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Evaluation of an alternative statistical decision rule for site remediation

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EVALUATION OF AN ALTERNATIVE STATISTICAL DECISION

RULE FOR SITE REMEDIATION

by

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ABSTRACT

A method proposed by the Environmental Protection Agency to optimize site remediation, called the CRG approach states that soil cleanup levels are typically misapplied on a point by point basis rather than on average. The effectiveness of the CRG approach will be evaluated for lognormal, exponential, uniform, and normally distributed data by first simulating data sets for each distribution. Next the CRG approach will be applied to the simulated data sets. Finally, the results of the CRG approach will be analyzed.

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CHAPTER 1

INTRODUCTION

The U.S. EPA has developed guidance documents for statistical evaluation of contaminant concentration data collected from Superfund and RCRA (Resource Conservation and Recovery Act) sites. These guidance documents are not regulatory in nature, i.e., these do not establish a rule with the force of law. At many Superfund and RCRA sites, the remediation decisions are made on the basis of whether the mean contaminant concentration exceeds a specified target cleanup limit (TCL) or not. One statistical approach recommended by the U.S. EPA for this decision problem is to compute an upper confidence limit (UCL) of the mean, and then compare it to the TCL: if the UCL exceeds the TCL, then the site is declared contaminated, otherwise it is declared to be clean. Bowers *et al.* (1996) developed an alternative statistical method, called the Confidence Removal Goal (CRG) method, for making remediation decisions at Superfund sites when the observed concentrations follow the lognormal distribution. The use of this approach was recently proposed for the cleanup activities at a Superfund Site. The CRG approach computes a threshold value, c^* (the CRG) from observed data in cases when (i) the data follow a lognormal distribution, and (ii) the upper confidence limit(UCL) of the population mean exceeds the Target Cleanup Level (TCL), which has been called the CleanUp Goal (CUG) by Bowers *et al.* (1996) [2]. Of course, if the UCL is smaller than the TCL value, then no cleanup is required and the CRG need not be computed. The proposed approach removes site matrices (i.e., soils) from those locations which exhibit higher contaminant concentrations than the CRG value, c*, and replaces those contaminated soils by almost clean soils with concentration, c_0 [2]. In other words, if the maximum detected concentration at the site exceeds the CRG value, then no cleanup is required for that site.

The CRG approach is based on the belief that soil cleanup levels are typically misapplied on a point by point basis, rather than on average [2]. This misapplication is not cost-effective because it results in post-remedy conditions that overshoot the target risk goals. Because environmental contamination is characterized by a distribution of concentrations, some exceedances of target averages, average risk, or average concentration, can be allowed in the post-remediation distribution. The CRG approach presents a mathematical model for calculating this allowable higher than average concentration, termed the Confidence Removal Goal (CRG), which places a limit on concentrations requiring remediation while still ensuring that target average concentrations are satisfied overall. The CRG is site specific because it depends on the contaminant concentration distribution [2]. This is the reason for checking the successfulness of this approach for the normal, uniform, and exponential distributions as well as the lognormal, which is the standard distribution for spatially related data. The strength of this approach was believed to be its ability to handle typical data uncertainties quantitatively because it relies on the Upper Confidence Limit as a measure of the mean concentration, hence the term "confidence" in the CRG. The monetary significance of the CRG approach is easily seen since the approach reduces the amount of remediation that needs to be done.

In the risk assessment it is recognized that there is some uncertainty attached to environmental sampling that attempts to provide a measure of the true average contaminant concentration of the site. Agency risk assessment guidance specifies that this uncertainty addressed by use of an Upper Confidence Limit (UCL) on the mean contaminant concentration of the samples, instead of simply the mean of the samples, in the risk equation[2]. The UCL is a function of the sample size. The UCL specifies a value that we are confident the true mean is beneath, and the value of the UCL increases with decreasing sample size. Since the UCL is used to assess exposure in an EPA-guided risk assessment, a higher calculated risk ensues than if simply the sample mean were used in the equation. This is a conservative approach that increases the likelihood that a contaminated site will be characterized as having unacceptable risk [2].

The Cleanup Goal is obtained from risk assessment and the result is given as:

$$\text{CUG} = \frac{\text{Permissible Risk}}{\text{Toxicity} \times \text{Other exposure factors}} \text{ [2].}$$

The value of the CUG derived in this manner is an average permissible concentration because it is directly analogous to the mean, or a statistical estimate of the mean (i.e. the UCL), value used in the risk equation. Thus an exposure unit (i.e. site) will require remediation if its average contaminant concentration, as represented by the UCL, exceeds the CUG, not if individual data points exceed the CUG. As long as the UCL < CUG, no remediation is required.

This same logic can be applied to those exposure units or sites where some remediation is required. The attainment of acceptable risk does not necessitate remediating every location in an exposure unit where the contaminant concentration exceeds the CUG. Rather, enough remediation must be done so that the CUG and the target risk are met on average across the exposure unit. This can be done by specifying a "removal goal", a higher contaminant concentration than the CUG, such that if remediation of all areas with contaminant concentrations exceeding the removal goal are carried out (or a "confidence" removal goal if the target is the UCL rather than the mean), then the CUG would be met on average across the site. This approach in no way compromises the original risk-based goal; in fact, anything more would not be cost effective for it would exceed the goal [2].

Given the premises that risk-based CUGs can tolerate individual point exceedances in the field as long as the CUG is satisfied on average, the challenge is to predict how many or how high the exceedance is allowable while still meeting the average condition, and taking into consideration the uncertainty imposed by limited sampling schemes [2]. The results will be different form each data distribution, thus each analysis is site specific. However, the methodology is robust in its applicability.

For the Superfund Site at which the CRG method was being considered , cadmium and nickel were among the main contaminants of concern. Soil samples were taken from several areas of the site including the perimeter surface (30 observations) and the fenced surface area (67-68 observations). All of the statistics were computed for the combined data set obtained from the perimeter and fenced area with 97 observations for cadmium and 98 observations for nickel.

Using the Shapiro-Wilk's W-test of normality, it was concluded that nickel concentrations failed the lognormality test. However, both cadmium and nickel data passed the Kolmogorov-Smirnov test for lognormality for the perimeter and fenced areas combined together. The H-statistic based UCL of the mean for cadmium and nickel turned out to be 1733 mg/kg and 33,458 mg/kg, respectively; both of these values exceeded their respective EPA CUG values of 700 mg/kg and 13,000 mg/kg. The CRG values were computed next with the EPA CUG as one of the input parameters. Using these CUG values, the CRG values for cadmium and nickel concentrations turned out to be 15,508 mg/kg and 1,038,592 mg/kg, respectively. The maximum detected concentrations for cadmium and nickel at this site were 14,450 mg/kg and 69,800 mg/kg, respectively. In both cases, the CRG values turned out to be much higher than the respective maximum detected values, leading to the conclusion that no cleanup was needed at the METCOA Superfund Site. At the request of the U.S. EPA, Singh and Singh (1996) used Monte Carlo simulation to investigate the performance of the CRG approach, and showed that the CRG approach recommended very little cleanup a large number of times, even when the true population mean was much higher than the CUG value [8].

The lognormal distribution is very commonly used to model environmental data. Various methods for estimating the parameters of the lognormal distribution exist in the literature [5]. U.S. EPA guidance documents recommend the use of H-statistics to compute the UCL of the arithmetic mean of lognormally distributed data [4]. This method is described in detail in Gilbert (1987). For data sets with non-detects, estimation methods developed for censored data from a lognormal distribution may be used [7]. A recent work by Gilbert (1993), however, indicates that a statistical test of hypothesis based on the H-statistics yields unusually high false positives, which would result in unnecessary cleanup [6]. The situation may reverse when dealing with estimation of the mean background level. If the H-statistic based formula for the UCL of the mean is used for the observed background concentrations, the background mean may be over-estimated, which may result in not remediating a contaminated block of the site. Stewart (1994) showed that incorrect usage of lognormal distribution may lead to disastrous results [9]. Singh and Singh (1996) have shown that non-parametric estimation methods such as the method of jackknife or the method of bootstrap yield better UCLs for lognormal data than the H-statistic based UCL [8].

The CRG approach itself seems quite reasonable, but does not seem effective for the lognormal distribution. In this thesis, the performance of the CRG approach is investigated for the continuous uniform distribution, the exponential distribution, and also the normal distribution. Formulas for calculating the CRG value for a given data are derived, and Monte Carlo simulation is used to investigate the performance of the approach.

CHAPTER 2

DERIVATION OF CRG VALUE FOR VARIOUS PROBABILITY MODELS

In this section, we describe the probability models mentioned above and derive the formulas for calculating the CRG values in each case.

The Lognormal Distribution

Contaminant concentration follows a lognormal distribution if the log-transformed concentrations follow a normal distribution. This can be mathematically described as follows:

If $\ln(X)$ follows a normal distribution, $N(\mu,\sigma^2)$, with mean μ and variance σ^2, then X is lognormally distributed with paramaters μ and σ^2: we will use $LN(\mu,\sigma^2)$ to denote this distribution. Figure 1 shows the plot of a normal and a lognormal distribution with $\mu = 0$, $\sigma^2 = 0.5$. Figure 2 shows a plot of lognormal distributions, each with $\mu = 0$, and varying σ^2 values.

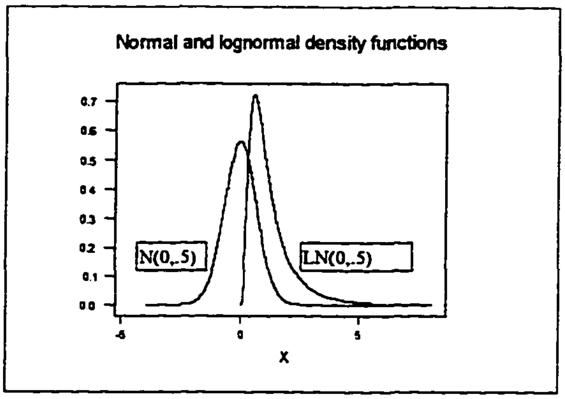

Figure 1: Graph of normal N($\mu = 0, \sigma^2 = 0.5$)
and lognormal LN($\mu = 0, \sigma^2 = 0.5$)
density functions

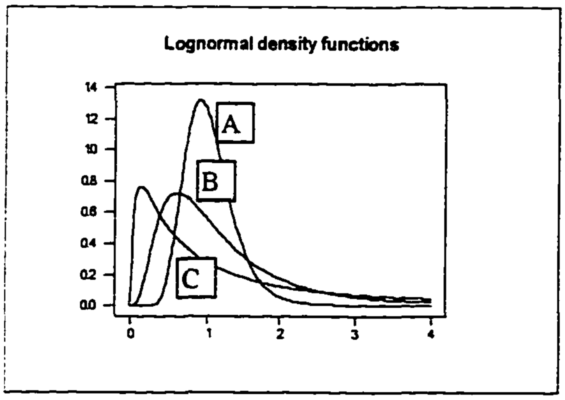

Figure 2: Graph of lognormal A: LN($\mu = 0, \sigma^2 = 0.1$),
B: LN($\mu = 0, \sigma^2 = 0.5$) and C: LN($\mu = 0, \sigma^2 = 2.0$)
density functions

The lognormal distribution LN(μ, σ^2) has the following population parameters:

$$\text{Mean} = \mu_1 = e^{\mu + 0.5\sigma^2} \quad (1)$$

$$\text{Median} = e^{\mu} \quad (2)$$

$$\text{Variance} = \sigma_1^2 = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1) \quad (3)$$

$$\text{Coefficient of Variance } (CV) = \eta = \sqrt{V(X)}/E(X) = \sigma_1/\mu_1 = \sqrt{e^{\sigma^2} - 1} \quad (4)$$

$$\text{Skewness} = \eta^3 + 3\eta \quad (5)$$

The removal goal (c^*) is calculated in the CRG approach by first obtaining the post-remediation mean, and then setting this equal to the clean up goal. Once we have this equation we can solve for c^*, the removal goal. Thus, we will first derive the post-remediation mean of the lognormally distributed data. The pre-remediation distribution of contaminant concentration (c) is lognormal with parameters μ and σ^2. Since remediation consists of removing all soil from the site with a contaminant concentration greater than or equal to c^* and replacing it with clean fill which has a contaminant concentration of c_0, our post-remediation distribution is a mixture of a discrete degenerate distribution at $c = c_0$ and a continuous distribution.

The continuous part of our mixture is a lognormal distribution for values of $c < c^*$, and is given by:

$$f(c;\mu,\sigma^2,c_0) = \begin{cases} \dfrac{1}{c\left(\sigma\sqrt{2\pi}\right)} e^{-[(\ln c-\mu)/\sigma]^2/2}, & c < c^* \\ 0, & c \geq c^* . \end{cases}$$

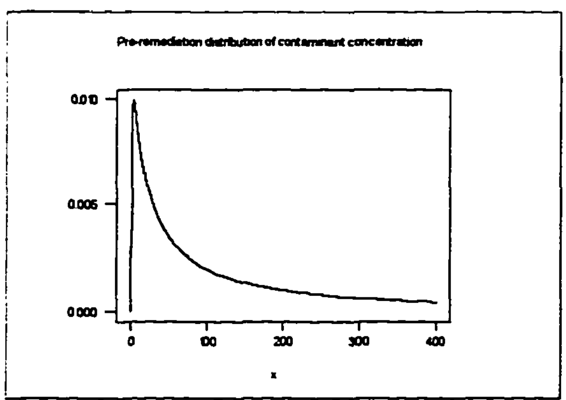

Figure 3 - Pre-remediation distribution of contaminant concentration

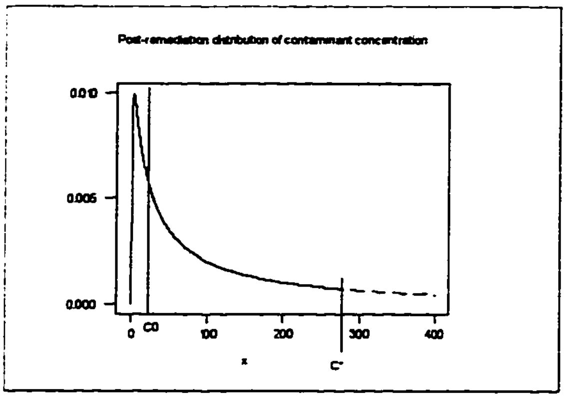

Figure 4 - Post-remediation distribution of contaminant concentration

Note that $f(c;\mu,\sigma^2,c_0)=0$ when $c \geq c^*$ since $c \geq c^*$ requires remediation. The discrete degenerate part at $c = c_0$ has probability

$P(c > c^*) = 1 - P(c < c^*) = 1 - \Phi\left(\frac{\ln c^* - \mu}{\sigma}\right)$, where $\Phi(z)$ is the standard normal cumulative density function. Using this probability, we will calculate the mean (expected value) of the post-remediation distribution. The discrete degenerate part of our mixture gives rise to the term $c_0\left[1-\Phi\left(\frac{\ln c^* - \mu}{\sigma}\right)\right]$ and the continuous part gives rise to

$E(c) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^{c^*} c\frac{e^{-[(\ln c-\mu)/\sigma]^2/2}}{c}\,dc$. Hence the expected value of our post-remediation distribution is given by:

$$c_0\left[1-\Phi\left(\frac{\ln c^* - \mu}{\sigma}\right)\right] + \frac{1}{\sigma\sqrt{2\pi}}\int_0^{c^*} c\frac{e^{-[(\ln c-\mu)/\sigma]^2/2}}{c}\,dc.$$

To evaluate the contribution of the continuous part we make the substitution $u = \frac{\ln c - \mu}{\sigma}$.

Then

$$\frac{du}{dc} = \frac{1}{c}\cdot\frac{1}{\sigma}$$

or

$$du = \frac{1}{c\sigma} dc.$$

Thus we obtain

$$\frac{1}{\sigma\sqrt{2\pi}} \int_0^{c^*} c \frac{e^{-[(\ln c-\mu)/\sigma]^2/2}}{c} \; dc$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln c^*-\mu}{\sigma}} e^{\mu+\sigma u} e^{-u^2/2} \; du$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln c^*-\mu}{\sigma}} e^{\mu} e^{-(u^2-2\sigma u)/2} \; du$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln c^*-\mu}{\sigma}} e^{\mu+\sigma^2/2} e^{-(u-\sigma)^2} \; du$$

$$= e^{\mu+\sigma^2/2} \Phi\left(\frac{\ln c^* - \mu}{\sigma} - \sigma\right).$$

Combining the discrete degenerate and the continuous parts of our mixture distribution, we obtain

$$c_0\left[1 - \Phi\left(\frac{\ln c^* - \mu}{\sigma}\right)\right] + e^{\mu+\sigma^2/2} \Phi\left(\frac{\ln c^* - \mu}{\sigma} - \sigma\right)$$

as our post-remediation distribution.

As prescribed by the CRG method, we now set CUG equal to the post-remediation distribution and solve for c^*, the value that achieves our clean up goal.

To test the effectiveness of the CRG approach, we generated samples from a lognormal distribution with a given mean that was larger than the assumed CUG. This ensures that remediation should be done at the given site. Then we went through the CRG approach for the lognormal data. We calculated the post-remediation distribution, and set this equal to our given CUG. Now using the equation

$$\text{CUG} = c_0\left[1-\Phi\left(\frac{\ln c^*-\mu}{\sigma}\right)\right]+e^{\mu+\sigma^2/2}\Phi\left(\frac{\ln c^*-\mu}{\sigma}-\sigma\right)$$

we calculated c^* numerically using the 95% CI UCL for μ and σ from our lognormal distribution. Once we obtained c^* we analyzed our data sets and observed that for data sets consisting of 25 samples, the max observed value of c, the contaminant concentration, was less than the removal goal, c^*, in 41 out of 100 trials [see Table 2]. We then changed the sample size to 50 and ran 100 more trials, observing that 29 out of 100 gave us maximum values of c that were less than c^* [Table 2]. Finally, we ran 100 trials of 100 samples each and observed that 19 of the 100 trials gave us a result that we would not remediate at all since none of the sample values were larger than the confidence removal goal. These results conflict with what we know about the distribution, we know that the mean for a lognormal distribution is $e^{\mu+\sigma^2/2}$, thus for our samples our mean was approximately 1100, also our CUG was set as 700, so we should have some remediation at the site. Since we observed 41, 29, and 19 times for the trials with sample size 25, 50, and 100 respectively, we can see that a large percentage of our trials gave us results that we would not remediate at all. These are false results or "false negatives". This shows us that our calculated methodology is giving us a value for c^*, the confidence removal goal, that is too large.

The results do improve for larger sample sizes as expected, but the CRG method is still largely ineffective since it gives false negatives too often. Also, this doesn't even take into account the number of times when some remediation is done, but the UCL of the mean remains higher than our CUG.

The Continuous Uniform Distribution

Since the analysis is site specific we want to analyze the CRG method with other possible data distributions. The first distribution that we consider is the uniform distribution, for which we will now derive the clean up goal using the CRG methodology.

Once again, the removal goal c^* is calculated in the CRG approach by first obtaining the post-remediation mean, and then setting this equal to the clean up goal. Once we have this equation we can solve for c^*. Thus, we will first derive the post-remediation mean of the uniformly distributed data.

The pre-remediation distribution of contaminant concentration (c) is uniform with parameters a and b, the upper and lower bounds of our data set. The distribution of uniform data is given by:

$$f(c;a,b) = \frac{1}{b-a}, \quad a < c < b \ [1].$$

Now, since we want to calculate the post-remediation mean, we need to look at our post-remediation distribution, which will be a mixture of a discrete degenerate distribution at $c = c_0$ and the continuous uniform distribution.

The continuous part of our mixture is a uniform distribution for values of c<$c*$, given by:

$$f(c;a,b)=\begin{cases}\frac{1}{b-a}, & a<c<c*\\ 0, & c\geq c*\end{cases}.$$

The discrete degenerate part of our mixture at $c=c_0$ has probability $P(x>c*)=\frac{b-c*}{b-a}$, which is derived from uniform methodology.

Now that we have the probability density functions we can calculate the mean of this distribution by calculating E(c), the expected value of c. The expected value of the discrete part is given by $c_0\left[\frac{b-c*}{b-a}\right]$, while the expected value of the continuous part of the distribution is given by $\int_a^{c*}c\frac{1}{b-a}dc$. Simplifying the continuous part, we obtain

$$\int_a^{c*}c\frac{1}{b-a}dc=\frac{c^2}{2}\frac{1}{b-a}\Big|_a^{c*}$$

$$=\frac{(c*)^2-a^2}{2(b-a)}.$$

Finally by combining the continuous and discrete parts we determine the post-remediation mean:

$$c_0\left[\frac{b-c*}{b-a}\right]+\frac{(c*)^2-a^2}{2(b-a)}.$$

Next consider $\alpha=\frac{\mu_{post}}{\mu_{pre}}$. Since the data is uniform, the pre-remediation mean is given by $\frac{b+a}{2}$. Thus,

$$\alpha = \frac{\mu_{post}}{\mu_{pre}} = \frac{\left[\frac{c_0(b-c^*)}{b-a} + \frac{(c^*)^2 - a^2}{2(b-a)}\right]}{\frac{b+a}{2}}$$

$$= \frac{2}{b+a}\left[\frac{c_0(b-c^*)}{b-a} + \frac{(c^*)^2 - a^2}{2(b-a)}\right]$$

$$= \frac{2c_0 b - 2c_0 c^* + (c^*)^2 - a^2}{b^2 - a^2}$$

so that

$$\alpha(b^2 - a^2) = 2c_0 b - 2c_0 c^* + (c^*)^2 - a^2$$

$$(c^*)^2 - 2c_0 c^* + 2c_0 b - a^2 - \alpha(b^2 - a^2) = 0$$

$$c^* = \frac{2c_0 \pm \sqrt{4(c_0)^2 - 4[2c_0 b - a^2 - \alpha(b^2 - a^2)]}}{2}$$

$$c^* = c_0 \pm \sqrt{(c_0)^2 - 2c_0 b + a^2 + \alpha(b^2 - a^2)}\ .$$

The Exponential Distribution

Again, the removal goal c^* is calculated in the CRG approach by first obtaining the post-remediation mean, and then setting this equal to the clean up goal. Once we have this equation we can solve for c^*. Thus, we will first derive the post-remediation mean of the exponentially distributed data.

The pre-remediation distribution of contaminant concentration c is exponential with parameter θ. The distribution of exponential data is given by:

$$f(c;\theta) = \frac{1}{\theta}e^{-c/\theta},\ \ c > 0\ \ [1].$$

Now, since we want to calculate the post-remediation mean, we need to look at our post-remediation distribution, which will be a mixture of a discrete degenerate distribution at $c = c_0$ and the continuous exponential distribution.

The continuous part of our mixture is an exponential distribution for values of c<c^*, given by:

$$f(c;\theta) = \begin{cases} \frac{1}{\theta} e^{-c/\theta}, & 0 < c < c^* \\ 0, & c \geq c^* \end{cases} .$$

The discrete degenerate part of our mixture at $c = c_0$ has probability $P(x > c^*) = 1 - P(c < c^*) = 1 - (1 - e^{-c^*/\theta}) = e^{-c^*/\theta}$ which is derived from exponential methodology.

Now that we have the probability density functions we can calculate the mean of this distribution by calculating E(c), the expected value of c. The expected value for the discrete part is given by $c_0 e^{-c^*/\theta}$. The continuous part of the distribution is given by

$$\int_0^{c^*} c \frac{1}{\theta} e^{-c/\theta} \, dc .$$

To simplify the contribution of the continuous part we make the substitution $u = \frac{-c}{\theta}$.

Then

$$\frac{du}{dc} = -\frac{1}{\theta}$$

or

$$du = -\frac{1}{\theta} dc$$

so that

$$\int_0^{c^*} c\frac{1}{\theta}e^{-c/\theta}\,dc$$

$$=\frac{1}{\theta}\int_0^{c^*} -u\theta e^{u}(-\theta)du$$

$$=\theta\int_0^{c^*} ue^{u}du$$

$$=\theta\left[ue^{u}-e^{u}\right]_0^{c^*}$$

$$=\theta\left[-\frac{c}{\theta}e^{-c/\theta}-e^{-c/\theta}\right]_0^{c^*}$$

$$=-c^*e^{-c^*/\theta}-\theta e^{-c^*/\theta}+\theta.$$

Combining our continuous and discrete parts we get as our post-remediation mean:

$$c_0e^{-c^*/\theta}-c^*e^{-c^*/\theta}-\theta e^{-c^*/\theta}+\theta.$$

Now that we have calculated the post-remediation mean for exponential data, the CRG method obtains a value for c*, the removal goal, by setting the clean up goal equal to the post-remediation mean. Our resulting equation is

$$\text{CUG}=c_0e^{-c^*/\theta}-c^*e^{-c^*/\theta}-\theta e^{-c^*/\theta}+\theta.$$

This equation cannot be simplified easily, so we must calculate c* numerically.

The Normal Distribution

Next, we will analyze the CRG method for normally distributed data. Again, we must calculate the post-remediation mean of the normally distributed data. The pre-remediation distribution of contaminant concentration c is normal with parameters μ and σ^2. The distribution $f(c;\mu,\sigma^2,c_0)$ is given as:

$$f(c;\mu,\sigma^2,c_0)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(c-\mu)^2}{2\sigma^2}},\quad -\infty<c<\infty\ [1].$$

Since remediation consists of removing all soil from the site with a contaminant concentration greater than or equal to c^* and replacing it with clean fill which has a contaminant concentration of c_0, our post-remediation distribution is a mixture of a discrete degenerate distribution at $c=c_0$ and a continuous distribution.

The continuous part of our mixture is a normal distribution for values of $c<c^*$ and is given by

$$f(c;\mu,\sigma^2,c_0)=\begin{cases}\frac{1}{\sqrt{2\pi}}e^{-[(c-\mu)/\sigma]^2/2}, & c<c^*\\ 0, & c\geq c^*.\end{cases}$$

Note once again that $f(c;\mu,\sigma^2,c_0)=0$ when $c\geq c^*$ since $c\geq c^*$ requires remediation. The discrete degenerate part at $c=c_0$ has probability $P(c>c^*)=1-P(c<c^*)=1-\Phi\left(\frac{c^*-\mu}{\sigma}\right)$, where $\Phi(z)$ is the standard normal cumulative density function. Using this probability, we will calculate the mean (expected value) of the post-remediation distribution. The discrete degenerate part of our mixture gives rise to the term $c_0\left[1-\Phi(\frac{c^*-\mu}{\sigma})\right]$, while the continuous part of our post-remediation mixture is gives rise to the term $\int_{-\infty}^{c^*}c\frac{1}{\sqrt{2\pi}\sigma}e^{-[(c-\mu)/\sigma]^2/2}dc$. Next we want to simplify the continuous part of the post-remediation distribution, so we use a change of variables by defining $z=\frac{c-\mu}{\sigma}$. Thus

$$\int_{-\infty}^{c^*}c\frac{1}{\sqrt{2\pi}\sigma}e^{-[(c-\mu)/\sigma]^2/2}dc$$

$$=\frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\frac{c*-\mu}{\sigma}}(z\sigma+\mu)e^{-\frac{z^2}{2}}\sigma dz$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{c*-\mu}{\sigma}} z\sigma e^{-\frac{z^2}{2}}+\mu e^{-\frac{z^2}{2}}dz$$

$$=\sigma\int_{-\infty}^{\frac{c*-\mu}{\sigma}}\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}zdz+\mu\int_{-\infty}^{\frac{c*-\mu}{\sigma}}e^{-\frac{z^2}{2}}dz$$

$$\sigma\int_{-\infty}^{\frac{c*-\mu}{\sigma}}z\phi(z)dz+\mu\int_{-\infty}^{\frac{c*-\mu}{\sigma}}\phi(z)dz,$$

where $\phi(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$. Since $\phi'(z)=-z\phi(z)$, we obtain

$$\sigma\int_{-\infty}^{\frac{c*-\mu}{\sigma}}z\phi(z)dz+\mu\int_{-\infty}^{\frac{c*-\mu}{\sigma}}\phi(z)dz$$

$$=-\sigma\int_{-\infty}^{\frac{c*-\mu}{\sigma}}\phi'(z)dz+\mu\Phi\left(\frac{c*-\mu}{\sigma}\right)$$

$$=-\sigma\phi(z)\Big|_{-\infty}^{\frac{c*-\mu}{\sigma}}+\mu\Phi\left(\frac{c*-\mu}{\sigma}\right)$$

$$=-\sigma\phi\left(\frac{c*-\mu}{\sigma}\right)+\mu\Phi\left(\frac{c*-\mu}{\sigma}\right).$$

The mean of the post-remediation distribution is the sum of the discrete and continuous portions, so we obtain

$$c_0\left[1-\Phi(\frac{c*-\mu}{\sigma})\right]+-\sigma\phi\left(\frac{c*-\mu}{\sigma}\right)+\mu\Phi\left(\frac{c*-\mu}{\sigma}\right)$$

$$=c_0+(\mu-c_0)\Phi\left(\frac{c*-\mu}{\sigma}\right)-\sigma\phi\left(\frac{c*-\mu}{\sigma}\right).$$

Now that we have calculated the post-remediation mean for normally distributed data, the CRG method obtains a value for c*, the removal goal, by setting the clean up goal equal to the post-remediation mean. Our resulting equation is:

$$\text{CUG}=c_0+(\mu-c_0)\Phi\left(\frac{c^*-\mu}{\sigma}\right)-\sigma\phi\left(\frac{c^*-\mu}{\sigma}\right).$$

This equation again cannot be solved for c* analytically, so we must calculate c* numerically.

CHAPTER 3

DESCRIPTION OF SIMULATION EXPERIMENTS

In order to evaluate the performance of the CRG method for site remediation, we used Monte Carlo simulation experiments described in the steps given below and generated using the FORTRAN programs adapted from Bronson (1990) and given in appendix 2 [3]:

INPUT PARAMETERS:

Clean fill concentration = 10

Clean up goal (CUG) = 80

Number of iterations =100

Number of samples n =25, 50, and 100

Step 1: Generate pseudo-random numbers from the probability model under investigation - uniform U(0,200), exponential EXP(100) with mean $\theta = 100$, or normal distribution N(100, 10) with mean 100 and standard deviation 10.

Step 2: Use the derived equations for c* to calculate c* for each distribution.

$c^* = c_0 \pm \sqrt{(c_0)^2 - 2c_0 b + a^2 + \alpha(b^2 - a^2)}$, for the uniform U(a,b) distribution

CUG = $c_0 e^{-c^*/\theta} - c^* e^{-c^*/\theta} - \theta e^{-c^*/\theta} + \theta$ for the EXP(θ) distribution.

CUG = $c_0 + (\mu - c_0)\Phi\left(\frac{c^* - \mu}{\sigma}\right) - \sigma\phi\left(\frac{c^* - \mu}{\sigma}\right)$ for the normal $N(\mu,\sigma^2)$ distribution.

We then substituted the UCL of the mean for θ, c_0=10, and CUG = 80 to obtain an equation with one unknown, namely c*. Since this equation cannot be solved for c* easily, we calculated c* numerically, by testing c* values over an interval and obtaining the c* value which best satisfies this equation.

Step 3: We then calculated the UCL of the mean for μ and the sample standard deviation and substituted the values in for μ and σ respectively. Again this substitution leads to an equation with one variable, c*, so we solved the equation numerically for c*.

Step 4: After obtaining c*, we observed the number of times the maximum of the concentration values was less than c* since this would indicate no remediation was to be done at all. This is a false assumption since we generated the data purposefully to obtain a pre-remediation mean that was greater than the CUG. In our data sets the pre-remediation mean is 100.

Step 5: We replaced the contaminant values that were greater than c* with our clean fill value of 10.

Step 6: We calculated the UCL of the mean of the adjusted data set and compared it to the CUG, counting the number of times that the CRG method did not remediate enough to bring the UCL of the mean down below the CUG.

CHAPTER 4

SIMULATION RESULTS AND CONCLUSIONS

For each of the three distributions; normal, exponential, and uniform, we generated 100 iterations of data sets with sample sizes of 25, 50 and 100.

From the uniform data we obtained the following results (see Tables 3-5, Appendix 1).

size	max<c*	UCL>CUG
n=25	3	95
n=50	0	96
n=100	0	99

From the exponential data we obtained the following results (see Tables 6-8, Appendix 1).

size	max<c*	UCL>CUG
n=25	12	92
n=50	9	96
n=100	1	92

Finally, from the normal data we obtained the following results (see Tables 9-11, Appendix 1).

Size	max<c*	UCL>CUG
n=25	1	100
n=50	0	100
n=100	0	100

The preceding tables give the sample size in column one. The second column gives the number of times the maximum concentration of a data set was less than the calculated c* value, indicating that no remediation should be done. Column three gives the number of times the UCL of the mean after remediation is greater than the CUG as observed in the results contained in Appendix 1.

The reason for observing the number of occurrences in which the maximum concentration of each data set was less than the c* value was to obtain an initial percentage for which the CRG method gave a false reading. The readings are said to be 'false negatives' because in these cases there would be no remediation done. None of the concentration values exceed the calculated c*, however, we generated the data sets with a true mean that was higher than the CUG to ensure that some remediation must be done. Thus, we know that every time we fail to clean one of these sites, the CRG method has failed.

As observed in Tables 3-5 and 9-11, Appendix 1, the CRG method is relatively effective for data from a uniform or normal distribution, but gave false readings in 12, 9, and 1 trial out of 100 for exponential data with sample size of 25, 50, and 100 respectively. These values are much better than the results collected from the lognormal data, but in comparison with the other distributions they are moderately worse.

Column three of the preceding tables shows the number of times out of 100 that the UCL of the mean of the post-remediation data is greater than the CUG. All three distributions gave readings of at least 92 out of 100 for all of the sample sizes. According to current standard U.S. EPA convention, the UCL of the mean should be less than the

CUG in order to be confident that the site is sufficiently clean. According to this convention only a very small percentage of sites would be sufficiently remediated using the CRG method, as observed in column 3 of the preceding summary tables.

In conclusion, It appears that the CRG method is very ineffective for data that follows a lognormal distribution. The method is greatly improved for the other data distributions, in the sense that it reduces the number of initial false readings. However, in all of the data sets generated from uniform, exponential, or normal distributions not enough remediation is done to obtain an UCL of the mean contamination level that falls below the CUG. Thus, we cannot be confident that, after applying the CRG method, the contamination level of the site would be down below the calculated 'safe' level of contamination for the site.

APPENDIX I

RESULTS FROM THE EXAMPLES

Table 1 - Input Values for Simulation Experiments for fhe Lognormal Distribution (Singh and Singh (1996))

Run No.	Sample Size N	μ	σ	C_0	CUG	True Mean	True Median
1	25	5.00	2.00	5	700	1096.6	148.4
2	50	5.00	2.00	5	700	1096.6	148.4
3	100	5.00	2.00	5	700	1096.6	148.4
4	25	5.00	3.00	10	13000	13359.7	148.4
5	50	5.00	3.00	10	13000	13359.7	148.4
6	100	4.33	2.16	5	700	782.7	75.9
7	100	6.00	1.00	5	700	665.1	403.4
8	100	5.00	1.00	5	700	244.7	148.4

Table 2 - Summary of CRG Calculations for 100 Iterations (Singh and Singh (1996))

Run No.	Simulated Site Condition	Sample Size n	# Of Times CRG>Sample Max
1	Highly elevated Cd	25	41
2	Highly elevated Cd	50	29
3	Highly elevated Cd	100	19
4	Elevated Ni	25	68
5	Elevated Ni	50	72
6	Slightly elevated Cd	100	55
7	Clean for *Cd*	100	28
8	Clean for *Cd*	100	0

Table 3 - Results from Uniform Data- Sample Size 25

xbar	ucl	c*	max	max<c*	ucl(p)	ucl(p)>cug
84.60	102.38	177.63	196.03	no	93.34	yes
95.57	120.62	177.63	198.22	no	86.92	yes
103.68	127.98	177.63	197.53	no	98.27	yes
91.96	115.79	177.63	194.87	no	91.05	yes
114.92	137.67	177.63	198.74	no	99.75	yes
102.33	119.77	177.63	170.59	yes	119.73	yes
98.83	120.06	177.63	182.24	no	113.09	yes
121.08	141.35	177.63	196.74	no	105.66	yes
105.84	130.44	177.63	197.42	no	99.52	yes
96.96	119.02	177.63	190.60	no	96.23	yes
112.30	137.69	177.63	198.47	no	107.50	yes
85.10	111.17	177.63	196.09	no	85.13	yes
104.70	128.39	177.63	194.42	no	97.60	yes
86.70	109.26	177.63	197.59	no	92.70	yes
82.80	105.05	177.63	198.17	no	78.51	no
100.61	119.24	177.63	179.79	no	105.56	yes
100.15	121.15	177.63	196.24	no	106.28	yes
114.15	139.22	177.63	197.77	no	92.91	yes
85.44	110.43	177.63	194.76	no	94.02	yes
99.75	122.74	177.63	198.59	no	106.34	yes
95.50	121.25	177.63	195.18	no	79.84	no
93.42	117.06	177.63	181.61	no	109.87	yes
84.45	109.64	177.63	199.56	no	92.71	yes
97.07	119.98	177.63	186.85	no	97.47	yes
118.81	141.61	177.63	199.42	no	121.15	yes
97.81	122.59	177.63	185.31	no	92.21	yes
101.02	126.47	177.63	199.30	no	102.24	yes
90.24	113.39	177.63	188.64	no	89.77	yes
84.49	105.20	177.63	191.17	no	88.36	yes
107.31	131.78	177.63	192.20	no	102.16	yes
111.73	137.81	177.63	197.74	no	101.04	yes
104.32	126.00	177.63	189.68	no	111.41	yes
95.24	117.59	177.63	187.63	no	95.11	yes
78.23	97.84	177.63	173.41	yes	97.81	yes
120.26	139.70	177.63	199.93	no	120.34	yes
103.62	125.63	177.63	187.00	no	118.59	yes

103.97	128.20	177.63	190.88	no	113.48	yes
107.36	130.96	177.63	179.67	no	117.88	yes
106.11	127.80	177.63	194.79	no	120.37	yes
103.03	123.96	177.63	197.24	no	108.29	yes
102.61	125.65	177.63	197.97	no	109.73	yes
81.88	102.68	177.63	177.77	no	95.13	yes
120.02	141.18	177.63	199.48	no	97.72	yes
99.67	121.75	177.63	195.94	no	98.01	yes
89.01	111.76	177.63	195.28	no	103.38	yes
109.81	132.29	177.63	199.79	no	95.61	yes
85.99	108.67	177.63	197.62	no	99.95	yes
114.85	134.09	177.63	190.97	no	127.46	yes
100.48	125.27	177.63	199.98	no	94.11	yes
103.13	127.76	177.63	190.67	no	105.41	yes
94.13	117.84	177.63	186.82	no	102.97	yes
95.95	116.22	177.63	186.58	no	108.78	yes
89.10	114.39	177.63	198.01	no	97.73	yes
94.14	119.08	177.63	195.46	no	103.35	yes
99.50	123.56	177.63	196.67	no	91.46	yes
95.56	119.69	177.63	195.35	no	87.90	yes
104.63	129.43	177.63	192.97	no	90.53	yes
94.21	116.28	177.63	193.49	no	92.59	yes
82.19	106.43	177.63	195.16	no	89.24	yes
111.33	134.14	177.63	193.71	no	105.37	yes
84.11	106.88	177.63	193.20	no	90.07	yes
96.33	120.09	177.63	183.01	no	112.95	yes
107.90	130.46	177.63	185.30	no	110.05	yes
96.64	119.48	177.63	196.17	no	86.89	yes
88.81	110.38	177.63	195.28	no	101.94	yes
88.51	105.45	177.63	190.70	no	89.18	yes
83.67	106.68	177.63	190.02	no	90.25	yes
119.69	141.88	177.63	198.98	no	97.57	yes
108.63	129.26	177.63	190.87	no	115.71	yes
105.48	124.56	177.63	198.69	no	116.79	yes
105.31	126.56	177.63	187.02	no	112.81	yes
91.60	116.23	177.63	194.82	no	108.09	yes
105.96	126.71	177.63	199.62	no	103.98	yes
102.14	127.71	177.63	191.96	no	80.97	yes
97.40	120.75	177.63	188.82	no	113.28	yes
98.05	122.20	177.63	194.88	no	98.36	yes

116.55	141.08	177.63	199.69	no	98.45	yes
120.60	139.93	177.63	199.07	no	103.03	yes
93.72	116.05	177.63	193.16	no	108.06	yes
96.84	118.47	177.63	197.19	no	102.78	yes
94.69	114.50	177.63	186.34	no	107.00	yes
96.30	119.91	177.63	187.40	no	105.06	yes
100.07	123.49	177.63	191.60	no	93.35	yes
99.72	123.39	177.63	196.59	no	107.86	yes
115.19	138.96	177.63	197.51	no	101.99	yes
83.88	105.61	177.63	194.44	no	79.96	no
76.78	97.87	177.63	195.41	no	70.70	no
103.93	129.45	177.63	199.80	no	81.62	yes
114.08	138.83	177.63	198.02	no	110.90	yes
99.93	121.50	177.63	198.24	no	113.48	yes
85.67	110.46	177.63	198.29	no	75.18	no
91.00	113.76	177.63	189.16	no	105.93	yes
84.07	103.79	177.63	174.65	yes	103.75	yes
82.15	101.82	177.63	186.09	no	85.22	yes
106.84	129.35	177.63	197.22	no	99.26	yes
88.55	111.22	177.63	197.41	no	86.40	yes
99.05	120.77	177.63	194.39	no	105.70	yes
91.15	116.25	177.63	191.73	no	100.44	yes
105.76	127.94	177.63	196.52	no	104.69	yes
89.50	111.53	177.63	185.78	no	103.83	yes

Table 4 - Uniform Data- Sample Size 50

xbar	ucl	c*	max	max<c*	ucl(p)	ucl(p)>cug
110.75	128.51	177.63	199.28	no	79.47	no
96.36	110.83	177.63	192.62	no	95.86	yes
90.60	105.80	177.63	194.31	no	86.55	yes
111.61	125.74	177.63	198.61	no	95.13	yes
91.43	106.31	177.63	189.45	no	95.04	yes
100.98	116.14	177.63	198.87	no	105.18	yes
105.95	121.90	177.63	197.69	no	95.54	yes
87.13	102.38	177.63	195.48	no	86.67	yes
85.23	101.90	177.63	198.90	no	77.05	no
101.95	119.11	177.63	198.78	no	91.83	yes
95.15	110.01	177.63	193.37	no	102.62	yes
84.75	100.36	177.63	195.28	no	88.48	yes
101.91	116.55	177.63	197.72	no	93.79	yes
95.93	113.10	177.63	195.10	no	97.92	yes
91.45	106.25	177.63	197.62	no	94.41	yes
90.06	105.55	177.63	192.37	no	94	yes
100.91	114.17	177.63	194.82	no	99.77	yes
98.02	113.36	177.63	197.06	no	98.43	yes
92.93	107.79	177.63	198.12	no	92.32	yes
100.15	115.33	177.63	195.30	no	100.45	yes
97.07	115.10	177.63	199.07	no	92.25	yes
91.49	106.76	177.63	198.27	no	91.07	yes
98.31	114.10	177.63	198.29	no	95.4	yes
102.60	116.26	177.63	198.18	no	101.66	yes
97.26	111.77	177.63	192.57	no	96.55	yes
93.85	109.85	177.63	197.96	no	90.41	yes
84.02	100.44	177.63	197.94	no	75.07	no
97.05	114.83	177.63	197.93	no	87.36	yes
93.82	110.10	177.63	190.62	no	87.52	yes
101.85	118.19	177.63	199.09	no	107.21	yes
95.92	111.46	177.63	198.11	no	92.11	yes
105.54	121.33	177.63	198.53	no	99.12	yes
114.21	129.92	177.63	199.05	no	100.88	yes
104.04	118.55	177.63	199.22	no	100.12	yes

95.82	109.91	177.63	197.15	no	105.92	yes
98.35	113.13	177.63	197.27	no	105.79	yes
98.23	114.20	177.63	197.45	no	102.82	yes
76.61	90.83	177.63	184.20	no	86.87	yes
94.25	111.23	177.63	195.53	no	83.84	yes
112.94	129.49	177.63	199.25	no	95.68	yes
107.72	122.48	177.63	192.80	no	104.53	yes
100.70	116.57	177.63	199.30	no	101.56	yes
100.42	116.33	177.63	199.15	no	108.65	yes
90.43	106.41	177.63	195.70	no	94.75	yes
101.68	114.99	177.63	197.08	no	103.43	yes
114.39	130.70	177.63	198.94	no	101.75	yes
94.13	110.02	177.63	188.95	no	91.32	yes
80.71	98.40	177.63	199.60	no	68.96	no
115.95	131.74	177.63	199.02	no	95.75	yes
85.66	102.18	177.63	192.60	no	90.4	yes
94.40	111.40	177.63	194.99	no	87.98	yes
95.11	111.26	177.63	198.35	no	91.45	yes
98.91	113.55	177.63	198.12	no	94.61	yes
105.58	122.98	177.63	198.43	no	86.18	yes
91.31	106.78	177.63	199.37	no	95.07	yes
92.39	111.12	177.63	197.83	no	87.3	yes
103.15	119.15	177.63	196.26	no	100.94	yes
93.91	108.66	177.63	196.74	no	96.85	yes
98.33	113.30	177.63	194.98	no	94.74	yes
91.07	106.60	177.63	195.70	no	94.91	yes
89.44	105.04	177.63	188.73	no	93.88	yes
109.38	125.91	177.63	198.90	no	101.11	yes
89.28	105.80	177.63	196.67	no	86.17	yes
99.78	117.31	177.63	199.78	no	85.94	yes
92.76	109.24	177.63	190.58	no	98.01	yes
85.96	99.04	177.63	182.82	no	91.58	yes
91.90	106.82	177.63	198.10	no	95.1	yes
88.36	102.91	177.63	193.39	no	95.13	yes
100.55	117.37	177.63	197.41	no	90.66	yes
89.59	106.27	177.63	191.49	no	91.09	yes
119.18	134.43	177.63	196.08	no	102.93	yes
86.97	103.81	177.63	194.42	no	92.19	yes
103.88	119.50	177.63	197.70	no	101.04	yes
89.30	103.98	177.63	199.74	no	88.23	yes

117.26	133.92	177.63	199.19	no	101.4	yes
104.73	121.97	177.63	196.03	no	88.09	yes
97.78	113.97	177.63	198.40	no	91.31	yes
103.56	119.17	177.63	197.78	no	96.54	yes
95.01	111.59	177.63	196.39	no	85.08	yes
107.63	125.66	177.63	196.62	no	83.85	yes
108.23	124.48	177.63	199.15	no	102.08	yes
103.24	120.12	177.63	199.70	no	90.4	yes
108.79	124.39	177.63	198.83	no	98.59	yes
100.27	114.95	177.63	199.17	no	92.38	yes
109.04	125.86	177.63	193.83	no	89.35	yes
111.66	127.52	177.63	194.36	no	105.71	yes
95.39	109.59	177.63	192.26	no	94.51	yes
112.65	129.45	177.63	198.73	no	96.68	yes
93.75	111.04	177.63	195.18	no	88.2	yes
99.48	116.15	177.63	196.04	no	86.55	yes
106.12	122.38	177.63	199.96	no	103.75	yes
98.46	113.82	177.63	198.59	no	90.01	yes
79.90	96.62	177.63	199.58	no	80.27	yes
107.77	123.44	177.63	199.06	no	105.3	yes
105.69	121.90	177.63	195.86	no	99.58	yes
103.60	119.41	177.63	196.43	no	96.81	yes
94.94	111.16	177.63	199.88	no	87.32	yes
95.76	110.64	177.63	191.72	no	91.47	yes
101.04	117.59	177.63	197.20	no	91.47	yes
103.46	117.49	177.63	195.56	no	103.04	yes

Table 5 - Uniform Data - Sample Size 100

xbar	ucl	c*	max	max<c*	ucl(p)	ucl(p)>cug
97.70	108.62	177.63	199.52	no	90.12	yes
89.56	99.91	177.63	193.02	no	88.81	yes
101.23	113.23	177.63	199.88	no	78.91	no
88.91	99.89	177.63	199.89	no	85.98	yes
97.65	109.80	177.63	198.48	no	82.05	yes
95.41	106.45	177.63	198.96	no	90.93	yes
107.67	118.39	177.63	199.93	no	98.26	yes
94.10	105.00	177.63	195.72	no	86.17	yes
101.11	112.83	177.63	198.12	no	94.37	yes
108.50	119.04	177.63	198.68	no	93.66	yes
102.94	114.35	177.63	197.64	no	96.2	yes
102.06	114.04	177.63	198.41	no	89.92	yes
103.03	114.37	177.63	198.84	no	89.87	yes
91.07	103.03	177.63	197.90	no	86.13	yes
102.40	114.03	177.63	199.47	no	84.06	yes
95.43	107.01	177.63	199.93	no	83.96	yes
105.91	117.87	177.63	199.99	no	86.59	yes
105.21	116.61	177.63	198.00	no	89.36	yes
103.44	115.16	177.63	196.97	no	93.67	yes
95.05	106.85	177.63	197.08	no	86.29	yes
93.36	104.97	177.63	198.29	no	86.1	yes
91.95	103.21	177.63	193.67	no	93.64	yes
105.06	115.96	177.63	197.79	no	94.39	yes
92.95	105.21	177.63	199.49	no	80.39	yes
92.92	104.09	177.63	198.67	no	84.94	yes
99.62	112.00	177.63	198.93	no	80.24	yes
99.54	110.97	177.63	198.02	no	92.13	yes
103.34	115.26	177.63	198.89	no	91.38	yes
94.45	105.84	177.63	199.74	no	81.12	yes
106.40	117.89	177.63	199.63	no	92	yes
103.79	115.21	177.63	199.75	no	89.03	yes
115.33	126.22	177.63	199.90	no	106.41	yes
101.16	113.38	177.63	199.27	no	83.61	yes
103.43	114.71	177.63	199.06	no	92.86	yes

93.54	104.42	177.63	197.04	no	93.25	yes
97.33	108.46	177.63	199.19	no	89.73	yes
99.85	110.66	177.63	195.72	no	92.1	yes
97.43	109.50	177.63	199.31	no	86.73	yes
100.93	112.35	177.63	198.58	no	92.22	yes
98.57	110.38	177.63	199.94	no	83.54	yes
99.41	111.03	177.63	200.00	no	86.74	yes
106.74	117.45	177.63	197.52	no	97.46	yes
93.83	105.53	177.63	196.54	no	88.9	yes
102.94	115.03	177.63	197.57	no	91.13	yes
89.09	100.39	177.63	194.00	no	85.43	yes
98.35	109.79	177.63	199.41	no	89.73	yes
97.89	108.76	177.63	199.46	no	90.24	yes
99.89	110.46	177.63	197.60	no	95.96	yes
90.64	100.67	177.63	192.08	no	95.02	yes
105.50	117.34	177.63	198.81	no	90.02	yes
106.73	118.32	177.63	199.38	no	94.2	yes
102.21	112.78	177.63	199.99	no	92.88	yes
108.79	119.67	177.63	199.51	no	94.42	yes
98.74	110.74	177.63	199.62	no	88.07	yes
102.24	113.91	177.63	198.87	no	89.82	yes
97.10	108.59	177.63	194.92	no	91.85	yes
99.65	110.42	177.63	194.40	no	93.71	yes
95.92	107.51	177.63	198.26	no	88.68	yes
102.65	113.90	177.63	198.97	no	92.06	yes
97.52	108.54	177.63	196.52	no	82.74	yes
102.73	113.26	177.63	198.68	no	94.71	yes
99.84	112.24	177.63	199.69	no	87.51	yes
96.53	107.18	177.63	199.93	no	92.39	yes
96.21	107.44	177.63	198.56	no	88.69	yes
104.94	116.71	177.63	198.61	no	85.38	yes
95.47	107.28	177.63	198.30	no	88.64	yes
112.02	122.41	177.63	199.16	no	102.88	yes
104.36	115.80	177.63	197.43	no	93.78	yes
89.22	99.17	177.63	199.96	no	87.35	yes
107.33	119.27	177.63	198.85	no	91.06	yes
92.76	103.56	177.63	199.71	no	90.55	yes
94.11	105.22	177.63	196.92	no	84.51	yes
102.00	112.99	177.63	199.73	no	90.97	yes
97.23	108.75	177.63	197.55	no	93.77	yes

91.62	103.53	177.63	196.29	no	90.23	yes
98.93	110.73	177.63	199.41	no	84.21	yes
103.49	115.38	177.63	197.18	no	85.96	yes
100.43	111.36	177.63	195.28	no	95.06	yes
98.27	110.30	177.63	198.22	no	87.66	yes
95.41	106.62	177.63	199.99	no	89.36	yes
97.75	108.70	177.63	196.62	no	88.77	yes
93.45	104.74	177.63	196.05	no	82.16	yes
94.55	105.53	177.63	198.30	no	88.96	yes
105.17	116.79	177.63	197.41	no	96.95	yes
102.00	113.62	177.63	199.19	no	95.21	yes
98.57	109.45	177.63	188.09	no	100.57	yes
103.77	115.34	177.63	199.98	no	89.44	yes
98.47	108.98	177.63	192.38	no	94.48	yes
96.18	107.54	177.63	198.06	no	83.06	yes
92.24	103.31	177.63	199.43	no	86.62	yes
99.40	111.37	177.63	197.60	no	88.94	yes
95.36	107.42	177.63	199.79	no	82.71	yes
91.91	104.07	177.63	195.48	no	83.56	yes
99.11	110.38	177.63	199.77	no	89.71	yes
93.98	105.65	177.63	197.55	no	87.06	yes
92.34	103.65	177.63	197.38	no	82.9	yes
95.30	107.10	177.63	193.16	no	90.45	yes
100.10	111.43	177.63	197.23	no	96.74	yes
94.21	106.34	177.63	198.53	no	81.83	yes
100.12	112.03	177.63	199.92	no	81.96	yes

Table 6 - Exponential Data - Sample Size 25

xbar	ucl	c*(theta)	c*(ucl)	max	max<c*	ucl(p)	ucl(p)>cug
81.89	105.24	296.01	285.62	219.12	yes	105.2	yes
87.03	121.23	296.01	269.48	283.49	no	91.95	yes
128.26	163.28	296.01	262.58	301.87	no	121.96	yes
116.39	163.08	296.01	262.56	467.00	no	99.66	yes
89.70	125.26	296.01	267.39	330.08	no	73.85	no
86.72	125.46	296.01	267.30	492.64	no	87.96	yes
110.12	148.92	296.01	262.35	463.49	no	118.72	yes
68.81	96.40	296.01	306.08	286.53	yes	96.36	yes
116.33	148.86	296.01	262.35	337.71	no	116.16	yes
95.63	125.78	296.01	267.16	284.93	no	96.19	yes
90.75	129.81	296.01	265.60	345.94	no	73.66	no
125.75	162.99	296.01	262.56	341.00	no	117.59	yes
82.92	118.93	296.01	270.92	385.42	no	94.49	yes
80.92	108.35	296.01	281.06	251.63	yes	108.32	yes
110.58	150.22	296.01	262.30	368.17	no	95.75	yes
96.72	138.89	296.01	263.37	494.80	no	104.7	yes
117.48	159.70	296.01	262.38	373.72	no	78.13	no
86.60	113.51	296.01	275.28	321.65	no	93.66	yes
102.26	131.45	296.01	265.08	278.20	no	117.76	yes
71.54	94.19	296.01	314.10	208.43	yes	94.13	yes
81.79	110.86	296.01	278.00	223.45	yes	110.83	yes
96.02	134.43	296.01	264.28	415.48	no	93.35	yes
130.87	197.90	296.01	267.38	733.00	no	67.84	no
78.77	106.07	296.01	284.30	307.04	no	87.33	yes
85.67	125.60	296.01	267.24	377.85	no	69.62	no
106.18	138.57	296.01	263.43	318.91	no	105.48	yes
104.56	146.59	296.01	262.48	457.07	no	104.15	yes
85.36	114.51	296.01	274.36	273.74	yes	114.47	yes
106.08	148.84	296.01	262.35	338.25	no	68.04	no
79.61	106.66	296.01	283.43	267.35	yes	106.6	yes
95.93	134.91	296.01	264.16	355.10	no	93.51	yes
74.37	106.13	296.01	284.21	342.29	no	84.43	yes
100.79	135.11	296.01	264.12	330.97	no	117.32	yes
100.05	148.01	296.01	262.40	487.97	no	75.61	no

122.92	177.66	296.01	264.05	644.70	no	115.87	yes
108.51	141.31	296.01	263.01	339.81	no	122.98	yes
85.47	116.77	296.01	272.49	301.58	no	100.13	yes
81.40	118.08	296.01	271.52	329.74	no	83.23	yes
107.51	141.80	296.01	262.94	313.48	no	109.93	yes
84.26	115.38	296.01	273.61	304.17	no	98.37	yes
92.26	137.85	296.01	263.55	394.36	no	73.13	no
115.45	179.82	296.01	264.35	844.39	no	107.22	yes
101.88	140.15	296.01	263.17	335.80	no	108.6	yes
105.12	149.20	296.01	262.34	529.92	no	98.39	yes
106.57	157.21	296.01	262.29	552.99	no	94.2	yes
100.51	131.05	296.01	265.20	274.15	no	117.73	yes
105.33	141.80	296.01	262.94	380.38	no	103.57	yes
146.87	202.50	296.01	268.27	417.49	no	86.77	yes
123.05	164.46	296.01	262.66	342.29	no	98.01	yes
117.73	158.36	296.01	262.33	437.04	no	103.38	yes
94.20	131.95	296.01	264.93	415.61	no	82.94	yes
115.60	160.46	296.01	262.41	433.03	no	104.18	yes
133.79	181.54	296.01	264.59	433.93	no	111.01	yes
107.50	146.82	296.01	262.47	386.59	no	90.46	yes
127.98	171.00	296.01	263.25	397.58	no	112.2	yes
88.35	119.82	296.01	270.34	284.45	no	104.97	yes
65.59	96.52	296.01	305.70	294.11	yes	96.48	yes
114.76	154.45	296.01	262.25	396.69	no	111.27	yes
125.33	173.01	296.01	263.47	448.24	no	106.03	yes
101.70	141.11	296.01	263.04	475.52	no	108.66	yes
100.59	136.15	296.01	263.89	413.50	no	109.87	yes
109.57	141.68	296.01	262.96	324.51	no	124.74	yes
95.97	129.23	296.01	265.80	267.63	no	116.41	yes
127.31	174.10	296.01	263.60	474.56	no	81.79	yes
118.46	150.87	296.01	262.28	313.33	no	122.52	yes
102.99	142.59	296.01	262.85	402.02	no	98.64	yes
90.86	122.29	296.01	268.88	319.73	no	104.31	yes
110.77	148.41	296.01	262.38	363.78	no	116.11	yes
86.03	117.16	296.01	272.19	323.99	no	98.29	yes
102.12	137.08	296.01	263.70	302.01	no	93.22	yes
145.61	211.94	296.01	270.21	783.33	no	115.1	yes
156.20	211.49	296.01	270.12	530.86	no	134.17	yes
137.36	184.49	296.01	265.04	544.82	no	113.48	yes
119.14	165.42	296.01	262.73	472.23	no	100.41	yes

101.65	137.97	296.01	263.53	393.20	no	100.73	yes
110.09	160.71	296.01	262.43	529.78	no	106.6	yes
121.92	171.73	296.01	263.33	579.39	no	101.02	yes
76.18	112.69	296.01	276.07	441.77	no	80.25	yes
84.35	118.43	296.01	271.27	279.56	no	88.71	yes
84.35	116.67	296.01	272.56	292.06	no	101.02	yes
91.16	123.85	296.01	268.06	280.92	no	109.63	yes
72.71	108.24	296.01	281.20	362.72	no	85.36	yes
89.19	131.70	296.01	265.01	518.42	no	93.33	yes
123.81	164.36	296.01	262.65	438.58	no	123.29	yes
96.27	138.19	296.01	263.49	376.64	no	97	yes
91.70	120.48	296.01	269.93	293.21	no	104.62	yes
101.44	125.60	296.01	267.24	213.30	yes	125.56	yes
114.70	153.06	296.01	262.25	344.27	no	104.39	yes
107.29	144.57	296.01	262.64	273.27	no	119.32	yes
84.58	114.82	296.01	274.09	342.43	no	93.5	yes
122.10	164.81	296.01	262.68	382.71	no	125.11	yes
87.35	122.92	296.01	268.54	330.49	no	89.43	yes
137.84	193.92	296.01	266.64	500.46	no	99.15	yes
90.03	127.15	296.01	266.59	366.21	no	89.65	yes
58.86	81.99	296.01	458.06	212.07	yes	81.94	yes
77.76	107.86	296.01	281.71	267.14	yes	107.82	yes
127.66	173.30	296.01	263.50	461.69	no	121.46	yes
80.61	113.41	296.01	275.37	326.06	no	94.32	yes
79.86	103.44	296.01	288.74	216.69	yes	103.4	yes
96.99	127.84	296.01	266.32	297.67	no	112.29	yes

Table 7 - Exponential Data - Sample Size 50

xbar	ucl	c*(theta)	c*(ucl)	max	max<c*	ucl(p)	ucl<cug
105.82	137.64	296.01	263.59	645.37	no	94.51	yes
100.43	129.13	296.01	265.84	529.07	no	88	yes
83.36	118.75	296.01	271.05	757.4	no	59.34	no
95.53	121.89	296.01	269.1	458.29	no	94.1	yes
100.61	130.04	296.01	265.53	456.23	no	87.28	yes
72.22	90.73	296.01	330.92	267	yes	90.73	yes
107.27	141.61	296.01	262.97	751.45	no	95.18	yes
91.16	107.83	296.01	281.76	260.15	yes	107.83	yes
75.27	97.3	296.01	303.27	425.76	no	84.11	yes
67.9	84.58	296.01	390.79	271.31	yes	84.58	yes
90.34	115.85	296.01	273.22	439.23	no	94.94	yes
95.17	118.74	296.01	271.06	281.31	no	92.87	yes
117.01	157.42	296.01	262.3	758.66	no	88.07	yes
105.8	131.92	296.01	264.94	475.83	no	109.03	yes
99.78	123.45	296.01	268.26	435.36	no	111.04	yes
114.35	143.1	296.01	262.79	420.49	no	96.15	yes
116.35	145.74	296.01	262.54	489.77	no	111.73	yes
82.5	106.28	296.01	283.99	407.38	no	94.63	yes
103.19	141.21	296.01	263.02	853.44	no	86.18	yes
97.91	124.41	296.01	267.79	422.29	no	104.3	yes
106.31	143.53	296.01	262.75	773.05	no	91.88	yes
94.11	114.43	296.01	274.43	285.86	no	107.67	yes
100.9	129.77	296.01	265.62	628.62	no	99.72	yes
113.05	143.91	296.01	262.71	570.18	no	91.48	yes
114.57	145.42	296.01	262.57	611.24	no	109.33	yes
102.75	128.26	296.01	266.15	387.7	no	99.78	yes
92.09	121.13	296.01	269.54	428.03	no	92.74	yes
105.68	128.7	296.01	265.99	410.07	no	117.53	yes
111.23	137.15	296.01	263.68	363.57	no	107.52	yes
107.45	137.43	296.01	263.63	479.68	no	104.55	yes
91.39	115.06	296.01	273.88	359.23	no	96.23	yes
96.86	124.95	296.01	267.54	450.79	no	92.01	yes
118.73	147.75	296.01	262.41	450.86	no	103.58	yes
94.87	121.64	296.01	269.24	455.12	no	84.71	yes

104.11	142.33	296.01	262.88	729.1	no	77.94	no
100.12	128.97	296.01	265.9	462.14	no	93.42	yes
88.14	108.51	296.01	280.85	285.72	no	101.63	yes
91.51	117.68	296.01	271.8	424.32	no	86.42	yes
98.64	128.07	296.01	266.22	468.54	no	95.89	yes
123.89	160.27	296.01	262.4	535.19	no	73.68	no
113.78	153.75	296.01	262.25	835.43	no	89.59	yes
91.31	118.56	296.01	271.18	542.89	no	91.33	yes
79.21	97.89	296.01	301.54	327.54	no	88.88	yes
98.82	123.46	296.01	268.26	368.05	no	97.86	yes
97.87	126.75	296.01	266.75	416.78	no	86.04	yes
90.67	113.6	296.01	275.2	424.99	no	101.22	yes
66.92	86.35	296.01	366.6	331.61	yes	86.35	yes
102.21	129.57	296.01	265.69	426.45	no	82.39	yes
75.61	95.84	296.01	307.97	282.02	yes	95.84	yes
115.32	150.01	296.01	262.31	475.87	no	86.65	yes
79.54	98.9	296.01	298.77	342.23	no	89.35	yes
110.3	141.62	296.01	262.97	655.85	no	106.76	yes
92.69	116.98	296.01	272.33	381.56	no	91.32	yes
93.47	120.34	296.01	270.01	487.37	no	95.82	yes
92.07	111.43	296.01	277.38	248.12	yes	111.43	yes
125.85	156.52	296.01	262.27	486.94	no	101.72	yes
92.98	119.98	296.01	270.24	393.23	no	84.61	yes
72.93	90.84	296.01	330.24	268.14	yes	90.84	yes
88.1	110.31	296.01	278.64	346.58	no	101.23	yes
79.93	99.43	296.01	297.42	291.11	yes	99.43	yes
109.56	142.99	296.01	262.8	556.46	no	84.11	yes
79.89	100.29	296.01	295.31	350.22	no	90.57	yes
103.93	131	296.01	265.22	398.04	no	86.72	yes
99.76	120.97	296.01	269.63	305.52	no	113.68	yes
80.91	103.76	296.01	288.16	361.45	no	86.27	yes
97.1	119.06	296.01	270.84	304.47	no	91.49	yes
109.25	143.5	296.01	262.75	757.08	no	92.25	yes
111.18	145.62	296.01	262.55	671.29	no	92.58	yes
115.19	145.97	296.01	262.53	546.02	no	101.66	yes
110.61	149.72	296.01	262.32	854.98	no	91.62	yes
108.19	135.33	296.01	264.07	449.89	no	105.85	yes
89.21	115.66	296.01	273.37	440.42	no	92.81	yes
106.73	135.97	296.01	263.93	424.61	no	90.71	yes
96.39	122.61	296.01	268.71	403.66	no	80.04	yes

113.21	144.45	296.01	262.65	517.11	no	81.47	yes
97.97	128.38	296.01	266.11	528.66	no	81.35	yes
110.77	133.93	296.01	264.4	317.78	no	107.06	yes
119.65	154.93	296.01	262.25	677.87	no	106.73	yes
85.41	107.66	296.01	281.99	336.34	no	82.45	yes
106.52	135.26	296.01	264.08	496.45	no	102.62	yes
110.17	140.99	296.01	263.05	594.89	no	84.36	yes
118.44	153.07	296.01	262.25	715.91	no	101.23	yes
82.9	104.68	296.01	286.56	389.53	no	85.58	yes
81.06	101.5	296.01	292.61	275.74	yes	101.5	yes
100.71	134	296.01	264.38	722.78	no	102.03	yes
103.38	127	296.01	266.64	358.62	no	111.81	yes
110.41	140.62	296.01	263.1	501.72	no	100.76	yes
106.07	130.88	296.01	265.26	403.43	no	91.05	yes
79.95	103.32	296.01	288.97	384.81	no	81.76	yes
72.57	97.68	296.01	302.16	516.7	no	70.05	no
99.35	127.42	296.01	266.48	520.5	no	90.2	yes
108.85	136.99	296.01	263.72	529.73	no	93.22	yes
98.42	125.93	296.01	267.09	455.47	no	98.55	yes
90.25	115.04	296.01	273.9	463.09	no	92.89	yes
107.27	134.89	296.01	264.17	497.57	no	110.9	yes
117.18	148.57	296.01	262.37	644.39	no	114.41	yes
87.34	110	296.01	278.99	370.45	no	92.2	yes
101.9	131.12	296.01	265.18	505.97	no	89.92	yes
96.81	121.62	296.01	269.25	491.24	no	99.56	yes
124.69	155.45	296.01	262.26	386.56	no	93.64	yes

Table 8 - Exponential Data - Sample Size 100

xbar	ucl	c*(theta)	c*(ucl)	max	max<c*	ucl(p)	ucl(p)>cug
90.35	107.82	296.01	281.77	531.62	no	86.82	yes
118.77	142.36	296.01	262.88	504.53	no	87.41	yes
92.19	108.00	296.01	281.52	434.78	no	91.12	yes
105.80	127.06	296.01	266.62	596.46	no	79.2	no
105.73	127.92	296.01	266.28	537.19	no	88.93	yes
94.69	112.03	296.01	276.74	433.24	no	93.46	yes
104.72	127.80	296.01	266.33	654.95	no	78.11	no
87.82	107.72	296.01	281.90	643.62	no	85.66	yes
88.90	104.72	296.01	286.49	469.94	no	93.64	yes
88.93	104.70	296.01	286.52	352.61	no	92.93	yes
99.46	116.28	296.01	272.87	434.97	no	90	yes
88.43	103.82	296.01	288.06	393.79	no	91.81	yes
107.68	129.16	296.01	265.83	535.48	no	90.37	yes
108.68	128.81	296.01	265.95	537.40	no	91.08	yes
92.13	109.90	296.01	279.12	379.07	no	98.24	yes
106.67	129.73	296.01	265.63	711.30	no	88.53	yes
90.97	106.82	296.01	283.19	367.45	no	94.51	yes
101.47	121.16	296.01	269.52	568.93	no	93.25	yes
93.86	110.43	296.01	278.50	308.97	no	93.64	yes
89.72	104.75	296.01	286.43	383.78	no	95.49	yes
97.90	113.63	296.01	275.16	353.41	no	98.65	yes
86.12	104.50	296.01	286.86	476.84	no	80.28	yes
93.83	112.35	296.01	276.42	421.93	no	88.96	yes
97.16	116.19	296.01	272.94	634.10	no	89.47	yes
103.76	123.97	296.01	268.00	590.51	no	88.77	yes
110.80	134.17	296.01	264.34	534.27	no	85.6	yes
97.13	114.38	296.01	274.47	374.40	no	92.26	yes
106.58	124.36	296.01	267.81	428.88	no	105.47	yes
114.79	135.59	296.01	264.01	663.35	no	106.66	yes
106.29	126.10	296.01	267.02	468.19	no	92.57	yes
113.81	131.79	296.01	264.98	455.23	no	100.55	yes
89.02	104.80	296.01	286.35	365.55	no	92.41	yes
92.72	110.79	296.01	278.08	583.38	no	90.58	yes
102.63	122.80	296.01	268.60	542.38	no	93.7	yes

95.07	114.17	296.01	274.67	436.07	no	91.55	yes
123.68	147.07	296.01	262.45	753.40	no	98.1	yes
96.57	116.88	296.01	272.40	609.43	no	93.74	yes
93.21	107.87	296.01	281.71	381.94	no	96.06	yes
92.22	110.85	296.01	278.02	523.43	no	87.12	yes
117.86	141.43	296.01	262.99	586.00	no	86.11	yes
93.29	113.83	296.01	274.98	558.13	no	88.24	yes
94.85	110.80	296.01	278.07	405.68	no	99.21	yes
105.69	125.50	296.01	267.29	606.37	no	105.53	yes
103.08	125.15	296.01	267.44	664.77	no	91.55	yes
112.09	134.64	296.01	264.23	649.32	no	88.02	yes
109.39	132.27	296.01	264.84	522.45	no	76.78	no
87.05	103.02	296.01	289.53	404.25	no	89.65	yes
100.09	119.93	296.01	270.27	518.53	no	90.19	yes
96.62	115.90	296.01	273.18	667.54	no	89.28	yes
93.31	111.73	296.01	277.05	499.59	no	87.21	yes
91.55	113.14	296.01	275.63	721.05	no	76.19	no
94.45	113.58	296.01	275.21	535.74	no	92.4	yes
129.85	155.35	296.01	262.25	731.34	no	98.05	yes
89.06	107.09	296.01	282.79	604.78	no	83.91	yes
102.79	122.76	296.01	268.62	430.52	no	91.43	yes
109.68	131.96	296.01	264.93	591.70	no	84.76	yes
84.71	100.72	296.01	294.32	446.17	no	90.76	yes
93.16	113.63	296.01	275.17	578.03	no	76.76	no
83.11	98.83	296.01	298.97	397.44	no	81.8	yes
99.79	119.99	296.01	270.23	575.98	no	86.32	yes
98.25	116.41	296.01	272.77	361.92	no	87.91	yes
111.94	135.35	296.01	264.06	895.68	no	93.86	yes
115.87	137.44	296.01	263.63	512.22	no	86.43	yes
89.57	107.85	296.01	281.73	492.95	no	84.42	yes
104.22	125.55	296.01	267.26	479.06	no	92.73	yes
112.92	134.02	296.01	264.38	540.78	no	86.91	yes
128.49	158.74	296.01	262.34	1231.04	no	99.65	yes
99.50	118.44	296.01	271.26	441.94	no	85.09	yes
129.35	155.80	296.01	262.26	669.49	no	91.55	yes
105.12	124.64	296.01	267.68	496.61	no	89.72	yes
105.85	131.38	296.01	265.10	903.81	no	77.96	no
114.79	135.75	296.01	263.97	616.48	no	92.87	yes
88.45	103.90	296.01	287.91	385.53	no	86.85	yes
105.82	125.78	296.01	267.16	516.30	no	88.54	yes

94.25	119.18	296.01	270.76	765.42	no	83.55	yes
108.56	127.89	296.01	266.29	367.83	no	100.24	yes
91.86	105.82	296.01	284.70	317.83	no	102.05	yes
106.71	125.99	296.01	267.07	489.08	no	88.01	yes
99.69	121.41	296.01	269.37	637.60	no	78.36	no
98.17	114.86	296.01	274.05	349.49	no	92.49	yes
95.80	114.11	296.01	274.72	480.76	no	90.52	yes
77.52	91.24	296.01	328.01	311.29	yes	91.19	yes
104.65	123.26	296.01	268.36	469.06	no	95.5	yes
81.70	98.53	296.01	299.76	445.24	no	77.84	no
100.38	120.58	296.01	269.87	465.28	no	85.67	yes
95.17	114.19	296.01	274.65	504.62	no	81.24	yes
89.56	108.54	296.01	280.81	471.42	no	80.09	yes
93.62	113.50	296.01	275.28	697.83	no	88.97	yes
103.67	124.43	296.01	267.78	538.91	no	89.08	yes
106.64	127.29	296.01	266.53	475.91	no	94.75	yes
100.59	119.85	296.01	270.32	587.77	no	96.61	yes
103.26	122.28	296.01	268.88	444.23	no	90.38	yes
101.03	120.85	296.01	269.70	489.51	no	85.46	yes
96.21	115.40	296.01	273.59	462.17	no	80.4	yes
109.62	130.86	296.01	265.27	602.68	no	91.92	yes
107.01	130.31	296.01	265.44	664.29	no	85.75	yes
112.01	132.01	296.01	264.92	442.39	no	87.83	yes
105.59	126.29	296.01	266.94	543.23	no	90.74	yes
95.24	115.66	296.01	273.37	670.44	no	90.23	yes
95.17	113.48	296.01	275.30	489.97	no	90.25	yes

Table 9 - Normal Data - Sample Size 25

xbar	ucl	c*(mu)	c*(ucl)	max	max<c*	ucl(p)	ucl(p)>cug
99.73	101.04	102.57	103.51	105.66	no	94.95	yes
100.32	101.41	102.57	103.85	105.01	no	102.52	yes
99.93	101.09	102.57	103.56	106.36	no	102.04	yes
100.75	102.06	102.57	104.44	107.08	no	104	yes
100.83	102.19	102.57	104.56	107.13	no	100.94	yes
99.99	101.43	102.57	103.87	109.06	no	99.86	yes
99.77	101.30	102.57	103.75	109.14	no	99.84	yes
99.08	100.40	102.57	102.93	106.23	no	101.12	yes
98.48	99.77	102.57	102.36	106.70	no	100.45	yes
100.41	102.09	102.57	104.47	108.82	no	98.02	yes
99.34	100.60	102.57	103.11	104.55	no	101.49	yes
99.40	100.70	102.57	103.20	105.29	no	99.63	yes
99.24	100.66	102.57	103.17	105.24	no	99.38	yes
100.33	101.37	102.57	103.81	104.30	no	100.71	yes
100.29	101.70	102.57	104.11	105.42	no	100.56	yes
100.16	101.15	102.57	103.61	104.60	no	103.44	yes
100.25	101.41	102.57	103.84	107.37	no	103.42	yes
100.07	101.17	102.57	103.63	104.14	no	102.29	yes
100.58	101.78	102.57	104.18	106.22	no	98.41	yes
98.79	100.35	102.57	102.89	107.09	no	98.72	yes
100.21	101.31	102.57	103.75	105.01	no	100.51	yes
100.60	101.85	102.57	104.25	106.35	no	100.81	yes
99.57	100.83	102.57	103.32	105.42	no	99.71	yes
99.73	100.81	102.57	103.30	104.65	no	101.91	yes
100.87	102.05	102.57	104.43	105.92	no	98.76	yes
99.59	100.67	102.57	103.17	103.91	no	102.87	yes
99.37	100.52	102.57	103.04	104.05	no	99.65	yes
99.57	100.95	102.57	103.43	105.57	no	101.66	yes
100.57	101.77	102.57	104.17	107.39	no	102.66	yes
99.88	101.34	102.57	103.79	108.11	no	97.55	yes
99.22	100.74	102.57	103.24	107.07	no	99.15	yes
101.14	102.40	102.57	104.75	107.39	no	101.39	yes
101.25	102.45	102.57	104.79	108.01	no	101.44	yes
99.32	100.37	102.57	102.91	106.56	no	101.36	yes

100.98	102.28	102.57	104.64	106.99	no	98.85	yes
99.72	100.93	102.57	103.42	104.97	no	101.84	yes
99.14	100.62	102.57	103.13	105.53	no	101.15	yes
100.93	102.33	102.57	104.68	108.86	no	103.02	yes
99.54	100.77	102.57	103.27	104.74	no	97.45	yes
100.12	101.36	102.57	103.80	104.88	no	103.43	yes
99.55	101.02	102.57	103.49	106.52	no	99.7	yes
99.76	101.12	102.57	103.58	110.00	no	102.79	yes
100.25	101.41	102.57	103.85	104.27	no	103.58	yes
100.13	101.31	102.57	103.76	106.73	no	97.97	yes
99.68	101.18	102.57	103.64	107.83	no	94.7	yes
99.16	100.21	102.57	102.76	104.05	no	99.46	yes
100.78	102.10	102.57	104.47	107.73	no	100.92	yes
99.71	100.80	102.57	103.29	106.33	no	101.76	yes
100.22	101.63	102.57	104.05	107.31	no	95.39	yes
99.80	100.77	102.57	103.26	104.58	no	103.05	yes
99.87	101.05	102.57	103.52	106.50	no	101.87	yes
99.07	99.99	102.57	102.56	102.36	yes	99.92	yes
100.11	101.60	102.57	104.02	107.44	no	97.7	yes
99.09	100.00	102.57	102.57	104.62	no	102.26	yes
100.55	101.90	102.57	104.30	106.89	no	100.67	yes
100.28	101.44	102.57	103.87	105.45	no	102.41	yes
99.33	100.46	102.57	102.99	104.17	no	101.49	yes
99.67	100.88	102.57	103.36	105.39	no	99.85	yes
100.17	101.14	102.57	103.61	106.38	no	102.29	yes
100.53	101.46	102.57	103.90	104.37	no	102.78	yes
99.78	101.11	102.57	103.57	106.34	no	99.95	yes
100.84	101.76	102.57	104.17	105.17	no	103.03	yes
99.44	100.79	102.57	103.28	108.00	no	101.42	yes
99.56	100.49	102.57	103.02	104.23	no	102.79	yes
101.12	102.30	102.57	104.66	106.85	no	103.29	yes
100.13	101.48	102.57	103.91	104.85	no	100.45	yes
99.14	100.12	102.57	102.68	104.34	no	102.34	yes
100.40	101.73	102.57	104.14	108.13	no	103.58	yes
99.61	100.68	102.57	103.18	105.91	no	101.65	yes
100.07	101.24	102.57	103.69	106.23	no	102.17	yes
99.51	100.91	102.57	103.39	107.42	no	97.23	yes
99.35	100.60	102.57	103.11	104.62	no	101.47	yes
100.40	101.87	102.57	104.26	108.76	no	100.54	yes
99.47	100.72	102.57	103.22	106.04	no	99.71	yes

100.27	101.38	102.57	103.82	107.36	no	102.35	yes
98.15	99.47	102.57	102.09	104.84	no	100.16	yes
99.72	101.07	102.57	103.54	106.11	no	99.92	yes
99.32	100.99	102.57	103.47	104.98	no	97.27	yes
99.49	100.82	102.57	103.31	105.04	no	99.74	yes
100.13	101.59	102.57	104.02	107.21	no	95.22	yes
99.12	100.33	102.57	102.86	105.66	no	99.27	yes
100.28	101.39	102.57	103.83	104.44	no	102.5	yes
101.00	102.32	102.57	104.68	105.27	no	101.36	yes
97.60	99.21	102.57	101.86	103.37	no	95.46	yes
99.85	101.59	102.57	104.01	109.93	no	101.61	yes
100.69	102.17	102.57	104.54	106.04	no	100.93	yes
100.68	101.85	102.57	104.24	106.06	no	102.82	yes
100.20	101.45	102.57	103.88	107.35	no	100.32	yes
100.79	102.06	102.57	104.44	106.79	no	101.04	yes
98.78	100.34	102.57	102.87	105.40	no	96.6	yes
100.07	101.42	102.57	103.86	106.00	no	97.95	yes
101.21	102.19	102.57	104.56	105.50	no	103.45	yes
99.41	100.64	102.57	103.15	104.55	no	97.33	yes
100.15	101.56	102.57	103.99	107.06	no	97.99	yes
99.25	100.62	102.57	103.13	107.42	no	99.39	yes
100.23	101.45	102.57	103.88	104.67	no	95.6	yes
100.54	101.66	102.57	104.07	107.38	no	100.66	yes
99.34	100.80	102.57	103.29	106.43	no	94.36	yes
99.92	101.09	102.57	103.56	104.35	no	100.25	yes
99.69	101.12	102.57	103.58	106.75	no	99.72	yes

Table 10 - Normal Data - Sample Size 50

xbar	ucl	c*(mu)	c*(ucl)	max	max<c*	ucl(p)	ucl(p)>cug
99.57	100.43	102.57	102.95	106.02	no	96.29	yes
100.38	101.19	102.57	103.65	105.81	no	95.83	yes
100.02	101.00	102.57	103.47	107.94	no	93.93	yes
100.45	101.32	102.57	103.77	107.12	no	97.09	yes
99.95	100.92	102.57	103.40	107.53	no	93.99	yes
99.48	100.29	102.57	102.84	105.73	no	97.45	yes
100.38	101.07	102.57	103.54	105.29	no	98.44	yes
100.06	101.07	102.57	103.53	108.79	no	93.82	yes
100.56	101.46	102.57	103.89	110.33	no	95.81	yes
100.68	101.55	102.57	103.98	106.31	no	94.63	yes
99.54	100.44	102.57	102.97	105.29	no	94.93	yes
99.78	100.60	102.57	103.11	104.14	no	100.11	yes
99.40	100.27	102.57	102.81	105.49	no	97.41	yes
99.20	100.15	102.57	102.71	106.12	no	97.23	yes
100.31	101.22	102.57	103.68	106.83	no	95.63	yes
101.18	101.97	102.57	104.36	108.28	no	93.62	yes
99.69	100.50	102.57	103.02	105.82	no	97.67	yes
99.55	100.36	102.57	102.89	104.24	no	97.66	yes
100.33	101.25	102.57	103.71	108.42	no	97.03	yes
100.16	100.96	102.57	103.44	105.36	no	95.56	yes
100.65	101.44	102.57	103.87	106.96	no	99.83	yes
100.11	100.75	102.57	103.25	105.36	no	99.35	yes
100.14	100.92	102.57	103.40	106.82	no	94.2	yes
100.01	100.82	102.57	103.31	105.73	no	92.58	yes
99.85	100.78	102.57	103.27	106.32	no	93.79	yes
99.78	100.81	102.57	103.30	109.87	no	92.2	yes
99.81	100.79	102.57	103.28	109.51	no	93.49	yes
99.87	100.77	102.57	103.26	107.27	no	93.88	yes
99.62	100.46	102.57	102.98	105.97	no	95.02	yes
100.67	101.59	102.57	104.01	109.32	no	95.98	yes
100.30	101.36	102.57	103.80	109.39	no	88.09	yes
99.70	100.60	102.57	103.11	106.50	no	95.04	yes
100.52	101.33	102.57	103.78	107.52	no	95.86	yes
100.28	101.22	102.57	103.68	106.25	no	92.76	yes

100.19	101.08	102.57	103.55	107.57	no	98.02	yes
100.17	101.08	102.57	103.55	108.18	no	95.47	yes
100.44	101.25	102.57	103.70	106.79	no	97.14	yes
100.45	101.30	102.57	103.75	106.00	no	98.51	yes
100.02	101.07	102.57	103.54	106.49	no	89.37	yes
99.75	100.63	102.57	103.14	105.72	no	96.52	yes
100.72	101.65	102.57	104.07	107.37	no	96.03	yes
99.93	100.81	102.57	103.30	106.19	no	92.38	yes
100.18	101.07	102.57	103.54	106.90	no	96.89	yes
99.24	100.11	102.57	102.67	108.00	no	97.16	yes
99.90	100.73	102.57	103.23	106.86	no	95.26	yes
100.53	101.37	102.57	103.81	105.96	no	97.25	yes
99.91	100.83	102.57	103.32	107.09	no	90.96	yes
100.78	101.73	102.57	104.14	107.31	no	96.1	yes
99.55	100.33	102.57	102.87	106.55	no	96.36	yes
100.51	101.52	102.57	103.95	108.34	no	94.35	yes
99.68	100.43	102.57	102.96	104.85	no	97.8	yes
99.51	100.48	102.57	103.00	106.04	no	94.91	yes
100.14	100.95	102.57	103.43	107.63	no	98.11	yes
99.88	100.63	102.57	103.14	106.05	no	94	yes
99.41	100.42	102.57	102.95	106.18	no	94.71	yes
100.01	100.75	102.57	103.25	105.13	no	100.29	yes
100.17	101.08	102.57	103.55	106.61	no	95.55	yes
99.69	100.51	102.57	103.03	106.03	no	95.1	yes
100.02	100.85	102.57	103.34	105.97	no	94.06	yes
100.86	101.76	102.57	104.16	107.26	no	97.54	yes
101.00	101.90	102.57	104.29	108.77	no	98.9	yes
100.64	101.46	102.57	103.89	106.70	no	97.41	yes
99.85	100.88	102.57	103.36	108.51	no	94.93	yes
100.04	100.96	102.57	103.44	109.26	no	96.7	yes
99.83	100.78	102.57	103.28	107.57	no	92.35	yes
99.79	100.71	102.57	103.21	107.18	no	96.42	yes
99.89	100.84	102.57	103.33	106.94	no	96.55	yes
99.38	100.47	102.57	102.99	109.53	no	91.7	yes
99.04	99.91	102.57	102.49	104.87	no	94.54	yes
99.33	100.15	102.57	102.70	105.38	no	94.8	yes
100.02	101.00	102.57	103.48	106.81	no	96.83	yes
100.96	101.76	102.57	104.17	108.77	no	96.27	yes
99.20	99.99	102.57	102.56	105.06	no	95.98	yes
100.23	101.15	102.57	103.61	109.00	no	95.48	yes

99.89	100.96	102.57	103.44	109.74	no	94.94	yes
99.60	100.29	102.57	102.84	105.06	no	98.84	yes
99.54	100.35	102.57	102.89	105.92	no	97.54	yes
99.94	100.82	102.57	103.31	105.65	no	90.95	yes
100.69	101.54	102.57	103.97	106.70	no	94.7	yes
100.05	100.92	102.57	103.40	108.44	no	93.98	yes
99.91	100.70	102.57	103.20	105.75	no	96.68	yes
100.29	100.99	102.57	103.47	106.20	no	99.51	yes
100.02	101.12	102.57	103.58	107.18	no	90.85	yes
100.38	101.31	102.57	103.75	107.48	no	95.66	yes
99.76	100.64	102.57	103.15	106.24	no	95.09	yes
100.19	101.11	102.57	103.58	108.71	no	95.51	yes
99.70	100.62	102.57	103.13	108.47	no	97.7	yes
100.24	100.98	102.57	103.46	106.31	no	99.47	yes
100.35	101.18	102.57	103.64	107.18	no	97.08	yes
99.38	99.98	102.57	102.55	104.50	no	99.66	yes
99.15	100.09	102.57	102.65	108.78	no	94.36	yes
99.53	100.50	102.57	103.02	106.37	no	94.82	yes
100.00	100.87	102.57	103.35	105.62	no	95.42	yes
99.26	100.03	102.57	102.60	105.30	no	93.32	yes
99.53	100.51	102.57	103.03	108.13	no	96.06	yes
100.37	101.27	102.57	103.72	109.15	no	97.05	yes
100.60	101.61	102.57	104.03	108.77	no	95.84	yes
99.78	100.76	102.57	103.25	108.07	no	93.52	yes
99.82	100.76	102.57	103.25	108.22	no	93.67	yes
99.80	100.69	102.57	103.19	106.37	no	92.34	yes

Table 11 - Normal Data - Sample Size 100

xbar	ucl	c*(mu)	c*(ucl)	max	max<c*	ucl(p)	ucl(p)>cug
99.81	100.39	102.57	102.92	106.40	no	95.65	yes
99.64	100.27	102.57	102.81	107.74	no	90.89	yes
99.89	100.47	102.57	103.00	107.27	no	93.44	yes
100.40	101.03	102.57	103.50	107.89	no	91.58	yes
100.17	100.82	102.57	103.31	108.19	no	89.76	yes
100.17	100.69	102.57	103.19	107.35	no	95.94	yes
99.81	100.37	102.57	102.91	108.05	no	92.66	yes
100.47	101.06	102.57	103.53	106.82	no	90.2	yes
99.62	100.26	102.57	102.81	106.09	no	92.47	yes
100.07	100.68	102.57	103.18	107.80	no	91.31	yes
100.57	101.15	102.57	103.61	108.29	no	94.09	yes
99.64	100.25	102.57	102.79	106.53	no	91.67	yes
100.04	100.58	102.57	103.09	105.97	no	91.39	yes
100.30	101.00	102.57	103.47	108.60	no	90.64	yes
99.82	100.49	102.57	103.01	109.08	no	94.05	yes
99.60	100.24	102.57	102.79	107.53	no	89.27	yes
100.36	100.97	102.57	103.45	109.26	no	93.77	yes
99.73	100.43	102.57	102.96	107.36	no	88.47	yes
99.88	100.44	102.57	102.97	106.83	no	92.74	yes
99.94	100.56	102.57	103.07	107.38	no	92	yes
100.40	101.02	102.57	103.49	109.71	no	92.38	yes
100.12	100.72	102.57	103.22	106.31	no	87.4	yes
99.52	100.14	102.57	102.69	106.14	no	90.9	yes
99.57	100.14	102.57	102.70	107.39	no	92.4	yes
100.22	100.86	102.57	103.35	109.08	no	91.37	yes
100.02	100.67	102.57	103.17	107.16	no	91.29	yes
99.53	100.18	102.57	102.73	109.11	no	91.55	yes
99.78	100.44	102.57	102.96	108.75	no	90.13	yes
99.87	100.58	102.57	103.10	108.48	no	86.3	yes
99.43	99.96	102.57	102.53	106.41	no	94.54	yes
100.70	101.31	102.57	103.76	108.17	no	93.51	yes
100.18	100.76	102.57	103.26	105.52	no	89.97	yes
99.61	100.27	102.57	102.81	106.28	no	90.96	yes
100.40	100.98	102.57	103.45	106.20	no	90.13	yes

100.06	100.72	102.57	103.22	107.38	no	92.07	yes
100.08	100.73	102.57	103.23	109.62	no	92.01	yes
99.71	100.40	102.57	102.93	109.20	no	88.47	yes
99.71	100.34	102.57	102.88	108.63	no	90.15	yes
100.45	101.12	102.57	103.58	107.91	no	87.57	yes
100.08	100.68	102.57	103.18	106.43	no	90.57	yes
100.25	100.86	102.57	103.35	107.46	no	93.06	yes
99.81	100.50	102.57	103.03	107.42	no	87.01	yes
100.12	100.73	102.57	103.23	109.02	no	92.11	yes
100.53	101.17	102.57	103.63	108.02	no	90.14	yes
99.69	100.32	102.57	102.86	109.13	no	91.66	yes
99.94	100.57	102.57	103.09	108.71	no	90.45	yes
100.52	101.15	102.57	103.61	108.00	no	90.2	yes
100.06	100.65	102.57	103.16	107.94	no	94.28	yes
99.77	100.39	102.57	102.92	110.70	no	93.28	yes
99.61	100.21	102.57	102.76	108.64	no	91.65	yes
99.94	100.61	102.57	103.12	108.72	no	89.61	yes
100.34	101.02	102.57	103.49	108.51	no	89.88	yes
100.36	101.04	102.57	103.51	112.53	no	93.82	yes
99.42	100.04	102.57	102.61	105.57	no	88.38	yes
99.86	100.52	102.57	103.04	109.35	no	90.3	yes
99.57	100.18	102.57	102.74	110.02	no	90.82	yes
99.60	100.24	102.57	102.79	109.91	no	93.12	yes
100.42	101.13	102.57	103.59	109.76	no	89.91	yes
100.37	100.94	102.57	103.42	108.50	no	95.38	yes
100.19	100.79	102.57	103.28	107.56	no	90.69	yes
99.56	100.16	102.57	102.72	108.32	no	92.34	yes
100.08	100.71	102.57	103.21	106.88	no	93.62	yes
100.22	100.82	102.57	103.31	107.82	no	94.53	yes
99.63	100.27	102.57	102.82	108.22	no	90.11	yes
99.70	100.28	102.57	102.83	106.16	no	92.52	yes
99.39	99.99	102.57	102.56	107.28	no	94.42	yes
100.44	101.06	102.57	103.53	107.48	no	90.86	yes
100.23	100.93	102.57	103.41	109.21	no	89.74	yes
100.58	101.27	102.57	103.72	109.92	no	88.46	yes
100.25	100.89	102.57	103.37	107.58	no	92.25	yes
100.09	100.71	102.57	103.22	106.56	no	90.53	yes
99.94	100.51	102.57	103.03	108.67	no	92.77	yes
100.22	100.87	102.57	103.35	106.86	no	89.87	yes
99.82	100.44	102.57	102.97	106.02	no	91.89	yes

100.27	100.87	102.57	103.36	105.99	no	89.18	yes
99.68	100.36	102.57	102.89	106.76	no	87.74	yes
100.25	100.85	102.57	103.33	107.13	no	89.88	yes
100.62	101.24	102.57	103.69	107.74	no	91.8	yes
99.71	100.36	102.57	102.89	108.69	no	90.9	yes
99.86	100.47	102.57	103.00	108.85	no	92.66	yes
100.09	100.81	102.57	103.30	109.95	no	90.38	yes
100.31	100.96	102.57	103.44	109.77	no	89.86	yes
100.07	100.66	102.57	103.17	108.23	no	91.3	yes
100.15	100.82	102.57	103.32	107.91	no	90.48	yes
100.00	100.56	102.57	103.08	105.73	no	92.13	yes
100.62	101.22	102.57	103.68	106.55	no	93.48	yes
100.01	100.68	102.57	103.19	108.44	no	89.67	yes
99.70	100.32	102.57	102.86	108.64	no	91.75	yes
99.95	100.56	102.57	103.07	107.20	no	90.41	yes
100.21	100.81	102.57	103.31	106.75	no	93.77	yes
99.76	100.34	102.57	102.88	107.04	no	93.33	yes
100.21	100.84	102.57	103.33	109.65	no	93.69	yes
99.95	100.58	102.57	103.10	108.70	no	91.96	yes
99.67	100.37	102.57	102.90	109.77	no	89.24	yes
100.13	100.75	102.57	103.25	107.92	no	93.7	yes
100.12	100.76	102.57	103.26	108.06	no	90.5	yes
100.45	101.08	102.57	103.55	107.62	no	91.64	yes
99.69	100.40	102.57	102.93	109.13	no	89.97	yes
99.71	100.32	102.57	102.86	108.33	no	91.72	yes
100.58	101.16	102.57	103.62	106.98	no	94.89	yes

APPENDIX II

FORTRAN PROGRAMS

Program 1 - Uniform Data -Sample Size 25

```
      program main

c     Define variables as real, integer, or character.

      real czero,alpha,r(25),s(25),cstar,bsq,asq,czsq,ucl,xbar
      real sd,sumdev,max1
      integer a,b,y,z,x,d,e
      character*12 file1

c     Start naming data files.

      file1(1:5) = 'w(25)'
      file1(9:12) = '.dat'

c     Set parameter values.

      czero=10
      a=0
      b=200

c     Enter value for alpha (alpha=0.8)

      write (6,*) 'enter a value for alpha'
      read (5,*) alpha

c     Open output file

      open(unit=2,file='v(25).dat')
      write (2,*) 'xbar ucl c* max max<c*'

c     Start main do loop for data file generation
```

```
      do i = 1,100
      write(file1(6:8),fmt='(i3.3)') i
      open(unit=1,file=file1,status='new')

c       Generate unif(a,b)
      call rnun (25,r)
      y=(b-a)
      do 10 z=1,25,1
      s(z)=(y*r(z))+a
10    continue

c       Call subroutine to calculate UCL

      call uclimit(s,ucl,xbar)
c       Calculate maximum of the data set

      max1=0
      do 12 y=1,25,1
      max1=max(max1,s(y))
12    continue

c       Calculate c*

      bsq=((b)**2)
      asq=((a)**2)
      czsq=((czero)**2)
      cstar=czero+(sqrt(czsq-2*czero*b+asq+alpha*(bsq-asq)))
      if (max1.lt.cstar) then
      write (2,14) xbar,ucl,cstar,max1
14    format(' ',4f10.2,' yes')
      else
      write (2,15) xbar,ucl,cstar,max1
15    format(' ',4f10.2,' no')
      endif

c       Output data to file

      write (1,*) 'c'

      do 20 z=1,25,1
      write (1,16) s(z)
16    format(' ',f10.2)
20    continue
```

```
close(unit=1)

enddo
close(unit=2)
end
```

Program 1 - Subroutine 1

```
c       Calculate ucl

      subroutine uclimit(s,ucl,xbar)
      real czero,alpha,r(25),s(25),cstar,bsq,asq,czsq,ucl,xbar
      real sd,sumdev
      integer a,b,y,z,x,d,e

c       Calculate sample mean

      xbar=0
      do 12 d=1,25,1
      xbar=xbar+s(d)
 12   continue
      xbar=xbar/25

c       Calculate sample standard deviation

      sumdev=0
      do 13 e=1,25,1
      sumdev=sumdev+(s(e)-xbar)**2
 13     continue
      sd=sqrt(sumdev/24)

c       Calculate UCL for mean

      ucl=xbar+1.96*sd/5

      return
      end
```

Program 2 - Uniform Data - Sample Size 25

```
program main
    real s(25),t(25),bsq,asq,czsq,czero,cstar,alpha
    real ucl,xbar
    integer z,y
    character*3 c
    character*12 file1
    file1(1:5) = 'w(25)'
    file1(9:12) = '.dat'
    open(unit=2,file='pu25.dat')
    write(2,*)'ucl(p) ucl(p)>cug'
c      Calculate c*

    alpha=.8
    czero=10
    bsq=((200)**2)
    asq=((0)**2)
    czsq=((czero)**2)
    cstar=czero+(sqrt(czsq-2*czero*200+asq+alpha*(bsq-asq)))

c      Read data from files

    do i = 1,100
    write(file1(6:8),fmt='(i3.3)') i
    open(unit=1,file=file1,status='old')
    read(1,5) c
5   format(a1)
    do 20 z=1,25,1
    read(1,10) s(z)
10   format(f10.2)
20   continue

c      Remediate samples

    do 25 z=1,25,1
    if (s(z) .ge. cstar) then
    t(z)=czero
    else
    t(z)=s(z)
    endif
25  continue
```

```
c       Calculate ucl for posterior distribution

        call uclimit(t,ucl,xbar)

c       Determine whether ucl>cug

        if (ucl.gt.80) then
        write(2,35) ucl
35      format(' ',f10.2,'  yes')
        else
        write(2,36) ucl
36      format(' ',f10.2,'  no')
        endif

        close(1)
        enddo
        close(2)
        end
```

Program 1 - Normal Data - Sample Size 25

```
      program main

c     Define variables as real integer, or character

      real czero,r(25),s(25),cstar,ucl,xbar,a,mval,max1
      real sd,sumdev
      integer z
      character*12 file1

c     Start naming data files

      file1(1:5) = 'w(25)'
      file1(9:12) = '.dat'

c     Set parameter values

      write (6,*) 'enter a value for cug'
      read (5,*) cug
      write (6,*) 'enter a value for czero'
      read (5,*) czero
      write(6,*)'enter a value for the lower limit of cstar'
      read(5,*) llim
      write(6,*)'enter a value for the upper limit of cstar'
      read(5,*) hlim

c     Open output file

      open(unit=2,file='v(25).dat')
      write (2,*) 'xbar ucl cstar(mu) cstar(ucl) max max<cstar'

c     Start main do loop for data file generation

      do i = 1,100
      write(file1(6:8),fmt='(i3.3)') i
      open(unit=1,file=file1,status='new')

c     Generate normal(mu=100,sd=10)

      call rnnor (25,r)
```

```
      a=sqrt(10)
      do 10 z=1,25,1
      s(z)=(a*r(z)+100)
 10   continue
c        Calculate ucl

      call uclimit(s,ucl,xbar)

c        Calculate maximum of the data set

      max1=0
      do 12 y=1,25,1
      max1=max(max1,s(y))
 12   continue

c        Calculate c*

      call cstarcalc(llim,hlim,cstar,cug,czero,mrslt,mval)
      call cstarcalc2(llim,hlim,cstarhi,cug,ucl,czero)
      if (max1.lt.mval) then
      write (2,14) xbar,ucl,mval,cstarhi,max1
 14   format(' ',5f10.2,'  yes')
      else
      write (2,15) xbar,ucl,mval,cstarhi,max1
 15   format(' ',5f10.2,'  no')
      endif

      write (1,*) 'c'
      do 20 z=1,25,1
      write (1,16) s(z)
 16   format(' ',f10.2)
 20   continue

      close(unit=1)

      enddo
      close(unit=2)
      end
```

Program 1 - Subroutine 1

```
c       Calculate ucl

      subroutine uclimit(s,ucl,xbar)
      real czero,r(25),s(25),cstar,ucl,xbar,t,p,df
      real sd,sumdev
      integer d,e


c       Calculate sample mean

      xbar=0
      do 12 d=1,25,1
      xbar=xbar+s(d)
12    continue
      xbar=xbar/25

c       Calculate sample standard deviation

      sumdev=0
      do 13 e=1,25,1
      sumdev=sumdev+(s(e)-xbar)**2
13    continue
      p=.975
      df=24
      t=tin(p,df)
      sd=sqrt(sumdev/25)

c       Calculate ucl
      ucl=xbar+(t*sd/sqrt(25))

      return
      end
```

Program 1 - Subroutine 2

```
c       Calculate c* using true mean

        subroutine cstarcalc(llim,hlim,cstar,cug,czero,mrslt,mval)
        real czero,cstar,ucl,xbar,sd,sumdev,cug
        real llim,hlim,mrslt,mval,a,b,c,d,e,f,g
        integer loopval

        cstar=llim

c       Start loop to calculate best c*

        loopval=0
        do while(cstar.le.hlim)
        a=sqrt(10)
        b=100-czero
        c=(cstar-100)/a
        d=anordf(c)
        e=(1/sqrt(2*3.14159265))*(exp((-1/2)*(c**2)))
        f=czero+b*d-a*e
        g=abs(f-cug)
        loopval=loopval+1
        if (loopval.eq.1) then
        mval=cstar
        mrslt=g
        else if (g.lt.mrslt) then
        mval=cstar
        mrslt=g
        end if
        cstar=((llim*100)+loopval)/100
        enddo
        write(6,*) mrslt, mval

        return
        end
```

Program 1 - Subroutine 3

```
c       Calculate c* using the ucl of the mean

        subroutine cstarcalc2(llim,hlim,cstarhi,cug,ucl,czero)
        real czero,cstar,ucl,xbar,sd,sumdev,cug,cstarhi
        real llim,hlim,mrslt,mval,a,b,c,d,e,f,g
        integer loopval
        cstar=llim

c       Start loop to calculate best c*

        loopval=0
        do while(cstar.le.hlim)
        a=sqrt(10)
        b=ucl-czero
        c=(cstar-ucl)/a
        d=anordf(c)
        e=(1/sqrt(2*3.14159265))*(exp((-1/2)*(c**2)))
        f=czero+b*d-a*e
        g=abs(f-cug)
        loopval=loopval+1
        if (loopval.eq.1) then
        cstarhi=cstar
        mrslt=g
        else if (g.lt.mrslt) then
        cstarhi=cstar
        mrslt=g
        end if
        cstar=((llim*100)+loopval)/100
        enddo
        write(6,*) mrslt, cstarhi

        return
        end
```

Program 2 - Normal Data - Sample Size 25

```
      program main
      real s(25),t(25),d(100)
      real ucl,xbar
      integer z,y,czero
      character*10 c
      character*13 file1
      file1(1:5) = 'w(25)'
      file1(9:12) = '.dat'
      open(unit=2,file='pe25.dat')
      write(2,*)'ucl(p) ucl(p)>80'

c        Read c* values from file

      open(unit=3,file='cs(ucl).txt',status='old')
      do 3 y=1,100,1
      read(3,2) d(y)
2      format(f10.2)
3      continue

      czero=10

c        Read data from files

      do i = 1,100
      write(file1(6:8),fmt='(i3.3)') i
      open(unit=1,file=file1,status='old')
      read(1,5) c
5     format(a1)
      do 20 z=1,25,1
      read(1,10) s(z)
10    format(f10.2)
20    continue

c        Remediate samples
      do 25 z=1,25,1
      if (s(z) .ge. d(i)) then
      t(z)=czero
      else
      t(z)=s(z)
```

```
      endif

   25 continue

c        Calculate ucl for posterior distribution

      call uclimit(t,ucl,xbar)
      if (ucl.gt.80) then
      write(2,35) ucl
   35 format(' ',f10.2,'  yes')
      else
      write(2,36) ucl
   36 format(' ',f10.2,'  no')
      endif

      close(1)

      enddo
      close(2)
      close(3)
      end
```

Program 2 - Subroutine 1

```
c       Calculate ucl of the posterior mean

        subroutine uclimit(t,ucl,xbar)
        real t(25),ucl,xbar
        real sd,sumdev
        integer d,e

c       Calculate sample mean

        xbar=0
        do 12 d=1,25,1
        xbar=xbar+t(d)
12      continue
        xbar=xbar/25


c       Calculate sample standard deviation

        sumdev=0
        do 13 e=1,25,1
        sumdev=sumdev+(t(e)-xbar)**2
13      continue
        sd=sqrt(sumdev/24)
        ucl=xbar+1.96*sd/sqrt(25)

        return
        end
```

Program 1 - Exponential Data - Sample Size 25

```
      program main

c       Define variables as real, integer, or character

      real r(25),s(25),czero,cstar,ucl,xbar,sd,sumdev,cug,max1
      real llim,hlim,mval,cstarhi
      integer z,theta
      character*12 file1

c       Start naming data files

      file1(1:5) = 'w(25)'
      file1(9:12) = '.dat'
      open(unit=2,file='v(25).dat')
      write (2,*) 'xbar ucl cstar(theta) cstar(ucl) max max<cstar'

c       Set parameter values

      write (6,*) 'enter a value for cug'
      read (5,*) cug
      write (6,*) 'enter a value for czero'
      read (5,*) czero
      write(6,*)'enter a value for the lower limit of cstar'
      read(5,*) llim
      write(6,*)'enter a value for the upper limit of cstar'
      read(5,*) hlim

c       Start main do loop for data generation

      do i = 1,100
      write(file1(6:8),fmt='(i3.3)') i
      open(unit=1,file=file1,status='new')

c    generate exponential(100)

      call rnexp (25,r)
      theta=100
      do 10 z=1,25,1
      s(z)=(100*r(z))
```

```
10  continue

c       Calculate ucl

    call uclimit(s,ucl,xbar)

c       Calculate maximum of the data set

    max1=0
    do 12 y=1,25,1
    max1=max(max1,s(y))
12   continue

c       Calculate c* using true mean and ucl of the mean

    call cstarcalc(llim,hlim,cstar,theta,cug,czero,mrslt,mval)
    call calc2(llim,hlim,cstarhi,cug,ucl,czero)
    if (max1.lt.cstarhi) then
    write (2,14) xbar,ucl,mval,cstarhi,max1
14  format(' ',5f10.2,'  yes')
    else
    write (2,15) xbar,ucl,mval,cstarhi,max1
15  format(' ',5f10.2,'  no')
    endif

c       Output data to file

    write (1,*) 'c'
    do 20 z=1,25,1
    write (1,16) s(z)
16  format(' ',f10.2)
20  continue

    close(unit=1)
    enddo
    close(unit=2)
    end
```

Program 1 - Subroutine 1

```
c       Calculate ucl

        subroutine uclimit(s,ucl,xbar)
        real czero,alpha,r(25),s(25),cstar,ucl,xbar
        real sd,sumdev
        integer a,b,y,z,x,d,e

c       Calculate sample mean

        xbar=0
        do 12 d=1,25,1
        xbar=xbar+s(d)
12      continue
        xbar=xbar/25

c       Calculate sample standard deviation

        sumdev=0
        do 13 e=1,25,1
        sumdev=sumdev+(s(e)-xbar)**2
13      continue
        sd=sqrt(sumdev/24)
        ucl=xbar+1.96*sd/sqrt(25)

        return
        end
```

Program 1 - Subroutine 2

```
c       Calculate c* using the true mean

        subroutine calc2(llim,hlim,cstarhi,cug,ucl,czero)
        real r(25),s(25),czero,cstar,ucl,xbar,sd,sumdev,cug
        real llim,hlim,mrslt,mval,a,b,c,d,f,cstarhi
        integer theta,loopval

        cstar=llim

c       Start loop to calculate best c*

        loopval=0
        do while(cstar.le.hlim)
        a=(cug-ucl)
        b=(czero-cstar-ucl)
        c=exp(-cstar/ucl)
        d=b*c
        f=abs(d-a)
        loopval=loopval+1
        if (loopval.eq.1) then
        cstarhi=cstar
        mrslt=f
        else if (f.lt.mrslt) then
        cstarhi=cstar
        mrslt=f
        end if
        cstar=((llim*100)+loopval)/100
        enddo
        write(6,*) mrslt, cstarhi

        return
        end
```

Program 1 - Subroutine 3

```
c       Calculate c* using the ucl of the mean

        subroutine cstarcalc(llim,hlim,cstar,theta,cug,czero,mrslt,mval)
        real r(25),s(25),czero,cstar,ucl,xbar,sd,sumdev,cug
        real llim,hlim,mrslt,mval,a,b,c,d,f
        integer theta,loopval


        cstar=llim

c       Start loop to calculate best c*

        loopval=0
        do while(cstar.le.hlim)
        a=(cug-theta)
        b=(czero-cstar-theta)
        c=exp(-cstar/theta)
        d=b*c
        f=abs(d-a)
        loopval=loopval+1
        if (loopval.eq.1) then
        mval=cstar
        mrslt=f
        else if (f.lt.mrslt) then
        mval=cstar
        mrslt=f
        end if
        cstar=((llim*100)+loopval)/100
        enddo
        write(6,*) mrslt, mval

        return
        end
```

Program 2 - Exponential Data - Sample Size 25

```
      program main
      real s(25),t(25),d(100)
      real ucl,xbar
      integer z,y,czero
      character*10 c
      character*13 file1
      file1(1:5) = 'w(25)'
      file1(9:12) = '.dat'
      open(unit=2,file='pe25.dat')
      write(2,*)'ucl(p) ucl(p)>80'

c     Read c* values from file

      open(unit=3,file='cs(ucl).txt',status='old')

      do 3 y=1,100,1
      read(3,2) d(y)
2      format(f10.2)
3      continue

      czero=10

c     Read data from files

      do i = 1,100
      write(file1(6:8),fmt='(i3.3)') i
      open(unit=1,file=file1,status='old')
      read(1,5) c
5     format(a1)
      do 20 z=1,25,1
      read(1,10) s(z)
10    format(f10.2)
20    continue


c     Remediate samples

      do 25 z=1,25,1
      if (s(z) .ge. d(i)) then
```

```
      t(z)=czero
      else
      t(z)=s(z)
      endif

25    continue

c       Calculate ucl for posterior distribution

      call uclimit(t,ucl,xbar)
      if (ucl.gt.80) then
      write(2,35) ucl
35    format(' ',f10.2,'  yes')
      else
      write(2,36) ucl
36    format(' ',f10.2,'  no')
      endif

      close(1)

      enddo
      close(2)
      close(3)
      end
```

Program 2 - Subroutine 1

```
c       Calculate ucl of the posterior mean

        subroutine uclimit(t,ucl,xbar)
        real t(25),ucl,xbar
        real sd,sumdev
        integer d,e

c       Calculate the sample mean

        xbar=0
        do 12 d=1,25,1
        xbar=xbar+t(d)
12      continue
        xbar=xbar/25

c       Calculate sample standard deviation

        sumdev=0
        do 13 e=1,25,1
        sumdev=sumdev+(t(e)-xbar)**2
13      continue

        sd=sqrt(sumdev/24)
        ucl=xbar+1.96*sd/sqrt(25)

        return
        end
```

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