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The impact of interactive factors on Romanian students' understanding of place value

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THE IMPACT OF INTERACTIVE FACTORS ON ROMANIAN STUDENTS' UNDERSTANDING OF PLACE VALUE

by

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ABSTRACT

The Impact of Interactive Factors on Romanian Students' Understanding of Place Value

By

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Students' mathematics achievement is believed to be influenced by a variety of factors (Brenner, M.E., Herman S., Ho, H.Z. & Zimmer, J.M., 1999; Cai, C. 2000; Huntsinger, C. & J., P.E., 2000; Ma, 1999; Miura, 1987; Stevenson, L. & Stigler, J.W. 1986). As such, the performance gap in mathematics between students from different countries was attributed in turn, to the teachers' subject matter and pedagogical knowledge (Ma, 1999, Perry, 2000), curriculum development (Li, 2000; Valverde, Bianchi, Wolfe, Schmidt, Houang, 2002), native language (Miura, 1987), as well as parental raising and teaching strategies (Dornbush, Ritter, Leiderman, Roberts, & Fraleigh, 1987; Huntsinger, Jose, Larson, Krieg, & Shaligram, 2000).

While these studies provide reasonable explanations for the performance gap, their limitation was to only analyze isolate factors rather than look into how these factors interact. This approach might have provided a more in-depth understanding of what enables Asian students to perform better than other nations in the international mathematics and science comparisons, and what prohibits U.S. students from performing at a similar level. Complexity (Maturana & Varela, 1984; Senge, 1990;Waldrop, 1992) may hold the answer to this dilemma, by analyzing how these factors work with each other in order to produce the end result, student mathematics understanding.

This study aimed at gaining understanding about the way different factors interact in the Romanian educational system, by examining teaching strategies, curriculum, parental teaching styles, as well as teachers' interaction with students and parents and parents' interaction with their children. Participants were four first-grade teachers, their students and their students' parents.

Findings revealed that overall, teachers who had both a conceptual and a procedural knowledge of place value concepts and who created the best learning opportunities for their students both in school and at home had students who possessed a more in-depth understanding of place value concepts. Moreover, the quality of home interaction was another success indicator success, as parents who were more knowledgeable developed their own assessment rubrics and reinforced classroom concepts more than parents who lacked the conceptual understanding of the topics and limited themselves to only modeling what the teacher did in class.

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CHAPTER 1

INTRODUCTION

The general assumption regarding student mathematics performance is that it depends on a series of factors, such as: teacher content knowledge, instructional practices, parental teaching strategies, curriculum, and language (Brenner, Herman, Ho, & Zimmer, 1999; Cai, 2000; Huntsinger, Jose, Larson, Krieg, & Shaligram, 2000; Ma, 1999; Miura, 1987; Stevenson & Stigler, 1986). As such, the success or lack of success of students in the international mathematics and science competitions came to be regarded as a direct result of these aforementioned factors. What makes Asian students outperform other nations in the Programme for International Student Assessment 2000, Third International Mathematics and Science Study 1994, and Third International Mathematics and Science Study-Repeat 1999 mathematics and science international competitions as reported by the National Center for Educational Statistics (1994; 1999a; 1999b), while American students' performance is merely average?

Teacher and comparative literature both provided thorough analyses of potential factors believed to be responsible for the performance gap among students from different countries. Some researchers affirmed that the school factors widened the gap in performance (National Center for Educational Statistics, 1994, 1999a; 1999b; Perry, 2000; PISA 2000). First teacher content knowledge is believed to influence the way teachers transfer this knowledge into classrooms (Ball, 1990; Ball & McDiarmid, 1990; Kennedy, 1991; Schulman, 1996) and further impact student learning and achievement. Hence, the stronger mathematical content knowledge and pedagogical content knowledge of the Chinese elementary school teachers in Ma's (1999) study may lead to a better understanding of basic mathematics of Chinese students and in turn, it may ultimately lead to their better performance in the international mathematics and science competitions. This advantage, coupled with a more cohesive Chinese curriculum (Li, 2000), which exposes Chinese students to fewer but more in-depth topics than their American peers, may lead to the better performance of Chinese students in the international mathematics competitions.

Secondly, researchers (Miura, 1987) advocated that differences in mathematics performance are innate in the language we speak: speakers of Asian languages, seem to have an advantage over speakers of other languages (English, French) due to their numerical number characteristics congruent with the Base-10 system. The Base-10 system impacts the acquisition of concepts like place value, for example, which in turn impacts all other mathematics concepts, like addition, subtraction, multiplication.

Thirdly, other researchers (Chen & Stevenson, 1995; Huntsinger et al., 2000) stated these differences are perpetuated by the cultural milieus in which students live, such as more liberal/authoritative households (Caucasian American), more traditional/authoritarian households (Asian American). As such, students in more traditional households were exposed to more formal types of interactions (drills, more interaction time, worksheets and rubrics) that seemed to benefit the Asian American students in Huntsinger's (2000) longitudinal study.

These studies assumed particular factors may hold the answer for the performance gap in mathematics and provided evidence to support their assumptions. Nevertheless, the main limitation they share is that these studies only look at this performance gap by analyzing these factors in isolation, which may provide a limited explanation of the Asian students' success. A closer look at the TIMSS 1994 and TIMSS-R 1999 scores reported by the National Center for Educational Statistics (1994; 1999a; 1999b) revealed that if Asian students were compared to students from Romania, although the latter shared many of the characteristics believed to render the Asian students successful, as assumed by the afore-mentioned studies, the Romanian students performed significantly lower than the Asian students and the U.S. students. For example, Romanian curriculum was found to be as cohesive as the Chinese curriculum (Schmidt, McKight, Houang, Wang, Wiley, Cogan & Wolfe, 2001), while the overall school climate in Romania was again similar to the Chinese school climate (TIMSS, 1994, TIMSS-R 1999).

This study seeks to investigate how both schooling factors (teacher knowledge, instructional strategies, and curriculum) and non-schooling factors (parental teaching techniques) interact and impact students' mathematics learning following the complexity theory (Maturana & Varela, 1984; Senge, 1990; Waldrop, 1992) by drawing on surveys, observations, and interview data from Romanian first-grade teachers, students and their parents in relation to student learning of place value in mathematics at the first-grade level.

Need for the Study

The value of this study lies beyond the explanations it may provide for the students' place value understanding and the factors that may be responsible for this understanding. More importantly, this study strives to challenge previous research findings that held particular factors responsible for the mathematics achievement of Asian and United States students, by introducing in the equation a third variable: the Romanian educational system. Due to its similarities and differences to both China and United States, Romania has the potential to challenge the previous assumptions that either home or schooling factors are responsible for the better performance of Asian students.

For example, by analyzing the way national curriculum is organized one may conclude that the Chinese curriculum is more cohesive than the United States curriculum, a fact which may enable the Chinese students to learn fewer concepts but more in-depth (Li, 2000; Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001). A closer look at the way Romanian curriculum is organized (Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001) may reveal a surprising number of similarities to the Chinese curriculum, in terms of sequential themes that make curriculum more cohesive. Moreover, Romanian textbooks had less content breaks than the Chinese textbooks, which might lead to even more cohesion between topics in the Romanian curriculum. Despite this apparent curriculum advantage, Romanian students were outperformed by both Chinese and United States peers in the TIMSS 1995 and 1999 studies. This performance gap cannot justify the national curriculum as a single cause of the better performance of Chinese students.

Some researchers tried to relate the better results of Chinese students to instructional strategies (Ma, 1999; Perry, 2000). On the other hand, a look at the way Romanian teachers participating in the TIMSS 1994 study (National Center for Educational Statistics, 1994) stated they organized their classrooms revealed striking similarities to the way the Chinese teachers participating in the same study stated they designed their mathematics classes, namely giving students more practice by themselves in class if they were having difficulties, having students work more independently without assistance from the teacher. However, this fact alone may not justify the better results obtained by the Chinese students, opposed to the very low results obtained by the Romanian students.

Miura (1984) stated that Asian students enter school with an advantage in their language, as the number naming systems of Asian languages are believed to impact the understanding of place value concepts and the Base-10 system for the speakers of these languages (Saxton & Towse, 1998). Consequently, Asian students learn to easily manipulate numbers, a fact that may lead to a better understanding and performance of mathematics of these students. On the other hand, the numeration system of Romanian language is also consistent with the Base-10 system, yet to a lesser degree than the Chinese language, but to a greater degree than the English language (Wang, Lin, Tanase, Sas, 2008). If Romanian students were to be at an advantage due to their number naming system, this is not reflected in their results in the TIMSS 1994 and 1999 international competitions, as their performance was rated as below average (National Center for Educational Statistics 1994).

The above assumptions that Chinese students perform well due to their exposure to better teaching strategies, a more cohesive curriculum, and the advantage of the native language seem to make sense in the China-United States comparison. It then holds true that Chinese students are better at mathematics because Chinese classrooms are differently organized than the United States classrooms in terms of mathematics instruction (National Center for Educational Statistics, 1994; Perry, 2000). The Chinese mathematics curriculum is more cohesive (Li, 2000; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), and parents and students hold different standards about mathematics in China than they do in the United States. (Chen & Stevenson,1995; Hess, Chih-Mei, & McDevitt,1987; Miura, 1987).

However, as seen in the PISA (2000) and TIMSS 1994 and 1999 studies (National Center for Educational Statistics, 1994), Romania is similar to China in terms of classroom organization, school atmosphere, and curriculum organization. Both Chinese and Romanian students spend an equal number of mathematics instruction hours yearly (118 vs 114), the Romanian curriculum is seen as cohesive, if not more cohesive than the Chinese curriculum in that is has a fewer number of breaks. On the other hand, Romania shares similarities in terms of scores with the United States, which raises the question the accuracy of previous research findings. This study analyzes students' understanding of place value concepts in a country that supposedly has an advantage (in terms of language, classroom organization, educational standards, and curriculum), but still performs poorly in international competitions, offering yet another interpretation in terms of relationship between different factors and student mathematics learning. Researcher focused on place value concepts as these concepts represent the cornerstone of mathematics, as understanding of these concepts influences the understanding of further mathematics topics such as addition, subtraction, multiplication and division, to only name a few (Ho & Cheng, 1997).

Conceptual Framework

In order to account for the multitude of interactions between factors considered responsible for student learning, this study relies on the complexity theory for a more in-depth understanding of the interrelatedness of these factors, as the above cited studies showed that relying solely on one factor at a time may not justify why Romanian students perform lower than both United States and Chinese students when they have a more cohesive curriculum, are assigned more homework and have more positive attitudes towards mathematics than do their counterparts from both United States and China.

What is complexity theory, and how may complexity theory explain the performance gap in mathematics to a greater extent than the theories used in the previous studies? Defined as a science of emergence (Waldrop, 1992), complexity is a class of behaviors in which the components of a (living) system constantly organize and reorganize themselves into larger structures. Furthermore, in a complex system many independent agents interact with each other in many ways. The outcomes of such interactions are, according to Waldrop (1992), complex systems that are adaptive, "in that they just don't positively respond to events the way a rock might roll around in an earthquake. They actively try to turn whatever happens to their advantage" (Waldrop, 1992, p. 11).

According to Waldrop (1992), each of these systems is a network of many agents acting in parallel. To exemplify, think of the composition of the brain: in a brain, the agents are cells; in a cell the agents are organelles. If we maximize the context, in ecology, the agents are species, in an economy, the agents are individuals or households. However we might define them, "each agent finds itself in an environment produced by its interactions with the other agents in the system, and because of the constant reaction to the other agents' action, nothing in the environment of the complex systems is fixed" (Waldrop, 1992, p. 145).

These complex systems have also been named autopoetic systems (Maturana & Varela, 1984). These systems, or machines, are in essence:

> A network of processes of production (transformation and destruction) of components that produces the components which i) through the interactions and transformations continuously regenerate and realize the network of processes (relations) that produced them; and ii) constitute it (the machine) as a concrete unity in the space in which they (the components) exist by specifying typological domain of its realization as such a network (p.76).

Similar to Waldrop's (1992) interpretation of adaptive systems, Maturana and Varela (1984) describe the autopoetic systems as a composite unity, existing in a space defined by its components, and the relations between these components constitute the organization of the system. In other words:

> If a component "A", through its interaction with a component "B" triggers an interaction of "B" with "C" that triggers a reduction in the production of "D,"

> then we may say that "A" controls the production of "D," "A," "B," "C," and "D" interacting through relations of contiguity (p. xxi).

Another term used for the same concept is "systems thinking" (Senge, 1990), defined as the fifth discipline, the discipline that integrates all the other disciplines, fusing them into a coherent body of theory and practice. The conceptual framework beyond the fifth discipline is the way in which individuals, businesses and all other organizations perceive themselves: from seeing themselves as "separate from the world to connected to the world, and from seeing problems as caused by someone or something "out there" to seeing how our own actions create the problems we experience" (Senge, p.12). That is to say that both our success and lack of success is not influenced by us alone, but by the actions of everyone else in the system.

Systems thinking, then, has the capacity to render individuals and organizations able "to make a shift from seeing the world primarily from a linear perspective to seeing and acting systematically (Senge, 1990, p.135). Consequently, Senge advocated that everything that happens in a living system is caused by the actions of all the factors involved in that particular action. The major assumption in complexity theory is therefore acknowledging the fact that in any system, there are no independent agents, and that, on the contrary, each agent is part of a team, and that disregarding this fact may only provide limited understanding of how systems evolve.

Complexity theory has been used to explain the way systems interact with each other not only in biology, (Freeland, 1979; Kauffman, 1991; Kauffman, 1992; Maturana & Varela, 1984) but also in economy (Arthur, 1989; Arthur, 1990), and computer science (Axelrod, 1984; Goldberg, 1989; Holland, 1975; Holland, Holyoak, Nisbett, & Thagard, 1986), allowing for possibilities to be applied to other fields. The field of education then, seemed a viable option: what happens when complexity theory is applied to the field of education? How may complexity and the study of interactive factors help gain a more profound understanding of students' mathematics achievement?

From this perspective, investigating the interactions between the multiple factors believed responsible for student learning may provide a more in-depth understanding of student mathematics learning. The following questions provide a basis for inquiry:

1. What knowledge do students possess about place value concepts?
2. How do classroom interactions influence students' mathematics understanding of place value?
3. How do home interactions influence students' understanding of place value?
4. What interactions exist between teachers and parents that influence students' understanding of place value?
5. How do parents-teachers-students interactions influence students' understanding of place value?
6. What is the relationship among all the interactions that take place?

Assumptions and Limitations of the Study

The major assumptions underlying this study were as follows: if Romanian students perform poorly in the international comparisons, this poor performance may be caused by a gap in the system, namely either teachers lack the conceptual and/or procedural understanding of the topic, or classroom or home interactions between teachers, students and parents are weak. Acknowledging the fact that any living organism is made up of the interactions between its agents (Waldrop, 1992), and that in order for learning to occur students must interact with teachers, teachers must interact with parents who, in turn, interact with their children, what causes the gap in the system in the case of Romania that prevents our students to perform at their best in international comparisons?

The major limitation of this study is the small sample size of teachers. Only four first-grade teachers were observed and interviewed in this study, which made generalization to the larger population difficult. In order to reduce this limitation, the researcher collected different types of data from different sources. As such, besides conducting two interviews with each of the teachers and observing them teach place value concepts to their students, the researcher also interviewed five students from each class regarding their understanding of place value, sent questionnaires to all the students' parents and also interviewed two to three families from each class regarding their involvement in their children education, along with analyzing student tests on place value. This triangulation enabled the researcher to verify data obtained from the three sources (parents, teachers and students) and served to make a stronger case for the interactions among the multiple factors assumed responsible for the students' understanding of place value.

An additional limitation of the study may have been the researcher's influence on the environment of each participant. The researcher's presence in the classroom settings (observing the teacher and students) and at home (interviewing parents) may have prompted the participants to behave in ways they would not normally behave. In order to reduce this limitation, the researcher explained to the participants the nature of the data collection, specifying to the students that there were no right or wrong answers to the interview questions, to the teachers that their real names would not be used in future articles or presentations related to this study and that these classroom observations would in no way be used to judge their performance by the school administration, and to the parents that the information gathered from them through questionnaires and interviews would not be reported to the school (teacher or principal).

An incumbent limitation of the study was the fact that this study was conducted in Romanian, with the researcher being the one transcribing and translating the documents from Romanian into English. As in any translation, facts may be lost and words or statements may be given a different interpretation. In order to prevent this from happening and to provide accurate data, the researcher asked four native speaking Romanians, who have a working knowledge of English, to verify the accuracy of the translations.

Definitions of Terms

The following list of words provides an understanding of the concepts used most frequently in this study, enabling the reader to better grasp the meaning behind these concepts and interpret the data.

<u>Apprenticeship of observation</u>: Informal learning about what teaching is that occurs when we are students in school and as students we are exposed to different ways of teaching and learning (Lortie, 1975).

<u>Assisted performance</u>: The process of paring up novices with mentors in order for experienced teachers to induct novices into the intellectual and practical challenges of reform-minded teaching (Feinman-Nemser & Beasley, 1997).

Autopoetic systems: A composite unity existing in a space defined by its components, the relations between these components constituting the organization of the system (Maturana & Varela, 1984).

Big ideas: The central ideas characteristics to a discipline, ideas that form the foundation of that discipline and without which teachers only have a limited understanding of their subject matter (Graeber, 1999).

Cognitive representation of number: Through language, numbers are mentally represented and stored, and for those languages that are rooted in ancient Chinese numerical names are organized so that they are congruent with the traditional Base-10 numeration system (Miura, Kim, Chang, & Okamoto, 1988).

Complexity theory: a science of emergence advocating a class of behaviors in which the components of a (living) system constantly organize and reorganize themselves into larger structures (Waldrop, 1992).

Conceptual knowledge of mathematics: Understanding the "why" mathematical concepts are solved in a particular way informs the way teachers teach these concepts to their students (Ma, 1999).

Content knowledge: The amount and organization of knowledge per se in the mind of the teacher (Schulman, 1986, p.9). In order for teachers to teach subject matter to their students, they need to possess content knowledge, which is seen as a prerequisite for teaching (Schulman, 1986).

Cultural influences: The home environments students grow up in and that are influencing their school behavior, the attitudes and motivations they develop towards particular school subjects. These influences may be parental teaching techniques, as well as standards and expectations parents hold towards education (Chen & Stevenson, 1995).

Elementary/middle-school: Primary schools in Romania are called "general schools" and they comprise both elementary and middle school grades (grades 1^{st} –9^{th}).

Freeze the frame: To provide prospective teachers with opportunities to stop the action and analyze what is happening in the case studies. Teacher education classes can enable prospective teachers in analyzing such cases with the help of mentors (Kennedy, 1991).

Knowledge packages: Procedural and conceptual topic are interwoven, these packages having a sequence in the center and a circle of linked topics connected to the topics in the sequence. As such, a mathematics teacher will know the entire field of mathematics and the whole process of learning it (Ma, 1999).

Learning on-site: Teacher learning situated in the context of practice (Ball and McDiarmid, 1990).

Multi-tier program design: A program focusing on the interacting development of students, teachers and researchers, in which both students and teachers are engaged in problem-solving situations that challenge them to revise previous mathematics concepts (Schorr & Koellner-Clark, 2003).

Non-schooling factors: Factors believed to influence student learning outside of the school context: parental teaching strategies, parents' content knowledge, and parents' interaction with students.

Pedagogical content knowledge: Ways of representing the subject that make it comprehensible to others, such as illustrations, examples, demonstrations, explanations (Schulman, 1986).

PISA: Programme for International Student Assessment first conducted in 2000, and repeated in 2003 and 2006. It assesses students' analysis, reasoning and communication skills.

Place value: The positions of the digits in any numeral affecting their value. Thus, each digit and numeral in a place-value system conveys far more information than it would in a system without place value (Sovchik, 1989).

Procedural knowledge of mathematics: Knowing how to solve mathematics problems, but not knowing why they are solved in a particular way (Ma, 1999).

PUFM: Profound understanding of fundamental mathematics, teachers' understanding of elementary mathematics topics that are the cornerstone for understanding all complex mathematics principles (Ma, 1999).

Schooling factors: Factors believed to influence student learning in the school context: teachers' content knowledge and their use of instructional strategies, curriculum, teachers' interaction with students.

TIMSS: Third International Mathematics and Science Study conducted in 1994 and repeated in 1999 and 2003. Students from 41 countries were included in this study, which was the first largest study of this kind.

Organization of the Study

Chapter one introduces the stated problem, namely the types of interactions that exist between school and home environments and the ways these interactions influence student learning. The research questions of the study are also exposed in this chapter, and the most commonly used terms are defined.

Chapter two reviews related literature on both schooling and non-schooling factors in an attempt to understand the better performance of Asian students in the international mathematics and science comparisons. The review points to the gap in the existent literature and justifies the need for further research.

Chapter three describes the theoretical framework that inspired the research design of the study. The methodology, data sources and collection, and procedures for data analysis are described in this chapter. Data sources and analysis are explained in chapters four (student findings), five (teacher findings), and six (parent findings). Findings are reported and research questions are answered. Chapter seven draws the conclusions of the study and proposes recommendations for future research. References and appendices conclude the document.

CHAPTER 2

REVIEW OF RELATED TEACHER EDUCATION AND COMPARATIVE EDUCATION LITERATURE

This chapter provides a review of literature on the performance gap between students from different countries, about teacher knowledge and classroom practice, and other factors that are assumed to influence student mathematics achievement, such as curriculum, language, home influences, student motivation, and student and parental expectations. Through this review, an attempt is made to identify and analyze the gaps in the literature in explaining this relationship. Important research questions will be proposed for this dissertation study.

Teacher Education Literature: Teachers' Knowledge of Mathematics and Its Impact on Students' Mathematics Achievement

The Rhetoric for Mathematics Education Reform: The United States Case

Starting from the late 1990s advocates of reform blamed the poor results of the United States students in mathematics on the traditional teaching practices and fragmented mathematics curriculum and suggested reform should start particularly in these areas (Carnine, 1991; Nicol, 1999; Spungin, 1996; Stedman, 1997a; Stedman, 1997b). Almost a decade later, Ball (2003), Lawrenz (2003) and Manoucheri &

Goodmman (2000) still advocated reform in an effort to improve the teaching and learning of mathematics. These reform proposals seemed quite logical. Since the gap in the mathematics performance between United States students and students from the top-performing countries was mainly explained through large discrepancies in national curriculum and teaching practices, it was deemed important that the reform of the United States educational system start in these two areas (Ball, 2003; Senger, 1999; Stedman, 1992).

Curricular Reform

United States mathematics curriculum was described over the years as facts-based and memory driven (Stedman, 1997b), with fragmented textbooks (Flanders, 1994) that are covering too many topics without providing an in-depth approach (Schmidt et al., 1996). All these facts indicated an acute need for reforming mathematics curriculum. The assumption is that since all top-performing countries in the international mathematics competitions had a more cohesive curriculum, covering fewer but more in-depth topics, unlike a denser United States curriculum, students in top performing countries will be at an advantage over the United States students in mathematics learning (Stedman, 1997b; Westbury, 1993). These assumptions should, however, be regarded with some caution. Wang & Lin (2005) stated that one of the limitations of these international comparisons was that the United States has been constantly compared to top-performing countries, without much attention being directed to the lower-performing countries. Such comparisons may prove interesting, as in the case of Romania, which although having a centralized curriculum like the top-performing countries, had below-average results at all levels of competition. Whether a centralized or decentralized curriculum may better serve the needs of a nation remains the subject of further investigation.

What would happen to teaching as United States mathematics curriculum changed from the traditionalists' textbooks abundant in information (Greenes, 1995) into more standards-oriented textbooks that are focused on enabling teachers to reflect on their own knowledge and student thinking and learning? Different results of this reform are presented in the studies below. Drawing on data from classroom observations and teacher interviews, Rodriguez (2000) investigated the implementation of new curriculum in an experienced teacher's second-grade classroom, and found a change in Maria, moving from traditional instruction to more-reform oriented instruction. This change in the teacher's practice was attributed to her participation in the Mathematics Project, which impacted her understanding of how to apply reform-oriented curriculum in the classroom. This case study supported the view that in order for teachers to change their practice and implement new curriculum in their classrooms, they need to be provided with appropriate teacher development in conjunction with the implementation of new curricula.

In a different case, drawing from classroom observations and interview data, Manoucheri & Goodman (2000), investigated two mathematics teachers using a Standards-based mathematics textbook in their seventh-grade classrooms. Findings show that while one teacher was able to help students establish connections between important concepts in the curriculum by using engaging activities, another teacher did not possess a strong mathematics knowledge for using the textbook and her teaching resembled her level of understanding of the content. The study suggests that curriculum alone is not enough for teaching change and it is important for teachers to possess a deeper conceptual and procedural understanding of the concepts in the curriculum to reform their teaching.

Other researchers (Cohen, 1991) also showed discrepancies in teachers' implementation of new curriculum materials while heavily relying on traditional teaching techniques, as in the case of Ms. Oublier's second-grade classroom. Findings indicated a superficial nature of reform, as the teacher fell back on the traditional methods of instruction when there was no support to facilitate reform implementation. This study suggests that reforming curriculum without providing support for teachers to change their practice will not have the anticipated impact in classrooms.

These studies together indicated that reforming curriculum alone is unlikely to solve the nation's problems related to achievement in mathematics. It is necessary to provide the teachers with the support needed to implement the more standards-based textbooks, as two out of four teachers in the above studies fell back on traditional ways of teaching the curriculum. Their significance lies in the fact that they indicate a need for reform for the curriculum and unveiled successful examples of standards-based curriculum implementation in the classroom. However, these studies failed to link curriculum development to theoretically good teaching and student learning. Attention should be thus directed to the ways teachers use instructional practices in the classrooms along with the curriculum to teach mathematics for understanding.

Instructional Practice Reform

Along with curriculum, teachers' instructional practice has been deemed essential in improving the quality of mathematics teaching and learning in United States classrooms. The discrepancies in teachers self-reports between the United States and top-performing countries about their envisions for and descriptions of their instruction at all three grade levels: fourth-, eighth-, and twelfth-grades were analyzed, which lead to the assumption that if United States teachers were to perform more like the Asian teachers, then the performance gap may be diminished (PISA, 2000; TIMSS 1994; TIMSS-R 1999).

Just as in the case of the curriculum, the assumptions regarding instructional practices are not without question: again the practices of United States teachers were compared to the practices of the teachers from the top performing countries, without looking into what teachers from other countries might do in their classrooms. If United States policy makers were aware, for example, of the similarities between the practices of Romanian and Chinese teachers in the TIMSS-R 1999 study, as reported by the National Center for Educational Statistics (1999a; 1999b) and yet the large discrepancies between the scores of the students in the two countries, would they still be eager to advocate a change in the practices of the United States teachers?

In an effort to improve classroom practice, small steps have also been taken in the area of instructional strategies, which again brought about mixed results. Drawing on data from classroom observations, Schorr & Koellner-Clark (2003) looked at the ways one middle school teachers attempted to change his practice as well as the conditions that led to these changes in a multi-tiered program design focusing on the development of students, teachers and researchers. Following discussions with his peers, the teacher realized that his way of asking questions did not enable him to acknowledge for his students' conceptual understanding of the concepts, and he decided to change his approach from procedures to conceptually based ideas. Despite its obvious strength, this study is limited in that the teachers' use of new instructional strategies (more standards-oriented), although enabling the teacher to better understand and teach the concept, is not related to student performance.

Similar attempts to reform teaching practices in a six-grade mathematics class were analyzed by Brown, Stein & Foreman (1996), drawing data from interviews, classroom observations and documentation of project activities as part of the QUASAR Project, a national mathematics reform project aimed at studying the implementation of mathematics instructional programs for middle school students. Findings indicated that one of the teachers, Ms. Jackson, relied heavily on what she saw modeled in the staff development sessions provided by the resource partners, creating a classroom in which students felt free to take risks and benefited from the framework of assistance. While this study suggested that there is a strong connection between teacher practice, professional development and student understanding, one isolated case cannot allow one to generalize the results to the rest of the teachers in the remaining five schools. Moreover, was this an isolated incident in Ms. Jackson's teaching, or did she continue to implement these novel strategies all through the program? More studies were needed to analyze teachers' predispositions to change their practice by contrasting more cases.

One such study (Senger, 1999) analyzed the change patterns three fourth-grade teachers faced for the duration of a school year. Drawing data from interviews with the teachers and classroom observations, and group meetings with the researcher and the principal throughout the school year, the researcher found that the change process varied from teacher to teacher, even thought the same interaction and support were provided to each of the participants in the meetings and interview with the researcher. While one of the teachers was open to constructivist teaching suggestions and experimented on a daily basis, the second teacher did not want to implement the novel teaching strategies that clashed with his ideas of mathematics being very structured and rigorous. The third teacher initially believed the teacher was the authority, yet these beliefs conflicted with the newer mental images of constructivism, and he made attempts to change some of the classroom activities to allow more space for student creativity. One of the major limitations of this study, however, was the lack of connection between the implementation of novel teaching approaches and student achievement. However, the major significance of this study lies in the finding that even when support is provided, reform of teaching practices may vary across teachers. Consequently, teacher reform, even when support is provided, may or may not impact all teachers in an equal manner and hence, may not alone be the answer to improving the nation's students' mathematics performance.

The above studies focused on factors in isolation, looking either at the way new curriculum was implemented or how standards-based teaching was implemented without looking at how both these factors may influence each other, and moreover without linking the better curriculum or better teaching techniques to student learning. More research is needed to investigate how teachers' mathematics knowledge may influence their use of curriculum and instructional strategies, and it might ultimately impact student learning.

Improvement of Teachers' Subject Matter Knowledge

Being able to use curriculum effectively and adopt teaching strategies that enhance learning are deeply rooted in understanding the subject matter (Ball, 2003).

Subject matter knowledge refers to the "amount and organization of knowledge per se in the mind of the teacher" (Schulman,1986, p.9). From this perspective teachers must not only understand that something is so, but they must also understand why something is so.

Since teacher knowledge was assumed to be essential to student learning (Ball & Bass, 2001), scholars (Cochran-Smith & Lytle, 1999; Feinman-Nemser & Remillard, 1996; Schulman, 1986) suggested that the presence or absence of this kind of knowledge affects the learning opportunities teachers provide to their students. Consequently, what teachers know about their subject and how they teach in class may determine what students learn. Understanding what type of knowledge teachers possess, as well as how they use this knowledge to use the classroom curriculum and develop the best learning environment that meets the needs of all their students may shed more light into the impact some schooling factors may have on student learning.

Learning in Situ vs. Learning in the Context of Teacher Education Classes

The significance of knowing one's subject matter well in order to teach it was largely explored by Ball and McDiarmid (1990). According to the researchers, "if teaching entails helping others learn, then understanding what it is to be taught is a central requirement of teaching" (Ball & McDiarmid, p.437). Conversely, a teacher's ignorance of the subject matter can harm students (Conant, 1963). Conant's argument is reinforced by Ball and McDiarmid, who further stated:

> When teachers possess inaccurate information or conceive knowledge in narrow ways, they may pass on these ideas to their students. They may fail to challenge students' misconceptions; they may use texts uncritically or alter them inappropriately...teachers' conceptions of knowledge shape their practice-the

> kinds of questions they ask, the ideas they reinforce, the sorts of tasks they assign. (p.437)

According to these researchers the absence of subject matter knowledge or the presence of inaccurate knowledge informs the choice of instructional strategies, and applied in the classroom context, it may harm student learning. In other words, teachers must have the math knowledge unique to teaching, which may enable them to explain and represent ideas in various ways to students (Hill, Sleep, Lewis & Ball, 2007).

This is the case of Schonfeld's (1988) longitudinal study of teaching and learning in a tenth-grade geometry class. Drawing data from classroom observations of the target class and other eleven mathematics classes, interviews with students and teachers and questionnaire analyses regarding students' understanding of mathematics, the study revealed that exposed to a mathematics class where mathematics was based on memorization and drill leading to a procedural understanding of the concepts covered, students failed to develop the conceptual understanding of the topics, believing that accuracy is what counts and not the understanding of the concepts. This study suggests that mathematics teaching and learning needs to focus on transmitting both a procedural and a deep conceptual understanding of the mathematics concepts, otherwise students and prospective teachers will not learn and teach mathematics for understanding.

In a similar vein, Ball (1990) examined in a longitudinal study what knowledge teacher candidates bring with them to formal education, showed that even if mathematics teachers possessed the procedural knowledge to solve the problems, they lacked the conceptual understanding that enabled them to teach the topic effectively to their students. Drawing data from questionnaires and interviews with 252 prospective teachers participating in a large study of teacher education (TELT), results showed that both elementary teachers and secondary teachers majoring in mathematics had problems "unpacking" the meaning of division with fractions. This study also suggests that if future teachers possessed a compartmentalized knowledge, where procedural and conceptual understanding of the concepts were seen as separate entities, their understanding of fundamental mathematics concepts was narrow. Findings of the above studies reveal that teachers lacked the conceptual understanding of the notions, even if they possessed the procedural knowledge to solve the problems. This procedural knowledge of mathematics in most cases informed the preservice teachers' choice of instructional strategies: if knowing mathematics means knowing how to do it, teaching mathematics is realized by following a set of step-by-step procedure to arrive at answers (Ball and McDiarmid, 1990, Maestre & Lockhead, 1983; Schonfeld, 1985).

If students are exposed to this limited understanding of mathematics in their mathematics precollege classes (Schonfeld, 1988), behavior which is reinforced further by the college mathematics classes (Ball, 1990; Graeber, 1999), future mathematics teachers begin teaching with a fragmented understanding of mathematics which may be the results of a fragmented curriculum (Greenes, 1995; Schonfeld, 1988) and it may ultimately lead to a learning environment that does not enhance student learning.

More recent studies still advocate the need for reform. The Final Report of the National Mathematics Advisory Panel (2008) calls for a more coherent pre K-8 mathematics curriculum with a more logical progression from less difficult topics to more sophisticated subject matter may serve better the needs of today's classrooms. At the instructional level, conceptual along with procedural knowledge of mathematics is still heralded. In order for teachers to teach for understanding, they need to have a strong grasp of mathematics for teaching. In a similar vein, Hill, Sleep, Lewis & Ball (2007) stress the significance for teachers to know to do the math they are teaching, but also to explain and represent ideas in a various ways to students. Key to mathematics learning seems to also be teachers' use of formative assessment, if teachers use assessment to design and to individualize instruction (Final Report of National Mathematics Advisory Panel, 2008). The conclusions of the above studies are as follows: teachers' subject matter knowledge has an impact on student learning. Since teachers cannot teach what they have not learned (National Mathematics Advisory Panel, 2008), teachers who have a procedural understanding of the topics will teach procedurally, the end result being students who only have a procedural understanding of the topics. What are the solutions then, how may this fragmented understanding of teacher subject matter knowledge be remedied?

Learning More

To improve teachers' subject matter knowledge, the usual solution has been to require teachers to study more mathematics. Studies (Good & Grouwns, 1987; Taylor, 1987) were conducted with the purpose to show the relationship between teachers' knowledge of subject matter and student achievement. In an effort to understand more about this relationship, drawing data from student tests, Tooke (1993) investigated the different types of mathematical knowledge of twenty-three student teachers enrolled in different mathematics classes, and whether or not this knowledge related to student achievement. Findings indicated that teachers' course work beyond calculus had an impact on their students in algebra classes. This study suggests that an increase in the students' mathematical achievement may be associated with an increase in the number of mathematics courses completed by the teachers. Tooke's theory, that acquiring more mathematical knowledge in college will produce more prepared mathematics teachers, who, in turn will help prepare better students relates to what Cochran-Smith and Lytle (1999) described as "knowledge-for-practice," namely that knowing more (subject matter, instructional strategies), leads to more effective practice.

Findings from a larger data base seem to contradict Tooke's (1993) findings: Ma's (1999) study, conducted in conjunction with the TELT database, indicated that even if United States elementary teachers had more formal training than Chinese teachers, their understanding of mathematics was more limited than in the case of their Chinese counterparts and this limited understanding might translate into poor student results in the international mathematics competitions. That learning more mathematics may not be a solution to remedy the mathematics performance of United States students in the international comparisons was also discussed by Ball (2003), while addressing the need for reform of the United States education. Ball stated that while many researchers suggest to improve by requiring teachers to study more mathematics, "increasing the quantity of teachers' mathematics coursework will only improve the quality of mathematics teaching if teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work," (Ball, p.1) opposing thus the increase in the mathematics classes. The goal of reform, as envisioned by Ball is not to produce teachers who know more mathematics, but to improve students' learning.

Moreover, the conclusion of the National Mathematics Panel (2008) was that current research studies investigating the relationship of teachers' content knowledge to their students' achievement at the elementary and middle school level, using the mathematics courses that teachers have taken as a proxy for their mathematical knowledge produced mixed results.

The above studies provide arguments against and in favor of increasing the number of mathematics classes future teachers should attend. Learning more may or may not have an impact on student understanding, but the nature of teacher knowledge is more likely to impact student achievement. If future teachers possess a profound understanding of fundamental mathematics (Ma, 1999), they are more likely to transpose this understanding in their classroom practice and enable their students to possess a conceptual and procedural understanding of the mathematics topics. Learning more for both teachers and their students may not be then a viable solution to diminish the performance gap in mathematics. Learning "big" may be one.

Learning Big

Some researchers (Greaber, 1999; Greenes, 1995) attempted to show that it is the quality of what is being taught in college mathematics classes, and not the quantity that produces meaningful learning. In order for teacher education programs to prepare better teacher candidates who will teach mathematics for understanding, one solution is to expose teachers to the "big ideas" in their college mathematics classes (Graeber). In reviewing her students' work in mathematics, education and chemistry, Graeber was struck by the difficulty all these students had in identifying the "big ideas" of a course, which lead her to conclude that maybe "faculty are unsure of what the big ideas might be or do a poor job of conveying them" (Graeber, p.190). This study suggests that in order for prospective teachers to possess these big ideas before entering the classroom, faculty need to identify the central ideas of their courses and find ways to convey those ideas to their students consistent with constructivist views of learning (Brooks & Brooks, 1993; Steffe, 1990). Otherwise sending the teachers into classrooms without owning these ideas would be "doing the profession a disservice" (Graber, p.204).

In a similar vein, Greenes (1995) discussed the significance of teaching big ideas, which she paralleled with constructivist teaching, the key to good teaching being according to her "knowing what students know and their degree of understanding, and using that information to plan appropriate learning experiences" (Greenes, p.87). This idea is distanced from the traditional way to teach and learn mathematics, which has been criticized (Greenes & Fitzgerald, 1991) for rote learning without understanding.

Both Graeber (1999) and Greenes (1995) advocated for teaching prospective teachers the "big ideas" in college mathematics classes, as understanding student thinking and being able to make the distinction between conceptual and procedural knowledge would enable teachers to design the best lessons for their students.

Learning in Situ

The above findings imply that teacher education classes should provide multiple opportunities for college students to understand the substance of their subject matter knowledge that would enable them to teach it effectively to their students, but in most cases, they fail to do so. Attention should also be directed to the learning on site, as teachers may develop content knowledge while teaching. Ball and McDiarmid's (1990) argument is grounded in and supported by cognitive apprenticeship theories, which advocate for teacher learning situated in the context of practice. In this way, knowledge of subject matter is reinforced and informed by the pedagogical content knowledge

(Schulman, 1986). Findings (Ball & Feinman-Nemser, 1988; Wilson, Schulman & Richert, 1987; Winerberg, 1987) reveal an increase in the understanding of the content knowledge of the teachers by their practice.

However, other lines of research (Feinman-Nemser & Buchman, 1985) show that some situated experiences, like student teaching, may be more detrimental than beneficial to students, raising the question whether or not learning on situ is the optimal solution to remedy the lack of subject matter knowledge teacher education classes seem to perpetuate. The solution to this dilemma, as envisioned by Kennedy (1991), is to "freeze the frame," namely to provide prospective teachers with opportunities to stop the action and analyze what is happening. Teacher education courses can provide prospective teachers with opportunities to analyze such cases with the help of mentors. Kennedy's hypothesis was that subject matter ideas could be best acquired in the context of practice, through a process of situated learning in the teacher education classes.

The above studies present the following controversy: although teacher education classes do not provide a solid knowledge base for prospective mathematics teachers (Ball and McDiarmid, 1990), teachers may enhance their subject matter knowledge through their practice (Ball & Feinman-Nemser, 1988; Wilson, Schulman & Richert, 1987). On the other hand, research studies showed that some situated apprenticeships do not enhance teacher learning (Feinman-Nemser & Buchman, 1985) and that teachers need to possess subject matter knowledge prior to possessing pedagogical knowledge (Kennedy, 1991). In order for curricular and instructional reform to function, teachers need to possess a solid knowledge base (Ball, 2003). Whether this knowledge base should be developed in tertiary education or much earlier (primary and secondary education), like in the case of the Chinese teachers in Ma's (1999) study, is still open for consideration. What is an obvious limitation in all the studies discussed in this review is the lack of a direct connection between teachers' subject knowledge and student learning. More research needs to investigate the relationship between teacher subject matter knowledge, use of curriculum materials and teaching strategies and their impact on student learning.

In an attempt to fill the gap in the teacher education literature, and link teachers' knowledge to student achievement, Hill, Rowan and Ball (2005) conducted a longitudinal study involving 115 elementary schools, drawing their data parent interviews, student assessments and teachers' logs and questionnaires. Findings revealed a pale positive correlation between teachers' content knowledge and the number of years teaching mathematics, teaching certification or the number of mathematics courses teachers took, challenging previous research findings (Graeber, 1999) stipulating that "learning more" may increase teachers' content knowledge. However, researchers did find significant correlations between teachers' mathematics knowledge and student achievement, but this correlation might be caused either by the content-specific knowledge for teaching or by the teachers' general knowledge and/or aptitude for teaching. This study shows a direct connection between teachers' knowledge of mathematics and student achievement, advocating that teachers' knowledge of mathematics plays a significant role in the teaching of elementary mathematics content (Hill et al., 2005.

Working to fill the gap in the literature and show the connection between teachers' cognition, their attitudes towards mathematics, and students' understanding of mathematics, Ma (1999) designed a study comparing American and Chinese elementary teachers' mathematics knowledge and attitudes and their impact on classroom instruction, drawing data from interviews with the Chinese teachers and using the TELT data on United States teachers previously collected by Ball (1988). Results showed that that despite an uneven teacher preparation in China and United States, the seventy-two Chinese teachers seemed to possess a Profound Understanding of Fundamental Mathematics (PUFM) than did their twenty-three United States counterparts. This study suggests that the better capacity of Chinese teachers to provide correct and diverse explanations for the topics was embedded in their attitudes towards mathematics and their view that mathematics is not rigid. Overall findings of Ma's (1999) study revealed that the American teachers tended to be procedurally focused, while the Chinese teachers possessed both procedural and conceptual understanding of the topics investigated. Analyzing the overall teachers' understanding of the concepts, Ma reported that the knowledge of the Chinese teachers was coherent while that of the United States teachers was fragmented, and her assumption was that the fragmentation in the United States teachers' understanding was the effect of the way curriculum is designed and teaching is approached in United States

The conclusion that can be drawn from Ma's (1999) study is that Chinese teachers displayed a more comprehensive knowledge of mathematics taught in elementary school, and this knowledge might influence their students' understanding of mathematics concepts and their better scores in the mathematics competitions. While this study is very significant for linking teachers' content knowledge and their attitudes about mathematics and providing arguments for the Chinese teachers' superior mathematical knowledge, a major limitation of Ma's study is the lack of direct connections between the teachers' understanding of mathematics, their instructional practice and the students' scores.

Consequently, further research needs to be conducted to investigate the direct relationship between content knowledge, instructional strategies and students' mathematics achievement.

Reflection

This review of the teacher education literature aimed at uncovering a few of the perspectives on learning to teach, and more specifically, how content matter knowledge is acquired and further applied into the classroom. Need for curricular and instructional reform of the United States educational system was also addressed. United States reform continues to be heralded as necessary (Hill, Sleep, Lewis, & Ball, 2007; National Mathematics Advisory Panel, 2008), with the caution that it also takes into consideration factors beyond teacher knowledge, curriculum and teaching.

This is reminiscent of Wang and Lin (2005), who stated that mathematics teaching and curricula can be culturally scripted and what may work in one country, may not work in another country. In other words, before assuming that teaching or curriculum may be the causes of success of some countries and trying to implement new practices into the United States educational system, more research needs to be conducted and comparisons made with other countries that may provide viable answers. One such case is Romania, as it shares similarities with top-performing countries in terms of curriculum and teaching practices (Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001), but the Romanian students' scores are below averages, even below the results of the United States students, as shown in the TIMSS 1994 and TIMSS-R 1999 studies (National Center for Educational Statistics 1994; 1999a; 1999b).

If a more cohesive curriculum and better instructional practices seem not to advantage Romanian students in the PISA (2000) and TIMSS (1994, 1999) studies, what factors prevent the United States students from performing well in the international competitions? Moreover, Wang and Lin drew our attention toward the fact that Asian American students tend to outperform their American peers in the international comparisons, despite their exposure to the same curriculum and teaching strategies as the United States students. These findings seem to accentuate even more the need to take into consideration other factors when examining the performance gaps and suggest that reform is more complex than initially assumed and not just limited to changes in curriculum and instruction. In addition, future studies should investigate schooling and non-schooling factors that may directly affect the better performance of students from the Asian countries and a lower performance of students from United States and other countries such as Romania.

Researchers (Stedman, 1997a; Stedman, 1997b; Wang & Lin, 2005) suggest policy makers and educators adopt a wider view on reform, going beyond schooling factors and analyzing potential non-schooling factors that may impact student achievement. Complex problems have complex solutions, and a look into the way teachers acquire knowledge, what knowledge they posses, how they make use of this knowledge in the classroom, along with studying parental and student attitudes may provide more in-depth answers to the deep achievement problems of the United States educational system.

Comparative Education: On Teachers' Knowledge, Classroom Strategies, Home Influences and Student Learning of Mathematics

By now, it is a fact that Asian students tend to outperform all other countries in international comparisons of mathematics and science, while the American students' performance has been described as merely average by Stevenson & Stigler (1992): "when twelfth-grade American students were compared to students from fourteen other countries, they were the lowest quarter in geometry, and in algebra they were second from the bottom" (Stevenson & Stiegler, 1992, p.31). These results come from the data collected on the occasion of the Second International Mathematics Study (Stevenson & Stiegler, 1992) and they further show that:

> Average students in other countries often learn as much mathematics as the best students learn in United States. Data from the Second International Mathematics Study show that the performance of the top 5 percent of United States students is matched by the top 50 percent students in Japan. Our very best student-the top 1 percent scored lowest of the top 1 percent in all participating countries (Stevenson & Stiegler, 1992, p.31).

Although these data were collected in the 1980s, the situation had not changed much over the past decade. The Third International Mathematics and Science Study (TIMSS) was conducted in 1994 and repeated in 1999. Results still showed that in both mathematics and science Asian students outperformed the United States students at all levels of assessment: fourth-, eighth-, and twelfth-grades (National Center for Educational Statistics, 1999). Other cross-cultural comparisons of mathematics and science conducted on a lower scale also revealed the better performances from Asian students at different levels (Brenner et al., 1999; Cai, 2000; Huntsinger, Jose,Larson, Krieg & Shaligram, 2000; Stevenson, & Stigler, 1986), attempting to justify this better performance in the light of a series of factors that may impact student mathematics learning.

Language and Mathematics Achievement: a Theory of Linguistic Relativity

Over the years, researchers advocated pro and against the advantages of a national language for student learning of mathematics. One of the groundbreaking researchers in this field is Irene Miura (1987), who argued in favor of the linguistic influence on the mathematics achievement of young students. In an attempt to look at how children from different cultures speaking different languages construct numbers, Miura tested Japanese and American first graders residing in the United States in two consecutive trials, asking them to construct five numbers using Base-10 blocks. Findings indicate a significant difference in the two groups' cognitive representation of number: the Japanese students in this study were more likely than the American students to use canonical Base-10 constructions to represent number correctly, a fact that is assumed to further impact students' understanding of other mathematical concepts.

How does language increase/decrease the understanding of mathematical concepts? According to Miura et al., (1988) "through language, numbers are mentally represented and stored, and for those languages that are rooted in ancient Chinese (Chinese, Japanese and Korean), numerical names are organized so that they are congruent with the traditional Base-10 numeration system" (Miura, p.1446). In these languages, the value of a given digit or multi digit numeral depends on "the face value of the digit (0 through 9) and on its position on the numeral, with the value of its position increasing by powers of 10 from right to left" (Miura, p.1446). Conversely, English speakers learn the words to 20 by rote, many of the number words having no initial meaning (Fucson, 1991).

Similar findings accounting for the preference of Asian students for canonical constructions occurred in the third study conducted by Miura & Okamoto (1989) on United States and Japanese students residing in Japan. Results again revealed that the Japanese students showed a significantly greater understanding of place value than did the United States children, being able to construct each number in two different ways and using more canonical and noncanonical constructions than did the United States students.

To explore further the impact of language on students' mathematics achievement, Miura et al., (1988) repeated the first study on a more diverse population: American, Chinese, Japanese and Korean first graders and Korean kindergartners and results again revealed a greater mental flexibility for the Asian students who were able to construct numbers in two ways on a larger scale than did American students.

In an attempt to investigate whether or not other non-Asian speakers would perform similarly to United States students, Miura et al., (1994) replicated the same study including students from France and Sweden along with the Asian and American first grade students. Results again showed a preference of Asian speakers for canonical and noncanonical constructions of tens and ones rather than one-to-one unit constructions. Differences were also found in the ability of Asian students to construct two correct constructions for the same number, which may reveal a greater flexibility with number quantity.

Acknowledging the differences in cognitive representations of numbers between speakers of Asian and non-Asian languages, researchers further investigated the impact the cognitive representations of number have on specific mathematical concepts such as counting. Miller et al., (1995) tested their assumption that differences in counting ability between Chinese and American pre-school students should focus on areas in which languages differ, namely the "teen" names. Results confirmed the hypothesis that United States students had trouble learning the numbers between 10-20, which is a clear indicator of their failure to understand the Base-10 system, and consequently, they start school with a disadvantage that arises from the way English counting system is constructed.

While investigating the practices used in American schools to remedy the disadvantage represented by the lack of verbal support in the English language for multi-unit Base-10 representations, Fucson, & Kwon (1991) found out that children are taught multi-digit addition and subtraction as step-by-step procedures of adding and subtracting single-digit numbers, and as such, students view multi-digit numbers as composed of single-digits placed to each other. School does little to provide students with the support needed in order to fill the performance gap believed to be the direct result of language. One solution proposed by Miller et al., (1995) in order to remedy this disadvantage is to familiarize American children with Arabic numerals at an earlier age, as these numerals provide a consistent Base-10 representation for numbers.

While all these studies provided another explanation of why Asian students performed at a superior level in the mathematics competitions, their major limitation was the exclusion of a series of different factors, which, along with language, may justify the better mathematics results of the Asian students in the international comparisons in later grades (i.e. kindergarten exposure, family influences). Understanding the influence of language on mathematics achievement and the fact that non-Asian speakers may begin school with a disadvantage may impact the way curriculum is designed and teaching is approached.

This theory of linguistic relativity was however debated by scholars who attempted to show that the influence of language on the cognitive representation of number is less direct than it was previously suggested. For this purpose, Saxton & Towse (1998) partly replicated Miura's (1987) study on 93 English-speaking children and 50 Japanese-speaking children (aged 6 and 7 years), changing the methodology. By introducing variations in the testing procedures the researchers were trying to test the hypothesis that every time Japanese speakers hear a multi-digit number name, they would be able to generate a representation of the number helped by the Base-10 structure, and any variations included in the test procedure should not influence their responses. Findings revealed that the children's performance on Base-10 tasks was influenced by the type of instruction provided, as speakers of both English and Japanese improved their performance when the experimenter demonstrated the use of tens and units cubes (Prompt), as opposed to units cubes alone (No Prompt). This study implies that language might not be the central component in the students' mathematics achievement, researchers urging to investigate other factors that may provide a deeper understanding for the better mathematics performance of Asian students, such as attitudes of parents and children about mathematics, the quality of schooling in different countries, teacher preparation, etc.

In an effort to probe further for linguistic relativity and to investigate whether or not language alone is responsible for the gap in performance between different groups of students, a new study was conducted by Wang, Lin, Tanase & Sas (2008). This study replicated Miura's (1987) study in terms of methodology, testing Chinese, Romanian and American students. Romanian language has been identified as a very important variable to test linguistic influences on students' mathematics achievement due to its similarity to both Chinese and French, languages previously tested in Miura et al.'s (1994) study. Romanian matches the ancient Chinese-based languages in its numerical language characteristics, but not in its linguistic roots. Its roots are Latin, like French. Romanian language is, therefore, unique in its similarity and difference to the Chinese-based languages in that it allows for an isolation of the numerical language characteristics variable.

Results showed no statistical significant differences between Romanian students' performance and that of United States students', which is a surprising factor since English number naming is not consistent with base-10 system. On the other hand, Chinese children outperformed both Romanian and United States students in using base-10 systems. Moreover, the Chinese students' performance increased substantially as they progressed from the first to the second trial. The importance of this study lies in the fact that it showed significant differences in the number manipulation of students from three different countries, and it implies a connection between language and other factors that may impact students' mathematics achievement. As such, despite the semi-consistency and transparency with the base 10 numbers of the Romanian language, the Romanian students performed closer to their American peers than to the Chinese peers, as initially believed. If the language advantage did not seem to be a factor in the way Romanian students manipulated numbers, then what other factors may be involved in the Chinese students' better manipulation of numbers? Can it be that the better subject matter knowledge of the Chinese teachers as well as a possible better home support in the Chinese families may reinforce the language advantage? This study's major limitation was, however, the small sample size used and the experimental treatment itself, which may prevent researchers from directly exploring the relationship between number-naming language structure and mathematics.

Home Influences on Students' Mathematics Achievement: Theories about Motivation, Level of Expectations and Teaching Strategies

Cultural influences on mathematics achievement were the subject of numerous researchers. The purpose of this literature review investigating cultural factors' influences on mathematics achievement is to present both the arguments these studies use to support their approach and their limitations.

Child Rearing Methods

In an attempt to understand the performance gap in mathematics between students from different countries, Dornbush et al., (1987) extended Baumrind's (1971) conceptualization of family impact on the adolescent school performance. Drawing data from questionnaires and Caucasian, Hispanic and Asian-American student grades, and perceptions of parental attitudes, researchers found three parenting styles: authoritative (parents admit that sometimes youth knows more), authoritarian (parents are correct and should not be questioned), and permissive (parents are tolerant, they don't care if students get good or bad grades). While authoritarian and permissive parenting styles were associated with lower grades, authoritative style was associated with good grades. However, despite the authoritarian style's association with poor grades, as a group, the Asian students were receiving good grades in school. Consequently, the proposed parenting styles could not explain the better mathematics performance of the Asian students in this particular study.

In further trying to explain the Confucian roots of home training in the Asian culture and link home training based upon Confucian beliefs to student mathematics success, a series of researchers (Lin & Fu, 1990; Chao, 1994; Jose et al., 2000) argued that the concepts used to describe Chinese parenting style as authoritarian were misleading, and that a new interpretation of this parenting style was needed. Chao's (1994) study offers a plausible interpretation to the paradox raised in the Dornbush et al. (1987) study, by providing an alternative interpretation of terms such as "authoritarian" and "restricting," namely the concept *chiao shun*, which in Chinese means "training." Data were drawn from questionnaires sent to fifty immigrant Chinese mothers and their children, and fifty European-American mothers and their children, in which participants were asked to categorize themselves as either being authoritative (encouragement of independence, expression of affection, rationale guidance) or authoritarian (authoritarian control, supervision of the child, control by anxiety), factors originally derived by Block (1981).

Findings show the Chinese mothers distinguished between authoritarian parenting style and training (chiao shun), which has a positive meaning, emphasizing parents' involvement in their children education. Due to the United States cultural context rooted in the Puritan and evangelical religious influences, the term "training," has negative connotations for the American mothers, meaning a stricter or more rigorous teaching or education. Therefore, the parenting style described as authoritarian by the American mothers does not apply to the Chinese mothers, who share a very different cultural background with the United States mothers. Future research needs thus to be conducted to explore the relationship between this training and school achievement, in order to explain the paradox in Dornbush et al.'s (1987) study. A look at another country's home influences may thus provide more insights into the ways parents interact at home with their children and in what ways these different interactions may influence student learning. If the term training has then a positive connotation in the Chinese context, how is training experienced in the Romanian households, if the TIMSS s' 1994 and 1999 results show Romanian and Asian parents holding high standards and expectations for their children?

Parental Strategies and Mathematics Achievement

Another line of research attempted to fill the gap in the literature regarding what teaching practices are most common at home among parents with diverse cultural backgrounds, a fact that was believed to offer a better explanation for the better results obtained by Asian students in international comparisons. In a longitudinal study, Huntsinger et al. (1993) investigated parental practices in 40 Asian-American and 40 Caucasian-American homes and correlated these practices to the better mathematics performance of Asian-American students. Findings revealed that Chinese American students outperformed their Caucasian peers in mathematics, and this better performance of Chinese students might be due to the more formal approaches used by the Chinese American parents. Formal methods of teaching were reported to be: longer duration of interactions between parents and students, paying more attention to the written representation of a problem, expecting children to spend greater amounts of time in studying mathematics, using memorization, drills, and worksheets. Not only Chinese students who were taught by their parents with more formal strategies performed higher, but also the Caucasian students who were taught with the same approaches scored higher than those who were taught using more informal techniques.

The findings of this study are very significant not only in that they shed light on parental teaching approaches at home, but they also question similar teaching approaches at school. Would then students benefit more form being exposed to formal teaching approaches both at home and at school? More research is however needed to correlate teaching practices at home and at school and investigate the impact of the two factors on the students' understanding of mathematics. In this vein, a look at the way Romanian parents interact with their children at home may challenge or reinforce previous research findings: are formal interactions more likely to lead to a better understanding of the base 10 concepts, or o the contrary, is the authoritarian style associated with a poorer understanding of place value concepts?

Parental Expectations, Student Motivation and Mathematics Achievement

Chen & Stevenson (1995) compared and contrasted beliefs and attitudes about education of 1,958 American students, 2,600 East Asian students from China and Japan, and 304 Asian American students. Data were drawn from a student questionnaires and a test with open-ended questions. Overall findings revealed that Asian-American students outperformed their Caucasian colleagues in all the trials, but they performed lower than both the Chinese and the Japanese students. This study suggests that parental expectations and student standards, along with student motivation impact mathematics achievement. As such, students who set higher standards for themselves (going to college, for East Asian students) scored generally higher than students who were motivated to get a better job (Asian-American students). On the other hand, other achievement-related behaviors (i.e. time allocated to studying mathematics at home) do not provide a clear explanation for the better results in mathematics achievement of different groups, since the students who scored the highest (both Chinese and Japanese) dedicate less time to mathematics than do Asian American students.

This study supports the cultural-motivational theory of academic achievement, namely that the beliefs and attitudes of students lead to high motivation and that high parental standards represent a cultural heritage characteristic in Asian students in general. However, immigration and acculturation to new settings, as in the case of Asian-American students, may produce differences in this cultural heritage, differentiating these students from both their Asian and Caucasian peers. This study only analyzed students' attitudes about mathematics, and it hints at what parents expect of their children without looking more in-depth at what parental practices may account for gaps in performances across cultures and among groups with the same cultural background.

In the same vein, Hess et al., (1987) argued in favor of family beliefs held by parents in different cultures and their impact on students' performance, specifically the role of success in the students' mathematics achievement. Data were drawn from interviews with fifty-one Chinese-American mothers and their children, forty-seven Chinese mothers from People's Republic of China (PRC) and their children, as well as forty-eight American mothers and their children. Findings revealed significant differences in the factors perceived to impact mathematics achievement. The PRC families tended to attribute failure to causes under their control; the other two groups believed mathematics failure was due to factors over which they had no control (i.e. ability, luck, school training). Cultural variations were also present in the groups' beliefs about the better mathematics performance. Mothers and children from PRC gave most of the credit to schools, Chinese Americans gave credit to home, while their Caucasian peers regarded home training almost as significant as school training. An interesting conclusion arises: school, more than home training is believed to be responsible for the better results of both the Asian students and the Caucasian American families (Chen & Stevenson, 1995).

On the other hand, if school (teachers and curriculum) is assumed to impact the students' success in mathematics positively even more so than home training, how may the gap in mathematics performances of students from different cultural groups be explained, if Romanian and Asian students in the TIMSS 1994 and 1999 study seem to share similarities in terms of school culture (hours of mathematics instruction, instructional practices, curriculum)? A more extensive cultural study of mathematics performance, TIMSS 1994 and TIMSS-R 1999 (National Center for Educational Statistics, 1994; 1999a; 1999b) also analyzed differences in beliefs about success and failure in mathematics between students from 41 countries. Significant findings revealed that there was no correlation between parental and student expectations to perform well in mathematics and students' mathematics achievement. To exemplify, note that 98% of the United States parents expected their children to do well in mathematics (and they were situated in 28th place out of 41 countries), whereas 93% of the Chinese parents expected their children to do well in mathematics and they were situated in fourth position. Interestingly enough, the same percentage of Romanian parents, 93% expected their children to do well in mathematics, but they were situated in 34^{th} place.

Are these beliefs about mathematics supported by home study? Paradoxically, Romanian students tend to spend more time at home studying mathematics than do their counterparts from Hong Kong and United States While the Chinese students spent 0.9 hours a day studying mathematics, and the United States students 0.8 hours per day, Romanian students dedicated an average of 1.8 hours per day to the study of mathematics, which may be correlated to their expectation to perform well in mathematics.

Very interesting findings are also revealed from the investigation of students' attitudes about mathematics. Discrepancies have been found in the students' interest in mathematics and their mathematics results. Only 48% of the Asian students liked mathematics, while 23% disliked it. Compare these findings to the attitudes of United States students (47% liked mathematics and 17% disliked it). Interestingly, more Romanian students liked mathematics (52%) and fewer disliked it (18%) than their peers from the other two countries. The overall attitudes about mathematics were also in favor of Romanian students: 60% of the students had positive attitudes, and only 25% of the students had negative attitudes about mathematics. Comparatively, less Asian students (57%) had positive attitudes and more students (31%) had negative attitudes towards mathematics.

The paradox lies in the fact that, despite the Romanian students' positive attitudes about mathematics, parental expectations and time dedicated to the study of mathematics outside of class, 33 countries out of 41 in the TIMSS 1994 study (National Center for Educational Statistics, 1994) outperformed them. Consequently, factors that may explain the Asian students' better mathematics performance (i.e. hard work, parental expectations, student attitudes), fail to explain the poor performance of students from other countries like Romania. All studies investigated above provide only limited explanations and a weaker understanding for the success of a particular group of students. Future research needs to explore the relationship between the different factors assumed to impact student success in order to provide a more plausible explanation regarding the mathematics achievement of students.

Curriculum

Another very important factor believed to impact students' mathematics achievement was the national curriculum. Researchers (Li, 2000; Valverde et al., 2002) conducted a thorough analysis of content, topic coverage, page space dedicated to each topic and types of problems that are presented in the books, and found correlations between the notions comprised in the textbooks and the mathematics achievement of the students.

A cross-cultural curriculum analysis was also conducted by Valverde et al., (2002). Using TIMSS1994 database (National Center for Educational Statistics, 1994), researchers analyzed the structure of the textbooks, as they believed this influenced classroom experiences. They called these patterns the morphology of the book and argued that this would enable them to uncover the pedagogical model advanced by the book. There were three emergent types of books: textbooks with one dominant content theme, textbooks with more than one dominant content theme, and textbooks with fragmented content coverage. Findings reveal that United States books (58%) had a fragmented content structure, based on the repetition of the same topic spread across the book, while books from China tended to have more of a progression of sequential themes (50%).

Besides analyzing the overall structure of the textbooks, researchers looked into the content coverage of the textbooks. Results indicated that while the eighth-grade United States textbooks covered more topics than did the books from Hong Kong. On the other hand, 39% of the topics covered in the Chinese books had no complex performance expectations, while the median percent of the complex performance expectations was 2. For the United States books, only 5% of the covered topics had no complex performance expectations, and 15% was the median percent of the complex performance expectations (Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001).

Researchers also found that analyzing the number of content strands would allow them to identify how often content themes change (that is, the number of times within each textbook that a content strand ends and a new one begins). United States textbooks were again found to have the largest number of breaks (215), while books from Hong Kong and China had fewer breaks: 53 and 75, respectively. Findings indicated that the more topics changed and the larger number of topics covered (i.e. like in the case of United States textbooks), the poorer the mathematics performance of students. On the other hand, textbooks with more cohesion between topics and textbooks that covered fewer topics but more in-depth, were assumed to impact positively the mathematics achievement of the students, as in the case of China/Hong Kong textbooks.

However, this hypothesis does not provide a clear explanation for the results of the Romanian students in the TIMSS 1994 study (National Center for Educational Statistics, 1994): if curriculum were, indeed, a determining factor in what happened in the classrooms in terms of instruction and student learning, how can one explain why, despite similar curricular characteristics to Hong Kong (Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001), Romanian students' mathematics performance is closer to that of United States students rather than that of the Asian students? Similar to the Chinese books, Romanian books have a progression of sequential themes (50%) and cover fewer topics than do United States books (32%). Most importantly, Romanian textbooks had only 20 content breaks, less content breaks encountered in the books from the United States and China, which implies more cohesion between topics even when compared to the books from Hong Kong.

Consequently, a mere analysis curricular structure may not provide a sufficient explanation for the better performance of the Asian students in the international mathematics comparisons. A different curriculum analysis and interpretation is necessary to provide more insights regarding the types of topics covered and their impact on the students' achievement in mathematics. With this purpose, Li (2000) took a different approach in curriculum analysis, investigating the types of problems presented in several middle school textbooks from the United States and China. Li's hypothesis was that the difference in mathematics achievement was not made by the number of pages dedicated to each topic, but the types of problems addressed in the textbooks. To support his hypothesis, Li compared five American textbooks intended for use in the United States (in various settings and with diverse populations) and their equivalents in China.

The researcher developed a three-dimensional framework for analyzing the problems: mathematical feature, contextual feature and performance requirements, and he used this system to code all the problems that did not have accompanying solutions presented. Results showed no statistical differences for the two items (mathematical and contextual features) across the curriculum of the two countries, but significant differences in the problems' performance requirements. Findings revealed that 26% of the United States books and only 16% of the Chinese books required a conceptual understanding of the solutions, and 63% of the United States books and 72% of the Chinese books required a procedural understanding of the problems presented. Due to this study's limitation (the small number of lessons selected), the researcher recommended a future larger scale investigation of textbook problems across grade levels and content topics, viewed with dual lenses: textbook content analysis and problem analysis, that would provide better opportunities to study the effect of curriculum on students' mathematics performance.

Instructional Practice

The above studies attempted to link language, home practices and curriculum with student mathematics learning. However, the language advantages and the way a curriculum is used may only benefit students in the teacher transfers this information into the way he/she plans for instruction and implements the lesson, as knowing one's students (Greenes, 1995) is assumed to be key to learning. As such, the low results of Romanian students in the TIMSS 1994 and 1999 and PISA 2000 studies may or may not be explained by the weaker learning opportunities in the Romanian classrooms, despite the language advantage and a more cohesive curriculum. Some other factors, besides instruction may be responsible for the weak performance of Romanian students in international comparisons.

What exactlty goes on in mathematics classrooms in China and the United States, how is instruction different and how may these differences explain the better performance of the Asian students in the international comparisons? Perry (2000) examined some of the classroom practices she believed would be responsible for the outstanding performance of the Asian students in the international mathematics comparisons. Starting with the hypothesis that good instruction should make a difference in children's developing understanding of mathematical concepts, and drawing data from classroom observations, Perry investigated mathematical explanations given by teachers in 80 United States, 40 Taiwanese and 40 Japanese first- and fifth-grade classrooms. Mathematical explanations were chosen because important information about mathematical concepts are transmitted through teacher explanations and are assumed to impact the development of mathematical knowledge. Findings revealed discrepancies in the explanations offered by the teachers in the three countries at each of the three grade levels observed, Asian students hearing more complex explanations than did their United States peers. The conclusion of this study is that if students are exposed to more mathematics explanations, as well as more complex explanations, they may tend to believe that explanations are an appropriate form of discourse in mathematics classes, and will be more likely to understand a concept than if they received explanations infrequently.

These findings may explain why Asian students obtain better results in the international mathematics and science comparisons: if teachers possess a more conceptual understanding of the topics, they are better able to convey the information to their students. What happens though if other countries are introduced in the equation? Romania may serve as an important variable in looking at the performance gap, since Romanian students' scores are similar to United States students' scores in the PISA 2000 and TIMSS 1994 and TIMSS-R 1999 international comparisons (National Center for Educational Statistics, 1994; 1999a; 1999b) yet teachers' classroom strategies are similar to the teachers in the high performing countries (Hong Kong).

In this vein, the TIMSS 1994 study (National Center for Educational Statistics, 1994) provided more complex examples of how mathematics classes are organized and what classroom practices are predominant in 41 different countries. As background information, the tested population for the United States was 7,087 students, for Hong Kong it was 3,339 students, and from Romania is was 3,725 students. The total number of participating schools was the following: 183 schools in the United States, 163 schools in Romania, and 85 schools in Hong Kong. Of the above three countries only the United States and Hong Kong satisfied the guidelines for sample participation rates, grade selection and sampling procedure, whereas Romania did not meet the age/grade specifications having a higher percentage of older students, which would lead one to assume they would perform better than the other countries, not worse.

When comparing the average of instructional days in school year, it was noticed that the three countries spent similar number of days teaching mathematics in the school year (Hong Kong: 171; Romania: 173; and the United States: 178) as well as a similar number of yearly mathematics instruction in hours, in the case of Romania (114 on average), and Hong Kong (118 on average). On the other hand, United States teachers spent 146 hours on average teaching mathematics in a school year. Paradoxically, despite the longer exposure to mathematics, United States students only performed average in the TIMSS 1994 and TIMSS-R 1999 mathematics tests (National Center for Educational Statistics (1994; 1999a; 1999b). It is worth mentioning that despite a longer exposure to mathematics instruction during the school year and a slightly longer school year than in Hong Kong and Romania, more schools from the United States (60 %) reported that at least 5 % of their students were absent in a typical school day than Hong Kong (2 %) and Romania (22 %). As seen in Chen and Stevenson's (1995) study, significant differences were found regarding the school attendance of Caucasian American, Asian American, Chinese and Japanese students. Results showed that the higher the number of days students were absent from school (Caucasian Americans), the lower the number of scores and the lower the number of days students were absent from school (Japanese and Chinese students), the higher the scores in mathematics.

When comparing instructional strategies in the three countries, teachers from Hong Kong and United States were again found to approach instruction differently at the 8^{th} grade. For example, if students were having difficulties, 79% of the Chinese teachers but only 21% of the United States teachers agreed to give students more practice by themselves during class. Note that Romanian teachers had similar beliefs to the Chinese teachers, as 80% of them would enable students to work more in class. The structure of the classroom differs between United States and Hong Kong: fewer students worked together as a class to respond to one another in Hong Kong (11%) and Romania (12%) than do in United States (22%), but more students worked individually without assistance from the teacher in Hong Kong (62%) than in the United States (50%).

Paradoxically, some of the practices encountered in the United States classrooms should place the United States students at an advantage against students from other countries: more United States students (56%) than Chinese students (19%) discussed completed homework in their mathematics lesson almost always, and teachers checked for mathematics homework and assigned homework more frequently in the United States than in Hong Kong and Romania. When investigating classroom related practices, findings again place United States students at an advantage: more eighth-grade United States students (39%) were tested in their mathematics lessons than students from Hong Kong (37%) and Romania (35%), and they worked more from worksheets and textbooks alone in their mathematics lesson. As such, instruction seemed to be more rigorous in the United States classrooms, and it should lead to better mathematics performances.

In a similar vein, the PISA (2000) study revealed similar results to the TIMSS 1994 and TIMSS-R 1999 studies (National Center for Educational Statistics, 1994; 1999a; 1999b): Chinese students still outperformed most of the countries in reading, mathematics and science, with the United States students showing an average performance and Romanian students being behind in all three content areas. Overall results show that out of the 41 performing countries, Romania was surpassed by 31 countries and only performed similar or slightly better than 10 countries. Why does Romania perform so poorly in all international mathematics and science competitions? Despite similar instructional practices and similar school organization as the Chinese, Romania does not even perform at the United States level, which has been shown to differ significantly from the Chinese context. Consequently, more research needs to be conducted to analyze the impact of different factors on student mathematics learning, to offer plausible explanations to the performance gap evident in the three countries discussed above.

Conclusions

All the above studies showed, to a greater or lesser extent, the impact of various factors on students' mathematics achievement. Researchers investigated teachers' content knowledge (Ma, 1999), mathematics curriculum (Li, 2000; Valverde et al., 2002), the impact of native language (Fuscon, 1991, Miura et al., 1994, Saxton & Towse, 1998), as well as different home influences on student achievement, such as parental teaching strategies, parental standards and expectations, and child rearing practices in some countries. Overall findings of all the above studies may shed some light on student mathematics performance. However, this understanding is limited, when comparing countries sharing similar characteristics in terms of curriculum, language influences, parental practices, classroom instruction (China and Romania) but with significant differences in terms of their students' mathematics achievement results in international comparisons.

No study has been conducted to analyze the interaction of these various factors on students' mathematics achievement. Accordingly, the need for a complexity theory of interrelated factors appears to be important to providing a more in-depth understanding of student mathematics achievement. Looking at the way multiple factors interact and influence student learning may suggest why some students perform better than other students even when exposed to a similar curriculum and teaching practice. The current study may suggest a better explanation for the performance gap of first-grade Romanian students, looking at home-school interactions.

CHAPTER 3

RESEARCH DESIGN AND INSTRUMENTATION

The goal of this study is to provide a better understanding of students' mathematics performance from the perspective of complexity theory. Defined as a science of emergence (Waldrop, 1992), complexity is a class of behaviors believed to exist due to the interactions between the independent agents in this class. According to Waldrop (1992), disregarding the fact that these agents interact may only provide a limited understanding of the system. Consequently, when applied to the field of mathematics education, disregarding the fact that these factors (school, parents, students) interact and that the end result, student learning is the outcome of this interaction, will only provide a limited understanding of how learning occurs. The interactive factors, in my case, are school, family, and students, and the outcome of this interaction is the way students understand and perform in mathematics.

Caution must be made regarding the way complexity theory is applied to this study: while analyzing the interactions between some of the factors believed to have a very strong impact on student learning (teachers' content matter knowledge, knowledge of students and curriculum, as well as parental teaching strategies and communication with teachers, this study may reveal how these particular factors influence student learning of place values, without holding the ultimate answer regarding gaps in student learning performance.

The researcher investigated how these factors worked together: How school (teachers and curriculum) might impact both students and parents, how parents might impact both teachers and their children, and finally, how students might impact both teachers and parents. Borrowing Maturana's (1984) terminology, if in biology component "A", through its interaction with component "B" triggers an interaction of "B" with component "C" that triggers a reduction in the production of component "D, where "A," "B," "C," and "D" are interacting through relations of contiguity, can this be true for the field of education, and specifically, to the field of learning mathematics? If we consider the learning mathematics as our autopoetic (or complex adaptive) system, can we look at the interaction of "A" with "B" and "C" and their impact on "D," where "A" represents school (teachers, curriculum, classroom practice), "B" represents home environment (parents, culture), "C" represents the students and "D" represents the mathematics learning? Figure 1 shows this interaction more clearly.

Research Context

The literature covered in Chapter Two herein uncovers the impact both schooling and non-schooling factors have on children's mathematics achievement in the international context of education (Chen & Stevenson, 1995; Dornbush et al., 1987; Hess, et al., 1987; Huntsinger, et al., 2000; Li, 2000; Ma, 1999; Miller et al., 1995; Miura et al., 1994). Cross-national comparisons show very interesting similarities and differences in terms of curriculum development and implementation, teacher knowledge and classroom practice, as well as the impact of language on students' mathematics achievement between students from high performing countries (China, Korea, and Japan), and students from a low performing country (United States).

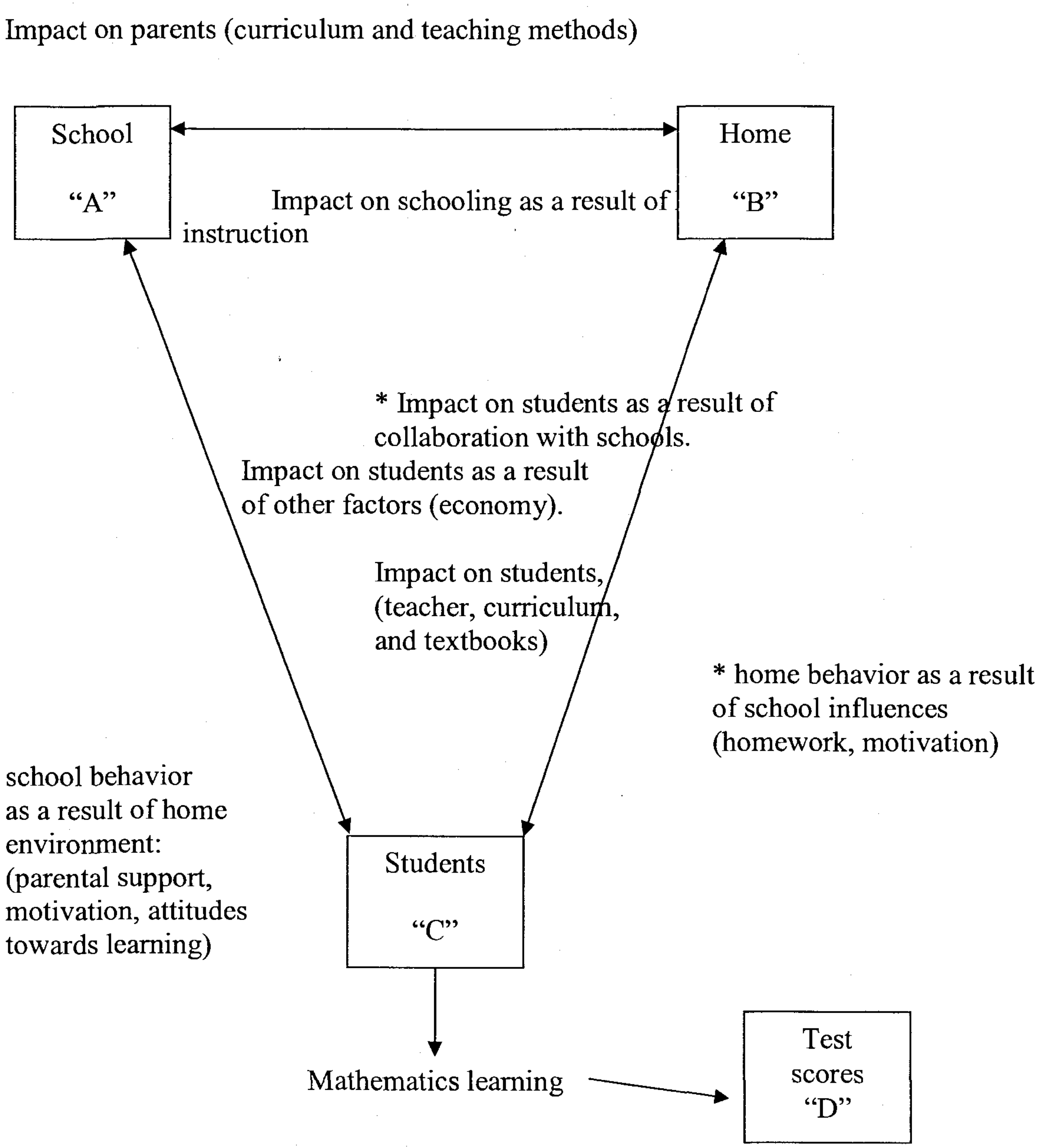

Figure 1. Interaction between School, Home and Students.

This study was conducted with the purpose to understand some of the factors contributing to the mathematics success of students, with an emphasis on the interrelatedness of these factors. For this purpose, two elementary schools in Romania were selected from a large, southeastern city with a population of 345, 000, hence referred to as Tomis (a pseudonym). The names of the schools and all the participants throughout this study will be changed in order to preserve anonymity. The selection of only one school district out of the 41 school districts in the country represents one of the major limitations of this study. To minimize this limitation, Tomis was selected because of its representative sample of students: most elementary students go to school in large cities in both average and high performing schools. Students with different ethnicities go to school in Tomis, the city population being represented by 25 different ethnicities, among which Romanians (91.2%), Turks (3.4%), Tartars (3.3%), Rrhoms (0.9%), Russians (0.8%), Hungarians (0.1%), Greeks (0.08%), Armenians (0.06%), and others (under 0.05%). From this perspective, Tomis accounts for a larger cultural variety than any other city in Romania.

The Tomis urban school district is comprised of 224 general schools, which house both elementary school (grades first through fourth) and middle school (grades fifth through ninth) with an enrolment of 19,740 students in the elementary classes for the 2004- 2005 academic year. Each elementary grade classroom is regularly taught by an elementary teacher who teaches all subjects, except special classes for religious education, computers, and foreign languages). Elementary school teachers are with the students along the course of the four years of elementary school, until students move up to middle school. In the academic year 2004-2005, there were 905 elementary school teachers in the Tomis school district.

In this context, two elementary schools were selected to account for both average and higher performing schools. Two elementary teachers from each school were also purposefully selected to represent both more experienced teachers as well as less experienced teachers. This context allowed for a determination of whether children in an average performing school were exposed to similar instructional strategies, curricular choices and teacher knowledge as students in a better performing school. Thus, the impact of interrelated factors on student mathematics achievement could be explored.

The first school, Iorga Elementary, is one of the oldest schools in Tomis, founded in 1879 and functioning ever since. With a total of 1,356 students, of which 724 students at the elementary level and 632 students at middle school level, this school is one of the few schools in Tomis offering four types of educational opportunities. Besides the regular elementary and middle schools levels functioning according to a normal schedule where elementary levels meet in the mornings from 8:00-12:00 and middle school in the afternoons from 12:00-6:00 or 7:00. The school provides the Step alternative at the elementary level, where students are exposed to a more Western type of education. The Step students spend the whole day in school, taking part in regular classes in the morning, having lunch at school and doing homework and other activities in the afternoon. At the middle school level, the school offers the opportunity for continued education through remedial night classes for those students who for various reasons had to drop out of middle school.

Seventy-five licensed teachers teach the students in twenty-three classrooms. The school is well endowed with different resources; four different laboratories, fourteen computers, and other electronic devices (VCR/DVSs, tape recorders, etc.). The school library provides a rich reading environment with students being able to choose from 11, 892 book volumes. That all these resources are put to a good use is attested by the impressive results of the students at different school competitions, at both the county level and national level. In order to maintain the scope of this paper, only the results of the students in mathematics competitions will be reported.

As such, in the 2006-2007 school year, middle school students participated in the mathematics competition held at the state level and won eight awards, four second and four third prizes. At the national level, following the national mathematics test compulsory for all eighth-grade graduates, out of the 124 eighth-grade students in this school, three students achieved the maximum score possible (10), 41 students received scores of 90%, 32 students received scores of 80%, 46 students had an average performance scoring between 50% and 70%, with only two students failing the national tests. Based on the scores from this national test, this school will be considered as high performing throughout this paper. This rich learning environment, with overall better opportunities for students to be engaged in learning may have a strong impact on student mathematics learning, producing students who have both a conceptual and procedural mathematics learning.

The second school, Delavrancea Elementary School, was founded in 1934 and has been functioning ever since. Twenty-eight licensed teachers teach the 350 students of this school in eleven classes during two educational cycles: elementary grades study in the mornings from 8:00-11:20 and middle school grades study in the afternoons from 12:30-6:20. The school is endowed with different resources; three laboratories, fourteen computers, one overheard projector, and other electronic devices (VCR/DVSs, tape recorders, etc.). The school library provides a rich reading environment, with students being able to choose from 7,000 book volumes. There were no awards for the students of this school at mathematics state-level competitions during the 2006-2007 school year. There were twenty-nine eight-grade students taking part in the compulsory national mathematics competition, scoring as follows: two students achieved over 90%, six students received 80%, twenty students had an average performance, scoring between 50% and 70%, and one student failing the national tests. Based on the scores from this national test, this school will be considered as average performing throughout this paper. The more reduced learning opportunities provided by this school (fewer instructional materials) may ultimately inhibit student learning.

Participants

The subjects of this study were four first-grade teachers, their students, and the students' parents.

Teachers

Four first-grade elementary teachers were purposefully selected to participate in this study. These teachers were: Ms. Ali and Ms. Reiz (the less experienced teachers) and Ms. Ionescu and Ms. Popescu (the veterans). The selection of these subjects was made with two major goals in mind. Both veteran and less experienced teachers were selected in order to account for the impact the level of expertise had on student understanding of place value concepts. Secondly, teachers were selected in two different elementary schools, in order to account for the impact the level of school and different curriculum had on student understanding of place value concepts.

Two teachers from each school were asked to be a part of the study and agreed to be observed on at least two occasions and be interviewed twice, once prior to the beginning of the lesson and once at the conclusion of the lesson. All teacher participants were females, two veterans (one from each school) and two novices (one from each school). The veteran teachers in this study had been teaching for at least twenty years, while the less experienced teacher had been teaching for nine years and the novice teacher for four years at the time this study took place.

Ms. Ionescu, a veteran teacher at Iorga Elementary School, had been a teacher for twenty-one years and her highest education degree was the Baccalaureate degree, awarded at the end of high-school. However, Ms. Ionescu was enrolled in college classes to become a middle school teacher, her subject of study in college being Romanian language, which happened to be her favorite subject to teach in class. In contrast, the novice teacher at the same school, Ms. Ali, had only been an elementary school teacher for four years, yet her highest degree of education was a college degree (a three-year-college degree specifically designed for elementary school teachers, as opposed to a regular four-year college degrees in Romania). Ms Ali stated that her favorite subject to teach in school was mathematics.

Ms. Popescu, a veteran teacher at Delavrancea Elementary School, had been a teacher for thirty-three years, and just like her colleague at Iorga, her highest education degree was also a Baccalaureate diploma. Ms. Popescu did not list either Romanian language or mathematics as her favorite subject. Ms. Reiz, the less experienced teacher at Delavrancea had been a teacher for the past nine years. Just like her colleague from the same school, her highest level of education was a high-school degree, and just like her novice colleague in the other school her favorite subject to teach was mathematics.

At this point, some similarities and differences emerge from the data. Both teachers in the better performing school (Iorga) had a higher education degree; the novice had a college degree and the veteran was in the process of getting a college degree, unlike their colleagues at Delavrancea, who only had a high school degree. Both less experienced teachers preferred to teach mathematics to other subjects, while both veteran teachers preferred to teach other subjects to teaching mathematics.

Students

The first-grade students of the four teachers were the other subjects of this study. Sixty-four first-grade students were tested on their knowledge of place value following instruction on this topic. In order to account for the effect of both schooling factors on student mathematics achievement, (such as teaching strategies, choice of curricular activities, teacher knowledge) and non-schooling factors (such as parental influences), students' test scores were analyzed from the perspective of students' interaction with both the teacher and the parents. All parents gave their permission for their children to be part of the study, be observed during regular classes on at least two occasions and be interviewed twice by the researcher. Researcher randomly selected five students from each class for further interviews.

The student participants of this study came from two equally old and consecrated general schools in Tomis. One school has produced better student results than the other school, based on awards won in mathematics competitions, as well as the results its students had at the national mathematics competition held yearly for students finishing the 8^{th} grade and moving on to high school. Two first-grade classrooms were selected from each school, one taught by a veteran teacher and the other one taught by a novice or less experienced teacher.

Overall, sixty-four students participated in this study. There were no students who wished to withdraw from the study nor were there parents who did not allow their children to take part in the study. There were thirty-six students in both first-grade classes at the better performing school, Iorga (twenty-four students in the veteran teacher's class and twelve students in the novice teacher's class), and twenty-eight students in both classes at the average performing school, Delavrancea (eighteen students in the veteran teacher's class and ten students in the less experienced teacher's class).

Most of the student participants came from small and average-size families, and only a few students came from large families. A small family is considered in this study to be a family consisting of two adults and one child, while an average family consists of two adults and two children. For this study, families with more than two adults and three or more children were considered large. Overall, 45 % of the students came from small families, 40 % of the students came from average-sized families, and 15 % of the students coming from large families. A more thorough break-down shows more students came from small families and less from average-sized families in the under-performing school, and less students came from small families and more from average-sized families in the better performing school. This is an interesting fact, as all the teachers seemed to believe that one of the major impediments in student learning was whether or not there were siblings in the household, which may lessen the time parents could spend helping each child with homework.

In terms of social-economic status (family income), more students came from families with an average income than from families with large incomes. In this study, an average income is considered between \$50- \$100 per capita monthly, while a large income is considered over \$150 per capita monthly. Overall, there were 70 % students from families with an average income and 30 % from large income-families. A school break-down shows an equal amount of students coming from average and large income families, namely 70 % average and 30 % large income families.

A look into the participants' ethnicity shows students belonging to the three major ethnicities in Tomis: Romanian, Turkish and Macedonian. The majority (62%) of the students were Romanian, with Turkish students (25 %), and Macedonian students (13 %) rounding out the demographics. A school break-down shows the ethnicities represented as follows: a large majority of Romanian students (80 %) in the average-performing school, with only 10 % Turkish students and 5 % Macedonian students, while the ethnicity break-down in the better performing school is more equal, with 45 % Romanian and 40 % Turkish students, and only 15 % Macedonian students.

In terms of gender, there was an almost equal number of male and female participants: 51 % of the students were male and 49 % were female. More male students (60 %) than female students (40 %) were in the better performing school and more female students (55 %) than male students (45 %) were in the average performing school. However, this difference is not relevant in student performance nor is it the object of this investigation.

Overall, more students from small-sized families and average-income families participated in this study, which provides for a representative sample of student population in Tomis School District.

Parents

The parents of the sixty-four students were sent questionnaires to determine demographic information; income and education level, as well as their involvement with their children's homework and home-school interactions. The inclusion of these subjects in the study was meant to take into consideration any external factors that may impact student achievement in mathematics as well as account for the interactive effect both school and family might have on students' performance in mathematics. Thirty-eight of the families who were sent the questionnaires returned them and agreed to be further interviewed by the researcher. At the better performing school, Iorga Elementary, eighteen parents returned questionnaires in the veteran teacher's class, while only five parents returned questionnaires in the novice teacher's class. At the average performing school, Delavrancea Elementary, eleven parents returned questionnaires in the veteran teacher's class, while only four parents returned questionnaires in the novice teacher's class. Only nine parents were randomly selected by the researcher from the total of 38 families who returned the questionnaires for further interviews: three parents were selected from the veteran teacher's class at Iorga Elementary, two parents from the novice teacher's class at the same school, and two parents in each of the two teachers' classes at Delavrancea Elementary.

A look into parents' level of education shows that more parents in the better performing school, N. Iorga, had a higher degree of education than parents in the average performing school. For this study, higher education will be considered a college degree or any post high school degree (a two or three-year college degree), while average education will be considered a high school degree. A school break down shows that 43 % of the parents in the better performing school held a higher education degree, with the most concentration in Ms. Ionescu's class, where 85 % of the parents had at least a post high school and at most a college degree. On the other hand, fewer parents in the average performing school held a higher education degree (5 %), with the most concentration in Ms. Popescu's class, where 10 % of the parents held at least a post high school and at most a college degree.

Data Collection and Analysis

Gathering data is a discovery process: talking to people, observing actions and interactions will provide a deeper understanding of the educational setting, namely the first-grade. According to Rossman and Rallis (2003), interviewing, observing, and studying material culture are the primary ways to learn in the field:

> Through observing, interviewing and documenting material culture, qualitative researchers capture and represent the richness, texture and depth of what they study. Data gathering is accomplished by practicing these techniques...The techniques provide structure; the resulting complex tapestry-the final product-is a unique expression woven by the researcher. (p.153)

The data in this study were drawn from teacher interviews and questionnaires, classroom observations, curriculum analysis, students' tests and homework, and parent questionnaires and interviews. The selection of the multiple instruments served for

Table 1

Putney Data Table

Research Questions	Type of Data to be collected	Process of Analysis	Related literature	Time of collection
What knowledge of place value concepts do students possess?	Field notes Student interviews Student tests Homework analysis	Content analysis Discourse analysis		During the months of February-March 2007 (field notes, interviews with students, tests and homework analysis)
How do classroom interactions influence 1st grade students understanding of place value?	Field notes Teacher Interviews Teacher Questionnaires Curriculum Analysis Student Interviews Artifacts used in teaching Student tests	Content analysis Discourse analysis	Li, 2000 Ma, 1999 Perry, 2000 Schmidt et al., 2001 Valverde et al., 2002	During the months of February-March 2007 (field notes, curriculum analysis, student tests) Pre and post interview with the four teachers
How do home interactions influence students' understanding of place value?	Parental questionnaires Parent interviews Student homework	Discourse analysis Content analysis	Dornbush et al, 1987 Hess et al., 1987 Huntsinger et al., 2000 Miller et al., 1995	During the months of February-March 2007 (questionnaires, student homework)
What interactions exist between teachers and parents that directly or indirectly influence students' understanding of place value?	Teacher interviews Teacher Questionnaires Parental questionnaires Parental Interviews	Discourse analysis		During the months of February-March 2007 (questionnaires, student homework, interviews)
How do all these factors interact and impact students' understanding of place value?	Field notes Interviews With parents, teachers and students Curriculum analysis Artifacts used in teaching Student tests and homework	Content analysis Discourse analysis Taxonomic analysis	Maturana & Varela, 1984 Waldrop, 1992	During the month of February-March 2007 (questionnaires, student homework, teacher interviews, student tests, curriculum analysis, artifact analysis)

triangulation purposes, enabling the researcher to interpret the results in the light of the complex-theory approach. Data in table 1 show connections between the research questions, the type of data collected as well as process of analysis.

Teacher Surveys

All first-grade teachers in the four schools were sent a survey and asked to fill it out. The survey consisted of ten questions asking for background information, as well as teachers' feelings and beliefs about teaching mathematics. Four teachers were chosen based on their willingness to participate in the study, their qualifications and experience in the field, in order to meet the participation criterion, namely two novice and two experienced teachers were observed at each school. Based on the amount of years teachers had been teaching at the time this study was conducted, teachers were categorized as veterans if they had been teaching for more than twenty years and as novices or less experienced if they had been teaching for less than nine years. Moreover, teachers' education degree was accounted for, as it was believed it might impact student learning. As such, both teachers at the better performing school had a higher education degree than their colleagues at the average performing school: the veteran teacher was enrolled in higher education classes at the time this study was conducted, while her novice peer already had a three-year college degree. Survey data served to gain a better understanding of teachers' education degree and years of experience. Surveys were written in Romanian and they were further translated into English by the researcher.

Teacher Interviews

The special strength in interviewing in qualitative research is the opportunity to learn what you cannot see and to explore alternative explanations of what you do see (Glesne, 1999). Interviews are therefore conducted with the purpose of collecting more data, as well as searching for opinions, perceptions and attitudes about the same topic. With this goal in mind, two open-ended interviews were conducted with the four teachers: a pre interview and a post interview.

Pre-interview

Before the teachers taught the first lesson on place value concepts, they were asked to describe how they would teach the lesson, the resources they would make use of, and the activities they would implement. The pre- interview aimed at obtaining information regarding teachers' subject matter and pedagogical content understanding. The interview was comprised of 20 open-ended questions: the first eight questions addressed the teachers' education background and teaching experience, their comfort in teaching mathematics, and their beliefs about teaching and learning mathematics. The next 12 questions aimed at obtaining information regarding the teachers' understanding of place value concepts, as well as the goals and objectives of the lesson that was taught and the teaching strategies that were used to increase student understanding of the topic. The researcher coded the pre- interview data and assigned them to the following categories:

a) knowledge of subject matter: based upon the nature of the definition the teachers provided for base 10 numbers, the objectives they set for the lesson, the tasks they developed for the lesson as well as the manipulatives the students were enabled to use in the class, teacher content knowledge was coded as strong or weak. Teachers with a strong knowledge base were found to expose their students to richer learning opportunities.

b) knowledge of students: based upon the knowledge teachers believed their students should possess before informal instruction of base 10 numbers as well as the way they made use of their past teaching of the same lesson, teachers' knowledge of students was coded as strong or weak. Teachers with a stronger knowledge of students could use their past teaching experience to address student past misconceptions and teach for understanding.

c) knowledge of curriculum: based upon subject matter knowledge and student knowledge, teachers' knowledge of curriculum was coded as strong or weak. Teachers who used their subject matter knowledge and curricular knowledge to use the curriculum in a way that would benefit their students were believed to have a strong curricular knowledge. If teachers had, on the contrary a more limited understanding of both subject matter knowledge and their students, then they exposed their students to either too complex notions or they kept their students at a level they had already exceeded, inhibiting thus student learning.

Post-interview

After the lesson was taught, teachers were asked again questions about the outcomes of the lesson. The post interview consisted of two parts: The first eight questions addressed the goals and objectives upon which the lesson was based, and asked the teacher to briefly describe what happened during instruction and state whether or not the objectives were covered, and what could have been done to increase student understanding of the topic. In the second part of the interview teachers were asked about their relationships with the parents of their students. Both interviews contained open-ended questions, allowing the researcher to further probe as needed. The pre interview was conducted the day before the lesson was taught, while the post interview was conducted immediately following the lesson.

The researcher coded the post- interview data and assigned them to the following categories:

a) knowledge of subject matter: based upon the nature of the definition the teachers provided for base 10 numbers, the objectives they set for the lesson, the tasks they developed for the lesson as well as the manipulatives the students were enabled to use in the class, teacher content knowledge was coded as strong or weak. Teachers with a strong knowledge base were found to expose their students to richer learning opportunities.

b) knowledge of students: based upon the knowledge teachers believed their students should possess before informal instruction of base 10 numbers as well as the way they made use of their past teaching of the same lesson, teachers' knowledge of students was coded as strong or weak. Teachers with a stronger knowledge of students could use their past teaching experience to address student past misconceptions and teach for understanding.

c) knowledge of curriculum: based upon subject matter knowledge and student knowledge, teachers' knowledge of curriculum was coded as strong or weak. Teachers who used their subject matter knowledge and curricular knowledge to use the curriculum in a way that would benefit their students were believed to have a strong curricular knowledge. If teachers had, on the contrary a more limited understanding of both subject matter knowledge and their students, then they exposed their students to either too complex notions or they kept their students at a level they had already exceeded, inhibiting thus student learning.

d) knowledge of instructional strategies: based upon the types of assessment tools developed by the teachers as well as the overall classroom atmosphere (whether or not students were engaged in interactions among themselves as well as more hands-on activities), teachers' instructional practice was considered stronger if teachers exposed students to more complex topics in class, used a variety of assessment tools to check student understanding and engaged students in both teacher-student and student-student interactions. On the contrary, teachers who did not engage their students in these types of interactions and only exposed them to less complex notions had students with a weaker understanding of place value concepts.

e) interactions with parents: based on the nature of interactions between teachers and parents, a stronger interaction was considered one in which parents met with teachers not only to discuss their child's progress, but also to be informed about the topics that were being taught and the approach the teacher was using to teach these topics in order to increase their won understanding of these topics and their children's understanding. A weaker parent-teacher interaction was therefore one in which parents were only informed about their children's progress, with little or no reference regarding the teaching and understanding of place value concepts, as well as their significance for student learning.

Data from the interviews enabled the researcher to understand the nature of subject matter knowledge of the teachers, as well as how this knowledge impacted the way teachers used the curriculum to plan for instruction. Moreover, the purpose of the interviews was to understand how teachers used their past experience in teaching the same lesson as well as what type of student knowledge they possessed and how these influenced their teaching. Both interviews were conducted in Romanian and were further translated into English by the researcher. Both interviews were semi-structured, allowing the researcher to ask the structured questions and further probe for deeper understanding when needed. All teacher interviews were audio taped.

Classroom Observations

Observation is fundamental to all qualitative research (Merriam, 1998), as it takes the researcher inside the setting, helping him/her discover complexity in a social setting by being there. Observation entails systematic noting and recording of events, actions and interactions, the challenge being to identify the "big picture" among the vast amount of exchanges. The researcher generally observes to understand the context, to see tacit patterns people are unwilling to talk about, to provide direct personal experience and knowledge, and to move beyond the selective perceptions of both researcher and participants. According to Merriam, observation is a research tool "when (1) it serves a formulated research purpose, (2) is planned deliberately, (3) is recorded systematically, and (4) is subjected to checks and controls on validity and reliability" (Merriam, p.95).

In this case, both veteran teachers were observed three times, while the novice and less experienced teacher were each observed twice while teaching place value concepts. This topic was purposefully selected, as researchers (Ho & Cheng, 1997) found Base-10 knowledge to be crucial for children's understanding of other mathematical concepts, like addition, multiplication, etc. The researcher took notes and collected artifacts developed or used by the teacher (i.e. worksheets). Notes focused on the content and nature of instruction, the teacher-student and student-student interactions, and the types of activities implemented by the teacher. Data from the notes and artifacts were used to construct categories of the types of problems solved in class.

A more in-depth explanation of the different types of exercises is provided in the section on curriculum study. Researcher also coded the type and duration of teacher-student and student-student interactions. As such, if students were exposed to more opportunities to work collaboratively for a longer time and if the teacher was more a facilitator of instruction than the supreme authority, student learning of place value concepts was enhanced. Teacher as facilitator was generally helping students make sense of what they learned, guiding them to discover the meanings, while teacher as supreme authority was mainly concerned to have students arrive at the correct answer. The classroom observations served to gain an understanding of the types of interactions occurring in school between teacher and students and students and students. Both veteran teachers were observed three times and both the novice and the less experienced teacher were observed two times, each of these observations lasting for fifty minutes. All notes were taken in Romanian, the language of instruction, for the sake of accuracy, and were later translated into English by the researcher.

Curriculum Study

Researcher analyzed the curricular materials teachers used to plan and deliver the lesson. Classroom instruction mainly focused on the use of textbooks, so a thorough analysis of curriculum might provide a deeper understanding of how textbooks were structured and used in the classroom. Schools have a choice in the selection of the books they are to use, so different schools may operate using different textbooks. The selection of the two schools enabled the researcher to account for different curricular materials and artifacts used by teachers in their classes. Materials specifically developed by the teacher were also analyzed. Three mathematics textbooks were used by the teachers in this study: both teachers at Iorga used the same textbook, while the teachers at Delavrances used different textbooks. The researcher conducted a thorough analysis of the three textbooks by looking specifically at the chapters on place value concepts, counting the pages and the numbers of exercises dedicated to place value concepts, and analyzing the types of these exercises (some addressed the place of tens and units, some had students compare, compose and decompose numbers, some asked students to find neighbors of certain given numbers, etc.) and their nature (whether they were lower order thinking or higher order thinking problems). The type and nature of the exercises were then used as codes to categorize the exercises encountered in the curriculum, the tests, and home and class work, following the model of three United States' First Grade mathematics: Everyday Math (2004), Investigations (2004), and Math Advantage (1998). For example, based on its type an exercise could be either lower order T/U (tens and units) or higher order T/U. See Table 2 for a more accurate description of types and nature of each exercise.

The classroom textbooks were used as the main source to classify exercises into different types. In total, there were seven types of exercises encountered in the textbooks and in the class and homework: T/U (tens and units), C (counting), C/D (composition/decomposition), CP (comparison), N (neighbors), CB/CF (counting backwards and forwards), and C by 2, 3, 5, 10 (counting by 2, 3, 5, 10). Here are a few examples of each types of exercise discussed above.

a) T/U (tens and units): Given a group of 15 elements, how many are T and how many are U? Or given that a number has 3 T and 2 U, what number is it?

b) C (counting): Count from 10 to 20, or given the following exercise: 50, 51, 52, …, …, …, 56, 57, …, …, …, 61, …, …, …, 65, please count from 50 to 65.

c) C/D (composition/decomposition): Decompose the following numbers in tens and units, given their tens: 30; 50; 67; 90 and 87, or given the tens and units find the number (10 and 8; 10 and 5).

d) CP (comparison): Compare the following numbers: 40 and 50; 35 and 32; 56 and 59; 70 and 60; 43 and 45; 98 and 96, or another exercise, given the axis with numbers from 0-100, and the number of girls on the axis being 80 and that of the boys being 70, were there more girls or more boys at the cinema?

e) N (neighbors): Number 42 is closer to number X than to number 50, or another type of exercise given numbers 10 and 12, what is their neighbor?

f) CB/CF (counting backwards and forwards): Count from 31 to 62 and from 77 to 33, or another type of exercise, given the numbers 19, 7, 12, 10, 9, 20, 6. 3 count them both forwards and backwards.

g) C by 2, 3, 5, 10 (counting by 2, 3, 5, 10): Count by 2 from 80 to 100, or by 5 from 0 to 100, or another type of exercise given the numbers 6, 8, 10, count by two to find out the following three numbers.

As far as the nature of these exercises, they were considered lower order exercises if they asked students to perform simple computations, as shown in the preceding examples, and higher order thinking problems if they asked students to perform more complex computations, as shown in the following examples:

a) T/U (tens and units): Write all numbers between 30-100 where the units are equal to the tens, or given the tens are triangles and units are circles, write the following numbers made of triangles and circles: 35, 68, 80.

b) C (counting): Find X, if X is higher than 10 and lower than 18.

c) CP (comparison): Compare the following numbers 62, 74, 66, 71

d) N (neighbors): Given the numbers: 15, 17, 13, 19, which is the closest to 18?

e) C by 2, 3, 5, 10 (counting by 2, 3, 5, 10): Discover the rule and continue the counting: 66, 67, ..., ...; 93, 92, ..., ..., 42, 44, ..., ..., 80, 70, ..., ...

f) 2 D (two digits): Write all numbers made of two digits that have the sum of the digits 10.

Student Tests

Artifacts were also collected from students. In order to account for the impact of instruction, curriculum and teacher knowledge on student mathematics achievement, students were tested on their understanding of place value concepts post formal classroom instruction. Formal testing included asking students to take a test a few

Table 2

Mathematics Exercises Found in United States Textbooks

Types of Exercises	Lower order thinking problems (called "Practice Items" or "Basic Computations")	Higher order thinking problems (called "Problem of the Day" or Challenging Problems")
T/U (tens and units)	1. Given the numbers 17, 18, 19, 20, 10, 14, write in 2 columns how many tens and units you have (Math Advantage, 1998, p.205 A). 2. Students work with partners in an activity called "Copying cubes." Each builds something with 10-15 interlocking cubes. They exchange and try to make a copy of each other objects. When both students have finished, they check that the copies are identical. (Investigations, 2004, p.14) 3. Given the tens and units, guess what number am I: 3 Us and 2 Ts; 5 Us and 6 Ts; 2 Ts and 7 Us; 4 Us and 6 Ts (Everyday Mathematics, 2004, p.336).	1. Write problems involving numbers 10-20. Then exchange your problem with your partner's and solve each other's problems (Math Advantage, 1998, p.206 A). 2. Given the tens and units, what number am I: 3 Ts and 19 Us (Everyday Mathematics, 2004, p.338).
Counting	1. Write the missing numbers: 1, 2, ..., 4, 5, 6, 7, 8..., 9, 10, 11, ..., 1, ..., 15, 16, 17, ..., 19, 20 (Math Advantage, 1998, p.204). 2. Given the 100 numbers chart, find the following numbers on the chart: 10, 20, 50, 15, 63 (Investigations, 2004, p.83). 3. On the 100 numbers chart, count and find out the missing numbers. Ex. 1, ..., 3, ..., 5, ... (Investigations, 2004, p.99). 4. Write one more: 36, ...; 45, ...; 61, ..., 83, ...(Everyday Mathematics, 2004, p. 337).	1. Given the following groups of 10 elements, color groups of tens to show the numbers: 30, 60, 50, 20 (Math Advantage, 1998, p.204). 2. Count adding 10: 4, ..., ..., ..., ..., ... (Everyday Mathematics, 2004, p.343).
Counting by 2, 3, 5, 10	1. How can you count by 10 to 100 (Math Advantage, 1998, p. 201)? 2. Count by 2 from 2-20, and 2-24 (Math Advantage, 1998, p.237). 3. Count to 25 in numbers other than 1 (Investigations, 2004, p.96). 4. Continue the sequence, counting by 2: 17, ..., ..., ..., ..., 27. Same for 5: 20,...50. Same for 10: 62, ...22. (Everyday Mathematics, 2004, p. 337).	1. Look for a pattern in each group of numbers. Write missing numbers: 5, 10, 15..., 25 10, 20, ...40 60, 65, ..., 75 70, 80, ...100 (Math Advantage, 1998, p.237). 2. Count backwards by 10 from 100 (everyday Math, 2004, p.7)
CP (comparison)	1. Compare numbers 11 and 19, circle	How do you know which number is

	the greater and explain why it is greater (Math Advantage, 1998, p.221 A). 2.Which number is greater, 14 or 41, 47 or 57, 43 or 34, 86 or 68 (Math Advantage, 1998, p. 227)? 3. Compare the following numbers: 11 and 7; 29 and 42; 21 and 25; 35 and 15; 37 and 37 (Everyday Mathematics, 2004, p.349). 4. Playing the compare the dots game, the player who has the card with more dots says "me". Compare 5 and 7 dots, 3 and 4, 5 and 6, 7 and 8, 9 and 10 (Investigations, p.8).	greater: 47, 37; 13, 31; 53, 35 (Math Advantage, 1998, p.221)? 2. Playing the double compare dots game, players determine the total number of dots on both cards and player with the higher number of dots says "me" (Investigations, 2004, p.9). 3. Fill in the numbers to make these correct: a number lower than 46; a number lower than 155 (Everyday Mathematics, 2004, p.363).
N (neighbors)	1. Given the following numbers, find the numbers before and after them: 47, 98, 32, 20, 26, 61, 84, 90 (Math Advantage, 1998, p.225). 2.Which number comes before 72, which number comes after 72, which number comes between 75 and 77, and which number comes between 78 and 79 (Math Advantage, 1998, p.226)? 3. On the 100 number chart, what is the number coming after 18, before 10, between 14-16, 85-87 (Investigations, 2004, p.84). 4. Find all numbers that come before and after the following numbers: 14; 49; 71; 88 (Everyday Mathematics, 2004, p.381).	1. Guess the number: it is between 70 and 90, it is greater than 80, it has 8 ones (Math Advantage, 1998, p.225). 2.On the 100 number chart, remove consecutive numbers and ask students to identify numbers in the middle of a set of empty spaces (Investigations, 2004, p. 84).
C/D (Composition/ Decomposition)	1. Suppose I have 12 pets. How many cats and how many dogs can I have (Investigations, 2004, p.38).	1. Make your own problem in which you have two different kinds of things (Investigations, 2004, p. 40).
CB/CF (counting backwards/forwards)	1. Order the following numbers from lowest to greatest: 86, 17, 21, 5, 43 (Math Advantage, 1998, p.228). 2. What is wrong with the following counting sequences? a. 9, 10, 11, 12, 13, 14, 16, 17, 18, 19? b. 9, 10, 11, 12, 13, 41, 15, 16, 17, 18, 19? c. 25, 26, 27, 28, 29, 20, 21, 22, 23 (Investigations, 2004, p.104). 3. Count backwards and forwards from 100 (Everyday Math, 2004, p.7).	1. Write numbers 1-8. Cross out the 2 numbers that come before 3. Cross out the number that comes before 7. Cross out the numbers between 3-7. Cross out the number that is less than5. What number is left (Math Advantage, 1998, p.227)? 2. Count backwards from 100 by 10 (Everyday Math, 2004, p. 7).

days following the initial school instruction of the topic, to account for the impact of both schooling and non-schooling factors. The researcher coded the types of exercises students were asked to solve in the test in higher order and lower order thinking exercises, respecting the model discussed in the above curriculum study. The following were all the types of exercises encountered throughout the four tests:

a) lower order thinking: T/U (tens and units); C/D (composition/decomposition); CB/CF (counting backwards/forwards); N (neighbors); C (counting).

b) higher order thinking: C (counting); 2D (exercises involving two digits). All tests were teacher-made, being consistent in the number of exercises the students were tested on but differing in the degree of exercises students were tested on.

Researcher also looked into the most frequent mistakes students made in the final test and then linked these mistakes to the type of reinforcement students received both in school and at home in order to correct these mistakes. As such, students in both veteran teachers' classrooms tended to have more problems with T/U (tens and units) exercises, as evidenced by their tests, while students in the novice teacher's classroom had more problems with CB/CF (counting backwards/forwards) exercises, and students in the less experienced teacher's classroom had more problems with CP (comparisons) exercises.

Teachers were the ones who graded the tests. Only perfect tests were considered for the grade of A, a complication of the study that needs to be acknowledged is the fact that A may not mean the same across the four classrooms, and one way to address this complication was by analyzing individual items (going beyond tests and also looking at homework, pre and post interviews). The data from the tests served to understand what/if any impact both teaching and home strategies had on student learning of place value concepts.

Student Homework

In order to acknowledge the existence of external factors on students' understanding of place value, the researcher collected work students had completed at home. Since parental involvement is more prevalent in early school years (Epstein, 1990), parents' influences on students' understanding of the topic should be present. Looking at homework, the researcher analyzed and categorized the types of exercises students were assigned to do at home, coding them into lower order think and higher order thinking concepts, according to the model encountered in the curriculum study discussed above. The following were all the exercises encountered in student homework: C/D (composition/decomposition); C (counting); CB/CF (counting backwards/forwards); CP (comparisons); T/U (tens and units); N (neighbors); C by 2, 3, 5, 10 (counting by 2, 3, 5, 10).

Moreover, homework was considered as correct and coded as such when all the problems were solved with no error and incorrect when there were errors in the workbooks. Also, according to the teacher's notes in the textbooks and comparing the workbooks among themselves, homework was coded as complete and incomplete. Artifacts from homework expanded over a month's period, the researcher considering only the homework assigned following the first teaching of the base 10 numbers up until and including the last teaching of base 10 numbers. Researcher then linked the mistakes made by students in tests and classroom work to the reinforcement of these topics provided by the teacher through homework.

Student Interviews

In order to account for the influence of external factors on student learning of place value concepts, five students were randomly selected from each class and tested on their understanding of tens and units. A few days after the teacher taught the place value concepts the same five students were re-tested on the same concepts, in order to account for the impact of schooling factors on student learning. The first interview consisted of fourteen questions in which students were asked to solve three place value exercises designed by the researcher. Students were asked to solve the same problems in both interviews, the only difference being, in the post interview students were asked seven questions about the home support they received with mathematics and asked to state their feelings about mathematics. Here are the questions students were asked in the pre-interview and asked again following the lesson on base 10 numbers:

1) Identify the following numbers and then indicate how many tens and how many units they have: 23, 17, 24, 19, 15, 20, and 13.

2) Given the following tens and units (corresponding to the above numbers) can you find out the number?

3) Given the following tens and units (corresponding to numbers 13, 21, 48, 15, 32, 14) can you find out the numbers? Answers were coded as correct is students identified the numbers and correctly described how many tens and units composed the number, and coded as incorrect if students had difficulties in identifying the number or describing how many ten and units composed the number.

The purpose of these data enabled the researcher to understand better what kind of knowledge of place value students had prior to formal instruction, as this knowledge may be related to parental support. The interviews were conducted in Romanian and then translated into English by the researcher. Student interview were audio taped.

Parent Questionnaires

Sixty-four families were sent questionnaires and asked to answer the open-ended questions, and thirty-eight families returned the questionnaires. The questionnaires consisted of two parts: The first comprised of seven questions addressing demographic information, which enabled the researcher to account for socio-economic factors (such as family income, parents' level of education, and current job); and the second part consisted of five questions about the degree of involvement in their children's education and the nature of help provided in order to enhance their children's understanding of the topics taught at school. For the demographic part of the questionnaires, the researcher divided the families in the following categories:

a) income: small income (under $200 monthly), average income (between $200-$600 monthly), and large income (over $1000 monthly)

b) level of education: high school and below; high school and above; college and above

c) comfort with mathematics and base 10 numbers: very comfortable; comfortable; not quite comfortable; uncomfortable

For the second part of the questionnaire, parents were asked to describe their involvement in children learning at home and at school, and the researcher devised the following categories for the data:

a) how many times they go to school to talk to the teacher: very often; often; sometimes; rarely

b) how many times they go to school to discuss the concepts taught with the teacher: very often; often; sometimes; rarely

c) how many times they get involved in math classes at school: very often; often; sometimes; rarely

Questionnaires served the researcher to understand what type of knowledge parents had about place value concepts as well as how this knowledge and their involvement in their children's education at home impacted children learning. Questionnaires were written in Romanian and later translated into English by the researcher.

Parent Interviews

Three families were randomly selected from the veteran teacher's class at Iorga Elementary and two families from the other three teachers' classrooms and they were further interviewed. The interview consisted of two parts: The first consisting of five questions in which parents were asked about their children's understanding of place value at school and the explanations they provided their children at home in order to enhance the understanding of these concepts; and the second consisted of an interview where parents were questioned about their relationship with the school and the teacher. The second part included eleven questions.

For the first part of the interview, researcher coded parental definition and understanding of place value concepts as strong if parents understood what base 10 numbers were and why they were significant for student learning. Likewise, parental definition of base 10 concepts was coded as weak if parent could not provide a definition for base 10 numbers and/or they lacked the understanding why these numbers were significant for student learning. Researcher also asked parents to describe the types of opportunities they provide for their children both pre and post formal classroom instruction, coding their answers as either strong home interaction if parents went beyond help with homework and following the teacher's model and devised their own assessments, and weak home interaction if parents were limited in only checking their children's homework.

For the second part of the interview researcher coded parent-teacher interaction into strong (if teachers and parents met to discuss hat and how would certain topics be addressed in class and what parents needed to do at home to enhance learning) and weak (if teachers and parents met only to discuss students' progress or lack of progress. Parent interviews were audio taped and later translated into English.

This multitude of data collection and analysis strategies were meant to triangulate the findings obtained throughout this study. However, a major limitation encountered in the data collection and analysis was the limited number of participants (four teachers) included in this study that may not account for a representative sample of the Romanian teacher population. The researcher attempted to reduce this limitation by drawing data from a multitude of sources (interviews, observations, tests, questionnaires, artifacts, and curriculum analysis).

The Role of the Researcher

Merriam (1998) distinguished between four roles researchers may have while conducting observations: complete participant, participant as observer, observer as participant, and complete observer. The researcher's role in this study was that of a participant as observer, gathering data from field-note observations and brief interaction with the teacher (formal interviews) and students (tests and homework collection). This allowed for a peripheral membership role which enabled the researcher to "observe and interact closely enough with members to establish an insider's identity without participating in those activities constituting the core of group membership" (Merriam, p.101).

Flexibility is very important for any researcher. The secret is, as Merriam (1998) affirms, "to combine participation and observation so as to become capable of understanding the program as an insider while describing the program for outsiders" avoiding to fall into the trap of either being too involved or not being involve enough (p.102). In the role of participant as observer, the researcher directly interacted with the participants (the four teachers, the selected twenty students, and the selected ten parents) during the formal interviews, and indirectly with the teachers and students via the classroom observations and their parents through the questionnaires. The limited direct interaction with the classroom students allowed for observation of all the actions and interactions that were taking place during the lesson, which could then be recorded as field notes.

Conclusions

Due to the theoretical framework guiding the design of this study (complexity theory) and the proposed instrumentation (interviews, questionnaires, classroom observation, tests, and curriculum analysis) applied to a diverse population (teachers, students, and parents), this proposed study has the potential to shed more light on students' performance in mathematics and the factors that influence this performance. As discussed in the literature review, the analysis of separate factors provides only limited explanations for the Asian students' success, and only hinted at causes that may prohibit students from other countries to perform at the same level.

Among the most cited factors to contribute to mathematics failure were teachers' knowledge and classroom practice, curriculum design and implementation, and cultural variations in students and parents' beliefs and expectations. These factors seem plausible when students from the Asian countries (China, Japan, and Korea) are compared to students of Asian or European origin from United States, as significant differences in curriculum, home and classroom practices were noted between these groups of students. When other countries are included in the equation (Romania), the explanations these research studies provide seem insufficient to account for the gap in mathematics performance.

A significant challenge in this study, including Romanian students, parents and teachers in the analysis is: if the above factors justify differences in performances between two different cultures with different home and classroom practices, should the same factors be held responsible for the poor results of the Romanian students, who, paradoxically, are taught with similar instructional strategies and from similar curricular materials as are their Chinese peers, and yet perform at the level of their American peers?

The following three chapters will present and discuss the research findings as well as offer explanations for the questions driving this study, namely the types of interactions existent between teachers, students and parents and the outcome of these interactions on student learning.

CHAPTER 4

FINDINGS OF THE STUDY

Students

The aim of this chapter is to discuss student knowledge of place value concepts as well as any differences that may appear in the students' performance, raising questions about the causes for these differences that will be further addressed in Chapters 5 and 6. In order to present an accurate description of student performance, a variety of sources were analyzed, including test scores as well as both pre-teaching and post-teaching interviews, class work and homework analysis.

Tests

The four teachers used different final evaluation tests in the four classrooms. All tests were self-made, with teachers using different resources to create the tests such as the classroom textbook, other textbooks, as well as their own knowledge of subject matter and their students. In order to provide an accurate analysis of these different tests, and in consistency with the categories encountered in the textbooks, the researcher divided problems in two main categories: higher order thinking problems and lower order thinking problems. Higher order thinking problems were those concepts requiring students to discover the rule and then count, as well as apply their prior knowledge to new concepts. The following is an example of a higher order thinking problem: Write all two-digit numbers that have 3 in the place of tens. Another problem is asking students to compare the following four numbers among themselves: 62, 74, 66, 71 (in other words, how is 62 in relation with 74, in relation with 66 and in relation with 71). For these types of exercises, students were not provided with the rule, they had to discover the rule in order to be able to solve the exercise.

On the other hand, lower order thinking problems were considered in this study those problems requiring students to do simple operations, such as counting, composing/decomposing base 10 numbers, finding neighbors to numbers, comparing numbers, etc. Examples of lower order thinking problems follow below: count from 10 to 20 by 2; decompose the given numbers in tens and units: 67, 87, 99, 60; compare the following pairs: 30 and 54, 12 and 22, 15 and 65, etc. For these types of exercises the rule was given by the teacher and the students had to only follow the rule to complete the problem.

Each category is comprised of 2-10 different kinds of exercises, all relating to place value concepts. The following are examples of the higher order thinking problems that appeared across the four tests: for counting problems (HOC), students were given the following pairs of numbers: 65, 66,…; 31, 34, …; 42, 44,…;80, 70, … and were asked to discover the rule and continue the counting. To solve this type of exercises, students had to know if it was forwards or backwards counting, and then that it was counting by 1/3/2/10 respectively. Another type of higher order thinking problem was the two-digits problem (HO2D), for which students were asked to write all numbers made of two digits that have the sum of the digits 10.

The lower order thinking problems comprised problems dealing with tens and units (T/U), composition/decomposition of base 10 numbers (C/D), neighbors (N), counting (LOC), counting forwards and backwards (CF/CB), and comparisons (CP). Here are a few examples from each category: find out the number made of 4 tens and 1 unit (T/U); decompose the following numbers: 15, 17, 14, 11, 19, 16, 12, 18, 13 (C/D); write the neighbors of the following numbers: 46 and 48, 38 and 40, 50 and 52, 47 and 49, 74 and 72, 59 and 61 (N); order the following numbers from the lowest to the highest: 78, 37, 19, 90, 34, 28, 85, 43 and the following numbers from the highest to the lowest: 37, 30, 48, 17, 2, 60, 73, 32 (CB/CF); compare the following numbers: 18 and 13, 15 and 16, 17 and 17, 12 and 16, 14 and 11, 18 and 20 (CP).

Students in the veteran teacher's class at Iorga Elementary were assessed on seven items (two higher order thinking and five lower order thinking problems), while the novice teacher at the same school, Ms. Ali only tested her students on five lower order thinking problems. A similar test was administered at the other school, Delavrancea Elementary, where the less experienced teacher, Ms. Reiz administered a test comprising two higher order thinking and seven lower order thinking problems, while Ms. Popescu's test only comprised five lower order thinking problems.

Test results show both similarities and differences across and within the same school. As anticipated, overall scores show better student results in the better performing school, where 60 % of the students scored A, and only 12.5 % of the students scored D and below. In contrast, less students in the average-performing school scored A (47.5 %) and more students scored D or below (17.5 %). A complication of this test was the fact that being teacher-made, students across the four classrooms were tested on different concepts, hence an 'A' may not mean the same across the tests. This complication was however compensated including in the 'A' comparison only the perfect papers.

The test results included in Table 3 indicate that students at the top-performing school scored higher than the students at the average-performing school, even when tested on more difficult concepts. Moreover, students at the former school also tended to make less mistakes throughout the test, even if they were tested on both higher and lower order thinking problems. At each school, one teacher only tested her students on lower order thinking problems, and as expected student test scores should have revealed differences due to degree of complexity in test items. However, students in the top performing school who were tested on more complex concepts outperformed their peers at the same school who were only tested on less difficult items, which indicates a significant impact of schooling factors (curriculum, teaching), as well as non-schooling factors (parental involvement in children learning).

On the other hand, the better results of Ms. Popescu's students in the average school could be justified by the lack of complexity of the test items, while their peers scored lower overall but were also tested on more difficult items. In order to understand better what constitutes student knowledge, it is important not only to note the number of mistakes students made in tests across the four schools, but also look into the types of exercises students seemed to have more problems with and look into the ways in which these more problematic topics were addressed in class by the teacher and reinforced at home. This cross-analysis aims at providing a better understanding of the impact both schooling and non-schooling factors may have on students.

Table 3

Test Comparisons Within and Across Schools

Participants	LO (lower order thinking problems in test)	% of students making mistakes in LO problems in test	HO (higher order thinking problems in test)	% of students making mistakes in HO problems in test	A (grades)	D or below (grades)
Ms. Ionescu's students	5	25 %	2	37.3 %	65 %	15 %
Ms. Ali's students	5	75 %	0	NA	50 %	10 %
Ms. Popescu's students	5	50 %	0	NA	65 %	5 %
Ms. Reiz's students	7	50 %	2	90 %	30 %	30 %

Classroom Reinforcement

In order to better account for the factors responsible for student mistakes, researcher looked into the most typical mistakes students in four classrooms had problems with in the final test and connected these mistakes to the degree of reinforcement or lack of reinforcement these students received from school and home. Classroom reinforcement refers to the support provided in class by the teacher (i.e. the level of problems solved in class, the degree of stress teacher laid upon the concepts students had problems with). Looking at data gathered from teacher interviews and student final tests, the researcher was struck by some discrepancies between the concepts teachers initially thought their students might have difficulties with as stated in the teacher interviews and the concepts students had difficulties with as reflected in their tests. It seemed therefore interesting to analyze both hypothetical problems and real problems in student learning and pay attention to the degree of reinforcement students received, as this may provide clear answers regarding student knowledge/lack of knowledge of place value concepts.

Data in table 4 indicate that in most cases, teachers' beliefs regarding the topics their students would have problems with were erroneous. One out of the four teachers (Ms. Popescu) was right about the concepts her students faced difficulties with, but in her case, she did not seem to provide a sufficiently strong reinforcement of these concepts, as only 25 % of the problems solved in class dealt with this T/U concepts. The final test shows that Ms. Popescu's students had difficulties in understanding the T/U concepts, as 33 % of the students made mistakes in exercises dealing with T/U. These misunderstandings may be justified, on one hand by the average reinforcement provided by the teacher in class, and on the other hand by the lack of reinforcement of these topics at home.

On the other hand, Ms. Ionescu's expectations of her students' struggles did not match the reality, yet Ms. Ionescu provided more support to her students on the topics they struggled with in the final test than on the anticipated shortcomings (50 % of the topics covered in class dealt with T/U and only 5 % with counting concepts), which may reflect her flexibility in thinking and learning from her students and using this knowledge in providing her students with the best learning opportunities, as stated in her first interview. The better results of Ms. Ionescu's students may be then due to the teacher's strong pedagogical content knowledge, as evidenced in her choice of class activities. It is

Table 4

Data Gathered from Teacher Interviews and Student Final Tests

Participants	Hypothetical problems	Classroom reinforcement (% of problems solved in class)	Real problems	Classroom reinforcement (% of problems solved in class, gathered from classroom observations)
Ms. Ionescu's students	Counting by 2, 3, 5, 10	5 %	T/U	50 %
Ms. Ali's students	T/U	45 %	CB/CF	11 %
Ms. Popescu's students	T/U	25 %	T/U	25 %
Ms. Reiz's students	T/U	15 %	CP	15 %

nevertheless interesting to investigate whether the assigned homework also stressed T/U concepts.

The other two teachers, Ms. Ali and Ms. Reiz, both novices, had also erroneous beliefs regarding the topics their students would face problems with, yet the reinforcement provided on both anticipated and real struggles was limited, as only 11 % of Ms. Ali's classroom activities and respectively 15 % of Ms. Reiz's reinforced the topics their students struggled with in the final test. Were the homework opportunities created by the teachers better than the classroom opportunities in providing students with a more in-depth understanding of the concepts they struggled with in the test, or, on the contrary, the assigned homework provided students with a limited amount of learning opportunity? A look into home reinforcement may help us understand what opportunities were present/missing in/from student learning.

Home Reinforcement

An overall analysis of homework across the four classrooms reveals a better home support and reinforcement in one class (Ms. Ionescu's class, Iorga Elementary), as opposed to an average home reinforcement by homework in the other three classes. An in-depth look into the quantity and quality of homework provided by Ms. Ionescu shows not only a larger number of problems her students had to solve at home, but also a larger variety of topics covered by homework and degree of complexity encountered in these exercises.

Looking at homework exclusively for the lessons observed, researcher found that Ms. Ionescu's students had to solve between 4-9 different types of exercises per homework, among which the more common were tens and units (T/U), comparison (CP), counting backwards and forwards (CB/CF), neighbors (N), counting by 2, 3, 5, 10 etc. Ms. Ionescu's students had to solve both higher order and lower order thinking exercises, and their higher test scores are the proof that all the support they were provided with both after school assignments and in class impacted student learning of place value concepts. Because Ms. Ionescu initially believed her students would struggle with counting by 2, 3, 5, 10 concepts, 30 % of the exercises assigned in homework dealt with these concepts. Examples of these types of exercises as they appear in homework follow: given the number 41, count forwards by 2 to number 53, and given the number 31, count backwards by 2 to number 15. Other exercises asked students to count backwards and forwards by 3, 4, 5, and 10 following the model for counting by two described above.

Looking at the test problems students had more difficulties with, namely the T/U concepts, 20 % of the exercises assigned at home by the teacher dealt with T/U concepts, which translates into good homework reinforcement of the concepts students had difficulties with. Examples of these types of exercises follow: given the numbers 42, 57, 69, 96, color the tens place with blue and the units place with green. Other exercises asked students to find out all two-digit numbers that have 10 as their sum, or to find out all numbers composed of 3 and 5. These are only a few examples of the exercises Ms. Ionescu assigned as homework. Had it not been for this strong classroom and homework reinforcement of T/U concepts, it is likely that students in Ms. Ionescu's class made more mistakes in this type of exercises in the test.

On the other hand, Ms. Ali's students, unlike their peers from the same school were only assigned a limited number of exercises per homework, with only 20 % of these exercises involving higher order thinking concepts and an equally small number of diverse exercises, with only 4 different types of exercises per homework. While the teacher believed her students to have problems with T/U concepts, an analysis of Table 5 homework shows no such exercise assigned for homework, nor does it show any counting backwards and forwards exercises assigned, which were the exercises students had problems with in the final test.

Most of the exercises Ms. Ali stressed in homework were lower order counting (count from 10-20 or 10-100 by 1 and 10), comparisons (compare the following numbers: 11 and 12, 12 and 13, 13 and 14, etc), and composition/decomposition (decompose the following numbers: 11...20). As stated previously, Ms. Ali's reinforcement of these topics in class was only average, and with a poor reinforcement of counting topics through homework students were left with a poor understanding of the topics and

Table 5

Homework Reinforcement -Data Gathered from Homework Analysis

Participants	Hypothetical problems	Home reinforcement	Real problems	Home reinforcement (gathered from homework analysis)
Ms. Ionescu's students	Counting by 2...	30 %	T/U	20 %
Ms. Ali's students	T/U	NA	CB/CF	NA
Ms. Popescu's students	T/U	20 %	T/U	20 %
Ms. Reiz's students	T/U	NA	CP	15 %

performed average in final tests. The conclusion that can be drawn from the student data from Iorga Elementary is as follows: the more students had a diverse and more complex homework requirement, the more reinforcement was allotted to more difficult concepts both at home and at school, the better students understood place value concepts.

A look at student homework at the average-performing school shows that through the homework assigned, none of the teachers provided their students with the best learning opportunities. As such, Ms. Popescu's students were only assigned lower order thinking exercises and only 4 different types of exercises per homework. However, like in Ms. Ionescu's case, Ms. Popescu provided students with some homework opportunities meant to help them understand better T/U concepts (namely the concepts they had problems with in the final test), as 20 % of the homework dealt with these concepts. Examples of such exercises asked students to find out certain numbers, given that tens were triangles and units were circles, or to write numbers formed only from tens on the axis number. As seen in the classroom reinforcement analysis, the reinforcement the teacher provided her students with in class regarding T/U units was only average. Consequently, just like in the case of Ms. Ali's students, Ms. Popescu's students had difficulties in understanding certain concepts, and the way the teacher covered these topics in class and assigned homework to strengthen student understanding of these topics at home was not strong enough to help students in understanding place value concepts to the best of their ability.

The situation is very similar in Ms. Reiz's classroom, where although students had more complex exercises to solve and more diverse types of exercises per homework (6), the comparison problems they had difficulties with in the final test constituted only 15 % of all the homework. Here are some examples of comparison problems: compare the following numbers: 42 and 34, 65 and 56, 78 and 76. Other comparison examples included other numbers students had to compare, but only compare them two at a time and not four at a time like in Ms. Ionescu's class (i.e. compare 34, 56, 76, and 12). If we consider the same percentage of exercises on comparison solved in class with the help of the teacher, we can conclude once again that the degree of both home and school reinforcement of these topics was not strong enough and the student results in the final test are a proof.

Pre- and Post-Interviews

In order to have a more complete understanding of the knowledge students had in the beginning and at the end of the lesson on base 10 numbers, and to account for any impact classroom and homework reinforcement may have had on student understanding of place value, researcher also randomly selected five students from each class and interviewed them on two occasions, before the lesson was taught (to account for any knowledge on place value students might have had from home) and a few days after the lesson was taught (to account for the impact of both schooling and non-schooling factors on student understanding of place value concepts). The pre-interview contained 3 different types of problems, all focusing on place value concepts (T/U), while the post-interview tested student knowledge on the same topics comprised in the first interview, and also asked students to talk about the significance of studying numbers as well as what support they had from teacher and family in learning numbers. Here are the questions students were asked in the pre-interview and asked again following the lesson on base 10 numbers:

1) Identify the following numbers and then indicate how many tens and how many units they have: 23, 17, 24, 19, 15, 20, and 13.
2) Given the following tens and units (corresponding to the above numbers) can you find out the number?
3) Given the following tens and units (corresponding to numbers 13, 21, 48, 15, 32, 14) can you find out the numbers? (A more accurate description of the questions along with the worksheets can be found in the Appendix section at the end of the manuscript).

Results of the pre-teaching interview show some interesting findings. When comparing the results of students in Ms. Ionescu's class to those of students in Ms. Ali's class, both teachers at Iorga Elementary, more correct answers were given by students in Ms. Ionescu's class than in the novice teacher's class. Similar findings emerged from the second school, Delavrancea Elementary, where students in Ms. Popescu's class tended to make fewer mistakes than students in Ms. Reiz's class. Answers were considered correct and coded as correct if the student recognized the number (i.e. recognizing number 23) and was able to differentiate between its tens and units (i.e. 23 is formed of 2 tens and 3 units), and if student was able to find out a number, given its tens and units (give 1 ten and five units, the number is 15). Answers were considered incorrect and coded as such if the students could not recognize the number (i.e. saying 32 instead of 23), if they could not distinguish between its tens and units (i.e. 23 is formed of 3 tens and 2 units) or if they could not find out a number given its tens and units (i.e. saying 84 instead of 48). Overall findings indicate that veteran teachers had students who had a stronger knowledge of base 10 numbers pre- formal instruction than did the novice and the less experienced teacher.

A look at post-interview findings shows improvements mainly in students who performed poorly in the previous interview at both schools. To exemplify, the answers of students in Ms. Ali's class were 65 % correct and those of students in Ms. Reiz's class were 55 % correct (see only 25 % in the previous interview). Less students made less mistakes in both of these classrooms, and this may be the result of both teaching and parental influence. A slight increase in the number of correct answers can be noticed in the results of students from the other two classrooms, with an increase of 15 % and more of correct answers. This increase in student knowledge can be the impact of both teaching and parental factors, as student knowledge of place value concepts improved across the duration of this study.

A few conclusions can be drawn from the above findings that attempt to answer the first research question regarding student knowledge of place value concepts: not only more, but also more diversified and complex homework seemed to produce a better

Table 6

Results of Pre and Post-Interview with Students

Participants	Pre-Interview Correct Answers	Pre-Interview Incorrect Answers	Post-Interview Correct Answers	Post-Interview Incorrect Answers
Ms. Ionescu's 5 students	75 %	25 %	90 %	10 %
Ms. Ali's 5 students	25 %	75 %	65 %	35 %
Ms. Popescu's 5 students	75 %	25 %	95%	5 %
Ms. Reiz's 5 students	25 %	75 %	55%	45 %

student understanding for students in Ms. Ionescu's class as shown in the final student tests, as 65 % of her students scored A. Also, more reinforcement both at school and at home of the concepts students struggled with produced a better understanding of place value topics, as seen in the final test results of Ms. Ionescu's students. Students were generally more knowledgeable of place value concepts when they were provided with home support both before and after the topics were covered in class, as seen in the results of both student interviews, as an increase in student understanding of T/U could be seen across the four classrooms. Moreover, students in both veteran teachers' classrooms had a better understanding of the topics before formal instruction, which may be an indicator of strong parental support at home.

The above data indicate that both schooling factors (teacher, curriculum) and non-schooling factors (parental interactions) seem to positively impact student learning, as students across the four classrooms produced a better understanding of T/U after the lesson was taught and they had time to revise it at home with parents, which is reminiscent of the complexity theory (Maturana & Varela, 1984; Waldrop, 1992), according to which what makes a system work is the interaction of multiple agents. On the other hand, the final tests also show differences in student learning across the four classrooms, which may be due to the diverse learning opportunities students were provided with by the teacher and their parents.

It seems essential therefore to have a look at the way both these factors influence student learning, as researcher described and analyzed what occurred in the four classrooms and talked to parents to find out how instruction occurred at home. The following two chapters cover classroom and home interactions, aiming at answering the other research questions driving this study, namely what types of interactions exist between teachers and students, children and parents, teachers and parents and analyzing the ways these multiple interactions impact student understanding of place value concepts.

CHAPTER 5

FINDINGS OF THE STUDY

Teachers

The aim of this chapter is to describe these teachers' knowledge of subject matter, students, and curriculum, as well as how this knowledge influenced their classroom practice. Moreover, the teacher-parent interaction is monitored and discussed. Special attention is paid to the impact this interaction had on student learning. Conclusions are drawn about the impact schooling factors (teachers, curriculum, assessment) have on student learning of place value concepts. This chapter is divided into two main parts, teachers' knowledge and teachers' practice, each of these parts having further subparts.

Teacher Knowledge

This section deals with teacher knowledge of subject matter (namely their understanding of base 10 numbers), of students (the design of the activities to address their students' diverse learning styles), and of curriculum (the implementation of curriculum in the classroom).

Teachers' Subject Matter Knowledge

This section investigates teachers' understanding of base 10 concepts, looking at teachers' definition of these concepts and significance of learning these concepts as reflected in the tasks they assigned their students both in class and at home. Teachers' subject matter knowledge is compared both across and within same school.

Knowledge of the subject matter, as stated previously, may impact the way teachers prepared for the lesson as well as represent this knowledge in class. In order to account for preparation for the lesson, teachers were interviewed about the objectives they set for the lesson, as well as their flexibility in learning from past experiences. In this vein, results show that teachers who set up more complex objectives possessed a more in-depth understanding of place value concepts and produced better student results than their colleagues who set up less complex objectives. All teachers got their objectives from the school curriculum, however they all had their input in exceeding the objectives or staying at the level set by the curriculum. This is where the difference in objectives between the four teachers lies as well as whether or not teachers set higher standards for their students.

In the first interview conducted with Ms. Ionescu, the veteran teacher at Iorga Elementary, the teacher defined base 10 numbers as "a group of numbers comprised between 10-100," and stated that these concepts were important "for their use in real life, as we don't learn to stay at an abstract level but to apply what we learn." Ms. Ionescu's determination to convey this information to her students in a way in which her students would understand not only what the base 10 numbers were but also why it was significant to learn them is reflected in the complex objectives the teacher set, the thoroughness with which she prepared for the lesson, the concepts she learned from teaching this lesson in the past and adapting them to the current lesson.

For example, Ms. Ionescu stated in the pre-interview that the main objectives she had set for the base 10 numbers lesson were to:

> Understand that these numbers correspond to a group of elements that have as many elements as these elements, to read, write, compare, order and eventually for some kids who have a more developed intellectual ability and are better thinkers than others, to discover numbers from tens and units respecting certain requirements: given the ten and unit, the sum of numbers is a certain number.

Ms. Ionescu's statement shows her preoccupation with the different learning styles of her students and the desire to address these differences in learning by providing her students with a challenge appropriate for their individual rhythm of development, which is key to learning (Greenes, 1995). Moreover, the teacher stated that she followed the curriculum requirements, but did not limit herself to only using the curriculum. Observation notes expose a large variety of exercises used by the teacher in order to supplement the classroom textbook. For example, the teacher introduced fifty-three new types of exercises, both lower and higher order thinking for her students to solve in class. While both types of exercises were assigned to all students, the more complex ones requiring students to discover certain rules and continue counting or discover two-digit numbers respecting certain requirements (see quote from above teacher interview), were designed in particular to challenge those students described by the teacher as "better thinkers than others." Data from table 7 may suggest that understanding the significance of base 10 numbers, Ms. Ionescu introduced these numbers in different types of exercises and spent time making sure her students mastered the easier concepts first and gradually increased the difficulty level, reminiscent of Ma's (1999) theory that if teachers spent more time on fundamental mathematics, students will have a stronger knowledge of mathematics. Ms. Ionescu further reinforced all types of exercises introduced in class

Table 7

Ms Ionescu - Exercises Assigned in Class, at Home, and in Final Test

	LO T/U	LO C	LO CP	LO C/D	LO CB/CF	LO C by 2, 3, 5, 10	LO N	HO 2D
Class work	Given the numbers 17, 3, 9, 12, 5, arrange them in the tens and units place.	Let's count from 10-20, taking turns.	Compare the followin g pairs: 15-18; 16-19; 20-17.	Compose and decompose the numbers: 49, 62, 71.	Order forwards the numbers: 34, 12, 59, 61, 3.	Count by 2 from 10-20 and by 10 from 10-100.	Write the neighbors of the numbers: 5, 18, 16, 14.	Which is the highest number formed of 2 digits?
Home work	Given the numbers 42, 57, 69, 96 color the tens in red and the units in green.	Count from 21 to 26 and 39 to 43.	Compare the followin g pairs: 12-18; 10-14; 16-11; 13-19; 20-15; 17-11.	Decompose the following numbers: 16, 19, 14, 12, 11, 13, 15, 20.	Order forwards the numbers: 11, 13, 20, 17, 15, 18, 16.	Count by 5 from 25 to 60	Find the neighbors of the pairs: 14-16; 12-14; 16-18; 18-20.	Find all two digits number s that have the sum 10.
Tests	Given the tens and units, find the numbers: 8 T, 3 U; 4 T, 1 U; 1 T, 5 U; 6 T, 8 U.	N/A	N/A	Compose and decompose : 46; 57; 28; 16, 3 and 30.	Order forwards (95, 8, 43, 17, 62) and backwar ds (70, 89, 63, 85, 72).	See HO exercise	Find the neighbors of the numbers: 29-31; 69-70; 33; 57.	Discove r the rule and count: 57, 59...; 31, 34, 37...; 95, 90, 85...

with homework assignments and she probed her student understanding of the same topics in the final evaluation. Teachers' knowledge of base 10 numbers is therefore reflected in the way she designed her lesson as well as the opportunities she created for learning these concepts both in class and at home.

On the other hand, Ms. Ali's definition of base 10 numbers was shorter, "students learned about single digits with numbers 0-9, and now the passage from 10-100 is the formation of numbers from tens and units," and her understanding of the significance of these concepts was fragmented: as she was not able to link the learning of base 10 numbers with concepts that were going to be studies further on: "learning about numbers was significant because mathematics itself was significant." As stated in the interview, Ms. Ali's objectives were less complex and were entirely dictated by the school curriculum (same curriculum as in the case of Ms. Ionescu). Ms. Ali stated that she wanted her students to "understand ten concepts, the formation of base 10 numbers, 10-20, 11, 12, etc., decomposition, ordering. And for them to know to represent the tens, to order backwards and forwards, even and uneven." Unlike Ms. Ionescu's objectives, Ms. Ali's objectives were simple and quite uniform for all students, assuming all students were at the same rhythm and only introducing more simple base 10 concepts. Although this teacher had students who were also better thinkers and strong independent learners, she did not address the various needs of these better students and prepared a lesson that would reach most of her students, without setting up additional challenges for the gifted ones. Moreover, all additional extra-curriculum materials used by the teacher (a total of twenty-five exercises) were of the same reduced difficulty level as all the exercises solved from the classroom textbook.

Table 8 presents types of exercises used by Ms. Ali to teach place value concepts, in which we can see Ms. Ali being consistent in exposing her students to less complex notions both in class and at home.

Table 8

Ms. Ali - Exercises Assigned in Class, at Home, and in Final Test

	LO T/U	LO C	LO CP	LO C/D	LO CB/CF	LO N	LO C by 2, 3, 5, 10	HO
Classwork	Given the numbers, use your manipulatives to form them: 12, 14, 17, 20.	Let's take turns counting 10-100.	Compare the pair; 12-16.	Decompose the numbers: 34, 47, 77, 48 and 53.	Let's count forwards from 10-20 and backwards from 29-10.	What are the neighbors of the numbers: 44, 66, 32.	N/A	N/A
Homework	N/A	Count from 10-20.	Compare numbers: 11-12, 12-13, 13-14, 14-15, 15-16, 16-17, 17-18, 18-19, 19-20.	Decompose numbers: 11 to 20.	N/A	N/A	Count by 10 from 10-100.	N/A
Tests	N/A	N/A	Compare the numbers: 18-13; 15-16; 17-17; 12-16; 14-11; 12-20.	N/A	Order forwards and backwards: 19, 15, 18, 13, 12, 10, 17.	Find the neighbors of the numbers: 16, 10, 14, 17, 19, 13, 18, 11, 9.	N/A	N/A

A look at teachers' knowledge of subject matter in the average performing school also reveals more similarities in teachers' understanding of base 10 numbers but differences in the way they conveyed these concepts to their students. The veteran teacher, Ms. Popescu defined base 10 numbers in the pre-interview as "numbers 10-100, the numbers formed by adding a unit to each number, having thus the next number, " much in the same style as her colleague at Iorga Elementary, clear and to the point. Her understanding of why numbers were significant resembled the answer given by her veteran peer: "students need to learn these numbers because they will need them further for addition and subtraction and later on for multiplication and division."

When interviewed for the first time and asked about base 10 numbers, the less experienced teacher at Delavrancea Elementary, Ms. Reiz, defined numbers as both single and double digits, "from 10 up to an infinity all of them are formed with the help of these numbers from 1-9." Ms. Reiz, just like her colleague at the same school saw learning about these concepts important as they further related to other mathematics concepts.

As revealed by their initial interviews, both Ms. Popescu and Ms. Reiz had equally simple and uniform objectives set for all their students. Notice how Ms. Popescu's objectives matched Ms. Ali's objectives in their lack of complexity: "to know how to count backwards and forwards, composition and decomposition of numbers, ordering and comparing numbers." Same for Ms. Reiz's objectives, who seemed to focus more on less significant aspects, like number pronunciation for example: "to know how to count correctly from 10-100 and especially to pronounce correctly the numbers 10-20. The second objective is their formation, composition and decomposition."

Both teachers at this school had objectives dictated by the curriculum and they followed these objectives entirely, exposing their students to less complex notions as they wanted their students to master these concepts before moving on to more complex notions. There were no extra-curricular materials used by Ms. Popescu, and while Ms. Reiz did introduce seventeen new types of exercises in class, only one of them was of a

Table 9

Ms. Popescu – Exercises Assigned in Class, at Home, and in Final Test

	LO T/U	LO C	LO CP	LO C/D	LO CB/CF	LO N	LO C by 2, 3, 5, 10	HO
Class work	Given that triangles are tens and circles are units, please write the following numbers: 80, 41, 54, 63.	Let's count from 30-40.	Compare the numbers: 45-42; 36-86; 42-12; 89-84; 71-77.	Decompose the numbers: 30, 45, 46.	Count from 77 to 33 and from 31-62.	Write the neighbors of the pairs: 47-49; 38-40; 69-71; 31-29.	Count by 10 to 100 using your sticks.	N/A
Homework	Given the triangle is a ten, write the numbers: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.	Count from 30-100.	N/A	N/A	Count from 63 to 74 and from 92 to 86.	N/A	Write all numbers formed of tens from 10-100.	N/A
Tests	Given a certain number of triangles and circles, find the numbers: 14, 10, 11, 16, 15.	N/A	N/A	Decompose the numbers: 26, 29, 15, 13.	N/A	N/A	N/A	N/A

more complex difficulty level, requiring students to discover the rule and continue the counting. Uniform learning seemed to be the case for both teachers at this school, not providing all their students with challenge nor designing more challenging activities for their stronger students. Table 9 presents example of exercises used by Ms. Popescu while teaching place value concepts. Data in table 9 show an emphasis solely on more simple base 10 concepts, this teacher also treating all her students uniformly. Moreover, not all the concepts covered in class was reinforced at home, or assigned in the final evaluation,. which comes in contrast with the way the other veteran teacher monitored student understanding of place value concepts, which might have led to the better understanding of place value concepts by Ms. Ionescu's students, as shown by test scores. What types of exercises did the less experienced teacher at the same school use to teach place value concepts? Table 10 shows examples of exercises used by Ms. Reiz.

Data in table 10 indicates that while introducing students to a large variety of exercises, Ms. Reiz also mostly emphasized less complex topics, not addressing any complex topics in class yet assigning them for homework. Moreover, some of the concepts addressed in class were not reinforced with homework, yet students were tested on these concepts, which may have lead to the weaker understanding of Ms. Reiz's students as indicated by her student test scores.

Overall findings show thus that the teacher who set up more complex and diverse objectives for her students and challenged her students more produced better student understanding of place value concepts, as is the case of Ms. Ionescu. A look at her student test scores reveals that more than half of her students scored A on the final test, which checked understanding of both easier and more complex place value topics. On the

Table 10

Ms. Reiz - *Exercises Assigned in Class, at Home, and in Final Test*

	LO T/U	LO C	LO CP	LO C/D	LO CB/CF	LO N	LO C by 2, 3, 5, 10	HO
Class work	Given the numbers 48, 71, 56, 15, 90, 23, 58, 95, underline all tens.	Count form 10-20.	Compare the numbers: 16-15; 14-11; 12-14; 15-13.	Decompose 17, 16, 19, 15, 46, 63.	Order the numbers forwards: 9, 13, 20, 15, 3, 11.	Write the neighbors of: 18, 15, 46-48, 88-90, 75-77, 60-62.	Count by 2 from 0-30.	N/A
Home work	Color the tens with blue and the units with yellow for the numbers: 42, 90, 87, 99, 53, 100, 31, 75.	Count from 30-39, 60-69 and 90-99.	N/A	N/A	N/A	Write the neighbors of 40, 70, 90, 27-29, 30-32, 63-65.	Count by 2 from 30-40 and by 5 from 30-50.	Write all two digits numbers that can be formed with numbers 7, 4, 9.
Tests	Count the triangles and circles and write down the numbers: 13, 30, 32, 48.	Write numbers 45-51.	Compare the numbers: 7-75; 27-29; 56-46; 20-50.	Compose and decompose: 24, 35, 57, 76, 40 and 8.	Order the numbers forwards: 78, 37, 19, 90, 28, 34, 85, 43.	Write the neighbors: 46-48; 38-40; 50-52; 47-49; 74-72.	See HO exercises.	Discover the rule and continue counting: 65, 66, ...; 80, 70, ...; 42, 44, ...

other hand, the other teachers who set up simple objectives for their students and did not provide their students with more challenge obtained good and average student results in the final tests, but noteworthy is the fact that their students were merely tested on less complex topics.

Lesson planning appears to be a significant factor in student learning. In planning the current lesson, teachers were not only influenced by the curriculum, they were also more or less influenced by their previous experience teaching the base 10 numbers lesson, as all of them had taught the lesson at least one time in the past. Teachers who used their past teaching to plan better lessons for their students, acknowledging differences in student learning styles and exposing students to both HO and LO concepts had students who possessed a stronger knowledge of mathematics. Data in table 11 accounts for differences in teachers' mathematics subject matter knowledge as well as their knowledge of students and curriculum.

Consequently, the factors that were present in Ms. Ionescu's lesson plan, were less represented in the other teachers' preparation for lesson. These are: the teachers' flexibility to learn from past experiences, their desire to apply what they had learned to the present lesson, and their openness to learn from and with the students (also present in Ms. Ali's case) in an attempt to create the best learning opportunities for their students. Drawing from their previous experience with the base 10 numbers, the four teachers assigned tasks that required students to perform more or less complicated computations, fact that seemed to ultimately benefit Ms. Ionescu's students and inhibit learning in the other three teachers' students.

The direct impact of teacher knowledge of mathematics and instructional strategies is shown through the within school comparison, focusing on the differences in student final test scores: at Iorga Elementary, 65 % of all Ms. Ionescu's students vs. 50 % of all Ms. Ali's students scored A in the final test. Noteworthy is the fact that Ms.

Table 11

Teachers' Knowledge of Content, Students and Curriculum

Teachers	Challenge	Support	Manipulatives	Beyond curriculum	Differentiated instruction	Involvement with parents
Ms. Ionescu	Engaged students in LO and HO exercises	Provided a lot of opportunities for students to be engaged in class in both HO and LO exercises, assigned both types of exercises as homework and tested students on both types of exercises	Abacus Cards with numbers Charts	Teacher used curriculum and went beyond curriculum Developing other HO and LO exercises	Created differentiated worksheets for students For weaker independent learners she designed individualized worksheets with a richer intuitive support	Invited parents to participate in her classes and encouraged them to make suggestions about improving the classes and their children's performance Discussed how she would approach the concepts in class and what parents can do to enhance learning at home
Ms. Ali	Engaged students only in LO exercises and spent a lot of time making sure students master the easy concepts	Provided support for only mastering the LO concepts in class and at home	Sticks Slide rules Charts with numbers	Extracurricular activities exposed students to the same LO exercises as the classroom textbook	Uniform instruction that matched the uniformity of her objectives Did not create differentiate worksheets even if she had brighter students	Open to parental suggestions Mainly discussed students' progress and taught the parents base 10 concepts
Ms. Popescu	Engaged students only in LO exercises and spent a lot of time making sure students master the easy concepts	Provided support for only mastering the LO concepts in class and at home	Axis with numbers 0-100 Charts with numbers	Teacher limited to mainly using the curriculum and engaged her students in LO exercises	Uniform instruction that matched the uniformity of her objectives Did not create differentiate worksheets even if she had brighter students	Open to learning from parents Parental interactions were limited to students' progress No focus on how students learn, but on what students need to learn

Ms. Reiz	Exposed her students to mainly LO concepts in class but assigned them HO concepts as homework	Weak support of HO concepts in class, yet the teacher assign those topics both as homework and tested students on them	Objects from the class, sticks, Charts with elements	Teacher limited to using the curriculum without exposing students to more complex notions	Assigned individual worksheets at the end of the class to understand where her students were The sheets were not individualized though, assessing all her students on the same concepts	Parental interactions were limited to report on student progress

Ionescu's test checked student understanding of both easier and more complex topics, while Ms. Ali tested her students only on less complex notions. Moreover, significantly more correct answers were given by students in Ms. Ionescu's class (87.5 % in the post interview) than by students in Ms. Ali's class (65 % correct answers in the post interview), which leads to the conclusion that teachers' own understanding of base 10 numbers as reflected in her choice of activities impacted the way students learned about numbers.

On the other hand, differences also appear in students' test scores at Delavrancea Elementary, where 65 % of students in Ms. Popescu's class scored A in the final test, as opposed to 30 % students in Ms. Reiz's class, and 93 % of the answers given by students in Ms. Popescu's class in the post interview were correct, vs. only 55 % answers correct given by students in Ms. Reiz's class, which shows a big gap in student understanding of base 10 numbers. Although students in Ms. Popescu's class were tested on less complex concepts than students in Ms. Reiz's class, it is noteworthy that even when tested on the same concepts in the interview by researcher, Ms. Popescu's students scored significantly higher than their peers at the same school which again may be due to the way teacher reinforced the concepts taught in class and at home.

Across-school comparisons show similarities in veteran teacher understanding of place value concepts as well as their student results: the more in-depth understanding of place value concepts, the better student learning of the concepts as reflected by student results in both final tests and interviews. In other words, if teachers possessed both a procedural and conceptual understanding of the topics, the better learning opportunities they provide their students (Ma, 1999). On the other hand, if teachers possessed the procedural understanding but lacked the conceptual understanding of place value concepts (Ball 1990), their students were exposed to a limited understanding of the concepts, as is the case of the students in Ms. Ali and Ms. Reiz's classes as evidenced by their student final tests and interview results.

Moreover, learning more mathematics in college did not seem to produce better teachers (Tooke, 1993), as Ms. Ali is the only teacher with a college degree, and she seemed to possess the most limited knowledge of place value concepts out of the four teachers interviewed and observed in the study, as shown by interviews, classroom observations as well as students test scores and interviews. This is again reminiscent of the "learning more" (Tooke, 1993) vs. "learning big" (Ball, 2003; Graeber, 1999; Greenes, 1995; Ma, 1999) dispute according to which not exposing future teachers to more mathematics concepts, but exposing them to these concepts in a conceptual way will produce better teachers.

A reasonable conclusion drawn from the above data would then be that teacher knowledge of subject matter generally influences student learning, but how is subject matter knowledge linked to student and curriculum knowledge? Moreover, research (Schulman, 1986) showed that it is not enough for teachers to know subject matter, it is important they also possess pedagogical content knowledge. It is therefore important to look into the ways teacher subject matter knowledge impacts teacher's knowledge of students and use of curriculum, and, in turn, how they all impact classroom practice.

Teachers' Knowledge of Students

The way teachers used student knowledge in designing and implementing their activities had a strong impact on student learning, as demonstrated by this study's findings. As such, teachers who used their past experience in teaching the same lesson and applied what they learned about this lesson and their students in the current lesson, displayed a stronger knowledge of students and were better able at creating opportunities for their students to understand better base 10 numbers.

For example, the veteran teacher at the better performing school, Ms. Ionescu, stated in her interview that her past experience with teaching this lesson taught her the importance of treating students differently and using for some of her current students individualized worksheets with lower difficulty levels and a richer intuitive support than with other students. For her current lesson, Ms. Ionescu implemented differentiate worksheets, "developmental sheets for some and improvement and catching-up sheets for others." The teacher also diversified her use of manipulatives in class, adding to the usual abacus, sticks and slide rules, more hands-on objects (cards with numbers, logical game with group-diagrams, charts with groups of elements, as well as chips with numbers 10-30) in an effort to create more opportunities to engage her students in learning numbers hands-on.

While talking about the knowledge she thought her students should possess, the teacher stated that her students needed to know very well numbers 0-10, their formation, writing, reading, and to be able to correctly compare them, as well as add, subtract, group and classify them. With this goal in mind, and using the manipulatives she thought would be beneficial for student learning, Ms. Ionescu asked her students to notice that numbers 0-9 were written with one figure on the chart, and then by adding a unit to each number, a ten could be obtained. Students had numerous opportunities to then practice with sticks making sets of tens, grouping elements in tens and units throughout the class. Ms. Ionescu stated that the activities she designed were helpful for her students, reinforcing again the need to learn from past experiences and apply this new knowledge into present teaching:

> I learn new things from one series of students to the other, so I can always improve my activities and I can say I learn from experience. One can better adjust the manipulatives to a new series of students, I would say that knowing this series I managed to select the best manipulatives and activities for them.

The teacher stated in her post-interview that overall, there were no major problems with this lesson, as her students had a strong knowledge base from the beginning. Some minor problems she associated with parental explanations at home, as she stated that she had to readdress some concepts in class to make sure her students were not confused by the misunderstandings created by their parents. That Ms. Ionescu's students understood base 10 concepts is proven by her students' final test scores and interview questions, as 65 % of her students scored A in the final test. This teacher had a strong knowledge of her students, as her activities, use of manipulatives and learning opportunities exposed her students to both complex and less complex notions, as the teacher knew how much to challenge her students and what type of support to provide in order to increase student understanding of the concepts, taking into consideration their individual rhythm of development.

The novice teacher at the same school, Ms. Ali had a more limited understanding of both student knowledge as well as how to apply what she learned from past experience into her teaching. Ms. Ali stated in her initial interview that what she had learned from teaching the same lesson in the past was mostly the base 10 concept, and how a ten was formed, without any specifics about how this might impact her students or how this could contribute to her developing a better lesson for her present students. In her present teaching, the teacher stated that she made a point of planning the lesson and teaching with her current students, as she believed this would help them understand better base 10 numbers. No dramatic change in the teacher's use of manipulatives were made, as she continued to use the same manipulatives she had used in her past teaching, namely slide rules and sticks, used to count numbers and form groups of tens and units. These manipulatives provided more reduced learning opportunities for Ms. Ali's students, as compared to Ms. Ionescu's students, who were exposed to a larger spectrum of manipulatives and could use them in a larger variety of activities, as shown in the tables above.

As for her students' knowledge, Ms. Ali, like her colleague Ms. Ionescu, believed her students should master numbers 0-9 very well before proceeding to more complicated notions, and consequently stated that she spent more time with these numbers as she wanted her students to have a strong understanding of base 10 numbers. The teacher felt she did a good job teaching the lesson, and was confident that the activities she selected were the best, as she did not see any problems overall with the way she taught the lesson and student understanding. Her students' final test results as well as the interview questions cannot fully prove that Ms. Ali's students understood the lesson as well as Ms. Ionescu's students did, as less students scored A in the final test and answered correctly in the pre- and post-interviews in Ms. Ali's class than in Ms. Ionescu's class. This teacher's understanding of her student knowledge was not as strong as Ms. Ionescu's as the teacher believed there were no problems with this lesson, and students understood the concepts really well, while their test scores showed obvious problems with their understanding of base 10 number concepts that may be due to the way teacher reinforced these concepts in class and at home, as shown by Ms. Ali's exercises table data.

Were students at the average-performing school exposed to similar learning opportunities as students at Iorga Elementary? The veteran teacher at Delavrancea Elementary, Ms. Popescu, stated that what she learned from her past teaching about this lesson that helped her prepare the current lesson on base 10 numbers was the fact that if her students could count up to 10 and if they could count by 10 to 100 they could count easily, and the only difficulty they might face would be going over the threshold. In her present lesson, the teacher spent a lot of time on numbers 0-9 to make sure her students had a solid understanding of these numbers, and also created opportunities to learn by playing as the teacher was enrolled in methodology classes that stressed out the importance of game at such an early age. The manipulatives she continued using in her current lesson were the same as she used in the past, namely sticks, abacus, the shapes, as well as the big axis with numbers from their textbook, as she stated that students had to use concrete materials which would provide them with the support needed to understand more complex notions. However, no complex notions were taught or addressed in Ms. Popescu's teaching, leaving students with only using these concrete materials for simple computations, as shown in the previous section.

All three teachers discussed so far identified as vital for future learning student understanding of numbers 0-9, and they all stated that in order for students to understand these concepts, they dedicated a lot of instruction time to teaching them. Some teachers, as Ms. Popescu, stated that she could tell the difference between students who went to Kindergarten and students who did not, as those who attended K had a better understanding of these numbers and required less assistance than their peers who did not attend K. Like her colleagues, Ms. Popescu also believed that her lesson was successful, and she stated that overall there were no major problems, as she exposed her students to a lot of different activities (including games) and she created countless opportunities for them to use concrete support. Findings reveal that Ms. Popescu's students had indeed no problems understanding the less complex concepts they were exposed to, as demonstrated by their final test scores and interview questions. However, unlike Ms. Ionescu who created more complex learning opportunities for her students, Ms. Popescu reinforced concepts her students were good at and did not create any further challenges either in class or at home through homework. In this way, one could say that Ms. Popescu's understanding of her students was not as strong as Ms. Ionescu's, as this teacher did not provide her students with an appropriate degree of challenge based on their test and interview results. Ultimately, keeping students at a level they have exceeded may not prove very beneficial for student learning.

The less experienced teacher at the same school, Ms. Reiz, stated in her pre-interview that she learned a lot from teaching the same lesson in the past, as some mistakes or misunderstandings she discovered at her older students made her think of better ways to explain number formation and writing to her current students, and the way she wanted to apply this in her current teaching was through creating opportunities for her students to be engaged in learning through discovery. The teacher diversified and increased her use of manipulatives in class, adding to the regular drawings more concrete materials, like sticks, pens, groups of elements as she taught the lesson with the goal to have students understand that ten units form a ten.

In terms of student knowledge, this teacher too believed students needed to master numbers 0-9 very well before teaching more complex notions, and she believed that the activities she chose for the lesson on base 10 numbers reached their goal. Some students had problems understanding these concepts though, as stated by the teacher and shown by test results, as Ms. Reiz's students had the lowest scores of all four classrooms. Teacher's weaker knowledge of students may be the factor that impacted their student learning, as Ms. Reiz stated her present students could better learn by discovery yet she did not engage her students in discovery learning at all, and she tested them on complex notions although her coverage of more complex notions was not as well developed and implemented as was Ms. Ionescu's. This comes to reinforce Schonfeld's (1988) statement that if mathematics concepts were taught as step-by-step procedures to be memorized, then mastery of the physical rather than the intellectual skill was sought, and even if students could solve place value concepts, they would not be able to understand the mechanisms behind the procedures they used and would be at a loss at large.

Teacher knowledge of students, as shown by these data, had a big impact on student learning, as teachers who knew their students well created better learning opportunities for these students, opportunities that benefited their students at large which represents key to learning (Greenes, 1995). For examples, understanding that students have their individual rhythm of development, Ms. Ionescu developed differentiated worksheets, exposing her stronger students to more challenging concepts and enabling her weaker students to work more on lower order concepts with the use of manipulatives until they mastered them and could move on to more challenging concepts. On the other hand, teachers who did not know their students very well tended to expose their students to either too complicated or too easy concepts according to their level of understanding. This is reflected by the instructional choices of Ms. Ali and Ms. Popescu, who exposed their stronger students to the same learning opportunities they engaged all their students, risking to bore their stronger students while the weaker independent learners mastered the easier concepts. On the other hand, by exposing her student to higher order concepts at home, without reinforcing them in class, Ms. Reiz's students developed a more fragmented understanding of base 10 concepts.

Teachers' understanding of their students' level further enabled teachers to design activities building on what students already knew and further increase their difficulty level. Asked about the most difficult concepts she thought her students might struggle with, Ms. Ionescu stated that as her students could not read yet, the concepts they would struggle with would be counting backwards higher numbers. In order to address these concepts in class, the teacher said she would make use of manipulatives and use the analogy with numbers 0-10 as the foundation on which she would build student learning of counting backwards higher numbers. Knowing her students well and understanding that each had their own individual rhythm of development, the teacher developed differentiate worksheets and engaged her students in working with manipultives a lot, as stated in the initial teacher interview:

> I want more than anything for them to work with manipulatives, to observe and acknowledge how each number is formed, to write, to correctly place one digit and two digit numbers with a number, this is why I stress out writing numbers in tables and this is why I always go back to one digit numbers, so that they understand that some numbers are formed only from tens and some are formed from units.

Teacher's knowledge of her students is also reflected in her gradually increasing the difficulty level and limiting the use of manipulatives once she realized her students mastered base10 concepts, as she stated that students could not benefit if they were kept at a level they surpassed. This is reflected in the increased difficulty level of assigned homework, classroom work as well as test items, as Ms. Ionescu's students were exposed to both complex and easier concepts and proved they possessed a deeper understanding of base 10 concepts, as evidenced in the tables presented in the previous section. This finding comes to reinforce Perry's (2000) analysis of classroom practice of teachers from Japan, China and United States, according to which if students were exposed to more complex explanations about more complex topics they would have a stronger understanding of mathematics.

On the other hand, Ms. Ali's students, as stated in her initial interview, were likely to face problems with adding and subtracting numbers, and the teacher intended to help her students acquire these concepts by using a lot of examples with number decomposition and reinforcing numbers 0-10 a lot in order for students to be able to grasp double digit numbers. As shown by her classroom observations, Ms. Ali exposed her students to a lot of composition/decomposition exercises, and spent a long time on one-digit numbers, which slightly increased her students' understanding, but not to the extent of Ms. Ionescu's students, as shown by the final tests and interview questions. In her effort to help her students master easier concepts, Ms. Ali seemed to struggle to approach more complex notions, exposing her students mostly to basic computations with a reduced level of difficulty. Moreover, she did not create diverse learning opportunities for her better prepared students, keeping them at the same level with lower achieving students. Consequently, the design and implementation of her activities seemed to only benefit the average-performing students, as opposed to the learning opportunities created by Ms. Ionescu that benefited all her students, due to their different degree of complexity.

The same approach seemed to be present in Ms. Popescu's class at Delavrancea Elementary, who stated in her pre-interviews that her students would not face too many problems with the concepts being taught as long as they mastered composition/decomposition. The teacher's support with these concepts was manifested in the oral and written opportunities created for her students, but just like her colleague at Iorga Elementary, Ms. Ali, this teacher also exposed her students to only basic computations, disregarding the fact that some students were better able to grasp more difficult concepts and she could have challenged these students more. The learning opportunities that Ms. Popescu created in her lesson seemed thus to benefit more her lower achieving students, as her students scored as high as Ms. Ionescu's students in the final test but were mainly tested on simple concepts.

Another example of an inaccurate knowledge of students is the case of Ms. Reiz, who anticipated in her pre-interview that her students would have problems with pronouncing the numbers more and stated she would correct this by always correcting their pronunciation. No references were made to the way students learned or what base 10 numbers misunderstanding they might have and what opportunities she could create to prevent or correct these misunderstandings.

The above examples show how teachers' knowledge of students may help them create those learning opportunities that could benefit student learning, as shown in Ms. Ionescu's case. On the other hand, a weaker understanding of students may prevent the teachers from creating more complex learning opportunities for their students, exposing students to concepts they have already mastered or concepts that were too difficult for their level of understanding due to the inadequate support provided by both teacher and parents. More investigation needs to be conducted though into what teaching practices take place both at home and at school that enable some students to outperform their peers. As such, a look into the ways teachers make use of curriculum and assessment techniques in class could bring some clarification regarding teachers' practice and how practice is influenced by knowledge of subject matter, students, and curriculum.

Teachers' Knowledge of Curriculum

The ways teachers make use of curriculum is believed to influence their classroom practice (Li, 2000; Valverde et al., 2002). Analyzing content coverage as well as page space dedicated to mathematics topics may provide us with a better understanding of what learning opportunities are available throughout the curriculum. Moreover, looking at ways in which curriculum is used in the classroom, as a starting point for learning or as the main source of information may provide a clearer understanding of the gap in student performance.

A look into the data of this study reveals that relying solely or mostly on the classroom textbook is related to a poorer student understanding of the base 10 numbers. In opposition, using curriculum as a starting point and supplementing teaching with diverse other materials may benefit students more. For example, Ms. Ionescu stated in her interview that while the objectives of the base 10 numbers lesson were written in the school curriculum, she adjusted them to the class level, not limiting herself to only using the curriculum nor exceeding it too much, as she realized it was important "to respect students' individual rhythm of development." These objectives and the teaching methodology informed the teacher's choice of manipulatives, as the teacher stated that she needed a theoretical base before selecting the best materials that would make teaching and learning easier for a new series of students. She then supplemented the classroom textbook requirements with problems form different alternative textbooks, with a total number of 53 exercises, addressing both simple and more complex topics, as shown in the classroom observations.

Ms. Ali seemed to be satisfied with the opportunities the classroom textbook provided her students, as she stated in her interview that she found the objectives for the base 10 numbers lesson in the curriculum and mostly relied in the curriculum to teach the place value concepts. Although the classroom textbook, the same one used by Ms. Ionescu, provided some opportunities for complex topics, Ms. Ali did not cover these topics in her class, and even if she supplemented the materials she used in class with 11 exercises, all these newly introduced exercises were of a lower order thinking, not exposing her students to a variety of exercises and not providing them with challenge, as shown in her classroom observations.

On the other hand, the veteran teacher at Delavrancea Elementary also followed mostly the classroom curriculum objectives, designing activities using mostly the textbook and some other alternative books, yet she also exposed her students only to lower order thinking problems, using the classroom textbook as the main force to drive her teaching. The less experienced teacher at the same school, Ms. Reiz, also stated that she followed thoroughly the classroom curriculum, as she wanted to reach all the objectives written in this curriculum, yet she used different other textbooks in designing the activities, providing her students with a more diversified pool of problems addressing more complex and simple problems. The seventeen exercises the teacher introduced in the class from sources other than the textbook all addressed lower and higher order thinking concepts, yet the teacher did not go past exposing her students to a limited number of higher order topics in class, as home assignments mostly consisted of simple computations.

Another interesting finding emerging from these data is the fact that teachers who used textbooks rich in lower order problems tended to limit themselves to solely using the textbook to cover all these types of exercises, or if they used additional exercises, these would be of a similar difficulty level. The unit on base 10 numbers found in the textbook used by both teachers at Iorga Elementary contained thirty-three base 10 number exercises, both higher and order thinking, 94 % of the exercises being lower order and 6

% being higher order thinking exercises. Lower order thinking exercises, for example, required students to compose and decompose numbers up to 100, to form numbers given tens and units, count by 2, 3, 5, 10s, arrange numbers forwards and backwards, find neighbors to given numbers, etc, while higher order thinking exercises required students to count backwards or forwards by discovering the rule (i.e. 67, 69…; 31, 34, 37…; 95, 90, 85…).

In addition to the textbook, Ms. Ionescu used extracurricular activities addressing both topics, with 85 % of these exercises being lower order thinking and 15 % being higher order thinking, which seemed to provide her students with sufficient diverse learning opportunities. Examples of higher order extracurricular materials are provided below: write all two digits numbers that have 10 as their sum, or write the lowest/highest two-digit numbers, etc. Lower order thinking exercises involved computations such as asking students to count to 20, to find neighbors to numbers up to 20, to decompose and compose numbers up to 90, as well as to count by 2s, and 5s.

On the other hand, Ms. Ali's extracurricular materials only contained lower order thinking problems, not providing the students with the support needed for them to understand more complex notions, which may explain the weaker test scores and interview questions. Here are a few examples of such exercises: order numbers up to 20 backwards and forwards, find the neighbors to numbers up to 20, compose and decompose as well as compare numbers up to 100.

The two teachers at the other school used different textbooks, both of these textbooks comprising more problems than did the textbook used by the teachers at Iorga Elementary. For example, the unit on base 10 numbers on Ms. Popescu's textbook contained forty-seven base 10 number exercises, with 92 % of these exercises being lower order thinking and 8 % being higher order thinking. Examples of lower order thinking exercises are given below: given certain numbers, color the tens with red and the units with green; order forwards and backwards numbers up to 100, compose and decompose numbers up to 100, while higher order thinking problems asked students to write numbers that had number 8 in the tens place and number 5 in the units' place. This teacher did not provide her students with other extra-curriculum materials.

The base 10 numbers unit in Ms. Reiz's textbook contained forty-five base 10 numbers exercises, of which 86 % were lower order thinking and 14 % were higher order thinking. Below are examples of higher order exercises in Ms. Reiz's textbook: given the numbers 10 and 20, find three numbers closer to 20 than to 10; write all numbers higher than 30 and lower than 100 that have the ten digit 4 and the units digit 8. On the other hand, lower order thinking problems asked students to find neighbors to given numbers, compare, compose and decompose numbers up to 100, count backwards and forwards numbers to 100, etc. In addition, Ms. Reiz also supplemented these materials with extra-curriculum materials, which covered 6 % of higher order topics. Examples of higher order thinking problems encountered in the extracurricular materials are as follows: given the table with numbers up to 30, complete the table with the missing numbers and then color all even numbers. Lower order thinking problems were similar to those encountered in Ms. Ionescu and Ms. Ali's extracurricular materials. Paradoxically, even if the textbook Ms. Reiz used created opportunities for students to be engaged in higher order thinking concepts, notes from observations show no such exercises solved in class. Although the assigned homework did include higher order thinking concepts, and students were tested on such concepts in the final test, they had problems mastering these more complex topics without the classroom support provided by the teacher. This comes to support Cohen's (1991) finding, according to whom even if teachers are provided with a standards-based curriculum, but if there is not support to facilitate the implementation of this new material, teachers tend to fall back on traditional methods of instruction, as was the case of Ms. Reiz, who although used a textbook that created opportunities for students to be engaged in more complex notions, she did not encourage her students to take advantage of these opportunities, and moreover, she approached teaching from a more traditionalist perspective, which seemed to be more detrimental to student learning as evidenced by her student test and interview scores.

Although classroom textbooks exposed students at Delavrancea Elementary to more exercises than did the textbook at Iorga Elementary, the teachers in the latter school supplemented their activities with problems of equal or greater difficulty level to a greater extent than their peers at Delavrancea Elementary. In the case of the classes where teachers used the same curriculum, the difference in test scores could be explained through the teacher's choice of more complex and diverse problems in the extra-curriculum materials and homework assignments. Consistency seemed to be the key, namely teachers maintaining higher requirements for students both through classroom practice, homework assignments and test items. Moreover, if teachers (i.e. Ms. Ionescu) possessed a strong knowledge of mathematics, they were able to help their students make connections between the important concepts in the curriculum (Manoucheri & Goodman, 2000), which seemed to ultimately benefit student learning.

For example, Ms. Ionescu's use of both higher order problems (15 %) and lower order problems (85 %) in class, additionally the opportunities she created for her students to explore these notions at home (40 % higher order problems and 60 % lower order problems), as well as her use of both higher order and lower order test items may explain her students' better test scores. On the other hand, Ms. Ali's classroom and homework exposure to mainly lower order exercises, and the use of only lower order problems in the final test justifies her students weaker test scores and hence a weaker understanding of base 10 concepts.

Similar findings are reveled at the second school, where the veteran teacher only reinforced lower order concepts in class and at home, depriving her students of more in-depth learning opportunities. Although 65 % of Ms. Popescu's students scored A in the final test (like Ms. Ionescu's students), Ms. Popescu's students were only tested on lower order thinking concepts and should have scored better than Ms. Ionescu's students who were tested on both complex and easy concepts.

The last teacher in this study, Ms. Reiz, also created no classroom opportunities of her students to be involved in higher order exercises, yet she created some opportunities for students to be engaged at home in both higher order and lower order exercises. However, not having the support provided by classroom instruction and implicitly the teacher, the students did not benefit from homework as much as did Ms. Ionescu's students, as they were outperformed by all other students in the final test.

Consequently, the textbook does make a difference in the student learning only if the teacher transforms the textbook to address better their students' needs. The textbooks that contained not more, but more complex problems seemed to influence teacher's practice and produced better student results, as in the case of Ms. Ionescu. Going beyond curriculum seemed also to be related to better student understanding of base 10 concepts. Ms. Ionescu is a successful example of a teacher whose strong mathematics knowledge base enabled her to adapt the curriculum to her students' needs, becoming a "co-constructor of knowledge and creator of curriculum" (Cochran-Smith & Lytle, 1999), exceeding the textbook expectations when necessary and providing her students with the best learning opportunities.

Curriculum alone, be it centralized (China), decentralized (United States), or semi-centralized (Romania) more cohesive (China, Romania) or less cohesive (United States), may not solve a nation's problems related to achievement in mathematics. A mere curricular structure may not provide a sufficient explanation for the better performance for Asian students (TIMSS, 1999), as Romanian textbooks were even more cohesive than both Chinese and United States textbooks, with less content breaks but covered the same amount of topics as Chinese books did. Consequently, although being worth looking into curriculum and analyzing the types of learning opportunities they provide, curriculum cannot provide in-depth explanations regarding the performance gap of students from different countries (as in the case of international comparisons) and students from Romania.

Teachers' Classroom Practice

In the best learning environment, teachers' classroom practice is informed by the teachers' knowledge of subject matter, students and curriculum. As such, when teachers based their instructional decisions, activities, and assessing techniques on students' individual rhythm of development, to what extent to rely on the classroom curriculum and whether or not to exceed it, as well as how well they understood the concepts they were about to teach, they were more likely to create a rich learning environment for their students by exposing them to concepts differing in degree of complexity, diverse teaching approaches (whole class, small group as well as individual instruction), which as shown by the above data work together to influence student learning.

Teachers who relied on a variety of assessment tools to check student understanding of the topics, had students who understood better base 10 numbers, as evidenced by Ms. Ionescu's students. The teacher designed multiple activities that enabled her to assess student understanding and, in turn better understand how well her students mastered the base 10 numbers, as shown by her classroom observations. As such, the independent sheets students had to fill out allowed students to work at their own pace and created feedback for the lesson, helping the teacher to spend more with the students who needed her help and designing better activities that addressed the needs of each individual student. The teacher also played games with her students, she had them recognize the numbers written on cards and raise the card with the appropriate number, played a logical game with group diagrams, used charts with groups on which students could group the elements by tens, assessed student understanding thought the homework opportunities she created as well as in a final examination that tested students on the concepts reinforced in class and at home.

Ms. Ali, on the other hand assessed student understanding using less varied tools, such as worksheets students had to fill out in class, as well as asking students to go to the board to solve problems individually, usually taking turns. Ms. Ali's students also formed numbers using sticks and slide rules, and composed and decomposed numbers, but were only involved in easier problems than were Ms. Ionescu's students. The less complex activities and highly uniform structure of class seemed to keep all the students at the same level, which may have contributed to their fragmented understanding of base 10 numbers, as shown by the test results. On the other hand, Ms. Ali's statement in the final interview strikes in that while acknowledging that she kept the standards lower, she stated that she did provide her students with challenge, considering the lesson requirement complex. Despite her belief that the lesson was quite complex, when compared to the lesson planned and implemented by her colleague at the same school, Ms. Ali's lesson lacked in complexity, challenge, diversity of resources used and activities implemented, as well as a high order thinking homework and class-work requirement.

Ms. Popescu, the veteran teacher at Delavrancea Elementary had also designed and planned multiple activities to assess student understanding. At a first glance though, all these activities offered a minimum of challenge and kept the students in their comfort zone, performing simple computations in which students used sticks, shapes, the axis with numbers to count and show the place of tens and units. Student assessment was both formal (through worksheets and the final test) and informal (asking students to come to the board and write numbers after dictation). The end result were students who understood base 10 numbers, but who performed at the same level with Ms. Ionescu's students, who were assessed on more difficult concepts.

Ms. Reiz, the less experienced teacher at the same school stated in the pre-interview that her students would learn through answering her questions, coming to the board and learning by discovery. While the teacher did provide ample opportunities for her students to work individually at the board and on workbooks in class, and asked them questions to check their understanding, as shown by her classroom observations, her students were not exposed to any opportunities that would enable them to learn by doing and discovering facts, as the learning environment she created in her class was entirely traditional. Students counted tens and units using different class objects, their sticks and abacus, they used groups of elements for numbers 10-100 as these tools were fun to use and helped them better understand base 10 numbers. Apparently, Ms. Reiz did not provide her students with enough challenge, only exposing them to learning by assimilating facts and thus depriving them of richer learning opportunities.

Of an equal significance in student learning, asides for using particular assessment tools seemed to be the time spent reinforcing the concepts students struggled with in the test. As such, Ms. Ionescu's students seemed to face more difficulties with tens and units in the final evaluation test. A look at the degree of reinforcement offered by the teacher reveals the fact that the teacher provided good reinforcement for these concepts, as 50 % of the total number of exercises solved in class related to concepts involving tens and units. Ms. Ionescu also introduced these concepts to her students via extra-curriculum activities, which again reinforces the idea that overall students tend to benefit from being exposed to more complex notions and explanations of more complex notions (Perry, 2000).

On the other hand, Ms. Ali's students seemed to struggle more with counting numbers forwards and backwards in the final test, and a look at the support provided by the teacher in class relating to these concepts is mere average, as only 25 % of the exercises dealt with concepts involving counting numbers. At the other school, Ms. Popescu's students also faced problems with concepts involving tens and units in the final test, yet, unlike Ms. Ionescu, Ms. Popescu' s reinforcement of these concepts in class was below average, as only 25 % of the problems solved involved tens and units. Likewise, Ms. Reiz's students faced problems with comparing numbers in the final test, and this can be explained by the fact that while reinforcing these concepts in class, only 15 % of the problems solved dealt with comparison issues.

An interesting finding is provided by the comparison between the teachers' perspectives of the concepts students would struggle with and the concepts students ended up having problems with, as well as the look into the type of reinforcement teacher provided students with for the concepts expected to create difficulties for them. As such, although Ms. Ionescu's students had problems with concepts involving tens and units in the final test, the teacher initially believed her students would have problems with counting backwards higher numbers, as they had not yet mastered reading these numbers. However, both the home and the class reinforcement the teacher created in order to provide her students with the needed support to overcome these problems, were merely average, exposing students to counting concepts in proportion of 6 %. The discrepancy between the concepts the teacher thought her students would struggle with and the concepts students struggled with did not affect student knowledge, as the teacher provided more support with the concepts students had problems than to the concepts she initially thought students would have problems with. This may be due to the teacher's stronger knowledge of students and ability to learn from her teaching, as she may have seen her students struggle with different concepts that the ones she anticipated they would struggle with and provided more support where needed.

Ms. Ali's students, although facing problems with counting numbers backwards and forwards in the final test, were expected to struggle with tens and units, as stated by the teacher in the initial interview. In order to prevent misunderstandings of these concepts, the teacher provided an average classroom reinforcement of these concepts, exposing students to over 25 % of problems with tens and units notions. The homework reinforcement of these concepts was very weak, as students did not have to solve problems with tens and units. In this case, just like in the case of Ms. Ionescu, there was a discrepancy between what the teacher initially thought her students would struggle with and the real problems students had in the final test. However, in this case this discrepancy seemed to be detrimental to student learning, as the teacher reinforced less the concepts her students struggled with and more those concepts her students mastered easily.

Ms. Popescu was the only teacher who was right in her estimate of what her students would struggle with, as her initial belief stated in the pre-interview matched the real problems students had, namely understanding tens and units. However, the type of reinforcement provided both in class and at home was below average, as only 15 % of the total number of exercises dealt with tens and units concepts. In this case, although there was no discrepancy between the teacher's initial belief and problems her students had in the final test, the average support and challenge provided by classroom activities seemed not to be enough to help students gain a deeper understanding of place value concepts.

Finally, Ms. Reiz' s students struggled with comparison concepts in the final test, although the teacher believed they would face problems learning about tens and units, and provided a good classroom and homework reinforcement of these concepts, as 50 % of the exercises solved dealt with tens and units. In this case, like in Ms. Ali's case, the discrepancy between the teacher's initial belief and the real problems students had in the final test did not help students gain a deeper understanding of base10 concepts.

Consequently, it is important for teachers to know their students well and know what problems they might face, but ultimately it is more important to act upon this knowledge and provide students with the challenge and support needed to overcome these struggles. Otherwise, we have teachers (Ms. Popescu) who know what to expect of their students but who do not create the best leaning opportunities for them, having students performing below their ability level, or we have teachers (Ms. Ali, Ms. Reiz) who do not know what to expect of their students and who stress out other concepts than the ones their students struggled with.

Besides creating somewhat richer learning opportunities for their students, some teachers had a different approach on teaching base 10 concepts, and this different approach also seemed to make an impact on student understanding of place value concepts. For example, teachers' use of more hands-on activities to reinforce base 10 numbers seemed to enhance student learning of place value concepts. In other words, teachers who were more open to learning through discovery developed more complex assessment techniques, exposing students to a variety of resources and manipulatives meant to help them better understand place value concepts. Some teachers were also more open than others to provide opportunities for student-student interactions. In this study, student-student interactions refer to those occasions in which students worked together to solve problems assigned by teachers. The difference in student performance seemed then to be given by those learning opportunities that were more present in some classes and seemed to lack in others.

Ms. Ionescu, for example, was more bent towards active learning, as she challenged all her students and constantly engaged them in learning through discovery, having them solve problems using figuring out the rules by themselves, while in small groups. Students were engaged in numerous student-student interactions (at least five per observation on average), and engaged in small groups 23 % of the instructional time (average time per observation). As a facilitator, the teacher helped students make meaning from learning, monitored group/pair work and provided support and explanations when necessary, asked students for multiple solutions for the same problems and guided learning by discovery. Overall Ms. Ionescu's students seemed to have benefited from her choice of instructional strategies, as reflected by the high percentage of students who scored A in the final tests and interview questions.

Did the novice teacher at the same school engage her students in active learning? As her classroom observations revealed, Ms. Ali created some opportunities for student-student interactions. Small group opportunities were also present in this teacher's class, although more limited than in Ms. Ionescu's class, although the teacher stated that her students seemed to benefit a lot from being exposed to group work. Her assessment tools were consistent with her teaching approach, as Ms. Ali mostly focused on oral assessment (through observing her students at the board and group work), without providing opportunities for differentiate instruction. More of a traditionalist teacher, Ms. Ali drilled her students more in front of the classroom and limited student-student interaction although she had previously stated that this kind of interaction seemed to benefit her weaker students.

The situation seemed to be similar at the other school, where the veteran teacher, more than the novice teacher created some opportunities for her students to learn through discovery through games, by asking a lot of questions and encouraging her students to help one another. In small groups, for example, students had to find the neighbors to certain given numbers and count backwards and forwards by 2 and 5. However, the level of teacher-student and student-student interactions and the activities Ms. Popescu designed were of a lower thinking order than those opportunities created by Ms. Ionescu, whose students seemed to have a more in-depth understanding of both complex and easier concepts. Moreover, the teacher created fewer opportunities for student-student interactions (one interaction on average per observation). Consequently, not only allowing students the chance to be engaged in discovery learning, but also exposing them to the more varied and complex learning opportunities enhanced student learning, as evidenced by student test results and interview questions.

On the other hand Ms. Reiz, the less experienced teacher at the same school, did not provide any type of hands-on activities, games or group/pair work that seemed to benefit the other teachers' students. In her role as supreme authority, this teacher would walk among her students checking their answers and naming students to answer and go to the board, and when they made a mistake, she would assign someone else to answer, not allowing the student to figure out where he/she went wrong by himself/herself.

Overall, teacher as facilitator seemed to have a better impact on student learning than teacher as supreme authority. The more time spent asking for multiple solutions to the problems, raising questions, providing explanations, helping students make meaning from learning the better the student results. Teachers who facilitated instruction more had students who scored generally higher than their peers who were constantly given directions and drilled. The three teachers who engaged their students in active learning more, Ms. Ionescu, Ms. Ali and Ms. Popescu had at least 50 % of the students scoring A in the final test and at least 65 % correct answers in the interview questions. On the other hand, Ms. Reiz's students, who did not benefit of the same learning opportunities as their peers, scored lower on both final test and interviews (only 30 % of the students scored A in the final test and only 55 % scored correct in the post interviews).

All the above data undoubtedly show the impact teachers' use of diverse resources and manipulatives, as well as instructional practices had on student understanding of base 10 numbers. Yet, however reasonable the explanations regarding the impact of instructional practice, instructional practice alone is not enough to explain the gap in the performance of these students as it is not enough to explain the gap in performance of Romanian students in the international comparisons. Despite similar instructional practice and similar school organization to China (TIMSS, 1999), Romanian students were outperformed by Chinese and American students. This international study reports that even if Chinese and Romanian students were exposed to more individual practice in class if they had difficulties with concepts, opportunities which were considerably higher than the ones United States students had, the difference in performance between Chinese and Romanian students and United States and Romanian students was obvious, the conclusion being that only instructional practice alone may not provide an in-depth understanding of why some students perform better than others, even when exposed to the same learning opportunities.

In the same vein, analyzing the results of the students in this study, a look into instructional practice alone (what learning opportunities are provided by the teacher in class) cannot alone explain why some students outperform others who are exposed to similar classroom learning opportunities. A deeper look into what interactions exist between parents and teachers and parents and their children at home, as well as how curricular decisions and assessment techniques teacher use, may provide better explanations for the performance gap of Romanian first-graders. This shall be the topic of the following chapter on parents.

CHAPTER 6

FINDINGS OF THE STUDY

Parents

In order to account for the impact both schooling and non-schooling factors have on student learning, and most importantly, for the ways these factors interact with one another and impact student learning, the researcher looked into teacher-student-parent interactions and analyzed classroom instruction as well as home instruction. This chapter aims to uncover home practices and their impact on children understanding of place value concepts. Moreover, parent-teacher interaction is described, and researcher analyzed whether or not this interaction had any influence over the way students apprehended place value concepts.

Families of sixty-four students participated in this study. They were all sent questionnaires and thirty-eight families returned the questionnaires and agreed to be further interviewed by the researcher. Eighteen parents in the veteran teacher's class and five parents in the novice teacher's class returned the questionnaires at Iorga Elementary, while eleven parents at the veteran teacher's class and four parents in the less experienced teacher's class returned questionnaires at Delavrancea Elementary. Nine families were randomly selected from these participants and they were further interviewed by the researcher. As previously stated in Chapter 3, data show that 43 % of parents in the better performing school, Iorga Elementary, had a higher degree of education, while only 5 %

of the parents in the average performing school had a higher education degree. Parental degree of education seemed to impact student learning, in terms of learning opportunities these parents created at home for their children. Not only did these parents list their favorite subject as mathematics, but parents who were more comfortable with mathematics in general and base 10 numbers in particular seemed to get more involved in their children's learning, and they had a stronger knowledge base of mathematics.

For example, more parents in the better performing classrooms at both schools had a clearer understanding of what place value concepts were and what was their significance for student learning than parents in the average performing classrooms at both schools. As such, parents in Ms. Ionescu and Ms. Popescu's classes could define base 10 numbers as "all positive and whole numbers, higher than 0," "all numbers comprised between 0-100," "the basis of mathematics," while all parents in the other two classrooms stated they did not know what these numbers represented. Moreover, similar responses were given to the question regarding the significance of learning these concepts, as more parents in the better performing classes than parents in average performing classes stated in the interviews they believed these numbers represented "the basis of mathematics…when they go past 10 they start to learn mathematics," they further believed it was important to learn these numbers as students "deal with these numbers all their life, using them with other subjects as well," and because these numbers will provide them with "a complete image of what follows, number order in general, negative numbers."

On the other hand, parents in the novice and less experienced teachers' classrooms did not seem to grasp the significance of learning these numbers, as they had a harder time describing how students would benefit from learning numbers. Most of the responses given by parents were very general, "they must learn them to do well in life," "it will be good for them later," "we must really learn them because without counting them we can't really get by." It seems therefore that parental limited understanding of place value concepts impacted their children's understanding of the place value concepts, as generally more students in both veteran teachers' classrooms whose parents had a stronger knowledge base performed better than did their peers whose parents had a more fragmented understanding of the base 10 numbers.

On the other hand, parents who had a stronger knowledge base of place value concepts also tended to feel more comfortable with helping their children before and after class. Survey findings show that while 40 % of the parents in Ms. Ionescu's class were very comfortable with mathematics and 60 % of the parents returning the questionnaires stated they were comfortable with base 10 numbers, they helped their children understand these concepts both before and after the teacher taught them, as the three parents interviewed stated they helped their children at home. For example, these parents stated that in order to make sure their children understood these concepts before class, they played games that involved counting (Monopoly), practiced all the time and gave children a lot of examples with numbers. After the teacher addressed the concepts in class, the same three parents stated they checked their children's homework to have an understanding of what happened in class, and devised their own worksheets and used different resources to consolidate the concepts taught in class. This more formal interaction, as defined by Huntsinger (1993), in which parents spent a longer amount of time with their children, devising worksheets and drilling their children more, resulted in a better student understanding of base 10 numbers in the current study, which is relevant in students' responses in the pre- and post-interviews, as only some students made mistakes when asked by the researcher to solve exercises involving base 10 numbers. Homework support was also present, a look at student homework found 75 % of the homework requirements correct. Parental support is also evidenced in the final test scores, as 65 % of the students in Ms. Ionescu's class scored A, and only 15 % scored D or below. That is to show that when present, parental support highly impacted student learning.

Parents in the novice teacher's class at the same school stated in the questionnaires they were as comfortable with mathematics in general as parents in Ms. Ionescu's class, but overall less comfortable with base 10 concepts, and this lack of comfort is also evidenced in the support they provided both before and after teaching, as gathered from the parent interviews. While no parents explained the place value notions to their children before teaching, all of them stated they got involved after the teacher approached these concepts in class, mostly following the teacher's model, using money as well as fingers to count numbers and reinforce classroom concepts. This more antiquated type of support was reflected in the way students learned numbers, as the students in Ms. Ali's class made more mistakes overall in both pre and post interviews as well as in the final test than did Ms. Ionescu's students. Moreover, even if their homework was generally correct, the notions that were reinforced at home were of the same lower difficulty level as the notions teacher addressed in class, not providing the students with challenge and keeping even the brighter students at the same level. In this case, too, parental support and the quality of support that was provided seemed to impact student learning of place value (Huntsinger, 1993). The lower the standards at home, and the more fragmented understanding parents had of place value concepts, the weaker students' understanding of the same concepts.

On the other hand, six parents in Ms. Popescu's class stated in the survey they were comfortable with both mathematics in general and with base 10 numbers in particular, and they tried to help their children both before and after the teacher addressed the concepts in class. The researcher interviewed two of the parents of these students and found out that while one mostly followed the teacher and textbook's model, using sticks, fingers, the other one developed her own examples with numbers. However, students in this class were not exposed to the same learning opportunities as students in Ms. Ionescu's class, as the standards were generally lower. The students did reach these standards, but not exceed them, as 65 % of students in Ms. Popescu's class scored A in the final test, just like in the case of Ms. Ionescu, even when these students were tested on concepts that had a different degree of complexity. On the other hand, if the same standards were maintained at home, student learning may have been inhibited and students missed being exposed to more complex notions as their peers in Ms. Ionescu's class were.

The parents in the less experienced teacher's classroom at the same school, were the least comfortable with both mathematics in general and place value concepts in particular, as only one of the surveyed parents stated she was very comfortable helping her child understand these notions. This lack of comfort is largely shown in their degree of involvement with homework and helping their children learn place value concepts before and after school, as neither one of the two interviewed parents stated they were engaged in helping out their children. This lack of support is also evidenced in the way their children answered in pre and post interview questions (most of the students made mistakes), and the final test grades, as only 30 % of the students scored A. On the other hand, the teacher did provide limited opportunities for students to be exposed to higher order thinking concepts, but did not seem to provide the students with the support needed to understand these concepts. Being that parents did not feel as confident about helping their children with these concepts at home, the students in Ms. Reiz's class were then at a loss and only developed a fragmented understanding of base 10 concepts.

Overall, in answering the third research question I raised in chapter 3, namely, how do home interactions influence students' understanding of place value, parents who possessed a stronger knowledge base of mathematics were more comfortable with their ability to help their children master more complex notions, and were more engaged at home with their children, creating their own teaching resources. Conversely, parents who lacked the confidence they could provide their children with the support needed tended to get less involved at home before the teacher addressed the concepts in class for fear they would make mistakes, and only followed the teacher's pattern when helping at home. The way these different types of home interactions impacted student learning is obvious in the way students performed in the final test and interview questions. In those rare cases where parents went beyond the teachers' explanations and designed their own instructional activities, student understanding seemed to be better. For example, parents in Ms. Ionescu's class stated they made their own rubrics and worksheets and played games involving counting with their children, providing their children with the knowledge base needed to understand better the concepts taught in class. These parents also seemed to be confident in the degree of support provided as they stated they believed they did a good job helping their children master place value concepts before and after class. On the other hand, the parents in the other three classrooms mostly followed the teachers' and the textbook's model and were less confident of the level of support provided, as they stated they did "as much as they could" or "not a good job" in explaining the concepts. Consequently, the parents who got involved more at home in their children's learning, going beyond the teacher's explanations and model, tended to provide their children with the support needed to understand base 10 numbers.

When asked about their involvement in their children learning at home and at school, all parents who were interviewed stated they had not made any suggestions regarding the base 10 numbers lesson, as they felt comfortable with the way the teacher addressed the concepts in class. All parents also stated that the only involvement in their children learning took place at home, and was conducted with the purpose "to strengthen and diversify the concepts taught in class." More parents in the better performing class, than in all other classes, stated that the teacher urged them to get involved in their children learning at home, provided support with the homework, assigned general rules and told parents to push their children as much as they could. This impulse from the teacher might have helped better motivating the parents of the students in her class to get involved and help with homework, however this support was not enough to enhance student learning, as 35 % of Ms. Ionescu's students scored B or below in the final test.

Consequently, opportunities were not always created by the teachers to involve parents in classroom instructions, leaving the parents with a limited understanding of what goes on in class in terms of teaching and learning mathematics. However, even in the cases where teachers (Ms. Ionescu) were open to have parents in her class and invited them to participate in math classes, due to time constraints and other personal reasons no parents took advantage of the teacher's offer. This mentality is representative to the general parental population in Romania, who tend to respect teacher's authority in the classroom and do not believe that their presence in the classrooms would benefit their children. In all of the above cases, the parent-teacher interactions were not as strong as to benefit student learning, even in the best-case scenario, Ms. Ionescu's class.

This reluctance to get involved in children learning past support with homework, may be due, in general to parental beliefs that they could not impact curriculum and school instruction in any way, as all of the parents who were interviewed stated they had no impact on schooling factors. Even in the case of the parents with a strong knowledge base of mathematics and who felt comfortable with their ability to help their children master place value concepts, they too stated they did not want to get involved as it was not their profession and they knew they could not impact curriculum or teaching, although one of the teachers stated parents were welcome to her class to help her better address the concepts, in an effort to make learning more meaningful for their children. This seems to be a major gap in the Romanian educational system, as parents tended to see their role limited to home instruction and even if provided with opportunities to be part of their children learning at school, they still did not believe they could impact schooling factors. This trend seems to remind one of the beliefs of Asian parents in Chen

& Stevenson's (1995) study according to whom school, more than home training is believed to be responsible for the better results of the Asian students. However, in this study, a better home-school interaction might provide all parties involved in the learning process, students, teachers and parents with better learning opportunities, as parental interaction with both teachers and students seemed to highly impact student learning, as evidenced by the above data.

Parental involvement in children's education, should, however, not be limited to home interactions, as this is not enough to help their children. Without understanding how the teacher approached the concepts in class, parents mostly relied on their own instructional methods, which were not appropriate in most of the cases and deprived their children of richer learning opportunities. Note, for example, how parents in both the novice teacher and the less experienced teacher's classroom stated they helped their children learn how to count by using their fingers, as opposed to parents in both veteran teachers' classrooms who used games (Monopoly) or created their own assessment rubrics to help their children learn how to count. In an attempt to answer the fourth and fifth research questions, namely what types of interactions exist between teachers and parents, respectively what types of interactions exist between teachers, parents and students that influence student understanding of place value, researcher asked teachers to describe the opportunities they had to meet with parents to discuss student learning. In other words, were teachers educating the parents in an effort to increase their understanding of the topics and enable them to help their children at home to better understand place value concepts? In other words, were all factors involved in the learning process working together in an effort to increase student learning of place value concepts (Marturana & Varela, 1984; Senge, 1990; Waldrop, 1992)?

This study shows that in general, teachers who created opportunities to meet with parents and discuss the concepts being taught had parents who better understood the learning process and who, in turn, could impact their children's learning at home. As stated by the teacher in her post-interview, Ms. Ionescu's formal meetings with parents took place once a month (for Parent-Teacher Conferences), but more opportunities for informal meetings existed, as the teacher invited parents to call her at all times with questions and she saw some weekly or daily for pick up/drop off. In order to help parents understand how their children learned, during PT conferences the teacher usually discussed with parents the concepts that were being taught and how she would approach these concepts in class, "so that I can help them understand these notions and explain them correctly to their children." Ms. Ionescu stated that it was very important she informed parents about their children's progress/lack of progress, as she believed lack of progress to be dangerous for student learning:

> At our last PT conference I handed them the final test, we looked through them together to see if there are any problems with all the children, I gave them the tests at home to analyze them with their children and tell me if the mistakes they made were real or rather they were due to causes like lack of attention, maybe they were disturbed by other kids, etc.

Ms. Ionescu's concern that parents understand how their children learn and how they perform in class was obvious in her above statement, and her openness to inviting parents to get involved in their children's learning was meant to benefit all parties involved in learning: students, teacher and parents.

Ms. Ali also stated in her post-interview that she created opportunities to meet with parents both formally (monthly at PT conferences) and informally (for pick up/drop off, phone calls at home). Ms. Ali's interaction with parents was though different in nature from Ms. Ionescu's as Ms. Ali stated that she did her best to inform parents about their children's learning yet her students' parents had a limited understanding of the concepts being taught and could unwillingly harm student learning more than benefit it:

> I meet with them quite often, but I cannot ask for more. Even if I insist a lot, this is all they can do, a lot of them have regrets about not taking school more seriously. They come to school and ask me things, and I have to explain them what to do, every day I have to tell them this is what we need to do in this workbook.

At a fist glance, the discrepancy in the quality of teacher-parent interactions in the two teachers' cases is obvious, and if it were to judge from their statements parental intervention at home and interaction with teacher were vital for student learning. Moreover, the nature of parental interaction with the teacher and home support seemed to make a big difference in student learning, as both teachers created opportunities for parents to meet with them and ask questions, but the parents in Ms. Ali's class seemed to have a weaker understanding of the concepts and were not very able to guide their children at home and provide them with the needed support.

Ms. Popescu, the veteran teacher at Delavrancea Elementary also stated in her post-interview that she met with her students' parents regularly for PT conferences (monthly) and informally every week with the ones who came to school for pick up, drop off or to meet with the teacher. The teacher stated that parents were very helpful in the beginning an provided both her and their children with a lot of support at home, as some of the students in Ms. Popescu's class did not go to K and were a little behind the rest of the students:

> We had PT every other week, and every morning parents who came to school had my support and I had their support, because I explained to them for every lesson, this is what they need to know today for math class, and parents helped me. Now I ask for help quite rarely.

The teacher deemed essential to always informing parents about their children's progress, as she kept a file on each student with tests and classroom observations, and parents checked the folder and were being kept up to date about progress/lack of progress. Ms. Popescu is therefore another successful example of a teacher who worked well with parents, communicating with them orally or via student work files, encouraging them to get involved more in their children's learning at home.

What is the situation in the less experienced teacher's class, did she create enough learning opportunities for parents and, equally significant, were parents as supportive as parents in Ms. Popescu's class? According to the less experienced teacher, she met with parents monthly for PT conferences and also daily with some of them, and she took every opportunity she got to explain the concepts she was teaching and the way their children learned: "I explain to them which ones are the units, which are the tens, and they understand easier than the kids do anyway. I ask them to work at home with these notions." Opportunities are then created in this case, too, but at a different level than those created by both Ms. Ionescu and Ms. Popescu, due to the parents' fragmented understanding of place value concepts. Noteworthy is the fact that both Ms. Ionescu and Ms. Popescu's parents were very confident they helped their children understand the concepts very well ("I am sure I did," "As much as I could") as opposed to parents in Ms. Reiz and Ms. Ali's class ("Well enough," "I don't know"), as stated by parents in the interviews.

The above data suggest that parents are a vital link in student learning, and the more involved parents got in their children education both at home and at school and the more knowledgeable they were, the better student learning, as only teacher support in class was not enough to enhance student learning (see the case of some of Ms. Ionescu's students, who scored average in the test possibly due to less richer learning opportunities provided by their parents). Consequently, looking back to the complexity theory terminology (Maturana & Varela, 1984) in Chapter 3, we have the following relationships between the agents of an autopoetic system: component A, through its interaction with component B, triggers an interaction of B with C that triggers a reduction in the production of D, where A (teachers), B (parents), C (students) are interacting through relations of contiguity, and these multiple interactions impact D (learning mathematics). In other words, the system is changed by the interactions of all its agents (A,B,C) that work together rather than in isolation.

In this study, teachers' interactions with students at school had a positive impact on student learning, stronger in the case of the teacher who had a better knowledge of base 10 concepts and overall planned and delivered a more challenging lesson that ultimately seemed to benefit her students. Parental interactions with their children at home seemed also to enhance student learning in these cases where parents had a stronger understanding of base 10 concepts and helped their children beyond homework. Moreover, the nature of parent-teacher interactions had a positive impact on student learning if both sides meet to not only discuss student progress, but also to gain an understanding of student learning and support one another in enhancing student learning at school and at home.

CHAPTER 7

CONCLUSIONS AND IMPLICATIONS FOR RESEARCH

Conclusions

This study was started with the assumption that student learning is generally influenced by a variety of factors, both of a schooling and non-schooling nature. As discussed by researchers in the fields of comparative education and teacher education literature, student mathematics achievement is believed to be impacted by teacher subject matter and pedagogical content knowledge (Ball, 2003; Cochran-Smith & Lytle, 1999; Feinman-Nemser & Remillard,1996; Schulman, 1986) that eventually transposes in their use of curriculum and instructional practices (Cohen, 1991; Manoucheri & Goodman , 2000; Rodriguez, 2000; Schorr & Koellner-Clark's , 2003; Senger, 1999). Moreover, student learning seemed to be influenced by the quality and quantity of parental interactions with their children at home (Chen & Stevenson, 1995; Hess et al., 1987; Huntsinger et al., 1993; TIMSS 1999).

In order to understand better a nation's success and failure in mathematics, researchers, policy makers and practitioners in United States guided their attention to comparative international studies, in a hope to learn what practices work in other countries and how United States students perform in comparison with students from other countries. Comparative studies were deemed as essential (Howson, 1999;

Romberg, 1999) as they would inform policy makers of the changes that need to occur in United States in order to improve mathematics teaching and learning. Extensive comparative international studies were conducted in the fields of mathematics and science, involving over forty countries from all over the world and testing students of different age levels. Studies like PISA (2000) or TIMSS (1999) revealed acute performance gaps in student mathematics learning; top ranking students were from the Asian countries while United States students' performance was described as merely average (Stevenson & Stiegler, 1992).

More comparative studies were conducted and the implications of these studies were that due to differences in the educational systems of countries from Asia and United States, Asian students were exposed to richer learning opportunities both in class and at home. The direct result of these different opportunities were that Asian students had an advantage over the other countries participating in the PISA (2000) and TIMSS (1999) studies. Factors assumed to impact student learning were analyzed, and as a direct result curricular and instructional reform started to occur in United States. However, some researchers (Wang & Lin, 2005) urged that mathematics teaching and curricula can be culturally scripted and what may work in a country may not work in another country. In other words, before making drastic changes in a country's curriculum and instructional strategies, implementing new practices into the United States educational system, more research needs to be conducted and comparisons made with other countries that may provide a more in-depth understanding of student performance in mathematics. Another country present in both PISA (2000) and TIMSS (1999) was Romania, which, due to its similarities and differences with Asian countries and United States may suggest a more in-depth understanding for the better performance of Asian students and the average performance of United States students in international mathematics comparisons. In this respect, Romanian and Chinese educational systems are quite similar in terms of teacher knowledge, curriculum, curriculum implementation and instructional strategies (National Center for Educational Statistics, 1994, 1999a; 1999b; Perry, 2000; PISA 2000; Schmidt, McKnight, Houang, Wang, Wiley, Cogan, & Wolfe, 2001), yet Romanian students were outperformed by both Asian and American students, challenging thus previous findings according to which Asian students tended to perform better because they either had a stronger curriculum or because the Asian teachers knew more and had a stronger pedagogical content knowledge than did United States teachers.

The current study was then born in an effort to offer a better understanding of why some students perform better than others if they are provided with similar learning opportunities at school or at home, while analyzing the factors believed to influence student learning (both schooling and non-schooling) working together rather than in isolation and describing the impact the interrelatedness of these factors may have on the way students understand mathematics in general and place value concepts in particular.

The results of this study have contributed to the larger fields of teacher education and comparative and international education, as they uncover the subject matter knowledge teachers possess and the ways in which this knowledge impacts the use of curriculum and practice in class. Moreover, this study speaks about those practices common at home and their impact on student learning, as well as on the interaction that takes place between parents and teachers, and teachers and students.

Overall Findings

The three data chapters discussed above attempt to shed more light into the factors that influence student learning by describing the types of interactions that occur both at school and at home between teachers, parents and students. One of the most significant factors impacting student mathematics learning was the teacher. In the best classroom environments, teachers' strong knowledge of mathematics and students informed better their lesson design and use of curriculum, enabling them to create rich learning opportunities for their students, which were meant to enhance learning. Of an equal importance to student learning was parental involvement, as parents who possessed a stronger understanding of base 10 concepts got involved more in their children's learning at home beyond homework and created more meaningful learning opportunities for their children. A more detailed interaction of these factors follows.

Teachers who possessed a more in-depth understanding of place value concepts were generally better at planning the lesson while taking into consideration what they learned from their experience of teaching a similar topic, and teaching the lesson in a way that addressed the needs of their current students, building on what students already knew and increasing the difficulty level. On the other hand, the more complex objectives teachers set for their class, as well as the higher standards teachers held for teaching and learning place value concepts had a positive impact on student learning and understanding of place value concepts, as the students who were engaged in both higher order and lower order thinking problems had a more in-depth understanding of place value concepts.

These teachers were also more likely to design and implement the best learning activities for their students, using different resources to prepare the lesson and allowing opportunities for students to work with a multitude of manipulatives, be engaged in whole class, individualized and small group instructions from which students seemed to benefit, as stated by one of the teachers in the final interview. That teachers' knowledge of subject matter and students impacted teachers' instructional approaches is evidenced in students' test results, as teachers who were more bent toward constructivism and engaged students in learning by doing and discovery, as well as by exposing students to both higher and lower order interaction in class (questions or requirements coming from the teacher) had students who generally performed better in the final test and seemed to possess a stronger understanding of numbers.

Equally significant in the process of teaching and learning about place value concepts were the ways teachers' knowledge of subject matter and students influenced their use of curriculum, as the more in-depth subject matter knowledge and the better the teachers knew their students the better they were able to modify the curriculum and add new materials to teaching to benefit their students. One interesting fact that is worth mentioning here is that even in the case of the teachers who used the same curriculum (Ms. Ionescu and Ms. Ali), their students scored differently in the final test and seemed to have different levels of understanding of the topic, which may lead to the conclusion that curriculum created certain learning opportunities and that if teachers supplemented the textbook with any additional materials they found beneficial, student learning was increased. As such, those teachers who used curriculum as a starting point, and exceeded its requirements, had students who possessed a deeper understanding of base 10 numbers.

Moreover, teachers who used textbooks richer in both higher and lower order thinking problems, and not only more problems but more diversified, and who created opportunities at home for students to be engaged in similar exercises had students who mastered more complex place value concepts.

To further address the research question addressing the impact classroom interactions might have on student learning, if teachers possessed a stronger mathematics knowledge base and prepared thoroughly for the lessons, if they used their past experience with teaching that lesson to inform the design of the activities for the new lesson, if they knew their students' level of understanding and addressed their students' individual needs, they seemed to have a positive impact on the way their students learned about base 10 numbers. If, moreover, these teachers' subject matter knowledge and their student knowledge informed their curriculum use and, if in turn the use of curriculum impacted the design of their activities, this enabled the teachers to create not only teacher-student but also student-student interactions from which students seemed to benefit overall. Of significant importance appeared to be not only the diversity of classroom interactions students were exposed to (be it with peers or with the teacher), but also the nature of these interactions, as the more students had to work with peers and benefited from feedback from the teacher (i.e. not only telling students they were wrong but also helping them discover why they were wrong and find solutions to the problems), the better they seemed to master place value concepts.

While these data suggest a reasonable explanation towards why some Romanian students perform better than others, generalizations cannot be made to all Romanian teachers. On the other hand, looking into the larger paramount umbrella of international comparison, can the same data explain why Romanian students in general do not perform at the same level with Chinese and United States students? More explicitly, if Romanian students were exposed to similar instructional practices and curriculum demands as Chinese and United States students, and if Romanian teachers' subject matter knowledge was as good as that of Chinese and United States teachers, what factors intervene in the learning process that inhibit student learning?

A cross-analysis of subject matter knowledge in the three countries frequently mentioned in this study, China, United States and Romania reveals similarities in what teachers, researchers and policy makers consider good subject matter and average subject matter. Teacher knowledge is assumed to be essential for student learning (Ball & Bass, 2001) and its absence further impacts student learning (Cochran-Smith & Lytle, 1999; Feinman-Nemser & Remillard, 1996; Schulman, 1986), it thus seemed important for this study to have a definition of what good subject matter knowledge involves. In China, for example, Ma (1999) described good subject matter knowledge as profound understanding of fundamental mathematics, enabling teachers who possessed PUFM to transpose this understanding in their classroom practice and having thus students who possessed both a conceptual and a procedural understanding of the mathematics topics.

In the United States, researchers described teachers who had good subject matter knowledge as those teachers who were exposed to learning the "big" ideas in their college mathematics classes (Ball, 2003; Graeber, 1999; Greenes, 1995) and who, in turn, transposed this knowledge into their practice and possessed both a conceptual and procedural understanding of the topics. In turn, their students were geared towards possessing a conceptual and procedural understanding of the topics. On the other hand, average subject matter knowledge of teachers in United States was related to memorization, drills and teaching students that mathematics constructions were step-by-step procedures to be memorized, which might ultimately lead students to gain a procedural understanding of the topics, but not a conceptual understanding of the topics (Schonfeld, 1988).

If the distinction between good and average subject matter knowledge is dictated by teachers possessing "knowledge packages" (Ma, 1999) and teaching for understanding rather than for the mastery of the "physical skill" (Schonfeld, 1988), which is namely the difference between possessing conceptual vs. procedural understanding of the concepts, than the case in Romania should be similar. Findings of the present study show that if teachers drilled more and focused on transmitting ideas in a step-by-step manner, those teachers (Ms. Ali, Ms. Reiz) had mostly a procedural understanding of the topics and conveyed a similar understanding to their students. If, on the contrary teachers possessed a strong knowledge base of place value concepts and their students and constantly tried to teach these concepts in a manner in which all their individual students would benefit, these teachers were more likely to have students who understood not only how to count and manipulate numbers, but why learning about these numbers were important, as in the case of Ms. Ionescu's students. Teacher subject matter knowledge seems thus vital for student learning, as evidenced by the above findings of international and Romanian studies. Consequently, if teacher subject matter knowledge is strong, student knowledge is strong. However, the mathematics performance gap in international studies as well as the present Romanian study cannot solely be explained by differences in teachers' subject matter knowledge. Other factors need also be examined in order to provide a more in-depth understanding of why some Romanian students outperform other Romanian students in the present study and why both Chinese and United States students outperform Romanian students in international studies.

Therefore, a look at what researchers and policy makers consider to be good curriculum may further provide explanations regarding the better performance of students. In the case of China, Li, 2000 and Valverde et al., (2002) analyzed the curriculum of the countries taking part in the TIMSS (1999) study, and concluded that in the case of Taiwan, their curriculum was considered more cohesive than the United States curriculum due to the less content strands (the number of times themes changed) and fewer number of topics covered. This was associated with better student performance for Chinese students. However, the Romanian curriculum was as cohesive as the Chinese curriculum, yet student results were lower than the results of the Chinese students.

Following these results, in an attempt to improve United States education, the curricular reform that took place in United States aimed at changing the more traditionalist textbooks abundant in information (Greenes, 19995; TIMSS, 1999) and adopting more standards-oriented textbooks. Case studies (Manoucheri & Goodman, 2000; Rodriguez, 2000) related the implementation of standards-based textbook to better student learning. In this light, covering fewer but more in-depth topics seemed to be correlated to a stronger and more cohesive curriculum. Hence, Romanian textbooks that contained fewer topics but covered them more in-depth would produce better student results. As is the case in the present study, the teacher whose textbook contained fewer topics but were richer in degree of complexity had students who possessed a deeper understanding of these concepts, as is the case of Ms. Ionescu. Although her colleague at the same school used the same textbook, her student results were lower than Ms. Ionecu's, fact that may be explained by her reliance mostly on the classroom curriculum and lack of exposure of her students to higher order thinking problems.

Consequently if a standards-base cohesive curriculum is associated with better student learning in international comparisons, its presence and better use in the Romanian classes can also be associated with better student performance, as in the case of Ms. Ionescu. Similarly, a textbook more abundant in information and the teachers' attempt to cover all the topics in the textbook is linked to a poorer student performance, as in the cases of Ms. Ali, Ms. Popescu and Ms. Reiz. If teachers' subject matter knowledge impacts the way they make use of curriculum, as previously stated, and if curriculum further impact instructional learning, it is important to look into what is considered good teaching and how good teaching impacts student learning.

In this vein, good teaching in Taiwanese classrooms was associated with exposing students to higher order thinking explanations (Perry, 2000), as well as giving students more practice in classrooms by themselves (TIMSS, 1999). In an effort to improve United States students' mathematics learning, reform was also geared towards instructional practice, and results in the field showed that teachers who were provided with support from colleagues and researchers, and who worked closely with them tended to produce better student results by approaching topics both conceptually and procedurally (Senger, 1999; Schorr & Koelner-Clark, 2003). As such, not only teaching concepts but teaching for understanding and engaging students in higher-order interactions seemed to produce better students results in both Taiwan and United States. Similarly, Romanian students in the present study who were exposed to higher order explanations, engaged in higher order interaction in class both between themselves and with the teacher, as well as were allowed opportunities to learn by discovery, possessed a more in-depth understanding of place value concepts than their peers who were engaged in lower order interactions and were mostly drilled.

Consequently, good teaching seems to mean the same thing across the three countries, and its presence seemed to have a strong impact on student learning. Moreover, if Romanian students were exposed to equally valuable learning opportunities, the teachers possessed a strong knowledge base that was transposed into teaching and made good use of the curriculum, in a word, if they have the same advantages their peers from China/Taiwan and United States had, what factors prevented them from performing well in international comparisons? The answer to this question may lay in the investigation of another similarly important factor impacting student understanding, namely parental influence, as the way parents interact with their students was shown to be beneficial towards their children learning (Huntsinger et al, 1993). Parents could therefore be the missing link in providing Romanian students with a better understanding of the concepts, as home reinforcement of the concepts is equally important to classroom exposure (Hess, 1987).

A look at parental involvement in their children's education at home and at school may shed more light onto the understanding of student performance gap: if parents possessed a strong mathematics knowledge base and were highly involved in their children's learning at home, designing their own activities and going beyond teachers' explanations and curriculum, providing their children with additional challenge, they seemed to have a positive impact on the way their children learned about base 10 numbers. If, in turn, parental understanding of place value concepts was fragmented, they lacked the ability to provide their children with richer learning opportunities at home to supplement and build on to what the teacher taught in class. Exposed to these different learning opportunities, some children tended to outperform their peers and possess a better understanding of place value concepts. Parental influence, seems thus to be another determinant factor in student learning, building onto teacher content knowledge and classroom instructions. Like in the case of teachers, these data suggest a reasonable explanation towards why some Romanian students perform better than others. However, when revisiting the larger paramount umbrella of international comparisons, can the same data explain why Romanian students in general do not perform at the same level with Chinese and United States students? More explicitly, if Romanian students were exposed to similar parental instructional practices to Chinese students, why don't they perform at the level of what the Chinese students?

A cross-analysis of parental interventions in the three countries referenced in this study, China, United States and Romania reveals similarities in what researchers describe as good parental support. Considered yet another factor contributing to the better performance of Chinese students, more formal methods of parental practices were linked to a better student learning in the case of Chinese American students, as shown by Huntsinger et al. (1993), who investigated parental practices in 40 Asian-American and 40 Caucasian-American homes. Findings revealed that Chinese American students outperformed their Caucasian peers in mathematics, and this fact is largely due to those practices they were exposed to: longer duration of interactions between parents and students, paying more attention to the written representation of a problem, expecting children to spend greater amounts of time in studying mathematics, using memorization, drills, and worksheets. These approaches seemed to also benefit the Caucasian students in the study, who scored higher than their peers who were exposed to less formal approaches.

On the other hand, a less rigorous parental support at home, as described by Chao (1994), and different parental approaches (permissive and authoritative, praising students when they get good grades and allowing them to make decisions), is associated with lower scores for United States students, in comparison with their Chinese peers whose parents' approach towards home instruction was more authoritarian (taking more control over their children's education). Consequently, the more parents were involved at home, designing worksheets and going beyond classroom instruction, the better student learning. This seems to also the situation in Romanian households described in the current study, as the more parents were engaged in helping their students both before and after teaching and designed their own activities, the more in-depth understanding of base 10 numbers their children seemed to have. On the other hand, the fewer challenging opportunities created by parents, in addition to parents' less rigorous understanding of place value concepts, the poorer student performance.

This could thus explain why Romanian students in the international comparisons were outperformed by their Chinese and American peers: if poor home support was added to a fragmented teacher knowledge and an equally average knowledge of students, the Romanian students were deprived of the richer learning opportunities their peers had through both classroom and home instruction. If, on the other hand, some Romanian students were exposed to more challenging learning contexts in the classroom, and if concepts were also reinforced in similar manners at home, as is the case of Ms. Ionescu's students, they tended to outperform their peers who were either exposed to more challenges in class but not at home, or who were not challenged enough both at school and at home.

The above findings suggest a close connection between a series of factors assumed to impact student learning, as both parents and teachers create opportunities for student learning. These opportunities depend upon individual teacher and parental knowledge of subject matter, knowledge of students/children as, through their nature, they may enhance or inhibit student learning (see quantity vs quality, as referenced throughout this study). Moreover, the type of interactions that take place between teachers and parents may also impact student learning, as shown by the above findings. Unfortunately, this study could not present a good example of close teacher-parent interactions, as no parents were part of the classroom instruction, the interaction being limited to what parents could do at home to help their children.

In those cases where parent-teacher interaction existed (Ms. Ionescu) and when parents met with teachers to discuss not only their children's progress/lack of progress, but also the way the teacher was going to approach the concepts in class, overall student learning increased. Through these types of interactions the teacher helped parents understand better not only the place value concepts but also how their children learn and what home learning opportunities would benefit them the most. On the other hand, if parent-teacher interactions were limited to only discussing children's progress (Ms. Reiz, Ms. Ali), parents who had a more fragmented understanding of the place value concepts

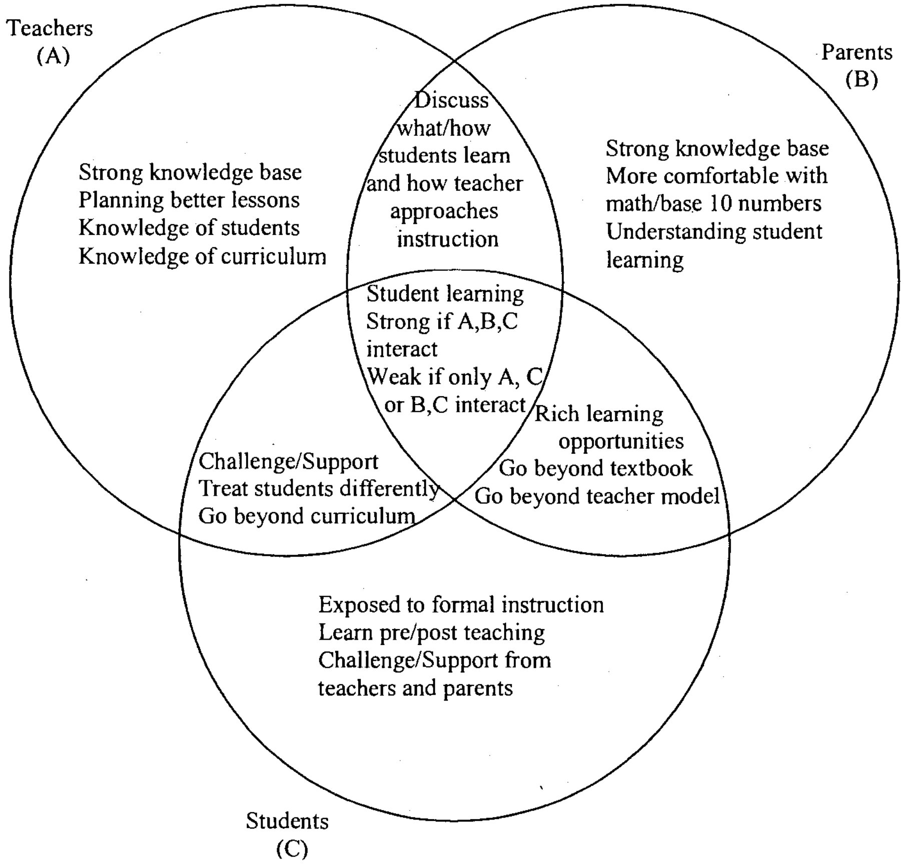

Figure 2. The Impact of Interactive Factors on Student Learning

were not able to provide their children with the same rich learning opportunities, inhibiting student learning.

Implications for the Future

Research

I started this study with the hope that through its nature and purpose, it would provide a clearer explanation of why some students outperform their peers in the international comparisons, by looking at the different learning environments they grow up and are being educated in. The research questions raised in the methodology section of this study may help provide a better understanding of the Romanian educational system, as the final assumption is that some Romanian students outperformed their peers in both interview questions and the final tests. As such, by being exposed to a better curriculum and an equally good use of curriculum, a stronger teacher subject matter and student knowledge, better classroom practices, a better communication between teachers and parents, as well as a stronger parental involvement in their education, some students (see Ms. Ionescu's students) developed a more in-depth understanding of place value concepts and performed better than their peers when tested on these concepts.

While these data seem to provide a reasonable explanation for the performance gap between the four classrooms analyzed in this study, are they enough to answer the questions raised in the second chapter, namely in the larger context of teacher education and comparative education? Namely, can these data provide plausible explanations regarding the performance gap of students in international comparisons? Moreover, can these data pinpoint the factors deemed as responsible for student learning?

A quick look back at both teacher education and comparative education revealed the following factors that had an impact on student learning and lead to the better performance of Asian students in the international comparisons: firstly, Chinese students perform better because of a more solid teacher subject matter knowledge (Ma, 1999); secondly, Chinese students perform better due to a more cohesive curriculum (Li, 2000; Valverde et al., 2002). Next, these students also tended to outperform all their peers due to better instructional strategies (Perry, 2000), as well as a better structure of the school year (TIMSS, 1999). Parental influences were also considered, and the better performance of Chinese students in international comparisons was also explained by the impact setting higher standards as well as developing more formal teaching approaches at home (Hess, 1987; Huntsinger et al., 1993).

Due to the theoretical lenses of this study, namely complexity, and the interrelatedness of multiple factors (Maturana & Varela, 1984; Waldrop, 1992), the assumption emerging from all the above research study is that Chinese students tend to perform better in international comparisons due to all these factors working together, since, according to Waldrop (1992) disregarding the fact that these factors interact may only provide a limited understanding of the system. Further on, the average and below average performance of United States and, respectively Romanian students in the international comparisons may be due to poorer interactions between both schooling and non-schooling factors. These are, however, mere assumptions, as no comparative study has been conducted to date to analyze the interactions between these factors in different educational systems.

In the years to come, I can see myself involved in this type of comparative education research, analyzing the interactions between factors held responsible for student learning in the contexts of Chinese and United States education and comparing these results to the Romanian context. And maybe later on, expend my area of interest to other countries as well.

Policy Makers

What relevance might this study have on policy makers? At a first glance, this study reinforces the idea that curriculum and instruction are indeed culturally scripted (Wang & Lin, 2005), that what may work on a large scale in an educational environment

(China), may not be applicable to another educational context (United States). On a smaller scale, as is the Romanian educational system, findings indicate that even when using the same textbook, teachers' use of curriculum may vary and impact student learning on different degrees. Hence, by changing a country's curriculum (see the need for curricular reform in United States as discussed in the beginning of Chapter Two) without providing the teachers with the needed support to understand the significance of the new curriculum and the impact it might have on student learning is not likely to improve student performance. Curricular and instructional practice reform may not be enough to produce better teachers unless "teachers learn mathematics in ways that make a difference for the skill with which they are able to do their work," (Ball, 2003, p.1).

Moreover, policy makers should be aware that not only one, but more factors are responsible for student learning, and by only looking at schooling factors without considering the home support provided by parents at home and acting on these considerations (implementing a new curriculum, pushing teachers to change their practice) may not necessarily lead to a better student understanding.

Practitioners

Being a teacher and now on the verge of becoming a teacher educator, my concern is not only to improve student learning, but also to improve teacher understanding of the factors that lead to success/lack of success of students. As such, I am hoping that this study might raise teachers' awareness of the impact their knowledge upon their use of curriculum and classroom practice. Secondly, this study should also make teachers think more about the impact the opportunities they create for students not only in class, but also at home might have on student learning.

Moreover, teachers should become more aware about the quality of their interaction with parents on student learning, namely that if the only interaction teachers and parents have is to discuss student progress, this type of interaction is unlikely to help parents provide their children with the support needed at home. If, on the contrary, teachers explain to parents how they would address mathematics concepts in class and create opportunities for parents to attend mathematics classes and experience teaching and learning mathematics first hand, in an effort to understand better how students learn and provide better home support, this type of interaction is more likely to help parents develop a better understanding of the mathematics concepts and be better able at helping their children reinforce these concepts at home. Last but not least, this study aims at helping teachers develop a better understanding of the interaction of all these factors and their impact on student learning.

If the ultimate goal of education is to enhance student learning and improve student performance, as teachers and teacher educators we are responsible to provide our students with the best learning opportunities, to offer an equal amount of challenge and support both in school and at home, to reinforce the significance of parental engagement in children education both in school and at home, and mostly, raise the awareness of teachers and parents along that it is the collaborative work of these factors that enhances student learning. And maybe then our students (American and Romanian) will outperform students from other countries in international comparisons of mathematics and science.

APPENDIX A

STUDENT PRE-INTERVIEW QUESTIONS

1. How many sets of tens and how many sets of ones are on the following worksheet?

2. How can we read those sets (i.e. 2 tens 4 ones)? SEE WORKSHEET 1

3. Can you guess the number? Ask students to read the numbers on the worksheet 2. The ask them: Why do you think that? Can you explain?

4. What does "29" mean to you? How can you show "29" using these blocks (Base 10 blocks)?

APPENDIX A

STUDENT PRE-INTERVIEW QUESTIONS WORKSHEET 1

Place value chart: How many sets of tens and how many sets of ones are there? How else can you read these sets?

Number	**Tens**	**Ones**
23	**2**	**3**
17		
24		
19		
15		
20		
13		

APPENDIX A

STUDENT PRE-INTERVIEW QUESTIONS WORKSHEET 2

Place value chart: What numbers are constituted from the sets of tens and ones below?

Tens	**Ones**	**Numbers**
●	● ● ●	**13**
● ●	●	?
● ● ● ●	● ● ● ● ● ● ● ●	?
●	● ● ● ● ●	?
● ● ●	● ●	?
●	● ● ● ●	?

APPENDIX A

STUDENT POST-INTERVIEW QUESTIONS

1. What did you learn in class today?/What did your teacher teach you in class today?

2. How many sets of tens and how many sets of ones are on the following worksheet? How can we read those sets (i.e. 2 tens 4 ones)? SEE WORKSHEET 1

3. Can you guess the number? Ask students to read the numbers on the worksheet 2. The ask them: Why do you think that? Can you explain?

4. What does "29" mean to you? How can you represent "29" using these blocks (Base 10 blocks)?

5. Why do you think it is important to know these concepts?

6. Who in your family usually helps you with your homework?

7. Who in your family usually comes to school to talk to your teacher?

8. Is mathematics easy to learn?

9. Is mathematics interesting?

10. Is mathematics useful in life outside school?

11. If you have a difficulty in understanding what is being taught during a mathematics lesson, do you usually a) ask a teacher questions during class; b) read textbook or notes; c) ask classmate/friend; d) ask teacher after class; e) keep quiet.

APPENDIX B

TEACHER SURVEY

1. What is the highest educational degree held? Please check all that apply:

a. high school b. institute c. college d. other (please specify)

2. What was your favorite subject in school?

3. How long have you been an elementary school teacher?

4. What content area do you feel most comfortable teaching?/ What is your favorite subject to teach?

5. How comfortable do you feel about teaching mathematics compared with teaching other content areas?

6. How long have you been teaching in your present position?

7. How many hours a week do you teach mathematics?

8. If you could go back in time and start over again, would you become a teacher or not? Why/why not?

9. How would you like to learn to teach mathematics?

10. How long do you plan to remain in teaching?

APPENDIX B

TEACHER INTERVIEW QUESTIONS (PRE-TEACHING)

1. Could you briefly define place value/ What is place value?

2. Did you teach this lesson before and how many times did you teach it?

3. What resources did you use for teaching this lesson before?

4. What did you learn from your past teaching experiences about the content of this lesson that helped you prepare the present lesson?

5. What are the common misunderstandings that your students used to have about place value? How did you learn about these? How are you going to cope with such situations?

6. What are your objectives for this lesson on place value? How did you come up with these objectives/goals? Why do you think these objectives are necessary?

7. Could you briefly describe how you are going to teach this lesson on place value? What examples are you going to use to teach your students and why?

8. What materials, including textbook, did you use to plan this lesson?

9. How much time did you spend preparing for the lesson you are going to teach today?

10. What will your students be doing during this lesson? Why?

11. Did you discuss the lesson with anyone in the school and what did you talk about?

12. Why do you think it is important for the students to develop an understanding of the place value concept?/ or to learn these concepts?

13. What would you say students need to be able to understand or be able to do before they could start learning about place value/number naming systems and why?

14. What do you anticipate will be the most difficult concepts your students will struggle with and why do you think this will be the case?

15. How will you approach these difficult concepts? Why?

16. How will you be assessing your students' understanding of place value and why do you think these assessments are useful with this particular lesson?

17. Research (Sovchik, 1989) advocates the importance of using a several kinds of concrete materials while teaching the Base 10 system to the students. What concrete materials do you mostly use to teach these concepts and why?

Scenarios:

1. Some first grade teachers noticed that several of their students were making the same mistakes in the following place-value task: children were asked to count 26 candies and then to place them into cups of 4 candies each, with two candies remaining. When the "2" of the "26" was circled and the children were asked to show it with candies, the children typically pointed to the 2 candies. When the "6" of "26" was circled and asked to be pointed out with candies, the children typically pointed to the 6 cups of candy. What would you do if you were teaching these concepts and noticed some of your students were doing this?

2. Teachers seem to have different approaches to teaching concepts involving place value. To make this meaningful for kids, something that many teaches try to do is relate mathematics to other things. Sometimes they try to come out with real-life situations or story problems to

show the application of some particular piece of concept. What would you say would be a good story or model for teaching place value?

APPENNDIX B

TEACHER INTERVIEW QUESTIONS (POST-TEACHING)

About the lesson

1. Can you tell me three important things that you learned about teaching this lesson on place value and how did you learn these?

2. What major problems, if any, did you face while teaching this lesson?

3. How did your students count or estimate quantities? Did they spontaneously use sets of tens? What is your evidence for that?

4. How flexible were children with their thinking about numbers? Could they take them apart and combine them in ways that reflect an understanding of ones and tens? What is your evidence for that?

5. What materials did you use to represent one and tens in your classroom instruction?

6. When materials were already arranged in groups of tens, did students use these structures to tell how many? What is your evidence for that?

7. How does understanding of place value help students develop skills in reading and writing numbers?

8. How did you help your students discover the relationship between tens and ones? What is your evidence for that?

9. To what extent do you think your students have reached the goals and objectives that you set up for this lesson?

10. Can you describe one of you best students and his or her learning in this lesson and explain why you think his or her performance matched or exceeded your expectation for this lesson?

11. Can you describe one of you average students and his or her learning in this lesson and explain why you think his or her performance matched or not your expectation for this lesson?

12. Can you describe one of you below-average students and his or her learning in this lesson and explain why you think his or her performance did not match your expectation for this lesson?

13. What did you think about the lesson procedures that you developed in this lesson? To what extent did you think the major procedures that you used in your teaching were useful for your student learning in this lesson?

14. If you are going to teach this lesson again, are you going to use the same examples that you used in this lesson and why and why not?

15. If you are going to teach this lesson again, are you going to use the same assessment to assess your student learning in the lesson and why and why not?

Teacher-Parent Relationship

16. Do you help parents understand the ways students learn? How did you help parents learn about place value concepts and why?

17. Do you usually seek the view of parents and take account of their suggestions and concerns? What about place value concept?

18. Do you communicate to parents the expectations that they talk with their children about their schoolwork? How did you communicate to the parents the expectation they should be involved in enhancing their children's understanding of place value concepts?

19. Do you encourage parents to help their children establish daily routines of activities (time for mathematics homework)? How do you do this?

20. How often do you visit with parents to discuss their children's progress (weekly, monthly, once a semester)? Did you inform parents about their children's progress on place value understanding? How?

APPENDIX C

PARENT QUESTIONNAIRES

Demographic Information

1.How many children do you have? What grades are they in?

2. How many people live in your household?

3. What is your educational background? What is your highest and/or most recent academic degree?

4. What was your favorite subject in school?

5. What is your current job?

6. Would you describe your (family) income as high, average or low?

7. What is your ethnicity?

8. What is your relationship with the student (mother, father, etc.)?

Involvement in Children's Education

1. Who is helping your children at home with homework?/Specify which subject you are helping your children with at home.

2. Do you have any difficulty in helping your child with the mathematics lessons? What about the lesson on place value?

3. How comfortable do you feel helping your child with mathematics lesson compared to the other subject areas? Please select one:

4 Very comfortable 3 Comfortable 2 Little comfortable 1 Uncomfortable

4. How comfortable do you feel about your own understanding of place value concepts?

Please select one:

4 Very comfortable 3 Comfortable 2 Little comfortable 1 Uncomfortable

5. How well do you believe you can help your child learn?

Home-School Relationship

1. How often do you visit school/keep in touch with the teacher? Please select one:

4 Very often 3 Often 2 Sometimes 1 Rarely

2. How often do you discuss the concepts taught in the mathematics classes with the teacher?

Please select one:

4 Very often 3 Often 2 Sometimes 1 Rarely

3. How often do you get involved in activities occurring in mathematics classes? Please select one:

4 Very often 3 Often 2 Sometimes 1 Rarely

4. What influence, if any, do you feel you might have on any of the schooling factors that impact your child's learning in school (i.e. curriculum, teaching methods)? Please explain.

APPENDIX C

PARENT INTERVIEWS

Understanding of Place Value

1. Could you briefly define place value/ What is place value?

2. Why do you think it is important for the students to develop an understanding of the place value concept?/ or to learn these concepts?

3. What do you think are the most common misconceptions students might have learning about place value and why do you think this might be the case?

Help with Children

4. Your child learned place value concepts in school the other day. What would you say he/she needs to be able to understand or be able to do before learning about place value/number naming systems? What did you do to help him/her learn this concept before the lesson and why?

5. Have you talked about his or her learning of this concept after the lesson and why? Do you know how the teacher taught this concept to your child?

6. What would you say your child struggled the most when learning these concepts and why do you think this is the case?

7. Did you re-explain place value concepts to your child at home? How did you re-explain the place value concept to your child and why?

8. When explaining this topic to your child, what instructional materials did you use and why?

9. How well do you believe you can help your child learn place value?

School-Home Relationship

10. Are you kept well informed about your child's progress in school? What about the place value concept?

11. Do you feel comfortable about approaching the school with questions or a problem or complaint? Have you contacted the school/teacher regarding the learning of place value concepts? Why?

12. Does the school seek the view of parents and takes into account their suggestions and concerns? What about place value concept?

13. Do you believe your child and school want your involvement? What about teaching and learning about place value?

14. How often do you go to school for meetings and activities? Did you go to school for this concept learning and why?

15. Do teachers encourage parents to help their children establish daily routines of activities? Have you involved in any classroom activities related to this concept?

16. Do teachers expect parents to talk with their children about their schoolwork? How did teacher convey her expectations that you discuss place value with your child?

17. What influence, if any, do you feel you might have on any of the schooling factors that impact your child's learning in school (i.e. curriculum, teaching methods)? Please explain.

Views about Schooling

18. Does the teacher generally provide appropriate homework? What about the place value concept?

19. Is students' work closely reviewed by teachers? How did the teacher review your child's understanding of place value concepts? Do you think this method was effective? Why?

20. Does the teacher assist the student when it is needed? What kinds of assistance did the teacher offer for your child in learning about place value?

21. Do you think teachers help students see that learning is their most important goal in school? How might have the lesson on place value reflected that?

REFERENCES

Arthur, W.B. (1989). *Emergent Structures: A Newsletter of the Economic Research Program*. Santa Fe: The Santa Fe Institute. Detailed accounts of the various projects undertaken during the program's first 18 months.

Arthur, W.B. (1990). Positive feedbacks in the economy. *Scientific American*, 92-99.

Axelrod, R. (1984). *The Evolution of Cooperation*. NY: Basic Books.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, 90(4), 449-466.

Ball, D.L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 93(4), 373-397.

Ball, D. L., & Feinman-Nemser, S. (1988). Using textbooks and teachers' guides: A dilemma for beginning teachers and teacher educators. *Curriculum Inquiry*, 18, 401-423.

Ball, D. L. & McDiarmid, G. W. (1990). The subject matter preparation of teachers. In W.R. Houston, M. Haberman, & J. Sikula (Eds.). *Handbook of Research on Teacher Education*, (pp. 437-439). New York: Macmillan.

Ball, D.L., & Bass, H. (2001). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple* perspectives on mathematics teaching and learning (pp.83-104). Wesport, CT: Ablex Publishing.

Ball, D.L. (2003). *What mathematical knowledge is necessary for teaching mathematics*. Secretary's Summit on Mathematics: U.S. Department of Education. February 6, 2003.

Baumrind, D. (1971). Current patterns of parental authority. *Developmental Psychology monograph*, 4, 1-103.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T. (1996). Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study. *Introduction*. Retrieved May 17, 2006 from http://isc.bc.edu/timss5i/MathB/html.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T. (1996). Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study. *Chapter four: Students Backgrounds and Attitudes about Mathematics*. Retrieved February 04, 2006 from http://isc.bc.edu/timss5i/MathB/html.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T. (1996). Mathematics Achievement in the Middle School Years: IEA's Third International Mathematics and Science Study. *Chapter five: Teachers and Instruction*. Retrieved February 04, 2006 from http://isc.bc.edu/timss5i/MathB/html.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T. (1999). School Contexts for Learning and Instruction. *Chapter four: Organization for Learning Mathematics and Science*. Retrieved May 17, 2006 from http://isc.bc.edu/timss5i/schrpt.html.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T.

(1999). School Contexts for Learning and Instruction. *Chapter six: School Atmosphere.* Retrieved May 17, 2006 from http://isc.bc.edu/timss5i/schrpt.html.

Beaton, A.E., Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., Kelly, D.L., & Smith, T. (1999). School Contexts for Learning and Instruction. *Appendix A.* Retrieved May 17, 2006 from http://isc.bc.edu/timss5i/schrpt.html.

Block, J.H. (1981). *The Child Rearing Practices Report (CRPR): A set of Q items for the description of parental socialization attitudes and values.* Berkley: University of California, Institute of Human Development.

Brenner, M.E., Herman, S., Ho, H.Z., & Zimmer, J.M. (1999). Cross-national comparison of representational competence. *Journal for Research in Mathematics Education,* 30(5), 541-557.

Brooks, J., & Brooks, M. (1993). *In Search of Understanding: The Case for Constructivist Classrooms,* Association of Supervision and Curriculum Development, Alexandria, Virginia.

Brown, C., Stein, M.K., & Foreman, E.A. (1996). Assisting teachers and students to reform the mathematics classroom. *Educational Studies in Mathematics*, 31, 63-93.

Cai, J. (2000). Mathematical thinking involved in U.S. and Chinese students' solving of process-constrained and process-open problems. *Mathematical Thinking and Learning*, 2(4), 309-340.

Carnine, D. (1991). Reforming mathematics instruction: the role of curriculum materials. *Journal of Behavioral Education,*1(1), 37-57.

Carpenter, T.P., Linquist, M.M., Matthews, W., & Silver, E.A. (1983). Results of the third NAEP mathematics assessment: Secondary School. *Mathematics Teacher,* 79(9), 652-659.

Carpenter, T.P., & Moses, J. (1983). The acquisition of addition and subtraction concepts. In R. Lesh and M. Landau (Eds.), *The Acquisition of Mathematics Concepts and Processes*, New York: Academic Press, pp.7-44.

Chao, R.K. (1994). Beyond parental control and authoritarian parenting style: understanding Chinese parenting through the cultural notion of training. *Child Development*, 65, 1111-1119.

Chen, C., & Stevenson, H.W. (1995). Motivation and mathematics achievement: A comparative study of Asian-American, Caucasian-American and East Asian high school students. *Child Development*, 66(4), 1215-1234.

Cochran-Smith, M., & Lytle, S. (1999). Relationship of knowledge and practice: teacher learning in communities. *Review of Research in Education*, 24, 249-298.

Cohen, D.K. (1991). Revolution in one classroom (or, then again, was it?). *American Educator: The Professional Journal of the American Federation of Teachers*, 15(2), 16-23 & 44-48.

Cohen, D. K., & Spilane, J.P. (1992). Policy and practice: The relations between governance and instruction. *Review of Research in Education*, 24, 249-298.

Conant, J. (1963). *The education of American teachers*. New York: McGraw-Hill.

Dornbush, S. M., Ritter, S.L., Leiderman, P.H., Roberts, D.F., & Fraleigh, M.J. (1987). The relation of parenting style to adolescent school performance. *Child Development*, 58, 1244-1257.

Epstein, J.L. (1990). School and family connections: theory, research and implications for integrating sociologies of education and family. In D.G. Unber & M.B. Sussman (Eds.). *Families in Community Settings: Interdisciplinary Perspectives*. New York: Haworth Press.

Everyday Mathematics (2004). The University of Chicago School Mathematics Project. Teacher's Lesson Guide1. The McGraw Hill Companies.

Feinman-Nemser, S., & Buchman, M. (1985). Pitfalls of experience in teacher preparation. *Teachers College Record*, 87, 53-65.

Feinman-Nemser, S., & Remillard, J. (1996). Perspectives on learning to teach. In Murray, E. (Ed.). *The Teacher Education Handbook (*pp.63-91). San Francisco: Jossey-Bass.

Fisher, R.A. (1930). *The Genetical Theory of Natural Selection*. Oxford: Clarendon Press.

Flanders, J. (1994). Student opportunities in Grade 8 mathematics: textbook coverage of the SIMS test. In I. Westbury, C. A. Ethnington, L. A. Sosniak & D.P. Baker (Eds.), *In search of more effective mathematics education: examining data from the IEA Second International Mathematics Study* (pp.61-93). Norwood, NJ.: Ablex.

Freeland, J.H. (1979). *The Eight Day of Creation*. NY: Simon and Schuster.

Fucson, K.C.(1991). Children's early counting: Saying the number-word sequence, counting objects, and understanding cardinality. In (Ed.). Dunkin K. & Shire B. *Language in Mathematical Education: Research and Practice*. Philadelphia, Open University Press.

Fucson, K.C., & Kwon, YT. (1991). Chinese-based regular and European irregular systems of number words: the disadvantages for English-speaking children. In (Ed.). Dunkin K. & Shire B. *Language in Mathematical Education: Research and Practice*. Philadelphia, Open University Press.

Glesne, C. (1999). *Becoming Qualitative Researchers. An Introduction*. Longman.

Goldberg, D. E. (1989). *Genetic Algorithms in Search, Optimization, and Machine* Learning. Reading, MA: Addison-Wesley.

Good, T., & Grouws, D. (1987). Researchers Advising Policy Makers: A word to the Wise. *Educational Psychologist*, 23(2), 282-293.

Good, T., & Grouws, D., & Ebmeier, H. (1983). *Active Mathematics Teaching*. New York: Longman.

Graeber, A. O. (1999). Forms of knowing mathematics: what preservice teachers should learn. *Educational Studies in Mathematics*, 38, 189-208.

Greenes, C. (1995). Mathematics learning and knowing: a cognitive process. *Journal of Education*, 177(1), 85-106.

Greenes, C., & Fitzgerald, W. (1991). Mathematical performance of non-math majors at the college level. In R. Underhill (Ed.), *Proceedings: Thirteenth Annual Meeting, Psychology Of Mathematics Education* (pp.98-104). Blackburg, VA: Virginia Tech.

Hess, R.D., Chih-Mei, C., & McDevitt, T. M. (1987). Cultural variations in family beliefs about children's performance in mathematics: comparison among People's Republic of China, Chinese-American, and Caucasian-American families. *Journal of Educational Psychology*, 79(2), 179-188.

Hill, H.C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406.

Hill, H., Sleep, L., Lewis, J., Ball, D. (2007). Assessing teachers' mathematical knowledge: What knowledge matters and what evidence counts? In F. Lester Jr. (Ed), Second handbook of research on mathematics teaching and learning (p. 111-155). Charlotte, N.C.: Information Age Publishing.

Ho, C.S., & Cheng, F.S. (1997). Training in place-value concepts improves children's additional skills. *Chinese University of Hong Kong*, 22, 495-506.

Holland, J.H. (1975). *Adaptation in Natural and Artificial Systems*. Ann Arbor: University of Michigan Press.

Holland, J.H., Holyoak, K, J., Nisbett, R.E., & Thagard, P.R. (1986). Induction: Processes of Inference, Learning, and Discovery. Cambridge, MA: MIT Press.

Howson, G. (1999). The value of comparative studies. In G. Kaiser, E. Luna, & L. Huntley (Eds.), *International comparison in mathematics education* (pp. 165-188). Philadelphia, PA: Falmer Press.

Huntsinger, C.S., Jose, P.E., Larson, S.L., Krieg, D.B. & Shaligram, C. (2000). Mathematics, vocabulary and reading development in Chinese American and European American children over the primary school years. *Journal of Educational Psychology*, 92(4), 745-760.

Inspectoratul Scolar Judetean Constanta. (2005). (Constanta School District). *Starea Învãtãmântului 2004-2005*. (The Situation of Romanian Education 2004-2005). Retrieved April 05, 2006, from http://www.isjcta.ro/.

Investigations in Number, Date and Space (2004). Building Number Sense. Grade 1. Scott Foresman.

Jose, P.E., Huntsinger, C., Huntsinger, P., & Liaw, F. R. (2000). Parental values and practices relevant to young children's social development in Taiwan and the United States. *Journal of Cross-Cultural Psychology*, 31(6), 677-702.

Kauffman, S.A. (1991). Antichaos and adaptation. *Scientific American*, 78-84.

Kauffman, S.A. (1992). *Origins of Order: Self-Organization and Selection in Evolution*. Oxford: Oxford University Press.

Kennedy, M. (1991). *An agenda for research on teacher learning* (NCRTL special report). East Lansing, MI: National Center for Research on Teacher Learning, Michigan State University.

Lambert, M. (1988a). What can research on teacher education tell us about improving he quality of mathematics education? *Teaching and Teacher Education*, 4, 157-170.

Lawrenz, F. (2003). Relationship between professional development, teachers' instructional practices, and the achievement of students in science and mathematics. *School Science and Mathematics*, 103(8), 378-387.

Li, Y. (2000). A comparison of problems that follow selected content presentations in American and Chinese mathematics textbooks. *Journal for research in Mathematics Education*, 31(2), 234-241.

Lin, C. Y.C, & Fu, V.R. (1990). A comparison of child-rearing practices among Chinese, Immigrant Chinese and Caucasian-American parents. *Child Development*, 61, 429-433.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, New Jersey: Lawrence Erlbaum associates, Publishers.

Manoucheri, A., & Goodman, T. (2000). Implementing mathematics reform: the challenge within. *Educational Studies in Mathematics*, 42, 1-34.

Math Advantage (1988). Grades 1-8. Teachers' Edition, 1. Hartcourt Brace.

Maturana, H., & Varela, F. (1984). *Autopoesis and Cognition*. D. Reidel Publishing Company.

Merriam, S.B. (1998). *Qualitative Research and Case Study Applications in Education*. San Francisco: Jossey-Bass Publishers.

Mestre, J. P., & Lochhead, J. (1983). The variable-reversal error among five culture groups. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp.181-188), Montreal, Canada.

Miller, K.F., Smith, C.M., Zhu, J., & Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: the role of number-naming systems. *Psychological Science*, 6 (1), 56-60.

Miura, I.T. (1987). Mathematics achievement as a function of language. *Journal of Educational Psychology*, 79 (1), 79-82.

Miura, I.T., Kim, C.C., Chang, C.M., Okamoto, Y. (1988). Effects of language characteristics on children's cognitive representation of number: cross-national comparisons. *Child Development*, 59, 1445-1450.

Miura, I.T., & Okamoto, Y. (1989). Comparisons of US and Japanese first-graders' cognitive representation of number and understanding of place value. *Journal of Educational Psychology*, 81 (1), 109-113.

Miura, I.T., Okamoto, Y., Kim, C.C., Chang C.M., Steere, M., Fayol, M. (1994).

Comparisons of children's cognitive representations of number: China, France, Japan, Korea, Sweden and the United States. *International Journal of Behavioral Development*, 17(3), 401-411.

National Center for Educational Statistics. (1994). *The Condition of Education 1994.* Washington, DC: U.S. Department of Education.

National Center for Educational Statistics. (1999a). *Highlights from TIMSS: Overview and key findings across grade levels* (NCES 1999-081). Washington, DC: Office of Educational Research and Improvement, U.S. Department of Education.

National Center for Educational Statistics. (1999b). *Highlights from TIMSS: Study-repeat (TIMSS-R)* (NCES 1999-027). Washington, DC: Office of Educational Research and Improvement, U.S. Department of Education.

Nicol, C. (1999). Learning to teach mathematics: questioning, listening and responding. *Educational Studies in Mathematics*, 37, 45-66.

Perry, M. (2000). Explanations of mathematical concepts in Japanese, Chinese and U.S. first- and fifth-grade classrooms. *Cognition and Instruction*, 18(2), 181-207.

PISA (2000). *Documente de Analiză, Sinteză, Diagnoză si Prognoză.* (Analysis, Synthesis, Diagnosis and Prognosis Documents). Retrieved May 16, 2006 from http://www.edu/ro/index.php/articles/c531.

Rodriguez, B.T. (2000). An investigation into how a teacher uses reform-oriented mathematics curriculum. Paper presented at the Annual Meeting of the American Educational Research Association New Orleans.

Romberg, T.A. (1999). School mathematics: The impact of international comparisons on national policy. In G. Kaiser, E. Luna, & L. Huntley (Eds.), *International comparison in mathematics education* (pp. 189-199). Philadelphia, PA: Falmer Press.

Ross, S.H. (1986). *The development of children's place-value numeration concepts in grades two through five.* Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Rossman, G.B., & Rallis, S.F. (2003). *Learning in the Field. An Introduction to Qualitative Research.* Thousand Oaks, CA: Sage Publications.

Saxton, M., & Towse, J.N. (1998). Linguistic relativity: the case of place value in multi-digit numbers. *Journal of Experimental Child Psychology*, 69, 66-79.

Schmidt, W., McKnight, C., & Raizen, S., in collaboration with six others. (1996). *A splintered vision: An investigation of U.S. science and mathematics education, executive summary.* Lansing, MI: U.S. National Research Center for the Third International Mathematics and Science Study, Michigan State University.

Senge, P. (1990). *The Fifth Discipline: The Art and Practice of the Learning Organization.* New York: Doubleday.

Senger, E.S. (1999). Reflective reform in mathematics: the recursive nature of teacher change. *Educational Studies in Mathematics*, 37, 199-221.

Schmidt, W., McKnight, C.C., Houang, R.T., Wang, H.C., Wiley, D.E., Cogan, L.S., & Wolfe, R.G. (2001). *Why Schools Matter. A Cross-National Comparison of Curriculum and Learning.* Jossey-Bass, CA.

Schonfeld, A. (1985). Metacognitive and epistemological issues in mathematical understanding. In E. A. Silver (Ed.), *Teaching and learning mathematical problem-solving: Multiple research perspectives.* Hillsdale, NJ: Lawrence Erlbaum.

Schonfeld, A. (1988). When good teaching leads to bad results: the disasters of "well-taught" mathematics courses. *Educational Psychologist*, 23(2), 145-166.

Schorr, R. Y., & Koellner-Clark, K. (2003). Using a modeling approach to analyze the ways in which teachers consider new ways to teach mathematics. *Mathematical and Learning*, 5(2 & 3), 191-210.

Schulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4-14.

Silver, E., Kilpatrick, J., & Schlesigner, B. (1990). *Thinking through Mathematics*. New York: The College Board.

Sovchik, R. (1989). *Teaching Mathematics to Children*. New York: Harper and Row Publishers.

Spilane, J. P., & Zeuli, J. S. (1999). Reform and teaching: Exploring patterns of practice in the context of national and state mathematics reforms. *Educational Evaluation and Policy Analysis*, 21(1), 1-27.

Spungin, R. (1996). Teaching teachers to teach mathematics. *Journal of Education*, 178(1), 73-84.

Stedman, L.C. (1997a). International achievement differences. An assessment of a new perspective. *Educational Researcher* 26(3), 4-15.

Stedman, L.C. (1997b). Deep achievement problems: the case for reform still stands. *Educational Researcher* 26(3), 27-29.

Steffe, L. (1990). On the knowledge of mathematics teachers. In R. B. Davis, C. Maher & N. Noddings (Eds.), *Constructivist Views on the Teaching and Learning of*

Mathematics, JRME Monograph # 4. National Council of Teachers of Mathematics, Reston, Virginia, pp. 167-184.

Steffe, L., & Cobb, P. (1988). *Construction of Arithmetical Meanings and Strategies*. New York: Springer-Verlag.

Stevenson, H.W. & Stigler, J.W. (1992). *Learning gap*. New York: Summit Books.

Taylor, A. R. (1987). *Relationships between classroom practices and student performance in mathematics: An analysis of cross-sectional data from the 1985 Provincial Assessment of Mathematics*. University of British Columbia, Canada, 1987. Dissertations Abstract International 49A: 1085.

The Final Report of the National Mathematics Advisory Panel, (2008). Retrieved April 20, 2008, from www.ed.gov/pubs/edpubs.html.

Tooke, D.J. (1993). Student teachers' mathematical backgrounds and attainment of their secondary students. *Clearing House*, 66(5), 273-277.

Valverde, G.A., Bianchi, L.J., Wolfe, R.G., Schmidt, W.H., Houang, R.T. (2002). *Using TIMSS to investigate the transition of policy into practice through the world of textbooks*. Boston: Kluwer Academic Publishers.

Waldrop, M.M. (1992). Complexity: The Emerging Science at the Edge of Order and Chaos. NY: Simon and Schuster.

Wang, J. & Lin, E. (2005). Comparative studies on U.S. and Chinese mathematics learning and the implication for standards-based mathematics teaching reform. *Educational Researcher*, 39(5), 3-13.

Wang, J., Lin, E., Tanase, M. & Sas, M. (2008). Revisiting the influence of numerical language characteristics on mathematics achievement: Comparison Among China, Romania, and US. International Electronic Journal of Mathematics Education, 3(1), 20-42.

Westbury, I. (1993). American and Japanese achievement…again: a response to Baker *Educational Researcher*, 22(3), 21-25.

Wilson, S. M., Schulman, L. S., & Richter, A. E. (1987). 150 different ways of knowing: Representations of knowledge in teaching. In J. Caldearhead (Ed.), *Exploring teacher thinking* (pp.104-124). London, England, Cassell.

Winerburg, S. S. (1987). *From fieldwork to classwork-Cathy: a case study of a beginning social studies teacher*. Palo Alto, CA: Stanford University, Knowledge Growth in Teaching Project.

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Presentations:
International Presentations

Tanase, M. (2006, December). A case study of constructivist learning in a teacher education class. Presentation at the Annual Meeting of the IADIS Cognition and Exploratory Learning in a Digital Age (CELDA) International Conference, Barcelona, Spain.

National Presentations

Tanase, M. (2008, March). Students' Understanding of Place Value: A Complex Theory Study of 1st Grade Romanian Students. Presentation at the Annual Meeting of the American Education Research Association (AERA), New

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Tanase, M. (2008, February). The impact of interrelated factors on Students' Understanding of Place Value: A Case Study of Romanian First-Graders. Presentation at the Eighty-Eight Annual Meeting of the Association of Teacher Educators (ATE), New Orleans, Louisiana.

Tanase, M. (2007, February). Preservice teachers' beliefs about knowledge, learning and teaching. Presentation at the Eighty-Seven Annual Meeting of the Association of Teacher Educators (ATE), San Diego, California.

Tanase, M., Leavitt, T., Sowder, M., & Smith, T. (2007, February). Roles of teacher educators in a constructivist learning environment. Presentation at the Eighty-Seven Annual Meeting of the Association of Teacher Educators (ATE), San Diego, California.

Smith, T., Leavitt, T., Sowder, M. & Tanase, M. (2007, February). Why are we here? The 18 plus year apprenticeship of 4 prospective teacher educators. Presentation at the Eighty-Seven Annual Meeting of the Association of Teacher Educators (ATE), San Diego, California.

Smith, T., Sowder, M., Leavitt, T., & Tanase, M. (2007, February). How exemplary teacher educators work to reinvent the educational landscape: A cross-case analysis. Presentation at the Eighty-Seven Annual Meeting of the Association of Teacher Educators (ATE), San Diego, California.

Tanase, M., Sas, M., Lin, E. & Wang, J. (2006, April). Comparing the influence of Chinese American numerical language characteristics on math achievement. Presentation at the Annual Meeting of the American Education Research Association (AERA), San Francisco, California.

Leavitt, T., Tanase, M., Sowder, M & Smith, T. (2006, April). Effects of a teacher education program on prospective teacher educators. Presentation at the Annual Meeting of the American Education Research Association (AERA), San Francisco, California.

Lin, E., Sowder, M., Smith, T. & Tanase, M., Leavitt, T. (2006, February). On becoming a teacher educator: reflections from for emerging teacher educators about practice and beliefs. Presentation at the Eighty-Six Annual Meeting of the Association of Teacher Educators (ATE), Atlanta, Georgia.

Plakthonick, M., Reis, D., Rajcopal, M. & Tanase, M. (November, 2001). Integrating New Technologies into Methods of Education (INTIME). Booth presentation at the 81st Annual Meeting of the National Council for the Social Studies, Washington D.C.

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