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Reactor physics studies for the Advanced Fuel Cycle Initiative (Afci) Reactor -Accelerator Coupling Experiments (Race) Project

Evgeny Yuryevich Stankovskiy
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REACTOR PHYSICS STUDIES FOR THE ADVANCED FUEL CYCLE INITIATIVE (AFCI) REACTOR-ACCELERATOR COUPLING EXPERIMENTS (RACE) PROJECT

by

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ABSTRACT

Reactor Physics Studies for the Advanced Fuel Cycle Initiative (AFCI) Reactor-Accelerator Coupling Experiments (RACE) Project

by

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In the recently completed RACE Project of the AFCI, accelerator-driven subcritical systems (ADS) experiments were conducted to develop technology of coupling accelerators to nuclear reactors. In these experiments electron accelerators induced photon-neutron reactions in heavy-metal targets to initiate fission reactions in ADS. Although the Idaho State University (ISU) RACE ADS was constructed only to develop measurement techniques for advanced experiments, many reactor kinetics experiments were conducted there. In the research reported in this dissertation, a method was developed to calculate kinetics parameters for measurement and calculation of the
reactivity of ADS, a safety parameter that is necessary for control and monitoring of power production.

Reactivity is measured in units of fraction of delayed versus prompt neutron from fission, a quantity that cannot be directly measured in far-subcritical reactors such as the ISU RACE configuration. A new technique is reported herein to calculate it accurately and to predict kinetic behavior of a far-subcritical ADS. Experiments conducted at ISU are first described and experimental data are presented before development of the kinetic theory used in the new computational method.

Because of the complexity of the ISU ADS, the Monte-Carlo method as applied in the MCNP code is most suitable for modeling reactor kinetics. However, the standard method of calculating the delayed neutron fraction produces inaccurate values. A new method was developed and used herein to evaluate actual experiments. An advantage of this method is that its efficiency is independent of the fission yield of delayed neutrons, which makes it suitable for fuel with a minor actinide component (e.g. transmutation fuels). The implementation of this method is based on a correlated sampling technique which allows the accurate evaluation of delayed and prompt neutrons. The validity of the obtained results is indicated by good agreement between experimental
data and simulated responses of neutron detectors. The accuracy (0.2% uncertainty) of the calculated effective delayed neutron fraction, together with the exponential decay of neutron population in the reactor, allows the estimation of the mean neutron generation time to be performed with acceptable uncertainty (1.5%). Because the multiplication constant is a standard result with MCNP, the difference between dynamic reactivity (which is measured in the experiment) and static reactivity is clearly shown.
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DEDICATION

I dedicate this work to the memory of my father.
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CHAPTER 1

INTRODUCTION

1.1. Transmutation

The United States Department of Energy anticipates that by about 2020 commercial utilities will have produced about 86,000 tons of spent nuclear fuel, which exceeds DOE's disposal limit of 63,000 tons of commercial spent nuclear fuel for the Yucca Mountain repository [1]. At present, besides direct spent fuel disposal, spent fuel reprocessing is proposed. The aim of this approach is optimized extraction of transuranics and fission products and, possibly, their transmutation by nuclear reactions [2,3].

The nuclear transformation of some long-lived nuclides into less troublesome ones can be performed only in strong neutron radiation fields [3]. Nuclear fission reactors and spallation sources are the existing sources of required fields for such transformation which is called trasmutation. The benefit of this approach is a reduction of mass of long-lived minor actinides and fission products in the high level radioactive wastes. In theory, the long-
lived fission products can be transformed into short lived or stable isotopes by neutron capture reactions. The practical implementation of this approach has been developed only for $^{99}$Tc, therefore transmutation cannot be a useful method to decrease the long-term risk which is formed by $^{135}$Cs, $^{126}$Sn, $^{79}$Se and $^{129}$I in geological repositories [2].

For this approach to be effective, one must also transmute actinides. The long-lived, highly radiotoxic actinides are transformed by fission reaction into less toxic fission products. Besides fission, such nuclear reactions as capture and (n,2n) take place, therefore some amount of fertile actinides is transformed into fissile actinides, which do not have fission threshold and can provide additional energy gain from transmutation. For practical purposes, the fuel fraction that can be transmuted in a single pass of the fuel through the burner cannot exceed the fuel burnup. For the maximum burnup of 25% and recycle intervals of 6 years, 100 times mass reduction is achieved after 96 years of operation. In other words the transmutation in an actinide burner allows the consolidation of fission products while producing electricity.
1.2. Accelerator-driven system (ADS)

Some actinide burners are based on the concept of accelerator-driven systems, which combine a particle accelerator with a subcritical core. "Subcritical" in this context means the system where the self-sustained fission chain reaction is not possible. Protons are injected onto a spallation target to produce source neutrons for driving the subcritical core. Spallation reactions in the target emit a few tens of neutrons per incident proton, which are introduced into the subcritical core to induce further nuclear reactions. The construction of a safe critical reactor that can utilize minor actinide fuel is difficult because of technical problems with the implementation of a control system. Therefore the core must be subcritical and the power control is performed by an accelerator. The core is very similar to that of a critical reactor, and can be designed to operate with a thermal or fast neutron spectrum.

The theoretical considerations of various accelerator driven systems are provided in Ref. 3. Accelerator-driven systems are subject to risks similar to those of critical reactors, such as solid fuel core melt-down, radioactive leaks to the environment, etc. The coupling between the accelerator and core is an additional risk factor since it
allows propagation of radioactivity through the accelerator in the case of an accident.

Criticality accidents are considered to be impossible with ADS. However, this is true only as long as one can monitor the effective value of reactivity. "Reactivity" in this context means the proximity to criticality. The reactivity is defined as the ratio \( (k_{\text{eff}}-1)/k_{\text{eff}} \), where \( k_{\text{eff}} \) is a multiplication factor. The multiplication factor is defined in Ref.6 as the ratio of the number of neutrons in one generation to the number of neutrons in the preceding generation. The reactivity monitoring cannot be done solely by relating the beam energy to the reactor output power: an increase of the reactivity of the subcritical part can be accompanied by poisoning of the spallation source. In this case the total power may stay constant or even be decreasing, since it consists of two parts: one is the power generated by the target and the other is the power coming from fission reactions in the subcritical core. The second one is proportional to the first one, but the coefficient of proportionality depends on the isotopic composition of the fuel and other factors, therefore it is not constant in time. As a result, the situation when output power does not increase until a critical situation appears is possible. Therefore it is necessary to devise
reliable methods to monitor the effective reactivity of the subcritical reactor.

1.3. Experimental programs with ADS

In the last ten years four experimental programs were performed with ADS prototypes (Ref. 4, 5, 6, 14). Every program had the reactivity measurements as one of the main subjects. The MUSE, TRADE, Yalina, and RACE programs are described briefly below.

1.3.1. MUSE

The MUSE (Multiplication avec Source Externe) experiments were launched in 1995, and the third stage of MUSE experimental program at the MASURCA (MAquette SURgénérateur de Cadarache) facility in Cadarache, France, was performed in 1998 [4]. The fourth stage was performed in 2001. A pulsed 14 MeV neutron source from the (d,t) reaction was coupled with a subcritical core with fast neutron spectrum. The source strength achieved $10^{10}$ neutrons/s. The subcriticality of the zero power system varied widely (the multiplication factor $k_{eff}$ ranged between 0.9 and 0.99). Different materials (sodium, steel, lead) in the diffusing buffer zone around the target were tested. The results of reactivity measurement experiments can be found in Ref. 4.
1.3.2. TRADE

To demonstrate the feasibility of stable operation of an ADS the experimental program called "TRADE" (TRIGA Accelerator Driven Experiment) started in 2001 at ENEA Casaccia Centre, Italy [5]. The original goal was to couple large-scale components of ADS, therefore an existing TRIGA (acronym of "Training, Research, Isotopes, General Atomics") reactor was made subcritical by removing the innermost ring of the fuel core. TRIGA reactor is a facility with a thermal neutron spectrum, i.e. fissions are caused by thermal neutrons (neutrons with a kinetic energy of about 0.0253 eV, which is the most probable energy at a temperature of 293 K). Originally a commercial proton cyclotron with proton energy 110 MeV and current up to 2 mA connected with solid tungsten target was considered for the neutron source. Only zero power experiments were conducted with pulsed neutron sources from (d,t) reactions and a spontaneous fission source $^{252}$Cf. The activity of the isotopic source was 0.4 Ci and the strength of 14.1 MeV pulsed source was $3.3 \cdot 10^7$ neutrons/s. The subcriticality of the core, $k_{\text{eff}}$ ranged between 0.95 and 0.995. The experimental results of the reactivity assessment were published in Ref. 5.
1.3.3. Yalina

One more experimental ADS program is the set of Yalina experiments, which has been performed at the Yalina facility outside Minsk, Belarus [6]. Yalina is similar to the MUSE program, since it uses the zero power core and external neutron generator which is based on (d,d) or (d,t) reactions. The principal difference between Yalina and MUSE is the neutron energy spectrum which is thermal in the Yalina facility. The strength of the neutron source created by the (d,t) generator is up to $2 \cdot 10^{12}$ neutrons/s, and the multiplication factor characterizing the subcriticality level of the core is around $k_{\text{eff}}=0.92$. The experimental results of reactivity measurements are presented in Ref. 6.

1.3.4. RACE

The RACE (Reactor-Accelerator Coupling Experiments) project originally had two parts: zero power experiments and high power experiments. Only zero power experiments were conducted. The major difference between RACE and other programs is the neutron source which was created as the result of photonuclear reactions induced by bremsstrahlung. The bremsstrahlung was created in the solid target of a linear electron accelerator. The determination of parameters of the zero power RACE program is the subject of
this dissertation, therefore the detailed description of experimental setup is provided in the text.

1.4. Outline of dissertation

This dissertation focuses on determination of kinetic parameters of subcritical facility at the Idaho State University Idaho Accelerator Center. Chapter 2 is devoted to theoretical background of reactivity determination methods. The description of the ISU ADS configuration and the results of experiments are provided in Chapter 3. Chapter 4 describes the main software used for analysis and outlines drawbacks of the standard way of calculating kinetic parameters. In Chapter 5 a new method of the calculation of effective delayed neutron fraction is described in detail. The simulations of neutron source and dynamic response to the neutron pulse, the comparison of simulations with experiments, and the determination of kinetic parameters are provided in Chapter 6. Chapter 7 presents conclusions and recommendations for further research and code development.
CHAPTER 2

REACTIVITY DETERMINATION METHODS

2.1. Variety of reactivity determination methods

Reactivity determination is the most important measurements conducted on reactors both at zero power level and with the presence of thermal feedback [7]. The reason is the key role of reactivity in the description of reactor behavior. Therefore it is required to know the dependence of reactivity on changing core loading, fuel composition due to a burnup, etc. Also many safety characteristics such as control and shim rod efficiencies are defined in terms of their influence on reactivity. There are many methods to assess reactivity and they can be divided into a few major groups: asymptotic period measurement, control rod drop method, source jerk method, pulsed neutron source methods, control rod oscillator measurements, and reactor noise methods.

There is no universal method which can be used in any situation. The problem is the absence of an analytic formula to describe the dependence of the detector count
rate on reactivity. The only available approximation that allows such dependence is a point kinetic model. In this case, a simple model is used to describe a neutron population as a product of two functions: one of them is time dependent and the other describes a spatial distribution. Such a simplification can be applied only in the case of stability of major integral kinetic parameters.

2.2. Basic equations and definitions

This section describes the theoretical considerations for the two methods used in the ISU ADS experiments: subcritical multiplication and pulsed neutron source.

Some important definitions must first be introduced. Let \( \vec{r} \) be a position vector; \( E \) is neutron energy; \( \hat{\Omega} \) is the direction of motion characterized by the unit vector \( \hat{\Omega} = \vec{v}/v \), where \( \vec{v} \) is neutron vector velocity and \( v \) is the absolute value of speed; \( t \) is the time; \( \Sigma_t \) is a macroscopic total cross section; \( \Sigma_s \) is a macroscopic scattering cross section; \( S \) is the neutron source; and \( \phi \) is the neutron flux. The neutron transport equation is [8]:

\[
\frac{1}{v} \frac{\partial \phi(\vec{r}, E, \hat{\Omega}, t)}{\partial t} + \hat{\Omega} \cdot \nabla \phi(\vec{r}, E, \hat{\Omega}, t) + \Sigma_t(\vec{r}, E) \phi(\vec{r}, E, \hat{\Omega}, t) =
\]

\[
\frac{\hat{\Omega}}{4\pi} \int dE' \cdot \Sigma_s(E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \phi(\vec{r}, E', \hat{\Omega}', t) + S(\vec{r}, E, \hat{\Omega}, t)
\]

(2.1)
The five terms in Equation (2.1) have the following physical meanings: \( \frac{1}{v} \frac{\partial \varphi(\vec{r}, E, \hat{\Omega}, t)}{\partial t} \) is the rate of change of neutron angular flux at the point \( \vec{r} \); \( \hat{\Omega} \cdot \nabla \varphi(\vec{r}, E, \hat{\Omega}, t) \) is the rate of change of the neutron angular flux due to motion of the neutrons in the straight direction without any collision; \( \Sigma_i(\vec{r}, E)\varphi(\vec{r}, E, \hat{\Omega}, t) \) is the number of neutrons disappearing as a result of nuclear reactions; 

\[
\int_{4\pi} d\hat{\Omega}' \cdot \Sigma_i(E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \cdot \varphi(\vec{r}, E', \hat{\Omega}', t)
\]

is the number of neutrons appearing as a result of scattering from energy and angle \( E', \hat{\Omega}' \) to \( E, \hat{\Omega} \); and \( S(\vec{r}, E, \hat{\Omega}, t) \) is the number of source neutrons appearing from both delayed neutrons and any external sources.

In the following text, shorthand notation and some derivation from Ref. 9 are used. The transport operator \( \hat{L} \) is used to simplify the notation and can be written as:

\[
\hat{L} \varphi = -\hat{\Omega} \cdot \nabla \varphi - \Sigma \varphi + \int_{4\pi} d\hat{\Omega}' \int_{0}^{\infty} dE' \cdot \Sigma' \cdot \varphi'
\]  

(2.2)

The adjoint to the transport operator is used and is defined by the requirement that

\[
(\varphi', \hat{L} \varphi) = (\hat{L} \varphi', \varphi)
\]  

(2.3)
Where \( \phi \) and \( \phi' \) are eigenfunctions of operators \( \hat{L} \) and \( \hat{L}^* \).

The introduction of eigenvalues allows one to write that
\[
\hat{L}\phi = \lambda \phi \quad \text{and} \quad \hat{L}^* \phi' = \lambda' \phi'
\]
and
\[
(\lambda - \lambda') \cdot (\phi', \phi) = 0
\]  
(2.4)

This is the condition of orthogonality of eigenfunctions.

Similarly to (2.2) the adjoint to the transport operator can be written as
\[
\hat{L}^* \phi^* = -\hat{\Omega} \cdot \nabla \phi^* - \Sigma \phi^* + \int_0^{2\pi} dE' \int_0^{\pi} d\Omega' \cdot \Sigma \cdot \phi'
\]  
(2.5)

The before-state and after-state of scattering, i.e. \( (E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \) in \( \hat{L} \) becomes \( (E \rightarrow E', \hat{\Omega} \rightarrow \hat{\Omega}') \) in \( \hat{L}^* \). The boundary condition of no incoming flux in the neutron transport equation is equivalent to the condition of no outgoing adjoint flux.

2.3. Source multiplication experiment

After introduction of the basic notation the idea of a source multiplication experiment can be explained. This method of determination of reactivity is based on the assumption that the reactivity of a subcritical reactor with constant source of neutrons is inversely proportional to detector count rate. The detector can be situated either inside the reactor or close to one. If the detector count
rate is proportional to the number of neutrons in the core, the following considerations are valid. Let equation

\[ \hat{F} \chi \varphi_s + \hat{L} \varphi_s + S = 0 \]  

(2.6)
describe a subcritical reactor with source \( S \) in a steady state condition. Here \( \chi \) is the spectrum of fission neutrons, and \( \hat{F} \varphi_s = \int dE' \int d\Omega' \sum_j \langle \vec{r}, E' \rangle \varphi_j(\vec{r}, E, \Omega, t) \) is the operator of neutron production in the fission process. Then the solution of equation

\[ \frac{1}{k_{\text{eff}}} \hat{F}^* \chi \varphi_i^* + \hat{L}^* \varphi_i^* = 0 \]  

(2.7)
is the importance function \( \varphi_i^* \) (the adjoint flux). The equation (2.7) is an adjoint to the critical equation for a reactor without an external source (this can be called the "conditionally critical" equation). The physical meaning of an adjoint flux can be described as the "importance" or some value directly proportional to the probability of the neutron with certain coordinates in phase space to induce fission in the core. For example in Ref. 10 \( \varphi_i^* \) is defined as a "function proportional to the asymptotic power level resulting from the introduction of a neutron in a critical system at zero power." After multiplication of Equation (2.6) by \( \varphi_i^* \) and Equation (2.7) by \( \varphi_s \) and subtraction of one from another the following result is obtained:
\[ \rho = \frac{(k_{\text{eff}} - 1)}{k_{\text{eff}}} = -\frac{\langle \phi_i^+, S \rangle}{\langle \phi_i^+, \chi \hat{F} \phi_i \rangle} \]  \hspace{1cm} (2.8)

The denominator represents the importance of the neutrons created in the fission, the numerator is a source strength weighted by the same importance function.

The detector registers both neutrons from fission events and neutrons from the external source. Therefore for detector counts \( N_d \) the following equation is valid:

\[ N_d = \epsilon_1 \langle \phi_i^+, \chi \hat{F} \phi_i \rangle + \epsilon_2 \langle \phi_i^+, S \rangle \]  \hspace{1cm} (2.9)

Where \( \epsilon_1 \) and \( \epsilon_2 \) are detector efficiencies corresponding to fission neutrons and source neutrons, respectively. With use of equation (2.9), equation (2.8) can be written as:

\[ |\rho| = \frac{\epsilon_1}{\epsilon_2} \left[ \frac{N_d}{\epsilon_2 \langle \phi_i^+, S \rangle} - 1 \right]^{-1} \]  \hspace{1cm} (2.10)

The simple inverse proportionality relation between detector count rate and reactivity can be observed only when 3 conditions are satisfied:

1) The efficiencies \( \epsilon_1 \) and \( \epsilon_2 \) are constant;

2) The effective source strength \( \langle \phi_i^+, S \rangle \) is constant; and

3) The detector registers significantly more neutrons from fission events than directly from the source.

When these conditions cannot be satisfied some additional computational work is required. The efficiencies of
detector and \( (\phi^+_\lambda, S) \) should be calculated for every particular configuration with different reactivity.

The most widely used practical application of the source multiplication method is reactivity monitoring during core loading. The "approach to criticality experiment" takes into consideration the variation of the effective multiplication factor in the range from 0 to 1. In this case the neutron source and detectors are installed in an unloaded core with multiplication equal to zero. Usually the detectors are installed outside of the core, while the source is located in the center of the reactor. Since the multiplication of the neutron population in fissions reactions increases detector count rates, the measurement of reactivity becomes possible in some relative units. As the reactor operates closer to criticality, the count rate increases. The curve representing the inverse dependence of detector counts on amount of fissile material will approach zero at the critical condition. If \( \Delta k = 1 - k_{\text{eff}} \), the general dependence (2.10) can be rewritten as:

\[
N_d^{-1} = \frac{|\rho|}{(\epsilon_1 + \epsilon_2 \rho)(\phi^+_\lambda, S)} = \frac{|\Delta k|}{\epsilon_1 (\phi^+_\lambda, S) \left( \frac{\epsilon_2 - 1}{\epsilon_1} \right) |\Delta k| + 1} \quad (2.11)
\]

Two simplified cases of this dependence are used in practice. One of them is when the detector is situated next
to the source and $\epsilon_1 = \epsilon_2 = \epsilon$, so the count rate is proportional to $\Delta k^{-1}$, i.e. multiplication:

$$N_d^{-1} = \frac{|\rho|}{(1+|\rho|)\epsilon^\langle \phi^*_i, S \rangle} = \frac{|\Delta k|}{\epsilon^\langle \phi^*_i, S \rangle} \quad (2.12)$$

The other case is when $\epsilon_1^\langle \phi^*_i, xF^R \phi_s \rangle \gg \epsilon_2^\langle \phi^*_i, S \rangle$, i.e. when the reactor is close to being critical and the multiplication is dominant in comparison to the source or when the detector is far away from the reactor, so $\epsilon_1 \gg \epsilon_2$. In this case $\frac{N_d}{\epsilon_2^\langle \phi^*_i, S \rangle} \gg 1$, therefore:

$$N_d^{-1} = \frac{|\rho|}{\epsilon_1^\langle \phi^*_i, S \rangle} \quad (2.13)$$

The count rate in this case is proportional to $\rho^{-1}$.

As can be seen from Equations (2.12) and (2.13), to obtain the explicit function $N_d(\rho)$ (or $N_d(k_{df})$) the calculation of detector efficiency and the $\langle \phi^*_i, S \rangle$ is required. During the core loading the curve of the ratio of the detector count rate in the current geometry to the detector count rate in the reference geometry as a function of fissile material mass is usually monitored. The value of $k_{df} = 0.7$ (when approximately $1/3$ of critical mass is loaded) is the usual choice for a reference reactor condition. The
monitored ratio for two cases (2.12) and (2.13) can be used in:

$$\frac{\epsilon_i}{\epsilon_0} \frac{N_{d,i} \langle \phi_{k,i}^*, S \rangle}{N_{d,0} \langle \phi_{k,0}^*, S \rangle} = \frac{\rho_i}{\rho_0} \frac{|1 + \rho_i|}{|1 + \rho_0|} = \frac{\Delta k_i}{\Delta k_0} \tag{2.14}$$

$$\frac{\epsilon_i}{\epsilon_0} \frac{N_{d,i} \langle \phi_{k,i}^*, S \rangle}{N_{d,0} \langle \phi_{k,0}^*, S \rangle} = \frac{\rho_i}{\rho_0} \tag{2.15}$$

Where $N_{d,i}$ and $N_{d,0}$ are count rates in the current and reference conditions, respectively. When the reactor is close to critical the difference between $\rho$ and $(1 - k_{\text{eff}})$ is negligible, but when $k_{\text{eff}} < 0.9$ the right choice of dependence should be made.

Therefore if a critical condition cannot be achieved, either the calculated or measured (by some different method) value of $\rho_0$ should be used to assess reactivity.

2.4. Differential pulsed neutron source method

The idea of this method, which is also called the "a method", is based on the point kinetic equation, namely on exponential dependence $(N(t) = \exp(-\alpha \cdot t))$ of neutron population in time, where $\alpha$ is the decay constant [11]. This behavior takes place after impingement of neutrons from a pulsed external source and establishment of exponential decay after a short transient. Figure 2.1 shows the typical
dependence of detector count rate on time in the case of a Dirac pulse.

A few different parts of typical response can be mentioned:
Region I - increase in response due to the impingement of source neutrons
Region II - transition to establishment of exponential decay
Region III - exponential decay, $N(t) \sim \exp(-\alpha \cdot t)$
Region IV - non-exponential dependence due to the influence of background
Region V - constant background created by delayed neutrons

Figure 2.1. The typical response of a detector to a neutron Dirac pulse.
To conduct measurements of the reactivity by the pulsed neutron source (PNS) differential method the following experimental setup is usually used. The pulsed neutron source is situated inside the core or very close to the core in the reflector. The detector is also placed in the core or in the reflector, but not close to the source. The detector counts are registered with a time analyzer which is turned on at the moment of the pulse. The time width of one channel of the time analyzer should be less than or equal to the pulse width. The time between two consecutive pulses should be long enough to allow the neutron population of prompt neutrons to die away.

The decay constant $\alpha$ may vary widely for different types of reactors and for different levels of subcriticality. But the general rule for period $\tau$ between two pulses is $\tau \geq 10 \cdot \alpha^{-1}$. In the differential $\alpha$ method the constant $\alpha$ is the parameter for measurement because the simple analytical relation between reactivity $\rho$ and $\alpha$ can be obtained. When the point kinetic model is valid for prompt neutrons (delayed neutrons are neglected in this method) the detector count rate can be written as:

$$N_d(t) = N_0 \exp(-\alpha \cdot t)$$  \hspace{1cm} (2.16)
where the constant \( \alpha = \frac{1}{l} \frac{k_{\text{eff}}(1-\beta_{\text{eff}})}{1-k_{\text{eff}}(1-\beta_{\text{eff}})} \); \( \beta_{\text{eff}} \) is the effective delayed neutron fraction which is defined in Ref. 8 as "the fraction of delayed neutrons which provides just enough extra multiplication to achieve criticality"; and \( l \) is the mean lifetime of prompt neutron in reactor. So when integral parameters \( \beta_{\text{eff}} \) and \( l \) (or the mean neutron generation time \( \Lambda = l/k_{\text{eff}} \) which is defined in Ref. 9 as "time between birth of neutron and subsequent absorption inducing fission") are known, the reactivity, measured in effective delayed neutron fractions is:

\[
\frac{\rho}{\beta_{\text{eff}}} = \frac{(k_{\text{eff}}-1)}{k_{\text{eff}} \beta_{\text{eff}}} = 1 - \frac{\alpha \cdot l \cdot k_{\text{eff}}}{\beta_{\text{eff}}} = 1 - \frac{\alpha \Lambda}{\beta_{\text{eff}}} \tag{2.17}
\]

Therefore to obtain the reactivity either reliable computational values of \( \beta_{\text{eff}} \) and \( \Lambda \) are required or their measured values at delayed criticality should be obtained. Then:

\[
\frac{\rho}{\beta_{\text{eff}}} = \frac{\alpha \Lambda}{\alpha_c \Lambda_c - \beta_{\text{eff}}} \tag{2.18}
\]

Where \( \alpha_c = \frac{\beta_{\text{eff}}}{\Lambda_c} \) is the decay constant at delayed criticality.

The equation can be further simplified that \( \beta_{\text{eff}} \) and \( \Lambda \) are the same in critical and subcritical reactors (according to
Ref. 15 this is robust approximation in the case of near-critical and thermal systems, such as the ISU ADS), then:

\[
\frac{\rho}{\beta_{\text{eff}}} = \frac{\alpha}{\alpha_c} - 1
\]

(2.19)

The ratios \(\frac{\beta_{\text{eff}}}{\beta_{\text{eff, c}}}\) and \(\frac{\Lambda}{\Lambda_c}\) should be calculated and must be equal to unity to allow the use of Equation (2.19).

Therefore to apply this method to assess experimentally the reactivity, one condition must be observed: the exponential dependence of the detector count rate on time (region III in Figure 2.1)

2.5. Integral method (area ratio method)

This method is based on the separation of the total detector response into two parts: (1) response to the prompt neutrons, created by multiplication of external source neutrons and (2) response created by multiplication of the approximately constant-in-time delayed neutron source. The ratio of these two values depends on reactivity and can approach zero when the reactor is close to delayed criticality. This may happen because the response to an external source is always limited and the power level, sustained by delayed neutrons, may be unconstrained. To derive the area-ratio estimator the neutron transport equation should be written separately for prompt and
delayed neutrons, together with the differential equation for delayed neutron's predictor concentration \( C_{pr} \):

\[
\frac{1}{\nu} \frac{\partial \varphi_p}{\partial t} = (1 - \beta) \hat{F} \varphi_p + \hat{L} \varphi_p + S \tag{2.20}
\]

\[
\frac{1}{\nu} \frac{\partial \varphi_d}{\partial t} = (1 - \beta) \hat{F} \varphi_d + \hat{L} \varphi_d + f_d \lambda C_{pr} \tag{2.21}
\]

\[
\frac{\partial C}{\partial t} = \beta \hat{F} (\varphi_p + \varphi_d) - \lambda C_{pr} \tag{2.22}
\]

The delayed \( f_d \) and prompt \( f_p \) neutron spectra are normalized to unity: \[\int_0^{E_{\text{max}}} f dE = 1.\] Equations (2.21) and (2.22) become the following after integration over period \( \tau \) (since delayed neutron population before each pulse is constant at the achieved "equilibrium" condition):

\[
0 = \int_0^\tau [(1 - \beta) \hat{F} \varphi_d + \hat{L} \varphi_d + f_d \lambda C_{pr}] dt \tag{2.23}
\]

\[
0 = \int_0^\tau [\beta \hat{F} (\varphi_p + \varphi_d) - \lambda C_{pr}] dt \tag{2.24}
\]

After multiplication of both of these equations by the adjoint flux (obtained from Equation (2.7)) and integration over energy and volume, the combined equation becomes:

\[
0 = \int_0^\tau \langle \varphi_k^*, \chi \hat{F} \varphi_d \rangle dt + \int_0^\tau \langle \varphi_k^*, \hat{L} \varphi_d \rangle dt + \int_0^\tau \langle \varphi_k^*, \beta f_d \hat{F} \varphi_p \rangle dt \tag{2.25}
\]
Where \( \chi = (1 - \beta) f_p + \beta f_d \) is the total fission spectrum.

Multiplying the adjoint Equation (2.7) by \( \varphi_d \) and integrating over space, energy and time:

\[
\frac{1}{k_{\text{eff}}} \int_0^r \langle \chi \hat{F}^* \varphi_k^*, \varphi_d \rangle dt = -\int_0^r \langle \hat{L}^* \varphi_k^*, \varphi_d \rangle dt \tag{2.26}
\]

The equation for reactivity can be obtained:

\[
\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = 1 - \frac{1}{k_{\text{eff}}} = 1 + \frac{\int_0^r \langle \hat{L}^* \varphi_k^*, \varphi_d \rangle dt}{\int_0^r \langle \chi \hat{F}^* \varphi_k^*, \varphi_d \rangle dt} \tag{2.27}
\]

After use of Equation (2.3): \( (\varphi^*, \hat{L} \varphi) = (\hat{L}^* \varphi^*, \varphi) \);

\( (\varphi^*, \hat{F} \varphi) = (\hat{F}^* \varphi^*, \varphi) \) combined with Equation (2.25) the reactivity is determined as:

\[
\rho = -\frac{\int_0^r \langle \varphi_k^*, \beta \hat{f}_d \hat{F} \varphi_p \rangle dt}{\int_0^r \langle \varphi_k^*, \chi \hat{F} \varphi_d \rangle dt} \tag{2.28}
\]

This mathematically correct expression cannot be used in an experiment because the numerator cannot be measured.

Instead of Equation (2.28), different approximations, valid with some assumptions, are used. The most practical is a simple area ratio formula, proposed by Sjöstrand in Ref. 12:

\[
\rho = \frac{A_p}{A_d} \beta \tag{2.29}
\]
Where $A_p$ and $A_d$ are the parts of total core response created by prompt and delayed neutrons. This method correctly represents Equation (2.28) when the spectral difference between delayed and prompt neutrons, weighted with the adjoint function, is negligible. Therefore, for example, for large thermal cores this method is a very good approximation. When the point kinetic model is valid, the reactivity determined by Equation (2.29) does not depend on detector position. Practical implementation of this method can be explained by the response presented in Figure 2.1. The prompt area is obtained as the difference between total integral under the curve and constant background created by delayed neutrons, and Equation (2.29) can be written as:

$$\rho = \frac{\int N dt - \tau \cdot N(\tau)}{\tau \cdot N(\tau) \beta} \quad (2.30)$$

Besides the simple area ratio method many modifications were proposed to describe specific assumptions, but there is no universal integral method reliable for every type of reactor. The description of some of approximations is provided in Ref. 7.
CHAPTER 3

ISU ADS EXPERIMENTAL CONFIGURATION AND CONDUCTED EXPERIMENTS

3.1. Materials and experimental setup

The Idaho State University (ISU) Reactor-Accelerator Coupling Experiments were conducted at the Idaho Accelerator Center between 2004 and 2006. The subcritical core was built from materials of an ISU subcritical assembly described in Ref. 13. This subcritical core was coupled with a linear accelerator to use an electron beam hitting the target to induce bremsstrahlung photon-neutron reactions and to generate initial source neutrons. The source neutrons were multiplied in fission chain reactions inside the assembly and created a neutron flux distribution which was monitored with different detectors. After some optimization the following design was chosen [14]. The fuel plates were equally spaced around a cylindrical target and were surrounded by a graphite reflector. The whole core was submerged in a water filled aluminum tank to use natural convection as a cooling system for the target and fuel
plates. The horizontal and vertical views of the ISU ADS subassembly along a beam line are presented in Figure 3.1 and Figure 3.2.

A total of 150 fuel plates are used in the core. Each plate is 2.032 mm thick, 76.2 mm wide, and 660.4 mm long. The plates are constructed with a fuel bearing region of 1.016 mm thick, 69.85 mm wide, and 609.6 mm long, that is contained inside the aluminum envelope. The fuel bearing region consists of a uranium-aluminum mixture with the uranium enriched to 20% $^{235}\text{U}$ by weight. The total amount of uranium used is 7615 g with 1510 g being $^{235}\text{U}$. It is assumed that all the plates are identical and that the uranium is distributed equally among them. This fuel was manufactured by M&C Nuclear Co.

![Figure 3.1. The horizontal cross section view of the ISU ADS assembly.](image)
Figure 3.2. The vertical cross section view of the ISU ADS assembly.

The plates are placed in the 6 trays with 25 plates in each tray. The distance between plates inside one tray is around 4 mm, but this value is not constant due to plates bending and non-ideal rectangular shape. The trays are installed in 3 rows forming the bottom, middle and top levels. The bottom and top trays are moved together, while the middle trays are arranged next to the target. A cross-sectional view of the trays with fuel plates is presented in Figure 3.3.
The target is cut from a solid piece of copper-tungsten composite (75% tungsten, 25% copper by weight). It is 89 mm long by 70 mm in diameter. It is welded on one side to a stainless steel flange which is connected to an aluminum beam guide tube. This tube has vacuum inside and it is attached to the aluminum vessel of the subassembly. This tube also is attached to the accelerator and its vacuum system, and it is part of the electron beam delivery system.

The reflector of the assembly was constructed from reactor grade graphite bricks. Unfortunately the origin of
this graphite is unknown. Since this graphite was used at ISU since the 1960's the most probable standard for this graphite is H-451. The description of this specification can be found in Ref. 15 or Ref. 16. The information regarding the most typical impurities of reactor-grade graphite is presented in Ref. 17. As a result the density of the ISU ADS graphite can be in the range 1.5-1.8 g/cm³.

The effect of different reflector densities may be significant because the probability of neutron interaction with nuclei is proportional to the density.

In the majority of experimental runs, the linear accelerator was used to provide source neutrons. In the ISU ADS, the linac with energy of electrons in the range of 20-40 MeV was used. These high-energy electrons created bremsstrahlung photons in the target and photonuclear production of neutrons was induced. Since the peak beam power occurs at less than the maximum energy, the accelerator during experiments was operated at 20 MeV. The pulse widths of the linac are variable from nanoseconds to 10 microseconds with pulse rates up to several hundred hertz.

Besides the accelerator, a Pu-Be sealed neutron source was used in some experiments. The description of a similar source can be found in Ref. 7. This particular cylindrical
source was manufactured by Monsanto Research Company - Mound Laboratory and contains 18 grams of commercial plutonium (91% $^{239}\text{Pu}$, 8% $^{240}\text{Pu}$, 1% $^{241}\text{Pu}$ and $^{241}\text{Am}$) with dimensions: 25 mm in diameter, 35 mm length. The strength of this source is $2.37 \times 10^6$ neutrons/s. The Pu-Be source was either attached to the target by a plastic holder or could be positioned along the beam line between the target and the graphite reflector. The target with attached holder and Pu-Be source are shown in Figure 3.4.

![Diagram of target with attached Pu-Be source](image)

**Figure 3.4.** Target with attached Pu-Be source.

Two LND 30763 fission chambers, manufactured by LND Inc. were used to measure neutron flux. The detail specification can be found on the manufacturer’s web site. The fission
chambers are argon-filled tubes with dimensions: 140 mm length, 12.7 mm diameter. The effective dimensions are: 63.5 mm length, 10 mm diameter. The neutron sensitive material is uranium with 93% enrichment of U\textsuperscript{235}. Each fission chamber contains 16 mg of uranium. Fission chambers were installed in the top layers of the graphite reflector, as shown in Figure 3.5. The distance from the back side of the target to the plane with the fission chambers is 45 mm.

![Figure 3.5. View of cross section with fission chambers in the reflector.](image)

The fission chambers were operated only in current sensitive pulse mode using the analog electronic CANBERRA
ADS7820 modules that feed the data acquisition system with a TTL (transistor-transistor logic) signal of 5 V amplitude and 100 ns width. The X-MODE data acquisition system was designed to operate on experimental nuclear reactors according to Ref. 18. It can process both digital and analog signals and can be utilized to perform state of the art neutronic experiments. The X-MODE has integrated in a single system all features needed for reactor measurements. It also provides tools to improve data processing such as online treatment and data reduction algorithms.

The main feature of the X-MODE system is a precise time marking capability. Time marking acts as a triggerless acquisition mode in which each event is counted and marked, so the maximum amount of available experimental information is stored. As a result, data processing is not limited by the acquisition settings. Such a triggerless system is widespread in the field of particle physics and has been used in noise measurements. Because of improvements in its storage capabilities it can now be used in other measurements. The mentioned time marking is a precise time stamping capability accurate to 25 ns.

A set of 40 gold foils were used as integral detectors dedicated to measure the spatial flux distribution. This common technique of neutron activation of samples placed in
the experimental assembly is followed by removal and counting. Due to the geometry of the assembly, only the distribution in the central vertical plane along the beam line could be measured.

3.2. Conducted experiments

The X-MODE data acquisition system was used only in the August and October 2006 series of experiments at the Idaho Accelerator Center (IAC). Approach to criticality experiments, pulsed neutron source (PNS) measurements with different positions of detectors and various parameters of accelerator, and beam trip measurements were performed in August. In October, in an attempt to improve statistics due to low count rates, similar PNS experiments were repeated with the Pu-Be source in the core. Measurements of count rates at different positions of the Pu-Be source along the beam line to obtain information about the importance function distribution were also performed. The flux distribution measurement with gold foils was performed during the final series of pulsed neutron source experiments.

3.3. Source multiplication experiment

The source multiplication record during an "approach to criticality experiment" is a required procedure according to the Nuclear Regulatory Commission (NRC). The source
multiplication measurement is a very inaccurate assessment of reactivity if a critical condition is not finally achieved. The theoretical basis of reactivity measurements by the source multiplication method is described in the previous chapter. The initial phase of the experiment was performed with the Pu-Be neutron source installed in the core without fuel. Then consecutive measurements of the neutron detector count rate were taken during core loading. The experimental data and calculated 95% confidence intervals for count rates for fission chambers A and B are presented in Tables 3.1 and 3.2.

<table>
<thead>
<tr>
<th>Step #</th>
<th>Number of fuel plates</th>
<th>Duration (s)</th>
<th>counts</th>
<th>count rate (cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>43977</td>
<td>217981</td>
<td>4.957±0.021</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>1326</td>
<td>15943</td>
<td>12.023±0.19</td>
</tr>
<tr>
<td>3</td>
<td>140</td>
<td>660</td>
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<td>24.9±0.4</td>
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<tr>
<td>4</td>
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<td>480</td>
<td>12496</td>
<td>26.0±0.5</td>
</tr>
<tr>
<td>5</td>
<td>144</td>
<td>606</td>
<td>16633</td>
<td>27.4±0.4</td>
</tr>
<tr>
<td>6</td>
<td>146</td>
<td>588</td>
<td>16720</td>
<td>28.4±0.4</td>
</tr>
<tr>
<td>7</td>
<td>148</td>
<td>568</td>
<td>16684</td>
<td>29.4±0.5</td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>560</td>
<td>16728</td>
<td>29.9±0.5</td>
</tr>
</tbody>
</table>
Table 3.2. Source multiplication data: fission chamber A

<table>
<thead>
<tr>
<th>Step #</th>
<th>Number of fuel plates</th>
<th>Duration (s)</th>
<th>counts</th>
<th>count rate (cps)</th>
</tr>
</thead>
<tbody>
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<td>245515</td>
<td>5.583±0.023</td>
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<tr>
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<td>1326</td>
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<tr>
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<td>140</td>
<td>660</td>
<td>15344</td>
<td>23.2±0.4</td>
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<tr>
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<td>142</td>
<td>480</td>
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<td>23.6±0.4</td>
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<td>588</td>
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<td>148</td>
<td>568</td>
<td>15083</td>
<td>26.6±0.4</td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>560</td>
<td>15150</td>
<td>27.1±0.4</td>
</tr>
</tbody>
</table>

The uncertainties of measured count rates were calculated by using the standard Poisson distribution law ($\sigma = \sqrt{N}$), where $\sigma$ is a standard deviation, and $N$ is the number of counts). During core loading a convenient way to plot the subcritical multiplication curve is by representing the ratio of count rate at the initial condition (without fuel) to the current count rate. The source multiplication curves for detectors B and A are presented in Figures 3.6 and 3.7.
Figure 3.6. Source multiplication curve for detector B. Dashed lines show the prospective number of plates at criticality.

Figure 3.7. Source multiplication curve for detector A. Dashed lines show the prospective number of plates at criticality.
The extracted values for the effective multiplication constant from this experiment are 0.834 for fission chamber B and 0.794 for fission chamber A. Both of them are related to the 150 fuel plates loaded into the core. Since at least two of the required conditions (described in previous chapter) for obtaining experimental values in the source multiplication experiment were not satisfied (e.g. detector efficiencies corresponding to fission neutrons were not constant, and the effective source strength changed during the loading), these values are unreliable.

3.4. Source importance measurements

The solution of the adjoint to critical equation is a weighting function used in the calculation of most functionals in reactor physics. Since it is an adjoint to the critical equation, the experimental measurements of this function have meaning only for a critical reactor. Therefore any measurements conducted in a subcritical core should be trusted with caution. Only the ratio (relative value) of importances for neutrons with different energies, angular distributions, etc. have physical meaning. Moreover, since this function is the solution of a homogeneous equation and it is one of a complementary function, the choice of importance function depends on normalization. Practically this function is defined as the
ability to reproduce neutrons in the core. Since there is no way to measure differential characteristics of the importance function, some functionals proportional to the importance function are measured in experiments.

In reactor physics, the average importance over the fission neutron spectrum is used. The technique of determination of this functional is based on the measurement of the neutron multiplication produced by the local source which should have a fission spectrum. When the source is placed at different positions inside the core, the detector count rate is changed, and relative importance is defined as the ratio of count rates at the current position to the maximum count rate. Therefore the theoretical considerations in the previous chapter regarding the source multiplication method of reactivity measurements are valid for source importance measurements.

To conduct "source importance" measurements the spectrum of source should be the same as the fission spectrum. In practice the $^{252}$Cf source is used, because it has neutron spectrum similar to prompt fission neutron spectrum. In the ISU ADS experiments, the $^{252}$Cf source was not available, therefore "source importance" measurements were conducted with a Pu-Be source instead of $^{252}$Cf source, and in a highly subcritical core instead of a critical reactor. Also only
the positions along a beam line behind the target were available, so the results of such measurements may be used only as approximate values of relative importance. The experimental results are presented in Table 3.3. Values of relative source importance are presented in Figure 3.8.

3.5. Pulsed neutron source measurements

A variety of pulsed neutron source experiments were performed during the August and October 2006 experimental campaigns. They differ by the number of loaded fuel plates, the presence of Pu-Be in the core, and the accelerator parameters.

Table 3.3. Count rates of fission chambers A and B at different positions of Pu-Be source along the beam line.

<table>
<thead>
<tr>
<th>Location from the target (mm)</th>
<th>FC A</th>
<th>FC B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R (cps)</td>
<td>σR (cps)</td>
</tr>
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<td>35-70</td>
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<tr>
<td>87-122</td>
<td>19.26</td>
<td>0.15</td>
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<tr>
<td>138-173</td>
<td>16.73</td>
<td>0.15</td>
</tr>
<tr>
<td>189-224</td>
<td>12.19</td>
<td>0.09</td>
</tr>
<tr>
<td>230-265</td>
<td>10.16</td>
<td>0.09</td>
</tr>
<tr>
<td>265-300</td>
<td>8.05</td>
<td>0.07</td>
</tr>
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</table>
A long accelerator run with stable parameters is required to perform area-ratio measurements therefore not all recorded data satisfied this condition. As explained in the theoretical background of the area-ratio method, the time required to develop the delayed neutron precursor population should be more than $10/\lambda_{\text{min}}$, where $\lambda_{\text{min}}$ is the minimal delayed constant for one of the precursor groups. For $^{235}\text{U}$ $\lambda_{\text{min}}=0.0133$ s$^{-1}$, therefore $t>12$ min is required to start measurements. Also due to low flux values, the required statistics can be accumulated only during a large number of repeated pulses, therefore the usual time frame
to perform a PNS experiment was around one hour. The stability of accelerator parameters (e.g., current, pulse width, and repetition rate) during this period was the main technical issue. The best stability was observed at the following values: beam current 10 mA, pulse width 1 μs, repetition rates 30 Hz. Experiments conducted with these parameters were considered as reference PNS measurements.

In August 2006, when the PNS experiments were performed, the positions of detectors differed from the setup described above, and they are presented in Figure 3.9.

Figure 3.9. A project view of assembly with fission chambers.
The data in August were collected with a data acquisition system sensitivity equal to one count per 1000 neutrons. The August experimental data are presented in Figures 3.10, 3.11, 3.12. Similar data for October experiments conducted in the presence of a Pu-Be source are presented in Figures 3.13, 3.14, 3.15. During this experiment, the data acquisition system with one count per $10^4$ neutrons was used. Detector counts have a Poisson probability distribution.

Figure 3.10. PNS histogram, collected in 2 hours run. The exponential die-away behavior is shown.
Figure 3.11. Front of PNS response.

Figure 3.12. Background level of PNS response.
Figure 3.13. PNS histogram with Pu-Be source in the core. The exponential die-away is shown.

Figure 3.14. Front of PNS response, Pu-Be source in the core.
The values of prompt integrals, delayed integrals, and confidence intervals for reactivity estimators extracted from experimental data are presented in Table 3.4. The bias of the reactivity estimator due to a non-stable pulse width is not required for thermal systems unlike for fast reactors [19]. The delayed area correction, which comes from the fact the delayed area cannot be obtained directly from the histogram background, is required only when the subcritical assembly is close to delayed criticality ($\rho < 0.5 \cdot \beta_{\text{eff}}$).
Table 3.4. Integrals extracted from experimental PNS histograms and 95% confidence intervals for reactivity in units of effective delayed neutron fraction.

<table>
<thead>
<tr>
<th>Fission chamber</th>
<th>Pu-Be source</th>
<th>$t_{max}$ (μs)</th>
<th>$A_{prompt}$ (counts)</th>
<th>$A_{delayed}$ (counts)</th>
<th>Reactivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>No</td>
<td>405±5</td>
<td>90442925</td>
<td>5749036</td>
<td>15.732±0.024</td>
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<tr>
<td>A</td>
<td>No</td>
<td>495±5</td>
<td>64476805</td>
<td>4094362</td>
<td>15.748±0.028</td>
</tr>
<tr>
<td>B</td>
<td>Yes</td>
<td>305±5</td>
<td>9049736</td>
<td>563132</td>
<td>16.070±0.035</td>
</tr>
<tr>
<td>A</td>
<td>Yes</td>
<td>525±5</td>
<td>9082547</td>
<td>577487</td>
<td>15.728±0.034</td>
</tr>
</tbody>
</table>

In Ref. 20 one can find the technical details of reducing experimental uncertainties during ISU ADS experiments caused by utilization of a linear electron accelerator to produce neutrons in photonuclear reactions. Such factors as detector saturation and dark current were treated in the following way. The detector saturation (the saturation of the signal delivered through the fission chamber and associated amplifier) caused by a significant photon flash, which was generated right after every accelerator shot, was successfully removed by use of current-sensitive fast amplifiers. The linac contains a microwave source (klystron), a waveguide, and series of cavities, where electrons are accelerated by the electric field of the microwaves traveling in the cavities. The generated
microwaves are likely to cause a parasitic signal (dark current) to appear on the signal going out of the detectors and transported by the cables. While this signal was observed during experiments, it was significantly reduced by special shielded cables (FILECA "Etudes") provided by CEA.

The confidence intervals presented in Table 3.4 were assessed with the use of a well known error propagation formula. If the area ratio reactivity is rewritten as:

\[ \rho(\$) = 1 - \frac{I - B_0 T}{(B - B_0) T} \]  \hspace{1cm} (3.1)

Where \( T \) is the mean value of the observed time \( T_p \) between two pulses. Its standard uncertainty \( \sigma_T \) is given by that of any mean value, that is, the sample standard deviation divided by the square root of the number of pulses \( P \).

\[ I = \sum_{n=1}^{N} H_n \bar{\lambda} \]  \hspace{1cm} is the total integral of the histogram \( H_n \). Its standard uncertainty \( \sigma_I \) is equal to \( \sqrt{I \bar{\lambda}} \) for the number of counts \( H_n \) in each bin following a Poisson distribution.

\( B_0 = P R_{\text{off}} \bar{\lambda} \) is the background level accounted for by the count rate \( R_{\text{off}} \) due to some neutron source when the neutron generator is off. Its relative standard uncertainty is
equal to that of $R_{\text{off}}$. In the ISU ADS experiments, $R_{\text{off}}$ is caused by the Pu-Be source.

$B$ is the overall background level accounted for by the delayed neutrons and the count rate $R_{\text{off}}$. It is the mean value of the M bin values belonging to the constant tail of the histogram. Its uncertainty is $\sqrt{B/M}$.

Then uncertainty is derived as:

$$
\sigma_{\rho(b)}^2 = \left( \frac{\partial \rho}{\partial l} \right)^2 \sigma_l^2 + \left( \frac{\partial \rho}{\partial B} \right)^2 \sigma_B^2 + \left( \frac{\partial \rho}{\partial B_0} \right)^2 \sigma_{B_0}^2 + \left( \frac{\partial \rho}{\partial T} \right)^2 \sigma_T^2 \\
+ 2 \left( \frac{\partial^2 \rho}{\partial l \partial B} \sigma_{l,B_0}^2 + \frac{\partial^2 \rho}{\partial B \partial B_0} \sigma_{B,B_0}^2 + \frac{\partial^2 \rho}{\partial B \partial T} \sigma_{B,T}^2 \right)
$$

Where the covariance terms are:

$$
\sigma_{B,T}^2 = \sigma_{B_0,T}^2 = \sigma_{B,l}^2 = 0 \\
\sigma_{l,B_0}^2 = \sigma_{l,T}^2 = B_0 \sigma_T^2, \quad \sigma_{B,B_0}^2 = \sigma_{B_0}^2
$$

and sensitivities are:

$$
\frac{\partial \rho}{\partial l} = -\frac{1}{(B-B_0)T} ; \quad \frac{\partial \rho}{\partial B} = 1 - \rho ; \quad \frac{\partial \rho}{\partial B_0} = \frac{\rho}{B-B_0} ; \quad \frac{\partial \rho}{\partial T} = \frac{1}{(B-B_0)^2 T} ;
$$

$$
\frac{\partial^2 \rho}{\partial l \partial B} = \frac{1}{B-B_0} ; \quad \frac{\partial^2 \rho}{\partial B \partial B_0} = 2 \frac{1}{B-B_0} - \frac{1}{(B-B_0)^2} ; \quad \frac{\partial^2 \rho}{\partial B \partial T} = -\frac{1}{B-B_0}
$$

3.6. Gold foil measurements

The spatial distribution of the neutron flux density in the ISU subcritical assembly was measured with use of the gold foil activation technique. The foils were previously attached to a plastic plate and installed into the core.
The plate was vertically oriented and coincided with the beam line. The foil positions can be described with the use of the following coordinate system. The X axis is along the beam line, the Z axis is vertically upward, and the Y axis creates right-hand system. The origin coincides with the middle of the line between the centers of the fuel plates next to the target. The sketch of the foil positions is presented in Figure 3.16.

Figure 3.16. Gold foil positions and coordinate system.
The gold foils were irradiated during the pulsed neutron source experiment without external Pu-Be source in the core. The round shape foils with 12.7 mm in diameter and 0.125-0.128 g mass were used (the mass was printed on each foil). The irradiation time was 2 hours and the accelerator parameters were the same as during the reference PNS measurements. The gold foils were removed from the assembly and activities were measured from 19 to 27 hours after irradiation. The intensity of gamma rays with energy 411.8 keV was measured, and correction coefficients were applied, taking into consideration the time between irradiation and measurement and the different foil masses. The individual measurement time for each foil depends on accumulated counts, and the final statistical uncertainty of counts was kept on an approximately constant level. The counting efficiency of 411.8 keV photons was 2.38 percent. The value of 411.8 keV is defined by the β decay of $^{198}\text{Au}$ created in a $^{197}\text{Au}(n,\gamma)^{198}\text{Au}$ reaction. The half life of $^{198}\text{Au}$ is 2.695 days, therefore correction for measurement time is required. The coordinates of the foils and calculated values of activities per unit of mass right after irradiation are presented in Table 3.5.
Table 3.5. The positions of the gold foils and results of the activity measurement.

<table>
<thead>
<tr>
<th>Foil</th>
<th>X (mm)</th>
<th>Z (mm)</th>
<th>411.8 keV peak (counts)</th>
<th>Count rate (cps)</th>
<th>Relative error (%)</th>
<th>Calculated initial activity (Bq/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
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<td>45</td>
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Table 3.5 (Continued)

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<th>Z (mm)</th>
<th>411.8 keV peak (counts)</th>
<th>Count rate (cps)</th>
<th>Relative error (%)</th>
<th>Calculated initial activity (Bq/g)</th>
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Table 3.5 (Continued)

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<th>Z (mm)</th>
<th>411.8 keV peak (counts)</th>
<th>Count rate (cps)</th>
<th>Relative error (%)</th>
<th>Calculated initial activity (Bq/g)</th>
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CHAPTER 4

MONTE CARLO SIMULATION WITH MCNP CODE

4.1. The Monte Carlo method of neutron transport equation solution

Neutron transport through matter can be considered as a stochastic process. In a general sense, the cross section can be considered as the probability of a particular event happening with a neutron. The Monte Carlo method directly simulates neutron transport as a chain of stochastic events. Such a method of neutron transport equation solution has both advantages and drawbacks in comparison with deterministic methods. Because in the Monte Carlo method the complete history of events from birth to disappearance is simulated, the result (the value of interest) can be assessed for any geometrical and material configuration. For a properly described problem, the run time is directly proportional to the number of interesting events $N_{res}$. The relative error of result depends on $N_{res}$ as $\frac{1}{\sqrt{N_{res}}}$. Therefore the time required to achieve an acceptable
level of statistical uncertainty depends on the average contribution of every simulated history to $N_{\mu}$. As a result, the Monte Carlo method is less effective than deterministic ones for problems requiring assessment of differential distributions, such as neutron flux per unit of energy, angle, etc., when events of interest happen relatively rarely. From an opposite point of view, deterministic codes require simplified geometry, such as spherical, cylindrical or slab, to correctly apply mathematical algorithms to solve the system of differential equations. Since the geometrical setup of ISU ADS experiments cannot be described with approximations required by deterministic codes, the Monte-Carlo method was chosen for the simulation of experiments.

One of the most elaborate and widely used Monte Carlo transport codes is the MCNP code (A General Monte Carlo N-Particle Transport Code) developed at Los Alamos National Laboratory [21]. The extended version with the same functionality regarding neutral particle transport is MCNPX [22]. MCNP can perform both analog simulation of neutron transport and biased simulation when different variance reduction techniques are applied. The analog simulation is implemented by tracing the path of an individual neutron through the matter and assessing the probability of
different processes that determine its history with use of predefined cross section data. In MCNP the data are supplied in separate data files, which are produced by NJOY from evaluated data files provided by different laboratories [23]. To perform biased simulations with MCNP, various variance reduction techniques are used for "biasing" probabilities of different events and consequently to increase the ratio of \( \frac{N_{\text{res}}}{N} \). Therefore the contribution from each simulated history to the result is higher than the contribution in the case of analog simulation, and less computer time is required to get a result with equal statistical uncertainty. All variance reduction techniques are based on the concept of weight adjustment. In this case, the particular weight is assigned to every particle at the start time. During the history flow this weight is changed and the contribution to the \( N_{\text{res}} \) is proportional to the current weight.

Because in the Monte Carlo method the histories of individual particles are simulated, the information regarding parameters of the starting particles is required. Therefore in general, the following distributions must be provided: distribution of positions in space, distribution of directions, and energy distribution of starting
particles. The simulation performed with a source described in terms of such distributions is called a fixed source calculation.

Besides a fixed source calculation, another type of calculation is applied to solve the neutron transport equation in a multiplicative media. In this case, Monte Carlo is used to calculate the eigenvalue (multiplication factor) and associated eigenfunction for the flux distribution. This criticality problem is started with some initial spatial distribution of neutrons and isotropical directional distribution of neutrons distributed in the fission energy spectrum. The method is based on the concept of neutron generation, i.e., when the history of a large number of neutrons in one generation is investigated in parallel. The history flow is terminated every time a neutron escapes or causes fission. The points where the fissions occur form a spatial distribution for neutrons in a consecutive generation. After simulation of some number of generations, the spatial neutron source distribution is stabilized and the ratio of the total number of fission neutrons in the current generation to the number of neutrons in the previous one is the statistical estimate of the multiplicative constant. This method requires weight adjustment to prevent the total neutron population from
increasing in a supercritical system or decreasing in a subcritical reactor.

To accumulate information regarding interesting events the concept of tallying is used. It can be implemented in two different ways: one is based on counting the probability of interesting events at the time of collision and the other one is based on counting the probabilities of prospective events. For example, the reaction rate can be calculated by either of these ways. In the first case, the probability of the sampled reactions of a required type is counted. In the second, the definition of the collision rate as the product of the cross section times the flux times the volume is used. The volume and cross sections are known from geometry and material setup of the problem. The flux in this case is calculated as the path length traversed by all particles through a volume per unit volume per unit time. Therefore the contribution to the \( N_{\text{res}} \) is made every time a particle probably may collide. The efficiency of calculations with tallies of the second type is therefore higher than the similar ones with the first type tallies. In MCNP the tallies can be represented as integrals:

\[
R_{\text{tall}} = C \int \varphi(E) f(E) dE
\]  

(4.1)
where $C$ is a normalizing constant, $\varphi(E)$ is neutron flux, and $f(E)$ is energy-dependent weighting function.

4.2. Calculation of kinetic parameters with MCNP

In MCNP all the concepts mentioned above are implemented. The first type of tallies is used in criticality calculations (or KCODE mode) to assess collision and absorption estimators of criticality eigenvalue $k_{\text{eff}}$ and prompt neutron lifetime, the tallies of the second type are called track length tallies and they are used both in fixed source calculations and KCODE mode. Since KCODE mode is used for calculation of kinetic parameters the standard estimators of this mode should be described in detail. The collision estimate of the prompt removal lifetime for one generation is the average time required for a neutron to be removed from the system by escape, capture (without fission) or fission:

$$\tau^c_r = \frac{\sum W_c T_c + \sum (W_c + W_f) \cdot T_s}{\sum W_c + \sum (W_c + W_f)} \quad (4.2)$$

Where $T_c$ and $T_s$ are the times from the birth of the neutron until escape or collision. $W_c$ is the weight lost at each escape, and the weight lost to absorption and fission is summed over all $k$ nuclides.
\[ W_c + W_f = W_i \sum_k f_k \left( \sigma_c + \sigma_f \right) / \sum_k f_k \sigma_{\tau_k} \]  \hspace{1cm} (4.3)

Where \( f_k \) is atomic fraction for nuclide \( k \); \( \sigma_c, \sigma_f, \sigma_{\tau} \) - are microscopic capture, fission, and total cross sections, respectively; and \( W_i \) is the weight entering the collision.

The absorption estimator of the prompt removal lifetime has the same meaning as the previous one, but it is calculated with use of slightly different probabilities:

\[ \tau_r^A = \frac{\sum W_i T_c + \sum (W_c + W_f) T_s}{\sum W_c + \sum W_f} \]  \hspace{1cm} (4.4)

for implicit absorption, i.e., when appropriate weight loss occurs instead of termination of the history.

For the case of analog absorption:

\[ \tau_r^A = \frac{\sum W_i T_c + \sum W_c T_c + \sum W_f T_f}{\sum W_c + \sum W_f} \]  \hspace{1cm} (4.5)

where \( T_c \) and \( T_f \) are the times from the birth of the neutron until capture or fission.

The track length estimator of prompt removal lifetime is a tally of the second type, and the contribution to the tally is proportional to the time span \( d \) of the track:

\[ \tau_r^n = \frac{\sum W_i d}{W_s} \]  \hspace{1cm} (4.6)
Where \( i \) is summed over all neutron trajectories, \( v \) is the velocity, and \( d \) is the trajectory track length from the last event. The normalizing weight \( W \) is the source weight summed over all histories in the generation. In Ref. 24 the concept of lifespan is explained and in the context of neutron-balance theory the lifetime is defined as the mean time from event to event and lifespan as the mean time from birth to event.

Besides prompt removal lifetime in the criticality calculation, the following lifespans are provided:

\[
\begin{align*}
    l_e &= \frac{\sum W_c T_e}{\sum W_c}, \\
    l_c &= \frac{\sum W_c T_c}{\sum W_c}, \\
    l_f &= \frac{\sum W_f T_f}{\sum W_f}, \\
    l_r &= \frac{\sum W_c T_e + \sum W_c T_c + \sum W_f T_f}{\sum W_c + \sum W_c + \sum W_f}
\end{align*}
\]

(4.7)

Where \( l_e, l_c, l_f, l_r \) are the escape, capture, fission, and removal lifespans, respectively. Here the summation is taken over all histories. Therefore \( l_r = \tau \) is valid only for the case when only one generation is simulated.

As it follows from definitions of lifespan and lifetimes for the criticality calculation mode, \( l_e \) is equal to \( \tau_f \) and they can be calculated as the mean time from fission to fission. This estimator (average intergeneration time) can be obtained as the ratio of two track length tallies (4.1):
\[ \bar{i}_{\text{fission}} = \frac{\int t \cdot P(t) \cdot dV \cdot dt}{\int P(t) \cdot dV \cdot dt} \]  

(4.8)

Where \( P(t) \) is the probability distribution function of fission event and \( t \) is the moment of the fission time. The contribution to the numerator and denominator is made only when fission is possible, i.e., when a particle traverses the media containing fissionable isotopes. Equation (4.8) is exactly the same as the one used in an early derivation of the point kinetic equation in Ref. 25. In Ref. 26 the relationships between neutron lifespans, reaction rate lifetimes, and neutron generation time are described.

Since the neutron generation model (in which the effective multiplication factor \( k_{\text{eff}} \) is defined as the ratio of the neutron population in successive generations) cannot be practically applied to describe reactor dynamics, the neutron balance model (in which \( k_{\text{eff}} \) is redefined to be the ratio of the neutron production rate divided by the neutron loss rate) is used in reactor physics theory. In Ref. 27 it is shown that neutron lifetime \( \tau \) is defined by the following equation:

\[ \frac{N}{\tau} = L + A \]  

(4.9)
where $N$ is the neutron population at some moment of time, $L$ is the neutron leakage rate, and $A$ is the neutron absorption rate. The neutron generation model and the neutron-balance model yield consistent equations that describe the time-dependent neutron population. In this case $\frac{1}{\tau}$ represents the probability per unit time that a neutron will be either absorbed or leak from the system.

In the neutron-balance model the expression for the neutron generation time was obtained in Ref. 28:

$$A = \frac{\int \int \int \int \frac{1}{V} \phi_i^* \phi \cdot d\Omega \cdot dV \cdot dE}{\int \int \int \int \phi_i^* \chi_j \Sigma_j \phi \cdot d\Omega \cdot dV \cdot dE \cdot d\Omega \cdot dE} \tag{4.10}$$

This equation using shorthand notation is: $A = \langle \frac{1}{\nu} \phi_i^* \phi \rangle / \langle \phi_i^* \chi_j \Sigma_j \phi \rangle$.

Similarly to this, the expression for neutron removal lifetime is: $\tau_i = \frac{\langle \frac{1}{\nu} \phi_i^* \phi \rangle}{\langle \phi_i^* \hat{L} \phi \rangle} \tag{4.11}$

The Equations (4.2-4.6) don't have spatial dependent weighting function (in other words the weighting function is equal to unity) and Equation (4.11) has spatial dependent weighting function ($\phi_i^*$, the solution of adjoint to transport equation). This difference clearly shows that
MCNP cannot provide the correct estimation of neutron kinetic time parameters. To implement the correct estimation of $\Lambda$ and $\tau_r$, the spatial and energy dependent weighting functions are required.

In addition to the neutron time parameters, the effective delayed neutron fraction must be calculated in order to be used in kinetic equations. The calculation of $\beta_{\text{ef}}$ is not a standard feature of the MCNP code. After introduction of delayed neutron physics in MCNP the most common approach of $\beta_{\text{ef}}$ calculation is based on the following approximation:

$$\beta_{\text{ef}} = \frac{k_{\text{eff}}^\text{total} - k_{\text{eff}}^\text{prompt}}{k_{\text{eff}}^\text{total}}$$  \hspace{1cm} (4.12)

In this case, two calculations are required: one is performed when in each fission event the total (prompt and delayed) number of neutrons is created, and the second simulation is performed with prompt neutrons only.

The calculation of $k_{\text{ef}}$ in MCNP is performed with use of the same tally types as the calculation of lifetimes. In all definitions of the $k_{\text{ef}}$ estimator the following notation is used: $i$ is a summation index for collisions where fission is possible (for track length, $i$ is the index of track); $k$ is a summation index for nuclides of the material involved in the collision; $\sigma_n$ is total microscopic cross section;
\( \sigma_f \) is microscopic fission cross section; \( \bar{V}_k \) is average number of prompt or total neutrons produced per fission; \( f_k \) is atomic fraction of nuclide; \( N \) is nominal source size for cycle; \( W_i \) is weight of particle entering collision; \( \rho \) is the atomic density in the cell; \( d \) is the trajectory track length from the last event. MCNP in the criticality mode uses a neutron generation model, so to assess the effective multiplication factor the ratio of numbers of neutrons in successive generations is calculated. The collision estimate for \( k_{\text{eff}} \) for one generation is:

\[
k_{\text{eff}}^c = \frac{1}{N} \sum_i W_i \left[ \frac{\sum_k f_k \bar{V}_k \sigma_f}{\sum_k f_k \sigma_n} \right]
\]

(4.13)

and represents the mean number of fission neutrons produced per cycle. The absorption estimator has the same meaning as the collision estimator, but only the nuclide involved in the collision is used for the absorption \( k_{\text{eff}} \) rather than an average of all nuclides in the material:

\[
k_{\text{eff}}^A = \frac{1}{N} \sum_i W_i \bar{V}_k \frac{\sigma_f}{\sigma_n}
\]

(4.14)

The track length estimator of \( k_{\text{eff}} \) is accumulated every time the neutron traverses fissionable material:
It is mentioned in the MCNP manual that the track length estimator tends to be the best estimator for optically thin problems, therefore for the ISU ADS core containing fuel plates with a thin fuel bearing region, this estimator gives the lowest variance.

The technique based on Equation (4.12) was applied to both ISU ADS configurations that were experimentally studied: without the Pu-Be source inside the core and with presence of the Pu-Be source. The configuration described in the previous chapter was simulated without any approximations. MCNP in KCODE mode was used to calculate $k_{\text{eff}}$ in fundamental (after convergence of spatial distribution of fission source) mode. Results of $k_{\text{eff}}$ and $\beta_{\text{eff}}$ calculations are presented in Figures 4.1-4.3.

The 95% confidence interval for $\beta_{\text{eff}}$ was assessed with the assumption of independent calculations of $k_{\text{eff}}^{\text{prompt}}$ and $k_{\text{eff}}^{\text{total}}$:

$$
\Delta \beta_{\text{eff}} = 2 \frac{k_{\text{eff}}^{\text{prompt}}}{k_{\text{eff}}^{\text{total}}} \sqrt{\left( \frac{\sigma_{k_{\text{eff}}^{\text{prompt}}}}{k_{\text{eff}}^{\text{prompt}}} \right)^2 + \left( \frac{\sigma_{k_{\text{eff}}^{\text{total}}}}{k_{\text{eff}}^{\text{total}}} \right)^2} \quad (4.16)
$$
Figure 4.1. $k_{\text{eff}}$ versus the number of simulated histories for two ISU ADS configurations.

Figure 4.2. $\beta_{\text{eff}}$ versus the number of simulated histories in the ISU ADS with the Pu-Be source.
Figure 4.3. $\beta_{\text{eff}}$ versus the number of simulated histories in the ISU ADS without the Pu-Be source in the core.

The calculations of $k_{\text{eff}}$ were performed for two configurations of the ISU ADS assembly with a different number of neutrons in one generation (or per one cycle) to check the quality of the result. Theoretically the results obtained with a different number of particles per cycle should converge to the same value. It can be seen from Figure 4.1 that such convergence is observed for $k_{\text{eff}}$, but not for the ratio of two values of $k_{\text{eff}}$, obtained from simulations with total and prompt only neutrons. This ratio defines the value of $\beta_{\text{eff}}$ in this simple technique.
Figures 4.2 and 4.3 show the lack of convergence of the $\beta_{\text{eff}}$ estimator. They indicate a very low accuracy for the obtained result. For more than 10 million simulated histories, the relative error of the result is around 10% for a 95% confidence interval. Because the reactivity is experimentally determined in units of $\beta_{\text{eff}}$, such a low accuracy of calculation is unacceptable.

The reason for this low accuracy is due to the application of a wrong approach: when the difference between two very close values is statistically determined with two independent, uncorrelated simulations. In this case the small value of interest is masked by the statistical error of uncorrelated calculations.

In addition to mean generation time and effective delayed neutron fraction, a third reactor kinetic parameter is static reactivity, which is defined as the ratio:

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \quad (4.17)$$

and can be calculated with use of the $k_{\text{eff}}$ estimator obtained in the criticality calculation.
CHAPTER 5

CALCULATION OF THE EFFECTIVE DELAYED NEUTRON FRACTION

5.1. Delayed neutron data

Because a simple calculation approach of effective delayed neutron fraction with MCNP gives unsatisfactory results, another method should be used. Some basic definitions and general considerations should be provided before the description of a new proposed method of $\beta_{\text{eff}}$ calculation.

The fundamental nuclear data characterizing the creation of delayed neutrons in a fission reaction are the yield of delayed neutrons per fission $\nu_d$ and energy spectrum of delayed neutrons $f_d(E)$. These data are measured experimentally and are distributed in evaluated data files for every fissionable isotope. The data are combined in a number of groups (usually 6 or 8) according to the neutron decay constant of delayed neutron precursors. This approach is used because delayed neutrons are not created directly in a fission process, rather they are the products of
radioactive decay of the isotopes created in the fission, therefore delayed neutrons are products of fission reactions. The yield of delayed neutrons per fission depends on the energy of the neutron causing the fission and usually is not provided separately in data files, rather the total number of neutrons (prompt plus delayed) and the number of prompt neutrons only are provided. In Figure 5.1 the yield of delayed neutrons, calculated as the difference between values mentioned above is shown.

Figure 5.1. Delayed neutron yield for $^{235}\text{U}$ and $^{238}\text{U}$ as a function of incoming neutron energy.
The data regarding numbers of total and prompt neutrons are extracted from the data file "endf6dn" distributed with MCNP and containing processed data from the LANL proposed ENDF-VI.2 data library, which contains data evaluated at 300 K [18]. The yield of a certain delayed neutron group is characterized by the product of the probability of a certain group and the total (for all groups) delayed neutron yield. The data extracted from the same data file "endf6dn" representing probabilities and decay constants of delayed neutron groups are presented in Table 5.1.

Table 5.1. The six group delayed neutron data.

<table>
<thead>
<tr>
<th>Group</th>
<th>Probability</th>
<th>Decay constant (s(^{-1}))</th>
<th>Half life (s)</th>
<th>Nuclide</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.035007555</td>
<td>0.013336</td>
<td>51.976</td>
<td>(^{235})U</td>
</tr>
<tr>
<td>2</td>
<td>0.180698227</td>
<td>0.032739</td>
<td>21.172</td>
<td>(^{238})U</td>
</tr>
<tr>
<td>3</td>
<td>0.172510226</td>
<td>0.12078</td>
<td>5.739</td>
<td>(^{235})U</td>
</tr>
<tr>
<td>4</td>
<td>0.386782158</td>
<td>0.30278</td>
<td>2.289</td>
<td>(^{235})U</td>
</tr>
<tr>
<td>5</td>
<td>0.158575224</td>
<td>0.84949</td>
<td>0.816</td>
<td>(^{235})U</td>
</tr>
<tr>
<td>6</td>
<td>0.06642661</td>
<td>2.853</td>
<td>0.243</td>
<td>(^{235})U</td>
</tr>
<tr>
<td>1</td>
<td>0.013937951</td>
<td>0.01363</td>
<td>50.855</td>
<td>(^{238})U</td>
</tr>
<tr>
<td>2</td>
<td>0.112796906</td>
<td>0.031334</td>
<td>22.121</td>
<td>(^{238})U</td>
</tr>
<tr>
<td>3</td>
<td>0.131027307</td>
<td>0.12334</td>
<td>5.620</td>
<td>(^{238})U</td>
</tr>
</tbody>
</table>
Table 5.1 (Continued)

<table>
<thead>
<tr>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
</tr>
<tr>
<td>Decay constant (s⁻¹)</td>
</tr>
<tr>
<td>Half life (s)</td>
</tr>
<tr>
<td>Nuclide</td>
</tr>
</tbody>
</table>

| 4 | 0.385142919 | 0.32373 | 2.141 | $^{238}$U |
| 5 | 0.253991613 | 0.90597 | 0.765 | $^{238}$U |
| 6 | 0.103103305 | 3.0487 | 0.227 | $^{238}$U |

The spectrum of each delayed neutron group for a certain isotope does not depend on the energy of the incoming neutron and also can be extracted from the data file. Spectra of six groups of $^{235}$U (extracted from data file “endf6dn”) are presented in Figure 5.2, where probabilities $P_d(E) = f_d(E) \cdot \Delta E$ of bins with 0.01 MeV widths are shown.

5.2. Delayed neutron production and effective delayed neutron fraction

The production of delayed neutrons in a nuclear reactor is characterized by the ratio of the delayed neutron yield to the total neutron yield, this value is referred to as $\beta_0$. 

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Figure 5.2 Delayed neutron energy spectra for six groups of precursors created in the fission of $^{235}$U.
This function is spatial-dependent since it depends on the isotope composition and the energy spectrum of the neutrons inducing fission at some particular point in the core:

\[
\beta_0(\vec{r}) = \frac{\sum_k \int \phi(\vec{r},E) \cdot f_k(\vec{r}) \cdot \sigma_{\text{f}}(E) \cdot \nu_{\text{d}}(E) \cdot dE}{\sum_k \int \phi(\vec{r},E) \cdot f_k(\vec{r}) \cdot \sigma_{\text{f}}(E) \cdot \nu_{\text{d}}(E) \cdot dE} \tag{5.1}
\]

Where \( k \) is a summation index of fissionable nuclides, \( f_k \) is atomic fraction for nuclide \( k \), \( \phi \) is neutron flux, \( \sigma_{\text{f}} \) is microscopic fission cross section, \( \nu_{\text{d}} \) and \( \nu_{\text{d}} \) are total and delayed neutron yields per fission for nuclide \( k \).

To characterize the production of every particular delayed neutron group \( i \), a similar expression for \( \beta_{0i} \) is used, in which \( \nu_{\text{d}i} \) is used instead of \( \nu_{\text{d}} \). This function can be integrated over the entire reactor volume, and the average value \( \bar{\beta}_0 \) is often provided as a characteristic of the reactor. In many practical cases the material composition of the fuel and neutron spectrum are constant and \( \bar{\beta}_0 = \beta_0 \). For the case of a thermal reactor with highly enriched uranium fuel, where only fission of \(^{235}\text{U}\) is considered, the value of \( \beta_0 \) is equal to the isotopic ratio of neutron yields which is 0.640% [8].
The impact of delayed neutrons on the reactor power (or the neutron population) is the subject governing the reactor behavior. Therefore in reactor kinetics the key role is not played by the fraction of created delayed neutrons \( \beta_0 \), the fraction of fissions induced by delayed neutrons \( \beta_{\text{eff}} \) is important. The difference between \( \beta_{\text{eff}} \) and \( \beta_0 \) is characterized by adjoint weighting of delayed neutron production and \( \beta_{\text{eff}} \) is defined as:

\[
\beta_{\text{eff}} = \frac{\langle \phi_d^*, f_d \nu f \Sigma_f \phi \rangle}{\langle \phi_d^*, \chi \nu \Sigma_f \phi \rangle}
\]

Where shorthand notation for integration, introduced in Chapter 1 is used; \( f_d \) and \( \chi \) are delayed and total neutron spectra, \( \Sigma_f \) is macroscopic fission cross section, \( \nu_d \) and \( \nu \) are delayed and total neutron yields, \( \phi \) is neutron flux, and \( \phi_d^* \) is adjoint flux for critical reactor.

5.3. Existing methods of effective delayed neutron fraction calculation

Recently two methods of \( \beta_{\text{eff}} \) calculation which take into consideration the efficiency of the delayed neutron to cause fission were introduced. One of them uses a Monte-Carlo method to solve the neutron transport equation [29]. In this paper, the authors use the theoretical approach.
introduced in Ref. 30, which considers the adjoint function as the iterated fission probability. This probability is defined as the number of fissions counted during particle history flow. Since the authors implemented their method in MCNP (which in criticality mode terminates the history flow in case of a fission), instead of using an iterated fission probability, the next fission probability is actually used. The modification of the source code is required to count fissions caused by delayed neutrons, and the ratio of the average number of fissions generated by delayed neutrons to the average number of fissions generated by all neutrons is considered as $\beta_{\text{ef}}$. In a subcritical system the iterated fission probability can be assessed (since the history cannot be indefinitely long). The results obtained with iterated fission probability were statistically indistinguishable from the results obtained with next fission probability. This method (NRG method) does not change the neutron generation of MCNP, therefore it is valid for any case when MCNP is valid. The NRG method requires only minor bookkeeping in the code, therefore it does not increase computer time required to obtain results with some predefined accuracy. The authors mentioned a 0.5% increase in computer time for their sample case. In comparison with the simple MCNP method described in the
previous chapter, which the authors of the NRG method call the "prompt method," the NRG method requires 40 times less computer time to achieve equal accuracy of the results.

The second recently proposed method of $\beta_{\text{eff}}$ calculation was implemented in a deterministic code in Ref. 31 and Ref. 32, and a theoretical basis was described in Ref. 33. In this method, the effectiveness of delayed neutrons is defined as the ratio of two eigenvalues and $\beta_{\text{eff}}$ is defined as:

$$\beta_{\text{eff}} = \beta_0 \frac{k_d}{k_i} \quad (5.3)$$

The calculation of $k_d$ is performed for the system in which all fission neutrons are created with a delayed neutron spectrum and the yield of delayed neutrons is equal to the total neutron yield. The calculation of $k_i$ is made properly. Since $k_d$ is an eigenvalue, its calculation is performed with a fundamental distribution of predictors and this distribution differs from a fundamental distribution in the real system. The difference will be bigger for the cases of compact thermal cores, where leakage of more energetic prompt neutrons is the main removal process, and in the fast reactors, where the majority of fissions is generated by neutrons in the resonance region. As a result, the test
performed by the authors of the NRG method has shown the biggest discrepancy between experimental data and calculated results to be for heterogeneous and fast systems [29].

In conclusion, the drawbacks of the reviewed methods are the following. The assumption made in the k-ratio method has no physical meaning, therefore this method failed in many cases. The implementation of the NRG method requires access to the MCNP source code, which is not possible for many users due to USA export control regulations. Also the NRG method is not efficient, it requires simulation of approximately 150 prompt neutrons per one delayed neutron history (for the case of minor actinide fuel this value can reach, for example, 2500 for $^{242}$Cm). The "prompt method," described in a previous chapter, is 40 times less efficient than NRG method.

5.4. New method of delayed neutron fraction calculation

For the present research, a new method of $\beta_{ef}$ calculation was developed that is based on two basic assumptions: 1) the shape of the spatial distribution of the fission events is formed by prompt neutrons only, and 2) the adjoint function is proportional to the next fission probability. The validity of the first assumption may be explained by the fact that there is no critical system with an effective
delayed neutron fraction exceeding 1%, therefore even for the worse case scenario (uranium fuel) only 1% of simulated histories are "biased," and it is the same "bias" that is performed for 100% of the histories in k-ratio method. Since the k-ratio method is considered as "trustworthy," the proposed method is even more "trustworthy," because it is at least 100 times more accurate in its description of real processes. The difference between reality and the model decreases with a decrease of delayed neutron fraction, so for example, for the case of $^{239}$Pu fuel only 1/500 part of the histories is simulated with the bias.

The second assumption is the same as in the NRG method: instead of the iterated fission probability the next fission probability is used as a weighting function. This is also a very accurate approximation, since the iterated fission probability can be expressed as the product of the first (next) fission probability $P_i$ and probability $P_{rest}$ representing fissions besides the first one in some fission reaction chains. Then the ratio of importances of two neutrons with the same parameters, introduced in a critical system in points C and D is: $R_{C/D} = \frac{P_i^C \cdot P_{rest}^C}{P_i^D \cdot P_{rest}^D}$, and in the case of the system, where fission occurs mostly in one energy
region (either thermal for thermal systems or resonance for fast systems) \( P_{\text{rest}}^c = P_{\text{rest}}^p \) and therefore \( R_{C/D} = \frac{R_{C/D}}{R_{C/D}} \).

The idea of the calculation of delayed neutron effectiveness in this new method can be described as follows. Let the system have constant-in-space ratio of delayed neutron yield to the total neutron yield (or production ratio) of delayed neutrons \( \beta_0(\bar{r}) = \text{const} \), then according to (5.2):

\[
\beta_{\text{eff}} = \frac{\beta_0 \cdot P_d}{(1 - \beta_0) \cdot P_p + \beta_0 \cdot P_d} = \frac{\beta_0 \cdot R}{(1 - \beta_0) + \beta_0 R}
\]

(5.4)

Where \( P_d \) is the probability for the fission event to be caused by delayed neutron, \( P_p \) is the probability for the fission event to be caused by prompt neutron, \( R = \frac{P_d}{P_p} \) is the ratio of these two probabilities or relative efficiency of delayed neutrons (which is referred to as \( \gamma \) in some textbooks, for example in Ref. 7). Since this method is based on a probability ratio it is called the "p-ratio method" in Ref. 34, where initial results obtained with this method were presented. The restriction of \( \beta_0(\bar{r}) = \text{const} \) can be removed with some additional computational efforts, as will be shown later.
So, to calculate $\beta_{\text{eff}}$ the accurate estimation of $R = \frac{P_d}{P_p}$ is required, and this approach has clear physical meaning: both prompt and delayed neutrons are born at the same location, and both of them have an isotropic angular distribution.

With MCNP this estimation can be done with the use of a correlated sampling technique. Correlated sampling allows evaluation of the small quantities that would be masked by the statistical errors of uncorrelated calculations. In MCNP the correlation of two runs is made by providing each new history in both problems with the same starting pseudorandom number. The sequencing of random numbers is done by incrementing the random number generator at the beginning of each history by a stride $S$ of random numbers from the beginning of the previous history. The control of the random number generator parameters (initial seed and stride size) is made by ordering required values in the input file with a special data card. The stride size should be bigger than the number of pseudorandom numbers required for simulation of one history.

The first step of calculation is the estimation of delayed and prompt neutron yields, which can be calculated...
with the standard MCNP track length tally for each fissionable isotope:

\[ Y_k = \rho_{ak} \int \sigma_{jk}(E) \cdot \nu_k(E) \cdot \varphi(E) \cdot dE \]  
(5.5)

Where \( k \) is fissionable isotope index; \( \nu_k \) is either prompt or delayed neutron yield per fission, \( Y_k \) is accordingly either prompt or delayed integrated neutron yield; \( \sigma_{jk} \) is microscopic fission cross section for isotope \( k \), and \( \rho_{ak} \) is nuclear density (number of nuclei per unit volume) of isotope \( k \).

In the simulation of the ISU ADS without Pu-Be source only two fissionable isotopes were presented in material configuration: \(^{235}\text{U}\) and \(^{238}\text{U}\). The energy dependent modifying functions \( \nu_k(E) \) are shown in Figure 5.1 for delayed neutrons, and in Figure 5.3 for prompt neutrons. The cross sections \( \sigma_{jk} \) of \(^{235}\text{U}\) and \(^{238}\text{U}\) are presented in Figure 5.4. All of these data are extracted from data file "endf6dn", which was used for MCNP simulations.

The calculation of \( Y_k \) is made in every cell with fissionable materials with use of mesh tallies, which allow getting the spatial distribution of the required value. MCNP performs spatial binning of values according to a predefined mesh.
Figure 5.3. Prompt neutron yield for $^{235}\text{U}$ and $^{238}\text{U}$ as function of energy of incoming neutron.

In the ISU ADS model each fuel bearing region in the fuel plate was uniformly divided into 64 bins in the longest direction and 8 bins in the vertical direction. After the calculation of prompt and delayed neutron yield, the spatial distribution of $\beta_0(\vec{r})$ is obtained and is presented in Figures 5.5 and 5.6 for some fuel plates.
Figure 5.4. Fission cross sections of $^{235}$U and $^{238}$U.
Figure 5.5. 2D spatial distribution of $\beta_0$ (%) for fuel plates with $y$ coordinates 6.35, 46.35, 74.9 mm in the ISU ADS.
Figure 5.6. 2D spatial distribution of $\beta_0$ (%) for fuel plates with y coordinates 115.0, 143.5, 183.5 mm in the ISU ADS.
Figure 5.7. 2D spatial distribution of $P_f$ (%) for fuel plates with y coordinates $6.35, 46.35, 74.9$ mm in the ISU ADS.
Figure 5.8. 2D spatial distribution of $P_j$ (%) for fuel plates with $y$ coordinates 115.0, 143.5, 183.5 mm in the ISU ADS.
It can be seen that $\beta_0(\bar{r})$ is almost constant (the variation of values is much smaller than statistical uncertainty), therefore $\beta_0(\bar{r}) \approx \bar{\beta}_0$ is a robust approximation.

The average value

$$\bar{\beta}_0 = \frac{\sum_k \int \phi(\bar{r}, E) \cdot f_k(\bar{r}) \cdot \sigma_{f,k}(E) \cdot \nu_{d,k}(E) \cdot dE \cdot d\bar{r}}{\sum_k \int \phi(\bar{r}, E) \cdot f_k(\bar{r}) \cdot \sigma_{f,k}(E) \cdot \nu_{k}(E) \cdot dE \cdot d\bar{r}}$$

(5.6)

can also be calculated as the sum of $\beta_0(\bar{r})$ weighted according to the probability of the fission. The spatial fission distribution for the same fuel plates is presented in Figures 5.7 and 5.8.

The relative fission probability $P_f$ is the probability of the fission to take place inside some particular cell of the mesh. The total probability, which is equal to the sum over all mesh cells, is equal to 100%.

Values of $\beta_0$ for each fuel plate (integrated over volume) with 95% confidence intervals, obtained after simulation of 2 million neutron histories, are shown in Figure 5.9. The slight difference in values of $\beta_0$ for each cell can be explained by the spectrum dependent yield of delayed neutrons from different isotopes. The relative yield of delayed neutrons from $^{238}\text{U}$: $Y_{d^{\text{238U}}}/(Y_{d^{\text{238U}}} + Y_{d^{\text{239U}}})$ is presented in Figure 5.10.
Figure 5.9. Average values of $\beta_0$ for different fuel plates as a function of plate position.

Figure 5.10. The relative yield of delayed neutrons from $^{238}\text{U}$ for different fuel plates as a function of plate position.
The calculated average values for the total core are:

\[ \bar{\beta} = (6889 \pm 8) \times 10^{-6} \]; relative yield of delayed neutrons from \(^{235}\text{U} \):

\[ \frac{Y_{d^{235}\text{U}}}{(Y_{d^{235}\text{U}} + Y_{d^{238}\text{U}})} = 0.9938 \]; relative yield of delayed neutrons from \(^{238}\text{U} \):

\[ \frac{Y_{d^{238}\text{U}}}{(Y_{d^{235}\text{U}} + Y_{d^{238}\text{U}})} = 0.0062 \].

The spectrum of delayed neutrons consists of a combination of delayed neutron spectra of \(^{235}\text{U} \) (99.38\%) and \(^{238}\text{U} \) (0.62\%). The data for the \(^{238}\text{U} \) are extracted from data file "endf6dn." These data are used in combination with data for \(^{235}\text{U} \), shown in Figure 5.2, to create delayed neutron spectrum, presented in Figure 5.11, where probabilities \( P(E) \) of bins with 0.01 MeV widths are shown.

Figure 5.11. Delayed neutron energy spectrum for the ISU ADS subassembly.
The relative yield of every delayed neutron group is the product of the relative yield from a certain isotope and the probability of the group, presented in Table 5.1.

To perform the calculation of relative efficiency of delayed neutrons \( R = \frac{P_d}{P_p} \) with a correlation sampling Monte Carlo technique the spectrum of prompt neutrons is required. To obtain this spectrum the parameters of every created neutron were recorded in the surface-source file, which is called "wssa/rssa" file in the MCNP manual. The record of coordinates of created neutrons is a standard feature of MCNP, available only in the criticality calculation mode. The "wssa/rssa" file is an unformatted sequential binary file, which can be divided in 3 major parts. The first one is the header with general information regarding problem identification, the second one contains information regarding geometry setup of the problem, and the third one consists of records with track data.

After simulation of a large number of neutron histories, the statistics regarding created neutrons can be analyzed. Since the spectrum of delayed neutrons is described in the data file as the probability of energy bins with 0.01 MeV widths, the same bin structure is chosen to describe the prompt neutron spectrum. The spectrum obtained after
simulation of 2 million histories is presented in Figure 5.12., where probabilities $P(E)$ of bins with 0.01 MeV widths are shown. The probability of a certain bin was defined as the ratio of the sum of track weights with energy in a certain bin to the total weights of all tracks. The number of required tracks depends on the quality of representation, since the statistical uncertainty in the bin is defined by a Poisson distribution.

![Figure 5.12. Prompt neutron energy spectrum for the ISU ADS subassembly.](image)

In correlated sampling the energy of every prompt neutron in the "wssa/rssa" file is substituted by the energy of the delayed neutron. This is performed using the method presented in Figure 5.13.
Figure 5.13. The substitution of prompt neutron energy by delayed neutron energy in "wssa/rssa" file as consequence of steps 1-2-3-4.
The cumulative probability functions (CPF) are obtained from the probability density functions for prompt and delayed neutrons. First the energy of the prompt neutron is read from the file (step 1); next the appropriate value of prompt neutron CPF is determined (step 2). This value of CPF is used for the delayed neutron function (step 3) to choose the energy of the delayed neutron (step 4), which is written in "wssa/rssa" file instead of the prompt neutron energy.

In this step the "wssa/rssa" file with the substituted delayed energy spectrum is prepared as the source file for simulation. But to obtain the next fission probability $P_d$, which is used in $R = \frac{P_d}{P_p}$, the simulation must be performed with the same number of histories per generation, as during simulation of prompt neutrons. Therefore the large "wssa/rssa" file must be cut to create many files with the required number of tracks written during simulation of one generation. All tracks in the "wssa/rssa" file belonging to one generation are written in series and have weight, which is equal to the ratio of number of particles in the cycle to the original number of neutrons in the first generation. Moreover the total weight of one generation is always
conserved, therefore the required cut can be easily performed with an auxiliary program.

The simulation of one generation can be performed in two ways: the first one is implemented in the fixed source mode with direct use of "wssa/rssa" file as the source of neutrons; the second one is implemented when "wssa/rssa" file is converted to the source tape file (which is called "srctp" in MCNP manual), and "stctp" file is used as the source file in the criticality calculation mode.

In the first case, a different history flow is realized for simulation of prompt neutrons (the KCODE mode) and delayed neutrons (the fixed source mode). The difference between them is explained by different implementation of event sampling in the MCNP code, they use different number of sampled random numbers to obtain neutron parameters in the collision.

In simple words this difference can be described as follows: in the case of correlated sampling (when initial parameters of simulated neutrons are the same), the history flow in fixed source mode coincides with the history flow in criticality mode until the first collision in a geometry cell containing fissionable material.

Therefore even for the case when the energy of the neutron is not altered (by substitution of an energy
spectrum) the history of the neutron cannot be repeated exactly. To provide the same random numbers at the start of the particle's history in criticality mode and in fixed source mode, the random number generation should be advanced not only by the appropriate number of previously simulated histories (or already used strides), but also the initial seed should be shifted by two random numbers. The reason for this is the following: when a particle starts in the criticality mode, two generated random numbers are used to sample direction (the cosines \( \cos(v_x) \) and \( \cos(v_y) \), where \( v \) is the velocity vector of starting particle), while in the fixed source mode (with "wssa/rssa" file as the source) the mentioned above cosines are read from file and are not sampled. Therefore to perform correct correlated sampling the simulation of prompt neutrons should be repeated in fixed source mode, and instead of two calculations for prompt and delayed neutrons, three of them are required: the first one to create source file, a second one to assess \( P_p \), and a third one to assess \( P_d \). Then the ratio \( R = \frac{P_d}{P_p} \) can be calculated for every generation. This approach has one important advantage: for subcritical systems the computation of iterated fission probability is available, and the adjoint function is used without any
approximations. The computation of next fission probability is also possible, in this case the treatment of fission as an absorption should be ordered in the MCNP input file.

The second method of computation of $R = \frac{P_d}{P_p}$ is performed in the criticality calculation mode and requires the use of "srctp" file for every neutron generation. The source tape file is required because the KCODE mode is not compatible with the "wssa/rssa" file. Therefore information from the "wssa/rssa" file regarding neutron tracks should be written in "srctp" file. The source tape file is an unformatted sequential binary file and has the structure different from one of "wssa/rssa" file. The "srctp" file is a buffer type file, which has a constant size during the MCNP run. The information is stored in two records: the first is the header, the second is an array with parameters of created neutrons. These parameters are different from the parameters in the "wssa/rssa" file, and are stored in a different order, but the coordinates and the energy of the neutron are present in both of them.

The updating of the "srctp" file is made at the end of the cycle, while the record with parameters is added in the "wssa/rssa" file right after the sampling of direction, before the simulation of neutron transport takes place. The
transfer of particle information from the “wssa/rssa” file to the “srctp” file can be implemented with an auxiliary program. To perform correlated sampling in KCODE mode the shift of the initial seed of random number generator is not required. Therefore to provide a starting delayed neutron with the same sequence of random numbers only the advancement of the random number generator by a previously simulated number of histories should be made. The exact repetition of a history flow in the case of an unaltered energy distribution is nevertheless not simple. The starting neutrons from the “srctp” file in KCODE mode always have weights equal to unity. Therefore, a weight adjustment technique in MCNP has different parameters at the moment of collision.

The history flow of a neutron with the same initial parameters in separate generations and in the original prompt problem is repeated exactly only until the first collision with a fissionable isotope. There are two ways to avoid this difference. First, the criticality mode card in the MCNP input file can be used. This card has a parameter which is mentioned as the initial guess for the multiplication constant and that parameter implicitly controls the weight adjustment at fission. The second way is to use separate generations in the original prompt
problem and use a proper guess for the multiplication constant in the input file to prevent the degeneration of neutron population. The latter approach avoids the requirement to cut a large "wssa/rssa" file into separate files for each generation. Therefore an auxiliary program is required only to substitute the energy of a prompt neutron by the energy of a delayed neutron in the "srctp" file.

5.5. Results obtained with the new method

To compare the efficiency of the proposed method of calculation with a simple MCNP approach, which was described in Chapter 4, the same MCNP model was used. Two problems with 2000 and 8000 neutrons per cycle were run. The ratio of the track length estimators of the multiplication constant was used as the efficiency of prompt and delayed neutrons to cause fission. For the case of delayed neutrons this value is not the eigenvalue, since it uses the fission source distribution from the prompt neutron criticality problem. The results of $k_{\text{eff}}$ calculation are presented in Figures 5.14-5.17. The estimations of $k_{\text{eff}}$ were obtained after the achievement of the fundamental spatial distribution of the fission source. The values of prompt $k_{\text{eff}}$ were extracted from the MCNP output files.
Figure 5.14. Single cycle track length estimation of $k_{eff}$ for the case with 2000 neutron histories per cycle.

Figure 5.15. Track length estimation of $k_{eff}$ (average over cycles) for the case with 2000 neutron histories per cycle.
Figure 5.16. Single cycle track length estimation of $k_{eff}$ for the case with 8000 neutron histories per cycle.

Figure 5.17. Track length estimation of $k_{eff}$ (average over cycles) for the case with 8000 neutron histories per cycle.
Values of \( k_{\text{eff}} \) for problems with a delayed neutron spectrum were calculated for every single cycle with a spatial distribution of the neutron source obtained during prompt \( k_{\text{eff}} \) calculations. Values of \( k_{\text{eff}} \) (average over cycles) both for prompt and delayed problems were calculated with the assumption of equal weights of every single cycle. The calculations of relative efficiency \( R \) were made after estimation of \( k_{\text{eff}} \) for every single cycle. The \( R \) (average over cycles) were also calculated and demonstrate convergence as is shown in Figures 5.18-5.21.

![Figure 5.18. Single cycle estimation of \( R \) for the case with 2000 neutron histories per cycle.](image)
Figure 5.19. Estimation of $R$ (average over cycles) for the case with 2000 neutron histories per cycle.

Figure 5.20. Single cycle estimation of $R$ for the case with 8000 neutron histories per cycle.
Figure 5.21. Estimation of $R$ (average over cycles) for the case with 8000 neutron histories per cycle.

To obtain values of $\beta_{\text{eff}}$ Equation (5.4) was used with calculated average relative efficiency $R$. The results are presented in Figure 5.22. The values of $\beta_{\text{eff}}$ for both studied cases (2000 and 8000 neutrons per cycle) practically coincided after approximately 2 million simulated histories.
Figure 5.22. $\beta_{\text{eff}}$ versus the number of simulated histories for the cases of 2000 and 8000 neutron histories per cycle.

The statistical uncertainties were assessed with the use of an error propagation formula, for $R$ this is:

$$\sigma_R = \sqrt{\sigma_{k_{\text{prompt}}}^2 + \sigma_{k_{\text{delayed}}}^2}$$  \hspace{1cm} (5.7)

Where $\sigma_{k_{\text{prompt}}}$ and $\sigma_{k_{\text{delayed}}}$ are standard deviations of $k_{\text{eff}_{\text{prompt}}}$ and $k_{\text{eff}_{\text{delayed}}}$, respectively, and $\sigma_R$ is the standard deviation of the relative efficiency $R$.

The uncertainty of $\beta_{\text{eff}}$ is calculated as:

$$\sigma_{\beta_{\text{eff}}} = \sqrt{\sigma_{\beta_{\text{eff}}}^2 + \sigma_R^2 + \frac{(\beta_0 \sigma_{\beta_0})^2 + \beta_0^2 R^2 (\sigma_{\beta_0}^2 + \sigma_R^2)}{(1 - \beta_0)^2 + \beta_0 R^2}}$$  \hspace{1cm} (5.8)
Where $\sigma_{\beta_0}$ is the standard deviation of $\beta_0$. Since the value of $\beta_0$ is calculated in the same problem as the value of $k_{\text{eff, prompt}}$, and $\beta_0$ is proportional to the production rate of delayed neutrons while $k_{\text{eff, prompt}}$ is the production rate of prompt neutrons, then both of them are calculated as a modified track length neutron flux tally. So the assumption that $\sigma_{\beta_0} \approx \sigma_{k_{\text{prompt}}}$ is valid. At the same time, during the calculations of $k_{\text{eff, prompt}}$ and $k_{\text{eff, delayed}}$ the same source distribution and the same number of simulated histories are used, therefore $\sigma_{k_{\text{prompt}}} = \sigma_{k_{\text{delayed}}}$, and $\sigma_R = \sqrt{2}\sigma_k$ are valid also. The dependence of standard deviations on the number of histories are shown in Figure 5.23.

![Figure 5.23](image)

Figure 5.23. Statistical uncertainties of $k_{\text{eff}}$, $R$, and $\beta_{\text{eff}}$ versus the number of simulated histories.
It can be seen from Figure 5.22 that values of $\beta_{\text{eff}}$ for both cases are almost equal after simulation of 2 million neutron histories, and the final 95% confidence interval is: $\beta_{\text{eff}} = 0.007782 \pm 8 \cdot 10^{-6}$. In Ref. 8 the values of $\beta_{\text{eff}} = 0.0065$ and $\beta_{\text{eff}} = 0.0075$ are used as examples for different uranium reactors.

Table 5.2. The 95% confidence intervals for values of $\beta_0$, $R$, and $\beta_{\text{eff}}$ for different number of simulated histories.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$ ($10^{-5}$)</th>
<th>$R$</th>
<th>$\beta_{\text{eff}}$ ($10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000 8000 2000 8000 2000 8000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1 \cdot 10^6$</td>
<td>689.3 689.0 1.1309 1.1315 779.5 779.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pm 1.4$ $\pm 1.4$ $\pm 0.0023$ $\pm 0.0033$ $\pm 2.8$ $\pm 2.8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2 \cdot 10^6$</td>
<td>689.0 688.8 1.1306 1.1305 779.0 778.7</td>
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<tr>
<td></td>
<td>$\pm 1.0$ $\pm 1.0$ $\pm 0.0023$ $\pm 0.0023$ $\pm 2.0$ $\pm 2.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4 \cdot 10^6$</td>
<td>688.9 688.9 1.1296 1.1298 778.2 778.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pm 0.7$ $\pm 0.7$ $\pm 0.0016$ $\pm 0.0016$ $\pm 1.4$ $\pm 1.4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$8 \cdot 10^6$</td>
<td>688.9 688.9 1.1297 1.1297 778.3 778.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pm 0.5$ $\pm 0.5$ $\pm 0.0012$ $\pm 0.0012$ $\pm 1.0$ $\pm 1.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$12 \cdot 10^6$</td>
<td>688.9 688.9 1.1297 1.1297 778.2 778.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\pm 0.4$ $\pm 0.4$ $\pm 0.0009$ $\pm 0.0009$ $\pm 0.8$ $\pm 0.8$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The comparison of Figure 5.22 and Figure 4.3 clearly shows the advantage of correlated sampling and the robustness of the $\beta_{\text{eff}}$ calculation method that has been developed in this work.

5.6. Application of new method for multizone reactors

For the case of $\beta_{\text{eff}}$ calculation for systems with variant fuel composition in different regions, for example the system where a highly enriched core is surrounded by a lower-enriched blanket, the $\beta_0$ is not constant in space, and additional calculations are required. In this case the original problem of the $k_{\text{eff}}$ calculation with prompt neutrons only is used to assess prompt and delayed neutron production rates in different regions, and for creation of wssa/rssa file with source distribution. After that, this created file should be post processed to two (or more) files with tracks created in regions with constant $\beta_0$. For each separated region the calculation of relative efficiency requires two correlated calculations: the first would be performed with a prompt neutron spectrum and the second would be performed with a delayed neutron spectrum. Finally, the $\beta_{\text{eff}}$ is defined as the ratio of the number of fissions induced by delayed neutrons to the total number of fissions:
Where \( f_j \) is a fraction of neutrons, produced in the \( j \) region, where \( \beta_{0j} \) is constant.

In the case of two regions (core and blanket), five calculations will be required. But for many systems, where \( \beta_0 \) is approximately constant (like the ISU ADS), only two calculations are necessary.

5.7. Calculation of multiplication constant

The last significant value which can be assessed after simulation of two systems with different spectra is a multiplication constant. The prompt multiplication constant is calculated explicitly in the original problem with a prompt neutron spectrum. It is the ratio of the prompt neutron production to the neutron source in every generation. The total multiplication constant has the same physical meaning, which can be calculated as:

\[
\beta_{\text{eff}} = \frac{\sum f_j \cdot \beta_{0j} \cdot P_{dj}}{\sum f_j \cdot (1 - \beta_{0j}) \cdot P_{pj} + \beta_{0j} \cdot P_{dj}} = \frac{\sum f_j \cdot \beta_{0j} \cdot P_{pj} \cdot R_j}{\sum f_j \cdot (1 - \beta_{0j}) \cdot P_{pj} + \beta_{0j} \cdot P_{pj} \cdot R_j}
\]

Equation (5.9)

Where \( f_j \) is a fraction of neutrons, produced in the \( j \) region, where \( \beta_{0j} \) is constant.

In the case of two regions (core and blanket), five calculations will be required. But for many systems, where \( \beta_0 \) is approximately constant (like the ISU ADS), only two calculations are necessary.

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\[
k_{\text{eff, total}} = (1 - \beta_0) \sum_k \left( Y_{k, \text{prompt}} + Y_{k, \text{delayed}} \right) + \beta_0 \sum_k \left( Y_{k, \text{prompt}} + Y_{k, \text{delayed}} \right)
\]

Equation (5.10)

Where \( Y_k \) is the yield of neutrons, as it is defined in Equation (5.5), per one source neutron. The values of \( Y_{k, \text{prompt}} \) and \( Y_{k, \text{delayed}} \) are yields of neutrons from the fission reaction.
on isotope $k$ calculated in the problems with prompt and delayed spectra, respectively.

For the fundamental mode of the ISU ADS subassembly, the following values were calculated (95% confidence intervals):

$k_{\text{eff}_{\text{prompt}}} = 0.8758 \pm 0.0005$; $\beta_0 = 0.006889 \pm 0.000004$;

$\beta_{\text{eff}} = 0.007782 \pm 0.000008$; $k_{\text{eff}_{\text{total}}} = 0.8828 \pm 0.0005$. The uncertainties include only those of the Monte Carlo method, and they may have greater values. The uncertainty also can be greater because of uncertainties in nuclear data. The non-MCNP uncertainties may increase confidence intervals, therefore the validation of the MCNP model is required and is provided in the next chapter.
CHAPTER 6

SIMULATION OF PULSED NEUTRON SOURCE EXPERIMENT

6.1. Simulation of the neutron source

The values of kinetic parameters (which are calculated in the previous chapter) are not constant during the performance of the pulsed neutron source experiment. They are valid for a fundamental mode, i.e. for a situation that may not be realized in a subcritical core. Therefore the simulation of pulsed neutron source experiments is required to validate the model that is used for the calculation of kinetic parameters, and to calculate the values of parameters that are valid for a subcritical core.

To perform the simulation of pulsed neutron source experiments the neutron source should be defined first. The photonuclear interaction is the only mechanism for the creation of source neutrons in the ISU ADS. Since high energy electrons create bremsstrahlung photons only in the target, the geometrical setup of the MCNP model can be simplified. The formalization of the neutron source has two main goals: first, the direct simulation of electron
transport (coupled with photon and neutron transport) is hundreds of times more time consuming than a neutron transport simulation; second, the MCNP criticality mode (which is used to calculate kinetic parameters) is compatible only with a neutron source. Therefore the spatial and energy dependent distributions of source neutrons should be obtained to simulate the propagation of the neutron burst in the core.

The MCNP model of this problem considers only the tungsten-copper target and is shown in Figure 3.5. An electron source is simulated as the circle with diameter 6.4 mm, representing the divergence of the electron beam at the point where the beam hits the target. The center of the electron source lies on the symmetry axis of a cylindrical geometry. All starting electrons have 20 MeV energy and a direction parallel to the symmetry axis. The distribution of source points over the disc is uniform. The result of the target simulation should be the spatial and energy distributions of neutrons and photons going out of the target. These neutrons and photons can create neutrons in the core via fission and photonuclear reactions.

Because the outgoing high energy photons can create neutrons in the core, the most reasonable way is to write
the surface-source file for consecutive simulations of photon transport and neutrons creation.

The majority of neutrons is created in the target. It is simpler, and therefore better, to write neutron parameters into a file. The neutron parameters are written into a file at the moment of their creation, therefore it is better to simply use these written values instead of obtaining the distribution of neutrons leaving the target.

This approach significantly reduces the complexity of the problem since the angular distribution of neutrons is not required and all created neutrons can be isotropically sampled in a problem with a full core geometry. The isotropy of created neutrons can be explained by the reasons provided in Ref. 35, where the MCNP model of a light particle creation is explained. The double differential cross section is the sum of two terms: one term is coming from a pre-equilibrium process and the other is a non-pre-compound contribution. The latter term is always isotropic. The fraction of neutron emission coming from the pre-equilibrium process has a forward-peaking angular distribution in the quasi-deuteron region. The same fraction is assumed to be isotropic in the giant-dipole region. Since the linac in the ISU ADS operates at 20 MeV, where the giant-dipole resonance is the dominant excitation
mechanism, the isotropic distribution of created neutrons is used in the MCNP simulation of the ISU ADS.

The parameters of created neutrons were written in a file to obtain the neutron spectrum and the spatial distribution of created neutrons. The required distributions were retrieved after the postprocessing of this file. This approach is better than the use of a modified photon tally (similar to Equation (4.5)), since the spectrum of created neutrons is unknown and it cannot be accumulated as a tally during the target simulation.

The results representing the spatial distribution of created neutrons are shown in Figures 6.1-6.2. The calculated spectra of created neutrons and neutrons leaving the target are shown in Figure 6.3, where probabilities of bins with 0.01 MeV widths are presented.

The calculated 95% confidence intervals of the neutron creation probability and probability of the neutron to leave the target are (for 20 MeV electrons)

\[(2.338 \pm 0.002) \cdot 10^{-3} \frac{n}{\text{electron}} \quad \text{and} \quad (2.290 \pm 0.002) \cdot 10^{-3} \frac{n}{\text{electron}},\]

respectively.
Figure 6.1. 2-D distribution of neutron creation density, $n/cm^3$ per one 20 MeV electron hitting the ISU ADS target.

Figure 6.2. 2-D distribution of relative neutron creation density, % of maximum value. $Y_{\text{max}} = 5.722 \times 10^{-3} \frac{n/cm^3}{\text{electron}}$
Figure 6.3. Energy spectra of neutrons created in the target and escaped from the target neutrons.

The probability of photonuclear neutron creation in the core is calculated in fixed-source MCNP mode with the "wssa/rssa" file as the source of photons and with the use of a full core geometry model. In the MCNP model photons are created in the target in two processes. One process takes place when electrons are deflected by atomic nuclei (bremsstrahlung). Another process is the creation of photons when neutrons (created in photonuclear reactions) interact with nuclei of target material. Tracks of all photons leaving the target are recorded in the "wssa/rssa" file. This file is used to calculate the probability of
neutron creation in the core in photonuclear reactions. The 95% confidence interval of neutron creation probability in the core in photonuclear reactions is $(7.63\pm0.01)\cdot10^{-6} \frac{n}{\text{electron}}$

(for 20 MeV energy of electrons hitting the target). Since this value is just 0.327% of the neutron creation probability in the target (which is equal to $(2.338\pm0.002)\cdot10^{-3} \frac{n}{\text{electron}}$), this contribution is neglected in further analysis.

As a result of neutron creation calculations, the fixed source for reactor analysis is defined as the set of the following probability distributions. The coordinates of starting neutrons are sampled from two distributions. First the X coordinate is sampled according to the probability density function presented in Figure 6.4.

After that the Y and Z coordinates are defined as coordinates of points, uniformly distributed on the circle with radius R, which are sampled from appropriate distributions $R(X)$, presented in Figure 6.5. In Figures 6.4-6.5 the probabilities of bins with 0.1 mm width are shown. The neutron directions are sampled from the isotropic distribution, and the energy is sampled from the spectrum, presented in Figure 6.3.
Figure 6.4. Probability density function of X coordinate of starting source neutron.

Figure 6.5. Probability density functions of the distance from beam line for the starting source neutron at different X.
6.2. Verification of MCNP model

The next step in the simulation of the pulsed neutron source experiment is a verification of the MCNP model. For this purpose the fixed source mode was used to simulate the response of detectors to the pulse. The simulation was performed with prompt neutron multiplication and results of the calculation were compared with the experimental prompt neutron response. Experimental data are obtained after subtraction of the delayed neutron background from experimental histograms presented in Figure 3.11. The histories of 2 million neutrons created in the target were simulated to compare uncertainty in the fixed mode with the uncertainty in the criticality mode (they should be the same). The response of the fission chamber was simulated as a track length tally modified by a $^{235}$U fission cross section.

The relative probabilities of the neutron detection at different moments are presented in Figures 6.6-6.11, where probabilities of bins with 10 μs width are shown. The calculated values were obtained by accumulation of tallies in 50 μs bins.
Figure 6.6. Prompt neutron PNS histograms in Fission Chamber B.

Figure 6.7. Front of prompt neutron PNS histograms in Fission Chamber B.
Figure 6.8. Slope of exponential prompt neutron response in Fission Chamber B.

Figure 6.9. Prompt neutron PNS histograms in Fission Chamber A.
Figure 6.10. Front of prompt neutron PNS histograms in Fission Chamber A.

Figure 6.11. Slope of exponential prompt neutron response in Fission Chamber A.
As can be seen from the figures, the experimental data and the calculated results are in good agreement for Fission Chamber B. For Fission Chamber A the time of the maximum probability and the slope are close to the experimental data, but the shape of the experimental response is different from the calculated shape. The discrepancy is observed during the stabilization before the fundamental exponential decay is achieved.

6.3. Evaluation of kinetic time parameters

The data characterizing the dynamic behavior of a reactor can be obtained from the fixed-source calculation of the prompt neutron response. If the single-exponential response is observed, the ratio of prompt removal lifetime to $\beta_{\text{eff}}$ can be evaluated. To check this condition, the fission rate in the total core and the fission rates in the different fuel plates were calculated. In Figure 6.12, fission rates per one neutron created in the target are shown.

Besides responses of fission chambers and total core, the responses of fuel plates in the middle trays are presented. The plates were situated next to the target, in the center of the trays, and far from the target.

It can be seen from Figure 6.12 that after approximately 2 ms the point kinetic behavior is observed in the ISU ADS simulation.
Figure 6.12. Simulated responses $\frac{\text{fission/µs}}{\text{source neutron}}$ to the neutron pulse in different parts of the ISU ADS subassembly.

The reactivity of the ISU ADS in the fundamental mode can be calculated using the results of the previous chapter:

$k_{\text{eff,stat}} = 0.8828 \pm 0.0005$. Since $\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$, the 95% confidence interval for static reactivity is: $\rho = -0.1327 \pm 0.0006$, or $\rho = (-17.06 \pm 0.08) \cdot \beta_{\text{eff}}$ if $\beta_{\text{eff}} = 0.007782 \pm 0.000008$. From a point kinetic equation the mean generation time is $\Lambda = \frac{\rho - \beta_{\text{eff}}}{\alpha}$ and
the prompt neutron lifetime is $l = k_{\text{eff}} \cdot \Lambda$. The values of $\Lambda$ and $l$ are presented in Table 6.1.

Table 6.1. Simulated results for different detectors.

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<thead>
<tr>
<th>detector</th>
<th>$\alpha^a$ (ms$^{-1}$)</th>
<th>$P^b$ (%)</th>
<th>$\Lambda^c$ (ms)</th>
<th>$l^d$ (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC B</td>
<td>-0.692</td>
<td>26.3</td>
<td>0.203</td>
<td>0.179</td>
</tr>
<tr>
<td>FC A</td>
<td>-0.660</td>
<td>34.0</td>
<td>0.213</td>
<td>0.188</td>
</tr>
<tr>
<td>Total core</td>
<td>-0.675</td>
<td>15.9</td>
<td>0.208</td>
<td>0.184</td>
</tr>
<tr>
<td>Next to target fuel plates</td>
<td>-0.685</td>
<td>13.5</td>
<td>0.205</td>
<td>0.181</td>
</tr>
<tr>
<td>Central fuel plates</td>
<td>-0.674</td>
<td>15.1</td>
<td>0.208</td>
<td>0.184</td>
</tr>
<tr>
<td>Peripheric fuel plates</td>
<td>-0.669</td>
<td>23.2</td>
<td>0.210</td>
<td>0.185</td>
</tr>
</tbody>
</table>

a. Slopes calculated by regression analysis in the period 2-4 ms after the pulse;
b. Part of fissions described by point kinetic model, $P$;
c. Calculated values of mean neutron generation time;
d. Prompt removal lifetime.

6.4. Simulation of area-ratio reactivity measurement

A common way to simulate the response to a delayed neutron source is the evaluation of the response as the difference between two fixed source calculations: one is
performed with total (prompt and delayed) yield of neutrons per fission and the other with simulation of prompt neutrons only. While this method has the same drawback as a simple MCNP method of the $\beta_{nf}$ calculation (which is described in Chapter 4), the difference between two values cannot be masked by the statistical uncertainty. This is due to the fact that in the calculation of the response a neutron multiplication takes place. Therefore the closer the system is to criticality, the bigger the difference between the two calculated values. In this case the results of the simulation are valid for the situation of delayed neutron equilibrium, when the number of created (during the source pulse) precursors is equal to the number of issued delayed neutrons. The results of simulation are presented in Table 6.2, where area-ratio reactivity estimator is calculated as the ratio of prompt response to the response created by delayed neutrons.

As can be seen from the table, the reactivity calculated by the area-ratio method is in good agreement with experimental data (Table 3.4). The low accuracy (high uncertainty) is explained by the inherent drawback of the method, when the denominator is defined as the difference between two uncorrelated Monte-Carlo calculations. Also it can be mentioned that the calculated reactivity depends on
the position of the detector, for example, different fuel plates give statistically distinguishable values.

Table 6.2. 95% confidence intervals of simulated responses to the neutron pulse in the case of $\bar{V}_{\text{prompt}}$ and $\bar{V}_{\text{total}}$ per fission and the reactivity area-ratio estimation for different part of the ISU ADS assembly.

<table>
<thead>
<tr>
<th>detector</th>
<th>$R_{\text{prompt}}$ (fission/\text{source neutron})</th>
<th>$R_{\text{total}}$ (fission/\text{source neutron})</th>
<th>$\rho = \frac{R_{\text{prompt}}}{R_{\text{total}} - R_{\text{prompt}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC B</td>
<td>$(544 \pm 3) \times 10^{-5}$</td>
<td>$(578 \pm 4) \times 10^{-5}$</td>
<td>$16.0 \pm 2.4$</td>
</tr>
<tr>
<td>FC A</td>
<td>$(512 \pm 3) \times 10^{-5}$</td>
<td>$(544 \pm 4) \times 10^{-5}$</td>
<td>$16.2 \pm 2.6$</td>
</tr>
<tr>
<td>Total core</td>
<td>$3.883 \pm 0.010$</td>
<td>$4.125 \pm 0.012$</td>
<td>$16.1 \pm 1.0$</td>
</tr>
<tr>
<td>Next to target fuel</td>
<td>$(8024 \pm 24) \times 10^{-5}$</td>
<td>$(8494 \pm 26) \times 10^{-5}$</td>
<td>$17.1 \pm 1.3$</td>
</tr>
<tr>
<td>plates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central fuel plates</td>
<td>$(4898 \pm 16) \times 10^{-5}$</td>
<td>$(5201 \pm 18) \times 10^{-5}$</td>
<td>$16.2 \pm 1.4$</td>
</tr>
<tr>
<td>Peripheric fuel plates</td>
<td>$(4492 \pm 15) \times 10^{-5}$</td>
<td>$(4795 \pm 17) \times 10^{-5}$</td>
<td>$14.8 \pm 1.2$</td>
</tr>
</tbody>
</table>

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6.5. Calculation of effective delayed neutron fraction in the pulsed neutron source experiment

To evaluate a reactivity in the units of $\beta_{\text{ef}}$, an estimation of the effective delayed neutron fraction should be performed for the case of pulsed neutron source. The direct calculation of the relative efficiency of delayed neutron to induce fissions (as it was performed in the case of the fundamental distribution of the delayed neutron source) is very time consuming. This can be explained by the fact that establishment of the fundamental mode requires the calculation of some number of consecutive generations leading to the fundamental mode. Therefore the distribution of precursors is formed not only by fission chains initiated by source neutrons, but also by chains initiated by delayed neutrons. As a result, the calculation of a steady-state distribution of precursors (which represents delayed neutron equilibrium) may require up to a few hundred MCNP criticality calculations. But from simple reasons the possible interval of $\beta_{\text{ef}}$ may be evaluated as follows. The value of the delayed neutron fraction for the pulsed neutron source experiment is between two relatively easily calculated values: one corresponds to the fundamental mode and is calculated in previous chapter
the other corresponds to the situation when a delayed neutron distribution is formed by prompt fission chains initiated by source neutrons. This situation occurs during the short period of time when the delayed neutron source is formed. Indeed the true distribution (achieved at delayed equilibrium) is formed as the second distribution, altered by the distribution of precursors formed by fission chains induced by delayed neutrons. The less subcritical the system is, the closer the true distribution is to the fundamental one. To evaluate $\beta_{\text{eff}}$ corresponding to such "initial" distribution the simulation of the prompt neutron response should be done in the criticality mode.

In the criticality mode, the time dependent response cannot be simulated, but the integral over time can be assessed. This integral is always converged in the case of a subcritical system. This approach requires less computer time than the fixed source calculation of close-to-critical systems, since very long chains of fission events are possible in the fixed source calculation. The calculation of the response in the criticality mode is performed by consecutive simulations of neutron generations, where source neutrons induce fissions and create the source
distribution the first fission generation and so on. When the fission source is stabilized the \( k_{\text{eff}} \) becomes constant and the tail of the response is computed as the sum of the geometrical series: 

\[ n_{\text{tail}} = \frac{n_{0,\text{fund}}}{1 - k_{\text{eff}}} \]

created by the first generation with a fundamental distribution. All previous generations contribute to the total response of the values which are proportional to the weight of a generation. These weights are products of multiplication constants.

The number of neutrons in one generation should be equal to the number of simulated histories in the fixed source mode to achieve a similar level of statistical uncertainty. In the case of a subcritical system, the degradation of the neutron population should be prevented by appropriate weight adjustment. The results of simulation of the ISU ADS are presented in Figure 6.13 and in Table 6.3.

In Figure 6.13 the probabilities of prompt neutron creation (neutron/neutron of previous generation) in fission reactions are shown for different fuel plates. The sum of the values shown in Figure 6.13 is equal to \( k_{\text{prompt}} \) for the appropriate generation.
Figure 6.13. Fission neutron distributions created by different generations of neutrons in three layers of fuel trays.
After 7 generations the $k_{\text{eff}}$, unweighted intergeneration time, and fission source distribution are stabilized, therefore the point-kinetic model can be applied.

In Table 6.3, the calculated fission probabilities (fission/neutron of previous generation) for different fuel plates and detectors, the average intergeneration time $\bar{t}_\text{fission}$ (the average time from birth to fission) and three different MCNP estimators of the multiplication constant are presented.

Since the parameters of the 7th generation are very close to the kinetic parameters of the fundamental mode, it is possible to conclude that the rest of the response can be simulated according to the single-exponential point kinetic model. For every generation, the same method of the $\beta_{\text{eff}}$ calculation as for the fundamental mode was used. Therefore 8 calculations with substitution of prompt neutron spectrum by the delayed neutron spectrum were made. The yields of prompt and delayed neutrons and $\beta_0$ for every generation were calculated.
Table 6.3. Stabilization of calculated parameters in criticality mode during eight consecutive generations.

<table>
<thead>
<tr>
<th>generation</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{\text{collision}}$</td>
<td>1.0293</td>
<td>0.9489</td>
<td>0.9096</td>
<td>0.8924</td>
</tr>
<tr>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0014$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{absorption}}$</td>
<td>1.0286</td>
<td>0.9486</td>
<td>0.9098</td>
<td>0.8925</td>
</tr>
<tr>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0014$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{track}_\text{length}}$</td>
<td>1.0308</td>
<td>0.9481</td>
<td>0.9091</td>
<td>0.8935</td>
</tr>
<tr>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0014$</td>
<td>$\pm 0.0014$</td>
<td></td>
</tr>
<tr>
<td>$\bar{t}_{\text{fission}}$, μS</td>
<td>99.8</td>
<td>125.9</td>
<td>133.4</td>
<td>136.8</td>
</tr>
<tr>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td></td>
</tr>
<tr>
<td>FC B</td>
<td>$(430\pm12) \times 10^{-6}$</td>
<td>$(574\pm14) \times 10^{-6}$</td>
<td>$(580\pm15) \times 10^{-6}$</td>
<td>$(549\pm15) \times 10^{-6}$</td>
</tr>
<tr>
<td>FC A</td>
<td>$(256\pm9) \times 10^{-6}$</td>
<td>$(491\pm13) \times 10^{-6}$</td>
<td>$(514\pm14) \times 10^{-6}$</td>
<td>$(513\pm14) \times 10^{-6}$</td>
</tr>
<tr>
<td>core</td>
<td>0.4267</td>
<td>0.3923</td>
<td>0.3762</td>
<td>0.3697</td>
</tr>
<tr>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td></td>
</tr>
<tr>
<td>target</td>
<td>$(1432\pm8) \times 10^{-5}$</td>
<td>$(814\pm6) \times 10^{-5}$</td>
<td>$(714\pm5) \times 10^{-5}$</td>
<td>$(693\pm6) \times 10^{-5}$</td>
</tr>
<tr>
<td>central</td>
<td>$(451\pm4) \times 10^{-5}$</td>
<td>$(479\pm4) \times 10^{-5}$</td>
<td>$(481\pm4) \times 10^{-5}$</td>
<td>$(476\pm4) \times 10^{-5}$</td>
</tr>
<tr>
<td>peripheric</td>
<td>$(185\pm3) \times 10^{-5}$</td>
<td>$(398\pm3) \times 10^{-5}$</td>
<td>$(457\pm4) \times 10^{-5}$</td>
<td>$(467\pm4) \times 10^{-5}$</td>
</tr>
<tr>
<td>generation</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>$k_{\text{collision}}$</td>
<td>0.8856</td>
<td>0.8816</td>
<td>0.8791</td>
<td>0.8782</td>
</tr>
<tr>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{absorption}}$</td>
<td>0.8856</td>
<td>0.8815</td>
<td>0.8794</td>
<td>0.8786</td>
</tr>
<tr>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td></td>
</tr>
<tr>
<td>$k_{\text{track}_\text{length}}$</td>
<td>0.8845</td>
<td>0.8795</td>
<td>0.8784</td>
<td>0.8759</td>
</tr>
<tr>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td>$\pm 0.0015$</td>
<td></td>
</tr>
<tr>
<td>$\bar{t}_{\text{fission}}$, μS</td>
<td>137.1</td>
<td>137.9</td>
<td>138.1</td>
<td>138.5</td>
</tr>
<tr>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td>$\pm 2.0$</td>
<td></td>
</tr>
<tr>
<td>FC B</td>
<td>$(538\pm15) \times 10^{-6}$</td>
<td>$(523\pm15) \times 10^{-6}$</td>
<td>$(510\pm15) \times 10^{-6}$</td>
<td>$(518\pm15) \times 10^{-6}$</td>
</tr>
<tr>
<td>FC A</td>
<td>$(513\pm14) \times 10^{-6}$</td>
<td>$(519\pm15) \times 10^{-6}$</td>
<td>$(515\pm15) \times 10^{-6}$</td>
<td>$(524\pm16) \times 10^{-6}$</td>
</tr>
<tr>
<td>core</td>
<td>0.3664</td>
<td>0.3644</td>
<td>0.3635</td>
<td>0.3629</td>
</tr>
<tr>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td>$\pm 0.0006$</td>
<td></td>
</tr>
<tr>
<td>target</td>
<td>$(688\pm6) \times 10^{-5}$</td>
<td>$(683\pm6) \times 10^{-5}$</td>
<td>$(683\pm6) \times 10^{-5}$</td>
<td>$(680\pm6) \times 10^{-5}$</td>
</tr>
<tr>
<td>central</td>
<td>$(476\pm4) \times 10^{-5}$</td>
<td>$(475\pm4) \times 10^{-5}$</td>
<td>$(472\pm4) \times 10^{-5}$</td>
<td>$(469\pm4) \times 10^{-5}$</td>
</tr>
<tr>
<td>peripheric</td>
<td>$(467\pm4) \times 10^{-5}$</td>
<td>$(471\pm4) \times 10^{-5}$</td>
<td>$(467\pm4) \times 10^{-5}$</td>
<td>$(467\pm4) \times 10^{-5}$</td>
</tr>
</tbody>
</table>
The relative efficiency of delayed neutrons versus prompt neutrons to induce fissions was finally assessed to calculate $\beta_{\text{eff}}$ for every generation. The calculated values of the mentioned parameters, the weights of every generation, and weighted contributions from each generation to the total response of different detectors are presented in Table 6.4.

**Table 6.4. Calculated values of parameters required for $\beta_{\text{eff}}$ calculation for eight consecutive generations.**

<table>
<thead>
<tr>
<th>generation</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{eff}}$</td>
<td>0.007663 ± 0.000022</td>
<td>0.007736 ± 0.000024</td>
<td>0.007756 ± 0.000024</td>
<td></td>
</tr>
<tr>
<td>$P_{\text{delayed}} / P_{\text{prompt}}$</td>
<td>1.1151 ± 0.0024</td>
<td>1.1230 ± 0.0024</td>
<td>1.1259 ± 0.0025</td>
<td></td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>0.006872 ± 0.000014</td>
<td>0.006888 ± 0.000015</td>
<td>0.006889 ± 0.000015</td>
<td>0.006889 ± 0.000016</td>
</tr>
<tr>
<td>weight</td>
<td>1.0308</td>
<td>0.9773</td>
<td>0.8885</td>
<td></td>
</tr>
<tr>
<td>FC B</td>
<td>(430±12) $\cdot 10^{-6}$</td>
<td>(592±14) $\cdot 10^{-6}$</td>
<td>(567±15) $\cdot 10^{-6}$</td>
<td>(487±13) $\cdot 10^{-6}$</td>
</tr>
<tr>
<td>FC A</td>
<td>(256±9) $\cdot 10^{-6}$</td>
<td>(506±13) $\cdot 10^{-6}$</td>
<td>(503±14) $\cdot 10^{-6}$</td>
<td>(456±12) $\cdot 10^{-6}$</td>
</tr>
<tr>
<td>core</td>
<td>0.4267 ± 0.0006</td>
<td>0.4044 ± 0.0006</td>
<td>0.3676 ± 0.0006</td>
<td>0.3285 ± 0.0005</td>
</tr>
<tr>
<td>target</td>
<td>(1432±8) $\cdot 10^{-5}$</td>
<td>(839±6) $\cdot 10^{-5}$</td>
<td>(698±5) $\cdot 10^{-5}$</td>
<td>(615±4) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>central</td>
<td>(451±4) $\cdot 10^{-5}$</td>
<td>(494±4) $\cdot 10^{-5}$</td>
<td>(469±4) $\cdot 10^{-5}$</td>
<td>(423±4) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>peripheric</td>
<td>(185±3) $\cdot 10^{-5}$</td>
<td>(411±3) $\cdot 10^{-5}$</td>
<td>(446±4) $\cdot 10^{-5}$</td>
<td>(415±4) $\cdot 10^{-5}$</td>
</tr>
</tbody>
</table>
Table 6.4 (Continued)

<table>
<thead>
<tr>
<th>generation</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{eff}}$</td>
<td>$0.007778 \pm 0.000026$</td>
<td>$0.007781 \pm 0.000027$</td>
<td>$0.007783 \pm 0.000027$</td>
<td>$0.007782 \pm 0.000027$</td>
</tr>
<tr>
<td>$P_{\text{delayed}} / P_{\text{prompt}}$</td>
<td>$1.1291 \pm 0.0026$</td>
<td>$1.1304 \pm 0.0027$</td>
<td>$1.1298 \pm 0.0027$</td>
<td>$1.1297 \pm 0.0027$</td>
</tr>
<tr>
<td>$\beta_0$</td>
<td>$0.006889 \pm 0.000017$</td>
<td>$0.006889 \pm 0.000017$</td>
<td>$0.006889 \pm 0.000017$</td>
<td>$0.006889 \pm 0.000017$</td>
</tr>
<tr>
<td>weight</td>
<td>$0.7938$</td>
<td>$0.7022$</td>
<td>$0.6175$</td>
<td>$4.3690$</td>
</tr>
<tr>
<td>FC B</td>
<td>$(427 \pm 12) \cdot 10^{-6}$</td>
<td>$(367 \pm 11) \cdot 10^{-6}$</td>
<td>$(315 \pm 9) \cdot 10^{-6}$</td>
<td>$(2263 \pm 66) \cdot 10^{-6}$</td>
</tr>
<tr>
<td>FC A</td>
<td>$(407 \pm 11) \cdot 10^{-6}$</td>
<td>$(364 \pm 10) \cdot 10^{-6}$</td>
<td>$(318 \pm 9) \cdot 10^{-6}$</td>
<td>$(2291 \pm 62) \cdot 10^{-6}$</td>
</tr>
<tr>
<td>core</td>
<td>$0.2909 \pm 0.0005$</td>
<td>$0.2558 \pm 0.0005$</td>
<td>$0.2245 \pm 0.0005$</td>
<td>$1.5854 \pm 0.0027$</td>
</tr>
<tr>
<td>target</td>
<td>$(546 \pm 4) \cdot 10^{-5}$</td>
<td>$(480 \pm 5) \cdot 10^{-5}$</td>
<td>$(422 \pm 4) \cdot 10^{-5}$</td>
<td>$(2973 \pm 22) \cdot 10^{-5}$</td>
</tr>
<tr>
<td>central</td>
<td>$(378 \pm 3) \cdot 10^{-5}$</td>
<td>$(333 \pm 4) \cdot 10^{-5}$</td>
<td>$(292 \pm 3) \cdot 10^{-5}$</td>
<td>$(2051 \pm 18) \cdot 10^{-5}$</td>
</tr>
<tr>
<td>peripheric</td>
<td>$(371 \pm 3) \cdot 10^{-5}$</td>
<td>$(331 \pm 4) \cdot 10^{-5}$</td>
<td>$(289 \pm 3) \cdot 10^{-5}$</td>
<td>$(2038 \pm 18) \cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

In Table 6.5, the 95% confidence intervals for detector responses calculated by two different methods are presented: the first is obtained by the simulation of a fixed source time-dependent response and the second is based on the criticality calculation of separated generations.
Table 6.5. 95% confidence intervals of simulated responses in the fixed source mode and in the criticality mode.

<table>
<thead>
<tr>
<th>detector</th>
<th>$R_{prompt}$, fixed source</th>
<th>$R_{prompt}$, criticality source neutron fission source neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC B</td>
<td>(544±3) $\cdot 10^{-5}$</td>
<td>(549±8) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>FC A</td>
<td>(512±3) $\cdot 10^{-5}$</td>
<td>(514±8) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>Total core</td>
<td>3.883±0.010</td>
<td>3.886±0.009</td>
</tr>
<tr>
<td>Next to target</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fuel plates</td>
<td>(8024±24) $\cdot 10^{-5}$</td>
<td>(8022±30) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>Central fuel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plates</td>
<td>(4898±16) $\cdot 10^{-5}$</td>
<td>(4904±26) $\cdot 10^{-5}$</td>
</tr>
<tr>
<td>Peripheric fuel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plates</td>
<td>(4492±15) $\cdot 10^{-5}$</td>
<td>(4497±24) $\cdot 10^{-5}$</td>
</tr>
</tbody>
</table>

The results of the criticality mode simulation coincide with the direct fixed-source simulation, therefore it can be concluded that calculation of the delayed neutron fraction (which is possible in the criticality mode) gives the correct result. A special case is the accounting of fissions which are directly induced by source neutrons.
According to their physical meaning, $\beta_0$ and $\beta_{eff}$ are the ratios of values characterizing delayed neutrons to the total values. In this case $\beta_{eff}$ (average over the pulse) will be less than “weighted” value which can be calculated from Table 6.4. So the “weighted” value is equal to

$$\beta_{eff}^{fission} = \sum_i W_i \cdot \beta_{eff}^i$$

where $i$ is the generation number. The 95% confidence interval is $\beta_{eff}^{fission} = 0.007761 \pm 0.000014$ and it is the delayed neutron fraction among all fission neutrons in the pulsed response. The value (average over pulse) is

$$\beta_{eff}^{PNS} = f_{fission} \cdot \sum_i W_i \cdot \beta_{eff}^i$$

where $f_{fission}$ is the fraction of fission neutrons in the pulsed response. So the 95% confidence interval $\beta_{eff}^{PNS} = 0.006988 \pm 0.000012$, since $f_{fission} = 0.9004 \pm 0.0005$. Both of $\beta_{eff}^{fission}$ and $\beta_{eff}^{PNS}$ are less than fundamental $\beta_{eff} = 0.007782 \pm 0.000008$. The difference in the case of $\beta_{eff}^{fission}$ is explained by different relative efficiencies of delayed neutrons to induce fissions in comparison with prompt neutrons. The relative efficiency varies during the stabilization of the fission source distribution. In the case of $\beta_{eff}^{PNS}$, the source neutrons are taken into consideration and this decreases the value. Obviously, the less subcritical the system is, the closer $\beta_{eff}^{fission}$ and $\beta_{eff}^{PNS}$.
are to $\beta_{\text{eff}}$. For deep subcritical systems, source neutrons create a significant part of the response. Therefore the bounding value is $\beta_{\text{eff}}^{\text{fiss.PNS}} = f_{\text{fission}} \cdot \beta_{\text{eff}}$ i.e.

$$\beta_{\text{eff}}^{\text{fiss.PNS}} = 0.007006 \pm 0.000012.$$

According to the physical meaning of the term, the delayed neutron fraction is the fraction of fissions which is induced by delayed neutrons. In the PNS experiment this value is between $\beta_{\text{eff}}^{\text{PNS}} = 0.006988 \pm 0.000012$ and

$$\beta_{\text{eff}}^{\text{fiss.PNS}} = 0.007006 \pm 0.000012.$$ Therefore the reactivity of the ISU ADS $\rho = -0.1327 \pm 0.0006$ in the units of $\beta_{\text{eff}}^{\text{PNS}}$, which is between $-18.95 \pm 0.10$ and $-19.00 \pm 0.10$.

6.6. Simulation of gold foil activation

Besides calculation of kinetic parameters the flux distribution can be obtained using the time-dependent simulation of the response to the neutron pulse. Experimental data were obtained with a gold foil activation technique. Therefore to calculate the activation of gold foils the standard MCNP track length tally was modified by the absorption cross section of $^{197}\text{Au}$. Since this material was used for tallying only and not for an actual transport calculation, and since the cross section has a resonance, the discrete reaction cross section from data file "newxsd"
was used to decrease the variance of the result instead of the point-wise absorption cross section of $^{197}\text{Au}$.

![Graph showing neutron absorption cross section of $^{197}\text{Au}$](image)

**Figure 6.14.** Neutron absorption cross section of $^{197}\text{Au}$.

The maximum value of $^{198}\text{Au}$ creation probability is

$$Y_{\text{max}} = (0.00437 \pm 0.00014) \frac{\text{nucl/g}}{\text{source neutron}}$$

at the point

$$(X = 65\text{mm}, Z = -35\text{mm})$$. To compare calculated values with experimental data all values were normalized per the maximum experimental value at the point with coordinates

$$(X = 56\text{mm}, Z = 30\text{mm})$$ where calculated yield of $^{198}\text{Au}$ is
\[ Y_{\text{max}} = \left(0.00389 \pm 0.00014\right) \frac{\text{nucl/g}}{\text{source neutron}} \cdot \] The calculated and experimental values are presented in Table 6.6.

Figure 6.15. 2-D distribution of gold foil activation simulation results, % of maximum value.

Table 6.6. The results of gold foil activity calculation.

<table>
<thead>
<tr>
<th>Foil</th>
<th>( X, ) mm</th>
<th>( Z, ) mm</th>
<th>( A_{\text{exp}}, ) Bq/g</th>
<th>( A_{\text{exp}}, % ) of max. value</th>
<th>( Y_{\text{calc}}, % ) of max. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>20</td>
<td>45</td>
<td>3394±21</td>
<td>74.6±0.5</td>
<td>(291±15) ( \times 10^{-6} )</td>
</tr>
<tr>
<td>R1</td>
<td>-550</td>
<td>249</td>
<td>90±2.0</td>
<td>1.98±0.04</td>
<td>(82±5) ( \times 10^{-6} )</td>
</tr>
<tr>
<td>P1</td>
<td>-550</td>
<td>140</td>
<td>149±3.3</td>
<td>3.28±0.07</td>
<td>(171±20) ( \times 10^{-6} )</td>
</tr>
</tbody>
</table>
Table 6.6. (continued)

<table>
<thead>
<tr>
<th>Foil</th>
<th>X, mm</th>
<th>Z, mm</th>
<th>$A_{exp}$, Bq/g</th>
<th>$A_{exp}$, % of max. value</th>
<th>$Y_{calc}$, % of max. value</th>
<th>$Y_{calc}$, % of max. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>-305</td>
<td>248</td>
<td>420±8</td>
<td>9.24±0.17</td>
<td>(37±4)10^{-5}</td>
<td>9.4±1.1</td>
</tr>
<tr>
<td>U1</td>
<td>550</td>
<td>251</td>
<td>136±21</td>
<td>2.99±0.06</td>
<td>(81±15)10^{-6}</td>
<td>2.1±0.4</td>
</tr>
<tr>
<td>T1</td>
<td>307</td>
<td>248</td>
<td>532±10</td>
<td>11.7±0.23</td>
<td>(47±4)10^{-5}</td>
<td>12.1±1.1</td>
</tr>
<tr>
<td>E1</td>
<td>20</td>
<td>295</td>
<td>588±11</td>
<td>12.9±0.24</td>
<td>(49±5)10^{-5}</td>
<td>12.6±1.3</td>
</tr>
<tr>
<td>D1</td>
<td>20</td>
<td>245</td>
<td>1135±22</td>
<td>24.9±0.5</td>
<td>(86±7)10^{-5}</td>
<td>22±1.8</td>
</tr>
<tr>
<td>Cl</td>
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<td>195</td>
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<td>35.4±0.7</td>
<td>(133±9)10^{-5}</td>
<td>34.2±2.4</td>
</tr>
<tr>
<td>L1</td>
<td>550</td>
<td>140</td>
<td>252±4</td>
<td>5.54±0.09</td>
<td>(129±19)10^{-6}</td>
<td>3.3±0.5</td>
</tr>
<tr>
<td>K1</td>
<td>356</td>
<td>140</td>
<td>735±13</td>
<td>16.16±0.28</td>
<td>(55±5)10^{-5}</td>
<td>14.2±1.3</td>
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<td>I1</td>
<td>307</td>
<td>140</td>
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<td>(60±5)10^{-5}</td>
<td>15.4±1.3</td>
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<tr>
<td>G1</td>
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<td>140</td>
<td>1998±20</td>
<td>43.9±0.4</td>
<td>(147±9)10^{-5}</td>
<td>37.7±2.3</td>
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<tr>
<td>B1</td>
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<td>145</td>
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<td>48.0±0.5</td>
<td>(185±12)10^{-5}</td>
<td>48±3</td>
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<tr>
<td>M1</td>
<td>-120</td>
<td>140</td>
<td>1665±20</td>
<td>36.6±0.4</td>
<td>(134±9)10^{-5}</td>
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<td>N1</td>
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<td>681±8</td>
<td>14.98±0.17</td>
<td>(57±5)10^{-5}</td>
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<tr>
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<td>531±6</td>
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<td>21.1±0.3</td>
<td>(68±5)10^{-5}</td>
<td>17.4±1.4</td>
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<tr>
<td>F1</td>
<td>107</td>
<td>110</td>
<td>2482±30</td>
<td>54.6±0.7</td>
<td>(182±11)10^{-5}</td>
<td>46.8±2.8</td>
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<tr>
<td>A1</td>
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<td>68.1±0.6</td>
<td>(224±13)10^{-5}</td>
<td>57.6±3.3</td>
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<tr>
<td>R</td>
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<td>110</td>
<td>1848±25</td>
<td>40.6±0.6</td>
<td>(134±8)10^{-5}</td>
<td>34.5±2.2</td>
</tr>
</tbody>
</table>
Table 6.6. (continued)

<table>
<thead>
<tr>
<th>Foil</th>
<th>X, mm</th>
<th>Z, mm</th>
<th>$A_{\text{exp}}$, Bq/g</th>
<th>$A_{\text{exp}}$, % of max. value</th>
<th>$Y_{\text{calc}}$, $\frac{\text{nucl.}^{144}\text{Au/g}}{\text{source, neutron}}$</th>
<th>$Y_{\text{calc}}$, % of max. value</th>
</tr>
</thead>
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<td>P</td>
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<td>110</td>
<td>628±8</td>
<td>13.81±0.18</td>
<td>(57±5)$\times 10^{-5}$</td>
<td>14.7±1.3</td>
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<tr>
<td>O</td>
<td>-350</td>
<td>110</td>
<td>498±7</td>
<td>10.95±0.16</td>
<td>(45±4)$\times 10^{-5}$</td>
<td>11.5±1.1</td>
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<tr>
<td>J</td>
<td>353</td>
<td>30</td>
<td>927±15</td>
<td>20.4±0.3</td>
<td>(77±4)$\times 10^{-5}$</td>
<td>19.8±1.1</td>
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<td>I</td>
<td>207</td>
<td>30</td>
<td>2591±33</td>
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<td>(217±8)$\times 10^{-5}$</td>
<td>55.8±2.1</td>
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<td>(389±14)$\times 10^{-5}$</td>
<td>100±4</td>
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<td>(262±10)$\times 10^{-5}$</td>
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<td>-121</td>
<td>0</td>
<td>3427±29</td>
<td>75.3±0.6</td>
<td>(296±10)$\times 10^{-5}$</td>
<td>75.9±2.5</td>
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<tr>
<td>E</td>
<td>-172</td>
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<td>2589±25</td>
<td>56.9±0.5</td>
<td>(220±7)$\times 10^{-5}$</td>
<td>56.6±1.9</td>
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<tr>
<td>D</td>
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<td>42.1±0.4</td>
<td>(153±6)$\times 10^{-5}$</td>
<td>39.3±1.4</td>
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<td>C</td>
<td>-273</td>
<td>0</td>
<td>1286±13</td>
<td>28.3±0.3</td>
<td>(110±5)$\times 10^{-5}$</td>
<td>28.3±1.2</td>
</tr>
<tr>
<td>B</td>
<td>-305</td>
<td>0</td>
<td>950±10</td>
<td>20.89±0.21</td>
<td>(87±4)$\times 10^{-5}$</td>
<td>22.2±1.0</td>
</tr>
<tr>
<td>A</td>
<td>-350</td>
<td>0</td>
<td>614±7</td>
<td>13.50±0.15</td>
<td>(59±3)$\times 10^{-5}$</td>
<td>15.1±0.9</td>
</tr>
<tr>
<td>K</td>
<td>-350</td>
<td>-110</td>
<td>513±6</td>
<td>11.28±0.14</td>
<td>(46±4)$\times 10^{-5}$</td>
<td>11.7±1.1</td>
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<tr>
<td>L</td>
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<td>-110</td>
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<td>14.91±0.20</td>
<td>(71±6)$\times 10^{-5}$</td>
<td>18.1±1.5</td>
</tr>
<tr>
<td>M</td>
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<td>-110</td>
<td>1850±19</td>
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<td>(154±9)$\times 10^{-5}$</td>
<td>39.4±2.4</td>
</tr>
<tr>
<td>N</td>
<td>-80</td>
<td>-110</td>
<td>2230±21</td>
<td>49.0±0.5</td>
<td>(188±12)$\times 10^{-5}$</td>
<td>48±3</td>
</tr>
</tbody>
</table>
6.7. The main results of the simulation

The probability of neutron creation in the target is

\[(2.338\pm 0.002) \times 10^{-3} \frac{n}{\text{electron}}\]

for 20 MeV energy of electrons. The neutron source strength of the ISU ADS facility for reference experiments (with beam peak current 10 mA, pulse width 1 \(\mu\)s, repetition rate 30 Hz) is therefore

\[(4.378\pm 0.004) \times 10^{-7} \frac{n}{s}\].

The static reactivity for \(k_{\text{eff,nucl}} = 0.8828 \pm 0.0005\) is \(\rho = -0.1327 \pm 0.0006\). In the units of delayed neutron fraction (\(\beta_{\text{eff}} = 0.007782 \pm 0.000008\) for fundamental mode) the static reactivity is \(\rho = (-17.06 \pm 0.08)\). The dynamic reactivity obtained by simulation of the area-ratio method is \(\rho = (-16.1 \pm 1.0)\). Since the single-exponent response is observed, the values of the prompt removal lifetime and the mean generation time can be evaluated: \(\tau = (0.184 \pm 0.003)\) ms and \(\Lambda = (0.208 \pm 0.003)\) ms.
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1. Conclusions

Because of its deep subcriticality, the ISU ADS configuration presented unprecedented challenges to measure and compute subcriticality, subcritical multiplication, and associated reactor kinetic parameters and characteristics such as the effective delayed neutron fraction, neutron lifetime, and neutron die-away. Knowledge of these parameters is necessary for monitoring and controlling of accelerator-driven transmutation systems, for which the RACE project was intended to contribute insights. Because of the deep subcriticality level, the area-ratio method was the only experimental technique available to determine reactivity.

The results of the simulations are in good agreement with the experimental data, and thus the results of the computational analysis appear representative. These results demonstrated clearly that the values of the static reactivity and the dynamic reactivity are different.
Therefore it can be concluded that the results of measurements require calculation of correction factors, at least for such deep subcritical systems. The origin of the discrepancy between dynamic and static reactivity was outlined in Chapter 2.

The main feature of this dissertation is the development and application of a new method of effective delayed neutron fraction calculation that is described in Chapter 5. This method is based on the calculation of probabilities of delayed and prompt neutrons to induce fission. Calculations of the effective delayed neutron fraction are implemented with a standard version of the MCNP code, using the Monte Carlo method of the neutron transport equation solution. An application of correlated sampling in the implementation of this method provides high accuracy and good convergence of the results. The efficiency of the new method does not depend on fuel composition since equal numbers of histories are simulated both for prompt and for delayed neutrons. This approach makes this method different from existing methods, in which the number of traced delayed neutron histories is proportional to the yield of delayed neutrons in the fission reaction. Because of this feature the new method is suitable for the calculation of the effective delayed neutron fraction in systems with
minor actinide fuel. Application of the new method to the ISU ADS uranium fuel is shown in Chapters 5 and 6. This simulation demonstrates the difference between results obtained for the fundamental distribution of delayed neutron precursors and for the distribution which is formed in the deep subcritical core during the propagation of a neutron pulse.

7.2. Recommendations for future research

If reactor accelerator coupling experiments will be continued at ISU, the subcritical core should be redesigned to increase the value of the multiplication constant. The distribution presented in Figure 6.2 shows that the diameter of the cylindrical target might be significantly decreased with little impact on neutron creation probability. In this case the new core will lose fewer neutrons in the central zone with high importance. The distribution of fissions presented in Figure 5.7 shows that the position of the target should be moved along the beam line so that the maximum fission density is in the center of the core. Figure 6.13 shows that the condition to have the same number of fuel plates in each tray may not be optimal. The performance can be improved if the number of fuel plates in the middle layer is less than the number of plates in the top and bottom layers.
Some recommendations can be made regarding the new method of calculating effective delayed neutron fraction. The implementation of the new method of calculating delayed neutron fraction could be significantly simplified if it were possible to modify the MCNP code directly. After future modification, the prompt and delayed calculations will be required even for systems with different fuel types (core and blanket). The implementation of the new method for systems with a fast neutron spectrum or for systems with mixed oxide fuel also will be simpler if the source code is modified.
REFERENCES


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