sustenance, quality, and direction of subsequent effort are guided by the degree of efficacy and can determine potential performance outcomes.
Self-efficacy beliefs permeate all phases of self-regulation (Schunk & Ertmer, 2000). The current research aims to support these three aforementioned conclusions while clarifying the influence of self-efficacy upon the efficiency of problem solving.

Research on working memory and math problem solving

The present study required multiplication problems to be solved without the use of paper, pencil, or computational devices. Participants mentally calculated problem solutions. The process of deriving mental solutions to solve multiplication problems requires temporary preservation of partial solutions in memory, while processing other problem information, to reach a complete solution. This problem solving process is a widely accepted description of how working memory operates (Salthouse, 1996).

Working memory capacity (WMC), the temporary attention and storage of information that has been read or heard, is an individual differences factor that influences learning (Baddeley, 1998; Baddeley & Logie, 1999; Bruning, Schraw, Norby & Ronning, 2004; Engle, Kane, & Tuholski, 1999). Baddeley's (1998) multi-component memory model describes memory functioning as consisting of two subsystems: (a) an auditory component, the phonological loop, which is a speech-based mechanism; and (b) a visual component, the visuospatial sketchpad or a mental imagery device. Attentional resources and temporary storage of information of both systems is mediated by a coordinating central executive function.

Working memory, as a multidimensional construct, utilizes interrelated parallel processing. As a controlled attention and a rehearsal process, learners allocate and shift attention of resources using both the phonological loop and the visuospatial sketchpad (Engle, Tuholski & Laughlin, 1999). Learners use working memory to coordinate relationships between and among various pieces of information. The capacity of...
Working memory is limited and differs by individual (Miller, 1956; Swanson & Beebe-Frankenberger, 2004; Sweller & Chandler, 1994).

Previous research indicated individuals with higher levels of working memory capacity perform better on learning tasks (Daneman & Carpenter, 1980; Just & Carpenter, 1992; Mayer, 2001) and working memory contributes to math problem solving success (DeStefano & LeFevre, 2004; Logie, Gilhooly, & Wynn, 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004). Additionally, working memory capacity is positively correlated with general fluid intelligence, (g), (Engle, Kane & Tuholski, 1999) and speed of processing (Bjorklund, 2005; Salthouse, 1996). Based upon these previous findings and the current definition of efficiency as the ratio of performance over time, WMC should prove instrumental in an individual's ability to problem solve efficiently.

DeStefano and LeFevre (2004) reviewed the literature concerning the role of working memory in mental arithmetic. Although, there is “relatively little research on the role of working memory in mental arithmetic” (p. 354) and “much of the extant research seems contradictory” (p. 354), the review described pertinent factors relevant to solving cognitive arithmetic problems. Three primary conclusions were substantiated in the review. First, all three components of working memory are involved in the problem solving process. Secondly, even apparently simplistic single digit mental arithmetic is a cognitively demanding task requiring use of the central executive function, the processing component of working memory. Finally, solving mental arithmetic problems is related to how information is presented, problem complexity, task requirements, and solution procedures. Research related to these conclusions is described below.
Identification of which memory components, singularly or collectively, are responsible for temporary storage, processing, and controlling of information is important in determining how, and what factors influence math problem solving ability. Previous research indicated mathematical proficiency follows automatically from improvements in phonological processing (Ashcraft, 1992; Mabbott & Bisanz, 2003), and disruptions to this process serve as a useful foundation to determine when and if problem solving can be mediated by working memory ability.

The seminal study by Daneman and Carpenter (1980) provided evidence of the relationship between comprehension and limitations of working memory through use of a reading span. In a reading span-task, participants read and comprehend a sequence of unrelated sentences, and are required to remember the last word of each sentence. Participants read the sentences, made a judgment about the soundness of the sentence, a processing task, and concurrently remember the final word of each sentence, a storage task. Reading span correlated with reading comprehension skill, leading to the conclusion that reading comprehension depends on general processing capacity, not reading ability.

In a follow-up study, Just and Carpenter (1992) proposed a capacity theory to explain how working memory deficits influence cognition. Using reading span techniques described in the Daneman and Carpenter study (1980), college students were found to exhibit significant individual differences in working memory ability as measured by reading times and measures of comprehension. The capacity model explains differences as a function of both procedural and declarative knowledge, and a modulating component to reflect moment-to-moment resource demands. In the capacity model, task demands, which strain capacity, inhibit individuals with smaller working
memory capacity to perform computations quickly or store intermediate products. Task demands impact capability as "working memory capacities are smaller when the comprehension task is easy and larger when it is demanding" (p. 145). Capacity was marginally impacted by practice in the model, and deemed inconsequential in enhancing processing efficiency. According to Just and Carpenter, the individual working memory differences of participants best explains transient computational and storage demands, and should be instrumental in problem solving efficiency.

In the current study, it is important to know if math problem solving ability is a function of math expertise or WMC. Turner and Engle (1989) investigated the relationship between the nature of tasks and working memory to determine if working memory operates independently of the type of task being performed in the working memory measure. Turner and Engle hypothesized using a concurrent processing task that requires a different set of strategies than the skill being measured, i.e. using arithmetic when measuring word processing ability, can detect individual differences in working memory capacity. "If the correlation between the operation-word span and reading comprehension is simply due to good readers also having good and efficient quantitative skills, then the correlation between operation-word span comprehension should disappear when the quantitative skills are factored out" (p. 130). Conversely, if working memory is operationally independent of skills measured by the span task, the partial correlation between operation word spans and reading comprehension should remain significant. In multiple studies, participants completed four complex WMC span tasks: two simple span tasks, which require only rote memory skills, and the Nelson Denny, a measure of reading ability. Participants provided researchers with their SAT verbal and quantitative scores as additional measures of academic performance. Results
indicated “good readers remembered more words than poor readers regardless of whether the background task required reading or arithmetic skills” (p. 149). Complex span tasks, but not simple span tasks, predicted reading comprehension. Extraction of variance using partial correlations and regression analysis concluded the operations word span task was a unique predictor of reading ability, implying working memory transcends task capability, and working memory tasks do not need to be related to the criterion variable under scrutiny.

In another study using secondary tasks, Logie et al. (1994) studied the role of working memory in solving mental arithmetic problems. Using adding span techniques which require problem addition, while concurrently maintaining a running total, volunteer participants were required to solve either “single carry” or “multiple carry” (p. 399) mental arithmetic problems in both single and dual task conditions. Using articulatory suppression, irrelevant pictures, or random generation of alphabet letters to disrupt memory ability, participants’ performance was vastly inhibited, regardless of interference method. Evidence of this nature supports the contention that working memory is impaired by disruption. Results for task disruption indicated errors were surprising close to correct answers, implying participants have access to “a vocabulary of sums and totals that they can access relatively automatically” (p. 407). It is possible since automaticity evokes less working memory resources, available mental capacity can be dedicated towards problem solving. Thus, as response evokes automaticity, processing can become more efficient.

Specifically for mental arithmetic, deciphering the differences between the influence of arithmetical competence and the influence of working memory aptitude upon problem solving ability was examined by Adams and Hitch (1997). The study
investigated if limitations of competence, or working memory constraints inhibit mental addition. The primary research question addressed the role of problem presentation. Using a within-subjects design, children ranging in age from 7 years, 11 months to 10 years, 11 months solved addition problems by either listening to the experimenter read aloud the problem or via visual presentation of problems. Problem complexity was manipulated by the use of single or multiple digit problems, or carrying problems. Response times were also measured. Results indicated significant differences between oral and visual span conditions. Main effects for complexity level were found. Response latency increased with age, problem complexity, and oral presentation. Participants’ visual addition spans were consistently higher than oral presentation, leading to the conclusion that working memory, not arithmetical competence, constrained mental addition. Visualization of problems boosted problem solving success.

Swanson (2004) investigated the relative contributions of problem representation, knowledge of operations, phonological processing, reading, and math skill in a study involving children ranging in age from eight to eleven. In addition to solving math word problems, students were measured on multiple aspects of auditory, verbal, and processing components of WMC, calculation skill, reading comprehension, and fluid intelligence. Use of phonological resources was measured by digit span tasks, phonemic deletion, and digit naming tasks. Phonemic awareness was assessed by a deletion task requiring words to be read aloud after the final or initial sound of the word was deleted. Digit span tasks, consisting of remembering numbers of increasing length, were used to assess rote memory skills. Verbal memory was measured by presenting participants groups of sentences, read aloud, while simultaneously validating the sentences and remembering the last word of the sentence. Requiring children to name random ordered
digits measured speed of retrieval. The Ravens Colored Progressive Matrices and the Woodcock Johnson Psycho-Educational Battery assessed fluid intelligence. Swanson speculated, if after partialing variance from the analysis of measures related to phonological processing and long term memory a non-significant relationship between problem solving and working memory should result. If non-significance developed, then age related differences or knowledge of operations would be accountable for math problem solving skill, not WMC. Thus, the objective of the study was to determine if age related differences in processing operate independently of the phonological system and processing resources in long-term memory. Information of this nature can help determine the relative influence of the role of working memory in math problem solving.

Results suggested executive processes in working memory, not domain-specific knowledge, mediated problem solving. These findings support previous results (Engle & Turner, 1989; Engle et al., 1999) indicating “the correlation between WMC and high order tasks is not a result of skills in the specific component of the working memory task, but rather reflects distinct processes that draw upon a common system” (Swanson, 2004, p. 658). Additionally, a regression analysis was performed indicating WMC contributed unique variance explaining problem solving accuracy. Conclusions indicated individual differences in problem solving exist; those individuals with large capacity have “more resources available” (p. 659) and are better equipped to conduct fundamental aspects of cognitive arithmetic, such as problem representation and problem execution.

Another factor, which significantly influences WMC, is the nature of the material being learned. Since different materials vary in complexity levels, diverse demands
upon working memory capacity exist. A simpler learning task must be chosen to reduce
demands on working memory (Paas et al., 2003). As the degree of complexity of
materials is increased, greater demands are placed upon working memory that can
potentially inhibit learning (Pollack, Chandler & Sweller, 2002; Sweller & Chandler,
1994). Thus, the complexity of to-be-learned information, combined with working
memory capacity, may also affect learning efficiency.

A relationship may also exist between learning efficiency, working memory, and
problem difficulty. Solving basic multiplication problems, such as 3 x 4, involves
association and the retrieval of a calculation algorithm from long-term memory (Logie
et al., 1994) and requires minimal working memory resources. Solving problems of
multiple digits involves greater complexity (Hitch, 1978; Hoffman, Schraw & Hartley,
2005; Logie et al., 1994; Mabbott & Bisanz, 2003) and takes longer (Hitch, 1978;
Hoffman et al., 2005; Royer, Tronsky, Chan, Jackson, and Marchant III, 1999; Siegler,
1988) implying a greater demand on cognitive resources. According to Siegler (1988),
the product of a problem is the “best predictor of relative difficulty” (p. 263). Labeled
as the problem-size effect (PSE), “PSE is the virtually ubiquitous phenomenon that the
difficulty of simple arithmetic problems increases as problem size increases” (Campbell

PSE, determined by representational set size, or the number of digits in a total
problem, has been shown to influence problem solving in children as young as
preschoolers. Klein and Bisanz (2000) presented problems non-verbally, using
manipulatives, concluded the maximum number of units held in working memory was a
major constraint on elementary tasks of adding and subtraction. Set size accounted for
88% of the variation in accuracy.
Hitch (1978), in a series of multi-digit working memory studies, observed time differences related to solving progressively complex mental arithmetic problems. Participants were required to solve problems (e.g. 256 + 451) and verbally report problem solving strategies. Response latencies were directly proportional to the degree of sequential manipulation necessary to problem solve. No-carry problems were more readily solved, following by carrying in the tens column, carrying in the hundreds column, and carrying both the tens and hundreds. Error rates increased as problems became more complex. Working memory decay, equivalent to forgetting, was a function of strategic stage processing described as, “the retrieval of information held in working storage and its arithmetical transformation using long term knowledge” (p. 322).

Seitz and Schumann-Hengsteler (2000) studied the influence of problem complexity and the role of working memory subsystems in performing mental multiplication. A dual task methodology involving either irrelevant speech or figure tapping was used. Problem complexity was manipulated by using outcomes with either singular or multiple digits. Results indicated performance deficits for both auditory suppression and visuo-spatial disruption. Performance was impacted more on difficult sums than for easy sums. Specific subsystem effects were found which lead to a conclusion that easy sums involve central executive retrieval, approaching automaticity; however, more difficult sums require sequential processing, using working memory resources from multiple subsystems.

Problems of various difficulties have been linked to performance and efficiency outcomes. Hoffman et al. (2004) required students to complete a working memory task and solve abstract (hard) and concrete (easy) syllogisms. Results indicated participants...
spent significantly more time solving abstract syllogisms than concrete syllogisms. Furthermore, abstract syllogisms were solved less efficiently than concrete syllogisms. Working memory did not affect efficiency. The results support the conclusion that problem solving efficiency is situational and a function of the complexity of information. Working memory was posited not to affect results based upon a ceiling effect since the mental effort imposed by the syllogisms did not exceed working memory capacity.

What procedures an individual uses in problem solving can be related to problem solving efficiency. If working memory constraints influence subsequent choice of strategies, participants striving towards efficient problem solving may be impeded by personal limitations. Generally, three different strategies for problem solving in cognitive arithmetic have been recognized in previous research: (a) Structural models in which the structural features of the problem determine solutions and latency (Ashcraft, 1992); (b) Network retrieval models whereby associative strengths between numbers determine solutions (Siegler, 1988); and (c) Integrative/multiple procedure models (Ashcraft, 1992) where both strength of association or relatedness from learning, structure and confidence in problem solving dictate solution approach (LeFevre, Bisanz, Daley, Buffone, Greenham and Sadesky, 1996).

Ashcraft (1990, 1992) investigated how strategic solutions used by children varied as a function of problem complexity. A relational network was proposed to coordinate problem solving. The network was predicated upon a stored system of associations, memory traces, and problem-specific bonds within the individual. For example, finding the solution to 2 + 2 involves an association with four, which is likely far stronger than the association with other numbers, thereby increasing solution probability. As problem
complexity escalates, the number and strength of associations is limited and of lower intensity, resulting in competing associations which may result in a higher degree of errors. Associations of low strength require more time to process. According to Ashcraft, extracting solutions from this network was assumed to involve “a process of spreading activation, with the problem-size effect due to slower access to facts with lower strength” (p. 193).

Siegler (1988) studied digit multiplication strategies in children and indicated strategy choice depended critically on the tenacity of the individual arithmetic facts in memory. Younger children relied more upon associative solutions, while older children used sophisticated metacognitive strategies to solve problems. The model of strategy choice helps explain how problem difficulty, error formation, and strategies change as problem complexity increases. The variations in strategy choice have performance implications that were found to influence both accuracy and overall speed of performance when solving math problems.

Speed of response and subsequent problem solving efficiency is influenced by usage of direct versus derived processes in solving cognitive multiplication. LeFevre et al. (1996) investigated how university students solved single digit multiplication problems in an attempt to clarify which model of mental representation influences problem solving. Undergraduates solved 100 multiplication problems that included all possible combinations of single digit integers. For each problem, accuracy, latency, and participant self-report of problem solving procedures were recorded. Self-report responses of solution procedures were coded and consisted of retrieval (I just know it), derived facts (based upon numerical rules), repeated addition, or “other” uncategorized procedures.
Results indicated 45% of errors were found on problems with products greater than 40, while only 21% of errors occurred with products less than 21, providing continued support for problem effect size. Latency values also reflected the ubiquitous problem size effect with 62% of the variance in response time related to problem size. Participants reported using multiple procedures to solve problems based largely upon structural representation of the problem.

The pattern of procedure reports indicated greater response latencies and more errors attributed to the use of non-direct retrieval procedures. Conclusions indicated "models of simple arithmetic that discount the influence of multiple procedures do not adequately capture adult performance" (p. 287). Apparently, a dynamic continuum of solution procedures exists, which determine how accurately and quickly problems are solved. Thus, the variability in selection of procedures support the conclusion that as participants encounter problems of greater complexity, based upon problem size, it is more likely that strategies, besides retrieval, are being employed in problem solution. Strategies beyond retrieval take longer, and involve more cognitive resources, hence, we can infer greater demands on working memory leads to reduced efficiency in problem solving, especially when problem complexity is increased beyond single digit multiplication.

Individual differences in working memory have not been linked to differences in strategy selection. Hecht (2002) investigated the use of strategy selection in a within-subjects design using a verification task of single addition. University volunteer participants either solved problems silently, repeated letters during problem solving, an articulatory suppression condition, or verified equations while generating random letters, a process designed to disrupt central executive functioning. The objective of the
study was to determine whether availability of resources determines strategy choice. Participants verbally indicated if they used a strategy of retrieval, counting, or a "special trick" (p. 449).

Although working memory resources did not dictate choice of strategy selection, disruption of central executive processing resulted in the use of more retrieval strategies, which place a lower demand upon working memory. Working memory was found to correlate with general math computation. Accuracy and response latency was impacted by articulatory suppression. Retrieval strategies, which occur automatically, place the least demands on working memory. Therefore, when retrieval strategies are untenable, such as in complex cognitive multiplication, automaticity is limited and other more demanding strategies were used placing greater stress on WMC. Results support the conclusion that disruption of memory resources, which is unrelated to strategy choice, impairs performance in mental arithmetic and potentially limits problem solving efficiency.

Together, this body of research leads to some collective presumptions about the nature of math problem solving. First, working memory ability boosts math problems solving performance (Hitch, 1978; Logie et al., 1994; Passolunghi & Siegel, 2001; Seitz and Schumann-Hengsteler, 2000; Swanson & Beebe-Frankenberger, 2004). Siegler (1988) found that gradually effective representation of math constructs becomes closely tied to the development of procedural automaticity, freeing up working memory resources. If participants are unable to evoke automaticity, which is commonly the case with mental multiplication, WMC should affect individual learning efficiency.

Secondly, as complexity increases, more cognitive resources and more time are necessary to solve math problems (Hitch, 1978; Hoffman et al., 2004; Seitz &
Schumann-Hengsteler, 2000). In the current study, efficiency and problem solving latency should be impacted by problem complexity. Simpler problems should take less time to solve and use less working memory resources. When more challenging problems are presented, subjects should be less efficient.

Research on efficiency

Some learning conditions afford the luxury of unlimited instructional time; others are subject to rigid time constraints, such as post-secondary classroom learning. Time limitations for instruction oblige concern to learning efficiency. Efficient problem solving in the current study is defined as the ratio of problem solving accuracy to response time. This definition is adapted from the description of learning efficiency indicating that efficiency is the ratio of the amount of information learned to the amount of time needed to learn it (Mory, 1992). Research, although limited, concluded differences in efficiency are not always identical to differences in learning performance (Paas, Tuovinen, Tabbers, & Van Gerven, 2003). Identifying factors that contribute to learning under instructional constraints constitutes an important step in defining overall problem solving efficiency.

The precise definition of learning efficiency varies by theoretical orientation and domain of interest. Some studies confound the definition of efficiency with learning performance, or the ability to process information quickly. Kranzler, Whang and Jensen (1994) examined efficiency and speed by measuring reaction time to elementary cognitive tasks, such as hearing beeps and pressing buttons. Gounard and Hulicka (1977) investigated the effects of age-related performance and indirectly defined efficiency as cognitive processing. Sensory input, the rate of information acceptance and response time, was posited to influence “optimal performance on cognitive tasks”
(p. 420), but limited empirical support was found to distinguish the role of each respective construct towards defining efficiency.

Stanovich (1980) proposed interactive theory, described as the compensatory shifting of information processing resources in reading, as an efficiency effort. Readers changing between lower level reading strategies, such as word recognition, and higher-level strategies such as contextual processing are deemed more efficient, since higher-level strategies take less time. Perfetti (1985) developed verbal efficiency theory as a mechanism to describe the distinction between effortful and automatic processes in reading. Reading which is accurate and fast implies efficient execution of reading subcomponents, such as decoding.

Walczyk (1994) explored the relationship between the use of lexical processes and more sophisticated metacognitive strategies in children's reading. Fourth graders completed a word-naming latency task, semantic memory tasks, and sentence thematic relationship tasks. Efficiency was measured as response latency and recall accuracy, respectively.

The domain of math problem solving has addressed efficiency as speed, combined with the probability of performance success (Campbell & Xue, 2001). One study (Kaye, et al., 1989) directly investigated computational efficiency, defined as accuracy and speed of addition problem solutions, combined with being accurate in the context of more complicated problems. A task of two-term addition with true-false verification was used to measure efficiency. Simple problems (3 + 2) were presented on a computer followed by solutions on a subsequent screen. The primary task required verification of solution accuracy. A secondary task required detection of auditory probes, designed to inhibit processing efficiency. Reaction time (RT) and accuracy of verification were

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measured. Results concluded when participants are required to maintain constant sums in memory or attend to dual tasks, RT increased. If participants can effectively perform a secondary task while concurrently performing a primary math problem solving task, it is deemed as a measure of computational efficiency.

Kulhavy et al. (1985) considered the complexity of information, measured by type of feedback, as a determinant of “instructional yield” (p. 286). Four additively more complex types of feedback were provided to students to determine if type of feedback differentiated post-test performance. Students, after reading a passage about Navy operations, recorded reading times, rated the perceived correctness of their responses to questions based upon the text, received correctness feedback, and then answered the same questions again. Efficiency was measured by the proportion of correct responses on the post-test and reading time during the program. A related ratio concerning time spent on feedback was recorded to determine feedback efficiency. No significant effects for instructional efficiency were found; however, more complex feedback resulted in longer reading times and a proportional decrease in feedback efficiency.

Phye and Bender (1989) expanded the efficiency assumptions of Kulhavy et al. (1985) and investigated the effects of feedback complexity on tests of memory retention and near transfer. In multiple studies varying performance outcome measures, students took a word definition pretest, received feedback of varying complexity, and then completed a post-test. The objective of the posttest was to determine if learning, measured as the corrective efficiency of feedback, varied as a function of feedback complexity or type of task.

According to the Phye and Bender (1989) model, efficiency is the ability of feedback to correct inappropriate responses. Feedback and performance are deemed
more efficient if the findings between pre and post-test results are more accurate. Support for the ability of feedback to boost performance was found, however, complexity of feedback was irrelevant. Correctability of errors as a measure of efficiency did not include consideration of response time.

Paas and Van Merriënboer (1993) outlined an algorithmic approach to measure efficiency. According to the model, a major component of efficiency is self-reported investment of mental effort. Learning is considered more efficient if performance is higher than might be expected based on invested mental effort, or equivalent if invested mental effort is lower than might be anticipated based on performance.

A formula quantifies the relationship between effort and performance. Efficiency for each learner is determined by calculating standardized z scores for both performance and effort. The product is represented as an instructional efficiency score \( E \) using a coordinate system that plots a relationship to the Cartesian axis, depicting performance and effort. Relative conditional efficiency is determined by quadrant location and deviation from a line of best fit. The formula assumes effort exerts a direct causal relationship upon efficiency. Conspicuously absent is a time component, which is not included in the formula. Similarly, the formula assumes a linear relationship among mental effort and performance variables, which is incongruent with other research related to cognitive arithmetic (Kaye et al, 1989; Swanson & Beebe-Frankenberger, 2004) as effort may be a variable relationship dictated by algorithm and strategy use.

Both feedback efficiency and lesson efficiency were examined by Mory (1994). Undergraduate students were presented with computer-based verbal information tasks, or concept knowledge tasks, either in adaptive (feedback customized based upon response), or non-adaptive conditions, to determine the effects of feedback upon
performance. The ratio of total number of correct responses divided by study time was used to calculate lesson efficiency. Feedback efficiency, a product of feedback study time, indicated feedback in the adaptive group was significantly more efficient than feedback presented in the non-adaptive group. Lesson efficiency yielded significantly different results, in favor of the more efficient, non-adaptive group. Mory concluded feedback must be situationally appropriate to meet individualized needs, "not as a means to decrease lesson time" (p. 287).

Recent views of efficiency (Sweller, 1994; Van Gerven et al., 2002; Van Gerven et al., 2003) used the Paas and van Merriënboer (1993) efficiency calculation to examine learning outcomes coupled with optimization of instructional design. Learning efficiency can be inhibited if the design of instructional material does not recognize that individual processing capabilities are limited (Mayer, 2001). Poorly designed instructional material facilitates the unproductive expenditure of mental effort resulting in a proportional decrease in learning efficiency.

Van Gerven et al. (2002, 2003) investigated the efficiency of various problem types in either multi-media or textbook training environments to determine how instructional format optimized knowledge transfer. Problems, conventional, means-end analysis, worked examples, or worked examples with a multi-media format were presented to elderly and young adults. Efficiency and training time per problem were measured as separate, dependent variables. Results indicated main effects for age, type of problem, and transfer condition. An interaction between age group and training time suggested efficiency might be a situational variable influenced by processing capability, which varies by age. The results also suggested training time "might as well be used as a component in calculating training efficiency" (Van Gerven et al., 2002, p. 102).
The previously mentioned studies all have varying interpretations and operational definitions of efficiency. These examples do not adequately account for both processing ability and the accuracy of performance in problem solving situations of increasing complexity. In some cases, (Paas & Van Merriënboer; 1993; Van Gerven et al., 2002, 2003) invested mental effort is a critical component of the efficiency formula. A definition of this nature does not take into account individuals in problem solving situations may hold in reserve processing capability, and performance may be a reflection of other factors besides effort. Therefore, the current study will not measure the ambiguous factor of self-reported effort, but a concrete representation of performance; response accuracy and problem solving latency.

Results on efficiency research imply two main conclusions. First, a wide range of variability exists as to how efficiency is measured. Gounard and Hulicka (1977) indicated the volume of information processed was the most salient factor, while reading research (Perfetti, 1985; Stanovich, 1980; Walczyk, 1992, 1994) found that sophisticated use of strategies promote compensatory processing and efficiency. Kulhavy et al. (1985) and Phye and Bender (1989) emphasize error correctability as a measure of efficiency. Some researchers (Paas & Van Merriënboer, 1993; Van Gerven, 2002, 2003) focus upon invested mental effort compared to performance as the barometer of efficient learning.

These results indicate a precise operational definition of efficiency has yet to be established. Time on task, as a major component of efficiency, has been virtually ignored by previous research with only Mory (1994) and Kaye et al., (1989) using time as a direct criterion measure. Paas et al. (2003) concluded time on task has been neglected in the “calculation of mental efficiency” (p. 69). Thus, questioning the utility
of instructional methods, as well as the caliber of instruction, is an important step in defining overall learning efficiency. An effective measure of efficiency should include a time component and were used in the current study.

Secondly, the precise influence of individual difference factors on learning efficiency is uncertain. Previous research found efficient instruction is influenced by problem type (Sweller & Chandler, 1994; Van Gerven et al., 2003), processing ability (Gounard & Hulicka, 1977; Phye & Bender, 1989; Walczyk, 1992, 1994), mental effort (Paas & van Merriënboer, 1993; Sweller & Chandler, 1994), and time on task (Kulhavy et al., 1985; Mory, 1994; Webb, Stock & McCarthy, 1994). The paucity of studies and lack of convergent results may contribute to mono operation bias (Shadish, Cook and Campbell, 2002).

Singular assessment combined with a lack of standardized measurement methods can confound the ability to detect true group differences. For example, Van Gerven et al. (2002, 2003) investigated the influence of problem types in multi-media and textbook training environments to determine how instructional format optimized knowledge transfer. Perceived mental effort was calculated to measure efficiency. If a complementary measure, such as the ratio of performance to time, was used to calculate efficiency (Mory, 1994), a different outcome might result. Homogeneity of efficiency measurement, coupled with expansion of individual difference research, may yield different research conclusions.

Summary of research findings

Variability in the type of instructional environment, cognitive complexity of material, and individual learner characteristics all influence relative learning efficiency (Bruning, Schraw, Norby & Ronning, 2004). Individually, much research exists
concerning the influence of self-efficacy, working memory, problem complexity, and math problem solving. Collectively, research on these topics is limited, and few studies address these variables from the context of problem solving efficiency.

A wide range of variability exists as to how efficiency is measured. Efficiency is broadly interpreted, with previous research focusing upon speed of processing, strategy use, or the degree of effort exerted to solve problems. Individual difference factors are posited to influence learning efficiency, but the precise impact of each is uncertain.

Efficacy has a powerful and pervasive role in mediating math problem solving ability (Hackett & Betz, 1989; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Various learner and task variables have a differential impact upon efficacy assessments. These variables determine the importance of self-efficacy, which in turn influences how learners apply strategic solutions to solve problems (Butler & Winne, 1995).

Working memory ability, a second individual difference factor, is related to math problems solving performance (Hitch, 1978; Logie et al., 1994; Passolunghi & Siegel, 2001; Seitz & Schumann-Hengsteler, 2000; Swanson & Beebe-Frankenberger, 2004) and likely efficiency as well. As the complexity of problems increases, more cognitive resources, and more time are necessary to solve math problems (Campbell & Xue, 2001; Hitch, 1978; Hoffman et al., 2004; Seitz and Schumann-Hengsteler, 2000). In the current study the collective focus upon the interaction of these variables will provide additional information concerning the enigma of problem solving efficiency.

The current study

This research investigated the influence of self-efficacy, working memory, and item complexity on cognitive arithmetic problem solving accuracy, time, and efficiency. Problem solving efficiency is an important consideration when contextual factors pose
time constraints. Achieving comparable performance results requires greater efficiency compared to when instructional time is unlimited. Thus, measuring efficiency, as the ratio of problem solving accuracy to response time, is appropriate and advantageous.

The purpose of this research is three-fold. The first purpose was to determine the role of domain specific self-efficacy on problem solving efficiency. Previous research concluded the mediating effects of self-efficacy in many math achievement situations (Lent, Lopez, Brown & Gore, Jr., 1996; Pajares & Kranzler, 1995; Pajares & Miller, 1994). The application of the construct of efficacy has not been applied to situations of problem solving efficiency and therefore, is warranted.

Secondly, this research investigated the role of working memory capacity as a potential mediator of problem solving performance and efficiency. Previous research indicates individuals with higher levels of WMC perform better on learning tasks (Daneman & Carpenter, 1980; Just & Carpenter, 1992; Mayer, 2001), and working memory contributes to math problem solving success (DeStefano & LeFevre, 2004; Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004). Prior research supports the contention that problem solving efficiency should be influenced by WMC.

The third purpose of this research was to explore the influence of problem complexity on problem solving accuracy, time, and efficiency. Previous research indicated as the degree of complexity increases, individuals dedicate more cognitive resources, and take more time to solve math problems (Campbell & Xue, 2001; Hitch, 1978; Hoffman et al., 2004; Seitz & Schumann-Hengsteler, 2000). In the current study, efficiency and problem solving latency should be impacted by problem complexity, as problems become more complex, performance and efficiency should decrease, will time
to solve should increase.

The results of this research will help answer two important questions: What is the extent of influence of self-referent beliefs, such as self-efficacy, upon problem-solving outcomes that previous research indicates is influenced by processing constraints? Does self-efficacy, when accounting for an individual’s task working memory ability, have the potential to boost problem solving performance and compensate for processing limitations? It is possible the relationship between working memory and efficacy may produce lower levels of problem solving efficiency as highly efficacious learners persist and take more time to solve problems compared to those with lower efficacy or limited processing capacity. Conversely, efficacy may interact positively with working memory, overcoming processing deficits prompting highly efficacious learners to work harder and become more efficient problem solvers.

Secondly, does the influence of working memory and self-efficacy change as problem complexity increases? Some individuals may benefit from efficacy beliefs only when problems are easy; conversely, efficacy beliefs in learning situations have been determined to have a lesser effect as task difficulty increases. As complexity increases, working memory may become a more significant factor, or conversely, efficacy may provide a compensatory effect facilitating problem solving. As problem complexity increases, the performance of less efficacious learners should suffer (Hoffman et al., 2005; Stajkovic & Luthans, 1998).

Answers to these research questions have broad instructional implications. Additional knowledge concerning the relationship between self-efficacy and working memory under conditions of increasing problem complexity can help instructors personalize instruction to instill confidence aspirations in students commensurate with
perceived problem solving success. This research is warranted, as current literature is inconclusive as to the interaction effects between efficacy, working memory ability, and problem complexity.

Hypotheses

The association among the factors of self-efficacy, working memory and problem complexity upon math problem solving accuracy, time, and efficiency results in two competing hypotheses. The first hypothesis concerns the role of self-efficacy as a moderator of performance and efficiency. Two competing views of self-efficacy are referred to as the efficacy support hypothesis and the efficacy interference hypothesis. According to the efficacy support hypothesis, self-efficacy increases problem solving accuracy and efficiency. High efficacy learners are more efficient since they can easily comprehend the task and need less time to solve problems. According to this view, as problem complexity increases, the role of efficacy escalates in determining problem solving accuracy and efficiency. This hypothesis is consistent with previous findings (Pajares & Miller, 1994; Zimmerman et al., 1992).

In contrast, the efficacy interference hypothesis, predicts learners with higher degrees of domain specific self-efficacy may be accurate, but spend more time and dedicate more mental effort towards solving problems and are therefore less efficient. Low efficacy users are less accurate and need more time to solve problems, as the task is harder to understand. Problems with greater complexity should increase the amount of problem solving time, and decrease performance as well. According to this view as problem complexity increases, the role of efficacy becomes less important (Campbell & Hackett, 1986). The efficacy interference hypothesis is supported by social cognitive theory, which indicates higher levels of self-efficacy result in more effort, task
The second hypothesis concerns the role of working memory. Two competing views of working memory are referred to as the *processing support hypothesis* and the *processing neutral hypothesis*. According to the support hypothesis, WMC helps problem solving efficiency. Participants with higher WMC are more efficient since higher capacity problem solvers evoke processing strategies that are more automatic, and require less processing resources resulting in the ability to solve problems accurately and more readily. According to this view, as problem difficulty increases, the role of working memory escalates in importance. This hypothesis is consistent with previous findings (Campbell & Xue, 2001; Adams & Hitch, 1997).

In contrast, the processing neutral hypothesis, predicts problem solving performance and efficiency is not affected by WMC. Performance and efficiency are affected by other factors, such as problem complexity, and higher working memory capacity cannot mediate performance outcomes. According to this view as tasks become more complex, learners will devote more time and resources resulting in performance deficits. This hypothesis is consistent with previous findings indicating as complexity of problems increases, processing time increases and efficiency decreases (Kaye et al., 1989; DeStefano & LeFevre, 2004).

**Predictions**

It was anticipated that math problem solving efficiency is a complex relationship mediated by the factors of self-efficacy, working memory, and problem complexity. Interaction effects among self-efficacy, working memory, and problem complexity were expected. Additionally, main effects were anticipated for both self-efficacy and working memory.
As problem complexity increases, the mediating effect of self-efficacy upon learning performance and efficiency should be greater (Campbell & Hackett, 1986; Newman & Wick, 1987). Learners with high self-efficacy should be more accurate and efficient in their problem solving abilities (Kulhavy & Stock, 1989; Webb et al., 1994). As problem complexity increases, the performance of less efficacious learners should suffer (Hoffman et al., 2005; Stajkovic & Luthans, 1998).

It was predicted problem solvers with greater WMC would have enhanced problem solving performance and efficiency. The influence of WMC was expected to be greater for problems that are more complex and less critical for less complex problems (Campbell & Xue, 2001). The impact of WMC upon highly efficacious problem solvers should be less instrumental for easy problems, as the confidence of problem solvers should overcome any processing deficits. WMC should have the greatest impact upon problem solvers with higher self-efficacy than those that are less confident in their problem solving abilities. As complexity increases, the powerful combination of high efficacy and superior WMC should result in the greatest performance and efficiency (Hoffman et al., 2005). Thus, it was predicated that interaction effects based upon complexity should be observed for both efficacy and working memory.

Concerning main effects, it was further predicted individuals with high problem solving efficacy would achieve higher accuracy and efficiency on math problem solving than participants with low efficacy. Those in the high WMC group should have greater problem solving accuracy and efficiency than participants in the low WMC group. Lastly, as problem complexity increases problem solving efficiency should decrease; however, problem solving accuracy should increase, but only for the highly efficacious participant (Hoffman et al., 2004).
Summary of the current study

This research investigated the role of self-efficacy, working memory, and item complexity on problem solving accuracy, time, and efficiency. An operational span task of working memory capacity was used to account for effects of processing capacity on problem solving and efficiency. Participants assessed domain specific self-efficacy before solving problems. Individuals’ used a computer to solve mental multiplication problems of increasing complexity without the aid of calculators or paper and pencil. Accuracy and response time were recorded to measure problem solving ability. Efficiency, defined as the ratio of problem-solving accuracy over time, was calculated.

Findings of this research will help determine the potential interaction effects between the degree of self-efficacy and working memory, which has vast implications for individualized instructional. Knowledge concerning the influence of individual difference variables contributing to problem solving efficiency will allow instructors to tailor instruction compatible with either objectives relating to performance accuracy or efficiency. Some learning situations require brevity, with a greater emphasis on efficiency, such as a typical university classroom. Other learning situations, such as web-based instruction, afford the luxury of limited contextual constraints, allowing the learner to proceed at their own pace. It is significant to determine how, and under what conditions, instructors should instill levels of confidence in learners as a mechanism to facilitate performance accuracy and efficiency. Acknowledgment and cultivation of individual belief structures is critical to instructional effectiveness. Teachers willing to assess and instill the belief of confidence in their students will take significant strides towards facilitating an instructional environment conducive to problem solving.
CHAPTER 3

METHODOLOGY

Participants and design

Study participants were students enrolled in an introductory Educational Psychology courses from a large Southwestern university, and volunteered as partial fulfillment of a class requirement. The total number of participants were 81, consisting of 21 males and 60 females. Participants were asked to specify estimated grade point average (GPA). Overall, mean GPA was 3.27.

The study design employed a 2 (level of working memory capacity: high, low) X 2 (level of self-efficacy: high, low) X 2 (complexity of math problem: easy, hard). Groups based upon levels of working memory were created by using a median split based upon outcomes of the working memory task described below. Groups based upon levels of self-reported self-efficacy for cognitive arithmetic were created by using a median split based upon outcomes of the self-efficacy scale described below. Two levels of problem complexity were created based upon problem size; (20) 2 digit x 1 digit problems with three digit solutions; and (20) 2 digit x 2 digit problems with three digit solutions. A repeated measures, mixed model multivariate analysis of variance was used. The variables of WMC and self-efficacy were between-subjects factors, whereas the type of math problem variable was a within-subjects factor.
Materials and procedures

Standardized instructions were provided to all participants and can be found in Appendix B. First, each participant completed an informed consent form. The informed consent form indicated that participants would solve problems of mental arithmetic. Participants were informed that cognitive fatigue may result from their participation. Subsequent to providing consent, each participant completed three distinct tasks as part of the actual research study. First, an operational span working memory task was completed, employing the methodology and scoring procedure suggested by Conway, Kane, Bunting, Hambrick, Wilhelm and Engle (2005). Working memory involves the use of limited capacity, domain general resources involving both the processing and simultaneous preservation of information in consciousness (Swanson & Beebe-Frankenberger, 2004). Working memory capacity (WMC) identified as a significant contributor to the variance associated with problem solving and general intellectual ability (Engle et al, 1999; Conway et al., 2005) was used in the current study to account for participants’ problem solving processing ability.

The working memory task is consistent with Daneman and Carpenter’s (1980) seminal measures of working memory. The operation span task requires concurrent processing and storage demands upon participants (Swanson, 2004). Each of 42 computer based trials consisted of providing participants with a simple math equation such as \((6/3) + 2 = 5\), followed by a single syllable word. Participants, completed the task individually, were required to verbally articulate the equation, verbally verify to the researcher if the equation was correct or incorrect by saying “yes” if correct, or “no” if incorrect, and then attempted to remember the word following the equation. When
instructed participants recalled a series of words following the viewing of multiple
equations. Accuracy on the working memory task was measured by the correct number
of words, recalled in serial order, for each set of word-equation pairs. Equations were
counterbalanced with equal addition and subtraction tasks and equality in the
correctness or incorrectness of the equation. Twelve trials consisting of between two
and six equation-word pairs were presented to each participant. Presenting equations in
the same order to each participant provides a mechanism to control for order effects
(Seitz & Schumann-Hengsteler, 2000) and allows for random distribution of fatigue and
practice effects. The 12 equation-word pairs trials were presented in the same order to
each participant, and consisted of word-equation pairs in the following combinations;
three 2-item pairs, two 3-item pairs, three 4-item pairs and three 5-item pairs. Each trial
was untimed, however, the researcher monitored participants to ensure the verbal
verification of the equation and solution was followed by the verbalization of the to-be-
remembered words. After each trial, participants recalled and self-recorded the to-be-
remembered words in exact serial order on a worksheet. After completion of each trial
and recall, the participants proceeded to the next trial, at their own pace, until all trials
were completed.

Scoring of the operation span involved assessment of both the equation processing
component and the serial recall task. Partial credit unit scoring (Conway et al., 2005)
was used for the recall task. Partial credit scoring calculates the mean proportion of
items within a trial that are recalled in correct serial order. Words recalled inaccurately
or in the wrong serial position are counted as incorrect. Individuals scoring less than
85% accuracy on the equation-processing component of the operation span result in
removal of the subject from the research study, based upon likely inattention to the
processing component, and consistent with practices recommended by Conway et al. (2005). In the current study, all participants achieved greater than 85% accuracy on the equation processing component of the task. The aggregate number of word items recalled correctly determined performance on the operation span. Based upon magnitude of aggregate recalled words participants were grouped into a high WMC group (above median score) or a low WMC group (below median score). Based upon lack of participant availability, using extreme scoring (Conway et al, 2005), which recommends an appropriation of participants into three or four groups, was not feasible.

Next, students completed a self-report assessment of problem solving confidence. Self-efficacy for math multiplication problems was measured by participants’ responses as to their degree of confidence in solving eight different mental multiplication problems, identical in length and difficulty to items solved in the actual study. This method of measuring efficacy was substantively similar to Lopez et al. (1997). Students rated problems on a ten-point scale ranging from no confidence at all (0) in solving accurately, to total confidence in problem solving accuracy (100). Participants were required to rate their level of confidence for each problem. Cronbach’s alpha, designed to measure the degree of internal consistency between efficacy ratings, was measured. Based upon self-reported efficacy ratings, median splits were conducted to segregate participants into either the high or low self-efficacy groups.

In the third part of the study, individuals used a computer to solve 42 mental multiplication problems of two levels of complexity without the aid of computers, paper and pencil or any other calculate aid. The problems differed in complexity based upon number of digits in the equation and the number of digits in the solution. The first two items of the 42-item instrument were designated as practice problems designed to
familiarize each student with the process and content of solving mental arithmetic, and were not included in the statistical analysis. The scored trials consisted of (20) 2 digit x 1 digit problems with three digit solutions (49 x 9 = 441) and (20) 2 digit x 2 digit problems with three digit solutions (45 x 12 = 540). Solving problems of multiple digits involves greater complexity (Campbell & Xue, 2001; Hitch, 1978; Hoffman et al, 2005; Logie, Gilhooly, & Wynn, 1994; Mabbott & Bisanz, 2003) and takes longer (Hitch, 1978; Hoffman et al., 2005; Royer et al., 1999; Siegler, 1988). The problems, developed by the researcher, can be seen in Appendix A.

Each problem was presented individually, one appearing on the computer screen at a time. Order of problem presentation was determined randomly. The randomized order was presented in the same sequence to each student. Presentation of problems randomly and consistently to each participant controls for order effects (Seitz & Schumann-Hengsteler, 2000) Instructions were presented on the computer screen and also read to the students as a group. Instructions to the participants can be seen in Appendix B.

Students used the computer keyboard to input answers to individual problems in a data entry field immediately below each problem. After designating an answer to each problem, students clicked “continue”. Upon clicking, “continue” the next problem was presented and the computer recorded the completion time for providing an answer and submitting the response. Students were informed they could not view problems on previous screens once they advanced to the next screen. Before beginning, the researcher indicated participants should read at their normal rate and click “continue” when ready to read the next problem. Students were instructed to solve problems as accurately and as quickly as possible without sacrificing accuracy of response. Additionally, students were instructed not to use the computer to input and temporarily
store partial problem solutions before indicating the complete answer to the problem. Students were aware both accuracy and problem solving time were recorded. There were no completion time limits during any portion of the procedure, however, students were told to try as best as possible to arrive at the correct solution to the problem even if they thought the problem was not readily solvable. Inputting a result was required to advance to the subsequent problem. After completion of the first 20 problems, participants received a message on their computer screens indicating a two-minute break would elapse before students could complete the remaining 20 problems. A break was instituted to avoid fatigue effects between the first 20 and second 20 problems.

Three dependent measures were recorded: number of fully correct responses to each of 40 multiplication problems, aggregate time in milliseconds (converted to seconds) to complete and submit answers to the multiplication problems, and problem-solving efficiency, the aggregate number of correct responses divided by response time (multiplied by 1000 for ease of representation). Results were segregated for each level of item complexity to determine any differences between less complex problem solving, versus problems that were more complex.
CHAPTER 4

RESULTS

Three repeated measures ANOVAs were performed on three measures of cognitive multiplication: performance, time, and efficiency using the MANOVA routine in SPSS. Two between-subjects variables were used: level of self-efficacy, with participants grouped as either low or high, and working memory ability with similarly grouped participants, either low or high. The within subjects factor, complexity, was treated multivariately over two different occasions.

One case in the original data set was eliminated due to the inability of the participant to complete the required problem solving task. The data from four other participants warranted elimination due to implausible responses or disregard of researcher instructions, resulting in a final data set consisting of N=81 (males=21, females=60). The data set was screened to determine both multivariate and univariate outliers deviating greater than three standard deviations from any dependent variable mean. Two multivariate outliers in excess of four standard deviation units from the mean were removed from the data set, recommended by protocol (Tabachnick & Fidell, 2001). Removal of outliers did not change the overall multivariate significance of results compared to untrimmed data.

Results of the evaluation of assumptions for multivariate analysis of variance were satisfactory for all measures indicating the assumptions of normality, equality of covariance, and sphericity were met. Therefore, all multivariate data interpretation for

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the dependent variables were based upon Wilks' criterion.

Means and standard deviations for performance, time, and efficiency measured at the two complexity levels are presented by group for the two levels of efficacy and memory, in Table 1. Summaries of all significant results presented by independent variable are presented in Table 2. Summaries of means and standard deviations for main effects by group can be found in Table 3.

Due to unequal gender participation t-tests were performed to explore if results related to problem solving performance and problem solving response time were influenced by gender. Both analyses indicated that gender did not affect results on the dependant measures. Results indicated for problems of low complexity gender did not influence problem solving performance or problem solving time, $t_{(77)} = -1.986, p = .051$ and $t_{(77)} = -.137, p = .891$, respectively. For problems of high complexity, gender did not influence problem solving performance or problem solving time, $t_{(77)} = -1.922, p = .058$ and $t_{(77)} = .284, p = .820$, respectively.

Reliability coefficients using Cronbach's alpha were calculated to determine the reliability of the eight-item self-efficacy measure. Results indicated the measure was reliable, $\alpha = .944$.

Problem Solving Performance

Problem solving performance was determined by the number of problems answered correctly on the 40 problems of cognitive multiplication. Participants received one point for each problem answered correctly. Results were aggregated to provide a total score for each level of problem complexity.

The results of the multivariate repeated measures analysis indicated a statistically
significant interaction between complexity and self-efficacy on the combined performance measures $F_{(1, 75)} = 6.393, p < .02, \eta^2 = .079$, indicative of a medium association (Olejnik & Algina, 2000) between degree of self-efficacy and math problem solving ability. A statistically significant interaction was not found between complexity and WMC on the combined performance measures $F_{(1, 75)} = 3.61, p < .07, \eta^2 = .046$, suggesting the practical association between working memory and problem complexity was minimal.

Participants had higher math performance scores, solving more problems when having higher efficacy ($M_{\text{Easy}} = 17.70, \text{SE}_{\text{Easy}} = .347, M_{\text{Hard}} = 12.89, \text{SE}_{\text{Easy}} = .840$) respectively, which was consistently better than the low confidence group ($M_{\text{Easy}} = 16.13, \text{SE}_{\text{Easy}} = .360, M_{\text{Hard}} = 9.15, \text{SE}_{\text{Hard}} = .873$) regardless of the complexity of the math problem.

Between-group univariate analysis of variance revealed statistically significant, main effects for self-efficacy on the performance measure ($F_{(1, 75)} = 9.95, MSE = 22.39, p < .005, \eta^2 = .117$), indicating a large differences between the high and low efficacy participants. Students answered more math problems correctly when having high self-efficacy ($M = 15.29, \text{SE} = .530$), than when having low efficacy ($M = 12.88, \text{SE} = .551$). Univariate analysis of variance for the WMC variable indicated significant differences between high and low WMC participants, ($F_{(1, 75)} = 4.25, MSE = 22.39, p < .05, \eta^2 = .054$). Students answered more math problems correctly when having high WMC ($M = 14.87, \text{SE} = .565$), than when having low WMC ($M = 13.30, \text{SE} = .514$).

**Problem Solving Time**

Problem solving time was determined by the latency of response for each of the 40
problems of cognitive multiplication. Response time was recorded in milliseconds, and converted to seconds for ease of analysis. Results were aggregated to provide a total problem solving time for each level of problem complexity.

The results of the multivariate repeated measures analysis revealed a statistically significant interaction effect between complexity and self-efficacy on the combined problem solving time measures was not observed $F_{(1, 75)} = 1.39, p = .255, \eta^2 = .017$. Similarly, a significant interaction between complexity and WMC was not found, $F_{(1, 75)} = 1.77, p = .188, \eta^2 = .023$.

Since a significant interaction was not observed for problem solving time main effects were examined for self-efficacy and working memory capacity. Results indicated statistically significant differences for problem solving time did not exist between groups for either self-efficacy or WMC, $F_{(1, 75)} = .202, p = .655, \eta^2 = .003$ and $F_{(1, 75)} = .186, p = .867, \eta^2 = .002$, respectively.

Problem Solving Efficiency

Problem solving efficiency was determined by computing the ratio between problem solving performance and problem solving time for each of the 40 problems of cognitive multiplication. Results were aggregated to provide a total problem solving efficiency score for each level of problem difficulty. The ratio of problem solving performance to problem solving time was multiplied by 1000 for ease of reporting purposes.

The results of the multivariate repeated measures analysis indicated a statistically significant interaction between complexity and self-efficacy on the combined efficiency measures $F_{(1, 75)} = 4.188, p < .05, \eta^2 = .053$, implying a small association between degree of self-efficacy and math problem solving efficiency. A statistically significant interaction was not found between difficulty and WMC on the combined performance.
measures \( F_{(1, 75)} = .454, p = .503, \eta^2 = .006 \), suggesting there was almost no association between working memory and problem efficiency.

Participants with greater efficacy for cognitive multiplication had higher math efficiency scores (\( M_{\text{Easy}} = 64.69, SE_{\text{Easy}} = 3.40, M_{\text{Hard}} = 17.96, SE_{\text{Hard}} = 1.64 \)) respectively, consistently better than the lower efficacy group (\( M_{\text{Easy}} = 50.32, SE_{\text{Easy}} = 3.54, M_{\text{Hard}} = 11.07, SE_{\text{Hard}} = 1.71 \)) regardless of math problem complexity.

Between-group univariate analysis of variance revealed statistically significant, main effects for self-efficacy on the efficiency measure (\( F_{(1, 75)} = 9.86, MSE = 439.72, p = .002, \eta^2 = .116 \)), indicating large differences between the high and low efficacy participants. Participants had higher efficiency scores when having high self-efficacy (\( M = 41.33, SE = 2.347 \)), than when having lower efficacy (\( M = 30.70, SE = 2.440 \)).

Univariate analysis of variance for the WMC variable did not indicate significant differences between high and low WMC participants on efficiency scores, (\( F_{(1, 75)} = 1.50, MSE = 439.72, p = .225, \eta^2 = .02 \)).

Summary of findings

Findings supported the predictions concerning the positive influence of self-efficacy upon problem solving performance and efficiency. Individuals with higher levels of self-efficacy solved more problems of cognitive multiplication and were more efficient in the problem-solving process. An interaction effect between efficacy and complexity was observed, as the degree of problem complexity increased the role of positive self-efficacy was more instrumental. Problem complexity was found to significantly influence results on all dependant measures, problems that were more complex resulted in lower performance and efficiency. Both the self-efficacy variable and the working memory variable revealed significant main effects on problem solving performance,
indicating individuals with higher efficacy and higher WMC solved more problems correctly.

Significant main effects concerning group differences on the working memory variable were not found for efficiency. No significant differences were found on the dependent variable of problem solving time indicating that both efficacy and WMC did not influence response latency in the current study.
CHAPTER 5

DISCUSSION

The goal of the current study was to determine the influence of self-efficacy and working memory capacity (WMC) upon problem solving efficiency, while controlling for item complexity, when solving multiplication problems cognitively, in other words, without the aid of paper or calculation aids. The research was designed to help answer two important questions: Foremost, does self-efficacy and working memory, individually or collectively, have an impact upon problem solving efficiency? Previous research has found direct effects of self-efficacy in many math achievement situations (Lent, Lopez, Brown & Gore, Jr., 1996; Pajares & Kranzler, 1995; Pajares & Miller, 1994). Previous research also indicated WMC contributes to math problem solving success (DeStefano & LeFevre, 2004; Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004). Prior studies have investigated the factors of information complexity, working memory, and self-efficacy from an absolute performance perspective, but not from the perspective of efficiency.

Secondly, does the influence of efficacy and working memory change as problem complexity increases? Some individuals may benefit from efficacy beliefs only when problems are less complex; conversely, efficacy beliefs may have a diminished effect as complexity increases. Working memory may become more important as complexity increases, or perhaps, efficacy may provide a compensatory effect enhancing the ability to solve math problems cognitively regardless of WMC. Previous research has indicated
as problem complexity increases, the performance of less efficacious learners should suffer (Hoffman et al., 2005; Stajkovic & Luthans, 1998).

It was predicted as the degree of self-efficacy increased; a greater degree of math problem solving efficiency would result. Additionally, it was expected greater WMC would enhance math problem solving performance. Lastly, it was anticipated individuals with greater self-efficacy might compensate for lower levels of working memory ability, resulting in greater problem solving efficiency. This suggests an interaction between self-efficacy and problem complexity.

Two competing hypothesis were compared to explain the influence of self-efficacy, WMC and problem complexity. According to the efficacy support hypothesis, self-efficacy increases problem solving accuracy and efficiency. Participants with higher problem solving efficacy are more efficient since they can easily comprehend the task and therefore need less time to solve problems. According to this view, as problem complexity increases, the role of efficacy escalates in determining problem solving accuracy and efficiency. This hypothesis is consistent with previous findings (Pajares & Miller, 1994; Zimmerman et al., 1992).

In contrast, the efficacy interference hypothesis, predicted participants with higher degrees of domain specific self-efficacy may be accurate, but spend more time and dedicate more mental effort towards solving problems and are therefore less efficient. Low efficacy users should be less accurate and need more time to solve problems, as the task is harder to understand. Problems with greater complexity should increase the amount of problem solving time, and decrease performance as well. According to this view as problem complexity increases, the role of efficacy becomes less important (Campbell & Hackett, 1986). The efficacy interference hypothesis is supported by
social cognitive theory, which indicates higher levels of self-efficacy result in more effort and task persistence (Bandura, 1997; Bouchard-Bouffard, 1990; Schutz, 1993) and therefore more time to solve problems, lowering efficiency.

The second hypothesis concerns the role of WMC. Two competing views of working memory are referred to as the processing support hypothesis and the processing neutral hypothesis. According to the support hypothesis, WMC helps problem solving efficiency. Participants with higher WMC should be more efficient since higher capacity problem solvers evoke processing strategies that are more automatic, and require less processing resources resulting in the ability to solve problems accurately and more readily. According to this view, as problem difficulty increases, the role of working memory escalates in importance. This hypothesis is consistent with previous findings (Adams & Hitch, 1997; Campbell & Xue, 2001).

In contrast, the processing neutral hypothesis, predicted problem solving performance and efficiency are not affected by WMC. According to this view as tasks become more complex, individuals devote more time and resources resulting in performance deficits. This hypothesis is consistent with previous findings indicating as complexity of problems increases, processing time increases and efficiency decreases (Kaye et al., 1989; DeStefano & LeFevre, 2004).

It was predicted that individuals with higher self-efficacy and higher WMC should have greater problem solving performance and efficiency. As problems became more complex, the role of efficacy and WMC should become more important in determining problem solving ability.
Review of Results

The main results of the study can be summarized as follows: The results endorse the efficacy support hypothesis and the predictions that efficacy is a powerful individual difference variable that has the potential to affect cognitive multiplication performance and efficiency outcomes. Individuals with higher levels of self-efficacy were able to solve more problems correctly and had higher levels of problem solving efficiency. These findings add new knowledge concerning the role of efficacy on efficiency outcomes and complement previous math problem solving self-efficacy research (Bandura, 1997; Hoffman et al, 2005; Pajares & Graham, 1999; Pajares & Kranzler, 1995; Pajares & Miller, 1994).

Within-group comparisons indicated an interaction between self-efficacy and problem complexity on the performance dependant variable. As the degree of problem complexity increased, the influence of enhanced self-efficacy became more important. Those individuals with the combinatorial luxury of high self-efficacy and high working memory capacity performed best.

An interaction between self-efficacy and problem complexity was indicated for the dependent variable of problem solving efficiency, again sustaining the prediction of the efficacy support hypothesis. Higher degrees of domain specific self-efficacy may create a compensatory effect, overcoming working memory constraints, resulting in more efficient problem solving.

An interaction between working memory and problem complexity was not found for the dependent variable of problem solving efficiency. These findings support the processing neutral hypothesis, which indicates other variables besides WMC may be instrumental in problem solving efficiency.
For all dependant variables the role of complexity was pervasive as problems of greater complexity were more difficult to solve, took longer to solve and were solved less efficiently. These findings support the ubiquitous problem size effect, which states that problems of greater length require more cognitive resources and more problem solving time (Campbell & Xue, 2001, LeFevre et al., 1996). Results for each dependant variable are discussed separately below.

Problem Solving Performance

Results for problem solving performance were examined by a within-group analysis designed to compare mean number of problems solved correctly for low complexity problems to the mean number of higher complexity problems solved correctly. A within-group comparison was chosen to reflect differences within each subject on the variable of complexity. The results concluded that mean differences on less complex problems compared to problems that are more complex was influenced by both self-efficacy and complexity. The greater the degree of self-efficacy indicated by an individual the more cognitive multiplication problems were solved, regardless of difficulty level. Results of this nature suggest that self-efficacy is a strong predictor of performance when solving mental arithmetic problems. This finding was anticipated and is consistent with other studies indicating the persuasive role of self-efficacy in math achievement (Chen, 2003; Pajares & Graham, 1999; Pajares & Kranzler, 1995; Pajares & Miller, 1994).

A within-subject analysis was also conducted to determine the impact of WMC at different levels of problem complexity. Results indicated that WMC was not a statistically significant mediator of problem solving accuracy at different levels of
complexity. These results are likely due to the fact that differences between the complexity of problems between conditions was not substantial enough to discriminate differences in working memory capacity within individual, or possibly variables such as efficacy mediate the need to engage memory resources. Perhaps if the dichotomy of complexity were greater between levels of the within subject variable, an interaction would have been revealed.

Based upon moderate practical significance of the interaction effect, main effects for the role of self-efficacy and working memory were conducted. The overall role of complexity in solving problems was large, as 64.6% of the variability in problem solving performance was accounted for by problem complexity. Subsequently, main effects for self-efficacy were examined and the results indicated that those individuals with higher self-efficacy clearly outperformed those individuals indicating lower self-efficacy. These between group results, which accounted for 11.7% of the variability in performance, indicated that efficacy is a mediating variable that influences problem solving performance, regardless of the degree of problem complexity or working memory ability.

Main effects were observed for WMC, in the direction of prediction. These results indicated individual differences in WMC distinguished differences in performance of cognitive multiplication. This result is consistent with previous research concerning the role of working memory in math problem solving (Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004; Hitch, 1978; Seitz and Schumann-Hengsteler, 2000).
Problem Solving Time

Analysis of problem solving time was conducted on both a within-subject basis and a between-subject basis. The within-subject comparison was designed to determine if participants solved less complex problems faster than more complex problems. Results indicated problems that are more complex take longer to solve. This result was predicted and is consistent with prior findings indicating length of problem solution and problem complexity results in greater problem latency. No interaction effects were found between problem solving time and problem complexity. The lack of interaction effect is due to the substantial differences in problem solving time between conditions. Regardless of the level of self-efficacy or working memory, these individual differences cannot overcome the variation in problem complexity, which explained 83.1% of the difference in problem solving time.

Secondary analysis of problem solving time did not indicate main effects for the variables of self-efficacy or for the WMC variable, indicating that differences in efficacy and WMC did not explain differences in problem solving time between groups of participants categorized as either high or low. A finding of this nature is likely related to the fact that complex problems require multiple transactions, use more cognitive resources and effort and take longer (Ashcraft, 1992; Hecht, 2002). Maintenance of partial solutions for intermediate results may exceed working memory capacity, therefore inhibiting individual differences in memory ability (DeStefano & LeFevre, 2004). With regard to the lack of main effects for efficacy, the strength of efficacy beliefs can not singularly overcome environmental and ability obstacles (Bandura, 1997), such as the time constraints in the current study.
Problem Solving Efficiency

Problem solving efficiency was calculated by dividing the aggregate number of problems solved correctly at each level of complexity by the aggregate amount of problem solving time at each complexity level. Results were examined by a within-group analysis designed to compare mean efficiency scores for low complexity problems in relation to the mean efficiency of higher complexity problems. An interaction between problem complexity and self-efficacy was observed. Those individuals assessing their self-efficacy as high achieved greater efficiency scores, regardless of the complexity level of the problem. These results, which were predicted to occur, imply that the pervasive effects of efficacy assessments can assist in predicting problem solving efficiency.

An interaction effect for WMC and problem complexity was not observed, indicating that working memory at different levels of complexity does not result in significant differences in efficiency scores. Lack of interaction effects for WMC may indicate that the degree of working memory capacity does not transcend different levels of problem complexity, perhaps as a result of the vast differences in complexity between lower level and more complex problems, or as a result of the influence of other individual difference variables. Lastly, almost identical to the large effect size for problem solving performance, mean comparison within groups indicated that significant overall differences in efficiency scores were due to problem complexity.

Based upon small practical significance of the interaction effect between complexity and self-efficacy (.053), main effects for the role of self-efficacy were conducted. Main effects were observed and conclude that the variable of self-efficacy significantly differentiated problem solving efficiency. Those individuals with higher self-efficacy
outperformed those individuals indicating lower self-efficacy. These between-group results, accounted for 11.6% of the variability in performance, indicating efficacy is a mediating variable influencing problem solving efficiency, regardless of the degree of problem complexity or working memory ability.

A main effect for WMC was not observed. The lack of WMC main effect was likely a result of the time component differences described above. Overall, the lack of between subject differences for the WMC variable indicates that other individual difference factors account for variations in problem solving efficiency.

**Explanation of Results**

The current research was designed to answer two main questions: does self-efficacy and working memory, individually or collectively, have an impact upon problem solving efficiency, and does the influence, if any, change as problem complexity increases. Interaction and main effects supported one assumption of the efficacy support hypothesis; self-efficacy increases problem solving performance. In addition to previous findings, which indicated self-efficacy, enhances problem solving performance (Bandura, 1997; Hoffman et al., 2005; Pajares & Graham, 1999; Pajares & Kranzler, 1995; Pajares & Miller, 1994), the current research extends the pervasive influence of self-efficacy to problem solving situations when accuracy and speed of response are both important, namely, efficiency.

Findings did not sustain the processing support hypothesis indicating higher WMC results in greater problem solving efficiency. Although between-group differences for the working memory variable were observed, and expected for problem solving performance, interaction effects and main effects were predicted, but not indicated.
between WMC and problem solving efficiency. The results concerning WMC and math problem solving performance are consistent with prior findings (DeStefano & LeFevre, 2004; Logie et al., 1994; Passolunghi & Siegel, 2001; Swanson & Beebe-Frankenberger, 2004). Still unexplained is why these results were observed, therefore several plausible explanations are proposed.

The findings for problem solving performance and efficiency were likely observed due to the substantive and pervasive effect of self-referent beliefs, consistent with social cognitive research. The effect of efficacy judgments is most pronounced when measured on a domain specific basis, closely in time to demonstrated performance, and congruent with capability (Bandura, 1997, Bouffard-Bouchard, 2001; Stone, 2003). The methodology used in the current study closely followed self-efficacy measurement protocol. Students provided self-efficacy assessments for problems closely approximating the structure, length and complexity of problems solved during the actual task. According to Pajares (2002a), self-efficacy beliefs exert a powerful influence on human agency when individuals are certain about the task to be performed. “Tasks perceived as more difficult or demanding than they really are result in inaccurate low efficacy readings, whereas those perceived as less difficult may result in overconfidence” (p.1). Presumably, the congruence among efficacy judgments, problems and performance were closely aligned, helping to explain the current results.

Although complexity was hypothesized to exert a sizeable influence on problem solving performance and efficiency, and observed, an effect of greater interest is the profound influence of efficacy judgments across complexity levels. Regardless of complexity level, self-efficacy assessments were consistent in mediating efficiency outcomes, surpassing WMC as a viable predictor variable.
A tenable explanation of efficiency outcomes should reflect on the precise association between efficacy, WMC, complexity, and the amount of cognitive effort expended towards solving cognitive multiplication problems. Prior research has indicated students with higher efficacy expend more effort to solve problems (Bandura, 1997) and relationships between effort and efficiency exist (Paas & Van Merriënboer, 1997). Chen (2003) in a study assessing the accuracy of math self-efficacy calibration upon math achievement indicated a negative correlation between self-efficacy projections and post-performance perceptions of effort. Individuals with perceived high efficacy displayed decreased effort expenditures to solve problems. The strength of a student’s expectation of superior math achievement was a potential mediator of how much effort was reported as being devoted towards completing the task. Contrary to post performance efficacy indices, performance indicated a linear trend between efforts expended and item difficulty. The more difficult an item, the more effort was reported as being expended. Chen concluded, “self-efficacy positively correlated with effort when effort was assessed before completing the targeted math performance, conversely, self-efficacy negatively correlated with effort when effort was assessed after completing the targeted performance” (p. 90). The context and perceived difficulty of the task was positively related to efficacy beliefs and anticipation of effort extension, until the actual task was completed. These results are similar to those found by Zimmerman and Kitsantas (1999) in a writing task, indicating an inverse relationship between efficacy and effort attribution.

Presumably, in the current study, if a high efficacy participant believed less extension of effort was required to complete the problem solving task, processing resources, including working memory, may not have been fully engaged, nor exceeded
capacity thresholds, thereby pre-empting the probability of an effect for WMC. The high efficacy problem solver, not exhausting their resources was able to solve problems more efficiently. Thus, higher efficacy assessments may result in more efficient problem solving due to the perception of using less effort and the expectation of higher performance commensurate with prior research findings.

There are at least three explanations why self-efficacy may be predictive of problem solving efficiency. These reasons include cognitive savings, attentional diversion, and strategy choice. These explanations are not mutually exclusive; each may independently or collectively influence efficiency. All contend the highly efficacious individual clearly understands task requirements, has requisite skills, perceives control, and anticipates successful outcomes. Bandura (1997) indicated when both belief of confidence and expectancy of task are congruent; the affects of efficacy assessments are most compelling.

Primarily, individuals with high efficacy may be more parsimonious in their assessment, choice, and application of cognitive resources. High efficacy problem solvers confident in their ability, preempt the need to apply individual problem monitoring, planning, evaluation, assessment, and subsequent revision. The diminished need results in cognitive savings resulting in more efficient problem solving. Previous research indicates that self-regulated learners are judicious and resourceful processors of information, motivated and cognitively equipped to understand, monitor, and direct their own learning (Wolters, 2003). However, application of strategy requires cognitive resources, which might otherwise be directed towards problem solving. Walczyk (1994) advocates strategies such as establishing appropriate goals, selecting strategies for goal attainment, monitoring progress towards goal achievement, and pursuing remedial
action, are used only when warranted. The efficacious learner may not warrant the need, thereby freeing up resources directed towards problem solving and thus becoming more efficient in their problem solving endeavors. Likely, the high efficacy participant engages in self-regulation strategy only when the complexity of the problems dictates the need. Current results support this conclusion, as high efficacy participants took more time and were less efficient when solving problems that were more complex. The increased time and reduced efficiency is a result of both the problem size effect and the necessity to use more time consuming, cognitively draining resources to effectively reach accurate conclusions.

Secondly, when tasks are formidable, or seemingly insurmountable, such as when low efficacious problem solvers encounter a complex problem, attention may be diverted from the task. Resources normal directed towards problem solving may be focused upon perceived ineptitude, anxiety resolution or determination of heuristics necessary to solve the problem. The low efficacy participant, perceptually disadvantaged may be daunted by self-handicapping thoughts, usurping precious resources normally devoted towards problem resolution. As the perception of the task increases in complexity, the debilitating effect may be further exacerbated.

Inefficient allocation of attentional resources may be especially prominent in situations when the participant has not accurately assessed their capability. The potential overestimation of possible success proves especially detrimental as the problem solver realizes that an overestimation of ability is insidious in the long run. Individuals potentially believing they should solve a problem correctly, but cannot, may become frustrated. Bandura (1997) indicated, “Pursuits that have only a small chance of success consume large amounts of time, effort, and resources that offer better prospect
of benefit when applied to more realistic endeavors (p. 77). If the individual accurately assesses their capability, efficiency is enhanced, as limited attentional resources do not need to be directed elsewhere.

The participant with a higher degree of self-efficacy, confident in their problem solving ability, focuses attention primarily upon the task. Individuals with higher self-efficacy may have the ability to inhibit task irrelevant interference. The confident individual discounts consideration of alternate conceptions related to problem solving based upon lack of perceived need and anticipation of accuracy. In conjunction, less effort is needed, resulting in outcomes that are more efficient.

Thirdly, strategy choice may explain efficiency outcomes. High efficacy problem solvers likely evoke strategies, which are less calculational, more retrieval based, and automatic. If the retrieval process is more automatic, efficiency is enhanced. Since automaticity does not evoke precious working memory resources, available mental capacity can be dedicated towards explicit strategy use, only when necessary. Previous research indicates that individuals with greater WMC employ more automatic retrieval strategies (DeStefano & LeFevre, 2004; Hecht, 2002).

However, strategy selection is not contingent upon WMC (Hecht, 2002) and evidence supporting differential strategy use does not account for the relationship between WMC and problem solving (Hambrick et al., 2005). Although processing costs are associated with the use of non-retrieval strategies, individuals with high efficacy may be more automatic in their choice of selection strategies, resulting in less monitoring and reflection, facilitating problem solving efficiency.

The role of efficacy and the threat of ambiguous temporal precedence inhibits clarity as to the exact relationship between efficacy and the use of particular math
problem solving strategies. It is unclear if high efficacy evokes the use of certain strategies, or if availability of strategies results in greater problem solving confidence. Geary, Hoard, Byrd-Craven, & DeSoto (2004) reported that children, when solving math problems, used an intrinsic moderator to assess competence, which subsequently triggers their strategic approach. “The use of retrieval-based processes is moderated by a confidence criterion that represents an internal standard against which the child gauges confidence in the correctness of the retrieved answer” (p. 3). Seemingly, in the current study, the high efficacy participant based upon the unique relationship between confidence and strategic choice, became the more efficient problem solver.

In essence, a compensatory relationship between efficacy, WMC, and complexity exists. Solving cognitive multiplication involves maintenance of task relevant information, storage of partial solutions, and the application of algorithms (Baddeley & Logie, 2001) within a context dictating perseverance when the problem-solving environment becomes more complex. In circumstances of this nature, apparently the expectation of success is able to compensate for, or overcome the limitations imposed upon one or more of the components of working memory capacity, regardless of the degree of problem complexity. The results suggest the strength of beliefs may supersede complexity obstacles and processing capability, thereby boosting efficiency. These results are especially powerful considering the high concentration of females in the current sample. Typically, females are found to be less efficacious on self-report measures of math efficacy (Pajares, 2002). The performance results suggest efficacy assessments may transcend any potential impact of gender differences upon perceived math self-efficacy as well.

Individual differences underlying math problem solving indicate gender may be a
contributing factor. The current study did not observe gender differences likely due to
the nature of the problem solving task. Most studies revealing slight gender differences
involve problems that require quantitative reasoning ability, analytic spatial–
visualization ability or contextual constraints.

Royer found “Gender differences are most likely to be present in situations
involving complex, unrehearsed, performance under time pressure. These are precisely
the conditions that exist in most high-level math tests, but not in the classroom (p. 254).
Leahey and Guo (2001) examined a large data set (NELS & NLSY) and found 1.5%
differences, mostly for geometry performance. Neither of these situations existed in the
current study.

Concerning outright complexity of problems, as the degree of problem difficulty
increased, problem solving performance and efficiency, both decreased. The findings
support previous research (Ashcraft, 2002; Kaye et al., 1989), demonstrating as
computational complexity increases, problem solving latency and performance both
decrease. Labeled the problem size effect (PSE), and described as “the most studied
phenomenon in the history of mathematical cognition research” (Campbell & Xue,
2001, p. 300). PSE is engaged as multiple digit problems elicit additional problem
encoding, procedural incrementing and the maintenance of intermediate sums in order
to solve a problem (DeStefano & LeFevre, 2004). The additional component processing
takes longer, and is a deferential determinant of problem solving time. PSE is the
dominant factor explaining why performance and efficiency decrease for problems that
are more complex.

Interaction effects for the variable of WMC upon problem complexity were not
found. This result is likely a result of the incremental complexity differences associated

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between problems at the lower, less complex level, compared to those of greater complexity. The number of problems solved correctly for less complex problems was almost identical, regardless of WMC grouping, \((M_{\text{low mem}} = 16.87, M_{\text{high mem}} = 17.44)\), and proportionally large (out of 20 total correct answers) indicating a ceiling effect. The number of problems solved correctly for the more complex problems indicated greater variability \((M_{\text{low mem}} = 9.15, M_{\text{high mem}} = 12.31)\) suggesting a stronger influence of WMC on more complex problems and supported by the overall significant between-group findings. Apparently, the complexity of problems at the lower level was too simple to evoke diversity in WMC results.

The assumption that higher working memory capacity should improve problem solving efficiency was not supported as neither within group nor between group differences were observed. This assumption was likely not supported primarily as a result of PSE, as prolonged latency strongly constrains efficiency. A second explanation is elusive need to possess, and apply significantly greater processing resources, which are imposed by complex problems. 2 x 1 math problems with three digit solutions were solved faster, more accurately and more efficiently than 2 x 1 problems with three digit solutions. The degree of complexity was the primary variable influencing problem solving efficiency. Eighty-eight per cent of the variance in problem solving efficiency was explained by the degree of problem complexity. Once the threshold of the easier problems is exceeded, efficiency decreased remarkably. A floor effect may have been induced by the cognitive complexity of the materials, which inhibited the influence of working memory typically ascribed to other problem solving situations (Daneman & Carpenter, 1980; Mousavi, S. Y., Low, R., & Sweller, J., 1995; Mayer, 2001).

A main effect for working memory was found for performance accuracy. Findings
indicated WMC was a prevailing component in problem solving success, which is consistent with previous research. Two plausible explanations for this confirmation apply in the current study. Although different theoretical models of working memory provide assorted explanations of the impact of WMC on complex cognition (see Miyake & Shah, 1999, for a complete review), most models consider activation or monitoring, coordination/organization, and the use of procedural strategies as contributory components to problem solving success.

Application of problem solving skills necessitates using WMC resources. The general capacity hypothesis (Hambrick & Engle, 2003) advocates a domain general view, and contends WMC is the foundational processing derivative responsible for complex problem solving. This view transcends a variety of domains, including math problem solving, and contends that the strength of relationships between cognitive measures should vary depending upon the use of attentional resources and processing ability, not the cognitive task. Previous studies (Passolunghi & Siegel, 2001; Swanson, 2004; Swanson & Beebe-Frankenberger, 2004; Seitz & Schumann-Hengsteler, 2000), with the goal of disentangling variance associated with background knowledge used in operational span measures, support this contention. In the current study, solving the most complex problems required dedication of attentional resources in a highly activated state, and likely explains the main effect for WMC.

Secondly, differences in performance may be a result of the partial use of automatic retrieval strategies. Individuals demonstrating performance prosperity and efficiency likely employ limited calculational algorithms. Use of automatic strategies, which involves little or no encoding, and computational procedure, take less time and use less working memory resources. Previous studies (DeStefano & LeFevre, 2004; LeFevre et
al., 1996; Kaye et al., 1989; Mabbott & Bisanz, 2003) confirm that use of automatic strategies are quicker, less error prone and result in superior performance.

Results of the current research have delineated differences between performance and efficiency outcomes. Efficacy was found to elicit between-group differences for both performance and efficiency, while WMC only affected performance. Efficiency differences indicated the combination of accuracy and speed of response produced superior results. Greater efficiency was likely prompted by the use of less effortful strategies employed by participants with high self-efficacy, as previously described (see p. 79).

Deciphering the distinction of efficiency compared to performance supports creating a conceptual linkage between utility of performance and either application of effort or availability of resources. Adams and Hitch (1997) reported a linear relationship between speed of response and performance on an integer addition task as children encountered math problems of increasing complexity. Relationship decrement between the variables was a function of complexity; problems that were more difficult prompted inferior performance. The association was interpreted as reflecting underlying constraints imposed by working memory differences, with easier problems deemed “efficiency of processing” (p. 23).

In almost an identical task of single digit addition, Kaye et al. (1989) concluded the use of efficient computational processes across age differences induced a “cognitive savings” (p. 468) extending processing resources. Older individuals were determined to possess additional capacity to attend to supplementary tasks and had the potential “to execute computations in the context of more complex mathematical problems” (p. 468). Thus, if distinct differences in problem complexity can be controlled, it appears an
implied linkage between complexity and efficiency of processing is warranted.

Either constraints on processing ability (Adams & Hitch, 1997) or availability of additional resources (Kaye et al., 1989) are both representations of computational efficiency. Therefore, as demonstrated in the current research, if the absolute ratio of performance to speed of response is greater for more complex problems, those participants may be deemed more efficient.

Limitations

Understanding the cognitive processes associated with solving of problems is a robust area of research. The current study did not employ any qualitative methods to assess what cognitive activities participants employed, nor how strategy selection may have contributed to problem solving performance and efficiency. Additional variance in outcomes may be accounted for by measuring strategy usage in conjunction with perceptions of efficacy and measurement of WMC. Future studies should employ a mixed methodology to decipher the influence of strategy differences in efficiency outcomes.

Within-group measures indicated large effect sizes were a result of problem complexity. Complexity in cognitive multiplication is a naturally occurring continuous variable based upon problem size, operand composition (i.e. multiplying 6 x 5 vs. 6 x 7), presentation modality (visual vs. auditory) and format (DeStefano & LeFevre, 2004). The categorization of continuous variables inhibits verifying precisely at which thresholds complexity overrides either the influence of efficacy assessments or processing capacity. Measuring problem complexity precisely is important.

Problems that are too simple may have inhibited the influence of WMC, as most
participants solved problems correctly. Problems too complex may have surpassed the
capacity of many of the participants in the high WMC group. Ascertaining the proper
blend, degree and composition of problem complexity may result in differential effects
of individual difference variables. Additional research is necessary to examine the exact
influence of problem type on dependant outcomes.

The current study used a dichotomous split in allocating participants into groups
(high vs. low). Ideally, extreme scoring (Conway et al., 2005), which creates quartiles,
is desired to avoid misclassification of participants. The categorization was based upon
practical constraints due to subject availability and overall sample composition (N=79,
M=21, F=57). Despite this less than optimal categorization, medium effects sizes
suggested causal influence of the variables. The constancy and direction of the casual
effects across dependent variables suggests that potential threats to validity were
mitigated. Future sampling methods should strive towards larger samples, with
equivalence in gender, to allow for the use of extreme score methodology advocated by
Conway et al. (2005).

Finally, for precision in measurement, voice activated scoring of responses should
be instituted. The current measurement of response time, although consistent across
individuals, included a component, which required keyboard input of digits, and mouse
usage to record responses. Potentially, faster keyboarding skills might marginally affect
latency outcomes and should be avoided, if feasible.

Implications

There are at least four key implications from the current research, the explanatory
nature of efficacy in predicting efficiency outcomes, performance and efficiency
distinctions, operational clarify of efficiency measurement, and the apparent compensatory nature of efficacy.

Foremost, these results support the contention that problem solving performance is malleable and a situational outcome mediated by individual learner differences. Those individuals with the greatest degree of self-efficacy consistently solved more problems, more efficiently, than their peers with lower self-efficacy. Acknowledgment and cultivation of individual belief structures is critical to instructional effectiveness. Teachers willing to assess and instill the belief of confidence in their student’s problem solving ability should take significant strides towards facilitating an instructional environment conducive to performance. Recognition of the situational, domain specific and dynamic composition of the changing nature of self-referent beliefs is likely a prerequisite to achieving efficient learning outcomes (Pajares, 2005).

Secondly, the distinction between performance and efficiency is important. Techniques that facilitate knowledge in one learning situation may not be optimal in another (Hoffman, Schraw, McCrudden & Hartley, 2004; Kalyuga, Ayres, Chandler & Sweller, 2003; McNamara, Kintsch, Butler, Songer, & Kintsch, 1996). Based upon the results in this study, which indicated efficacy accounts for more variability in performance than the processing factor of WMC, teachers under instructional time constraints, may prosper by focusing upon the perceived ability of students as the salient criteria. Adaptation of a methodology which focuses upon student expectations of outcomes may be more important when the learning and problem solving situations involve time limitations, which demand more efficiency.

Thirdly, these results help clarify the operational morass concerning the definition of problem solving efficiency. Defining efficiency as a ratio relationship seems
tangible, and adds objective data to the current emphasis upon self-reported mental effort (Paas & Van Merriënboer, 1993). Defining efficiency exclusively as either using less than invested effort for anticipated results, or expected effort for superior results, ignores the precise measurable variable of response latency, an equally integral component.

Finally, acknowledging the compensatory nature of cognitive processing is a key implication of these findings. Individuals faced with instructional confines upon the amount of time available for problem solving are well served to be confident in their abilities. Even in cases were working memory ability is constrained; highly efficacious individuals were able to boost their performance and problem solving efficiency. Although the current study investigated only the individual difference of working memory capacity, other individual differences, such as epistemological, ontological or other entrenched beliefs may also be mediators of efficiency. The ability to situationally discern which factors dominate the instructional condition and which are trivial is a potential gauge for successful instruction. Recognition of the compensatory nature of beliefs promotes a baseline for differentiated instruction, responding to potential learner variability, and pertinent individual differences.

The ultimate relevance of this study is reflected in illustrating the collective interplay of available resources, personal beliefs and complexity of the relevant domain. The confluence of these constructs outlines an adaptive platform to help facilitate efficiency. Likely, the efficacious, self-reflective problem solver is in a constant state of evaluation and awareness using metacognitive monitoring while regulating strategy, all in a perpetual effort to optimize problem solving ability. Any efforts to explain the cognitive reverberation must inveterate inquiry into the entire stratum of moderating
variables. Effective problem solving involves more than just understanding factual knowledge and reasoning operations in a particular domain (Bandura, 1993). To this end, advocacy of the relentless pursuit of efficiency is important to optimize instruction.
## APPENDIX A

### STUDY PROBLEMS

Presented before break

1. \(37 \times 26 = 962\)
2. \(51 \times 19 = 969\)
3. \(73 \times 12 = 876\)
4. \(23 \times 8 = 184\)
5. \(64 \times 4 = 256\)
6. \(43 \times 19 = 817\)
7. \(26 \times 22 = 572\)
8. \(69 \times 13 = 897\)
9. \(55 \times 12 = 660\)
10. \(27 \times 26 = 702\)
11. \(45 \times 12 = 540\)
12. \(32 \times 22 = 704\)
13. \(17 \times 47 = 799\)
14. \(32 \times 8 = 256\)
15. \(15 \times 7 = 105\)
16. \(19 \times 9 = 171\)
17. \(31 \times 29 = 899\)
18. \(73 \times 4 = 292\)
19. \(36 \times 22 = 792\)
20. \(35 \times 7 = 245\)

Presented after break

21. \(77 \times 7 = 539\)
22. \(33 \times 8 = 264\)
23. \(61 \times 15 = 915\)
24. \(67 \times 7 = 469\)
25. \(54 \times 3 = 162\)
26. \(51 \times 6 = 306\)
27. \(51 \times 15 = 765\)
28. \(49 \times 9 = 441\)
29. \(38 \times 26 = 988\)
30. \(29 \times 9 = 261\)
31. \(17 \times 37 = 629\)
32. \(59 \times 13 = 767\)
33. \(61 \times 6 = 366\)
34. \(42 \times 9 = 378\)
35. \(22 \times 22 = 484\)
36. \(28 \times 26 = 728\)
37. \(82 \times 5 = 410\)
38. \(39 \times 9 = 351\)
39. \(92 \times 5 = 460\)
40. \(83 \times 4 = 332\)

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APPENDIX B

INSTRUCTIONS TO PARTICIPANTS

Welcome and thank you for your participation in the “In Your Head” study.

I am Bob Hoffman, lead researcher on this study. First, we will complete the Informed consent form. Do you have any questions concerning the informed consent form?

Over the course of the next hour, you will be completing three tasks. Each task will be prefaced by individual written and verbal instructions. I would appreciate your focused concentration by staying on task and completing each task using the best of your ability. After the completion of all tasks, I will let you know when the study is concluded.

The first task you complete will involve the use of both paper and pencil, and computer. The following two tasks will be done exclusively on the computer. The initial task is called an operational span and helps determine how your memory operates. In this task, you will see a simple numerical equation and a solution to the equation followed by a word. The problems involve single digit multiplication or division. For example you may see an equation that looks like $6/2 + 3 = 5$, your job is to determine if the solution indicated to the problem on the computer screen is correct or incorrect. As soon as you have determined the solution, you need to aloud indicate, “Yes” if the solution is correct or “No” if the solution is incorrect. You must state yes, or no, aloud. Immediately following your verbalization of the problem and accuracy of the problem
solution, you need to then read aloud the word that follows the equation. You must say this word aloud. After you verbalize the equation, solution and either yes or no and the word following the problem you may press the space bar on your keyboard to proceed to the next problem. Actually once the task begins you will read everything aloud you see on the computer screen. For example, you would say, “Is $6/3 + 3 = 9$, it is not, therefore you would say “no” and then read the word following the solution such as “dog”. Your goal is to remember the words following the problem and to recall the words in exact order when instructed. Remember, you must first read the problem aloud, solve the problem, indicate verbally if the solution is accurate or inaccurate by saying “yes” or “no” and then verbalize the word on the screen before pressing the space bar and proceeding to the next problem. You will have several practice trials to get used to the process. I will be monitoring and recording your verbalizations so please work as accurately and quickly as you can without making mistakes. OK, let’s begin by writing the last five digits of your SS # on the top right corner on the second sheet of paper in your folders.

Now turn on the screen at your PC and read the instructions on your computer screen. After reading the instructions, press the number one (1) key on your computer keyboard to proceed. It is very important that you following the directions exactly as you read on the screen. Are you ready?

*Student begins the task and is monitored closely for compliance to instructions.*

*Students’ are monitored to detect if less than 85% accuracy on verification of problem solutions occur.*

The next task two tasks were done on the computer. On the task bar your screen should indicate “Hoffman Studies” please maximize the program. Click on the link
entitled Math. In this next task, you will indicate the degree of confidence you have in solving a series of math problems in your head. Let's begin by indicating your estimated GPA, your sex, and the last five digits of your SS # in the first three boxes on your screen. For example look at the problem 19 x 19 and determine the degree of confidence you would have arriving at the accurate problem solution by calculating the answer in your head. When I say in your head I mean without paper, pencil, computer, calculator or any other assistive aid including using your fingers or drawing imaginary numbers in the air. How confident would you feel arriving at the correct solution in your head? Please indicate the degree of confidence by clicking on the appropriate circle (0-100%, in 10% increments). Now read the instructions on the screen and indicate your confidence ratings for each problem. When you are finished, click “Continue” Do not proceed. Please wait for further instructions.

_Students complete part two of the study._

The final task will be to complete a series of math problems in your head. You will not be able to use a calculator or paper and pencil. You focused attention is appreciated as your both your problem solving time and your problem solving accuracy will be recorded. Please work as quickly as you can without making mistakes. Before the actual study begins, you will be given two practice problems to become acclimatized with the study process. Please keep in mind the task requires solving problems in your head. Do not indicate your response in the answer box until you have reached a final solution. It is not appropriate to use the answer box as a place to provide a partial solution to the problem, similar to a piece of scratch paper. After you complete one-half of the problems, you will receive a timed two-minute break. After the break, you will complete the remaining problems. Upon completion of the task, you will be instructed
to click the “submit” button. After you click submit, your screen should provide a thank you message. Leave that screen on your computer. You will then be finished. When you are finished, please remain seated until I tell you the task is completed. Do you have any questions? If not, please read the instructions on the screen and then proceed.

*Participants complete part three of the study.*

Thank you for your participation. The objective of our study was to determine the influence of confidence on problem solving. Some individuals believe that the higher the degree of confidence the greater the ability to solve problems. Others believe that confidence is unrelated to problem solving ability in certain situations. Your participation will help advance the body of knowledge on these topics. Thank you again and please wait to receive your participation certificate.
APPENDIX C

TABLES
Table 1

*Means and Standard Deviations for each dependent measure by group*

<table>
<thead>
<tr>
<th>Measure</th>
<th>High SE/High WMC</th>
<th>High SE/Low WMC</th>
<th>Low SE/High WMC</th>
<th>Low SE/Low WMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance (number correct)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Easy</td>
<td>18.29 (2.03)</td>
<td>17.11 (1.94)</td>
<td>16.60 (2.23)</td>
<td>16.63 (2.41)</td>
</tr>
<tr>
<td>Hard</td>
<td>14.14 (4.02)</td>
<td>11.63 (5.60)</td>
<td>10.47 (5.34)</td>
<td>7.83 (5.99)</td>
</tr>
<tr>
<td>Response Time (milliseconds x 1000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Easy</td>
<td>327.35 (135.36)</td>
<td>289.54 (69.90)</td>
<td>323.19 (84.24)</td>
<td>397.12 (127.57)</td>
</tr>
<tr>
<td>Hard</td>
<td>911.19 (389.71)</td>
<td>839.03 (317.03)</td>
<td>882.78 (230.71)</td>
<td>842.92 (234.81)</td>
</tr>
<tr>
<td>Efficiency (performance/time x 1000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Easy</td>
<td>65.94 (28.52)</td>
<td>63.44 (22.51)</td>
<td>54.45 (15.04)</td>
<td>46.20 (16.08)</td>
</tr>
<tr>
<td>Hard</td>
<td>19.25 (11.51)</td>
<td>16.67 (13.31)</td>
<td>12.69 (8.09)</td>
<td>9.44 (7.56)</td>
</tr>
</tbody>
</table>
Table 2

Summary of Significant Interaction and Main Effects by Variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Performance</th>
<th>Time</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complexity/</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficacy</td>
<td>$F_{(1,75)} = 6.393, p &lt; .02, \eta^2 = .079$</td>
<td>NS</td>
<td>$F_{(1,75)} = 4.188, p &lt; .05, \eta^2 = .053$</td>
</tr>
<tr>
<td>Complexity/</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WMC</td>
<td></td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>Main Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficacy</td>
<td>$F_{(1,75)} = 9.95, p &lt; .005, \eta^2 = .117$</td>
<td>NS</td>
<td>$F_{(1,75)} = 9.86, p = .002, \eta^2 = .116$</td>
</tr>
<tr>
<td>WMC</td>
<td>$F_{(1,75)} = 4.25, p &lt; .05, \eta^2 = .054$</td>
<td>NS</td>
<td>NS</td>
</tr>
</tbody>
</table>
Table 3

*Means and Standard Deviations for Main Effects by group*

<table>
<thead>
<tr>
<th>Measure</th>
<th>High SE</th>
<th>Low SE</th>
<th>High WMC</th>
<th>Low WMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance (number correct)</td>
<td>15.29</td>
<td>.530</td>
<td>12.88</td>
<td>.551</td>
</tr>
<tr>
<td>Response Time (milliseconds x 1000)</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>Efficiency (performance/time x 1000)</td>
<td>41.33</td>
<td>2.347</td>
<td>30.70</td>
<td>2.44</td>
</tr>
</tbody>
</table>
REFERENCE


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