A novel, remote cleaning, electromagnetic air filter

Zhongyi Gu

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A Novel, Remote Cleaning, Electromagnetic Air Filter

by

Zhongyi Gu

A thesis submitted in partial fulfillment of the requirement for the degree of

Master of Science

in

Electrical and Computer Engineering

Electrical and Computer Engineering
University of Nevada, Las Vegas
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ABSTRACT

When the enamel-like top layer of the desert soil is compromised, natural and man-made forces result in airborne dust particulates. This is of specific concern in plutonium contaminated soil excavation. Conventional water spraying techniques are effective in preventing large airborne dust particles but are ineffective for dust particles on the order of a few micrometers in diameter and smaller. One means of extracting these fine radio nuclide particulates from the air is with a quasi-electrostatic air filter which charges, traps, transports, and collects them with the aid of electrostatic and quasi-electrostatic fields. Human intervention is virtually eliminated. The air filter is divided into four sections: the charging region, the electrostatic trapping region, the transport region, and the collection region. This work focuses on the first three regions of the air filter. The charging region employs a photo-ionization mechanism to ionize the sand particles just below the breakdown of air. Large electrostatic fields have been tailored to extract the particles from the charging region and direct them into the transport region. The dynamic fields in this region guide the particulate to a collection region. By combining a finite element method with an analytical theory to characterize the fields in the air filter, single particle dynamics in the charging, electrostatic and the transport regions of the air filter are examined. Design constraints and limitations are studied. Air flow velocities and air viscosity
contributions are incorporated into the simulation. Normalized expressions allow for a large host of upscale or downscale designs. The electrostatic and quasi-electrostatic forces always counteract the usually dominant viscous forces of air.
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CHAPTER 1

INTRODUCTION

The research, design, and construction of electrostatic precipitators have been conducted from as early as 1820 by M. Hohlfeld [1]. A history of the evolution of electrostatic precipitators up to and including the early 1970's may be found in the reference. Early, industrial precipitators have been used to filter flyash, sulfuric acid, gas pollutants, and metal oxides from furnace fumes and combustion gases. Electrostatic precipitators have found other applications as well. One recent set of experiments employed the use of electrostatic precipitation for the artificial dissipation of natural fog [2]. This electrostatic fog-liquefier may find application in land, air, and sea transportation. A recent discovery has shown that electrostatic precipitators operated with pulsed positive voltages instead of negative DC voltages significantly reduce the emission of SO$_2$, NO, and NO$_2$ [3,4]. Although not a true electrostatic precipitator, other novel techniques such as the pulse corona induced plasma chemical process and the surface discharge induced plasma chemical process [5] are being examined to control gaseous pollutants and air toxics. Electrostatic precipitators have also found application in the removal of fluorides in Soderberg aluminum reduction [1]. A more delicate application examined here is the use of an
electrostatic or quasi-electrostatic air filter to filter the air from airborne microdiameter radionuclide contaminated soil particles resulting from environmental soil excavation and clean-up processes.

Electrostatic precipitators designed before the 1980's appeared to be less efficient in removing fine particles (less than 3 \( \mu \)m in diameter) as indicated in a 1972 National Academy of Engineering-National Research Council report [1,6]. More recently, a 1993 report [7] indicated that standards for the control of particle emissions will increase. Submicron particles from combustion are naturally enriched in toxic metal compounds. When inhaled, this may result in a health risk. Very fine particles (0.2 to 2 \( \mu \)m in diameter) are responsible for the formation of smog and haze. Efforts are underway to extract undesired submicron particles from the environment that support them. A recent novel electrostatic precipitator combined with an electrostatic agglomeration apparatus (EAA) is expected to increase the collection efficiency of submicron size particles [8]. Laboratory techniques employing Physical Vapor Deposition (PVD) and Chemical Vapor Deposition (CVD) have developed advanced industrial materials as a result of their superior physical and chemical properties. Ultra fine (under submicron size) particles are employed in these processes. High temperature electrostatic precipitators have been found feasible in the separation of these fine particles synthesized by thermally activated CVD [9]. Another method of collecting submicron particles is the complementary use of electrostatic and
hydrodynamic effects [10]. This technique employs electrostatic coagulation aided by the hydrodynamic effect of a vibrating particle.

Many of the theories, designs, and practical principles on electrostatic precipitators have been developed toward barbwire or long wire anode and collection plate cathode designs [11-37]. Other novel designs are coming into existence. A recent new precipitator [38] makes use of moving charged copper spheres to attract and collect particles. The only precipitation mechanism for the removal of dust was confined to the scrubbing action of the charged copper spheres. The copper spheres captured dust particles between 1.6 and 5 \( \mu \text{m} \) in size. The disadvantage of this technique is its inherent cross-sectional size capability. Even so, this technique consumes low power compared to conventional precipitators, eliminates the slurry disposal of wet scrubbers, employs solid spheres that are reusable and eliminates the use of handling liquids, and allows for ease of control in the size, concentration, and dynamics of the target spheres as compared to liquids.

In each of the designs mentioned, the dust particles are attracted to the cathode or physically forced on the anode wire of the precipitator or come in contact with a key component of the precipitator [22]. This is undesired when radionuclides are present. Radiation hazards become an issue. In environmental excavation of radio-active contaminated soil, a need exists to protect the excavation worker from inhaling radionuclide dust particulate. Such a
need is crucial in an arid region where sand predominately covers the terrain. In the process of maintaining and cleaning conventional electrostatic precipitators, microdiameter dust particles may become airborne and consequently result in a possible health risk.

It has been reported [39] that when a charged particle is bombarded by a unipolar ionic flow in an alternating electric field, it has the tendency of oscillating without being precipitated. This allows the particle to charge close to its saturation value. Simulations and experimentation were conducted with particles having a radius of 75 μm. Microdiameter and submicron size particles were not investigated.

One novel method of filtering the air is with a quasi-electrostatic air filter which charges, traps, transports, and collects the dust particulate with the aid of electrostatic and quasi-electrostatic fields. Unlike all of the corona charging devices listed above, the dust particles here are charged using a photoionization mechanism. Further, the electrostatic and quasi-electrostatic fields not only trap and guide the charged dust particles to a collection region, they also suspend the particles away from the walls and electrodes of the device. This is of importance when handling microdiameter dust particles with radionuclides attached to them especially when addressing health issues resulting from resuspension. The air filter itself is divided into four regions: the charging region, the electrostatic trapping region, the transport region, and the collection region. A theoretical and
numerical treatment of the first three regions of the novel air filter are examined. The charging region consists of an inlet grid and an exhaust grid. A fan draws the particles into the charging region from an extension hose that is mounted in the operator's cab on land excavation equipment. As the dust particles enter the charging region, they collide with ultraviolet photons that have enough energy to cause significant electron emission. A pulsed ultraviolet excimer laser is commercially available to perform such a task. By adjusting the air flow and laser beam properties, the dust particle can be charged up to its saturation value before air breakdown occurs. If breakdown results at the particle surface, the particle discharges. The strong electrostatic fields on the walls of the open-ended electrostatic cavity guide the charged particles out of the charging region and into the transport region of the device. These particles are positioned near the center of the aperture opening to the transport region between the first set of bi-planar electrodes. The quasi-electrostatic fields of the transport region with loading effects of the electrostatic and charging chambers are described and analyzed [40]. Briefly, four consecutive bi-planar sets of electrodes each having a different potential generates a field configuration that tends to trap the charged dust particles and suspends them from the electrodes. The same voltage pattern exists on each succeeding set of four consecutive bi-planar electrodes. After a duration in time, the plate potentials shift one plate to the right. The trapped charged particles are pulled along. The pattern repeats until the particles have been transported to the collection region. Many scenarios exist for the collection region. Here it is assumed that the collection region is matched
to the transport region such that loading effects are not of importance. Further, to address the maintenance and cleaning issues of this device, it is anticipated that the collection region will be lined with a disposable electret dielectric fiber material. The electrical polarization properties of the fiber material will attract and contain the charged dust particles [28]. When disposing of the lining, it is anticipated that small impacts on the lining will not dislodge the contaminated particles.

This thesis is organized in the following fashion. An estimation of the maximum charge a dust particle can sustain is examined in Chapter 2. The logistics on the air flow velocity and laser beam geometry are also discussed. A theory is developed in Chapter 3 which characterizes the two dimensional electrostatic field configuration for multiple sets of bi-planar electrodes each having a common axis. Based on single particle orbit trajectories, quasi-electrostatic air filter designs are examined in Chapter 4. A conclusion is provided in Chapter 5.
CHAPTER 2

DUST PARTICLE CHARGING

2.0 Introduction to Charging

To effectively guide a dust particle with an electric field, the particle requires a sufficient amount of charge. A number of charging processes can be employed. Direct charging requires the charging electrode to come in direct contact with an object or entity being charged. The object takes on the same polarity as the charging electrode. Induced charging results when the object being charged is grounded. As the charging electrode is brought near the entity, electrons are drawn from or repelled to ground. The polarity of the entity is opposite to that of the charging electrode. Corona and plasma charging results when the background medium itself is first ionized. As the object passes through such a medium, the ionized medium causes the object to setup a dipole-like field. This field in turn appropriately attracts the background ion or electron thereby ionizing the object. Such charging especially in the case of plasma charging is also dependent on the background ion/electron mass and temperature. Charging by these techniques is limited by space charge effects and the recombination collision rate.
Another charging technique is by photo-ionization. Energetic photons collide with an atom or molecule resulting in the emission of an electron. If the photon energy is larger [smaller] than the first ionization potential of the molecule, there is a probability [is zero probability] upon interaction that an electron can be emitted from the molecule. The probability is a function of the photon energy. The probability of photo-ionization increases as the photon density and the target atom (molecule) density either or both increase. A one microdiameter dust particle has an abundant supply of molecules to ionize. This ionization process was chosen since it appears not to be limited by space charge effects. Even so, recombination processes and discharge breakdown affect the charging process. Since space charge effects are not of significance, dust particle charging is to be performed by photoionization.

For sake of clarity in this work, both recombination and space charge effects are defined. Recombination for atoms and molecules is defined as the Coulomb attraction between a free electron and an ionized atom or molecule causing the two to join into a single entity. Space charge is defined as the Coulomb repulsion effect when a charge of a polarity is repelled from a region in space where other charges of the same polarity reside. Space charge effects tend to limit the flow of current.

Photo-ionization occurs in the charging section of the electromagnetic air filter. Refer to Fig. 1. A suitable laser source emits photons of energy capable
of ionizing a dust particle immersed in a large electrostatic field. The external electrostatic field is generated by large plate potentials both in the charging region and the electrostatic region of the air filter. The external electrostatic field traps the charged dust particles and draws them into the transport section of the air filter by way of the electrostatic region. The electrons are forced in the opposite direction. The efficiency of the air filter is limited by the large viscosity forces of the air. Consequently, both wind speeds and machine motion must be taken into consideration when determining the laser beam configuration and the maximum charge a dust particle may attain.

This chapter is organized in the following fashion. Section 2.1 describes the properties of the aluminosilicate molecule. The breakdown condition of a dust particle is discussed in Section 2.2. The internal stress is considered in Section 2.3. Section 2.4 provides a heuristic treatment for recombination. Section 2.5 discusses air flow constraints and laser beam requirements. Key approximations employed are justified in Section 3.6. Results are provided in Section 3.7.

2.1. Molecular Properties

The laser source and maximum charge are determined by the molecular property of the dust particle. A typical sand particle is composed of aluminosilicate (Al$_2$SiO$_4$) molecules. Seven atoms make up its molecular structure. It is reasonable to assume that the dust particle is amorphous.

Each molecule in the dust particle has the capability of being ionized. The
minimum energy required to ionize a single molecule of Al$_2$SiO$_4$ is determined by
the first ionization potentials of the individual atoms composing the molecule. This is a first estimate since the first ionization potential of an atom and a molecule may be very different. The first ionization potentials of the Al, Si and O atoms are respectively 6.0 eV, 8.1 eV and 13.6 eV. This implies that the photon energy required to perform ionization based on the first ionization potentials must be at least 6.0 eV or preferably 8.1 eV. The latter is preferred since all dust or sand particles contain a silicate molecule. A 6.0 eV and a 8.1 eV laser have respective wavelengths of 207 nm and 150 nm. These lasers lase in the ultraviolet end of the light spectrum. A typical commercially available laser source that has the capability of ionizing the aluminum atom and almost ionizing the silicon atom is the Compex 110 Excimer Laser (made by Lambda Physik Lasertechnik, a Subsidiary of Coherent, Inc.) with a wavelength of 157 nm (corresponding to a photon energy of 7.91 eV).

A single photon is required to emit an electron from a molecule of Al$_2$SiO$_4$. To estimate the number of photons needed to ionize the dust particle, one must determine the number of aluminosilicate molecules in a 1 micron diameter spherical dust particle. The geometrical shape of an atom is assumed to be spherical. A typical estimate for the atomic diameter of an atom is approximately 3 Å. The volume of a single molecule of Al$_2$SiO$_4$ is approximated as
In a 1.0 μm diameter spherical dust particle there is a maximum of approximately $5 \times 10^9$ molecules. If every Al atom (Si atom) in the Al$_2$SiO$_4$ molecule in the 1.0 μm diameter spherical dust particle releases a single electron in the ionization process, a minimum of approximately $1 \times 10^{10}$ photons ($5 \times 10^9$ photons) need to be supplied in the ionization process. This implies that a net dust particle charge of 1.6 nC (0.8 nC) will be attained. Probability of photoionization, breakdown effects, recombination effects, etc. have not been considered. These effects will significantly limit the net charge of the dust particle.

The molecular and particle weights are determined from the atomic weights of the atoms composing aluminosilicate (Al$_2$SiO$_4$) molecule. The atomic weight of Al, Si, and O is respectively 26.98 g, 28.08 g, and 16 g. Two atoms of Al, one Si atom, and four O atoms compose aluminosilicate. Consequently, the atomic mass of the molecule is 146.04 g. Dividing by Avogadros number, the mass of one molecule of aluminosilicate is $2.4 \times 10^{-25}$ kg. A one micron diameter spherical dust particle contains as an upper limit approximately $5 \times 10^9$ molecules. Therefore, the mass of a 1 μm diameter dust particle is $1.2 \times 10^{-15}$ kg.

2.2. Breakdown Conditions of Surrounding Environment

The breakdown conditions of air surrounding the dust particle limits the
amount of charge the dust particle can sustain. Exceeding these conditions will result in dust particle discharging.

It is commonly known that the dielectric field strength of air is approximately [41]

\[ |\vec{E}_{bd}| \approx 3 \text{ MV/m}. \]  \hspace{1cm} (2)

Further, air breaks down when the voltage exceeds approximately

\[ V_{bd} = 300 \text{ V}. \]  \hspace{1cm} (3)

Both the electric field and the voltage at the surface of the dust particle must exceed their respective breakdown limits before breakdown actually results. These values are based on experimental studies of parallel plate arrangements and are not necessarily valid for spherical geometries [41]. Even so, they are employed for estimation purposes.

It is assumed that the spherical surface of the dust particle supports a uniformly distributed surface charge density. Gauss' law

\[ \iiint_S \vec{D} \cdot d\vec{S} = q_{mc}. \]  \hspace{1cm} (4)
is employed with a Gaussian surface constructed over the surface of the particle. For breakdown not to occur, the dielectric strength of air must exceed the magnitude of the electric field at the surface of the spherical particle

\[ E_{bd} > |\vec{E}| = \frac{q}{4\pi\varepsilon R^2} \] (5)

where \( R \) is the radius of the particle. Consequently, the maximum charge possible as a consequence of electric field breakdown is

\[ q_{max} = 4\pi\varepsilon R^2 E_{bd} = 8.3 \times 10^{-17} \, C \] (6)

On small scale sizes, it is typical that voltage breakdown is more prevalent than electric field breakdown. To determine the effects due to voltage breakdown, the spherical dust particle is assumed to be isolated in space with its surface acting as an electrode. Both the proximity and shape of a second electrode have little effect on the potential distribution near the surface of the particle if the minimal distance of separation among electrodes or particles is large compared to the radius of the particle. As a result, consider the second electrode to be spherical in geometry with infinite radius and concentrically oriented with the particle. The potential on the surface of the particle before breakdown results is
Consequently, the maximum charge possible as a consequence of voltage breakdown is

$$q_{\text{max}} = 4\pi \varepsilon RV_{bd} = 1.67 \times 10^{-14} \text{ C.}$$  (8)

In order for breakdown to occur, both the voltage and the electric field must exceed their respective breakdown constraints. A 1 \(\mu\text{m}\) in diameter spherical dust particle can therefore sustain at best a charge of approximately \(1.67 \times 10^{-14} \text{ C}\) that corresponds to \(1.04 \times 10^5\) electrons.

### 2.3. Internal Stress Considerations on the Charged Dust Particle

The maximum number of electrons emitted from the dust particle was determined without consideration of the effect this has on the internal forces holding the particle itself together. If the electrostatic force is stronger than the internal forces of the particle, the particle will split apart. This is undesirable for two reasons. First, the viscous forces on the split particle will be stronger than the original particle. Second, the amount of charge the split particle can sustain will be less than the original particle. Since an electrostatic and a quasi-electrostatic field are to guide the particle in a non-vacuum region, both effects
accumulate to making it more difficult to control the particle with an applied field. The mechanical properties of aluminosilicate are therefore examined.

The bending stress at failure (i.e., the modulus of rupture [MOR]) for aluminosilicate sometimes referred to as mullite porcelain is 69 MPa [69x10^6 N/m^2][42]. The modulus of rupture is generated by a bending test. This strength parameter is similar in magnitude to a tensile strength. Fracture usually occurs along the outermost sample edge relative to the point of contact where the force is applied. This edge is under a tensile load. Bending stress tests are employed to measure the stress that materials such as glasses and ceramics may support since it is difficult to attach the materials under test to standard stress measuring equipment.

The element of electrostatic force acting on a volume of charge distributed uniformly over the first molecular layer on the surface of the spherical dust particle is given by

$$d\vec{F} = \rho \, \vec{E} R^2 \, dr \, d\Omega$$

where $dr$ is the diameter of the aluminosilicate molecule (thickness of the first molecular layer) and $d\Omega$ is the element of solid angle. The electric field at the surface of the particle resulting from a uniformly distributed volume of charge is
Consequently the pressure exerted by the electrostatic forces of the dust particle over the first molecular layer of aluminosilicate is

\[ p = \frac{\rho_e R^3}{3 \varepsilon_0} \frac{Rdr}{3 \varepsilon_0} \]  

(11)

where the net charge contained in the uniformly charged spherical dust particle is given by

\[ q_{net} = \frac{4}{3} \pi R^3 \rho_e \]  

(12)

Since the maximum pressure that can be applied before the particle separates into two or more pieces is the modulus of rupture, the maximum net charge that the dust particle can support before material rupture results is

\[ q_{net, MOR} = \frac{4}{3} \pi R^3 \left[ \frac{3 \varepsilon_0 P_{MOR}}{Rdr} \right]^{\frac{1}{3}} \]  

(13)

Most glasses and ceramics are amorphous. It is therefore assumed that the atoms are stacked in layers consisting of three atoms on the first layer, one atom in the second layer, and three atoms in the third layer. The stacking is compact such that the diagonal dimension of the molecule is about 9 Å. Therefore, it will
be assumed that the thickness of the first molecular layer is \( dr = 9 \, \text{Å} \). The maximum net charge and the maximum number of electrons that a 1 μm diameter aluminosilicate dust particle can sustain before rupture are respectively \( 1.06 \times 10^{-12} \, \text{C} \) and \( 6.6 \times 10^6 \) electrons. It is observed that these values are about two orders of magnitude larger than respective values obtained for air breakdown. Consequently, air will breakdown around the dust particle long before the dust particle ruptures due to electrostatic forces.

### 2.4. Heuristic Treatment for Recombination

Although the first ionization potential gives an estimate on the amount of energy required for a single electron to be emitted from an atom or a molecule, this energy does not account for the energy needed to prevent recombination that results when one or more atoms or molecules are ionized in a dust particle. It is the intention here to determine what additional energy is required to overcome both the first ionization of a single atom in the aluminosilicate molecule and the Coulomb attraction resulting from an already partially ionized dust particle. The excess energy between the incident photon and the first ionization potential appears for the most part as kinetic energy of the emitted electron. From conservation of energy and momentum considerations, it can be shown that little kinetic energy is transferred to the aluminosilicate molecule or dust particle.
It is assumed for a worst case scenario that the spherical dust particle is uniformly charged throughout its volume with some net charge $q_{\text{net}}$. Therefore the charged particle is modelled as a uniformly distributed volume of charge isolated in free space. This approximation is not unreasonable as a first estimate since the diameter of the dust particle is assumed to be much larger than the distance of separation between the surface of the particle and the surface of the walls of the guide, the walls of the electrodes, or the wall of another neighboring dust particle. This may be considered as a low density approximation. The electric field at the surface of an isolated spherical volume of charge of radius $R$ is given by

$$
\hat{E}(r) = \frac{\rho_s R}{3\varepsilon_0 r^2}\hat{r} \tag{14}
$$

The amount of energy it takes to move an electron $q$ from the surface of the particle to infinity is

$$
W = \frac{q \rho_s R^2}{3\varepsilon_0} \tag{15}
$$

where the charge density is given by Eq. (12). Assume that a 1 μm diameter dust particle is charged up to its air breakdown limit. Then, the amount of energy required to overcome recombination resulting from Coulomb attraction is
4.81 \times 10^{-17} \text{ J} \text{ or } 300 \text{ eV}. \text{ Consequently, the net energy required to remove this electron from the aluminum atom and off to infinity is approximately } 306 \text{ eV. Decreasing the net charge supported by the dust particle by a factor of 100 and 1000 yields a net energy of 9 eV and 6.3 eV respectively. Each photon generated by the 157 nm (7.91 eV) Compex 110 Excimer Laser, upon giving up its entire energy in the collision process, will have enough energy to emit an electron from a dust particle if the net charge of the dust particle is decreased by a factor of 157 from its maximum air breakdown value. That is, the maximum charge and number of electrons the dust particle can sustain due to laser ionization is roughly } 1.06 \times 10^{-16} \text{ C \ and } 663 \text{ electrons respectively.}

\textbf{2.5. Air Flow Velocity and Laser Beam Logistics}

Up to this point, particle charging has been limited by the characteristics of the particle and the surrounding medium. The laser beam properties (energy, frequency of repetition, and duration), the laser beam geometry, and the machine velocity have not been considered. The photons generated in each pulse must sweep through enough volume in the charging region to charge the incoming dust particles. The rate of mass charging and beam energy will limit the machine and wind velocities. The specifications for the Compex 110 Excimer Laser (made by Lambda Physik Lasertechnik, a Subsidiary of Coherent, Inc.) are employed. The wavelength, maximum pulse energy, and the maximum repetition rate are respectively 157 nm (7.91 eV), 10 mJ, and 50 Hz. Although these are the specifications as printed in literature, the manufacturer has
indicated that the maximum pulse energy and the maximum repetition rate are not coincident. Therefore, a more practical 10 Hz repetition rate is employed in the estimations below.

A 10 mJ pulse of 7.91 eV photons will contain about $7.89 \times 10^{15}$ photons. In a single pass, it is assumed that one in every 1000 photons will suffer a collision with a dust particle and perform photo-ionization. This 0.1% probability of ionization will be justified in a later section. Therefore, about $7.89 \times 10^{12}$ photons are available for ionizing the dust particles in the beam path. Each of these photons will release a single electron from the dust particle. Recall that a fully charged 1 μm diameter dust particle may have a void of $1.04 \times 10^5$ electrons before the air surrounding the particle will breakdown. Therefore a single 10 mJ pulse of 7.91 eV photons making a single pass in the charging region has the capability of fully charging about $7.59 \times 10^7$ dust particles of 1 μm diameter in size. If overcoming recombination is considered, only 663 electrons may be emitted on a single dust particle. Consequently, about $1.2 \times 10^{10}$ dust particles will be ionized on a single pass.

It is anticipated that the quasi-electrostatic air filter will be attached to excavation machinery. An inlet hose with fan will hydro-dynamically draw dust particles from the environment or excavation cab and force them into the charging chamber. As a result, a non-zero air flow velocity will exist in the charging region between the inlet and the exhaust grids. It is desired to irradiate
all volumes of air in this region with the laser. Consider a single pass charging system as a worst case design. Figure 2 illustrates the laser beam geometry to be examined. The beam width and the rate of repetition limit the allowable air flow.

Assume that the beam width, laser beam repetition rate, and the relative air flow velocity with respect to the air filter are respectively denoted by $W$, $T$, and $v_{ra} = W/T$. The angle subtended by the edges of the beam in a plane is $180^\circ$. For a beam length $L$, the volume of space occupied by the beam is

$$V_{beam} = \left(\frac{\pi L^2}{2}\right)W = \left(\frac{\pi L^2}{2}\right)v_{ra}T$$  \hspace{1cm} (16)

The volume of an individual dust particle is given by $V_p = \frac{4}{3}\pi R^3$. The percentage of the laser volume occupied by the dust particles assuming that each dust particle needs only a void of 663 electrons to be fully charged is

$$\left(1.2\times10^6\frac{V_p}{V_{beam}}\right)\times100\%$$  \hspace{1cm} (17)

The cavity geometry of the charging section of the electromagnetic air filter is 0.12 m by 0.10 m. The beam length is to extend diagonally across the cavity from corner to corner. Consequently, the beam radius is 0.156 m. For an air
flow velocity of 1 MPH (≈0.445 m/s) and a laser beam repetition rate of 10 Hz., the percent volume occupied by $1.2 \times 10^{10}$, 1 μm, dust particles in the beam is $3.7 \times 10^{-4}$% for a beam width of 0.0445 m.

The number density of photons available for particle ionization in the fan shaped beam assuming a 0.1% probability of ionization is $\rho_{\text{vpi}} = 4.64 \times 10^{15} \text{photons/m}^3$. Note that the pulse duration of the laser beam is orders of magnitude small compared to the transit time of the dust particle through the beam and the photon velocity is orders of magnitude larger than the air flow velocity. Since the photon density is assumed to be uniformly distributed, let the laser beam be divided into $[W/(2R)]$ rows of circular wedges with a radius equal to the radius of the fan like geometry of the beam and a maximum chord length equal to the diameter of the dust particle. The volume of any one circular wedge is

$$V_{\text{wedge}} = 2RL^3 \sin^{-1} \left( \frac{R}{L} \right)$$

where $L (=0.156 \text{ m})$ is the length of the laser beam and $R (=0.5 \times 10^{-6} \text{ m})$ is the radius of the dust particle. For these parameters, $V_{\text{wedge}} = 7.8 \times 10^{-14} \text{ m}^3$. Consequently, the photons available for ionization in any one wedge is $\rho_{\text{vpi}} V_{\text{wedge}} = 362 \text{ photons}$. The number of circular wedges that compose the fan like pattern of the beam is approximately $2.18 \times 10^{10}$ wedges. Therefore, the
1.2\times10^{10} dust particles charged up to their maximum charge has plenty of room in the fan beam such that there are no particle overlaps. Consequently, assuming no overlap among dust particles and assuming the photons pass through the region only once, any one dust particle lying on the perimeter of the beam will lose 362 electrons.

The above calculations have been restricted to the case when the beam photons pass through the beam path only once and if not absorbed by the dust particles in the path, are completely absorbed by the walls of the charging cavity. With the aid of mirrors, multiple photon passes will be achieved. Some absorption loss will occur at the optics but this should be minor. Both the charging efficiency and the probability of collision increases. Consequently, not only will the 1.2\times10^{10} dust particles achieve their maximum recombination limited charge (note, two passes are required), a higher concentration of dust can be fully ionized. Further, the repetition frequency of the laser may be relaxed if reflective diffusers are employed.

2.6. Justification of Various Assumed Parameters

A variety of parameters where assumed without adequate justification. Since many of the calculations above are dictated by the parameters chosen, explanations regarding key assumed parameters are provided here. As long as the assumed parameters are within an order of magnitude realm of its exact value, the results above are considered reasonable.
The diameter of the atom was chosen to be about 3 Å. It is difficult to physically measure the size of an atom as a consequence of the Heisenberg uncertainty principle. The smallest atom is the hydrogen atom. Classical first Bohr orbit calculations of hydrogen yield a Bohr orbit radius of 0.53 Å. Consequently, diameter estimates of 3 Å for larger atoms are considered reasonable.

Empirical data in literature do not seem to exist for the first ionization potentials for aluminosilicate. Therefore, the first ionization potentials for the atoms composing the aluminosilicate molecule were employed. These values may not be representative of the molecule or the dust particle as a whole since the bonding among the atoms and molecules are not considered in this number. It is not clear if the first ionization potentials are larger or smaller than the values used. It is assumed that the potentials are reasonable and are not significantly larger.

A conservative 0.1% probability of photoionization is justified based on classical dynamic theory for collisions. The theory assumes that multiple collision interactions are neglected and that the interparticle distance is large relative to the particle dimensions. Let \( P(x) \) represent the probability of a photon traveling a distance \( x \) without photoionizing an Al atom in the dust particle.
Further, let $P(x+dx)$ be the probability of a photon moving a distance $dx$ without suffering the same collision. Assume that each target atom has a cross sectional scattering area, $\sigma$, for photoionization. Further assume that the number of these target atoms between $x$ and $x+dx$ is $Ndx$. The probability that a single particle suffers a collision within a distance $dx$ is $\sigma Ndx$. Therefore the probability that a target atom does not suffer a collision is $(1-\sigma Ndx)$. The probability that no collision is suffered at $x+dx$ is the probability that no collision is suffered at $x$ times the probability the no collision is suffered in a distance $dx$. Consequently, the probability that no collision is suffered can be shown to yield

$$P(x) = P(0)e^{-\sigma x}$$

(19)

where $P(0)$ is the probability that no collision is suffered in moving a zero distance. The cross-section of the Al atom is assumed to be the atomic cross-section (~3Å in diameter) roughly equal to $7.07 \times 10^{-20} \text{ m}^2$. This is reasonable in comparison to the photoionization cross section for Ar that is typically $3.7 \times 10^{-21} \text{ m}^2$ for 15.5 eV photons [43]. An upper most limit of Al atoms in a 1 μm diameter dust particle is $1 \times 10^{10}$ atoms. This value is based on the fact that the atoms of aluminosilicate are tightly fitted into the amorphous dust particle. A second estimate is to assume that a 1 μm diameter dust particle contains about $1.04 \times 10^5$ Al atoms. This is the number of atoms to required to charge the dust particle to its maximum charge before air breakdown results. Consequently for
the $1 \times 10^{10}$ and $1.04 \times 10^5$ Al atoms, the density of Al atoms uniformly distributed throughout the $1 \mu m$ diameter dust particle is respectively, $1.91 \times 10^{28}$ atoms/m$^3$ and $1.99 \times 10^{23}$ atoms/m$^3$. The realism of these particle densities is established by examining known density ranges of other phenomena such as plasmas. Common plasma densities [44] range from about $1 \times 10^5$ m$^{-3}$ for space and astrophysics plasmas to between $1 \times 10^14$ m$^{-3}$ and $1 \times 10^{18}$ m$^{-3}$ for gas discharges and gaseous electronics to roughly $1 \times 10^{22}$ m$^{-3}$ for fusion reactor and experiment plasmas. Solid state plasmas may have densities as high as $1 \times 10^{29}$ m$^{-3}$. Consequently, the estimated densities are reasonable. The only time photoionization can occur is if the photon passes through the dust particle. Assume the distance the photon will travel is the diameter of the dust particle. Hence, the probability that a photon will not suffer a collision traveling the diameter of the dust particle is 0 (for a density of $1.91 \times 10^{28}$ atoms/m$^3$) and 0.014 (for a density of $1.99 \times 10^{23}$ atoms/m$^3$). Consequently, the probability of suffering a collision (probability of photoionization) is respectively 1 and 0.986. The former result appears unreasonable since the approximation regarding large interparticle distances is violated and since the compactness of the atoms is an upper extreme. The latter result appears to be more realistic since the density of target Al atoms is comparable to the upper limits of experimental plasma densities. Therefore, the assumption of 1 collision for every 1000 photons (0.1% probability of photoionization) is very conservative and suggests that there are about $7.41 \times 10^3$ Al atoms in a $1 \mu m$ diameter dust particle. The collision cross-section and $1 \mu m$ distance the photon travels through the dust particle as
assumed above are employed. This conservative assumption will help augment some of the less conservative assumptions assumed elsewhere.

2.7 Results

A complete heuristic argument excluding the effects of secondary ionization mechanisms has been provided to describe the charging of a 1 μm diameter, aluminosilicate, dust particle with a pulsed 10 mJ, 157 nm laser having a repetition frequency of 10 Hz by means of photoionization. The laser specifications are based on the Compex 110, fluorine gas laser. It has been shown that recombination effects dictate the amount of charge a single micrometer dust particle will obtain if a 7.91 eV photon completely gives up all of its energy to removing an electron from the dust particle in the collision process. Here, first ionization potentials of the atom and not the molecule where employed as a first approximation in determining the amount of energy that is required in the photoionization process. Since the diameter of the dust particle is about an order of magnitude larger than the wavelength of the photon, absorption effects have been neglected. Based on these considerations with the aid of ultraviolet mirrors or diffusers, the above calculations indicate that it is possible to charge a significant volume of 1 μm diameter airborne dust for particle trapping and transport.

Secondary ionization effects have been neglected and can only assist in the charging process since recombination effects limit the amount of charge any one dust particle can attain thereby preventing dust particle discharging due to dielectric breakdown of the surrounding air. Although not examined, the
energetic electron may possibly have enough energy to cause further ionization in a neighboring dust particle. Moreover, the void of electron in the dust particle (an aluminosilicate molecule) as a result of ionization may be filled by an electron transition from a higher level. The accompanying photon emission may cause more ionization on its way out of the dust particle (e.g. Auger effect). It is anticipated that the initial photoionization process may be desirably enhanced as a result of secondary ionization processes not examined or considered here.
CHAPTER 3

THE TWO DIMENSIONAL ELECTROSTATIC FIELD CONFIGURATION

3.0 Introduction to Field Theory

Charged, microdiameter, radionuclide dust particles are to be transported along a straight section of a bi-planar arrangement of flat parallel plate electrodes using electrostatic fields. The guide is terminated by a closed collection region. The inlet to the guide is the electrostatic and charging chamber. The viscosity of air, the mass and charge of the particle, and the geometrical configuration of the guide constrain the use of mechanical and electromagnetic forces for transport purposes. Unlike electromagnetic forces, electrostatic forces can impart average energy to slow moving particles of varying velocity in a stationary, viscous environment. It is anticipated that quasielectrostatic forces generated by adjacent sets of parallel plate electrodes have the capability of transporting dust particulates over long distances in a straight section of guide.
The guide configuration, composed of these adjacent sets of electrodes, is assumed to be two dimensional and semi-infinite in length. To aid in focusing the dust particulates, parallel electrodes are at equipotential. Potential differences between adjacent sets of plates are large to direct the charged particles along the guide. To transport the dust particles long distances, the plate potentials are shifted in the direction of the desired particle motion. As a result, the spatial distribution of the potential on the plates must exhibit a periodic nature. The time between transitions is anticipated to be about a particle transit time along one or two electrodes. This is large compared to the field's transient time constant developed by the voltage change on the electrodes. Further, the transient time constant is anticipated to be very small compared to the time it takes for the charge to overcome the large inertial and viscous forces and move a substantial distance before the steady state electrostatic forces dominate. Therefore, as far as the charged dust particle is concerned, it only experiences electrostatic forces and damping mechanical forces. As a result, an electrostatic model of the fields in the guide is adequate in characterizing the dust particle orbit. The fields due to the end effects are as significant as the fields well inside the guide. The electrostatic fields of the semi-infinite guide terminated on one side with loading effects of the electrostatic and charging regions are to be examined.

Numerical techniques such as the moment method and the finite element method are common numerical tools employed to examine two and three
dimensional fields in irregular geometrical structures. The demand for accuracy
and speed have led to innovative irregular finite-element meshes and
transformations [45,46]. Other more recent numerical techniques cast Laplace's
equation into a variational boundary element formulation or a dual and a
complementary variational formulation [47]. In this case, the boundary and not
the interior is discretized. These numerical techniques are invaluable tools for
complex geometries. Even so, analytical relations when obtainable generally
require less computer memory and computational time. Conserving these
resources is necessary when examining particle motion in a complex system with
reasonable accuracy employing a personal computer.

Conformal mapping along with the Schwartz-Christoffel transformation
provides one analytical approach to study two dimensional electrostatic field
problems supported by an irregular polygonal configuration. Although obtaining
the transformation in integral form is straightforward, the integration itself is very
difficult. Analytical relations for even the classical problem of the parallel plate
has more than sixty years of history [48-51].

Other analytical techniques make use of Fourier series solutions. For
example, a Fourier-Bessel solution of an Einzel-type electrostatic field has been
obtained for geometries with rotational symmetry [52]. A combination of both
radial and axial eigenvalues is posed to determine the electrostatic fields. Matrix
inversion is not required to obtain the coefficients in the series.
A Fourier series solution of Laplace's equation with the method of superposition is used to characterize the electrostatic field in the parallel plate configuration under consideration here. For notation purposes, the guide itself is divided into two sections; the transport section and the transition section. It is assumed in the transport section that the end effects are not significant. Consequently, the geometry may be treated as being infinite in extent. The end effects are significant in the transition section of the guide. To properly evaluate the fields in the transition section, loading effects resulting from the regions external to this section of the guide must be carefully incorporated in the field analysis. Electrostatic approximations regarding boundary condition information have led to analytical solutions of the electrostatic fields with the inclusion of end effects. Field plots are obtained and explained. Interpolation and expansion techniques have been employed to correct for numerical error resulting from series truncations. Comparisons are made with the finite element method.

This chapter is formatted in the following manner. Section 3.1 provides the governing relations for the electric fields in the infinite in extent transport section. Employing the method of superposition, relations governing the potential in the transition region are developed in Section 3.2. All loading effects have been properly incorporated into the theory. Section 3.3 provides the resultant potential and electric fields in the transition region. Field plots and comparisons are provided in Section 3.4.
3.1 Governing Relations for the Fields in the Infinite in Extent Transport Region

The semi-infinite guide is divided into two sections or regions: the transport section and the transition section. The transport section of the guide is defined as that portion of the guide where end effects are negligible. Consequently, it is considered to be infinite in extent in geometry. As illustrated in Fig. 3, the surfaces of the perfectly conducting, planar, parallel electrodes are strips of length L extending over all y and located at \( x = \pm d/2 \). As shown in the figure, the electrodes extend to \( z = \pm \infty \). The \( z=0 \) plane exists between two adjacent sets of electrodes. The potential distribution over both the upper and lower electrodes is displayed in Fig. 4. The potential is periodic. Its Fourier series representation is

\[
\begin{align*}
V(x = \pm d/2, z) &= \frac{1}{4}(V_1 + 2V_2 + V_3) + \sum_{n=1}^{\infty} \frac{1}{n\pi} [(V_1 - V_3)\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2L}\right) \\
 &= \left[(V_2 - V_1)(-1)^n(V_2 - V_3)\right]\left[1 - \cos\left(\frac{n\pi}{2}\right)\sin\left(\frac{n\pi}{2L}\right)\right]
\end{align*}
\]

(20)

The fields within the transport section are characterized by the two dimensional Laplace's equation given as

\[
\frac{\partial^2 V(x,z)}{\partial x^2} + \frac{\partial^2 V(x,z)}{\partial z^2} = 0
\]

(21)
Due to the geometrical symmetry of the plates and the sinusoidal nature in the plate potential as exhibited in Eq. (20), the form of solution for the potential within the transport region is

\[ V(x, z) = [A_1 \cos(ka) + A_2 \sin(ka)] \cosh(ka) \quad (22) \]

where \( A_1 \), \( A_2 \) and \( k \) are constants to be determined by boundary conditions. Since the potential is continuous at the electrode boundary, the constants of integration are obtained by equating Eqs. (20) and (22) at the electrode surface yielding

\[ V(x, z) = \frac{1}{4}(V_1 + 2V_2 + V_3) + \sum_{n=1}^{\infty} \left( \frac{V_1 - V_2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(nkz) \right) - \frac{1}{n\pi} \left( V_2 - V_1 \right) + (-1)^n \left( V_2 - V_3 \right) \left[ 1 - \cos\left(\frac{n\pi}{2}\right) \right] \frac{\cosh(nkz)}{\cosh\left(\frac{nkd}{2}\right)} \quad (23a) \]

where

\[ k = \frac{\pi}{2L} \quad (23b) \]

The electric field components obtained from the negative gradient of the potential are
Using a large argument expansion for the hyperbolic functions it can be shown that the field solution converges away from the electrode's surface. At \( x = \pm \frac{d}{2} \) between coplanar electrodes \((z = nL)\), the electric field magnitude is singular since the potential change is assumed to be discontinuous.

### 3.2. Governing Relations for End Effects in the Transition Region

To examine end effects, let the guide terminate at the \( z=0 \) plane. Refer to Fig. 5. A coordinate system similar to that for the infinite geometry of Fig. 3 is employed. For clarity, the region of the guide where end effects are significant will be denoted as the transition region or section. This region consists of \( M \) plates of length \( L \). The resultant field internal to this region is the superposition
of the fields generated by the plate potentials in this region and the source contributions external to the guide. In the former case, the ends of the transition region are extended beyond its boundaries in order to include the loading effects due to the external geometries. All sources are suppressed external to the guide. For the latter case, external loading effects on one side of the transition region need not be considered when determining the fields generated by the external sources on the opposite side. This is a consequence of the spatial decay of the source fields and the appropriate choice of the length of the transition region. The end of the transition region opposite to the source is modelled as a perfectly conducting plate with zero potential. Relative to the resultant field, the overall error generated is negligible.

To determine the length of the transition section (ML) of the guide, the guide is assumed to be terminated by a uniform strip of charge in the z=0 plane between x=-d/2 and x=+d/2. The boundary data at z=0 and z=ML is then examined for relative size and identified as a minimum and a maximum value at each x. The absolute value of the ratio is then formed and related to the spatial variation of the uniform strip of charge. Using a two order of magnitude approximation suitable for engineering application, the length of the plates (L) and the number of plates (M) are determined such that the following inequalities are satisfied for all x between the plates.
\[ \frac{|E_{x,\text{max}}(x)|}{|E_{x,\text{max}}(x)|} > 50 \ln \frac{\left( x + \frac{d}{2} \right)^2 + (ML)^2}{\left( x - \frac{d}{2} \right)^2 + (ML)^2} \]

(25a)

\[ \frac{|E_{y,\text{max}}(x)|}{|E_{y,\text{max}}(x)|} > 31.8 \tan^{-1} \left( \frac{d + x}{2ML} \right) + \tan^{-1} \left( \frac{d - x}{2ML} \right) \]

(25b)

Observe that if the length of the transition region is large relative to the distance of separation between the upper and lower electrodes, \( ML >> d \), both inequalities are satisfied as expected.

Employing the method of superposition allows for the eigensolutions to be determined in a systematic manner with electrostatic end effects. Therefore, the analysis for the potential in the transition region is separated into the following two parts: the potential due to sources external to the guide and the potential generated by sources in the transition region. In each part, Laplace's equation must be solved. With the aid of Eq. (21), the potential takes one of the following two forms.
\[ V(x, z) = \sum_{n=0}^{\infty} \left[ A_{1n} \cos(k_n x) + A_{2n} \sin(k_n x) \right] \left[ A_{3n} \cosh(k_n z) + A_{4n} \sinh(k_n z) \right] \] (26a)

\[ V(x, z) = \sum_{n=0}^{\infty} \left[ A_{1n} \cosh(k_n x) + A_{2n} \sinh(k_n x) \right] \left[ A_{3n} \cos(k_n z) + A_{4n} \sin(k_n z) \right] \] (26b)

where \( k_n \) is the \( n^{th} \) eigenvalue of the system. The harmonic functions vary with respect to the distance between the set of parallel plates of zero potential. These plates lie in the coordinate equal constant planes which the harmonic functions are a function of.

### 3.2.1. Potential Due to Sources External to the Guide

The boundary data potential over the \( z=0 \) plane between the parallel plate electrodes is not symmetric. Therefore, in general, a combination of both odd and even symmetry boundary data is assumed to exist. To account for this, both harmonic solutions must be retained. Since the plate potentials in the \( x = \pm \frac{d}{2} \) planes are zero, Eq. (26a) is employed to represent the fields in the transition section. The length of the guide is chosen to satisfy both Eqs. (25a) and (25b). Therefore, with a small degree of error, the plane terminating the transition region at \( z=ML \) is modeled as a flat perfectly conducting plate with zero potential.

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Satisfying the boundary condition at $z=ML$ yields

$$V_1(x,z) = - \sum_{n=0}^{\infty} \left[ A_{2n} \sin(k_{2n}x) + A_{2n}\sin(k_{2n}x) \right] \left[ \frac{\sinh(k_{2n}(z-ML))}{\cosh(k_{n}ML)} \right]$$

(27)

The zero potential at $x = \pm \frac{d}{2}$ is satisfied if $k_{2n} = \frac{(n+1)\pi}{d}$ and $A_{2n}$ are respectively replaced by $\left[ \cos\left(\frac{n\pi}{2}\right) \right]^{2} A_{2n}$ and $\left[ \sin\left(\frac{n\pi}{2}\right) \right]^{2} A_{2n}$. Taking advantage of the orthogonality condition of the harmonic functions and representing the boundary data in the plane at $z=0$ as $(x,0)$, the potential in the transition region due to sources located at $z<0$ is

$$V_1(x,z) = \frac{2}{d} \sum_{n=0}^{\infty} \frac{\sinh(k_{n}(ML-z))}{\sinh(k_{n}ML)} \left[ \cos\left(\frac{n\pi}{2}\right) \right]^{2} \cos(k_{n}x) \int_{-\frac{d}{2}}^{\frac{d}{2}} i(x,0) \cos(k_{n}x) dx$$

$$+ \left[ \sin\left(\frac{n\pi}{2}\right) \right]^{2} \sin(k_{n}x) \int_{-\frac{d}{2}}^{\frac{d}{2}} i(x,0) \sin(k_{n}x) dx$$

(28)
The boundary data in the \( z=0 \) plane is determined by sources external to the guide with all remaining sources suppressed. For \( z>0 \), the potential is represented by a convergent series as seen by a large argument expansion of the hyperbolic functions. At \( z=0 \), the convergence of the series is questionable. This implies that a large number of terms in the series may be required to accurately represent the potential near the \( z=0 \) plane.

### 3.2.2. Potential Generated by the Sources in the Transition Region

The proximity of the plates in the transition region relative to the external geometries is small. As a result, the potential distribution and the field pattern generated by sources in the transition region are significantly affected by the external structure and its material composition. Therefore, the geometrical configuration and the material composition of the structure external to the guide \( (z<0) \) must be specified with all sources contained therein suppressed. As exhibited in Fig. 6, this air filled region is composed of perfectly conducting, rectangular shaped walls positioned at \( x=(d/2) \), \(-<z<0; -h<x<(d/2), z=->; x=-h, -<z<0; -h<x<-(d/2), z=0 \). As should be expected, a perturbation in the shape or material composition of a section of the wall or medium in far proximity of the end of the guide should have little effect on the potential distribution and field pattern internal to the transition region. As a rough estimate, one may employ Eqs. (25a) and (25b) to determine what proximities may be considered as far.
The structure is separated into regions A and B by the x=-d/2 plane extending between z=- and z=0. The fields are determined in these regions separately and then related through boundary conditions. The loading effect of the transport section is modeled by N pairs of electrodes of length L extending beyond the z=ML boundary. The potentials on these electrodes in the transport region are retained. At z=(M+N)L, the remainder of the transport section is modeled as a perfectly conducting flat plate of zero potential. For simplicity and due to the geometrical symmetry of the transport section, a flat plate boundary at this position is reasonable. The length of the region extending beyond the z=ML boundary is chosen such that the presence of the plate on the field distribution in the transition region is negligible.

The non-zero boundary data characterizing the potentials generating the electric fields in this geometry is given as

\[ V\left(\frac{d}{2},z\right) = V_{10} \left[ \sum_{s=0}^{\frac{M+N+1}{4}} \left( u(z - 4sL) - u(z - \{4s + 1\} L) \right) + \sum_{r=1}^{\frac{M+N}{4}} \left( u(z - \{2s - 1\} L) - u(z - 2sL) \right) + \sum_{i=0}^{\frac{M+N-1}{4}} \left( u(z - \{4s + 2\} L) - u(z - \{4s + 3\} L) \right) \right] \]

(29a)

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where $V_0(-d/2,z)$ is the boundary data in the plane between regions A and B. This boundary information is not known a priori but provides a connection between the fields in the two regions. For clarity, M and N are integers evenly divisible by four. Boundary conditions link the boundary data potential to the potential inside the two regions. The boundary conditions which need to be satisfied at $x = \pm d/2$ are

$$V\left(\pm \frac{d}{2}, z\right) = V\left(\frac{d}{2}, z\right) + V_0\left(-\frac{d}{2}, z\right)[u(z + \epsilon) - u(z)]$$  \hspace{1cm} (29b)

Continuity of the potential is enforced by Eq. (30a). Since the boundary between regions A and B is not a perfect conductor, a degree of freedom is gained. Consequently Eq. (30b) is required to ensure that the gradient of the potential is in this case continuous.
The form of solution of Laplace's equation in both regions A and B is given by Eq. (26b). Appropriately satisfying the zero potential boundary conditions at \( z = -a \), \( z = 0 \), \( x = -h \), and \( z = (M+N)L \) yields the potentials

\[
V_{2A}(x, z) = \sum_{n=0}^{\infty} \frac{1}{\cos(k_{2nA}^\prime)} \left[ A_{2n} \cosh(k_{2nA}^\prime x) + A_{2n} \sinh(k_{2nA}^\prime x) \right] \sin[k_{2nA}^\prime (z + \cdot)]
\]

(31a)

\[
V_{2B}(x, z) = \sum_{m=0}^{\infty} B_m \sinh[k_{2mB}(x + h)] \cosh[k_{2mB} h] \sin[k_{2mB}^2 z]
\]

(31b)

where

\[
k_{2nA}^\prime = \frac{n\pi}{(M + N)L}
\]

(32a)

\[
k_{2mB} = \frac{m\pi}{L}
\]

(32b)

A subscript A or B on the potential and the constant \( k_2 \) is used to designate the region being considered.

To determine the constants of integration in Eqs. (31a) and (31b), the boundary conditions Eqs. (30a) and (30b) along with the orthogonality principle are employed. Equations (29a), (29b), (31a), and (31b) are appropriately
substituted into Eq. (30a). These relations are then multiplied by 
\( \sin(k_{2z}[z + \bar{r}]dz \) and integrated between \( z=-\bar{r} \) and \( z=(M+N)L \). In a similar fashion, 
Eqs. (31a) and (31b) are substituted into Eq. (30b). This relation is multiplied by 
\( \sin(k_{2z}z)dz \) and integrated between \( z=-\bar{r} \) and \( z=0 \). Upon rearrangement the 
constants of integration in Eqs. (31a) and (31b) are

\[
A_{1n} = \frac{\cos(k_{2nA})}{[(M + N)L + \bar{r}] \sinh(k_{2nA} \frac{d}{2})} \left[ 2(V_{10 \cdot 1n} + V_{20 \cdot 2n} + V_{30 \cdot 3n}) + \right.
\sum_{m=0}^{\infty} B_m \frac{\sinh\left(k_{2mA} \left[ h - \frac{d}{2} \right]\right)}{\cosh(k_{2mA} h)} \left. \right]_{m}^{mn}
\]

(33a)

\[
A_{2n} = -\frac{\cos(k_{2nA})}{[(M + N)L + \bar{r}] \sinh(k_{2nA} \frac{d}{2})} \sum_{m=0}^{\infty} B_m \frac{\sinh\left(k_{2mA} \left[ h - \frac{d}{2} \right]\right)}{\cosh(k_{2mA} h)} \left. \right]_{m}^{mn}
\]

(33b)

\[
B_m = \frac{2 \cosh(k_{2mA} h)}{k_{2mA} \cosh(k_{2mA} \left[ h - \frac{d}{2} \right])} \sum_{n=0}^{\infty} \frac{k_{2nA} \cdot 4mn}{\cos(k_{2nA})} \left[ -A_{1n} \sinh\left(k_{2nA} \frac{d}{2}\right) + A_{2n} \cosh\left(k_{2nA} \frac{d}{2}\right) \right]
\]

(34)
where $1_n$, $2_n$, $3_n$, and $4_mn$ are integrals associated with the $m$th and/or $n$th coefficient that have been tabulated in Appendix 1.

In this form, the solution of the $m$th and $n$th coefficient is not transparent. To amend this, the three relations are placed in the following vector/matrix form

\[
\begin{align*}
\vec{a}_1 &= \vec{k} + \vec{d}_1 \cdot \vec{b} \\
\vec{a}_2 &= -\vec{d}_2 \cdot \vec{b} \\
\vec{b} &= \vec{e}_1 \cdot \vec{a}_1 + \vec{e}_2 \cdot \vec{a}_2
\end{align*}
\]  

(35a)  

(35b)  

(36)

where $\vec{a}_1$, $\vec{a}_2$, and $\vec{k}$ are column vectors containing $n$ terms, $\vec{b}$ is a column vector containing $m$ terms, $\vec{d}_1$ and $\vec{d}_2$ are $m \times n$ matrices, and $\vec{e}_1$ and $\vec{e}_2$ are $n \times m$ matrices. The vector and matrix elements are

\[
K_n = \frac{2 \cos(k_{2n4})}{[(M + N)L + \frac{d}{2}] \cosh(k_{2n4})} 
\]  

(37a)
\[ D_{1mn} = \frac{\cos(k_{2n\theta})}{[(M + N)L + \frac{d}{2}] \cosh(k_{2n\theta} \frac{d}{2})} \frac{\sinh(k_{2m\theta} [h - \frac{d}{2}])}{\cosh(k_{2m\theta} h)} \]  

(37b)

\[ D_{2mn} = \frac{\cos(k_{2n\theta})}{[(M + N)L + \frac{d}{2}] \sinh(k_{2n\theta} \frac{d}{2})} \frac{\sinh(k_{2m\theta} [h - \frac{d}{2}])}{\cosh(k_{2m\theta} h)} \]  

(37c)

\[ E_{1mn} = \frac{2k_{2n\theta} \cosh(k_{2m\theta} h)}{k_{2m\theta} \cosh(k_{2m\theta} [h - \frac{d}{2}])} \frac{\sinh(k_{2n\theta} \frac{d}{2})}{\cosh(k_{2n\theta})} \]  

(37d)

\[ E_{2mn} = \frac{2k_{2n\theta} \cosh(k_{2m\theta} h)}{k_{2m\theta} \cosh(k_{2m\theta} [h - \frac{d}{2}])} \frac{\cosh(k_{2n\theta} \frac{d}{2})}{\cosh(k_{2n\theta})} \]  

(37e)

Decoupling Eqs. (35a), (35b), and (36) yields

\[ \tilde{A}_1 = \left[ \tilde{I} + \tilde{D}_1 \cdot \left[ \tilde{I} + \tilde{E}_2 \cdot \tilde{D}_2 - \tilde{E}_1 \cdot \tilde{D}_1 \right]^{-1} \cdot \tilde{E}_1 \right] \tilde{K} \]  

(38a)
\[ \bar{A}_2 = -\bar{D}_2 \cdot \left[ \bar{I} + \bar{E}_2 \cdot \bar{D}_2 - \bar{E}_1 \cdot \bar{D}_1 \right]^{-1} \cdot \bar{E}_1 \cdot \bar{K} \] (38b)

\[ \bar{B} = \left[ \bar{I} + \bar{E}_2 \cdot \bar{D}_2 - \bar{E}_1 \cdot \bar{D}_1 \right]^{-1} \cdot \bar{E}_1 \cdot \bar{K} \] (39)

where \( \bar{I} \) is the identity matrix. The size of the matrices and the vectors are chosen based on convergence and on the numerical accuracy of performing the matrix inversion.

3.3. Resultant Fields in the Transition Region

The resultant potential in the transition region is the sum of the potentials due to each of the sources. Therefore, with the aid of Eqs. (28) and (31a), the total potential is

\[
V(x, z) = \frac{2}{d} \sum_{n=0}^{\infty} \frac{\sinh(k_{in}[ML - z])}{\sinh(k_{in}ML)} \left[ \cos \left( \frac{n\pi}{2} \right) \right]^2 \cos(k_{in}x) \int_{-\frac{d}{2}}^{\frac{d}{2}} (x,0) \cos(k_{in}x) dx \\
+ \left[ \sin \left( \frac{n\pi}{2} \right) \right]^2 \sin(k_{in}x) \int_{-\frac{d}{2}}^{\frac{d}{2}} (x,0) \sin(k_{in}x) dx \\
+ \sum_{n=0}^{\infty} \frac{1}{\cos(k_{2n+1})} \left[ A_{2n} \cosh(k_{2n+1}x) + A_{2n} \sinh(k_{2n+1}x) \right] \sin[k_{2n+1}(x + \cdot)]
\]

(40)
The electric field is the negative of the gradient of the potential. The resultant electric field in the transition region is

$$
\vec{E}(x,z) = E_x(x,z)\hat{x} + E_z(x,z)\hat{z}
$$

(41)

where

$$
E_x(x,z) = \frac{2}{d} \sum_{n=0}^{\infty} k_{in} \frac{\sinh(k_{in}[ML-z])}{\sinh(k_{in}ML)} \left[ \cos\left(\frac{n\pi}{2}\right) \sin(k_{in}x) \int_{-d/2}^{d/2} (x,0) \cos(k_{in}x) dx - \right.
$$

\[ \left. \sin\left(\frac{n\pi}{2}\right) \right] \cos\left(\frac{d}{2}\right) \int_{-d/2}^{d/2} (x,0) \sin(k_{in}x) dx \right] -
$$

\[ \sum_{n=0}^{\infty} \frac{k_{2n+1}}{\cos(k_{2n+1})} \left[ A_{2n} \sinh(k_{2n+1}x) + A_{2n} \cosh(k_{2n+1}x) \right] \sin[k_{2n}(z+\frac{d}{2})] \] 

(42a)

$$
E_z(x,z) = \frac{2}{d} \sum_{n=0}^{\infty} k_{in} \frac{\cosh(k_{in}[ML-z])}{\sinh(k_{in}ML)} \left[ \cos\left(\frac{n\pi}{2}\right) \cos(k_{in}x) \int_{-d/2}^{d/2} (x,0) \cos(k_{in}x) dx + \right.
$$

\[ \left. \sin\left(\frac{n\pi}{2}\right) \right] \sin(k_{in}x) \int_{-d/2}^{d/2} (x,0) \sin(k_{in}x) dx \right] -
$$

\[ \sum_{n=0}^{\infty} \frac{k_{2n+1}}{\cos(k_{2n+1})} \left[ A_{2n} \cosh(k_{2n+1}x) + A_{2n} \sinh(k_{2n+1}x) \right] \cos[k_{2n}(z+\frac{d}{2})] \] 

(42b)
3.4. Field Plots and Comparisons

At \( x = \pm \frac{d}{2} \), it is difficult to achieve convergence of the electric field solutions: Eqs. (24a), (24b), (42a) and (42b). This is not unreasonable since the potential on the plate surface is modeled as being discontinuous with \( z \). About 90 terms in the series over \( n \) in Eqs. (42a) and (42b) and about eight terms in the series over \( m \) in Eq. (33b) are required to guarantee series convergence near the surface of the plates. On the electrode surface, the tangential component of the electric field, \( E_z \), is forced equal to zero. To compensate for convergence and numerical errors for this field component in the neighborhood of the plate, roughly in the regions \( \frac{d}{2} > |x| > \frac{d}{2} - 0.06d \) and \( z \neq nL \), a linear interpolation scheme is employed. The \( \hat{x} \) component of the field is corrected with the use of a Taylor expansion in the regions \( \frac{d}{2} > |x| > \frac{7d}{16} \) and \( z \neq nL \). At \( z = nL \) and \( x = \pm d/2 \), the \( z \)-component of the field is not zero since this position is located between adjacent plates. As expected, numerical analysis has shown \( E_x \) to approach zero and \( E_z \) to increase as these locations are approached. These approximations to the field solution are adequate. It is anticipated that most of the dust particles are to be contained in a region along and near the \( z \) axis.

The electrostatic fields in a guide composed of \( L = 6 \) cm wide parallel plate electrodes with a \( d = 5.0 \) cm separation are examined numerically. Referring to Fig. 6, the external cavity extends beyond the end of the guide a distance of...
=26 cm. The base of the cavity is \( h = 8.25 \text{ cm} \) as measured from the center of the guide. Employing Eqs. (25a) and (25b), it was predetermined that the number of plates required in the transition region of the guide is \( M = 8 \). The boundary data used at \( z = 0 \) in these relations was supplied by a finite element code characterizing the potential and the fields in the cavity region connected to the guide containing two sets of four plates each at a plate potential of zero and terminated by free nodes. In applying Eqs. (25a) and (25b), these fields were compared to the fields obtained from Eqs. (24a) and (24b) at \( z = M L \).

In the superposition scheme, sources in the transition region are significantly affected by the geometrical configuration of the transport region. To properly incorporate the loading effects of the transport region, the minimum number of plates extending beyond the transition region needs to be determined. The extended region is terminated by a zero potential endplate. By a numerical comparison, the number of plates is chosen such that the endplate does not significantly alter the fields in the transition region. A minimum of \( N = 8 \) plates is required. The plate potential sequence as illustrated in Fig. 4 is \( V_1, V_2, V_3, \) and \( V_2 \) respectively held at the potentials -20 kV, -40 kV, -80 kV, and -40 kV. The field lines generated by this geometry are shown in Fig. 7a. A finite element technique was employed to verify the analytical approach. Refer to Fig. 7b. Excluding the field line structure beyond \( z = 0.85 \text{ m} \), excellent agreement is observed when comparing the field line plots. When comparing the field strengths obtained by the two different methods, errors between 1% and 15%
results in the regions where the change in a field component is not too large. In those regions where the field component changes rapidly the error is significantly high. Due to computer memory limitations, a crude mesh was employed in the finite element technique. It has been verified that the errors resulting from the field strength comparisons are a consequence of the crudeness of the mesh employed by the finite element method. The discrepancies in the region beyond z=0.85 m are due to the termination of the guide. In Fig 7a, the transport region is assumed to extend to infinity. Due to memory constraints using the finite element method in generating Fig. 7b, the guide is terminated by ballooning.

A speculative discussion of particle transport is qualitatively described from the field line plots. This appears reasonable if the space charge effects are weak relative to the fields generated by the plates. Figures 7a, 8a, 8b, and 8c illustrate the field plots due to the sequential shifting to the right of the potentials in Fig. 4 yielding respectively first plate potentials of V₁, V₂, V₃, and V₂. For clarity, Fig. 7a corresponds to the plate potential distribution shown in Fig. 4 whereas Fig. 8a corresponds to the potential distribution shown in Fig. 4 shifted to the right by one plate yielding a V₂ first plate potential. The field lines near the center of the guide are predominately parallel to the axis of the guide. Near z=0, the field lines from the external cavity draws the charged particle into the guide. Since the viscosity of the air is large and the dust particle size is small, the viscous drag dominates over the inertial forces. It is therefore anticipated that a particle will move slowly somewhat along a field line. Observe in Fig. 7a that the...
charge entering into the guide from almost any position in x at z=0 is forced towards the center of the guide. The field lines extend to and terminate at the center of the third plate. By shifting the potential on the plates (refer to Fig. 8a), the charge located predominately above the second or third plate encounters the lines of force generated by the potential difference between plates two (potential of $V_1$) and four (potential of $V_3$) and is forced to move down the guide. Note that the potential difference of the cavity and plate two and the potential difference of plate two and plate four is such to cause charge separation on the surface of plate two. Hence the field lines both terminate and originate from the same plate. This feature along with the inertial force of the charged dust particle is anticipated to be adequate to continue to draw dust particulates from the input cavity region into the particle guide past the second plate. The potential is again shifted when the dust particle is above plate four (refer to Fig. 8b). After the particle drifts a distance of about the length of a plate or so, the plate potential is shifted again (refer to Fig. 8c). With this, the period between shifts in potential appears to be about the transit time for a dust particle to travel a distance approximately equal to the length of one or two plates. It would appear that many particles entering the guide out of phase with the sequential switching of the potentials will collide with the guide walls. This may not be the case. It is anticipated that the mass of the particle and the viscous nature of the air medium will prevent the charge from following sudden directional changes in the fields. Field gradients will be experienced by the particle thereby altering the drift motion of the charge. These gradients with the time transition of the potentials will
undoubtedly cause the charge out of synchronization with the potential shifts to exhibit an irregular circular like motion with an overall drift down the guide. A particle code is presently being developed to examine the dust particle trajectories in the semi-infinite particle guide.

Based on the above assumption of weak space charge effects, one can speculate on the dust load density that can be transported by this device employing a crude order of magnitude approximation. Space charge affects will occur both in the $x$ and $z$ directions in the two dimensional geometry. The $x$ component of the field is the weakest at $z=nL$ where $n=1,2,3,...$. The distance between coplanar plates is very small compared to the length of the plate. Further, the $z$ component of the field in the neighborhood of this region is nearly at its maximum value. Consequently it stands to reason that the time the charged dust particle will spend in the $z=nL$ planes is small compared to that in the region above the electrodes. The $x$ component of the field in the $z=nL+L/2$ planes is largely responsible for focusing the charged dust particles near the $z$ axis of the device. Further, the $z$ component of the field and the $z$ component of the force in these regions are small. As a result, one anticipates that the charged particle will spend more time in this plane as compared to any other plane. Consequently, only the $x$ component of the field in the plane passing through the center of the plate with the least potential is employed in the space charge approximation. Near the plate surface, it is expected that the impact of the fields generated by neighboring plates will be small. Therefore, retaining a
single term in the summation in Eq. (7a), the approximate magnitude of the x component of the electric field in the absence of charge is

\[
|E_x| \approx \frac{|V_i|}{L} \frac{\sinh(kx)}{\cosh\left(\frac{k \frac{d}{2}}{2}\right)}
\]

(43)

Now assume that a uniformly distributed planar slab of charge of thickness 'a' exists centered between the bi-planar electrodes. The field generated by the charge at the surface of the slab is

\[
|E_x(x = \frac{a}{2}, z)| = \frac{\rho \varepsilon_o a}{2}
\]

(44)

This assumes that the plate potentials have been suppressed. For space charge effects to be considered small, the x component of the electric field generated by the plate must be at least one order of magnitude large compared to the field generated by the slab of charge. As a result, the charge density adequately handled by the device can be expressed by the inequality as

\[
|\rho| < \frac{\varepsilon_o V_i \sinh\left(\frac{\pi a}{4L}\right)}{5\varepsilon_o L \cosh\left(\frac{\pi a}{4L}\right)}
\]

(45)
Assuming $a=3\text{cm}$, $d=5\text{cm}$, $L=6\text{cm}$, and $|\nu|=20kV$, the charge density $|\rho|<6.47\times10^{-6}\text{ Cm}^{-3}$. A microdiameter spherical dust particle will support about 1000 electrons before the atmosphere near the particle surface breaks down [41]. Consequently, the dust particle density is roughly less than $4\times10^{10}$ particles $\text{m}^{-3}$.
CHAPTER 4

A SINGLE PARTICLE MOTION STUDY

4.0 INTRODUCTION

A single particle trajectory study is conducted to investigate the limitations of a family of quasi-electrostatic air filter designs. This study is limited by the fact that space charge effects are neglected. Even so, it provides insights on the operation of the device under low dust loads. Dust load densities have been examined in the previous chapter in relation to the transport geometry. It is anticipated that practical dust load densities in the excavation cab is for below this value. Consequently, single particle studies appear to be an adequate form of measure to evaluate the parameter constraints of the air filter design.

This chapter is organized in the following fashion. Section 4.1 describes the design of the air filter in detail. Section 4.2 characterizes the field configuration in all three regions of the air filter. Section 4.3 examines tendencies in the single particle orbits. The design as a whole is numerically studied in Section 4.4.
4.1 The Design

A two dimensional quasi-electrostatic air filter is exhibited in Fig. 1. The air filter is divided into four major regions: charging region (Region I), electrostatic region (Region II), transport region (Region III), and the collection region (Region IV). The latter region is not displayed in Fig. 1. As the labelling of the individual regions implies, the task of the air filter is to charge, trap, transport and collect the dust particles without human intervention.

Dust and air are drawn into an inlet tube by means of a fan (not shown in Fig. 1) and directed into the charging region of the air filter. The air flow passes through a metallic grid, grid A, upon entering the charging region. The air and untrapped dust exit through a metallic grid, grid B. Grids A and B are held at the same potentials. Large airborne dust particles entering the charging region may be pulled by gravity to the bottom of the charging chamber. The microdiameter particles are photoionized by a fan shaped, pulsed, ultra violet laser beam. The charging of microdiameter particles is limited by recombination effects, dust loading densities, and the beam's energy, pulse duration, and pulse frequency.

The electrostatic fields are configured by the wall potentials of the open ended charging and electrostatic cavities and the potential distribution of the base plate, plate C, in the charging region. The electrostatic region is terminated by set of plates that protrude and constrict the aperture opening to the transport
region. The large electrostatic field is tailored to extract the charged microdiameter particles from the charging region and position them in a region midway between the first set of bi-planar electrodes in the transport region. The viscous forces of air are in direct competition with the electrostatic forces the particles encounter. In a zero air flow case, this effect aids in causing the charged particle to follow the electric field lines of force.

The transport region is composed of identical sets of bi-planar plates aligned with a common axis. The transport region is assumed to be two dimensional and semi-infinite in length. To aid in focusing the dust particles, the bi-planar electrodes are at the same potential. Adjacent sets of parallel plates differ in potential. The plate potential sequence of any four consecutive sets of plates follows the pattern ... \( V_1, V_2, V_3, V_2, \ldots \) where \( \ldots \) represents the repeated nature of the potential pattern indicated. Consequently, excluding the region near the ends of the transport region, the spatial period of the quasi-electrostatic fields is equivalent to four times the length of any one plate. End effects such as the cavity structure of the electrostatic region significantly alters the spatial periodic nature of the field configuration near the ends of the transport region. A four plate series is employed due to its generated field's ability to trap charges over a relatively large region in space but not necessarily too large for practical application. To transport the dust particles long distances toward the collection region, the plate potentials are shifted in the direction of the desired particle motion. The time between transitions is anticipated to be about a particle transit
time along one or two electrodes. This is large compared to the field's transient
time constant developed by the voltage change on the electrodes. Further, the
transient time constant is anticipated to be very small compared to the time it
takes for the charge to overcome the large inertial and viscous forces and move
a substantial distance before the steady state electrostatic forces dominate.
Therefore, as far as the charged dust particle is concerned, it only experiences
electrostatic forces and damping mechanical forces. As a result, an electrostatic
modelling of the fields in the guide is employed. Magnetic field contributions are
ignored since they do not significantly affect the overall particle motion.

The collection region can be any number of different geometries and
materials. For the most part, it is isolated from the environment. One plausible
configuration is a fiber web of polarized electret material. The dust particles
floating in a relatively motionless air medium will be Coulomb attracted to the
electret fibers [28]. This provides for a simple means of cleaning and disposing
of the material. Depending on the dust load, the attached dust particles will not
dislodge when small forces are exerted on the fiber mesh. This allows for a
relatively safe means of disposing radionuclides without further resuspension
during machine maintenance and cleanup procedures.

4.2 The Field Configuration

A finite element method along with analytical theory are employed to
characterize the fields in the air filter. The field lines were carefully tailored under
the assumption that the viscous nature of the air medium to microdiameter particles will prevent the charged dust particles from exceeding speeds that cause the particles to drift significantly across the electric field lines. In effect, the electrostatic forces were assumed to be large enough to overcome gradual changes in the inertial forces. Loading effects of the charging, electrostatic, and transport regions are carefully handled by means of boundary conditions.

A standard finite element method was employed to characterize the fields in the charging and electrostatic regions of the air filter. Refer to Fig.1. To simulate the loading effects and fields of the transport section, the transport region was assumed to be composed of four sets of bi-planar plates at any one of the four sequential permutations of the following potential pattern: \( V_1, V_2, V_3, V_2 \). The nodal grid used to describe the geometry of the air filter consists of 13 by 10 points in the charging region, 13 by 10 points in the electrostatic region, and 84 by 10 points in the modelled transport region. The potentials on the walls of the electrostatic cavity and grid plates in the charging chamber were respectively constrained as \( V_A \) and \( V_B \). The transport region of the device was terminated by free nodes due to memory limitations of the computer. That is, numerical ballooning was employed. The mesh nature of the inlet and exhaust plates is modelled in the same fashion as the solid walls of the electrostatic chamber. This is reasonable if the two dimensional inlet holes are small with respect to the dimensions of the grid walls. The fields in the openings will be significantly different than computed. It is assumed that the particles entering
the charging region are nearly neutral or have a small charge such that with the aid of the hydrodynamic forces due to air flow through the inlet grid, the trajectories of the dust particles are not significantly affected. The dynamics in the holes in the exhaust grid are not of significant importance since particles in these locations are assumed to be lost to the walls of the guide. Referring to Fig. 9, the potential distribution on the bottom (\( \tilde{x} = 0 \) plane) planar plate of the charging region [plate C in Fig. 1] is parabolic in nature. This allowed for the fields to be shaped in such a fashion as to guide most of the charged dust particles from the charging region to transport region. Figure 10 yields a potential plot of the charging region. Guided by Fig. 10, the planar bottom plate in the charging region with parabolic voltage distribution may be replace by a parabolic-like plate configuration driven by a single plate potential. Both configurations are reciprocal and will yield the same electric field configuration. A typical field plot in the charging and electrostatic regions is displayed in Fig. 11.

The approximate location of the charging region and the electrostatic region are respectively \( 0<\tilde{z}<150 \) and \( 0<\tilde{x}<125 \) and \( 0<\tilde{z}<150 \) and \( 125<\tilde{x}<250 \) where superscript tilde, \( \sim \), implies the quantities are normalized unitless. The normalization employed is given in Sects. 4.3 and 4.4. Since the viscosity of the air is large and the dust particle size is small, the viscous drag dominates over the inertial forces. It is therefore anticipated that a particle will move slowly somewhat along a field line. Employing this line of thought, it is observed in Fig. 11 that charged dust particles located in the following approximate regions are guided into the transport region: \( 0<\tilde{z}<50 \) and \( 0<\tilde{x}<105, 50<\tilde{z}<90 \) and \( 0<\tilde{x}<150, \)
and \(90 < z < 140\) and \(0 < x < 50\). The plate restricting the aperture opening to the transport region aids in guiding the charged dust particles near the central region between the upper and lower electrodes. The field lines entering the transport region are nearly parallel to the electrode plates in this region.

The theory for the fields generated in the transport section has been described in the previous chapter [40]. Normalized field plots pertinent to the design of the air filter are shown in Figs. 12a-d. For convenience, the origin of the normalized coordinate system has been translated to the intersection point between the plane containing the aperture opening between the electrostatic and the transport regions and the bottom set of coplanar electrodes. The normalized length of each electrode is 87.4. The transport region is divided into two sections, a transition section and a transport section. The fields in these sections are obtained by two different types of theories. The fields in the transition section are configured as a result of: the loading effects of and the fields generated in the electrostatic and charging regions, the loading effects of the transport section, and the fields generated by the plate potentials in the transition section [40]. The plates in the transport section are far from the ends of the transport region. Loading effects are insignificant and therefore neglected when analyzing the fields [40]. Figures 12a, 12b, 12c, and 12d respectively represent the field mappings for the plate potential patterns \(\{\tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3, \tilde{\nu}_2, \ldots\}\), \(\{\tilde{\nu}_2, \tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3, \ldots\}\), \(\{\tilde{\nu}_3, \tilde{\nu}_2, \tilde{\nu}_1, \tilde{\nu}_2, \ldots\}\), and \(\{\tilde{\nu}_2, \tilde{\nu}_3, \tilde{\nu}_2, \tilde{\nu}_1, \ldots\}\) where the first potential
listed in each case is the potential on the bi-planar electrodes closest to the electrostatic/transport aperture opening. These four permutations of plate potentials yield, in the sequence ordered (Fig. 12a then 12b, etc.), the generated fields resulting after a plate potential shift has occurred within one temporal period. Note that the field lines in the region between adjacent pair of electrodes are nearly parallel to the electrode surface especially in the central region between bi-planar electrodes. The dust particles tend to be forced either toward the collection region or away from the collection region. It will be shown that the field gradients and the temporal period of the plate potential shift tend to trap the suspended particles. The overall field line pattern tends to shift towards the collection region pulling the already trapped charged dust particles with it. Examining the plate potential sequence of Figs. 12b and 12d, it is observe that the pattern differs by the placement of \( V_1 \) and \( V_3 \). Note the field patterns are nearly the same except for the sloping of the field lines. This is expected since the ordering of the potentials is reciprocal. The field lines terminate normal to the electrode surface as expected. Figures 12a and 12d have the tendency of pulling the charge particles at nearly any position in the electrostatic/transport aperture opening towards the central region between the first set of bi-planar electrodes. Based on the argument that a charge particle follows a field line, Figs. 12b and 12c tend to draw the particles in the aperture opening towards the first electrode. Therefore, the launch time delay between the time a charged particle is trapped to the time the plate potentials are turned on may affect the
transport properties of the air filter. This is carefully examined in the design study of this paper.

4.3 Single Particle Dynamics

A Lorentz particle code was developed to examine the charged particle dynamics in the air filter. Electrostatic forces and the viscous forces of air are characterized in the code. A fourth and fifth order Runge Kutta routine is employed to solve the system of force equations. The field distribution in each region of the air filter is established before the Lorentz particle code is started. An interpolation scheme is employed to determine the field the particle experiences.

The governing relation describing the dynamics of the particle is given by

\[
\frac{d^2 \vec{r}}{dt^2} = \frac{q}{m} \vec{E}(\vec{r}) - \frac{6\pi R \eta}{m} \frac{d\vec{r}}{dt}
\]  

(46)

where \( R \) is the radius of the spherical dust particle, \( \eta \) is the viscosity of air, \( m \) is the mass of the particle, and \( q \) is the particle charge. It is assumed that the drag of air obeys Stokes’ law. Therefore, the second term on the right hand side of Eq. (46) is the frictional force (aerodynamic force) due to the drag of air on a particle modelled as a sphere of radius \( R \) divided by its mass. To provide for a more general study of families of quasi-electrostatic air filters, the force equation
is normalized. Define the electric field, voltage, position, time, and velocity in
terms of their respective normalized quantities as 
\[ E = \frac{6\pi R \eta c_o \tilde{r}}{q}, \quad V = \frac{c_o^2 m \tilde{v}}{q}, \]
\[ \tilde{r} = \frac{c_o m}{6\pi R \eta}, \quad t = \frac{m}{6\pi R \eta}, \quad \text{and} \quad \tilde{v} = c_o \tilde{v}. \]
The superscript tilde quantities are the unitless normalized parameters. The parameter \( c_o \) has units of velocity. By varying this parameter within the limits of a nonrelativistic theory, one may change the scale factor of the air filter design without altering the normalized particle dynamics. The unitless, normalized equation of motion simplifies to

\[ \frac{d^2 \tilde{r}}{dt^2} = \tilde{E}(\tilde{r}) - \frac{d \tilde{r}}{dt} \quad (47) \]

where

\[ \tilde{E}(\tilde{r}) = -\tilde{\nabla} \cdot \tilde{V}(\tilde{r}). \quad (48) \]

Typical single particle trajectories are illustrated in Figs. 13a-c. The top and the left border and the dashed line in these figures represent the inside boundary of the air filter. In each case, the dust particle is assumed to be successfully trapped and transported. The dust particle’s and the environment’s parameters are: for Fig. 13a, \( n = 1, \bar{x} = \bar{z} = 0.125, \bar{v}_x = 0, \bar{v}_z = 0, \bar{v}_{\text{airflow}} = 0, \bar{T}_{\text{shift}} = 1.555 \times 10^4 \); for Fig. 13b, \( n = 0.7, \bar{x} = \bar{z} = 0.125, \bar{v}_x = 0, \bar{v}_{\text{airflow}} = 0, \bar{T}_{\text{shift}} = 7.069 \times 10^3 \); for Fig. 13c,
\( n = 0.13, \bar{x} = 0.125, \bar{v}_z = 0, \bar{v}_{\text{z airflow}} = 0, \bar{T}_{\text{shift}} = 1.13 \times 10^4 \); for Fig. 13d, \( n = 0.5, \bar{x} = 12.5, \bar{v}_z = \bar{v}_{\text{z airflow}} = 2.66 \times 10^{-4}, \bar{T}_{\text{shift}} = 1.27 \times 10^4 \) where \((\bar{x}, \bar{z}, \bar{v}_x, \bar{v}_z)\) are initial particle position and initial velocity, \( \bar{T}_{\text{shift}} \) is the period of the transport plate potential, \( n \) is the ratio of the net charge to the net maximum charge a dust particle sustains after being charged, and \( \bar{v}_{\text{z airflow}} \) is the air flow velocity in the \( z \) direction. The initial velocity in the \( x \)-direction \((\bar{v}_x)\) and the launch time delay \((\bar{t}_{\text{delay}})\) are zero in each case. In each case, the machine dimensions, particle mass and size, air viscosity, maximum charge, and plate potentials are the same. These are provided in the design study. Observe in each of the figures that the particle trajectory in the charging and electrostatic regions tends to follow a field line. Refer to Fig. 11. Upon entering the transport region, the charged dust particle exhibits a rather jagged circular trajectory that tends to drift towards the collection region. The irregular orbit pattern of the charge is due to the viscous effects of the air medium, the electrostatic field gradients, and the periodic plate potential shifts. Figure 13a shows that the orbital trajectory of the charge dampens out with time. Although not clearly shown in the figure, a one dimensional oscillation motion with drift is present in the region \( 800 < \bar{z} \). The dust particle is trapped in a potential well suspended between the bi-planar electrodes. Figure 13b exhibits a particle trajectory that differs from Fig. 13a. Observe that as the charge undergoes an irregular circular orbit, its drift velocity becomes significantly small about \( \bar{z} = 400 \) as indicated by the multiple overlaps of circular orbits as compared to its orbital motion elsewhere. For a duration in
time, the charge is locally trapped and suspended away from the electrodes. Note that the particle does not follow a trajectory to the surface of the electrodes in the transition section of the transport region. When the field in the transport region is at the appropriate phase, the charged particle is recaptured by the field for particle transport. Figure 13c depicts another typical trajectory. It is apparent that the orbital velocity is comparable to the drift velocity. Further observe that the overall circular motion of the particle is spatially damped with position in the z direction. The particle is approaching a stable position midway between the bi-planar electrodes similar to that shown in Fig. 13a. When air flow is incorporated into the model (Refer to Fig. 13d), it is observed that the particle orbit in the charging region is shifted towards the exhaust plates as expected. In the electrostatic region and the transport region where the air flow is assumed zero, the particle orbit is similar to the other cases examined. In each case, orbit stability is eventually achieved midway between the bi-planar electrodes. All trajectories that exhibit a similar stable trajectory or tendency to such a trajectory are labelled as being properly trapped and transported.

Space charge effects have been neglected in this analysis. It is observed in Figs. 13a-c that the single charge particle is suspended away from the plate electrodes in the transport region. In the two-dimensional geometry, space charge effects result both in the x and z directions. As long as the charge cluster is forced to move in the +z direction on average, space charge effects are not of great concern. The major concern is how space charge effects affect the dust
cluster's motion in the x direction. If a dust particle is lost to the bi-planar electrodes, the electrodes may become radioactivated. It was estimated [40] that uniformly distributed dust loads (particle densities) on the order of $10^{10}$ particles/m$^3$ in a slab beam thickness of 3 cm may be suspended between the bi-planar plates if the minimum magnitude of the plate potentials in the transport region is 20 kV.

4.4 Design Study

The overall design of the quasi-electrostatic air filter is evaluated. Due to the many possible collector designs, the collector is omitted from the design study. One may impose that the transport section of the air filter device is terminated by a collector that is matched to the transport section and therefore will not load down and hence not affect the overall design. All results are normalized. To provide for a practical design, one set of design parameters will be provided.

The ability of the air filter to trap and transport a charge properly is determined by a simple pass and fail system. A particle is launched at different points in the charging region under various particle and environmental conditions. The particle is then followed. If the particle collides with the walls of the air filter, it is assumed to be lost to the system and is therefore considered as a fail case. If the particle passes through the charging, electrostatic, and transport sections without colliding with the walls of the device, the particle is
assumed to be trapped and properly transported and therefore is considered as a pass case. All observations have shown that when a particle is properly trapped, its motion in the transport region tends to exhibit a damped oscillation motion about the center axis of the transport region. If some computational limit has been exceeded and this tendency in motion is observed before the particle has completely traversed the entire distance of the transport region, it is assumed that the particle is properly trapped. Figure 1 illustrates the air filter geometry with respect to a normalized coordinate system. The cross sectional area of the charging region is divided into 130 points; 13 equally spaced columns (in the z direction) and 10 equally spaced rows (in the x direction). This grid allows for worst case scenarios in launch positions.

The general parameters of the air filter, particle, and supporting environment are presented below and used throughout the design study. To examine the design, a single normalized parameter such as the period of the transport plate potential (\( T_{\text{shift}} \)), the initial particle position and initial velocity \((x, z, v_x, v_z)\), the ratio of the net charge to the net maximum charge a dust particle sustains after being charged \((n)\), air flow velocity \((v_{\text{airflow}})\), or the launch time delay \((T_{\text{delay}})\) is varied. The normalized plate potentials, \(\vec{V}_1\), \(\vec{V}_2\), and \(\vec{V}_3\), in the transport region of the air filter are respectively -1.007, -2.014 and -4.0286. The normalized potentials on appropriately the side and top walls of the electrostatic, \(\vec{V}_e\), and charging regions, \(\vec{V}_o\), are respectively -0.5036 and -0.05036. The net maximum
charge of a one microdiameter dust particle is calculated to be $1.67 \times 10^{-16}$ C ($n=1$) for air breakdown at the surface of the charged particle and $8.3 \times 10^{-17}$ C ($n=0.5$) when recombination effects are considered. The normalized dimensions of the air filter as depicted in Fig. 1 are $d = 124.8$, $L = 87.4$, $t = 150$, and $h = 188$. The air viscosity, $\eta$, particle size, $R$ (radius), and particle mass, $m$, are respectively $1.8 \times 10^{-5}$ Nt-s/m$^2$, 0.5 $\mu$m, and $1.2 \times 10^{-15}$ kg.

A series of zero air flow conditions ($\overline{v}_\text{airflow} = 0$) are first considered. Assume that a particle is initially launched from rest at the origin (lowest most left hand corner) of the air filter. As the charged particle is launch, the plate potentials in the transport region are turned on. Initially, the plate potential sequence on plates one through four (refer to Fig. 1) is respectively $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$, and $\overline{V}_4$. The normalized temporal period ($\overline{t}_{\text{transport}}$) of the transport plate potential and the ratio of particle charge to the maximum particle charge ($n$) are depicted in Fig. 14. The shaded areas indicate the parameter values for particles successfully trapped and transported. Just beyond the dark lined border, it has been verified that these parameters will result in charge loss to the walls of the air filter. Not all conditions outside of the dark lined border have been examined. Further, one section of the shaded region which is not bordered by a dark line implies that the parameters beyond the shaded region have also not been examined. A range of charge ratios exists for select constant periods. This is important since it can not be expected due to different dust loads that all charged dust particles will have
the same net charge. Figure 15 shows that for n=1, the trapping of the charge at the origin initially at rest is insensitive to nearly an order of magnitude change in the period. Here, p=1 implies that the charge is properly transported and p=0 implies that the charge is lost to the walls of the air filter. The single particle dynamics were examined for each point on the grid assuming normalized temporal transport periods of $\tilde{T}_{\text{shift}} = 7.07 \times 10^3$, $1.27 \times 10^4$, and $2.54 \times 10^4$. Comparing each case, it is observed that only minor changes result in the boundary separating those points successfully trapped from those lost. The shaded region of Fig. 16 yields the region in which all launched dust particles have been successfully transported for all three transport periods. The boundary shifts slightly upward for $\tilde{Z}$ approximately between 100 and 150. This further shows the insensitive nature of the particle trajectories to changes in the period for nearly all particles in the charging region. Extending the period slightly outside the range between approximately $5.65 \times 10^3$ and $2.54 \times 10^4$ will drastically affect the overall performance of the device to the extent that most if not all particles are lost. This is observed in Fig. 14.

Based on Fig. 15, a normalized transport potential period of $1.27 \times 10^4$ and charge ratio of n=1 was chosen. Assuming a zero air flow condition, a single dust particle at rest was launched at each position inside the charging region. Changing the normalized delay time ($\tilde{T}_{\text{delay}}$) by increments of $1.41 \times 10^3$ over the range between and including 0 and $1.13 \times 10^4$ [one normalized temporal transport

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period] yielded a nearly insensitive change to the boundary demarcating the particles that are collected from those that are lost. Figure 17 illustrates the worst case boundary. The shaded region is the locations in which particles launched at all time delays examined are successfully collected.

As indicated in Fig. 14, the particle launched at the origin from rest is properly collected for both cases n=1 and n=0.5. It is of interest to determine if this property extends to most of the charged particles launched at each position on the grid in the charging region. Figure 18 shows that as the net charge on a particle decreases, the boundary closest to the exhaust grid deteriorates. This is in agreement with intuition. The boundary closest to the inlet grid only slightly changes. When the normalized temporal transport period at $\tilde{\tau}_{\text{shunt}} = 1.27 \times 10^4$ is compared to $\tilde{\tau}_{\text{shunt}} = 2.54 \times 10^4$, the change in the demarcation between pass and fail particle trajectories for n=0.5 is insignificant. Refer to Fig. 19. This in agreement with the n=1 case further demonstrates an insensitive change in the temporal period of the transport plate potentials. This tends to indicate that the properties observed for the n=1 case follow for the n=0.5 case. Consequently, it is anticipated that the boundary between the pass and fail particle trajectories is nearly insensitive to changes in the normalized launch delay time. This was verified for two different delay times.

Since the viscosity of the air is large and the dust particle size is small, the viscous drag dominates over the inertial forces. As a result, the motion of all
initially dynamic particles experiencing an aerodynamic drag tends to quickly change toward the equilibrium motion of the surrounding air flow. Only those velocities complementing the velocities resulting from the electrostatic forces will increase. The drag forces will eventually equalize the electrostatic forces preventing the dust particle from further acceleration and hence from further increases in velocity. Except when air flow exists, these arguments indicate that non zero initial particle velocities are not significant in the design of the air filter. Similar conclusions were obtained from select simulations. As a result, no further discussions concerning these particle parameters in the device design will be made.

When the air flow between the inlet and the exhaust grids is not zero, a significant change in the boundary demarcating the region of properly trapped and untrapped particles results. Here, the air flow is assumed to be constant and directed only in the \( +z \) direction. It is assumed that the air flow is laminar so as not to invalidate the governing relation employed. Back flow due to the presence of the exhaust grids is neglected. Further, for simplicity, the air flow on the aperture opening between the charging region and the electrostatic region, in the electrostatic region, and in the transport region is assumed to be zero. A zero air flow assumption in the transport region is reasonable due to the plates restricting the aperture opening to this region. All turbulence conditions are neglected. Consider the case when the normalized delay time is zero and the normalized transport period is \( 1.27 \times 10^4 \) and \( n=0.5 \). The initial velocity of the
charged particle is equal to be the air flow velocity. It is observed in Fig. 20a that the electrostatic and viscous forces are comparable for low air flow velocities. The normalized air flow velocity is $2.66 \times 10^{-4}$. As a result, the boundary near the exhaust grid significantly deteriorates. The shaded region in the figure represents those positions in which a charged dust particle successfully becomes trapped and transported. Although this may seem to significantly affect the efficiency of the air filter, recall that charging is performed inside the laser fan beam. Theoretically, as the particles pass through the laser beam on a single pass, they are charged to a fraction of their saturation value. With the aid of mirrors or diffusers, multiple passes through the laser beam can be realized. The number of passes required for maximum charging is dependent on the laser beam geometry, the frequency of repetition, the air flow velocity, and the dust load densities. The beam width is usually small compared to the extent of the charging region where particles are successfully trapped and transported. Figures 20b and 20c show a significant deterioration of the region that properly transports the charged particles. The respective normalized air flow velocities are $8.825 \times 10^{-4}$ and $1.75 \times 10^{-3}$. For these constant air flow cases, the collection region of the precipitator can be redesign to be more compact in size. The equipotential curves in Fig. 10 can be used as a guide in redesigning the plate at the bottom of the collection chamber.

A practical design is now presented. A maximum charge of $1.67 \times 10^{-16}$ C is used. The parameters $n$ and $c_0$ are respectively chosen as 0.5 and 52.57. The
mass, radius, and viscosity of air are the same as above. Consequently, the machine dimensions are: \( d = 4.64 \) cm, \( L = 3.25 \) cm, \( l = 5.57 \) cm, and \( h = 6.96 \) cm. The plate potentials are: \( V_1 = -20 \) kV, \( V_2 = -40 \) kV, \( V_3 = -80 \) kV, \( V_a = -10 \) kV, \( V_b = -1 \) kV. The temporal transport period is \( T_{\text{transport}} = 0.09 \) s. Air flow velocities corresponding to the normalized velocities \( 2.66 \times 10^{-4}, 8.825 \times 10^{-4}, \) and \( 1.75 \times 10^{-3} \) [Refer to Figs. 20a-c respectively] are respectively \( 1.4 \) cm/s, \( 4.64 \) cm/s, and \( 9.2 \) cm/s. Assuming the laser has a repetition rate of 10 Hz, the laser beam widths for \( 1.4 \) cm/s, \( 4.64 \) cm/s, and \( 9.2 \) cm/s flow velocities are, respectively, \( 1.4 \) mm, \( 4.64 \) mm, and \( 9.2 \) mm. Since the region of successfully trapped and transported charged dust particles is limited for the higher air flows as illustrated in Figs. 20b and 20c, it is assumed that the laser beam will be directed parallel to the inlet grid. Assume a 39 cm radius for the fan-like laser beam. This is a conservative assumption implying that the two dimensional geometry is terminated by walls separated by about seven lengths of the charging region. The volume of a circular wedge in the semi-circular fan-shaped laser beam with a maximum cord length equal to the diameter of the dust particle is \( 1.95 \times 10^{-13} \) m\(^3\). The number density of photons and the number of photons in a circular wedge available for ionization assuming laser beam widths of \( 1.4 \) mm, \( 4.64 \) mm, and \( 9.2 \) mm are respectively \( 2.36 \times 10^{16} \) photons/m\(^3\) and \( 4,602 \) photons, \( 7.12 \times 10^{15} \) photons/m\(^3\) and \( 1,388 \) photons, and \( 3.59 \times 10^{15} \) photons/m\(^3\) and \( 700 \) photons. It is assumed that 663 photons are required to ionize a dust particle to its maximum charge. It is further assumed that the dust particles will be ionized to their maximum charge
in a single pass through the laser beam. Under these assumptions, the maximum dust load densities for the 1.4 mm, 4.64 mm, and 9.2 mm width beams are approximately $3.1 \times 10^{13}$ particles/m$^3$, $1 \times 10^{13}$ particles/m$^3$, and $5.13 \times 10^{12}$ particles/m$^3$. Although the design parameters are reasonable, the air flow velocities appear low.
CHAPTER 5

CONCLUSIONS

An analytical theory has been developed to characterize the fields in a semi-infinite guide composed of a periodic system of four sets of parallel plates with different potentials. The loading effects of an irregular shaped rectangular cavity at the input of the guide is incorporated in the theory. Good agreement has been shown with a numerical finite element code.

A novel air filter design has been presented based on single particle trajectories. Space charge effects have not been examined. The air filter employs a photo-ionization mechanism to charge the dust particles. Those particles which are properly trapped and transported do not collect on the air filter walls or electrodes. This is of importance when collecting dust particles contaminated with radionuclides. The study is broad enough that a variety of different designs may be obtained from the normalized plots to suit the conditions at hand.
It appears that the maximum charge a dust particle can sustain is limited by recombination ("electrostatic") forces and the laser photon energy. A typical dust load calculation for photoionization charging has been computed using commercially available specifications for an ultraviolet laser. For a suitable laser fan beam geometry, about $1.2 \times 10^{10}$ dust particles are charged to half of their maximum charge in a single pass through the laser beam. The air flow velocity is equal to the laser beam thickness divided by the laser frequency of repetition. With the aid of mirrors and/or diffusers maximum charging is theoretically possible.

It has been shown that the air filter design is insensitive to reasonable changes in many of the design parameters. Although the field lines are tailored to terminated on the electrodes in the transport region, the trapped charge upon entering the region becomes suspended between the bi-planar set of electrodes comprising the transport region. Trapped charge particle trajectories tend to exhibit stabilized trajectories near the center between bi-plane electrodes. Particle trajectories are insensitive to launch time delays. The boundary demarcating the region of the charging section in which launched charged particles are successfully trapped is insensitive to reasonable changes in launch time delays, particle charge, and temporal transport periods. Changes in the air flow velocity significantly affect the boundary region. A typical set of design parameters has been calculated.
The novel quasi-electrostatic air filter has some disadvantages. First, the system requires an ultraviolet laser with high power consumption. The laser is bulky and heavy. Portability becomes an issue. Further, the laser is expensive and makes use of hazardous gases. The viscous force of air is always in direct competition with the electrostatic and quasi-electrostatic forces. As a result, the efficiency of the device decreases significantly as the air flow velocity increases for practical electrode potentials.

In order to develop a more practical, less hazardous, economically feasible, energy efficient, and portable air filter for excavation purposes, there is a need to eliminate the laser as a charging source, make use of the viscous and electrostatic forces such that they complement each other, and decrease the plate loading effects on the electrostatic source. These objectives are currently being investigated.
Figure 1. An illustration of the quasi-electrostatic air filter.

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Figure 2. The fan-like laser beam geometry.
Figure 3. Infinite in extent, 2-D, transport section consisting of a series of parallel plate electrodes of length L located at \( x = \pm d/2 \).
Figure 4. The potential distribution over the parallel plate electrodes composing the particle guide.
Figure 5. The transition region of the particle guide. The transition region extends from $z = 0$ to $z = ML$ where $M$ is an integer evenly divisible by 4.
Figure 6. The transition region illustrated with a rectangular cavity located at the input of the guide and modeled loading effects of the transport region with endplate \((ML<z<[M+N]L)\). The rectangular cavity is subdivided into two sections, A and B to facilitate boundary conditions.
Figure 7a. The field lines of a particle guide composed of 16 sets of parallel plate electrodes. The distance of separation between the parallel electrodes is 5 cm and the plate length is 6 cm. The external cavity dimensions as shown in Fig. 6 are \( l = 26 \) cm and \( h = 8.25 \) cm. The plate potentials as shown in Fig. 4 are \( V_1 = -20 \) kV, \( V_2 = -40 \) kV, \( V_3 = -80 \) kV. The field line plot generated by the theory developed in this paper. Comparing the field plot and Figure 7b generated by a finite element method, excellent agreement is shown to exist for \( z \) less than 0.85 m. For \( z \) greater than 0.85 m, discrepancies exist due to memory constraints in the finite element method.
Figure 7b. The field lines of a particle guide composed of 16 sets of parallel plate electrodes. The distance of separation between the parallel electrodes is 5 cm and the plate length is 6 cm. The external cavity dimensions as shown in Fig. 6 are $l=26$ cm and $h=8.25$ cm. The plate potentials as shown in Fig. 4 are $V_1=-20$ kV, $V_2=-40$ kV, $V_3=-80$ kV. The field line plot generated by a finite element method.

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Figure 8a. A field line plot with vector orientation is exhibited. In the case, the voltage distribution in Fig. 4 is sequentially shifted to the right yielding the following first plate potential $V_2$. Note, Fig. 7a is the first field line plot in the series with $V_1$ as a first plate potential. The geometry of the guide with external cavity and the plate potentials is identical to that of Fig. 7.
Figure 8b. A field line plot with vector orientation is exhibited. In each case, the voltage distribution in Fig. 4 is sequentially shifted to the right yielding the following first plate potential $V_3$. Note, Fig. 7a is the first field line plot in the series with $V_1$ as a first plate potential. The geometry of the guide with external cavity and the plate potentials is identical to that of Fig. 7.
Figure 8c. A field line plot with vector orientation is exhibited. In each case, the voltage distribution in Fig. 4 is sequentially shifted to the right yielding the following first plate potential $V_2$. Note, Fig. 7a is the first field line plot in the series with $V_1$ as a first plate potential. The geometry of the guide with external cavity and the plate potentials is identical to that of Fig. 7.
Figure 9. The potential distribution along the base plate of the charging region. Note that the normalized base plate potential is $\bar{V} = VB \times 0.01$. 

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Figure 10. The equipotential surfaces in the charging region of the air filter. The planar base plate of the charging region may be shaped along an equipotential curve for compactness. Further, this simplifies the need for a complicated potential distribution along the plate.
Figure 11. The field lines in the charging and electrostatic regions. Assuming the charges follow the lines of electric force, it is observed that most of the particles entering the charging region through the inlet grid are guided into the transport region.
Figure 12a. Typical normalized field plot in the transport region for the plate potential sequence \( \tilde{V}_1, \tilde{V}_2, \tilde{V}_3, \tilde{V}_2, \ldots \). The potential on the bi-planar set of electrodes adjacent to the electrostatic/transport aperture is the first potential in each sequence. The normalized plate potentials are \( \tilde{V}_1 = -1.007, \tilde{V}_2 = -2.014, \text{ and } \tilde{V}_3 = -4.0286 \). The normalized potentials on the walls of the electrostatic and charging region are \( \tilde{V}_6 = -0.5036 \) and \( \tilde{V}_6 = -0.05036 \). The potential distribution on the base plate of the charging chamber is illustrated in Fig. 9.
Figure 12b. Typical normalized field plot in the transport region for the plate potential sequence $[\tilde{\mathcal{V}}_2, \tilde{\mathcal{V}}_1, \tilde{\mathcal{V}}_2, \tilde{\mathcal{V}}_3, \ldots]$. The potential on the bi-planar set of electrodes adjacent to the electrostatic/transport aperture is the first potential in each sequence. The normalized plate potentials are $\tilde{\mathcal{V}}_1 = -1.007$, $\tilde{\mathcal{V}}_2 = -2.014$, and $\tilde{\mathcal{V}}_3 = -4.0286$. The normalized potentials on the walls of the electrostatic and charging region are $\tilde{\mathcal{V}}_6 = -0.5036$ and $\tilde{\mathcal{V}}_6 = -0.05036$. The potential distribution on the base plate of the charging chamber is illustrated in Fig. 9.
Figure 12c. Typical normalized field plot in the transport region for the plate potential sequence \([\bar{V}_3, \bar{V}_2, \bar{V}_1, \bar{V}_2, \ldots]\). The potential on the bi-planar set of electrodes adjacent to the electrostatic/transport aperture is the first potential in each sequence. The normalized plate potentials are \(\bar{V}_1 = -1.007, \bar{V}_2 = -2.014, \text{ and } \bar{V}_3 = -4.0286\). The normalized potentials on the walls of the electrostatic and charging region are \(\bar{V}_4 = -0.5036\) and \(\bar{V}_6 = -0.05036\). The potential distribution on the base plate of the charging chamber is illustrated in Fig. 9.
Figure 12d. Typical normalized field plot in the transport region for the plate potential sequence $[\tilde{V}_2, \tilde{V}_3, \tilde{V}_4, \ldots]$. The potential on the bi-planar set of electrodes adjacent to the electrostatic/transport aperture is the first potential in each sequence. The normalized plate potentials are $\tilde{V}_1 = -1.007, \tilde{V}_2 = -2.014, \text{ and } \tilde{V}_3 = -4.0286$. The normalized potentials on the walls of the electrostatic and charging region are $\tilde{V}_a = -0.5036$ and $\tilde{V}_b = -0.05036$. The potential distribution on the base plate of the charging chamber is illustrated in Fig. 9.
Figure 13a. Typical 1mm diameter particle orbit trajectory is displayed for the initial normalized trajectory parameters $n = 1$, $\tilde{x} = \tilde{z} = 0.125$, $\tilde{v}_x = 0$, $\tilde{v}_{z\text{flow}} = 0$, $\tilde{r}_{\text{shift}} = 1.555 \times 10^4$. In the case, $h$, $\tilde{r}_{\text{delay}}$, $\tilde{v}_x$, and $m$ are respectively $1.8 \times 10^{-5}$ Nt-s/m², 0 s, 0 m/s, and $1.2 \times 10^{-15}$ kg. The outline of the air filter is bounded by the border on the left and the top sides of the plot and the thick dashed lines in the plot.
Figure 13b. Typical 1mm diameter particle orbit trajectory is displayed for the initial normalized trajectory parameters $n = 0.7$, $x = z = 0.125$, $\bar{v}_x = 0$, $\bar{v}_{z, \text{flow}} = 0$, $\bar{t}_{\text{shift}} = 7.069 \times 10^3$. In the case, $h$, $\bar{t}_{\text{delay}}$, $\bar{v}_x$, and $m$ are respectively $1.8 \times 10^{-5}$ Nt-s/m$^2$, 0 s, 0 m/s, and $1.2 \times 10^{-15}$ kg. The outline of the air filter is bounded by the border on the left and the top sides of the plot and the thick dashed lines in the plot.
Figure 13c. Typical 1mm diameter particle orbit trajectory is displayed for the initial normalized trajectory parameters $n=0.13$, $x = y = 0.125$, $v_z = 0$, $v_{x,flow} = 0$, $\tilde{T}_{delay} = 1.13 \times 10^4$. In the case, $h$, $\tilde{T}_{delay}$, $v_z$, and $m$ are respectively $1.8 \times 10^{-5}$ Nt-s/m$^2$, 0 s, 0 m/s, and $1.2 \times 10^{-15}$ kg. The outline of the air filter is bounded by the border on the left and the top sides of the plot and the thick dashed lines in the plot.
Figure 13d. Typical 1mm diameter particle orbit trajectory is displayed for the initial normalized trajectory parameters $n=0.5$, $\bar{x} = \bar{z} = 12.5$, $\bar{v}_x = \bar{v}_{airflow} = 2.66 \times 10^{-4}$, $\bar{t}_{\text{delay}} = 1.27 \times 10^4$. In the case, $h$, $\bar{t}_{\text{delay}}$, $\bar{v}_x$, and $m$ are respectively $1.8 \times 10^{-5}$ Nt-s/m$^2$, 0 s, 0 m/s, and $1.2 \times 10^{-15}$ kg. The outline of the air filter is bounded by the border on the left and the top sides of the plot and the thick dashed lines in the plot.
Figure 14. A single 1mm diameter particle trajectory located at the origin is examined for a range of charge ratios and transport potential periods. Note that $\tilde{T}_{\text{shift}} = T \times 10,000$. The shaded region represents the combinations of parameters that resulted in properly trapped and transported conditions. Just beyond the dark line border, the charges are lost to the wall of the air filter. The section of shaded region not bordered by a dark line implies that particle transport has not been verified for those parameters beyond the shaded region. In each case, $\tilde{v}_{\text{airflow}} = 0$, $\tilde{v}_z = \tilde{v}_x = 0$, and $\tilde{T}_{\text{delay}} = 0$. 

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Figure 15. For $n=1$, the range of transport potential periods which result in a properly trapped and transported particle is exhibited. Note that the range of the period is almost an order of magnitude. All other parameters are the same as in Fig. 14.
Figure 16. The two dimensional charging region is divided into a 13 by 10 grid. A single particle is launched at each grid point assuming the temporal transport potential period is $\bar{T}_{\text{vel}} = 7.07 \times 10^3, 1.27 \times 10^4, \text{ and } 2.54 \times 10^4$. The shaded region and the solid line connecting simulated data points are those regions and points in which a 1 mm diameter dust particle was properly transported in each case. A slight shift in the boundary occurred when comparing each region individually. This is not shown. Note, $n=1$, $\bar{v}_{\text{zflow}} = 0$, $\bar{v}_z = \bar{v}_x = 0$, and $\bar{\tau}_{\text{delay}} = 0$. 

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Figure 17. The shaded region and the solid line connecting simulated data points are those regions and points in which a 1 mm diameter dust particle was properly transported when the launch time delay was varied between 0 and $1.13 \times 10^4$ in increments $1.41 \times 10^3$. The range was conducted over a single period of the transport plate potentials. The change in the shaded region for each delay time examined was slight. In each case, $n=1$, $\vec{v}_\text{airflow} = 0$, $\vec{v}_x = 0$, and $\vec{v}_\text{shift} = 1.27 \times 10^4$. 

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Figure 18. The two sets of lines connecting simulated data points are the boundaries demarcating the region between those particles that were successfully and not successfully transported for $n=1$ (solid line) and $n=0.5$ (dashed line). The boundary near the exhaust grid is deteriorated as the dust particle charge decreases. The parameters simulated for the 1 mm diameter dust particle in both sets of cases are: $\tilde{\nu}_\text{carflow} = 0$, $\tilde{\nu}_z = \tilde{\nu}_x = 0$, $\tilde{t}_\text{delay} = 0$, and $\tilde{t}_\text{slight} = 1.27 \times 10^4$. 

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Figure 19. The shaded region and the solid line connecting simulated data points are those regions and points in which a 1 mm diameter dust particle with n=0.5 is properly transported when the transport potential periods are $\tilde{\tau}_{\text{slag}} = 1.27 \times 10^4$ and $2.54 \times 10^4$. 

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Figure 20a. The boundary data points and shaded region represent those positions in which successful particle trapping and transport occurred for the normalized air flow velocity $2.66 \times 10^{-4}$. A small change in the air flow velocity result in a significant change in the area of the regions where successful transport occurs. Here, $n=0.5$, $\bar{\tau}_{\text{shift}}=1.27 \times 10^4$, $\bar{\tau}_{\text{delay}} = 0$, $\bar{\nu}_z = \bar{\nu}_{\text{satflow}}$, and $\bar{\nu}_x = 0$. 

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Figure 20b. The boundary data points and shaded region represent those positions in which successful particle trapping and transport occurred for the normalized air flow velocity $8.825 \times 10^{-4}$. A small change in the air flow velocity result in a significant change in the area of the regions where successful transport occurs. Here, $n=0.5$, $\tilde{t}_{\text{shift}} = 1.27 \times 10^4$, $\tilde{t}_{\text{delay}} = 0$, $\tilde{\nu}_z = \tilde{\nu}_{\text{airflow}}$, and $\tilde{\nu}_x = 0$. 

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Figure 20c. The boundary data points and shaded region represent those positions in which successful particle trapping and transport occurred for the normalized air flow velocity $1.75 \times 10^{-3}$. A small change in the air flow velocity result in a significant change in the area of the regions where successful transport occurs. Here, $n=0.5$, $\tilde{r}_{\text{shelf}}=1.27 \times 10^4$, $\tilde{r}_{\text{delay}}=0$, $\tilde{v}_z=\tilde{v}_{\text{airflow}}$, and $\tilde{v}_x=0$. 

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APPENDIX II

Tabulated Integral Relations

Below are some tabulated integral relations that exist in Eqs. (33a), (33b), (34), and (37a)-(37e).

\[
\begin{align*}
I_n &= \sum_{s=0}^{M+N-1} (M+N) \int \left[ u(z - 4sL) - u(z - (4s + 1)L) \right] \sin[k_{2n}(z + \cdot)]dz = \\
&+ \sum_{s=0}^{M+N-1} \frac{1}{k_{2n}} \left[ \cos(k_{2n}[4sL + \cdot]) - \cos(k_{2n}[(4s + 1)L + \cdot]) \right] \\
J_n &= \sum_{s=1}^{M+N-1} (M+N) \int \left[ u(z - \{2s - 1\}L) - u(z - 2sL) \right] \sin[k_{2n}(z + \cdot)]dz = \\
&+ \sum_{s=1}^{M+N-1} \frac{1}{k_{2n}} \left[ \cos(k_{2n}[\{2s - 1\}L + \cdot]) - \cos(k_{2n}[2sL + \cdot]) \right]
\end{align*}
\]
\[ j_n = \sum_{s=0}^{M-N-1} \sum_{\nu=1}^{(M-N) L} \left[ u(z - \{4s + 2\} L) - u(z - \{4s + 3\} L) \right] \sin[k_{2nL}(z + i)] dz = \]
\[ \sum_{s=0}^{M-N-1} \frac{1}{k_{2nL}} \left[ \cos(k_{2nL}\{4s + 2\} L + i) - \cos(k_{2nL}\{4s + 3\} L + i) \right] \]

\[ j_{4mn} = \int_{-1}^{0} \sin[k_{2nL}(z + i)] \sin(k_{2mL}z) dz = \]
\[ \frac{1}{2(k_{2nL} - k_{2mL})} \left[ \sin(k_{2nL} i) - \sin(k_{2mL} i) \right] - \frac{1}{2(k_{2nL} + k_{2mL})} \left[ \sin(k_{2nL} i) + \sin(k_{2mL} i) \right] \]
%%FILENAME:PMMAIN
%%THIS IS A MAIN CONTROLL CODE TO COMPUTE ELECTRIC
%%FIELDS,
%%TO PLOT STATIC,
%%AND DYNAMIC SECTION ELECTRICAL FIELD LINES, TO PLOT
%%PARTICLE MOTION PATHES
%%THIS CODE WILL RUN ON A PC BECAUSE OF PC MEMORY
%%HERE THREE POINTS:
%% 1.clear space for enough memory in every step
%% 2.save data in every step
%% 3.load data when necessary
%% if you have enough space, delete 'save' and 'clear all' syntax
%% if you have no enough space you have to run the code separately
%% according to above 3 points
%%TO RUN THE CODE TAKES AT LEAST 2 HOURS TO GET
%%RESULTS ON A PC
%%TO RUN THE CODE:
%% 1.IN MATLAB "MATLAB Command Window" tpye:pmmain
%% 2.TO ANSWER ALL THE QUESTIONS, INPUT DATA.

%%%%% POTENTIAL DISTRIBUTION
%%%%% STATIC AND CHARGING REGION
%%%%%WALL POTENTIAL Va,Vb
%%%%%ST_FIELD TO FIND FIELD IN CHARGING AND STATIC REGION
%%%%%TRANSPORT REGION
%%%%%FIELD1_0,FIELD1_A,B,C,D TO FIND FIELD IN TRANSITION
%%%%%SECTION
%%%%%FIELD2_A,B,C,D TO FIND FIELD IN TRANSPORT SECTION
%%%%%CASE A
%%%%%POTENTIAL SEQUENCE V1,V2,V3,V2...
%%%%%CASE B
%%%%%POTENTIAL SEQUENCE V2,V1,V2,V3...
%%%%%CASE C
%%%%%POTENTIAL SEQUENCE V3,V2,V1,V2...
%%%%%CASE D
%%%%%POTENTIAL SEQUENCE V2,V3,V2,V1...

%%%%% PART1 STATIC SECTION %%%%  

%%%%% CASE A 
st_field
% SAVE DATA AS FILENAME.MAT FILE
save dataStA v2 Ex Ey xd yd n m;
clear all
%%% CASE B
st_field
% SAVE DATA AS FILENAME.MAT FILE
save datastB v2 Ex Ey xd yd n m;
clear all
%%% CASE C
st_field
% SAVE DATA AS FILENAME.MAT FILE
save datastC v2 Ex Ey xd yd n m;
clear all
%%% CASE D
st_field
% SAVE DATA AS FILENAME.MAT FILE
save datastD v2 Ex Ey xd yd n m;
clear all

%%%%%PART2 DYNAMIC SECTION %%%%%

%%% CASE A
%CALL THREE CODES TO COMPUTE ELECTRICAL FIELD FOR CASE
%A
field1;
field1_A;
field2_A;
save dataA Exa Eya Exat Eyat Exc Eyc

%FIND Exi Eyi FOR PLOTTING DYNAMIC SECTION FIELD LINES
dy_field
save dataAL Exi Eyi

%TO FIND ExiA AND EyiA FOR PLOTTING A PARTICLE MOTION
PATH
load datastA
to_field;
ExiA=Exp;
EyiA=Eyp;
save dataAp ExiA EyiA
clear all

%%% for case B
%CALL THREE CODES TO COMPUTE ELECTRICAL FIELD FOR CASE
%B
field1;
field1_B;
field2_B;
save dataB  Exa Eya Exat Eyat Exc Eyc

%FIND Exi Eyi FOR PLOTTING DYNAMIC SECTION FIELD LINES
dy_field
save dataBL  Exi Eyi
%TO FIND ExiB AND EyiB FOR PLOTTING A PARTICLE MOTION PATH
load datastB
to_field;
ExiB=Exp;
EyiB=Eyp;
save dataBp1 ExiB EyiB
clear all

%%%%% for case C
%CALL THREE CODES TO COMPUTE ELECTRICAL FIELD FOR CASE %C
field1;
field1_C;
field2_C;
save dataC1 Exa Eya Exat Eyat Exc Eyc

%FIND Exi Eyi FOR PLOTTING DYNAMIC SECTION FIELD LINES
dy_field
save dataCL  Exi Eyi

%TO FIND ExiC AND EyiC FOR PLOTTING A PARTICLE MOTION %PATH
load datastC
to_field;
ExiC=Exp;
EyiC=Eyp;
save dataCp  ExiC EyiC
clear all

%%%%% for case D
%CALL THREE CODES TO COMPUTE ELECTRICAL FIELD FOR CASE %D
field1;
field1_D;
field2_D;
save dataD  Exa Eya Exat Eyat Exc Eyc

%FIND Exi Eyi FOR PLOTTING DYNAMIC SECTION FIELD LINES
dy_field
save dataDL Exi Eyi

%TO FIND ExiD AND EyiD FOR PLOTTING A PARTICLE MOTION
%PATH
load datastD
to_field;
ExiD=Exp;
EyID=Eyp;
save dataDp ExiD EyID
clear all

%%%%%PLOTTINT 4 CASE A B C D FIELD LINES OF DYNAMIC
%%%%%SECTION
%%%%%CASE A
%SET GEOMETRY PARAMETERS
geo_data
%SET INITIAL CONDITION FOR LINE START HORIZANTLE AND VERTICLE NUMBER
horz_num=40;
vert_num=30;
count = 2*horz_num+2*vert_num;
%LOAD DATA
load dataAL;
%CALL PLOTTING CODE
plotline
clear all

%plot case B
%SET GEOMETRY PARAMETERS
geo_data
%SET INITIAL CONDITION FOR LINE START HORIZANTLE AND VERTICLE NUMBER
horz_num=40;
vert_num=30;
count = 2*horz_num+2*vert_num;
%LOAD DATA
load dataBL;
%CALL PLOTTING CODE
plotline;
clear all

%plot case C
%SET GEOMETRY PARAMETERS
geo_data
%SET INITIAL CONDITION FOR LINE START HORIZANTLE AND %VERTICLE NUMBER
horz_num=40;
vert_num=30;
count = 2*horz_num+2*vert_num;

%LOAD DATA
load dataCL;

%CALL PLOTTING CODE
plotline;
clear all

%plot case D
%SET GEOMETRY PARAMETERS
geo_data

%SET INITIAL CONDITION FOR LINE START HORIZANTLE AND VERTICLE NUMBER
horz_num=40;
vert_num=30;
count = 2*horz_num+2*vert_num;

%LOAD DATA
load dataDL;

%CALL PLOTTING CODE
plotline;
clear all

%%%%%PLOTTING A PARTICLE MOTION PATH %%%%
%SET GEOMETRY PARAMETERS
geo_data

%LOAD 4 CASE FIELDS
load dataAp;
load dataBp;
load dataCp;
load dataDP;

% plotpath WILL CALL forcelaw
plotpath

%%%%% end of the code
% FILENAME:ST_FIELD
% THIS CODE IS TO FIND STATIC AREA ELECTRIC FIELD
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CALL gen_fem TO SET UP THE INITIAL ARRAY
  gen_fem;
% CALL set_node SET BOUNDARY CONDITIONS
  set_node;
% FIND POTENTIAL v2
  fem_solv;
% FIND FIELD Ex AND Ey
  efield;
clear all

%%% INPUT INITIALIZATION GEOMETRY CONSTANTS
% GET THE NUMBER OF COLUMNS:
\texttt{n} = \texttt{input('Please enter the number of columns in the array, \#>1; n = ')};

% GET THE NUMBER OF ROWS:
\texttt{m} = \texttt{input('Please enter the number of rows in the array, \#>1; m = ')};

% GET THE SPACE B/W THE NODES OF EACH COLUMN:
\texttt{xd} = \texttt{input('Please enter the space between the columns, typical .2, xd = ')};

% GET THE SPACE B/W THE NODES OF EACH ROW:
\texttt{yd} = \texttt{input('Please enter the space between the rows, typical .2, yd = ')};

\texttt{ne} = (m-1)*(n-1)*2
\texttt{nd} = m*n

% GENERATE THE LIST OF LOCAL NODES FOR EACH ELEMENT:
for \texttt{j} = 1:m-1
  for \texttt{i} = 1:n-1
    \texttt{nl((j-1)*(n-1)*2+(i-1)*2+1,1:3) = [(j-1)*n+i (j-1)*n+i+1 j*n+i];}
    \texttt{nl((j-1)*(n-1)*2+(i-1)*2+2,1:3) = [(j-1)*n+i+1 j*n+i+1 j*n+i];}
  end
end

% GENERATE THE VECTORS WHICH DESCRIBE THE POSITION OF EACH NODE:
for \texttt{j} = 1:m
  for \texttt{i} = 1:n
    \texttt{x((j-1)*n+i) = (i-1)*xd;}
    \texttt{y((j-1)*n+i) = (j-1)*yd;}
  end
end
% THIS CODE IS USED TO GENERATE THE LIST OF FIXED NODES, THEIR
% CORRESPONDING VALUES
% AND THE TOTAL COUNT OF THE FIXED NODES.
% INPUT:
% DATA FROM gem_fem.m
% ANSWER ALL OF THE QUESTION FOR FIXED THE GEMOTRY
% OUTPUT: A GEMONTRY IS MADE
% np - NUMBER OF FIXED NODES.
% ndp - NODE # OF A PERSCRIBED POTENTIAL.
% val - POTENTIAL CORRESPONDING TO THE NODE DESCRIBED
% BY ndp: %ndp(i)<~> val(i).
% THESE VALUES ARE REQUIRED FOR fem_solv.m, FILES

clear row col again choice fixnode dummy

% INITIALIZE THE fixnode ARRAY WITH NaN's SO THAT A SET
% POTENTIAL OF ZERO CAN BE DETERMINED AT THE OUTPUT.
fixnode=zeros(m,n);
for i = 1 : m
    for j = 1 : n
        fixnode(i,j)=NaN;
    end
end

if (exist('np') == 1)&(exist('ndp') == 1  )&(exist('var) == 1)
    again = input('Some fixed nodes exist, edit or clear them? e/c ','s');
    if again == 'e'
        for i = 1:np
            for row = 1 : m
                for coi = 1:n
                    if (row-1)*n+col == ndp(i)
                        fixnode(row,col)=val(i);
                    end
                end
            end
        end
        clear np ndp val
    end
end

% EDIT THE ARRAY fixnode.
again = 'y';
dummy = input('Note that you may "free" a node by setting the potential to
"NaN". <rtn>');
while again == 'y'
i = 0; f = 0; row = 0; col = 0;
choice = input('Would you like to enter a node, row, column of fixed nodes or
quit? n/r/c/q ','s');
if choice == 'q'
again = 'n';
elseif choice == 'r'
while (row<1)|(row>m)
row = input('Enter the row # to be fixed ');
end
while (i<1)|(i>n)
i = input('Enter the column # to start at, the lower value ');
end
while (f<1)|(f>n)
f = input('Enter the column # to end at ');
end
pot = input('Enter the potential of these nodes ');
for col = i:f
fixnode(row,col) = pot;
end
elseif choice == 'c'
while (col<1)|(col>n)
col = input('Enter the column # to be fixed ');
end
while (i<1)|(i>m)
i = input('Enter the row # to start at, the lower value , ');
end
while (f<1)|(f>m)
f = input('Enter the row # to end at ');
end
pot = input('Enter the potential of these nodes ');
for row = i:f
fixnode(row,col) = pot;
end
elseif choice == 'n'
while (i<1)|(i>m)
i = input('Enter the row of the fixed node ');
end
while (f<1)|(f>n)
f = input('Enter the column of the fixed node ');
end
pot = input('Enter the potential of this node ');
fixnode(i,f) = pot;
else
    again = input('An improper input, would you like to enter more info? y/n ','s');
end

%CONVERT VALUES TO np ndp AND val WHICE fem_solv.m WILL
%RECOGNIZE.
np = 0;
for row = 1:m
    for col = 1:n
        if isnan(fixnode(row,col)) ~= 1
            np = np + 1;
            ndp(np) = (row-1)*n+col;
            val(np) = fixnode(row,col);
        end
    end
end

%CLEAR THE VARIABLES LOCAL TO THIS CODE.
%END
% FILENAME: FEM_SOLV
% THIS CODE IS TO EVALUATE COEFFICIENT MATRIX FOR EACH ELEMENT
% AND ASSEMBLE GLOBALLY
% INPUTS: n, m, nl, x, y, nd, ne ARE GENERATED BY "gen_fem.m" WHICH
% MUST BE RUN PRIOR
% TO RUNNING THIS CODE.
% ndp, np AND val MUST BE SET USING set_node.m IN ORDER FOR THIS
% CODE TO WORK.
% THE FINITE ELEMENT REGION IS AN n X m MATRIX OF NODES.
% nl CONTAINS THE LIST OF LOCAL NODES FOR EACH ELEMENT.
% x AND y CONTAIN THE POSITION OF EACH NODE IN THE RESPECTIVE
% DIRECTION.
% ne = NUMBER OF ELEMENTS, nd = NUMBER OF NODES.
%
% HOW TO RUN:
% INPUT: data from former codes
% OUTPUT:
% v2 - the potential matrix in geometry which is for finding fields
% PICTURE - Equipotential surfaces picture
%
% VARIABLES MUST BE CLEARED TO ENSURE PROPER OPERATION
% WHEN RUN SEVERAL TIMES.

clear b c v v1 v2 x4 y4

%% INPUTS VARIABLES FROM FORMER CODES
b=zeros(nd,1);
c=zeros(nd); ce=zeros(3,3);
xl=zeros(1,3);v=zeros(nd,1);v1=zeros(1,nd);
v2=zeros(m,n);x4=zeros(1,n);y4=zeros(1,m);
p=zeros(1,3);q=zeros(1,3);yl=zeros(1,3);
flag_r=0; flag_c=0;

% np = NUMBER OF FIXED NODES.
% ndp = NODE # OF A PERSCRIBED POTENTIAL.
% val = POTENTIAL CORRESPONDING TO THE NODE DESCRIBED BY ndp:
% ndp(i)<-->val(i).
% ce = LOCAL COEFFICIENT MATRIX.
% c = GLOBAL " " .
% b = RIGHT HAND SIDE MATRIX OF THE SYSTEM OF EQUATIONS WHICH
% GIVES THE SOLUTION.
% v = VECTOR OF SOLVED NODE POTENTIALS

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TO SOLVE RIGHT HAND SIDE MATRIX OF THE SYSTEM OF EQUATIONS \( b \)
AND GLOBAL COEFFICIENT MATRIX \( c \)

for \( i = 1 : ne \)
for \( j = 1:3 \)
    \( k = nl(i,j); \)
    \( xl(j)=x(k); \)
    \( yl(j)=y(k); \)
end

\( p(1)=yl(2)-yl(3); \)
\( p(2)=yl(3)-yl(1); \)
\( p(3)=yl(1)-yl(2); \)
\( q(1)=xl(3)-xl(2); \)
\( q(2)=xl(1)-xl(3); \)
\( q(3)=xl(2)-xl(1); \)
\( area = 0.5 * \text{abs}( p(2)*q(3)-q(2)*p(3) ); \)

% Determine coefficient matrix for element
for \( m1 = 1:3 \)
for \( n1 = 1:3 \)
    \( ce(m1,n1) = (p(m1)*p(n1)+q(m1)*q(n1))/(4.0*area); \)
end
end

%assemble globally - find \( c(i,j) \) and \( b(i) \)
for \( j= 1:3 \)
    ir=nl(i,j);
    %check if row corresponds to fixed node
    %If a fixed node is found then flag_r is set to 1.
    flag_r = 0;
    for \( k=1:np \)
        if \( ir == ndp(k) \)
            flag_r = k;
        end
    end
    if flag_r == 0 % free node found in row check
        for \( l=1:3 \)
            ic = nl(i,l);
            %check if column corresponds to fixed node
            %If a fixed node is found then flag_c is set to 1.
            flag_c = 0;
            for \( k=1:np \)
                if \( ic == ndp(k) \)
                    flag_c = k;
                end
            end
            if flag_c == 0 % free node found in column check
                \( c(ir,ic) = c(ir,ic) + ce(j,l); \)
else
    b(ir) = b(ir) - c(j,l)*val(flag_c);
end
else
    c(ir,ir) = 1.0;
    b(ir) = val(flag_r);
end
end
end

%%%%% SOLVE THE RESULTING SYSTEM OF SIMULTANEOUS
%%%%% EQUATIONS
% c=inv(c);
% v = c * b;

%%%%% PREPARE AN OUTPUT PLOT
% GENERATE AXIS VECTORS IN APPROPRIATE UNITS (meters).
% SINCE v IS A VECTOR AND A MATRIX IS REQUIRED FOR PLOTTING,
% CONVERT v INTO MATRIX v2.
% clear b c

% OUTPUT THE RESULT: A 3D PLOT OF THE POTENTIAL AND A PLOT OF
% THE EQUIPOTENTIAL LINES
figure
mesh(x4,y4,v2)
xlabel('x [m]');
ylabel('y [m]');
zlabel('Volts');
title('Potential of User's Geometry')

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figure
contour(v2,20,x4,y4)
xlabel('x [m]');
ylabel('y [m]');
axis([xd n*xd yd m*yd]);
title('Equipotential surfaces')
% END OF THE CODE

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%% FILENAME: EFIELD
%% THIS CODE IS TO GENERATE THE ELECTRIC FIELD COMPONENTS OF
%% THE POTENTIAL
%% VALUES IN THIS CODE MUST BE CLEARED TO ENSURE PROPER
%% OPERATION.

%% TO FIND Ex Ey
[Ex,Ey]=gradient(v2,xd,yd);
%% NOTE THAT -Ex AND -Ey ARE PLOTTED BECAUSE E=-gradient(V).

%% TO PLOT THE PICTURE
figure
quiver(x4,y4,-Ex,-Ey,10)
xlabel('x [m]')
ylabel('y [m]')
title('Plot of E from solution of V.')
% FILENAME: FIELD1_0
% THIS CODE IS USED TO GENERATE THE ELECTRIC FIELD USING
% ANALYTIC SOLUTION FOR THE TRANSPORT SECTION.
% THIS IS USED FOR THE SEQUENCE:
% V1, V2, V3, V2, ...
% dist = the distance (vertical) between the electrodes.
% plat_lng = the length of each electrode -- calculated in genfdyn.m
% n9 = an integer; n9>=1. Used because the solution is an infinite series.
% k9 = pi*n9/(2*plat_lng).
% choice(1:4) = the potentials in the sequence.

%%SET INITIAL CONDITION
n=118;
m=11;
xd=0.01;
yd=0.01;
d=(m-1)*yd;
V1=0;
V2=0;
V3=0;
L1=0.07;
M1=16;

% FROM NUMERICAL METHOD BOUNDARY CONDITION FIND THE A, B, C
a=-1.7e6; b=1.76e4; c=3.9e3;

%%CALCULATION OF FIELD
% TO CALCULATE E1
for i=1:m
  yposvect=linspace(-d/2,d/2,m);
  ypos=yposvect(i);
  for j=1:n
    xposvect=linspace(0,M1*L1,n);
    xpos=xposvect(j);
    for n9=1:50
      ky1=(2*(n9-1)+1)*pi/d;
      l12=(a*(d*d)/(2*ky1)-4/(ky1^3))+2*c/ky1*(-1)^(n9-1);
      if (ky1*M1*L1>75)
        E1x(n9)=2*ky1*exp(-ky1*xpos)*cos(ky1*ypos)*l12/d;
        E1y(n9)=2*ky1*exp(-ky1*xpos)*sin(ky1*ypos)*l12/d;
      else
        sum1x=ky1*(cosh(ky1*(M1*L1*xpos))/sinh(ky1*M1*L1))*cos(ky1*ypos);
        E1x(n9)=2*sum1x*l12/d;
        sum1y=ky1*sinh(ky1*(M1*L1-xpos))/sinh(ky1*M1*L1)*sin(ky1*ypos);
        E1y(n9)=2*sum1y*l12/d;
      end
    end
  end
end

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end
if n9>1
    E1x_gu(n9)=E1x_gu(n9-1)+E1x(n9);
    E1y_gu(n9)=E1y_gu(n9-1)+E1y(n9);
else
    E1x_gu(n9)=E1x(n9);
    E1y_gu(n9)=E1y(n9);
end
end
E1x_gu_tot(i,j)=E1x_gu(n9);
E1y_gu_tot(i,j)=E1y_gu(n9);
end
end

%MODIFYING THE E1 FIELD SIGN
for j=1:n
    xpos=xposvect(j);
    if abs(xpos/L1-round(xpos/L1))>10*eps
        E1x_gu_tot(1,j)=0;
        E1x_gu_tot(m,j)=0;
    else
        E1x_gu_tot(3,j)=0.5*E1x_gu_tot(2,j);
        E1x_gu_tot(m-1,j)=0.5*E1x_gu_tot(m-2,j);
    end
end
E1y_gu_tot(:,n)=0.0*E1y_gu_tot(:,n);
for i=1:m
    if (abs(E1y_gu_tot(i,n-1))>abs(E1y_gu_tot(i,n-2)) | sign(E1y_gu_tot(i,n-1))~=sign(E1y_gu_tot(i,n-2)))
        E1y_gu_tot(i,n-1)=0.5*E1y_gu_tot(i,n-2);
    end
end

%%%TO CALCULATE E11
%E11
for i = 1:m
    ypos=yposvect(i);
    for j = 1:n
        xpos=xposvect(j);
        for n9=1:50
            ky11=2*n9*pi/d;
            l11=b*d/ky11;
            if (ky11*M1*L1>75)
E11x(n9)=2*ky11*exp(-ky11*xpos)*sin(ky11*ypos)*l11/d;  
E11y(n9)=-2*ky11*exp(-ky11*xpos)*cos(ky11*ypos)*l11/d;

else

sum11x=ky11*(cosh(ky11*(M1*L1*xpos))/sinh(ky11*M1*L1))*sin(ky11*ypos);
E11x(n9)=2*sum11x*l11/d;

sum11y=ky11*(sinh(ky11*(M1*L1*xpos))/sinh(ky11*M1*L1))*cos(ky11*ypos);
E11y(n9)=-2*sum11y*l11/d;

end

if n9>1
E11x_gu(n9)=E11x_gu(n9-1)+E11x(n9);
E11y_gu(n9)=E11y_gu(n9-1)+E11y(n9);
else
E11x_gu(n9)=E11x(n9);
E11y_gu(n9)=E11y(n9);
end

E11x_gu_tot(i,j)=E11x_gu(n9);
E11y_gu_tot(i,j)=E11y_gu(n9);

end

%MODIFYING THE E11 FIELD SIGN
for j=1:n
xpos=xposvec(j);
if abs(xpos/L1-round(xpos/L1))>10*eps
E11x_gu_tot(1,j)=0;
E11x_gu_tot(m,j)=0;
if (abs(E11x_gu_tot(3,j)) < abs(E11x_gu_tot(2,j))) | 
sign(E11x_gu_tot(3,j)) ~=sign(E11x_gu_tot(2,j)))
E11x_gu_tot(2,j)=0.5*E11x_gu_tot(3,j);
E11x_gu_tot(m-1,j)=0.5*E11x_gu_tot(m-2,j);
end
end

E11y_gu_tot(:,n)=0.0*E11y_gu_tot(:,n);
for i=1:m
if (abs(E11y_gu_tot(i,n-1))>abs(E11y_gu_tot(i,n-2)) | sign(E11y_gu_tot(i,n-1))~=sign(E11y_gu_tot(i,n-2)))
E11y_gu_tot(i,n-1)=0.5*E11y_gu_tot(i,n-2);
end
end
%THE TOTAL FIELD Exat
Exat=E1x_gu_tot+E11x_gu_tot;
Eyat=E1y_gu_tot+E11y_gu_tot;

%%%END OF THE CODE
% FILENAME: FIELD1_A
% THIS CODE TO CALCULATE COEFFICIENT MATRIX AND POTENTIAL AND FIELD
% USING MATRIX METHOD FASTER THAN FOR... LOOP METHOD OVER % 15-20 TIMES
% matrix K(1,n1)
% matrix D1(n1,m1)
% matrix D2(n1,m1)
% matrix E1(m1,n1)
% matrix E2(m1,n1)
% matrix B(m1,1)
% matrix A1(n1,1)
% matrix A2(n1,1)
% matrix Ex(m,n),Ey(m,n)

%%%%%SET INITIAL CONDITION AND GEOMETRY VARIABLES
ma=11;
na=112;
M=16;
N=0;
mp=M+N;
yd=0.01;
xd=0.01;
d=(ma-1)*yd;
h=0.15;
l=0.12;
L=0.07;
V10=-1e4;
V20=-2e4;
V30=-4e4;
mp=M+N;

%%%%%TO CALCULATE THE POTENTIAL AND FIELD
i=1:1:na;
j=1:1:ma;
x=(i-1).*xd;
y=-d/2+(j-1).*yd;
n1=1:1:90;
m1=1:1:8;
s=0:1:((M/4)-1);

%TO FIND kxn, kxm
kxn=(pi.*n1)/(mp*L+1);
kxm=(pi.*m1)/l;
l1n=(cos((4.*s.*L+l)*kxn)-cos(((4.*s+1).*L+l)*kxn));
\[ I_{1n} = \text{sum}(I_{1n})/kxn; \]
\[ s = 1:1:(M/2); \]
\[ I_{2n} = (\cos(((2.*s-1)*L+l)'*kxn) - \cos(((2.*s-1)*L+l)'*kxn)); \]
\[ I_{2n} = \text{sum}(I_{2n})/kxn; \]
\[ s = 0:1:(M/4)-1; \]
\[ I_{3n} = (\cos(((4.*s+2)*L+l)'*kxn) - \cos(((4.*s+3)*L+l)'*kxn)); \]
\[ I_{3n} = \text{sum}(I_{3n})/kxn; \]

\textit{% TO FIND \( lmn1 \) and \( lmn \)}
\[ lmn = \text{sin}((kxn \times \text{ones}(1, \text{length}(kxm))) - \text{ones}(\text{length}(kxn), 1) \times kxm, \times 1)*kxm, \times 1) - \text{sin}((\text{ones}(\text{length}(kxn), 1) \times kxm, \times 1) - \text{ones}(\text{length}(kxn), 1) \times kxn, \times 1)); \]
\[ lmn1 = lmn1 - \text{sin}((kxn \times \text{ones}(1, \text{length}(kxm))) + \text{ones}(\text{length}(kxn), 1) \times kxm, \times 1) - \text{ones}(\text{length}(kxn), 1) \times kxn, \times 1)); \]
\[ mp1 = mp; \]
\[ mp = 0; \]
\[ lmn1 = \text{sin}((kxn \times \text{ones}(1, \text{length}(kxm))) - \text{ones}(\text{length}(kxn), 1) \times kxm, \times 1) - \text{sin}((\text{ones}(\text{length}(kxn), 1) \times kxm, \times 1) - \text{ones}(\text{length}(kxn), 1) \times kxn, \times 1)); \]
\[ lmn1 = lmn1 - \text{sin}((kxn \times \text{ones}(1, \text{length}(kxm))) + \text{ones}(\text{length}(kxn), 1) \times kxm, \times 1) - \text{ones}(\text{length}(kxn), 1) \times kxn, \times 1)); \]
\[ mp = mp1; \]
\[ lmn1 = lmn1; \]

\textit{% TO FIND MATRIX \( k, D1, D2, E1, E2 \)}
\[ K = 2.*\cos(l,'*kxn,'/((mp,'*L+1,'*\text{cosh}(kxn,'*d/2')),'*(V10,'*l1n+V20,'*l2n+V30,'*l3n)); \]
\[ D1 = (\cos(l,'*kxn,'/((mp,'*L+1,'*\text{cosh}(kxn,'*d/2')))','*(\text{sinh}(h-\text{-d/2})),'*kxm,,'/\text{cosh(h,'*kxm))),'*lmn, \]
\[ D2 = (\cos(l,'*kxn,'/((mp,'*L+1,'*\text{sinh}(kxn,'*d/2)))','*(\text{sinh}(h-\text{-d/2})),'*kxm,,'/\text{cosh(h,'*kxm))),'*lmn; \]
\[ E1 = ((2/l,'*\text{cosh(h,'*kxm))/((kxm,'*cosh((h-\text{-d/2})),'*kxm))))*(kxn,'*sinh(kxn,'*d/2)),'/\text{cosh(l,'*kxn))),'*lmn1; \]
\[ E2 = ((2/l,'*\text{cosh(h,'*kxm))/((kxm,'*cosh((h-\text{-d/2})),'*kxm))))*(kxn,'*cosh(kxn,'*d/2)),'/\text{cosh(l,'*kxn))),'*lmn1; \]

\textit{% TO FIND A1, A2, B AND MATRIX OPERATION \( zz = \text{size}(E1); \)}
\[ C = \text{eye}(zz(1)) + E2*D2-E1*D1; \]
\[ C = \text{eye(size}(C)); \]
\[ A1 = (\text{eye}(zz(2)) + (D1*C1)*E1)'*K; \]
\[ A2 = (D2*C1)*E1'*K; \]
\[ B = C1*E1'*K; \]
% TO FIND FIELD Exa, Eya
Vx=((ones(length(y),1)*(1./cos(kxn.*l))).*(ones(length(y),1)*A1').*cosh(y'*kxn)+(ones(length(y),1)*A2').*sinh(y'*kxn))*(sin(kxn*(x+l.*ones(1,length(x)))));
Eyax=(ones(length(y),1)*(kxn./cos(kxn.*l))).*(ones(length(y),1)*A1').*cosh(y'*kxn)+(ones(length(y),1)*A2').*sinh(y'*kxn))*(sin(kxn*(x+l.*ones(1,length(x)))));
Exay=(ones(length(y),1)*(kxn./cos(kxn.*l))).*(ones(length(y),1)*A1').*cosh(y'*kxn)+(ones(length(y),1)*A2').*sinh(y'*kxn))*(cos(kxn*(x+l.*ones(1,length(x)))));
Exa(i,j)=Exax(i,j);
Eya(i,j)=Eyay(i,j);

%%% TO FIX NUMERICAL ERRORS
% TAYLOR EXPANSION AT -2d/5 j=1:m/2 IN Y DIRECTION FIELD
%Exa1=((d/2-y)'*((kxn.'^2./cos(kxn.*l)).*(-A.*sinh(kxn.*d/2)+A2'.*cosh(kxn.*d/2))*(cos(kxn'*(x+l.*ones(1,length(x))))));
a11=((kxn./cos(kxn.*l))'.*A1)*ones(1,length(y));
a12=((kxn./cos(kxn.*l))'.*A2)*ones(1,length(y));
b11=ones(length(y),1)*sinh(kxn.*(7*d/16))-(-7*d/16-y)'*(kxn.*cosh(kxn.*(7*d/16)));
b12=ones(length(y),1)*cosh(kxn.*(7*d/16))-(7*d/16-y)'*(kxn.*sinh(kxn.*(7*d/16)));
Eya1=-(a11.'*b11+a12.'*b12)*sin(kxn'*(x+l.*ones(1,length(x))));

% TAYLOR EXPANSION AT 2d/5 j=m/2:m IN Y DIRECTION FIELD
%Exa2=((-d/2+y)'*((kxn.'^2./cos(kxn.*l)).*(A1'.*sinh(kxn.*d/2)+A2'.*cosh(kxn.*d/2))*(cos(kxn'*(x+l.*ones(1,length(x))))));
a21=((kxn./cos(kxn.*l))'.*A1)*ones(1,length(y));
a22=((kxn./cos(kxn.*l))'.*A2)*ones(1,length(y));
b21=ones(length(y),1)*sinh(kxn.*(7*d/16))-(-7*d/16-y)'*(kxn.*cosh(kxn.*(7*d/16)));
b22=ones(length(y),1)*cosh(kxn.*(7*d/16))-(7*d/16-y)'*(kxn.*sinh(kxn.*(7*d/16)));
Eya2=-(a21.'*b21+a22.'*b22)*sin(kxn'*(x+l.*ones(1,length(x))));

% TO FIX Y DIRECTION NUMERICAL ERRORS
% FOR LARGE NODE GEMEMORY NEED THE ERRORS ARE VERY SMALL
% IF YOU DO NOT FIX,
% DO NOT AFFECT THE PARTICLE MOTION, SO I BLOCK OUT
for i=1:1:na
    for j=1:1:5
        Eya(j,i)=(Eya1(j,i));
        Eya(ma-j+1,i)=(Eya2(ma-j+1,i));
    end
end

% TO FIX X DIRECTION NUMERICAL ERRORS
for i=1:1:na
  for j=2:1:ma/2
    if ((abs(Exa(j,i)) < abs(Exa(j+1,i)) & (abs(Exa(j+1,i)) < abs(Exa(j+2,i))))
      if (((Exa(j,i)>0 & Exa(j+1,i)>0 & Exa(j+2,i)>0) | (Exa(j,i)<0 & Exa(j+1,i)<0 & Exa(j+2,i)<0))
        count1(i)=j;
        for w=1:1:count1(i)
          Exa(w,i)=(w-1)*(Exa(count1(i)+1,i))/count1(i);
          Exa(ma-w+1,i)=(w-1)*(Exa(ma-count1(i),i))/count1(i);
        end
        break;
      end
      end
    end
  end
end
%%%END OF THE CODE
% FILENAME: FIELD2_A
% THIS CODE IS USED TO GENERATE THE ELECTRIC FIELD USING V1, V2, % V3, V2, ... IN TRANSPORT REGION
% dist= the distance (vertical) between the electrodes.
% plat_lng = the length of each electrode
% n9 = an integer; n9>=1. Used because the solution is an infinite series.
% k9 = pi*n9/(2*plat_lng).
% choice(1:4) = the potentials in the sequence

%%% TO SET SET GEOMETRY PARAMETER AND INITIANIAL CONDITION
mc=11;
nc=56;
plat_lng=0.07;
L=plat_lng;
xd=0.01;
yd=0.01;
dist =(mc-1)*yd;
poten_no=4;
choice(1)=-1e4;
choice(2)=-2e4;
choice(3)=-4e4;
a0=(choice(1)+2*choice(2)+choice(3))/4;

%%% TO CALCULATE POTENTIAL AND FIELD
for i = 1:nc
    xpos=(i-1)*xd;
    for j = 1:mc
        ypos = (j-1)*yd-dist/2;
        for n9 = 1:50
            k9 = pi*n9/(2*plat_lng);
            an=(choice(1)*sin(n9*pi/2)+choice(3)*sin(3*n9*pi/2))/(n9*pi);
            bn=-(choice(2)-choice(1)+(-1)^n9*(choice(2)-choice(3)))*(1-cos(n9*pi/2))/(n9*pi);
            anp=an/cosh(k9*dist/2);
            bnp=bn/cosh(k9*dist/2);
            xnu = n9*pi*xpos/(2*plat_lng);
            ynu = n9*pi*ypos/(2*plat_lng);
            if n9 > 1
                V_gu(n9)=cosh(ynu)*(anp*cos(xnu)+bnp*sin(xnu))+V_gu(n9-1);
                Ey_gu(n9) =-n9*pi*sinh(ynu)*(anp*cos(xnu)+bnp*sin(xnu))/(2*plat_lng)
                + Ey_gu(n9-1);
                Ex_gu(n9) = -n9*pi*cosh(ynu)*(-anp*sin(xnu)+bnp*cos(xnu))/(2*plat_lng) + Ex_gu(n9-1);
            else
                V_gu(n9)=cosh(ynu)*(anp*cos(xnu)+bnp*sin(xnu));
            end
        end
    end
end
Ey_{gu}(n9) = -n9*pi*sinh(ynu)*(anp*cos(xnu)+bnp*sin(xnu))/(2*plat_lng);
Ex_{gu}(n9) = -n9*pi*cosh(ynu)*(-
anp*sin(xnu)+bnp*cos(xnu))/(2*plat_lng);
end
end
Vc(j,i)=V_{gu}(n9);
Exc(j,i) = Ex_{gu}(n9);
Eyc(j,i) = Ey_{gu}(n9);
end
end
Vc=a0+Vc;

% TO FIX Exc ERRORS
for i=1:1:nc
    for j=2:1:mc/2
        if ((abs(Exc(j,i)) < abs(Exc(j+1,i))) & (abs(Exc(j+1,i)) < abs(Exc(j+2,i))))
            if ((Exc(j,i)>0 & Exc(j+1,i)>0 & Exc(j+2,i)>0) | (Exc(j,i)<0 & Exc(j+1,i)<0 &
                Exc(j+2,i)<0))
                countc(i)=j;
                for w=1:1:countc(i)
                    Exc(w,i)=(w-1)*(Exc(countc(i)+1,i))/countc(i);
                    Exc(mc-w+1,i)=(w-1)*(Exc(mc-countc(i),i))/countc(i);
                end
                break;
            end
        end
    end
end

% TO FIX Eyc ERRORS
for i=1:1:nc
    for j=1:1:mc/2
        if ((Eyc(j,i)>0 & Eyc(j+1,i)>0 & Eyc(j+2,i)>0 & Eyc(j+3)>0) | (Eyc(j,i)<0 &
                Eyc(j+1,i)<0 & Eyc(j+2,i)<0 & Eyc(j+3)<0))
            countc1(i)=j;
            for w=1:1:countc1(i)
                Eyc(w,i)=sign(Eyc(countc1(i),i))*abs(Eyc(w,i));
                Eyc(mc-w+1,i)=sign(Eyc(mc-countc1(i),i))*abs(Eyc(mc-w+1,i));
            end
            break;
        end
    end
end
for i=1:1:nc
    for j=1:1:mc/2
        Eyc(j,i)=sign(Eyc(mc/2,i))*abs(Eyc(j,i));
\[ E_{yc(mc-j+1,i)} = -\text{sign}(E_{yc(mc/2,i)}) \times \text{abs}(E_{yc(mc-j+1,i)}) \]
end
end
%END
% FILENAME: DY_FIELD
% FIND Exi Eyi FOR PLOTTING DYNAMIC SECTION FIELD LINES
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for i=1:1:56
    for j=1:1:11
        Exi(j,i) =Exa(j,i) + Exat(j,i);
        Eyi(j,i) =Eya(j,i) + Eyat(j,i);
    end
end
for i=57:1:112
    for j=1:1:11
        Exi(j,i) =Exc(j,i-56);
        Eyi(j,i) =Eyc(j,i-56);
    end
end
% FILENAME:TO_FIELD
% THIS CODE IS TO FIND TOTAL FIELD IN THE FILTER
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for j=1:1:20
    for i=1:1:12
        Exp(j,i)=-Ex(j,i);
        Eyp(j,i)=-Ey(j,i);
    end
end
for j=10:1:20
    for i=13:1:68
        Exp(j,i)=Exa(j-9,i-12)+Exat(j-9,i-12);
        Eyp(j,i)=Eya(j-9,i-12)+Eyat(j-9,i-12);
    end
    for i=69:1:124
        Exp(j,i)=Exc(j-9,i-68);
        Eyp(j,i)=Eyc(j-9,i-68);
    end
end
%END

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% FILENAME: GEO_DATA
% THIS CODE INCLUDES SOME GEOMETRY PARAMETERS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
num=1;
xd=0.01;
yd=0.01;
m=20;
n=124;
xi=linspace(0,xd*(n),num*n);
yi=linspace(0,yd*(m),num*m);
poten_no=4;
%END
% FILENAME: PLOTLINE
% THIS IS CODE TO PLOT THE ELECTRIC FIELD LINES OF A
% PARTICULAR % GEOMETRY.
% THE BASIC ALGORITHM IS:
%     j = 0;
%     WHILE (NOT DONE PLOTTING LINES)
%         i = 0; j = j + 1;
%             WHILE (STILL IN REGION OR (Exatpoint ~= 0 AND Eyatpoint ~= 0))
%                 i = i + 1;
%                 FIND THE FOUR NODES SURROUNDING THE PARTICULAR
%                 PARTICLE.
%                 USE THESE NODES, AND A LINEAR INTERPOLATION TO FIND THE
%                 EFIELD AT
%                 THE CURRENT POSITION (Exatpoint, Eyatpoint).
%                 USE THE EQUATION OF A LINE TO MOVE TO THE NEXT %POINT:
%                 y = mx + b ==> new y = (Eyatpoint/Exatpoint)*deltax + old y.
%                 SAVE ALL THE POINTS (X,Y) IN AN ARRAY CALLED
%                 (fieldline(i,j),fieldline(i,j+1)).
%             END WHILE
%         END WHILE
%     PLOT THE LINES

%%% INPUT THE STARTING POINT FOR THE FIELD LINE(S).
clear fieldline

% USE THIS CODE TO INPUT SPECIFIC POINTS TO START AT.
%count = input('Enter the number of starting points');
%for j=1:count
%    fieldline(1,j) = input('Enter the starting position x, in meters:');
%    fieldline(1,j+count) = input('Enter the starting position y, in meters:');
%end

% USE THIS CODE TO INPUT THE BOUNDARY OF THE REGION AS %STARTING POINTS.
if exist('horz_num') ~= 1
    horz_num = input('Enter the number of horizontal points to start field lines from: ');
    vert_num = input('Enter the number of vertical points to start field lines from: ');
    count = 2*horz_num + 2*vert_num;
end

% SEQUEN_NO=4
%%% BOTTOM, TOP ROW AND LEFT,RIGHT COLUMN INITIALIZATION
clear dummy;
dummy = ones(1, horz_num);

% BOTTOM ROW POINTS INITIALIZATION.
fieldline(1, 1:horz_num) = linspace(xd, xd*n, horz_num);
fieldline(1, 1:count + horz_num + count) = yd*dummy*0.01;

% TOP ROW POINTS INITIALIZATION.
fieldline(1, horz_num + 1:2*horz_num) = linspace(xd, xd*n, horz_num);
fieldline(1, 1:count + horz_num:count + 2*horz_num) = yd*(m - 0.01)*dummy;

clear dummy;
dummy = ones(1, vert_num);

% LEFT COLUMN INITIALIZATION.
fieldline(1, 2*horz_num + 1:2*horz_num + vert_num) = (0.001)*dummy;
fieldline(1, 1:count + 2*horz_num:count + 2*horz_num + vert_num) = linspace(yd+.01, yd*m-.01, vert_num);

% RIGHT COLUMN INITIALIZATION.
fieldline(1, 2*horz_num + 1 + vert_num:2*horz_num + 2*vert_num) = (xd*n-.001)*dummy;
fieldline(1, 1:count + 2*horz_num + vert_num:count + 2*horz_num + 2*vert_num) = linspace(yd+.01, yd*m-.01, vert_num);
clear dummy;

%%%%%%FOR INTERPOLATION
if exist('k') ~= 1
   k = input('Enter the number of divisions per interpolated square; 3-5 typical ');
end

% SET the number of divisions per interpolated square k=5
k = 5;
deltax = xd/(num*k);

%%%%%% COMPUTING FIELD LINE POINTS
%%%%%%"FOR" LOOP LOOPS THROUGH TO EACH STARTING POINT.
for j = 1:count
   i = 1; n3 = 1; m3 = 1;
   while (fieldline(i,j) > 0) & (fieldline(i,j) < xd*n) & (fieldline(i,j+count) > 0) & (fieldline(i,j+count) < yd*m) & (i <= 600)
      % FIND THE FOUR NODES SURROUNDING THE CURRENT POINT.
      n3 = 1; m3 = 1;
      while xi(n3) <= fieldline(i,j)
         n3 = n3 + 1;
      end
      while yi(m3) <= fieldline(i,j+count)
         m3 = m3 + 1;
   end

   % "WHILE" LOOP CONTINUES TO GENERATE AN E FIELD LINE UNTIL CONDITIONS ARE SATISFIED.
end

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end

% THUS THE CURRENT POINT IS BETWEEN (m3,n3) (m3,n3-1) (m3-1,n3) (m3-1,n3-1)

% NOW LINEARLY INTERPOLATE TO FIND THE EFIELD AT THE CURRENT
% POSITION BASED ON THE
% SURROUNDING NODES.

ypercent = (fieldline(i,j+count)-yi(m3-1))/(yd/num);
Ex31=-(Exi(m3,n3-1)-Exi(m3-1,n3-1))*ypercent - Exi(m3-1,n3-1);
Ey31=-(Eyj(m3,n3-1)-Eyj(m3-1,n3-1))*ypercent - Eyj(m3-1,n3-1);
Ex42=-(Exi(m3,n3)-Exi(m3-1,n3))*ypercent - Exi(m3-1,n3);
Ey42=-(Eyj(m3,n3)-Eyj(m3-1,n3))*ypercent - Eyj(m3-1,n3);

xpercent = (fieldline(i,j)-xi(n3-1))/(xd/num);
Exatpoint=(Ex42-Ex31)*xpercent + Ex31;
Eyatpoint=(Ey42-Ey31)*xpercent + Ey31;

% NOW THE E FIELD AT THE CURRENT POSITION IS KNOWN
% ACCURATELY.

if (Exatpoint < 0)
sign1 = -1;
end

% CHECK TO SEE IF THE DIRECTION/SLOPE CHANGED -- MOVING IN -X
% DIRECTION?

sign1 = 1;

if (Exatpoint < 0)
sign1 = -1;
end

% CHECK TO SEE IF THE DIRECTION OF Y HAS CHANGED?

sign2 = 1;

if (Eyatpoint < 0)
sign2 = -1;
end

i=i+1;

% CHECK FOR EXCESSIVE SLOPE.
% IF SLOPE IS TOO BIG DON'T FOLLOW IT, JUST GO A SMALL AMOUNT.

slope=abs(Eyatpoint/Exatpoint);

if (slope< 5)

% IF SLOPE NOT TO LARGE THE CALCULATE NEXT POSITION.

fieldline(i,j)=fieldline(i-1,j) + sign1*deltax;
fieldline(i,j+count)=(Eyj(fieldline(i,j))/Exj(fieldline(i,j))*sign1*deltax + fieldline(i-1,j+count);
else

% IF SLOPE TO LARGE, INCREMENT Y BY DELTA Y = yd/(num*k) IN THE
% PROPER DIRECTION.

fieldline(i,j)=fieldline(i-1,j)+sign1*deltax/slope;
fieldline(i,j+count)=fieldline(i-1,j+count)+sign2*yd/(num*k);
end
end

%TO ADD IF CONDITION TO TEST IF FIELD LINES WERE PLOTTED
%ABOVE. IF NOT, REINITIALIZE
% TO THE STARTING POINT AND FOLLOW THE FIELD LINE IN THE
%OPPOSITE DIRECTION

if i<3,i;i=1;end

%%%%%% "WHILE" LOOP CONTINUES TO GENERATE AN E FIELD LINE
%%%%%% UNTIL CONDITIONS ARE SATISFIED
%%%%%% BUT IN THE OPPOSITE DIRECTION ASSUMING THE FIELD LINE
%%%%%%WAS NOT PLOTTED ABOVE.

while
(fieldline(i,j)>0)&(fieldline(i,j)<xd*n)&(fieldline(i,j+count)>0)&(fieldline(i,j+count)<yd*m)&(i<=600)

n3=1;m3=1;
while xi(n3)<=(fieldline(i,j))
    n3=n3+1;
end
while yi(m3)<=(fieldline(i,j+count))
    m3=m3+1;
end

% THUS THE CURRENT POINT IS BETWEEN (m3,n3) (m3,n3-1) (m3-1,n3) (m3-1,n3-1)

%NOW LINEARLY INTERPOLATE TO FIND THE EFIELD AT THE CURRENT
%POSITION BASED ON THE SURROUNDING NODES.

ypercent =abs((fieldline(i,j+count))-yi(m3-1))/(yd/num);
Ex31=-(Exi(m3,n3-1)-Exi(m3-1,n3-1))*ypercent - Exi(m3-1,n3-1);
Ey31=-(Eyi(m3,n3-1)-Eyi(m3-1,n3-1))*ypercent - Eyi(m3-1,n3-1);
Ex42=-(Exi(m3,n3)-Exi(m3-1,n3))*ypercent - Exi(m3-1,n3);
Ey42=-(Eyi(m3,n3)-Eyi(m3-1,n3))*ypercent - Eyi(m3-1,n3);

xpercent =abs((fieldline(i,j))-xi(n3-1))/(xd/num);
Exatpoint=(Ex42-Ex31)*xpercent + Ex31;
Eyatpoint=(Ey42-Ey31)*xpercent + Ey31;

%NOW THE E FIELD AT THE CURRENT POSITION IS KNOWN
%ACCURATELY.

%CHECK TO SEE IF THE DIRECTION/SLOPE CHANGED -- MOVING IN -X
%DIRECTION?
sign1 = +1;  
if (Exatpoint > 0)  
    sign1 = -1;  
end  

%CHECK TO SEE IF THE DIRECTION OF Y HAS CHANGED?  
sign2 = +1;  
if (Eyatpoint > 0)  
    sign2 = -1;  
end  

i=i+1;  

%CHECK FOR EXCESSIVE SLOPE.  
%IF SLOPE IS TO BIG DON'T FOLLOW IT, JUST GO A SMALL AMOUNT.  
slope=abs(Eyatpoint/Exatpoint);  
if (slope<5)  
    %IF SLOPE NOT TO LARGE THE CALCULATE NEXT POSITION.  
    fieldline(i,j)=fieldline(i-1,j) + sign1*deltax;  
    fieldline(i,j+count)=(Eyatpoint/Exatpoint)*sign1*deltax + fieldline(i-1,j+count);  
else  
    %IF SLOPE TO LARGE, INCREMENT Y BY DELTA Y = yd/(num*k) IN THE  
    %PROPER DIRECTION.  
    %INCREMENT X BY deltax/slope IN THE PROPER DIRECTION.  
    fieldline(i,j)=fieldline(i-1,j)+sign1*deltax/slope;  
    fieldline(i,j+count)=fieldline(i-1,j+count)+sign2*yd/(num*k);  
end  

%END OF FOLLOWING FIELD LINE IN OPPOSITE DIRECTION  
end  

%CLEAR FILLER ZEROS OUT OF THE ARRAY GENERATED.  
[L,M]=size(fieldline);  

for j=1:M/2  
    for i=1:L  
        if (fieldline(i,j)==0)  
            fieldline(i,j)=NaN;  
            fieldline(i,j+count)=NaN;  
        end  
        if (fieldline(i,j)<0.001*xd)||(fieldline(i,j)>(xd*n))||(fieldline(i,j+count)<0.001*yd)||(fieldline(i,j+count)>(yd*m))  
            fieldline(i,j)=NaN;  
            fieldline(i,j+count)=NaN;  
        end  
    end  
end
end
end

%%%%%%PLOT THE FIELD LINES
%PLOT LINES
figure
plot(fieldline(:,1:count)*100,fieldline(:,count+1:2*count)*100)
grid off
%SET PICTURE COORDINATOR
axis([0 n*xd*100 0 m*yd*100])
xlabel('z [cm]');
ylabel('x [cm]');
%CHANGE GRAPHS LINEWIDTH
set(get(gca,'children'),'linewidth',[1]);
%CHANGE FRAME LINEWIDTH AND ITS LABEL
set(gca,'linewidth',[2],'fontname','helvetica','fontweight','bold','fontsize',[16]);
%SET LABEL FONDS
set(get(gca,'xlabel'),'fontname','helvetica','fontweight','bold','fontsize',[16]);
set(get(gca,'ylabel'),'fontname','helvetica','fontweight','bold','fontsize',[16]);
%XTICK
set(gca,'Xtick',[0 20 40 60 80]);
%YTICK
set(gca,'Ytick',[0 1 2 3 4 5]);
%END
% FILENAME: PLOTPATH
% THIS CODE TAKES THE CURRENT ELECTRIC FIELD SOLUTION AND
% DETERMINES THE
% PATH OF A SINGLE CHARGED PARTICLE IN IT.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% VARIABLES MUST BE CLEARED TO ENSURE PROPER
% OPERATION
% FOR MULTIPLE RUNS.
clear positn0 positn positnf n2 m2

% INITIALIZ VARIABLES:
tfinal = 0;
% USE tf = 4e-3 TO SOLVE WITH REAL VARIABLES AND SYSTEM OF EQU.
tf = 4e-3;
tol = 1.e-5;

% positn0 IS THE INITIALIZATION VECTOR,
% positn IS THE ITERATION ARRAY.
% positnf IS THE OVERAL ARRAY DESCRIBING POSITION AND
% VELOCITY.
positin0 = [xp, yp, zp, vx, vy, vz]';
positin = positn0';
positnf = positn;
n2 = 1; m2 = 1; L = 1;

% WHILE THE PARTICLE IS STILL INSIDE THE REGION OF
% ITERATE. ALSO, IF THE MORE THAN 5400 POINTS ARE
% CALCULATED, IT WILL STOP.

while
(positin(m2,1)>e)&(positn(m2,1)<=n*xd)&(positn(m2,2)>e)&(positn(m2,2)<=m*yd)&(L<5400*2)
  % FIND THE COORDINATES OF THE NODE TO USE TO SET Ex and Ey
  % FOR
  % NEXT ITERATION. (m3,n3) is the node.
  % USE THE INTERPOLATED ELECTRIC FIELD GENERATED BY efield2.
  n3 = 1; m3 = 1;
  while xi(n3)<=positn0(1)
    n3=n3+1;
  end
  while yi(m3)<=positn0(2)
    m3=m3+1;
  end

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% THUS THE CURRENT POINT IS BETWEEN (m3,n3) (m3,n3-1) (m3-1,n3) (m3-1,n3-1)
% DETERMINE THE E FIELD AT THESE POINTS -- DEPENDING ON
% WHICH SET OF FIELDS IS APPLICABLE AT THE CURRENT TIME.
temp = floor(to/period);
cycle_t = to-temp*period;
if poten_no == 4
    if cycle_t <= period/4
        Ex1 = ExiA(m3-1,n3-1); Ex2 = ExiA(m3,n3-1); Ex3 = ExiA(m3-1,n3); Ex4 = ExiA(m3,n3);
        Ey1 = EyiA(m3-1,n3-1); Ey2 = EyiA(m3,n3-1); Ey3 = EyiA(m3-1,n3); Ey4 = EyiA(m3,n3);
    elseif cycle_t <= period/2
        Ex1 = ExiB(m3-1,n3-1); Ex2 = ExiB(m3,n3-1); Ex3 = ExiB(m3-1,n3); Ex4 = ExiB(m3,n3);
        Ey1 = EyiB(m3-1,n3-1); Ey2 = EyiB(m3,n3-1); Ey3 = EyiB(m3-1,n3); Ey4 = EyiB(m3,n3);
    elseif cycle_t <= period*3/4
        Ex1 = ExiC(m3-1,n3-1); Ex2 = ExiC(m3,n3-1); Ex3 = ExiC(m3-1,n3); Ex4 = ExiC(m3,n3);
        Ey1 = EyiC(m3-1,n3-1); Ey2 = EyiC(m3,n3-1); Ey3 = EyiC(m3-1,n3); Ey4 = EyiC(m3,n3);
    else
        Ex1 = ExiD(m3-1,n3-1); Ex2 = ExiD(m3,n3-1); Ex3 = ExiD(m3-1,n3); Ex4 = ExiD(m3,n3);
        Ey1 = EyiD(m3-1,n3-1); Ey2 = EyiD(m3,n3-1); Ey3 = EyiD(m3-1,n3); Ey4 = EyiD(m3,n3);
    end
end

% NOW LINEARLY INTERPOLATE TO FIND THE EFIELD AT THE
% CURRENT POSITION BASED ON THE SURROUNDING FOUR NODES.
ypercent = (positn0(2)-yi(m3-1))/(yd/num);
Ex21= (Ex2-Ex1)*ypercent + Ex1;
Ey21= (Ey2-Ey1)*ypercent + Ey1;
Ex43= (Ex4-Ex3)*ypercent + Ex3;
Ey43= (Ey4-Ey3)*ypercent + Ey3;

xpercent = (positn0(1)-xi(n3-1))/(xd/num);
Exatpoint=(Ex43-Ex21)*xpercent + Ex21;
Eyatpoint=(Ey43-Ey21)*xpercent + Ey21;

Ex1g = Exatpoint;
Ey1g = Eyatpoint;
% DECLARE Ex1g AND Ey1g GLOBAL SO THAT THE frclwdyn CODE CAN
% "SEE" THEM.
global Ex1g Ey1g qm;

%%%%%% SOLVE THE FORCE EQUATION.
tf1=tf+to;
% USE flw2_dyn FOR THE SYSTEM OF EQUATIONS SOLVING FOR
% NORMALIZED VARIABLES.
% USE frclwdyn FOR THE SYSTEM OF EQUATIONS SOLVING FOR REAL
% VARIABLES;
% THIS IS THE STIFF SYSTEM HOWEVER.
[t,positn] = ode45('ftest',to,tf1,positn0,ti,0);
%%%%%% SET UP INITIALIZATION VECTOR FOR NEXT ITERATION.
[m2,n2]=size(positn);
positn0=positn(m2,:);
to=t(m2);
%%%%%% APPEND THE LAST ITERATION TO THE TOTAL POSITION
%%%%%% ARRAY.
tfinal=[tfinal; t(m2)];
positnf=[positnf; positn(m2,:)];
%%%%%% CHECK SIZE OF positnf, IF LARGER THAN 2*5400 x 6 STOP.
[L,M]=size(positnf);
end

[tfinal(L) positnf(L,:)]
%% PLOT THE OUTPUT TRACE THROUGH THE E FIELD IN TIME AND
%% SPACE.
figure
plot(positnf(:,1),positnf(:,2))
axis([0 n*xd 0 m*yd])
xlabel('x [m]')
ylabel('y [m]')
title('x-y view of particle motion')
%%END
% FILENAME: FORCELAW
% THIS CODE IS TO SOLVE THE FORCE EQUATION
% THIS FUNCTION CONTAINS THE FULL LORENTZ EQ. FOR A STATIC E
% AND B FIELD.
% THIS FUNCTION ALSO INCLUDES THE EFFECTS OF AIR DRAG ON THE
% PARTICLE MOTION.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function xvdot=ftest(t,xv)

xp=xv(1); yp=xv(2); zp=xv(3);
vx=xv(3); vy=xv(4); vz=xv(6);

%%%%%%%THESE ARE NOT BEING USED CURRENTLY, BUT ARE
%%%%%%%NECESSARY.

Ez1=0.0;
Bx=0.0;By=0.0;Bz=0.0;
% Velocity of the machine, -Velocity of air with respect to the machine
Vmy = 0; Vmz = 0; Vmx=0;

%%%%%%% r=PARTICLE RADIUS, mass=PARTICLE MASS, eta=AIR DRAG
CONSTANT.
eta=1.8e-5;r=5.e-7;pi=3.1415926;mass=1.2e-15;
global Ex1g Ey1g qm;

%%%%%%% FORCE = LORENTZ + AIR VISCOSITY
%%%%%%% mA = F = q*(E + V x B) + (6*pi*eta*r*V)
%%%%%%% THUS: A= q/m*(E + V x B) + 1/m*(6*pi*eta*r*V)
%%%%%%% THIS HAS BEEN DECOMPOSED INTO x,y,z COMPONENTS
%%%%%%% BELOW.
%%%%%%% xvdot(1,2,3) CORRESPOND TO VELOCITY IN THE x,y,z
%%%%%%% DIRECTION.
%%%%%%% xvdot(4,5,6) CORRESPOND TO ACCELERATION IN THE x,y,z
%%%%%%% DIRECTION.

xvdot(1)=vx;
xvdot(2)=vy;
xvdot(3)=vz;
xvdot(4)=qm*(Ex1g+(vy-Vmy)*Bz-(vz-Vmz)*By)-6*pi*eta*r*(vx-Vmx)/mass;
xvdot(5)=qm*(Ey1g+(vz-Vmz)*Bx-(vx-Vmx)*Bz)-6*pi*eta*r*(vy-Vmy)/mass;
xvdot(6)=qm*(Ez1+(vx-Vmx)*By-(vy-Vmy)*Bx)-6*pi*eta*r*(vz-Vmz)/mass;

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% Filename: PTEST
% This is main control code to calculate and plot a
% single charged particle path or simulation
% This code consist of three codes
% 1. forcelaw.m
% 2. ode45.m
% 3. plotpath.m
%
% How to run the code:
% In MATLAB "MATLAB Command Window" type: ptest
% Input:
% period - the plate potential shift period (0.04–0.1).
% to - the initial phase.
% q - the particle charges (q<e-11).
% vx - the initial horizontal velocity.
% vy - the initial vertical velocity.
% xp - the initial horizontal position.
% yp - the initial vertical position.
%
% Output:
% tfinal - final value of t.
% vx - the final horizontal velocity.
% vy - the final vertical velocity.
% xp - the final horizontal position.
% yp - the initial vertical position.
%
% The result is displayed by :plot(x,y).
%
% You can get particle motion each point value by typing:
% positnf(:,1) - the horizontal position.
% positnf(:,2) - the vertical position.
% positnf(:,4) - the horizontal velocity.
% positnf(:,5) - the vertical velocity.
%
% All variable must be cleared to ensure proper operation
clear all
%
% Load field data
% These data were calculated and were save as .mat files
load dataAp;
load dataBp;
load dataCp;
load dataDP;

%%%GEOMETRY CONSTANTS
%THESE CONSTANT WE USED IN FINDING FIELDS:dataAp, dataBp, dataCp, dataDp
m=20;
n=124;
xd=0.01;
yd=0.01;
num=1;
xi=linspace(0,xd*(n),num*n);
yi=linspace(0,yd*(m),num*m);
poten_no=4;

%%%INITIALIZATION VARIABLES
period = input('Enter the period in seconds of the potential cycle: ');
to=input('Enter the initial phase: ');
q=input('Enter the particle charges, the maximum charge is ?e-16: ');
vx=0; vy=0; vz=0;
vx = input('Enter the horizontal velocity: vx = ');
vy = input('Enter the vertical velocity: vy = ');
e=0.0001;
xp=0; yp=0; zp=0;
while (xp<e)|(xp>xd*n)
    xp = input('Enter the initial horizontal position, between xd and n*xd: xp = ');
end
while (yp<e)|(yp>yd*m)
    yp = input('Enter the initial vertical position, between yd and m*yd: yp = ');
end

%%% CHARGED PARTICLE PARAMETERS
% r=PARTICLE RADIUS r=5.e-7
% eta=AIR DRAG CONSTANT eta=1.8e-5
% mass=PARTICLE MASS
mass=1.2e-15;
%CHARGE AND MASS RATIO qm, MAXIMUM qm=64.5e-2
qm=q/mass;

%%% TO CALCULATE AND PLOT THE PATH OF A SINGLE CHARGED PARTICLE IN FIELDS
plottest;
%PARTICLE MOTION VELOCITY AND POSITION ARE CALCULATED AND PATH IS PLOTED
% END OF THE CODE
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