The effects of mathematics methods courses on the mathematical attitudes, content knowledge, and pedagogical beliefs of preservice teachers

Robert John Quinn
University of Nevada, Las Vegas

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The effects of mathematics methods courses on the mathematical attitudes, content knowledge, and pedagogical beliefs of preservice teachers

Quinn, Robert John, Ed.D.
University of Nevada, Las Vegas, 1993
THE EFFECTS OF MATHEMATICS METHODS COURSES ON THE MATHEMATICAL ATTITUDES, CONTENT KNOWLEDGE, AND PEDAGOGICAL BELIEFS OF PRESERVICE TEACHERS

by

Robert J. Quinn

A dissertation submitted in partial fulfillment of the requirements for the degree of

Doctor of Education

in

Instructional and Curricular Studies

Department of Instructional and Curricular Studies
University of Nevada, Las Vegas
August 1993
The Dissertation of Robert J. Quinn for the degree of Doctor of Education in Instructional and Curricular Studies is approved.

Chairperson, Virginia E. Usnick, Ph.D.

Examiner Committee Member, Richard R. Powell, Ph.D.

Examiner Committee Member, Marilyn Sue Ford, Ph.D.

Graduate Faculty Representative, Eun Sook Hong, Ph.D.

Dean of the Graduate College, Ronald W. Smith, Ph.D.

University of Nevada, Las Vegas
August, 1993
DEDICATION

This dissertation is dedicated to my parents, who instilled in me the importance of getting a good education, and who have always been supportive of my efforts to do so.
ABSTRACT

This study investigated the effects of mathematics methods courses on the mathematical attitudes, content knowledge, and pedagogical beliefs of preservice elementary teachers and preservice secondary mathematics teachers. The study was conducted during the fall semester of 1992 at the University of Nevada, Las Vegas. Forty-seven preservice teachers participated voluntarily. Twenty-eight were enrolled in an elementary mathematics methods course; nineteen were enrolled in a secondary mathematics methods course.

Attitudes toward mathematics were studied quantitatively by administering Aiken's Revised Attitude Scale on a pretest-posttest basis. The results indicated that the attitude toward mathematics of preservice elementary teachers improved significantly. The attitude toward mathematics of preservice secondary mathematics teachers improved, but not significantly.

Mathematical content was measured quantitatively by administering a test designed to measure knowledge of meaningful mathematical content. This test was administered on a pretest-posttest basis. The results indicated that the meaningful mathematical content knowledge of preservice
elementary teachers improved significantly. The meaningful mathematical content knowledge of preservice secondary mathematics teachers did not change significantly.

Preservice teachers' pedagogical beliefs were studied qualitatively. Specifically, beliefs concerning the use of manipulatives, technological aids, and cooperative learning were considered. Respondents wrote about their beliefs in each of these areas at the beginning of the semester and were interviewed at the end of the semester. Analysis of the qualitative data resulted in the generation of the following four working hypotheses: (a) Mathematics methods courses provide preservice teachers with important knowledge and experience concerning the use of manipulatives, technological aids, and cooperative learning; (b) preservice teachers learn mathematical content through their experiences in the mathematics methods course which involve the use of manipulatives, technological aids, and cooperative learning; (c) preservice teachers are concerned that difficulties might arise which will impede their ability to incorporate the use of manipulatives, technological aids, and cooperative learning in their classrooms; and (d) preservice teachers leave their mathematics methods course feeling they need to learn more about the use of manipulatives, technological aids, and cooperative learning.
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Chapter 1

Introduction

The United States is in the midst of a transformation from an industrial to an information-based society (National Council of Teachers of Mathematics [NCTM], 1989). The era when employees used their physical abilities together with shopkeeper arithmetic skills to carry out essentially the same job throughout an entire career is over. Instead, new entrants to the job market must be capable of applying new technologies to unfamiliar problem situations and to work cooperatively toward the solution of these problems (NCTM, 1989). This metamorphosis, precipitated by recent technological advances, predicates a redefining of mathematical competency for our society (NCTM, 1989).

"Mathematics has become a critical filter for employment and full participation in our society" (NCTM, 1989, p. 4). The mathematical knowledge that will be expected in the work place of the future will include the ability to understand the mathematics underlying complex, open-ended problems and the ability to work with others to solve these problems. Pollak (1987), a noted industrial mathematician, stressed the
importance of having a future work force which believes in the utility and value of mathematics as well as the ability to apply mathematics to real-world problems.

Currently, many American students lack the understanding of the conceptual bases for the mathematical skills that they have learned (Brown, Carpenter, Kouba, Lindquist, Silver, & Swafford, 1988). Consequently, they are often unable to apply these skills in the solution of problems (Brown et al., 1988). It is no longer sufficient for a small percentage of our population to be mathematically literate, rather, it is an economic necessity that ALL of our current school children develop the ability to reason mathematically, communicate mathematically, and gain facility in the problem-solving process in order that they will some day be capable of solving problems which are yet to be formulated (NCTM, 1989).

NCTM’s vision of mathematics includes the contention that mathematics should not be considered a hodgepodge of unrelated algorithms and problems which can be completed by one correct method, but rather a field rife with possibilities for discovery and exploration. This change in the philosophical underpinnings of mathematics requires a corresponding change in pedagogical practice. The teacher’s role must shift from one of dispenser of knowledge to facilitator of learning. Student centered activities
involving communication, exploration, examination, transformation, and application must replace teacher presentation as the predominant mode of instruction (Cooney, 1988; NCTM, 1989).

Statement of the Problem

One impediment to the implementation of a philosophy geared to improving the teaching of mathematics stems from the tendency of teachers to teach in the same manner in which they have been taught (Frank, 1990). Many preservice teachers have been educated primarily by pedagogies that are no longer considered effective by current experts in the field of mathematics education. These preservice teachers have never been exposed to the use of manipulatives, the use of computers and calculators, and group learning situations (Trueblood, 1986). Thus, one condition for the success of the reform of mathematics education is that our preservice teachers be taught by pedagogies which are considered appropriate by the National Council of Teachers of Mathematics. These pedagogies which include the use of manipulatives, the use of technological teaching aids, and the implementation of cooperative learning strategies are not usually employed in the pre-college mathematics education nor in the college content courses of preservice teachers. Ball (1990)
contended that mathematics methods courses can be an intervention which change the preservice teachers' knowledge, assumptions, and feelings about mathematics, as well as their beliefs concerning their role as teachers in the classroom. Consequently, mathematics methods courses might provide the vehicle by which preservice teachers gain knowledge of, and an inclination toward, the use of manipulatives, technological aids, and cooperative learning.

A second factor affecting the mathematics education of our youth is the mathematical competence of their teachers. Teacher education programs sometimes have difficulty attracting top students into their ranks, leading to criticism concerning the adequacy of content knowledge of preservice teachers. This problem is particularly acute among preservice elementary teachers who usually do not take as many mathematics content credits as their secondary counterparts (Book & Freeman, 1984). Further, the mathematical base of many preservice secondary mathematics teachers and preservice elementary teachers is rule-bound, severely lacking in the meaning necessary to provide adequate explanations to their future students (Ball, 1990). If reform in mathematics education is to be successful, future teachers must leave their preservice training with pedagogical knowledge and adequate mathematical content knowledge. Only
then will they become teachers who are capable of stressing conceptual understanding, reasoning, and mathematical communication, three areas considered essential by the National Council of Teachers of Mathematics (NCTM, 1989).

Lastly, teacher attitudes toward mathematics can have profound effects on their students. Schofield (1983) found that there was a positive relationship between teachers' attitude toward mathematics and the achievement of their students. Unfortunately, many elementary school teachers suffer from math anxiety which can be defined as "a combination of poor attitudes toward mathematics and low expectations for one's achievement in mathematics" (Tishler, 1980, p. 1). Tishler (1980) went on to contend that math anxiety is debilitating and can be transmitted from teacher to student. It is important that elementary students do not develop negative feelings toward mathematics, let alone math anxiety, as such feelings can be difficult to improve later (Kelly & Tomhave, 1985). An inadequate foundation exacerbated by a lack of confidence often severely inhibits students' future learning of school mathematics (NCTM, 1989; Paulos, 1988). Therefore, another condition for the success of reform in mathematics education is that preservice teachers leave their preservice training with a positive attitude toward mathematics, as well as an understanding of how their teaching
should foster and maintain such an attitude in their students.

Thus, it seems reasonable to conclude that one means of improving the state of mathematics education in this country would be to provide preservice programs for future teachers which provide them with: (a) meaningful knowledge of mathematical content, (b) a positive attitude toward mathematics, and (c) pedagogical knowledge consistent with NCTM's philosophy of mathematics as espoused in *The Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989).

**Need for the Study**

Preservice teacher education programs usually consist of some combination of content and pedagogy. Mathematics departments charged with teaching mathematical content and education departments charged with teaching pedagogies to preservice teachers have battled over the relative importance of these two types of courses. Although support can be found in the literature for stressing either over the other (White, 1987; Brodbelt, 1984), many studies call for some type of integration of content and pedagogy (Dossey, 1984; Isenberg, & Altizer-Tuning, 1984; Burger, Jenkins, Moore, Musser, & Smith, 1983; Kerr & Lester, 1982).

In an attempt to inform this debate, the present study
investigated the effects of an elementary mathematics methods course and a secondary mathematics methods course which taught mathematical pedagogy through modeling activities covering aspects of mathematical content that these preservice teachers will someday have to teach. The rationale behind these courses is embedded in the revamped philosophical underpinnings of the nature of mathematics suggested by NCTM's Standards (1989). If courses such as these are to be deemed effective, they must positively impact the attitudes of preservice teachers toward teaching methodologies which are generally considered effective by the majority of current mathematics educators. These methodologies include: the use of manipulatives, the use of technological teaching aids, and problem solving in cooperative groups.

Further, this study provided information concerning the current state of the meaningful knowledge of mathematical content and attitudes toward mathematics that preservice teachers bring to their respective methods course. It also investigated the potential benefits which could be derived from mathematics methods courses with regard to the attitudes toward mathematics and the meaningful knowledge of mathematical content of preservice teachers.
Purpose of the Study

The purpose of this study was to determine the effects of elementary mathematics methods courses and secondary mathematics methods courses on preservice teachers. The effects of mathematics methods courses on the meaningful knowledge of mathematical content, the attitudes toward mathematics, and the pedagogical beliefs of preservice elementary teachers and preservice secondary mathematics teachers were studied.

The overall intention of this study was to consider the effects of mathematics methods courses in a variety of areas. The quantifiable nature of meaningful knowledge of mathematical content and attitudes toward mathematics led the researcher to consider the effects of methods course on these variables quantitatively. The nature of preservice teachers' pedagogical beliefs led the researcher to use qualitative methods to analyze the effects of methods courses in this area.

Operational Definitions

Manipulatives - Manipulatives are objects that can be touched, moved about, arranged, and rearranged by children. Some manipulatives exist normally within the environment, such as
money or a yardstick. Other manipulatives are specifically designed for the purpose of teaching mathematical concepts (Kennedy, 1986). Examples of these include geoboards, base-ten blocks, algebra tiles, Cuisenaire rods, and tangrams.

Technological Aids - Technological aids refer to a variety of calculators, computers, and the software packages used on those computers. This definition is narrow but it covers the aspects of technology considered in this study.

Cooperative Learning - "Cooperative learning is an approach that involves a small group of learners working together as a team to solve a problem, complete a task, or accomplish a common goal" (Artzt & Newman, 1990, p. 448). Cooperative learning experiences in this study took place within the classroom. They were used to encourage mathematical communication as well as to develop a better understanding of the diversity of approaches that different students take toward a problem.

Beliefs - Beliefs are a form of knowledge which are context dependent, have a social component, and enable an individual to meet their goals (Tobin, Tippins, & Hook, 1992). Throughout this study, beliefs will refer specifically to pedagogical beliefs.
Meaningful Knowledge of Mathematical Content - Meaningful knowledge of mathematical content refers to a conceptual and intuitive understanding of mathematics. This knowledge transcends simply applying algorithmic procedures or memorizing facts in order to attain correct answers. For the purposes of this study, meaningful knowledge of mathematical content is defined as the score attained on the revised version of the Essential Elements of Elementary School Mathematics Test (White, 1986, see Appendix II).

Attitudes toward Mathematics - Attitudes toward mathematics refer to the level of like or dislike felt by an individual toward mathematics. For the purposes of this study, attitudes toward mathematics is defined as the score attained on the Aiken's Revised Mathematics Attitude Scale (Aiken, 1963, see Appendix I).

Pedagogical Knowledge - Pedagogical knowledge refers specifically to an understanding of teaching strategies which involve the use of manipulatives, the use of technological aids,
and the use of cooperative learning strategies in the teaching of mathematics.

Research Questions

1. Do preservice elementary teachers and preservice secondary mathematics teachers differ significantly with regard to their meaningful knowledge of mathematical content?

2. Does taking a mathematics methods course result in a significant change in the attitudes toward mathematics of preservice elementary teachers?

3. Does taking a mathematics methods course result in a significant change in the attitudes toward mathematics of preservice secondary mathematics teachers?

4. Does taking a mathematics methods course result in a significant change in the meaningful knowledge of mathematical content of preservice elementary teachers?

5. Does taking a mathematics methods course result in a significant change in the meaningful knowledge of mathematical
content of preservice secondary mathematics teachers?

6. How will preservice teachers' beliefs toward the use of manipulatives, the use of technological aids, and cooperative learning change as a result of a mathematics methods course?

Research Hypotheses

1. The meaningful knowledge of mathematical content of preservice elementary teachers will differ significantly from that of preservice secondary mathematics teachers upon enrollment in their respective mathematics methods course.

2. Completion of a mathematics methods course will significantly change the attitudes of preservice elementary teachers toward mathematics.

3. Completion of a mathematics methods course will significantly change the attitudes of preservice secondary mathematics teachers toward mathematics.

4. Completion of a mathematics methods course will result in a significant change in the meaningful knowledge of mathematical
content of preservice elementary teachers.

5. Completion of a mathematics methods course will result in a significant change in the meaningful knowledge of mathematical content of preservice secondary mathematics teachers.
Chapter 2

Literature Review

The literature review includes sections relating to: (a) attitudes toward mathematics, (b) mathematical content knowledge, (c) the use of manipulatives in mathematics education, (d) the use of technological aids in mathematics education, (e) cooperative learning, (f) teacher beliefs, and (g) teacher preparation. The chapter concludes with a summary of these areas as they relate to the present study.

Attitudes Toward Mathematics

According to the National Council of Teachers of Mathematics (1989), the study of mathematics has become extremely important in today's society. NCTM further stated that individuals who avoid mathematics are often severely limited with regard to career choice or college major. Even training in vocational-technical fields may be closed to these individuals.

Numerous studies have considered the effects of attitudes toward mathematics and mathematics anxiety on various aspects of teaching and learning (Harvin, 1982; Taylor & Brooks, 1986;
Prior to discussing these studies it is important to note that a negative attitude toward mathematics and a mathematics anxiety are not synonymous. Sandman (1980) considered anxiety toward mathematics as one of six subscales on his Mathematics Attitude Inventory. Wood (1988) contended that mathematics anxiety is extremely difficult to define and even more difficult to measure. His work indicated that separating the construct of math anxiety from test anxiety and general anxiety may not be possible. Further, he stated that more work must be done to determine the relationship between mathematics anxiety and mathematical attitudes. Throughout this review the terms mathematical anxiety and attitudes toward mathematics will be used as each individual study used them.

Misconceptions about the nature and importance of mathematics, combined with an anxiety toward the subject, often block effective learning in this area (Paulos, 1988). Overcrowded schools and incompetent teachers can exacerbate these attitudes and anxieties about mathematics (Harvin, 1982; Taylor & Brooks, 1986).

Avoidance of mathematics has become commonplace (Paulos, 1988). Some experts feel that math avoidance and math anxiety are fostered by the nature of mathematics education that
our children receive (Kelly & Tomhave, 1985). The emphasis on correct answers, the pressure of timed tests, an ambiguous vocabulary, and difficult word problems, create an atmosphere in which math anxiety develops and flourishes (Kelly & Tomhave, 1985).

In order to combat the proliferation of math anxiety and/or avoidance, many leaders in the field of mathematics education agree that the focus of mathematics education needs to be changed. The curriculum must move away from its current preoccupation with computational skills and rote activities and emphasize mathematical insight, reasoning, problem solving, and experiences which make sense of the connections between mathematics and the world in which students live (NCTM, 1989).

The most important contributor to the attitude which elementary students develop towards mathematics is the attitudes of their teachers towards this discipline (Meyer, 1980). As early as 1964, Banks identified the attitude of the teacher towards mathematics as the single most significant factor contributing to student attitude, beating out parental attitude, repeated failure, and attitude of peers (Meyer, 1980).

Tishler (1980) contended that math anxiety is both contagious and debilitating. He further stated that some teachers are carriers of this syndrome. Kelly and Tomhave (1986) found
that elementary education majors exhibited a higher degree of mathematical anxiety than most other groups. The only category of students who were more math anxious were a self-selected group that had enrolled in a math anxiety workshop.

On the positive side, other studies indicated that the attitudes of preservice elementary teachers can be improved by a mathematics methods course (Sovchik, Meconi, & Steiner, 1981). Battista (1986) found that preservice teachers who initially had an above average level of mathematics anxiety experienced a significant decrease in the level of that anxiety during a mathematics methods course.

Studies also indicated that the attitude of students towards mathematics can be improved (Pedersen, Bleyer, & Elmore, 1985). A plethora of studies indicated that students' newly-improved attitude towards mathematics correlated positively with an improved level of performance (McGlone, 1985; Mukherjee, 1978; Pedersen et al., 1985; McDaniel, Davis, & Browning, 1990).

Females and minorities have been particularly prone to the development of poor attitudes toward mathematics and consequently often perform at a lower level (Pedersen et al., 1985; Taylor & Brooks, 1986). Fairness and equity, however, necessitate that educators adopt the philosophy that all students are capable of learning and that any attempt to lower standards
or eliminate requirements because of the race, gender, or socioeconomic status of the student is misguided (Adler, 1984). The importance of females attaining confidence in their mathematical abilities is particularly important in light of the high percentage of elementary teachers who are female. If these female preservice teachers can attain confidence in their own mathematical abilities, they will have an opportunity to provide the positive role models that all students need, including young female students.

Mathematical Content Knowledge

The mathematical content knowledge possessed by a teacher is another important component to their success in teaching elementary mathematics (Isenberg & Altizer-Tuning, 1984). Similarly, it is critical that secondary mathematics teachers have strong mathematical knowledge to be successful (Kerr & Lester, 1982).

Brodbelt (1984) found that many elementary teachers were poorly trained in mathematics. Gibney, Ginther, and Pigge (1988) found that elementary teachers graduating in the 1980s were weaker mathematically than those of the 1970s. Book and Freeman (1984) found that elementary preservice teachers had a weaker mathematical background than preservice secondary
mathematics majors. Tirosh and Graeber (1989) contended that preservice elementary teachers relied too heavily on procedural knowledge which was often not connected to conceptual knowledge.

Ball and Wilson (1990) found that mathematics majors had not been exposed to enough alternative teaching methods to be capable of teaching mathematics with an emphasis on meaning. Ball (1990) found that preservice secondary mathematics teachers lacked sufficient mathematical understanding to teach the subject effectively.

**Manipulatives**

"Manipulative materials are objects that appeal to many senses and that can be touched, moved about, rearranged, and otherwise handled by children.... They can be objects from the environment, such as money or measuring instruments, or materials specifically designed to teach mathematical concepts, such as base-ten blocks and balances" (Kennedy, 1986, p. 6).

The National Council of Teachers of Mathematics contended that every classroom should be equipped with manipulatives, including geoboards, pattern blocks, tiles, calculators, and numerous other hands-on materials (NCTM, 1989). Use of manipulative materials helps students take advantage of an often
wasted resource, the physical experiences that they have had outside the classroom (Simon, 1989).

A plethora of studies support the use of manipulatives as a means of effectively teaching mathematics to elementary school children (Balka, 1983). Kennedy (1986) concluded that the use of appropriate manipulatives enhances learning, generates interest, and promotes computational as well as problem-solving skills. Simon (1989) stressed the importance of using manipulatives as a means of incorporating the students' own intuitive understanding as a foundation upon which mathematical knowledge and structure can be better understood.

Baroody (1989) warned that although much evidence exists which supports the use of manipulatives, teachers must realize that their use is neither necessary nor sufficient to guarantee mathematical learning on the part of students. He contended that uninformed or inappropriate use of manipulatives will often lead to poor results. Further, Baroody felt that in some cases pictures or video displays might be more effective learning tools for students than the manipulation of concrete objects.

The appropriateness of using manipulatives at the secondary level has also generated much interest. Moser (1986) contended that those who denigrate the use of manipulatives at the middle school or secondary level do so based on a definition
of manipulatives which is too narrow. The development of complex learning materials which can be used in teaching three-dimensional geometry, as well as the importance of coins, cards, and dice in the teaching of probability provide further support for his arguments.

Simply placing manipulative materials in a classroom, however, does not predicate their use. Trueblood (1986) felt that prospective teachers must be taught how to plan and manage the use of manipulatives in their classroom. He found that when such training was included in mathematics methods courses, teachers used these methods in their student-teaching experience in a manner similar to which they had been taught. However, Bush (1988) found that many teachers who have both access to as well as knowledge of manipulative materials still fail to incorporate them regularly into their teaching. Hollingsworth (1990) addressed the issue of preservice as well as inservice teachers not using manipulatives in their classroom despite having been trained to do so. She contended that improvement in this area is predicated on the integration of manipulatives into the mathematics methods classroom rather than their relegation to out of class projects.
Technological Aids

Technological advancements over the past two decades have vastly changed the feasibility of using calculators and computers in the teaching of mathematics including arithmetic. The National Council of Teachers of Mathematics lent its support to the use of technological aids in the teaching of mathematics. In their position statement on the use of calculators, the NCTM recommended their integration into school mathematics programs at all levels. They encouraged calculator use during class, on homework, and for evaluation of learning (NCTM, 1991).

Hembree and Dessart (1986) discussed a meta-analysis which included 79 studies concerning the effects of calculator use in mathematics instruction. They concluded that calculator use together with traditional instruction did not detrimentally affect the paper-and-pencil skills of students. They went on to say that students who use calculators have a better attitude toward mathematics as well as a better self-concept of their own mathematical abilities.

Lochhead (1988) examined numerous studies which detailed the effects of calculators and computers on mathematical learning. He concluded that computer use did not seem to hinder the attainment of traditional objectives, while in some cases greater conceptual knowledge seemed to have been fostered.
Brady (1991) contended that it is important to educate students in the use of calculators and computers, as these are tools that they will be required to use upon entering the job market. She found that students in geometry and algebra classes not only enjoyed working with technological aids, but felt that the use of technology enhanced their knowledge of the course material.

Marshall (1990) advocated employing computers as a means of revamping the traditional mathematics curriculum. She felt that to accomplish this goal computers must be used for more than just a means of providing drill work. Calculators can help teachers become facilitators of knowledge in a problem-solving environment rather than dispensers of knowledge (Demana & Waits, 1989). Waits and Demana (1989) advocated the use of computers or graphing calculators in pre-calculus and algebra classes. They felt that the appropriate implementation of these tools provided students with a better understanding of functions and more depth to the geometric meaning of algebraic solutions.

Dockweiler (1989) studied the use of the Explorer Calculator in grades three through six. He found that students regularly using this aid were not hindered in their acquisition of basic mathematical skills. Starkey (1989) contended that calculators have an important place, even in the first grade, as a teaching tool. She felt that not only does the use of calculators
enhance the learning of these young students, it also instills in them a positive attitude. Bitter and Hatfield (1991) reported extremely positive results of calculator use. They found that the students who had learned using calculators scored better on the Iowa Test of Basic Skills. Further, the attitude of these students toward their own mathematical abilities improved.

Schultz, Morrison, and Pruett (1989) carried out a survey of secondary mathematics teachers. They found that teachers felt that they did not have enough computer hardware available to them. Fifty-eight percent reported that they did not have a computer in their classroom. Further, teachers expressed a concern over the availability of software which was correlated to what they were teaching. With regard to the type of software being used, the survey found that 76% of the teachers use drill and practice programs, 58% use games, 56% use tutorials, 49% use problem-solving software, 24% use simulations, 11% use programming software, and 11% use some other type of software. Improvement in student motivation topped the list of teacher-perceived advantages of computer use, followed by the ability to provide a review of skills through drill and practice.

Becker (1990) warned that simply providing computers in the learning environment was not sufficient to insure improved achievement. He found that only negligible differences existed
between students who took a traditional math class and those using computer software regularly. He concluded that in order for technology to be truly beneficial, teachers must modify their ideas concerning pedagogy to allow computers to enhance the curriculum.

Cooperative Learning

"Cooperative learning is an approach that involves a small group of learners working together as a team to solve a problem, complete a task, or accomplish a common goal" (Artzt & Newman, 1990, p. 6). Sutton (1992) contended that the use of cooperative learning strategies extended the mathematical learning of the students in her secondary mathematics classes. Further, it helped students acquire the ability to work together, a skill that will be important to them as they enter the job market. Testimony concerning the efficacy of cooperative learning at the primary grades came from Rosenbaum, Behounek, Brown, and Burcalow (1989). They particularly stressed that the use of cooperative learning helps students to overcome the computational obstacles inherent in problem-solving. They also reported that cooperative learning fosters in students the value of working together. Duren and Cherrington (1992) found that students who participated in cooperative problem-solving groups
retained problem-solving strategies better than students who worked alone. They felt some of the reasons for this were that students in cooperative groups: (a) displayed more persistence in problem-solving, (b) communicated the strategies they were using during the solution process, and (c) varied their approaches to problem-solving more readily. Yackel, Cobb, and Wood (1991) argued that the use of cooperative learning groups at the second grade level provides children with learning opportunities that would not exist in the traditional classroom setting.

Slavin (1988) qualified his support of cooperative learning by stating that this strategy will not be efficacious unless both a group goal as well as individual accountability are present. Slavin further stated that even flawed cooperative learning techniques which did not produce higher student achievement were valuable due to the improvement of students' self esteem as well as their ability to work with others. Slavin (1991) using data gathered by Stevens, found that even the highest achieving students benefitted from cooperative learning experiences when compared to similar students in the control group. Davidson and Kroll (1991) found that the research regarding cooperative learning in mathematics does not conclusively determine whether cooperative models with group rewards are more effective than those which do not use group rewards. They went on to contend
that the nature of the mathematical task as well as the type of materials used could play a role in the efficacy of cooperative learning in mathematics.

Johnson and Johnson (1987) contended that despite the efficacy of cooperative learning, teachers are reluctant to implement it in the classroom. A possible reason for this lack of implementation could be teachers' competence to incorporate such strategies into their repertoire. Noddings (1989) contended that for teachers to be effective in using cooperative learning strategies their training should include working in small groups themselves. According to Noddings, the reasons for this are threefold: (a) to learn about cooperative learning through first-hand experience, (b) to get a wider view of different approaches to solving a problem, and (c) to gain a better understanding of the role of teachers in cooperative learning. Swineford and Holtan (1991) stated that teachers often fail to incorporate new strategies like cooperative learning because they feel that doing so will be more time consuming. Further, teachers often feel that these strategies are inflexible and must be implemented exactly as they were taught.

Teacher Beliefs

O'Loughlin (1990) found that preservice teachers hold
strong beliefs concerning knowing, learning, and teaching, which typically lead them to favor didactic approaches with the teacher serving as transmitter of knowledge. Such beliefs, he contended, represent a major obstacle in efforts to reform educational practices. Wilson (1990) found that students enrolled in an introduction to education course believed that teaching is exposition and learning is absorption. These students considered the use of alternative teaching pedagogies as a means of making learning palatable without effecting the fundamental assumption of the teacher as provider of knowledge.

Ball and Wilson (1990) found that students entering preservice programs felt that showing and telling students how to do mathematics was the most effective means of teaching this subject. Thompson (1984) contended that teachers' beliefs are an important factor in the selection of the instructional practices that they employ. Wilcox, Schram, Lappan, and Lanier (1991) provided evidence that the nature of preservice teacher education can have an effect on the belief systems of prospective teachers with regard to the nature of mathematics as well as the nature of mathematics education. Their study showed that fostering a community of learners and encouraging active engagement in mathematical activities in a non-threatening environment helped preservice teachers gain confidence in their own ability to solve
problems. Further, it helped them understand the value of varied solutions as well as the importance of mathematical discourse in the problem-solving process.

Book and Freeman (1984) found that the educational beliefs of preservice elementary teachers and preservice secondary teachers were quite similar. The one major difference they found between these groups was that secondary majors stressed the importance of subject matter while elementary majors considered their orientation to students as tantamount. Cronin-Jones & Shaw (1992) interviewed preservice elementary and secondary science teachers. They found that secondary majors tended to have a more complex belief system than elementary majors. Beliefs about skill development, assessment, and subject concern were held only by secondary majors. They concluded that in order to be successful, a methods course should begin with an investigation of the beliefs of the preservice teachers enrolled in it.

**Teacher Preparation**

Recently, much emphasis has been placed on reforming teacher education to facilitate a corresponding change in mathematics instruction in our schools. Cooney (1988) contended that it is essential for teachers to be provided with experiences
that allow them to construct the same mathematics that they will be teaching. Simon and Schifter (1991) found that mathematics education interventions can affect the pedagogies employed by mathematics teachers. Central to their program was the belief that teachers' mathematical learning should be considered in the same manner as students' mathematical learning. Their program emphasized the importance of conceptual understanding and problem solving. Teachers who participated in this study became more likely: (a) to integrate the teaching strategies that they had learned into their own classrooms, (b) to listen more to the ideas and understandings of their students, (c) to be aware of the importance of keeping their students active and responsible in their mathematical learning, and (d) to be at ease with mathematics and enjoy it more.

**Conclusion**

This literature review has uncovered a number of problems which face teachers of mathematics methods courses as they attempt to prepare preservice teachers. These problems include: (a) the disparity between the efficacy of using manipulatives, technological aids, and cooperative learning in the teaching of mathematics and the beliefs which preservice teachers hold concerning the nature of teaching mathematics; (b) the
preponderance of poor attitudes toward mathematics of preservice elementary teachers; and (c) the lack of conceptual knowledge concerning mathematics which pervades both elementary preservice teachers and preservice secondary mathematics teachers.

Fortunately, however, this review also found evidence that inroads can be made regarding each of these problems. The present study attempted to add to this body of knowledge by investigating the effects of mathematics methods courses on the mathematical knowledge, attitudes toward mathematics, and pedagogical beliefs of preservice elementary teachers and preservice secondary mathematics teachers.
Chapter 3

Methodology

This study considered the effects of mathematics methods courses on preservice elementary teachers and preservice secondary mathematics teachers. The six research questions in this study were developed with the intention of informing educational practice based on the varied effects of a mathematics methods course. Howe and Eisenhart (1990) contended that the selection of an appropriate methodology for data collection and analysis should be driven by the nature of the research question. With this criterion in mind, the researcher determined that research questions 1 through 5, which involve meaningful knowledge of mathematical content and attitudes toward mathematics, could be studied quantitatively. The nature of research question 6, concerning how preservice teachers' beliefs changed, led the research to consider this question qualitatively.

Glesne and Peshkin (1992) stated, "Rather than argue about which paradigm or methods are better, we... see virtue in a variety of approaches. Different approaches allow us to know and
understand different things about the world" (p. 9). Howe and Eisenhart (1990) stated that educational research should be judged on the worth of the conclusions that can be drawn from it rather than the strict adherence to a methodological convention. The inclusion of both qualitative and quantitative methodologies within this study allowed the researcher to investigate different aspects of mathematics methods courses and to provide a fuller picture of the effects of such courses than either method alone could have provided. Further, the use of both methodologies was deemed appropriate because it provided information upon which conclusions could be based concerning the diverse yet important effects of a mathematics methods course.

**Subjects**

The present study included 47 preservice teachers at the University of Nevada, Las Vegas, an urban university of approximately 20,000 students. All of the 19 preservice teachers enrolled in secondary mathematics methods agreed to participate in this study, as did 28 preservice elementary teachers enrolled in elementary mathematics methods. A copy of the consent to participate form can be found in Appendix III. The study took place during the fall semester of 1992. Approval for the research to be done in this study was granted by the Human Subjects
Committee in August 1992.

Twenty-five of the preservice elementary teachers were female and 3 were male. Nine of the preservice secondary mathematics teachers were female and 10 were male. The median age of the elementary group was 28. The median age of the secondary group was 29. Table 1 contains additional information concerning the ages of the respondents. This information indicates that many of the students who participated in this study were non-traditional college students. The number of respondents that considered their pursuits in education to be a career change corroborated their non-traditional status. Eleven secondary majors considered teaching a career change, 7 considered it a first career. Twelve elementary majors considered teaching a career change, 16 considered it a first career.

At the beginning of the semester, 6 secondary majors reported concurrent enrollment in mathematical content courses, 4 reported taking 1, 8 reported taking 2, and 1 reported taking 3. Although some of these courses were subsequently dropped during the semester, the effects of these mathematical content courses on the meaningful knowledge of mathematical content, as well as the mathematical attitudes of these students, represents a potentially confounding variable to this study. None of the 28
Table 1

Ages of Respondents

<table>
<thead>
<tr>
<th>Age</th>
<th>Secondary (N=19)</th>
<th>Elementary (N=28)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 24</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>25 - 29</td>
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<td>30 - 34</td>
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<td>35 - 39</td>
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<tr>
<td>40 - 44</td>
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<tr>
<td>45 - 49</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>50 - 54</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
elementary majors took any mathematical content courses during the semester in which this study took place.

**Instrumentation**

Attitudes toward mathematics were measured using Aiken's Revised Mathematics Attitude Scale (Aiken, 1963, see Appendix I). This instrument consists of 20 questions which were answered on a 5-anchor likert scale. Scores were attained by awarding 0, 1, 2, 3, or 4 points for each item based on the level of the response. Negatively stated items were reversed so that higher scores indicated a more positive attitude toward mathematics than lower scores. Possible scores on this scale ranged from a low of 0 to a high of 80.

Meaningful knowledge of mathematical content was measured by a test which included 25 of the 50 items on the Essential Elements of Elementary School Mathematics Test developed by M. A. White (1986). White's test was devised to measure conceptual and intuitive understanding of mathematics. Items were created which require more than simply applying algorithmic procedures or memorizing facts to attain correct answers. The 25 multiple-choice questions (see Appendix II) selected for the adapted version of this test covered a wide variety of mathematical topics including: place value, percent,
fractions, multiplication, division, estimation, area, perimeter, geometry, number lines, measurement, probability, and statistics. (Note that question #1 of the test shown in Appendix II was deleted due to a typographical error in the choices listed.) The content covered by this test was at approximately the sixth grade level. The validity of this test was based on a qualitative appraisal made by two experts in the field of mathematics education. Cronbach's Alpha provided a reliability estimate of .86 for this instrument.

Data Collection

The data collection for this study took place during the Fall semester of 1992. Aiken's Revised Mathematics Attitude Scale (Aiken, 1963) was administered during the first week of classes and again during the last week of classes. These administrations took place during class. Respondents were given as much time as they needed to complete this scale.

The revised version of the Essential Elements of Elementary School Mathematics Test (White, 1986) was administered during the first two weeks of classes and again during the last three weeks of classes. Each administration of this test took place during class in the secondary mathematics methods course. Elementary majors were administered the test outside of class
by the researcher at times determined by the respondent. No time
limit for the completion of this test was imposed on either the
secondary or the elementary respondents.

The researcher was the instructor of the secondary
mathematics methods course and observed the elementary
methods course regularly. Extensive field notes were kept for all
of these experiences. These notes described: (a) the types of
activities which took place, (b) the nature of the interactions
between professor and students, (c) the nature of interactions
among students, and (d) the researcher's perception of the
attitudes of the students toward their classroom experiences.

During the first three class meetings each participant was
asked to write for eight minutes in response to three questions.
The researcher instructed them to write based on their current
beliefs concerning each question. The question was read aloud
and each respondent was provided with lined paper with that
question printed at the top. Respondents answered one question
each class in the order listed below.
1. What are your beliefs concerning the use of manipulatives in
   the teaching of mathematics?
2. What are your beliefs concerning the use of technological aids
   (calculators, computers, etc.) in the teaching of mathematics?
3. What are your beliefs concerning the use of cooperative
learning in the teaching of mathematics?

The researcher conducted interviews of all 47 participants during the last three weeks of the semester. Interview times were set at the convenience of the respondent. Each of these thirteen-question interviews (see Appendix IV) was tape-recorded and transcribed.

During this interview each respondent was asked questions about their beliefs concerning the use of manipulatives, technological aids, and cooperative learning in the teaching of mathematics. The order of the questions allowed respondents first to answer the question concerning their current beliefs, then read the answer that they had written at the beginning of the semester to the same question, and to describe the changes that had occurred in their beliefs since that time. This alternating approach was used for questions 1-6. Respondents were also asked to discuss the ways in which these pedagogies had been used by their teachers during their own educational experiences and how their methods course would effect their use of these pedagogies.

Data Analysis

An independent groups \( t \) test was used for research question 1 and correlated groups \( t \) tests were used for questions 2 through
5. In addition descriptive statistics from the item analysis of the content test were used to inform the nature of the differences of content knowledge between preservice elementary teachers and preservice secondary mathematics teachers as well as the differences from pretest to posttest.

The three writings and the transcribed interviews were analyzed qualitatively in order to answer research question 6. Open coding (Strauss & Corbin, 1990) was employed to identify concepts within the qualitative data. Axial coding (Strauss & Corbin, 1990) was then employed until appropriate connections within the data were established. The data analysis process culminated in a conceptual framework (Glesne & Peshkin, 1992) which led to the development of working hypotheses (Glesne & Peshkin, 1992). The researcher attempted to create a qualitative report which "judiciously and effectively presents the most compelling evidence" (Yin, 1989, p. 149).

The Secondary Mathematics Methods Course

Nineteen students were enrolled in the secondary mathematics methods course referred to in this study. This course carried three semester hours of credit and met for two 75-minute sessions throughout its fifteen-week duration. The researcher acted as instructor for this course. The classroom
set-up included 6 tables with 3 to 4 students sitting at each. The classroom was stocked with a variety of mathematical manipulative materials and calculators. These materials were used regularly in problem-solving situations. An in-class computer was used for demonstrations and a computer laboratory was reserved for two full class meetings.

The philosophical underpinnings of this course (ESE 416) are apparent from the course overview which was a part of the course syllabus given to the students.

COURSE OVERVIEW: The nature of mathematics education in grades K-12 will be analyzed. Effective teaching techniques that are consistent with the Standards recommended by NCTM will be modeled by the instructor and practiced by the students. These skills include effective questioning techniques, centering mathematical learning around problems, and actively involving students in learning mathematics. Students enrolled in ESE 416 will develop skills in planning mathematics lessons for students in grades 7-12 that are consistent with NCTM's Standards, incorporating appropriate use of manipulatives, technology, cooperative learning, and allowing students to construct mathematics. Specifically the topical areas of computation, algebra, geometry, trigonometry, calculus, probability, statistics, and discrete mathematics will be explored. Students will be presented instructional strategies to develop more formal understanding and skill with the concepts. A variety of models of teaching, classroom management techniques, disciplinary procedures, motivational strategies, and assessment techniques will be presented in order to assist preservice teachers in their
initial teaching experiences. Strategies to increase and improve the involvement of girls and minorities in mathematics will be discussed.

Throughout the course, students engaged in cooperative group activities during class. These learning activities encouraged the use of mathematical communication and helped students to develop a better understanding of the diversity of approaches to any situation. These activities usually involved one or more of the following: (a) problem solving, (b) the use of manipulatives, or (c) the use of technology. Initially such activities were introduced by the instructor as models of appropriate lessons that these preservice teachers could later incorporate into their own classrooms. The students also led such lessons in their most important assignment, a class presentation. The description of this assignment as described on the course syllabus is included below:

CLASS PRESENTATION: Each student will be given the opportunity to present a lesson to their peers. The objective of these class presentations is to give students experience in communicating mathematically and to practice being in a 'teaching situation' where they are explaining or constructing the learning environment for students. Students should use these presentations to put into practice the theories of mathematics education advocated by the Standards and discussed in this course.
Most of the presentations, whether initiated by the instructor or by a student, were followed by a class discussion. The purpose of these discussions was to reflect on the lesson, considering the pedagogies involved as well as the appropriateness and usefulness of the lesson. Thus, students had the opportunity to consider the value of a variety of different types of lessons, most of which were consistent with the NCTM's Standards (1989).

The Elementary Mathematics Methods Course

The elementary majors participating in this study were enrolled in one of two sections of an elementary mathematics methods course. This course carried three semester hours of credit and met for two 75-minute sessions throughout its fifteen week duration. One section included 29 students, 19 of whom agreed to participate in the study; the other section included 22 students, 9 of whom agreed to participate. One instructor taught both sections of this course. The researcher was not involved in the teaching of this course, but observed on a regular basis throughout the semester.

The classroom set-up included 6 tables with 3 to 6 students sitting at each. The classroom was stocked with a variety of mathematical manipulative materials, as well as a variety of
calculators. An Apple IIe computer was also available.

The instructor created an open classroom atmosphere in which student questions were encouraged and valued. The course stressed the importance of instilling in children a conceptual understanding of mathematics. Students engaged in hands-on, cooperative group activities during almost every class session. Activities involving the use of calculators were also facilitated by the instructor. These lessons were usually cooperative in nature and emphasized ways in which calculators can be used to enhance the learning process. All of the lessons described above provided pedagogically appropriate modeling of the types of lessons recommended by the National Council of Teachers of Mathematics. Further, these lessons covered content that these teachers will someday be responsible for teaching in their own classrooms.

Limitations of the Study

1. The relatively small sample size, particularly with respect to the preservice secondary mathematics teachers, could limit the quantitative aspects of this study.

2. The voluntary nature of participation in this study could skew the data and limit generalizability to the entire population. Twenty-eight of the 51 preservice elementary teachers asked to
participate in this study agreed to do so, making this problem particularly acute within this group.

3. The researcher's role as instructor of the secondary mathematics methods course might encourage normative responses on the part of some participants, which could limit the validity of the qualitative aspects of this study.

4. The non-traditional nature of many of the students in the sample used for this study could limit the validity of generalizing these results to more traditional settings.

5. The high percentage of preservice secondary mathematics teachers who were concurrently enrolled in mathematical content courses could limit the findings concerning content knowledge and attitude toward mathematics of this group.
Chapter 4

Results

The purpose of this study was to consider the effects of a mathematics methods course on the meaningful knowledge of mathematical content, attitudes toward mathematics, and pedagogical beliefs of preservice teachers. In this chapter the results of the statistical tests performed on each of the five quantitative hypotheses are discussed as are the findings of the qualitative investigation. The use of both quantitative and qualitative methodologies in this study was deliberate, in order to give a more complete picture of the effects of a mathematics methods course.

Research Question 1

Hypothesis: The meaningful knowledge of mathematical content of preservice elementary teachers will differ significantly from that of preservice secondary mathematics teachers upon enrollment in their respective mathematics methods course.

An independent groups \( t \) test was performed comparing the
scores on the revised version of the Essential Elements of Elementary School Mathematics Test (White, 1986) of the preservice secondary mathematics teachers (mean = 20.8) and the preservice elementary teachers (mean = 15.2). One preservice elementary teacher was eliminated from analyses involving the content test due to an invalid pretest. The difference in means was found to be statistically significant, \( t(44) = -6.56, p < .001 \), which indicated that the preservice secondary mathematics teachers possessed a significantly higher degree of meaningful knowledge of mathematical content than the preservice elementary teachers. Table 2 lists the percentage of respondents who answered correctly by question on the pretest and posttest for each group.

The item analysis shows that a higher percentage of preservice secondary mathematics teachers answered correctly on each of the 24 questions when compared to the preservice elementary teachers. The biggest differences between percentage of preservice elementary teachers and preservice secondary mathematics teachers who answered correctly was found on questions 12, 15, 20, 24, and 25. These questions involved long division, percent, measurement, statistics, and probability, respectively.
Table 2

Percent of Respondents Answering Correctly on Revised Essential Elements of Elementary School Mathematics Test (by Question)

<table>
<thead>
<tr>
<th>Question #</th>
<th>Elementary (N = 27)</th>
<th>Secondary (N = 19)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest</td>
<td>Posttest</td>
</tr>
<tr>
<td>2</td>
<td>85.2</td>
<td>96.3</td>
</tr>
<tr>
<td>3</td>
<td>74.1</td>
<td>81.5</td>
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<tr>
<td>4</td>
<td>44.4</td>
<td>33.3</td>
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<tr>
<td>5</td>
<td>59.3</td>
<td>70.4</td>
</tr>
<tr>
<td>6</td>
<td>88.9</td>
<td>92.6</td>
</tr>
<tr>
<td>7</td>
<td>85.2</td>
<td>74.1</td>
</tr>
<tr>
<td>8</td>
<td>96.3</td>
<td>92.6</td>
</tr>
<tr>
<td>9</td>
<td>33.3</td>
<td>48.2</td>
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<tr>
<td>10</td>
<td>51.9</td>
<td>70.4</td>
</tr>
<tr>
<td>11</td>
<td>37.0</td>
<td>66.7</td>
</tr>
<tr>
<td>12</td>
<td>37.0</td>
<td>29.6</td>
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<tr>
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<td>92.6</td>
</tr>
<tr>
<td>14</td>
<td>77.8</td>
<td>88.9</td>
</tr>
<tr>
<td>15</td>
<td>44.4</td>
<td>63.0</td>
</tr>
<tr>
<td>16</td>
<td>88.9</td>
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<td>40.7</td>
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<tr>
<td>25</td>
<td>14.8</td>
<td>29.6</td>
</tr>
</tbody>
</table>
Research Question 2

Hypothesis: Completion of a mathematics methods course will significantly change the attitudes of preservice elementary teachers toward mathematics.

Attitudes toward mathematics were measured using Aiken's Revised Mathematics Attitude Scale (Aiken, 1963). This instrument consisted of 20 questions which were answered on a 5-anchor likert scale which ranged from strongly disagree to strongly agree. Scores were attained by awarding 0, 1, 2, 3, or 4 points for each item based on the response. Scoring on negatively stated items was reversed so that higher scores indicated a more positive attitude toward mathematics than lower scores. Possible scores on this scale ranged from a low of 0 to a high of 80.

A correlated groups t test was performed comparing the scores on the pretest of the Aiken's Revised Mathematics Attitude Scale (mean = 39.5) and the scores on the posttest of the Aiken's Revised Mathematics Attitude Scale (mean = 43.3) of the preservice elementary teachers. This difference was found to be statistically significant, t(27) = -2.32, p < .05. This result indicated that at the conclusion of the elementary mathematics methods course, the attitude toward mathematics of preservice elementary teachers had improved significantly from their
attitude toward mathematics at the beginning of the course.

**Research Question 3**

Hypothesis: Completion of a mathematics methods course will significantly change the attitudes of preservice secondary mathematics teachers toward mathematics.

A correlated groups *t* test was performed comparing the scores on the pretest of the Aiken's Revised Mathematics Attitude Scale (mean = 64.1) and the scores on the posttest of the Aiken's Revised Mathematics Attitude Scale (mean = 66.3) of the preservice secondary mathematics teachers. This difference was not found to be statistically significant, *t*(18) = -1.65, *p* = .117. This result indicated that the attitude toward mathematics of preservice secondary mathematics teachers did not change significantly from the beginning of the secondary mathematics methods course to its conclusion.

**Research Question 4**

Hypothesis: Completion of a mathematics methods course will result in a significant change in the amount of meaningful knowledge of mathematical content of preservice elementary teachers.

A correlated groups *t* test was performed comparing the
scores on the pretest of the revised version of the Essential Elements of Elementary School Mathematics Test (mean = 15.2) and the scores on the posttest of the revised version of the Essential Elements of Elementary School Mathematics Test (mean = 16.9) of the preservice elementary teachers. This difference was found to be statistically significant, \( t(26) = -4.1, p < .001 \). This result indicated that the meaningful knowledge of mathematical content of preservice elementary teachers increased significantly from the beginning of the elementary mathematics methods course to its conclusion.

A review of the item analysis indicated that the percentage of preservice elementary teachers who answered correctly increased on 16 of the 24 questions. Six of the 8 questions on which the percentage of elementary teachers who answered correctly decreased, showed drops of less than 10%. Questions 4 and 7 each showed a decrease of 11.1%. This decrease indicated that 3 fewer subjects answered each of these questions correctly at the end of the semester as compared to the beginning of the semester. Twelve of the 16 questions on which the percentage of preservice elementary teachers who answered correctly increased, showed gains of more than 10%. The largest increases occurred on question 11 (29.7%), question 18 (25.9%), and question 21 (22.2%). These questions involved division of
fractions, area/perimeter, and measurement, respectively.

Despite the significant improvement in the number of questions answered correctly at the end of the semester by preservice elementary teachers, less than half of this group answered correctly on questions 4 (33.3%), 9 (48.2%), 12 (29.6%), 19 (14.8%), 22 (48.2), and 25 (29.6), at the end of the semester. These questions involved fractions, multiplication of fractions, long division, geometry, circumference of a circle, and probability, respectively. Question 24 on statistics was answered correctly by only 51.9% of these preservice elementary teachers.

Research Question 5

Hypothesis: Completion of a mathematics methods course will result in a significant change in the amount of meaningful knowledge of mathematical content of preservice secondary mathematics teachers.

A correlated groups t test was performed comparing the scores on the pretest of the revised version of the Essential Elements of Elementary School Mathematics Test (mean = 20.8) and the scores on the posttest of the revised version of the Essential Elements of Elementary School Mathematics Test (mean = 20.9) of the preservice secondary mathematics teachers. This
difference was not found to be statistically significant, $t(18) = -0.24$, $p = .81$. This result indicated that the meaningful knowledge of mathematical of preservice secondary mathematics teachers did not change significantly from the beginning of the secondary mathematics methods course to its conclusion.

The item analysis indicated that the change in the percentage of preservice secondary mathematics teachers who answered correctly was low for most of the questions. Exceptions included question 11 which showed an increase of 21.1%, question 12 which showed a decrease of 21.1%, question 22 which showed an increase of 15.8%, and question 24 which showed a decrease of 15.8%. These questions involved division of fractions, long division, circumference of a circle, and statistics respectively.

**Research Question 6**

**Question:** How will preservice teachers' beliefs toward the use of manipulatives, the use of technological aids, and cooperative learning change as a result of a mathematics methods course?

**Initial Beliefs.** Manipulatives, technology, and cooperative learning are all topics that preservice teachers may have
encountered by the time they take a mathematics methods course. Although the knowledge base in these areas prior to the methods course is not clear, the context of their introductions are, for the most part, positive. Consequently almost all of the preservice teachers made positive remarks concerning the importance and efficacy of these techniques at the beginning of the semester. The remainder of this section will discuss the initial beliefs expressed by preservice teachers concerning each of these areas.

"Manipulatives" has been a buzzword in mathematics education over the past two decades. The nature of the initial beliefs of preservice teachers concerning manipulatives is captured by the following statements made by preservice teachers at the beginning of their respective methods course. Diane (pseudonyms have been used for all respondents throughout this study), a secondary major, stated that, "Manipulatives are very effective in helping to teach mathematics.... The information gained by a discovery from manipulation is much more valuable and meaningful." Judy, an elementary major, held similar views, "I believe in the use of manipulatives in the teaching of mathematics. I believe that hands-on experiences and the ability to manipulate objects increases the retention and understanding of a concept."

Similarly, cooperative learning is another phrase that
education majors have probably been exposed to during their college career. When asked to write about cooperative learning at the beginning of the course, the great majority of respondents wrote favorably on its use in mathematics education. Donald, a secondary major, wrote, "Using this technique [cooperative learning] is very beneficial in that students are usually active. They will be able to exchange ideas and work on problems as a group. They will be able to (hopefully) learn new ideas from other students." Steve, another secondary major, wrote, "I believe that cooperative learning is a great way for students to discover things."

Elementary majors also touted the use of cooperative learning. According to Karen, "Group cooperative learning activity allow [sic] an exchange of ideas and really foster growth of the students [sic] knowledge base." Laura wrote, "The cooperative learning experience creates a warm-friendly environment. This environment is one that is conducive to learning. When the child is comfortable with math, s/he will be interested in learning more about it."

Review of the initial written responses of respondents to the question, "What are your beliefs concerning the use of technological aids (calculators, computers, etc.) in the teaching of mathematics?" also indicated that most held favorable views
in this area. Many respondents felt that technology could provide an important learning tool within the mathematics curriculum. Clint, a secondary major, wrote,

With the accuracy of the computer, students will be able to formulate their own rules with further examples. This allows them to discover the concepts on their own without having the concepts and/or methods lectured to them. This makes the learning more meaningful.

Robin, an elementary major, wrote, "I feel that the instruments listed could help a child learn to understand or view math concepts in a different perspective. This new way of seeing information could help them process or learn math differently." Elementary major, Mary, provides another perspective on the use of technology as a learning tool, "Computers will be wonderful tools, especially for younger children because of the software available, example - drill and practice. This type of software is beneficial for it gives the students extra practice if needed."

Many other respondents expressed a positive view toward the use of technology based on its potential to motivate students. Cheryl, a secondary major, posited that, "[technological aids] make math more interesting to students, more fun, more productive, as well as relative to real-life. It gives a student a chance to explore avenues not as readily available without aids."
Robin, an elementary major, felt that, "The use of technology could also spark a new interest in the subject or motivate the child to learn."

In sharp contrast to the initial beliefs held concerning manipulatives and cooperative learning, more than three-quarters of the respondents indicated reservations concerning the use of technology in the mathematics classroom. The most negative response came from Peter, an elementary major, who wrote, "I don't agree with the use of calculators and computers in mathematics. The children use these items as thinking types of crutches.... They become mentally lazy and no longer have to think or reason through a problem." Most of the other reservations expressed were less forceful and were split between a concern for learning the basics first and the danger of students becoming overly dependent on technological aids. The following response by Mary Anne, an elementary major, epitomized the thoughts of many, "I think when teaching the basics it should be pencil, paper, and manipulatives, but once there is an understanding of the basic process... tech aids can be used for more complex work." Emily, a secondary major, wrote, "A dependency can be brought on by excessive use of calculators. When a person constantly uses a calculator there is a tendency to become unsure of yourself. Basic math skills tend to deteriorate when not used."
Another difference found in the initial beliefs concerning the use of technological aids involved the importance of teaching students about technology as an end in itself. None of the respondents made such remarks concerning manipulatives. A few respondents commented on the importance of learning to be able to work cooperatively as an added benefit of cooperative learning, but not nearly in the numbers who supported the use of technology for these reasons. More than half of the respondents indicated that they felt it was important for students to learn to use technology as it is critical to our society today as well as to the work force that they are preparing to enter. Andrea, an elementary major, wrote, "I believe we should embrace the technological advancement in the classroom. If students are to achieve in the society they need to be able to use the technology material fluently when they are out of school." Secondary major, Paula, felt similarly, writing, "The future will be full of technological advances and if our students are not aware of their uses... they will not be able to fully function in society." Richard, a secondary major, added,

I feel it would be a disservice to not instruct students on their [technological aids] use. I believe our job as educators is to prepare students for adult life - especially preparation for the business world. We cannot send these
kids out into the professional world without familiarizing them in computers and other technologies.

**Findings about Beliefs.** Throughout all three phases of the qualitative portion of this study it was clear that students seemed to minimize the changes that had occurred in their beliefs. Several possible explanations exist for this. One is that students responded only that the direction of their beliefs had remained unchanged. In other words, if a student had written a positive statement at the beginning of the semester and their beliefs at the end of the semester were still positive, they responded that their beliefs had not changed. Closer examination of the data, however, often indicated clear changes in belief systems even by students who claimed to the contrary. One way of noting this was the discrepancy between the small number of students who felt that their beliefs in a particular area had changed when compared with the larger number who felt that this course would have an effect on the way in which they incorporated that area into their teaching.

**Changed Beliefs?** Most respondents stated their beliefs concerning the use of manipulatives in the teaching of mathematics had not changed much since the beginning of the
semester. Many, however, went on to clarify their answer using words like "reinforced", "confirmed", or "strengthened" to address the "unchanged change" concerning their beliefs toward the use of manipulatives. Such responses are epitomized by Karen, an elementary education major, "I don't think they changed at all. I think they have grown stronger. I'm more acutely aware of the need for manipulatives." Similarly, secondary major, Steve, said, "They really haven't [changed] because I've always thought they [manipulatives] were really good, what we learned in class, and the experiences we've had just reinforced my belief." Laura, another elementary major who claimed that her beliefs hadn't changed much since the beginning of the semester, said, "The purpose of manipulatives is much more great than I anticipated now than when I wrote this. I didn't realize that through manipulatives how much more learning is involved."

Although for the most part respondents indicated that their beliefs concerning the use of technological aids had remained relatively unchanged, there were some notable exceptions to this rule regarding the concerns expressed initially about students becoming overly dependent on calculators. Clare, an elementary major, said,

In the beginning I thought of them [calculators] as being crutches, but I don't think that anymore. I think that
they are more of a help overall and I don't think it's so
important that kids learn the actual calculations as it is
that they gain the understanding of what they are doing.

Doug, a secondary major, said, "In the beginning of the semester I
still had the concern that students would use computers and
calculators as crutches.... My views have advanced farther.... It
[technology] should be used most of the time."

Another elementary major, Samantha, related the changes
in her beliefs,

In the beginning of the semester I thought only high school
or college should be able to use calculators, but now.... I
believe that it can be used in elementary school and used
for concepts. I thought that calculators were just used for
skills only.... But now, its [my beliefs] changed by knowing
that it can be used for concepts and it can be an aid.

Further testimony for calculator use came from Ellen, an
elementary major, "At the beginning of the semester I said I
thought they should know their basic skills first and now I really
don't believe that. I think that they could be using the calculators
to help them learn their basic skills."

When asked how their beliefs concerning the use of
cooporative learning in the teaching of mathematics had changed
since the beginning of the semester, respondents felt that little
change had occurred. Prior to their methods classes, however, it
seems that many lacked a true understanding of what was meant
by cooperative learning but, nevertheless, responded positively.
The responses written by Mary Anne and Peter at the beginning of
the semester illustrated this phenomenon. Mary Anne, an
elementary major, wrote, "I no [sic] very little about cooperative
learning. What I do know is that it is efficacious [sic]." Elementary
major, Peter, wrote, "I believe it's helpful because research has shown that children can learn by working with their peers."

At the beginning of the semester, Doug, a secondary major,
lamented, "Cooperative learning must be incorporated. How do we
do it in mathematics with much to cover and limited time?" Kim,
another secondary major who had written at the beginning of the
semester that cooperative learning was "great", admitted later
that, "In the beginning of the semester I didn't know what cooperative learning was, but I hid it pretty well." Thus, the
great majority of written responses gathered at the beginning of
the semester were positive but tended to lack foundation in a
true knowledge base concerning cooperative learning.
Consequently, many of the statements in which students
contended that their beliefs had not changed may be based simply
on an agreement with the directionality of their original beliefs.
Effects on Future Teaching. Preservice teachers made powerful statements, as illustrated below, in response to interview question 10, "Do you feel that this mathematics methods class will effect the way in which you use manipulatives in your own classroom?" Stephanie, an elementary major, said,

Definitely! Because... I didn't grow up with it. I didn't use it so I had never seen it in action, I probably wouldn't use it. I had seen it in my other practicum before I had this class, but I felt real [sic] apprehensive because I had never had any training. I definitely will want to use it.

Elementary major, Mary Anne, said,

Oh, definitely! I wouldn't have had any idea how to use a manipulative or really what the different types of manipulatives that were out there available to me as a teacher. Now, I know many different types of manipulatives that are available.... I feel so much more comfortable with them.... I just purchased a starter kit... so I'm planning on using them.

Secondary students also seemed to be strongly affected, as evidenced by Paula's statement,

Definitely! I've been introduced to so many different things this semester... the algebra tiles... I never would have even known what they were if I hadn't taken this course. It's
made me realize how important they are, how they can be used. You gave us a lot of different, good ideas that I can use in my classroom. I plan to use as many of them as I can.

Gina, another secondary major, said, "Definitely! I've been exposed to ideas on how to use different types of manipulatives, like algebra tiles, the geo-boards and I think it made me more open to want to try different approaches and different manipulatives." Secondary major, Leslie, indicated, "I don't know that I would say how I would use it would be the correct term, but I think whether or not I would use them and the answer is yes, and I know the reasons why... I would certainly mimic some of the lessons I had.

In response to interview question 11, "Do you feel that this mathematics methods class will effect the way in which you use technological aids in your own classroom?" Paula, a secondary major, responded, "Definitely! I had no idea of the software that was out there.... I really, really, like the Geometric Sketchpad and the algebra was good too."

Despite the relatively low number of secondary students who indicated that their beliefs had changed concerning the use of cooperative learning, more than three-fourths of them stated
that this methods class would affect the way in which they use cooperative learning in their own classroom. This preponderance represented a combination of changed beliefs, knowledge gained, and first-hand experiences. Indicative of this, Tina responded, "Definitely!... intuitively, I thought it was a good idea, but now I've actually experienced it... I have seen some of the strengths and weaknesses and I think I'll be much more inclined to use it and use it wisely." Personal enjoyment of classroom activities led Peggy to say, "Oh, absolutely! I think our entire class was cooperative learning and the experience itself was enjoyable." Paula added, "We liked the cooperative learning experiences and we did a lot of things at our tables, and I got to see different perspectives, and I really enjoyed that, and I definitely will use it in my classroom." Linda stated, "It makes math more fun if you are doing it with other students... you are learning new things and discovering new things."

Like the secondary preservice teachers, more elementary majors stated that the course would affect their future teaching than had said that their beliefs concerning cooperative learning had changed. More than two-thirds of them indicated that the methods class would affect the way in which they use cooperative learning in the teaching of mathematics. In response to this question, Jean said,
Definitely, because the way she had us in groups in that class... gave us a lot of modeling on how to use groups in your classroom, and what kinds of questions to ask and how they [students] should interact with each other in a group.

Heather added,

Yes.... I've really liked the way that she had us explain how we got to a certain answer and she would have us post it up on the wall. I've never seen that done before and that's something that I definitely want to use in my classroom because you can get the same answer in different ways, and I feel it's important that children can see that.

**Educational Experiences.** The interview data indicated that preservice teachers received little exposure to the use of manipulatives, technological aids, and cooperative learning during their own elementary and secondary mathematics education.

With regard to manipulatives, many of the older students in the study felt that this was a function of the length of time that had elapsed since they were in school. When asked how his teachers had used manipulatives in the classroom, elementary major, Peter, said, "They didn't. I was raised in the 60s." Leslie, another older student, added, "'Manipulatives' was an unknown word in mathematics when I was learning math." Despite the
belief espoused above, the lack of exposure to manipulatives proved not to be related to age. A more traditional student, Harry, appears to have been accurate when he said,

It seems like they [teachers] are more aware of them now [1992] than they were then [1980s], even though it hasn't been a long time ago. I think when I was growing up there really wasn't a great deal of use of manipulatives.

Thus, it appears that although future generations of preservice teachers may come to the college classroom with more experience in the use of manipulatives, currently this experience is lacking, regardless as to whether the student completed high school 4 to 5 years ago, or 20 to 30 years ago.

Similarly, the use of technological aids in the mathematics education of preservice teachers is minimal. Approximately three-fourths of the respondents reported that technology was not used in their mathematics education. The rest indicated that calculators or computers were used sparingly and in most cases this use occurred in only one of the courses that they had taken. No one reported that the use of technological aids was pervasive throughout their mathematics education. The reasons for this paucity of technological experience vary. David said, "We didn't have them. I'm too old!" Unfortunately, many teachers have been slow to incorporate the use of technology despite its
advancements as indicated by Mary Ann, "I got in trouble a couple of times with different teachers for taking my calculator out in class in grade school. It was really frowned upon." Similarly, according to Shelly, "I know we were told we could not use calculators, not in our homework, not anything. The teacher pretty much knew every time we did and we got in trouble for it."

The data also indicated that the educational experiences of preservice teachers included a dearth of cooperative learning activities. Bill, a secondary major, said, "We were never put into tables, it was all straight line, don't talk to your neighbor, be quiet, do your own work." Tina, another secondary major, stated, "Our teachers never put us in groups. It was always strictly lecture, the teacher delivered, you did notes, did your homework, and took a test." Clare, an elementary major, stated, "I don't think cooperative learning was even thought of then. You know, it was you do your own and if you got any help or discussed it with anybody it was considered to be cheating."

Working Hypotheses

The process of open coding and axial coding (Strauss & Corbin, 1990) was applied to the qualitative data and culminated in the generation of four working hypotheses from that data. These working hypotheses concerning the effects of a
mathematics methods course were:

1. Mathematics methods courses provide preservice teachers with important knowledge and experience concerning the use of manipulatives, technological aids, and cooperative learning.

2. Preservice teachers learn mathematical content through their experiences in the mathematics methods course which involve the use of manipulatives, technological aids, and cooperative learning.

3. Preservice teachers are concerned that difficulties might arise which will impede their ability to incorporate the use of manipulatives, technological aids, and cooperative learning in their classrooms.

4. Preservice teachers leave their mathematics methods course with a feeling that they need to learn more about the use of manipulatives, technological aids, and cooperative learning.

The degree to which each of these working hypotheses pervades preservice teachers' beliefs concerning the use of manipulatives, technological aids, and cooperative learning at the conclusion of the mathematics methods course will be discussed in the following sections.

**Increased Pedagogical Knowledge**

Mathematics methods courses provide preservice teachers...
with important knowledge and experience concerning the use of manipulatives, technological aids, and cooperative learning.

**Manipulatives.** This hypothesis is evident in preservice teachers' responses concerning manipulatives. Heather said, "I've always believed in using manipulatives, but I've gotten more experienced using them. I can see that I would feel more comfortable now." Robin said, "I've learned how to use them [manipulatives] differently in ways I never thought." Mary Anne said, "The major change is now I have some experience with them [manipulatives] where before I didn't have any experience." Shelly said, "I think this class has just kind of helped me to understand the different ways that you can use them [manipulatives]. I still believe that they are important." Similarly, Richard said, "I've seen more ways that you could implement them [manipulatives] and right ways and wrong ways to introduce them."

Other respondents indicated that the newly gained knowledge concerning manipulatives showed them that manipulatives have a much more comprehensive utility than they had previously believed. Cindy said,

They [my beliefs] haven't really changed a lot, except that I previously thought that you mostly used them [manipulatives] with really young children, but I can see how you can use [them]... all the way through... maybe even
into high school.

Similarly, Doug said,

I can see these elementary students using blocks all the time. They are using manipulatives. Does that mean that they're not acting their age when they are in junior high school and high school, if they are using blocks? I don't believe that is the case anymore. I think that we need to re-evaluate the situation. So, I believe my views have intensified.

Samantha said,

I thought that using manipulatives were only just a positive approach to get them from having anxiety. Now I believe it's more than the anxiety. It has to do with actual concepts and seeing the concepts in front of you.

Laura stated that, "The purpose of manipulatives is much more great [sic] than I anticipated now than when I wrote this [beginning of semester writing]. I didn't realize that through manipulatives how much more learning is involved." Kim said,

I thought it [manipulatives] was good to get them excited, it was more of a thing to get excited with, at the beginning, and not for them to actually use it as a learning tool, and now I think it is an actual tool of learning.

Another secondary major, Emily, said,
I thought they [manipulatives] would be good only in lower levels of mathematics.... Now I can understand how it can be used in higher levels, like with algebra tiles.... And we've learned a lot of different ways to use them now, as opposed to, I thought it was just very basic little things, putting blocks together.

Secondary major, Donald, added,
I'd first indicated that they'd most probably be [of] greater importance during the earlier stages. But as we continued, I think that, obviously, older adolescents can be just as well... suited for this and they [manipulatives] can have a great importance in their learning.

Elementary major, Shelly, said, "With this class I've just expanded my knowledge on how to use the certain manipulatives. They are not just limited to one area anymore. They can be used across the math curriculum."

Some students like Paula perceived certain dangers concerning the use of manipulatives, but the methods course allayed these fears,
In the beginning of the course I was in favor of using them [manipulatives], but I was afraid that they [students] might become too dependent upon them so I said to use them in later years, but now I think that they should be used at all
levels and as much as possible.
Similarly, Tina stated, "Now I think I feel more positive toward
them as a tool, still having that slight hesitancy about becoming
dependent upon them, but not to the degree that I was at the
beginning of the class."

**Technological Aids.** Preservice teachers also indicated that
the mathematics methods course provided them with a stronger
knowledge base concerning the use of technological aids in the
teaching of mathematics. Many students indicated that they had
experienced aspects of technology of which they were previously
unaware. Kim, a secondary major, said, "I'd never seen a program
like that [Math Connections]. I was totally amazed." Elementary
major, Cindy, said, "I'd never seen the calculator that she put on
the overhead before, and there were other people in the class too
that had never seen that before. We were going, 'What is this?'"

**Cooperative Learning.** An improved knowledge base with
regard to cooperative learning was also found among preservice
teachers. Doug, who had expressed concern over classroom
management while incorporating cooperative learning, said, "I
find that, as opposed to the beginning, I feel that in cooperative
learning, in the long run, you are getting more control because
they are interested in what's going on." Another significant
change in beliefs was reported by Gina,
Before... I would just think they would be working together to help each other on homework. But, now it's more like discussion within a group.... It's to try to bounce ideas off of each other, and that way they'll be more open to mathematics.

Carl, the only secondary major who indicated that his beliefs concerning cooperative learning had become more negative, said, "I was probably more in favor of cooperative learning in the beginning than I am now. I think that some people just don't learn that way and they are better off learning on their own." Even he deigned to admit, however, that, "I do think cooperative learning is helpful to some students."

On the elementary side of the ledger, most students felt that their beliefs had remained basically the same in this area. Some reported that the experiences and learning that took place during the semester were valuable, Erica said, "I enjoyed working in cooperative groups this semester. I got to see how the dynamics worked a lot better by participating." Robin added, "My beliefs haven't changed, I just have a greater understanding of how to use it [cooperative learning] in a classroom." Others in the unchanged category reported a reinforcement in their beliefs as stated by Mary, "My beliefs in the beginning of the semester are the same, if not strengthened."
Several elementary majors reported significant changes in their beliefs concerning cooperative learning since the beginning of the semester. Ellen said, "Cooperative learning is really hard, so maybe just having a peer tutor would be better." Carolyn indicated that the way in which she would implement cooperative learning had changed, "I wouldn't put all the low kids together as maybe I would have in the beginning."

A few elementary majors indicated that their beliefs had become much more positive concerning the use of cooperative learning in the teaching of mathematics. One such statement came from Laura who indicated that she had gained an, "awareness of how many things... can be done with cooperative learning. I didn't realize that it was such a great arena."

**Content Learned through these Strategies**

Preservice teachers learn mathematical content through their experiences in the mathematics methods course which involve the use of manipulatives, technological aids, and cooperative learning. The recognition that these techniques are helpful to their own learning process provides preservice teachers with an incentive to use similar techniques with their students.

The paucity of actual learning experiences with
manipulatives that preservice teachers bring with them to college intensifies the importance of their use in the mathematics methods course. The opportunity to actually learn with them becomes paramount. Testimony on the efficacy of manipulatives came from Harry, a secondary student, who said, "From what we've done in class I'm really surprised about how well they work, with me at least. I'm real [sic] surprised at how easy it is to use them." Laura, an elementary major, felt similarly,

I had never been introduced to counting sticks and things of that nature, but knowing that the tools are out there and being able to handle them myself, and seeing the developmental thought process, and seeing that I learned to understand how mathematics works, encourages me to provide them to students.

Another elementary major, Sarah, said, "I didn't even understand most of the [mathematical] concepts until this class, so this class has really helped to make me understand how this all fits in place by actually using manipulatives." Bill, a secondary major, said,

I never really worked with them [manipulatives]... we didn't have them in high school.... Now, I've seen how they [manipulatives] work. It's not just a theory anymore or
something I read. I've been able to experience manipulatives and saw how they affected me, and I can kind of adapt that to a student in their learning.

Ellen supported the notion that preservice teachers learn while using technological aids in their methods course, "It was the hands-on learning with the calculator in class when we did different computations of the calculator and it was really fun and we still learned." Similarly, Becky stated that, "After you start working with it [calculator] you realize that is when the learning takes place... you have just a little bit of instruction to lead you into creating your own thought patterns."

Jennifer found that cooperative learning was helpful to her learning process. She said,

I could pretty much see myself putting my students in groups a lot more than I would have at the beginning of the semester since we learned that way.... I think if it helped me at my age, it would be helpful to smaller children too.

A Concern with Reality

Preservice teachers are concerned that difficulties might arise which will impede their ability to incorporate the use of manipulatives, technological aids, and cooperative learning in
their classrooms. These difficulties include: (a) insufficient time available to adequately teach concepts using these strategies, (b) unavailability of materials, and (c) classroom management.

The incorporation of manipulatives into the real-world of the classroom concerns many preservice teachers. Karen, an elementary major, ruminated on her experiences in a field-based experience required by the methods course,

I had what I thought was a wonderful activity and the student I was working with in a matter of three minutes destroyed the set.... Not only will I be using manipulatives, but I will be more careful about the types of manipulatives I use and the design that I use and the durability.

Emily, a secondary major, said,

I'm thinking that this all sounds wonderful and dandy on paper, but once you are in a classroom with 30 kids with 50 minutes and you spend 5 minutes organizing, hand out manipulatives, do this and that, I think the one point you are trying to get across in your manipulatives is going to take three days, as opposed to write it on the board.... I think there is a time factor for manipulatives.

With regard to the incorporation of technology into the
classroom, preservice teachers indicated a concern over the accessibility of calculators and computers in their future schools. Steve, a secondary major, said, "I don't know how [much] the math teachers are allowed to work in the computer room. If I can get those programs... definitely, but, it's just a matter of availability." Elementary major, Amy, said, "The calculators that she introduced us to were very interesting. If they were available to me in a classroom setting, then I would use them. That's the problem. Not all this stuff is available to us." In a slight deviation on this theme, secondary major, Leslie, considered the philosophical underpinnings of her future school, "I think its [sic] [using technological aids] a function of the particular school you are in. If the school's for it, you'll do it, and if the school is against it, you won't." Linda, the only respondent who indicated that her beliefs toward the use of technology moved in the negative direction as a result of their methods course, said, "I think I was a lot more idealistic about the use of computers because I thought there would be more access to computers.... I don't think many people [teachers] have access to the computers."

The difficulty of implementing cooperative learning strategies in their classrooms was a concern of some preservice teachers. Donald, a secondary major, felt that, "The class size
may be a deterrent in incorporating such a measure [cooperative learning], also, the cultural make-up of the class. Some students may not wish to work with other students." Ellen, an elementary major, said, "It's hard using cooperative learning... you have to be realistic... 2nd grade and 1st grade, they're not ready for that. I've been trying and trying and they just grab and push and they are awful to each other."

**There is More to Learn**

Preservice teachers leave their mathematics methods course with a feeling that they need to learn more about the use of manipulatives, technological aids, and cooperative learning. Some students felt that the work with manipulatives that they experienced in the mathematics methods course was not as much as they actually needed. Peter, an elementary major, indicated this belief when he said, "I gained just enough information from this class to realize that I don't know enough about manipulatives." Leslie, a secondary major, said, "I think that there should be more classes that you can take as electives... where you could learn... how to use different manipulatives to demonstrate different things to kids." Hopefully, Tina, a secondary major, was correct when she said, "I'm concerned about needing a lot more practice to use them, but I think the desire is
there. I think that's the most important thing."

For many students the exposure to technology that occurred during their methods course made them realize that more learning would be necessary in this area also. Jean, an elementary major, said, "I haven't really had a chance to really use them [calculators/computers], so I wouldn't feel comfortable, like explaining it to my children, so I would have to learn more."

Secondary major, Peggy, said,

I think this class has given me something to think about, to realize that I need to go out and get more. I think that they [calculators/computers] are very good educational aids. I believe that I need to know more about them. I plan to know more about them.

**Conclusions from Qualitative Data**

The experiences of preservice teachers in a mathematics methods course helps them develop favorable beliefs toward the use of manipulatives, technological aids, and cooperative learning. Such beliefs might override their natural tendency to teach in the way in which they have been taught. This conclusion is supported by Emily, a secondary major,

Before this, I never even thought of using manipulatives. I was going to do just the way I was taught. You know, math
is math. I have the basic knowledge, these are the facts, write it down, memorize it, this is how you use it. Now I found out there are a lot of different ways to make the learning of math fun and keep them interested. This theory is also supported by Cheryl's statement, "In this class we used them [manipulatives] a lot, which I never did in school, and by me being exposed to them it's opened my mind and given me ideas to introduce students to them."

Although it remains unclear whether a one-semester course can effect changes in beliefs that have developed over a lifetime, the powerful statements of some of the participants are encouraging. Mary Anne said, "I used to think of math as worksheets and drill and solitary work. I don't think of it as that anymore. Definitely not!" Similarly, Donna stated that, "If it was not for this class... I would have been thinking more along the lines of the paper and pencil activities. I've seen a lot of creative ideas that I can use from this class." These statements seem to indicate that a mathematics methods course can have a profound impact on the preservice teachers enrolled in it. Further, it seems that such a course might be capable of providing part of what is necessary to effect the type of substantive reform that mathematics education in this country needs.
Chapter 5

Discussion and Implications

This chapter will present: (a) recommendations for the improvement of educational practice in mathematics which are suggested by the results of this study, (b) suggestions for further research on questions that this study did not adequately answer, and (c) suggestions for further research based on researchable questions that were identified by this study.

Preservice Elementary Teachers Heading in the Right Direction

At the conclusion of the elementary mathematics methods course, preservice teachers were capable of answering only 16.9 out of 24 questions correctly on a sixth-grade level test. Despite the significant improvement that this figure represents, much room for further improvement remains. Similarly, the post scores of attitude for preservice elementary teachers is less than scintillating. Despite the significant improvement in this area, an average of 43.3 on a scale ranging from 0 to 80 leaves these preservice elementary teachers feeling neutral toward mathematics.
Since both the attitudes toward mathematics as well as the meaningful knowledge of mathematical content of preservice elementary teachers improved significantly from the beginning of the course to its conclusion, it is clear that courses such as the elementary mathematics methods course described in this study are beneficial. It seems reasonable to conclude that more time spent in courses which combine mathematical content knowledge with the pedagogical strategies used in this course would be of further benefit to preservice elementary teachers. This can be accomplished either by increasing the number of hours of mathematics methods courses that preservice elementary teachers must take, or by insuring that the mathematical content courses that they are required to take are taught by appropriate pedagogies.

**Improve Problematic Areas of Preservice Elementary**

Preservice elementary teachers performed particularly poorly at the end of the methods course on questions: 4 (33.3% correct), 9 (48.2%), 12 (29.6%), 19 (14.8%), 22 (48.2), 24 (51.9%), and 25 (29.6%). Two of these questions involve fractions, one involves long division, two involve geometry, and the final two cover probability and statistics.

Long division and fractions are topics which have often
caused difficulty for students. More time spent on developing conceptual knowledge of these topics in the required course for preservice elementary teachers should be beneficial to them.

Geometry is often the course in high school which frustrates many otherwise successful math students. One possible reason for this is that many high school geometry courses are taught at a level that the students are unable to understand (Crowley, 1987). Many preservice elementary teachers may still not have attained the van Hiele level at which most of high school mathematics is taught. Therefore, attempting to re-teach this material to preservice elementary teachers at a level that they were unable to understand in the past may result in a similar lack of success during their preservice education. Thus, it is particularly important that mathematics methods courses and/or mathematics content courses for preservice elementary teachers cover geometric topics more informally using appropriate techniques. Such experiences might help these individuals move through the sequential levels of the van Hiele model of the development of geometric thought. Knowledge of the van Hiele model and the ability to incorporate informal geometry into their elementary mathematics agenda could help future elementary teachers better prepare their students for the geometry that they will study in
high school.

The final two areas which proved particularly difficult for preservice elementary teachers were probability and statistics. The traditional mathematics curriculum has often shortchanged both of these areas (NCTM, 1989). Thus, it is possible that many preservice elementary teachers have never studied probability and statistics in their own educational backgrounds. The National Council of Teachers of Mathematics' Standards (1989) recommended that these topics be integrated at all levels of the curriculum. Clearly, if elementary teachers are to incorporate probability and statistics into their classrooms, they must understand these topics and their connections to other mathematical content. Mathematics content courses and mathematics methods courses must, therefore, adapt to these new goals in mathematics education and provide preservice elementary teachers with this knowledge.

**Improve Problematic Areas of Preservice Secondary**

Preservice secondary mathematics teachers also fared poorly on the posttest of meaningful mathematical content on certain topics. Less than 80% of this group answered correctly on 7 of the sixth-grade level questions on this test. These relatively poor performances occurred on questions 4 (79.0%
correct), 9 (73.7%), 11 (63.2%), 12 (68.4%), 19 (52.6%), 24 (68.4%), and 25 (79.0%). Three of these questions involve fractions, two involve geometry, and the final two cover probability and statistics. Interestingly, most of these questions are the same ones which preservice elementary teachers had the most difficulty answering.

Teachers of secondary mathematics methods courses should be aware that their students are sometimes deficient in unexpected areas. The poor performance on questions involving fractions is an example. One reason for the poor results in this area could be that many preservice secondary mathematics teachers' knowledge of fractions is ruled-based while this content test stressed meaningful mathematical content. Covering topics such as fractions using geoboards and Cuisenaire rods will not only provide preservice secondary mathematics teachers with important pedagogical knowledge for teaching, it may also increase their meaningful knowledge of this topic.

As previously mentioned, geometry often proves difficult even for students that experience success in most other areas of mathematics. Results from this study suggest that preservice secondary mathematics teachers, like their elementary counterparts, need to learn geometry informally so that they can progress through the sequential levels of the van Hiele model of
geometric thought. Further, an awareness of this model by secondary mathematics teachers should be beneficial to them as well as to their future students.

Traditionally, calculus has been considered the capstone of high school mathematics (NCTM, 1989). Therefore, it is likely that many preservice secondary mathematics teachers were pushed through a curricula which did not include the study of probability and statistics. In contrast, the National Council of Teachers of Mathematics recommended that more time be spent exploring mathematical connections in courses which precede calculus (NCTM, 1989). A wealth of fascinating opportunities to integrate probability into algebra, geometry, and pre-calculus courses exist. Preservice secondary mathematics teachers must be encouraged to apply their mathematical knowledge to find such connections and, in the process, improve their knowledge base in probability and statistics. An interesting ramification for the secondary mathematics methods teacher is that the dearth of experience in the topics of probability and statistics allows for a more realistic modeling of potential lessons, as in many cases, the preservice mathematics teachers are actually learning this content for the first time.
Encourage Student Teachers

The responses of preservice elementary and preservice secondary mathematics teachers with regard to the potential effects of their mathematics methods course on the way in which they will teach mathematics is encouraging. These preservice teachers have just completed 15 weeks of mathematical learning which is quite different from the ways in which they learned mathematics for the previous 20 to 40 years of their lives. They are excited by many of the pedagogical techniques that have been modeled for them and are enthusiastic about implementing them in their own classrooms.

It is essential that these preservice teachers be encouraged to teach using the strategies they have seen in their methods course. It is vitally important that the teachers who serve as their mentors during student-teaching continue to foster and encourage the implementation of the pedagogical practices discussed above. Ideally these mentors should be knowledgeable of these techniques and use them in their classrooms during the initial weeks when the student teacher is observing. Seeing the strategies in action in a real classroom will reinforce the preservice teachers' beliefs concerning their use. They will feel more comfortable implementing these pedagogies knowing that their students have been exposed to them before, and that the
mentor teacher is capable of helping them.

Facilitating the goals described above can be accomplished by providing appropriate course work to mentor teachers to insure that they are sufficiently knowledgeable concerning the use of manipulatives, technological aids, and cooperative learning. Further, mentor teachers should be specifically selected for their expertise in these areas. The student-teaching experience is often referred to as the culmination of the preservice teacher's college experience. Only when this experience truly allows student teachers to apply and build on the things that they have learned in their methods course will this statement be accurate.

**Provide More Mathematics Pedagogy**

A plethora of comments made by preservice elementary teachers and preservice secondary mathematics teachers indicated their desire to learn more concerning certain aspects of mathematical pedagogy. Such comments were particularly common regarding the use of manipulatives in the teaching of mathematics and in the use of technological aids in the teaching of mathematics. A single three-credit mathematics methods course does not seem adequate to cover all of the mathematical pedagogy that preservice teachers should know.
Attempts to resolve the inadequacies of a single three-credit mathematics methods course must be grounded in the reality of adding course work to an already full complement of graduation requirements. At the secondary level, the development of a course specifically concerning the use of manipulatives in the teaching of mathematics, and a course specifically concerning the incorporation of technology in the teaching of mathematics, should be considered. The requirement of a general methods course for preservice secondary mathematics teachers must be seriously questioned in light of the amount of subject-specific pedagogy that must be learned.

At the elementary level, more activities involving the use of manipulatives and technological aids should be included in the practicum. The required elementary mathematics methods course should continue to integrate the use of manipulatives in the teaching of mathematics, and the use of technological aids in the teaching of mathematics, into its curricula.

Another means of improving the pedagogical knowledge of preservice teachers at both the elementary and the secondary level would be to insure that the mathematical content courses they are required to take employ manipulatives and technological aids. The pedagogical modeling that such courses would provide could be extremely beneficial to these preservice teachers.
The importance of additional training for preservice teachers with regard to technological aids is underscored by the recommendations of many experts that calculators be incorporated at every level (NCTM, 1991), and that students use calculators as often as textbooks (Bitter & Hatfield, 1991). Providing additional training for preservice teachers in the use of technological aids is consistent with the findings of Bitter and Hatfield (1991) who warned that the successful incorporation of calculators would probably not occur without such support.

Can Secondary Methods Improve Content Knowledge?

Despite the non-significant change of meaningful knowledge of mathematical content in this study, researchers should continue to consider whether or not a secondary mathematics methods course can effect such a change. A test designed to measure secondary mathematics content may be a more appropriate tool to use for this purpose than the one employed in this study. The number of students (19) participating in the secondary portion of the study was barely acceptable for quantitative purposes. Further, many of these students were non-traditional. The results of this study do not preclude that a larger sample from a more traditional population might yield a significant increase in mathematical content knowledge.
Can Secondary Methods Improve Attitudes?

The increase measured in this study for the attitude toward mathematics of preservice secondary teachers was not significant. As noted above, the size of the sample and its non-traditional nature must be taken into account when considering this result. Further, the number of students who were simultaneously taking high-level mathematical content courses may have confounded the variable being studied. Enrollment in such courses might have had a detrimental effect on the attitudes toward mathematics of these students.

The result of the present study does not preclude that a secondary mathematics course could significantly improve the attitudes toward mathematics of preservice secondary mathematics teachers in another setting. Studies of the attitude change caused by such a course should be carried out with a sample which: (a) includes more than 19 respondents, (b) is more traditional in nature, and (c) includes only respondents who are not taking a mathematical content course concurrently.

Actualization of Beliefs

Studies need to be devised and carried out which investigate whether or not preservice teachers who have completed mathematics methods courses similar to the ones
considered in the present study actually follow through on their revised beliefs and incorporate the use of manipulatives, the use of technological aids, and cooperative learning, into their teaching. Longitudinal studies could be carried out on a purposively selected sample of preservice teachers. The participants in such studies could be observed and interviewed throughout their student-teaching experience, as well as during their first several years of teaching.

Can Practicing Teachers Change?

Studies should also be designed to investigate the effects of graduate mathematics education courses on practicing teachers. Many of the same questions asked in the present study could be adapted to this new population. The researcher could then attempt to determine if the effects of the graduate course work has any impact on the teaching styles of the participating teachers. A multiple case study design could investigate the differences between the effects of undergraduate mathematics methods courses on preservice teachers and the effects of graduate mathematics methods courses on practicing teachers.
Conclusion

Evidence has been provided that an elementary mathematics methods course can improve the attitudes toward mathematics, as well as the meaningful knowledge of mathematical content, of preservice elementary teachers. Further, preservice teachers at both the elementary and secondary level have been shown to be receptive to the mathematical pedagogies recommended by the National Council of Teachers of Mathematics. These results provide hope and encouragement for teachers of mathematics methods courses as they face the challenge of preparing preservice teachers to reform mathematics education in the next millennium.
References


students: Implications for the classroom. Arithmetic Teacher, 32(7), 45-49.


Appendix I  AIKEN REVISED MATH ATTITUDE SCALE

Directions: Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to express, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your own personal feeling. The five points are: Strongly Disagree (SD), Disagree (D), Undecided (U), Agree (A), Strongly Agree (SA). You are to encircle the letter(s) which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU.

1. I am always under a terrible strain in a math class.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

2. I do not like mathematics, and it scares me to have to take it.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

3. Mathematics is very interesting to me, and I enjoy math courses.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

4. Mathematics is fascinating and fun.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

5. Mathematics makes me feel secure, and at the same time it is stimulating.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

6. My mind goes blank, and I am unable to think clearly when working math.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

7. I feel a sense of insecurity when attempting mathematics.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]

8. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.
   \[
   \begin{array}{cccccc}
   & SD & D & U & A & SA \\
   \end{array}
   \]
9. The feeling that I have toward mathematics is a good feeling.

10. Mathematics makes me feel as though I'm lost in a jungle of numbers and can't find my way out.

11. Mathematics is something which I enjoy a great deal.

12. When I hear the word math, I have a feeling of dislike.

13. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.


15. Mathematics is a course in school which I have always enjoyed studying.

16. It makes me nervous to even think about having to do a math problem.

17. I have never liked math, and it is my most dreaded subject.

18. I am happier in a math class than in any other class.

19. I feel at ease in mathematics, and I like it very much.

20. I feel a definite positive reaction to mathematics; its enjoyable.
Appendix II Revised Essential Elements of Elementary Mathematics Test

The purpose of this test is to determine whether you can explain mathematical facts and rules meaningfully, showing that you understand the principles involved. Read all answers carefully and choose the best one, keeping in mind the emphasis on meaning. Circle the letter which corresponds to the correct answer.

1. Of these four numbers, three are the same. Which one is different?
   A. 64 hundreds and 59 ones
   B. 6 thousands, 3 hundreds, 4 tens, and 9 ones
   C. 63 tens and 49 ones
   D. 634 tens and 9 ones

2. $6 = 2 \times 3$ shows 6 as a product of prime factors. Which of the following shows 60 as a product of prime factors?
   A. $60 = 3 \times 4 \times 5$
   B. $60 = 6 \times 10$
   C. $60 = 1 \times 60$
   D. $60 = 2 \times 2 \times 3 \times 5$

3. Three of the following numbers are equal. Which one is different from the other three?
   A. $2/100$
   B. 0.2
   C. $1/5$
   D. 20%

4. Of the following numbers, choose the smallest
   A. $2/3$
   B. $8/9$
   C. $8/11$
   D. $7/8$

5. The sum of two negative numbers is
   A. less than either addend
   B. greater than either addend, since two negatives make a positive
   C. possibly smaller or larger, depending on the two numbers
   D. equal to the difference of the two numbers if they were positive

6. In the sentence $18 \div 6 = 3$, 18 represents a number of cookies.
   A. Neither 6 nor 3 can represent a number of children.
   B. If 6 represents a number of cookies, then 3 represents a number of cookies.
   C. If 6 represents a number of children, then 3 represents a number of cookies.
   D. 6 must represent a number of cookies.
7. In the division $56 \div 624$, the first quotient digit is written

A. over the 2 because 56 won't go into 600.
B. over the 2 because $10 \times 56 = 560$ and $100 \times 56 = 5600$; therefore the quotient will be between 10 and 99.
C. over the 6 because the dividends first digit indicates 600, and 600 is larger than 56.
D. over the 4 because $1 \times 56 = 56$.

8. When a certain number is divided by 8, the quotient is a number we will call $n$. In other words, $X \div 8 = n$. If you know what number $n$ is, you can find $X$ by

A. dividing 8 by $n$
B. dividing $n$ by 8
C. multiplying 8 by $n$
D. adding 8 and $n$.

9. If this entire rectangle represents one unit, the cross-hatched part represents

A. $(2/6) \times (1/2)$
B. $(1/2) \times (2/3)$
C. $(2/3) \times (1/3)$
D. $(1/3) \times (1/2)$

10. The number $10 \div (1/2)$ represents the solution to the problem

A. How many boys came to the club meeting if half of the ten children present were boys?
B. With 10 sticks of gum, how many children can have gum if each gets 1/2 stick.
C. Give each of 2 children half of a box of 10 apples. How many apples will each get?
D. Divide 10 crayons equally between two boxes. How many crayons will be in each box?
11. To explain the meaning of \((\frac{1}{2})^{-1} \div (\frac{1}{4})\), you might say

A. Invert (turn over) the \((\frac{1}{4})\).

B. Suppose you have \(\frac{1}{2}\) yard of ribbon and you need to cut it into pieces each \(\frac{1}{4}\) yard in length.

C. Multiply \((\frac{2}{1}) \times (\frac{1}{4})\)

D. Suppose you have \(\frac{1}{2}\) of a candy bar and you want to give some to each of 4 children.

12. In the division \(1.3 \div 62.48\) when you move the decimal point in the divisor past the 3 and then move the decimal point in the dividend, you are

A. multiplying divisor by 10 and dividend by 100.

B. not changing either dividend or divisor.

C. dividing divisor and dividend by 100.

D. not changing the quotient.

13. To estimate \(43 \times 28\) by rounding to the nearest 10, think

A. \(40 \times 20\)

B. \(40 \times 30\)

C. \(45 \times 25\)

D. \(50 \times 30\)

14. Of the following, the best estimate for \(18.5 \div 1.52\) is

A. 0.19

B. 1

C. 12

D. 113

15. 5% of $170 indicates the amount you would have if you

A. divide $170 into 100 equal parts and take 5 of the parts.

B. divide $170 into 5 equal parts and take 1 of the parts.

C. divide $170 into 10 equal parts and take 2 of the parts.

D. take 5 times $170 and move the decimal point 2 places to the right.

16. You have travelled 240 miles since you filled your gasoline tank. Your car averages 16 miles per gallon. The cost of gasoline is $1.15 per gallon. You have a $20 bill. To determine what amount of money, if any, you will have left after filling the tank, you might perform the following operations in the given order:

A. divide, subtract, multiply

B. divide, multiply, subtract

C. multiply, divide, subtract

D. subtract, divide, multiply
17. If \(2n + 1 = 18\), \(n\) is
A. 7.55
B. 8
C. less than 8
D. greater than 8

18. Choose the correct statement:
A. The area of figure A is greater than that of figure B.
B. The perimeter of figure A is less than that of figure B.
C. The area of figure A is less than that of figure B.
D. The perimeter of figure A is greater than that of figure B.

19. Points F and C on line DG. ABCD is a rectangle.

The area of triangle ABG is:
A. less than half the area of parallelogram AFGB.
B. more than half the area of parallelogram AFGB.
C. half the area of rectangle ADCB.
D. not determined by the conditions.

20. A piece of tape 1.6 meters in length is to be cut in equal lengths measuring 20 centimeters each. How many pieces of tape can be produced?
A. 8
B. 0.8
C. 32
D. 12.5
21. A strip of paper 3 yards, 5 inches long is taped on a wall for a mural. Another piece 1 yard, 2 feet, 7 inches long is taped end-to-end with the first piece, giving a total length of:

A. 5 yards  
B. 4 yards, 3 feet, 2 inches  
C. 5 yards, 2 feet  
D. 5 yards, 2 inches

22. A circular flower garden has a diameter of 10 feet. Approximately how many feet of border are needed to enclose the garden?

A. 18 feet  
B. 32 feet  
C. 28 feet  
D. 22 feet

23. On a number line, the point halfway between 1.3 and 3.9 is

A. 2.2  
B. 2.6  
C. 2.8  
D. 3.0

24. The average (mean) of 4 whole numbers is 16. Two of the numbers are 32 and 2. The other two numbers are

A. both greater than 2  
B. both less than 32  
C. both 16  
D. equal

25. A bag contains some marbles. You reach in and pull out one marble without looking. The probability of drawing a blue marble is 2/5 if the bag contains

A. 2 blue, 5 red, 3 yellow  
B. 4 blue, 10 red  
C. 4 blue, 3 red, 3 yellow  
D. 2 blue, 5 yellow

26. Describe the types of thought processes that you went through in answering these questions. Are there differences between this test and other mathematical content tests that you have taken? Explain why or why not. (Use back if necessary.)
Appendix III

Consent to Participate in a Research Study
University of Nevada-Las Vegas

Title: The Effects of Mathematics Methods Courses on the Mathematical Attitudes, Content Knowledge, and Pedagogy of Preservice Teachers.

Purpose: You are being asked to participate in a research study. This study hopes to learn the effects of a mathematics methods course on the attitudes and content knowledge of preservice teachers.

Subjects: You have been selected because you are enrolled in one of the following methods courses designed for preservice teachers: CIE 452 or ESE 416.

Procedures: If you decide to volunteer, you will be asked to take a pretest and a posttest of your mathematical knowledge and an attitude survey toward mathematics. The pretests will be given during the first week of classes, and the posttests will be given during the last two weeks of classes. The attitude survey should take about 15 minutes, the content test about 25 minutes. You will also be asked to undergo a structured interview concerning your experiences in the mathematics methods course.

Benefits: The goal of this research is to make recommendations for improving the quality of preservice teacher education in mathematics.

Confidentiality: The researcher will maintain the confidentiality of the data which your tests provide.

Compensation: You will be provided with a packet of interesting mathematics educational materials as a token of appreciation for your participation in this project.

Rights: Your participation is entirely voluntary, and you may withdraw from the project at any time without consequence. Your participation in this study represents consent for the researcher to use the data collected for dissertation purposes, as well as future professional presentation or publication.

Questions: If you have any questions please feel free to ask Robert J. Quinn, ICS Doctoral student, office CEB 237A.

Respondent signature

________________________________________________________________________Date_________________
Appendix IV - Interview Questions

1. What are your current beliefs concerning the use of manipulatives in the teaching of mathematics?

2. How have your beliefs concerning the use of manipulatives in the teaching of mathematics changed since the beginning of the semester?

3. What are your current beliefs concerning the use of technological aids (calculators, computers, etc.) in the teaching of mathematics?

4. How have your beliefs concerning the use of technological aids in the teaching of mathematics changed since the beginning of the semester?

5. What are your current beliefs concerning the use of cooperative learning in the teaching of mathematics?

6. How have your beliefs concerning the use of cooperative learning in the teaching of mathematics changed since the beginning of the semester?
7. How did your teachers use manipulatives when you were learning mathematics?

8. How did your teachers use technological aids when you were learning mathematics?

9. How did your teachers use cooperative learning when you were learning mathematics?

10. Do you feel that this mathematics methods class will effect the way in which you use manipulatives in your own classroom? Please explain.

11. Do you feel that this mathematics methods class will effect the way in which you use technological aids in your own classroom? Please explain.

12. Do you feel that this mathematics methods class will effect the way in which you use cooperative learning strategies in your own classroom? Please explain.

13. The final question is an open ended one. Do you have anything that you would like to add concerning the use of manipulatives, technological aids, or cooperative learning which you feel that these questions did not cover?