A three-dimensional finite element model for diagnostic windfield assessment

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A 3-Dimensional Finite Element Model

for

Diagnostic Windfield Assessment

by

Zijiang Shi

A thesis submitted in partial fulfillment of the requirement for the degree of

Master of Science

in

Mechanical Engineering

Department of Mechanical Engineering
University of Nevada, Las Vegas
December, 1996
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University of Nevada, Las Vegas  
December, 1996
ABSTRACT

A 3-dimensional model that calculates windfields over irregular terrain based on meteorological tower data has been developed. The governing equations of atmospheric motion, the finite element method and atmospheric boundary layer concept have been introduced. A surface model of the Nevada Test Site (NTS) has been generated from DEM files which contain elevation data, and a 3-D mesh has been generated. Initial estimates of the velocity field are developed by interpolating surface and upper level wind measurements. A diagnostic objective analysis technique is used to generate 3-D winds above the surface layer. This algorithm incorporates NTS tower field measurements to initialize and generate the upper level windfield. A surface boundary layer technique is used to calculate the upper level windfield. Vertical velocities are developed from successive solutions of the continuity equation, followed by an iterative procedure which reduces divergence over the complete field.

The finite element method is used to solve a Poisson equation which is used to adjust the velocity components. The upper and lateral boundaries above the topography are assumed to be open, allowing mass flow through the boundaries. The bottom boundary is set by the topographic elevations of NTS region, and is assumed to be solid. Major advantages of the procedure are that it is computationally efficient and allows boundary values to adjust in response to changes in the interior flow. The method has

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been successfully tested using a 3-D cavity problem and using NTS field measurements. The correctness of this method was verified by testing 3-D cavity model. Results from NTS model were compared with the Mathew diagnostic model used by Lawrence Livermore National Laboratory (LLNL). The influence of windfield by tower data was also discussed. The NTS model formulation can readily be extended to include other relevant physical and dynamic solution constraints such as momentum and energy conservation.
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ACKNOWLEDGEMENTS

I would like to express special thanks to Dr. Darrell W. Pepper, who served as my advisor for this research. I would also like to thank Mr. David B. Carrington for his help throughout the course of this project.

Support from CIASTA is gratefully acknowledged. I would also like to thank Dr. Darryl Randerson, Dr. Marc Pitchford and Mr. Doug Soule from National Oceanic and Atmospheric Administration (NOAA) for their help.
CHAPTER 1

INTRODUCTION

Prediction of atmospheric flows and accompanying precipitation has been a consuming interest for the geophysical community for many years. Accurate prediction of windfields is vitally important in order to forecast both normal conditions as well as consequences associated with severe weather. Atmospheric modeling is difficult, not only from the mathematical and numerical aspects, but also because model validation is limited. Measurements of atmospheric flow are sparse and generally insufficient to resolve important flow phenomena on both microscale and mesoscale levels. This problem is particularly apparent in regions where complex terrain exists.

Many observational and theoretical studies have been made of airflow over complex terrain (Klemp and Lilly, 1977; Warner et al., 1977; Pepper and Baker, 1980; Chan et al., 1982; Pielke et al., 1983). Most atmospheric models are finite difference / finite volume based numerical approaches. Application of the finite element method for atmospheric flow simulation has seen only limited use. Early mesoscale flow predictions with finite elements are discussed in Chang et al. (1982), Paegle and McLawdon(1983), Lee et al.(1983), and Pepper and Brueckner(1992).

Numerous studies have been conducted to model and experimentally verify airflow
and diffusion in complex terrain. Experimental data are largely inadequate due to locations in which the data are obtained, the sparsity of data, the data collection methods, and the costs (Dickerson and Gudiksen, 1980).

In this study, a diagnostic windfield model is developed for the Nevada Test Site, which is located in Nevada (see Fig. 1.1). Experimental data is achieved and accessed through Digital Elevation Map (DEM) files which include elevation data of Nevada Test Site. Velocity data are obtained from meteorological towers located in the Test Site (see Fig. 1.2). The numerical algorithm builds on a proven concept for calculating fluid flow over complex surfaces utilizing sparse data obtained from measurements. An adjustment windfield model is developed based on a preliminary terrain following model developed by Pepper (1990).

The goal in this study is to verify and test a diagnostic numerical model for calculating 3-D wind flow over the Nevada Test Site Region using a 3-D finite element procedure.
Figure 1.1 Picture of Nevada with Location of Nevada Test Site

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Figure 1.2 2-D Topographic Contours of NTS with Tower Locations

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CHAPTER 2

NUMERICAL WINDFLOW PREDICTION

Governing Equations of Fluid Dynamics

The mathematical relations which describe atmospheric motion are the conservation of mass, momentum, energy, and species transport. The governing equations for three-dimensional atmospheric flow are based on (Pielke, 1984):

Conservation of Mass:
\[
\frac{\partial p}{\partial t} + \frac{\partial p u}{\partial x} + \frac{\partial p v}{\partial y} + \frac{\partial p w}{\partial z} = 0
\]  
(2.1)

Conservation of Momentum:

x-direction:
\[
\frac{\partial p u}{\partial x} + \frac{\partial p u^2}{\partial x} + \frac{\partial p u v}{\partial y} + \frac{\partial p u w}{\partial z} = \frac{\partial}{\partial x}\left(k_i \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_i \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_i \frac{\partial u}{\partial z}\right) + 2\nu \omega \sin \psi - 2w \omega \cos \psi - \frac{\partial p}{\partial x} 
\]  
(2.2)

y-direction:
\[
\frac{\partial p v}{\partial x} + \frac{\partial p u v}{\partial y} + \frac{\partial p v^2}{\partial y} + \frac{\partial p w v}{\partial z} = \frac{\partial}{\partial x}\left(k_i \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_i \frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_i \frac{\partial v}{\partial z}\right) + \frac{\partial}{\partial x}\left(k_i \frac{\partial v}{\partial x}\right) - 2w \omega \sin \psi - \frac{\partial p}{\partial y} 
\]  
(2.3)

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z-direction:

\[
\frac{\partial \rho w}{\partial t} + \frac{\partial \rho u w}{\partial x} + \frac{\partial \rho v w}{\partial y} + \frac{\partial \rho w^3}{\partial z} = \frac{\partial}{\partial x} \left( k_h \frac{\partial w}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_h \frac{\partial w}{\partial y} \right) +
\]

(2.4)

\[
\frac{\partial}{\partial z} \left( k_z \frac{\partial w}{\partial z} \right) - g - \frac{\partial p}{\partial z}
\]

Conservation of Energy:

\[
\frac{\partial \theta}{\partial t} + \frac{\partial u \theta}{\partial x} + \frac{\partial v \theta}{\partial y} + \frac{\partial w \theta}{\partial z} = \frac{\partial}{\partial x} \left( k_h \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_h \frac{\partial \theta}{\partial y} \right) +
\]

(2.5)

\[
\frac{\partial}{\partial z} \left( k_z \frac{\partial \theta}{\partial z} \right) + \left( \frac{\partial \theta}{\partial t} \right)_{\text{rad}}
\]

Specific Humidity:

\[
\frac{\partial q}{\partial t} + \frac{\partial u q}{\partial x} + \frac{\partial v q}{\partial y} + \frac{\partial w q}{\partial z} = \frac{\partial}{\partial x} \left( k_q \frac{\partial q}{\partial x} \right)
\]

(2.6)

Species Transport:

\[
\frac{\partial C_m}{\partial t} + \frac{\partial u C_m}{\partial x} + \frac{\partial v C_m}{\partial y} + \frac{\partial w C_m}{\partial z} = \frac{\partial}{\partial x} \left( k_h \frac{\partial C_m}{\partial x} \right) +
\]

(2.7)

\[
\frac{\partial}{\partial y} \left( k_h \frac{\partial C_m}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial C_m}{\partial z} \right) + S_{C_m}
\]

where u, v, w are the east-west (x), north-south (y), and vertical (z) components of velocity, respectively; \( \omega \) is the angular velocity of the earth; \( \psi \) is the latitude; \( \theta \) is the potential temperature; \( q \) is the specific humidity; \( C_m \) is the species concentration; \( g \) is the acceleration of gravity; \( [\partial \theta / \partial t]_{\text{rad}} \) is the radiative cooling / heating of the atmosphere; \( S_{C_m} \) is the source / sink term which includes changes of state, chemical transformations, precipitation, and sedimentation; \( k_h \) is the horizontal diffusion coefficient; and \( k_z \) is vertical
The potential temperature is defined as:
\[ \theta = T_v \left( \frac{1000}{p} \right)^{R_v/c_p} \]  
(2.8)

where the pressure \( p \) is in mb, \( T_v \) is virtual temperature, \( c_p \) is the specific heat at constant pressure, and \( R_v \) is the universal gas constant. The ideal gas law can be written as:
\[ p = \rho R_d T_v \]  
(2.9)

where the density \( \rho \) is defined as the inverse of specific volume. Virtual temperature is defined as:
\[ T_v = T(1 + 0.61 q) \]  
(2.10)

The pressure can either be obtained from solution of the "discrete" momentum equations and a simple Poisson equation, or from the hydrostatic assumption (normally used in many atmospheric models). A potential function, which is solved from the Poisson equation, is used to adjust the velocity components (Pepper and Breckner, 1992).

The modeling of turbulence and resulting forms of closure are quite varied; the gradient diffusion approach is typically used (Pepper and Breckner, 1992). Horizontal mixing is approximated using the relation proposed by Smagorinsky et al. (1965) and later used by Anthes and Warner (1978), e.g., the intensity of horizontal mixing is related to the strength of the horizontal wind shear, i.e.,
\[ k_h = \frac{1}{2} k_0^2 h_e^2 \left[ \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right]^{1/2} \]  
(2.11)

where \( h_e \) is the average element length and \( k_0 \) is von Karman's constant.
The vertical exchange coefficients of momentum, heat, and moisture are given in the surface layer by the relations:

\[ k_t = \frac{k_0 u^* z}{\phi_m(\zeta)} \]  
\[ k_h = k_q = \frac{k_0 u^* z}{\phi_H(\zeta)} \]  

where \( u^* \) is the friction velocity; \( \phi_m \) is the nondimensional wind profile; \( \phi_H \) is the mean vertical temperature profile of the surface layer; and \( \zeta = z/L \), where \( L \) is Monin-Obukhov length. According to Businger et al. (1971), the expressions for the nondimensional wind and potential temperature profiles are

\[ \phi_m = \begin{cases} (1 - 15\zeta)^{-1/4} & \zeta \leq 0 \\ (1 - 4.7\zeta) & \zeta > 0 \end{cases} \]  
\[ \phi_H = \begin{cases} 0.74(1 - 9\zeta)^{-1/2} & \zeta \leq 0 \\ 0.74 + 4.7\zeta & \zeta > 0 \end{cases} \]  

Above the surface layer, the exchange coefficients are defined according to McNider and Pielke (1981). The coefficients depend on the thermodynamical stability of the surface layer. When this layer is stable, local exchange coefficients suggested by Blackadar (1979) are used. When the surface layer is unstable, the profile function of O'Brien (1970) is used.

The Finite Element Method

While the derivations of the governing equations for most fluid flow problems are

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not unduly difficult, their solutions by exact methods of analysis are a formidable task. In such cases, approximate methods of analysis provide alternate means of finding solutions. Among these, the finite difference method, the variational method, and the finite element method are most frequently used.

As in simple finite difference schemes, the finite element method requires a problem defined in geometrical space (or domain) to be subdivided into a finite number of smaller regions (a mesh). In finite differences, the mesh consists of rows and columns of orthogonal lines; in finite elements, each subdivision is unique and need not be orthogonal. For example, triangles or quadrilaterals can be used in two dimensions, and tetrahedrons or hexahedrons in three dimensions. Over each finite element, the unknown variables (e.g., temperature, velocity, etc.) are approximated using known functions. These functions can be linear or higher-order polynomial expansions that depend on the geometrical locations (nodes) used to define the finite element shape. In contrast to finite difference procedures (conventional finite difference discretizations, as opposed to the finite volume method, which is integrated), the governing equations in the finite element method are integrated over each finite element and the solution summed ("assembled") over the entire problem domain. As a consequence of these operations, a set of finite linear equations is obtained in terms of a set of unknown parameters over each element. Solution of these equations is achieved using linear algebra techniques (Pepper and Heinrich, 1992).

The finite element method has its formal basis in the Galerkin procedure of weighted residuals. The Galerkin method is simple to use and is guaranteed to yield a
compatible approximation to the governing differential equation. In the Galerkin method, the dependent variable is expressed by means of a finite series approximation in which the "shape" of the solution is assumed known, and depends on a finite number of parameters to be determined. Replaced in the governing differential equation, the approximation generates a residual function, which is multiplied by weighting functions and is required to be orthogonal to the weighting functions in the integrated sense, i.e.,

\[ \int W(x) R(\Phi, x) d(x) = 0 \]  

(2.16)

where \( \Phi \) is the unknown variable, \( R(\Phi, x) \) is the residual error function (the function obtained when the approximation to the exact solution of \( \Phi^* \) is replaced in the differential equation), \( x \) is the length coordinate, and \( W(x) \) is the weight. A set of linear algebraic equations can be generated from these expressions that allows to determine the unknown parameter, \( \Phi \), and hence an approximation to the solution (Pepper and Heinrich, 1992).

Formulation of 3-D Shape Functions

Eight nodes are used to define the trilinear, three-dimensional hexahedron. The hexahedron is shown in Figure 2.1.

![Figure 2.1 Eight-Noded Trilinear Hexahedron Element](image)

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The interpolation function can be written as:

\[ \phi = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 z + \alpha_5 xy + \alpha_6 xz + \alpha_7 yz + \alpha_8 xyz \]  

(2.17)

Equation (2.17) can be expressed in matrix form using \( \phi = C\alpha \), as

\[
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\phi_4
\end{bmatrix}
= 
\begin{bmatrix}
x_1 & y_1 & z_1 & x_1y_1 & x_1z_1 & y_1z_1 & x_1y_1z_1 & \alpha_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
x_4 & y_4 & z_4 & x_4y_4 & x_4z_4 & y_4z_4 & x_4y_4z_4 & \alpha_8
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4 \\
\alpha_5 \\
\alpha_6 \\
\alpha_7 \\
\alpha_8
\end{bmatrix}
\]

(2.18)

Since \( \alpha = C^T \phi \), we can find the values for \( \phi \) from expression

\[ \phi = \begin{bmatrix} 1 & x & y & z & xy & xz & yz & xyz \end{bmatrix} C^T \phi \]

(2.19)

and the shape functions are obtained:

\[ N = \begin{bmatrix} 1 & x & y & z & xy & xz & yz & xyz \end{bmatrix} C^T \]

(2.20)

The element in physical coordinate space \((x,y,z)\) is transformed to natural coordinate space by the use of isoparametric transformation, as shown in Figure 2.2:

![Figure 2.2 Natural Coordinate System For The Hexahedron Element](image)

Figure 2.2 Natural Coordinate System For The Hexahedron Element

\( x, y \) and \( z \) are expressed as functions of \( \xi, \eta \) and \( \zeta \):
\[ x = x(\xi, \eta, \zeta) \]
\[ y = y(\xi, \eta, \zeta) \]
\[ z = z(\xi, \eta, \zeta) \] (2.21)

where \(-1 \leq \xi \leq 1\), \(-1 \leq \eta \leq 1\), and \(-1 \leq \zeta \leq 1\) with \(\xi = \frac{x}{a}\), \(\eta = \frac{y}{b}\) and \(\zeta = \frac{z}{c}\) (for brick).

The shape functions are also expressed in terms of \(\xi\), \(\eta\) and \(\zeta\):

\[ N_i = N_i(\xi, \eta, \zeta) \] (2.22)

The shape functions for an isoparametric trilinear hexahedron element are defined as (Pepper and Heinrich, 1992):

\[
\begin{bmatrix}
N_1 \\
N_2 \\
N_3 \\
N_4 \\
N_5 \\
N_6 \\
N_7 \\
N_8 \\
\end{bmatrix} = \begin{bmatrix}
(1-\xi)(1-\eta)(1-\zeta) \\
(1+\xi)(1-\eta)(1-\zeta) \\
(1-\xi)(1+\eta)(1-\zeta) \\
\frac{1}{8} \\
(1-\xi)(1-\eta)(1+\zeta) \\
(1+\xi)(1-\eta)(1+\zeta) \\
(1+\xi)(1+\eta)(1+\zeta) \\
(1-\xi)(1+\eta)(1+\zeta) \\
\end{bmatrix}
\] (2.23)

Equation (2.23) can be written in more concise form as:

\[ N_i = \frac{1}{8}(1 + \xi \xi_i)(1 + \eta \eta_i)(1 + \zeta \zeta_i) \] (2.24)

where \(i = 1, 2, \ldots, 8\); \(\xi_i\), \(\eta_i\), and \(\zeta_i = \pm 1\), depending on the node location.

The derivatives of shape functions are written as:
\[
\begin{align*}
\frac{\partial N_1}{\partial z} &= \frac{\partial N_1}{\partial x} \frac{\partial x}{\partial z} + \frac{\partial N_1}{\partial y} \frac{\partial y}{\partial z} + \frac{\partial N_1}{\partial z} \\
\frac{\partial N_1}{\partial \eta} &= \frac{\partial N_1}{\partial x} \frac{\partial x}{\partial \eta} + \frac{\partial N_1}{\partial y} \frac{\partial y}{\partial \eta} + \frac{\partial N_1}{\partial \zeta} \\
\frac{\partial N_1}{\partial \zeta} &= \frac{\partial N_1}{\partial x} \frac{\partial x}{\partial \zeta} + \frac{\partial N_1}{\partial y} \frac{\partial y}{\partial \zeta} + \frac{\partial N_1}{\partial \zeta}
\end{align*}
\] (2.25)

which can be rewritten as:

\[
\begin{bmatrix}
\frac{\partial N_1}{\partial z} \\
\frac{\partial N_1}{\partial \eta} \\
\frac{\partial N_1}{\partial \zeta}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial x}{\partial z} & \frac{\partial y}{\partial z} & \frac{\partial z}{\partial z} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial N_1}{\partial x} \\
\frac{\partial N_1}{\partial y} \\
\frac{\partial N_1}{\partial z}
\end{bmatrix} = J
\begin{bmatrix}
\frac{\partial N_1}{\partial x} \\
\frac{\partial N_1}{\partial y} \\
\frac{\partial N_1}{\partial z}
\end{bmatrix}
\] (2.26)

where \( J \) is the Jacobian. The inverse Jacobian \( J^{-1} \) becomes:

\[
J^{-1} = \frac{1}{J}
\begin{bmatrix}
\frac{\partial x}{\partial z} & -\frac{\partial y}{\partial z} & \frac{\partial z}{\partial z} \\
-\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
-\frac{\partial x}{\partial \zeta} & -\frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
\] (2.27)

The Cartesian derivatives of \( N_i \) can be found as:
Continuity between elements exists over an elemental face (area) of a three-dimensional element. The node values describe the variation of the unknown variable $\phi$ identically on the element face common to adjacent elements.

Numerical Integration

The advantage of using 3-D natural coordinates lies in the ability to evaluate the integral equations from integration formulae. The implementation of Gauss quadrature for the numerical integration can be applied directly to the integral equations without a change of limits. The general form of the integral solution utilizing Gauss quadrature can be written as:

$$
\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta, \zeta) |J| d\xi d\eta d\zeta = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} w_i w_j w_k J(\xi_i, \eta_j, \zeta_k) \int f(\xi_i, \eta_j, \zeta_k) |J(\xi_i, \eta_j, \zeta_k)|
$$

where $w_i$, $w_j$, $w_k$ are the weight factors associated with the respective Gauss points (Pepper and Heinrich, 1992).

For a trilinear element, the number of integration points is two per direction. Hence, eight weighting points ($2 \times 2 \times 2$) are required to evaluate the matrix. Higher integration accuracy can be achieved with more points of integration. Figure 2.3 shows...
the Gauss point locations for the integration scheme, where the weight factors are 1.0 and
Gauss point locations are $\pm 1/\sqrt{3}$ in each direction.

![Figure 2.3 Gauss Point Locations in 3-D Natural Coordinate System](image)

Applications to The Poisson Equation

The finite element method has been successfully applied to practically every area in
science and engineering. We discuss here the finite element formulation of the Poisson
equation in three dimensions.

The general Poisson equation is:

\[
-\frac{\partial}{\partial x}(k_1 \frac{\partial \phi}{\partial x}) - \frac{\partial}{\partial y}(k_2 \frac{\partial \phi}{\partial y}) - \frac{\partial}{\partial z}(k_3 \frac{\partial \phi}{\partial z}) = f \quad \text{in } \Omega \tag{2.30a}
\]

\[
\phi = \hat{\phi} \quad \text{on } \Gamma_1, \quad k_1 \frac{\partial \phi}{\partial n_x} + k_2 \frac{\partial \phi}{\partial n_y} + k_3 \frac{\partial \phi}{\partial n_z} = q \quad \text{on } \Gamma_2 \tag{2.30b}
\]

where $k_i = k_i(x,y,z)$ and $f = f(x,y,z)$ are given functions of position in a three-dimensional
domain $\Omega$, and $\hat{u}$ and $\hat{q}$ are specified functions of position on the portions $\Gamma_1$ and $\Gamma_2$, respectively, of the surface $\Gamma$ of the domain (see Fig. 2.4).
Applying the Galerkin procedure to Eq.(2.30), one obtains:

\[
\int_{\Omega} N_i \left[ -\frac{\partial}{\partial x} \left( k_1 \frac{\partial \phi}{\partial x} \right) - \frac{\partial}{\partial y} \left( k_2 \frac{\partial \phi}{\partial y} \right) - \frac{\partial}{\partial z} \left( k_3 \frac{\partial \phi}{\partial z} \right) - f \right] dx dy dz = 0
\] (2.31)

Utilizing Green's theorem, Eq.(2.31) becomes

\[
\int_{\Omega} \left[ k_1 \frac{\partial N_i}{\partial x} \frac{\partial \phi}{\partial x} + k_2 \frac{\partial N_i}{\partial y} \frac{\partial \phi}{\partial y} + k_3 \frac{\partial N_i}{\partial z} \frac{\partial \phi}{\partial z} \right] dx dy dz [\phi_i] = \int_{\Omega} N_i f dx dy dz + \int_{\Gamma} N_i q_n ds \quad (2.32a)
\]

where

\[
q_n = k_1 \frac{\partial \phi}{\partial x} n_x + k_2 \frac{\partial \phi}{\partial y} n_y + k_3 \frac{\partial \phi}{\partial z} n_z
\] (2.32b)

Clearly, the primary variable is \( \phi \) and the secondary variable is \( q_n \). Equation (2.32a) can be written in matrix equivalent form as:

\[
[K] \{\phi\} = \{f\}
\] (2.33)

where

\[
K = \int_{\Omega} \left[ k_1 \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + k_2 \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + k_3 \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dx dy dz
\] (2.34a)

and

\[
f = \int_{\Omega} N_i f dx dy dz + \int_{\Gamma} N_i q_n ds
\] (2.34b)

( J. N. Reddy, 1993).
Atmospheric Boundary Layer Concept

In the case of fluid motions for which the measured pressure distribution nearly agrees with the perfect-fluid theory, the influence of viscosity at high Reynolds numbers is confined to a very thin layer in the immediate neighborhood of the solid surface. In that thin layer the velocity of the fluid increases from zero at the surface (no slip) to its full value which corresponds to external frictionless flow. The layer under consideration is called the boundary layer. For atmospheric motion, an atmospheric boundary layer can exist, similar to boundary layers commonly found in engineering problems.

Figure 2.5 represents diagrammatically the velocity distribution in such a boundary layer along a surface, with the dimensions across it considerably exaggerated. In front of the leading edge of the surface the velocity distribution is uniform. With increasing distance from the leading edge in the downstream direction the thickness, \( \delta \), of the retarded layer increases continuously, as increasing quantities of fluid becomes affected.

![Figure 2.5 Neutrally Stratified Boundary Layer Over Uniform Terrain](image-url)
For laminar flow in the boundary layer, the thickness of a boundary layer which has not separated can be estimated in the following way for small scale flows (H. Schlichting, 1979).

\[ \delta = 5 \sqrt{\frac{v l}{U}} \]  

(2.35)

The dimensionless boundary-layer thickness, referred to the length of the surface, \( l \), becomes:

\[ \frac{\delta}{l} = 5 \sqrt{\frac{v}{U l}} = 5 \sqrt{\frac{v}{U l} R_i} \]  

(2.36)

where \( R_i \) denotes the Reynolds number related to the length of the surface, \( l \), \( v \) is kinematic viscosity, and \( U \) is the velocity outside the boundary layer.

Instead of the boundary-layer thickness, another quantity, the *displacement thickness* \( \delta_1 \), is sometimes used, Fig. 2.6. It is defined by the equation

\[ U \delta_1 = \int_0^\delta (U - u) \, dy \]  

(2.37)

![Figure 2.6 Displacement Thickness \( \delta_1 \) in A Boundary Layer](image)

The displacement thickness indicates the distance by which the external streamlines are shifted owing to the formation of the boundary layer. In the case of a plate in parallel...
flow and at zero incidence such as atmospheric flow, the displacement thickness is about 1/3 of the boundary-layer thickness $\delta$ given in eq. (2.35).

In the case of steady flow, the boundary layer equations for three-dimensional flow are written as:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  
\[ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]

with the boundary conditions:

\[ z = 0: \ u = 0, \ v = 0, \ w = 0; \quad \text{at} \ z = \infty : \ u = U_\infty, \ v = V_\infty \]  

It is necessary to prescribe, in addition, a velocity profile at the initial section, $x = x_0$, say, by indicating the function $u(x_0, y_0, z)$. The problem is thus seen to reduce itself to the calculation of the further change of a given velocity profile with a given potential motion.
Numerical solution of the full Navier-Stokes equations is not feasible in a production code for predicting 3-D windfields. The simplest approach for generating a gridded wind field is to use interpolation of sparse measurements as the first step, followed by use of objective analysis to adjust wind vectors at each grid point within the computational domain.

Early methods of divergence reduction involved point-iterative methods and/or variational calculus with Lagrangian multipliers to adjust velocities (Pepper and Kern, 1976; Sherman, 1978; and Pielke, 1984). Such techniques are relatively simple to employ, and allow quick estimates of the flow field to be generated. However, the flow field is critically dependent on empirically chosen constants, and velocities at region boundaries can force the nature of the interior flow solution (Pepper and Brueckner, 1992). Most of these early works did not account for surface topography accurately, and did not employ terrain-following coordinate systems.

In this thesis, the region boundaries of the Nevada Test Site, vertical extent, and basic grid cell size are first selected. Such cell sizes are dictated by the terrain irregularity, horizontal surface area, and vertical height (generally of the mixing layer). Once the grid
has been generated, surface level velocity measurements and upper level wind data are
interpolated to produce initial values at each computational node point. The final step is
to adjust the interpolated velocity field using the finite element method to minimize
divergence of the flow field.

Surface Mesh Generation

The surface topography is constructed from measured elevation data. The
topographical elevation data of any region within the U. S. can be found from DEM files.

The Nevada Test Site (NTS) is located in southern Nevada just north of Las
Vegas (see Fig. 1.1). The locations of Nevada Test Site tower stations are shown in Fig.
1.2, and listed in Table 3.1. The NTS is located within the area defined by west longitude
115.5° to 116.5°, and north latitude 36.5° to 37.5°. This region is located in the center of
DAM files 11636, 11736, 11637 and 11737, as shown in Fig. 3.1.

![Figure 3.1 The Nevada Test Site Location](image)
Table 3.1  Current Permanent Meda Stations of The Nevada Test Site

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<td>115 32.00</td>
<td>3100</td>
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For each DAM file, there are 1201x1201 nodal elevation values. The elevation data of the NTS region can not be obtained from one DEM file directly. A FORTRAN code was developed to open each of the DEM files, and then assemble them to yield 2401x2401 elevation data. The central region of 1201x1201 elevation data points (NTS region) was selected, and the coordinates changed to x-y coordinates where the x-direction is East-West and y-direction is North-South. Node (116.5°, 36.5°) is node (0, 0) in the x-y coordinate system. Because of memory limitation with UNIX, 1201x1201 elevation data points can not be fully utilized in the calculation. Instead, a region of 25x25 elevation data is selected for the preliminary surface topography of the NTS region. A three-dimensional terrain plot of the NTS region is shown in Figure 3.2a. A two-dimensional (top) view of the mesh of the NTS region is shown in Figure 3.2b. The horizontal surface grid consists of 25x25 nodes with approximately 3800 m spacing between nodes in the x-direction and 4700 m spacing between nodes in the y-direction.

Figure 1.2 shows two-dimensional topographic contours of the NTS with tower locations as indicated from the Air Resources Laboratory, National Oceanic and Atmospheric Administration - Nevada Operations Office (NOAA-NVOO). Based on Table 3.1 and Figure 1.2, 27 towers are located in the NTS region. Only 15 tower stations were available for use with the weather record in 1993, which was used for the test case in this study. These 15 towers are not located at the orthogonal mesh grid points within the NTS region. Hence, the surface mesh of the NTS region was altered so that the 15 towers will be located at nodal points. Figure 3.3 shows the top view of the two-dimensional mesh with the 15 towers at the nodal points.
Figure 3.2a A 3-D Terrain of NTS Region

Figure 3.2b A Top View of NTS Region Orthogonal Mesh
Figure 3.3 Non-orthogonal Mesh with 15 Towers As Grid Points in NTS Region
Three Dimensional Mesh Generation

Four separate horizontal layers are generated above the ground. Including the surface layer, a five layer, 3-D hexahedral mesh is required. Layer-1 is the surface. Layer-2 is the tower layer. Since the heights of the towers in the NTS region are 10 meters above the ground, Layer-2 is set 10 m above the surface layer. Layer-3 is set 50 m above the surface layer. Layer-4 is 300 m above the surface layer, and Layer-5 is 1000 m above the surface layer. A 3-D mesh generation FORTRAN code was developed to generate the nodal points and element connectivity. The surface mesh data file is used to initialize the mesh generation scheme. A total of 3125 nodes and 2304 elements is created for the 3-D mesh of the NTS region (see Fig. 3.4a). Fig. 3.4b shows a 2-D cross sectional view of the NTS mesh for perspective.
Figure 3.4b  2-D Cross Sectional View of The NTS Mesh for Perspective
Initial Surface Windfield Generation

The surface wind field is constructed from the measured data (converted to $u$ and $v$ components) by interpolation over the initial mesh using inverse distance-squared weighting (Pepper and Kern, 1976). A fixed radius of influence $R$ is specified which indicates the distance beyond which the influence of a station's value is no longer felt (Pepper and Brueckner, 1992). The influence of gross terrain features (mountain ranges, etc.) is easily accounted for utilizing simple differences to initially specify velocity components within the computational domain.

The Formulation of Tower Data

As a test of the surface wind field calculation procedure, the windfield record of tower station data for 1993 was selected as the reference data set for the interpolation over the mesh, as shown in Fig. 3.5. The meteorological data were obtained every fifteen minutes. The average of the measured data for each tower during the time interval from 1:00 am to 1:45 am on Jan. 1, 1993 was chosen. A FORTRAN code was developed to read these data records from magnetic tape and calculate the average. Table 3.2 lists the wind speeds and directions of the fifteen towers. A top view of the tower locations of the wind vectors within the NTS region is shown in Fig. 3.6.
Table 3.2  The Wind Speed Records of NTS Towers on Jan. 1, 1993

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<th>1:30</th>
<th>1:45</th>
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<td>Direction</td>
<td>Speed</td>
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<td>8</td>
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Figure 3.6 A Top View of Tower Wind Vectors In NTS Region.
Tower Layer (Layer-2) Velocity Generation

Once the tower data was read, an inverse weighting was performed around each tower for the tower layer (Goodin, et al., 1979). Velocities of the grid points near towers are calculated based on the value of the tower, which decays as $1/r^2$ with distance from the tower. Thus,

$$W(r) = \begin{cases} 
1/r^2 & r \leq R \\
0 & r > R
\end{cases} \quad (3.1)$$

The velocity at a grid point becomes:

$$u(x, y) = W(r)u_{\text{tower}}(x_t, y_t) \quad (3.2a)$$

$$v(x, y) = W(r)v_{\text{tower}}(x_t, y_t) \quad (3.2b)$$

where $r = \sqrt{(x - x_t)^2 + (y - y_t)^2}$, $r$ is the radial distance between the node $(x, y)$ and the tower $(x_t, y_t)$; if $r > R$, $u(x, y) = 0$, $v(x, y) = 0$ ($R$ is the radius of influence, $R = 24$ km in this model); $(x_t, y_t)$ is the tower location which is the closest tower to the node $(x, y)$; $u_t$ and $v_t$ are the tower velocity in the $x$-direction and $y$-direction.

Divergence Reduction of the Tower Layer

The tower layer flow field is established by using actual tower values. The interpolated layer velocities must be checked to minimize divergence, i.e., try to ensure that $\nabla \cdot \vec{V} \equiv 0$.

A slightly modified version of a simple five-point filter is used to smooth the tower layer values. The new value at a given point is the average of the value at the point and
the values at the four nearest points. The equation for smoothing is:

\[ u_{i,j}^{n+1} = 0.2\left(u_{i,j}^{n} + u_{i-1,j}^{n} + u_{i+1,j}^{n} + u_{i,j+1}^{n}\right)(1 - \alpha_k) + \alpha_k u_{i,j}^{n} \] (3.3)

\[ v_{i,j}^{n+1} = 0.2\left(v_{i,j}^{n} + v_{i+1,j}^{n} + v_{i-1,j}^{n} + v_{i,j+1}^{n}\right)(1 - \alpha_k) + \alpha_k v_{i,j}^{n} \] (3.4)

where \( \alpha_k \) is a parameter which allows the user to keep the measured velocity at station \( k \) fixed (\( \alpha_k = 1 \)) or keep only some of its original influence (\( \alpha_k < 1 \)); this parameter is zero at all non-measuring station points; \( u_{i,j} \) and \( v_{i,j} \) are horizontal velocities of the tower layer.

Note that \( u_{i,j} \) and \( v_{i,j} \) can not be used in the finite element method in a structured mesh mode; \( u(i) \) and \( v(i) \) are used instead, where \( i \) is node number, as defined by its nodal connectivity within an element. Equations 3.3 and 3.4 can be written:

\[ u(i)^{n+1} = 0.2(u(i)^n + u(i+1)^n + u(i-1)^n + u(i + ncolumn)^n + u(i - ncolumn)^n)(1 - \alpha_k) + \alpha_k u(i)^n \] (3.5)

\[ v(i)^{n+1} = 0.2(v(i)^n + v(i+1)^n + v(i-1)^n + v(i + ncolumn)^n + v(i - ncolumn)^n)(1 - \alpha_k) + \alpha_k v(i)^n \] (3.6)

where \( ncolumn \) is the column number within the tower layer (see Fig. 3.7).

![Figure 3.7 5 Point Stencil (numbers in parenthesis denote element)](image)

This first step is designed to reduce as much of the anomalous divergence as possible. The number of passes through the smoothing step is related to the relative...
atmospheric stability at that layer and is determined empirically. A check of the divergence is determined by the relative error, $\varepsilon$:

$$
\varepsilon = u(i)^{n+1} - u(i)^n \quad \text{and} \quad \varepsilon = v(i)^{n+1} - v(i)^n
$$

(3.7)

where $n+1$ is the unknown value, and $n$ is previous value.

If $\max \varepsilon \leq 10^{-4}$, the procedure is converged and smooth velocities for this layer are formed; else let $u(i)^n = u(i)^{n+1}$, $v(i)^n = v(i)^{n+1}$ and return to eqs. (3.5-3.6) until $\varepsilon \leq 10^{-4}$. Figure 3.8 shows the interpolated tower layer windfield for the NTS region.

![Figure 3.8 Interpolated Tower Layer (10 m) Windfield for NTS Region](image-url)
Upper Layer Windfield Generation

There has not been a very reliable method to calculate upper layer winds based on tower layer wind data. In the past, most have used $r^{-1}$ weighting to produce a smooth upper layer wind field (see Sherman, 1978; Pepper and Brueckner, 1992). In this project, a boundary layer technique is introduced to calculate the upper layer wind field. The top layer (1000 m) is assumed as the height of the atmospheric boundary layer. The velocity in the top layer is assumed constant, e.g. $U_{\text{top}} = 10 \text{ m/s}, V_{\text{top}} = 6 \text{ m/s}$ (based on tower velocities in Table 3.2). For an irregular terrain, the following approach is used to obtain upper layer wind flow based on tower layer values and top layer values.

Upper Layer Velocity Generation

The velocity at a grid point within the upper layer (Layer-3 and Layer-4) is calculated using the velocity of the grid point which has the same horizontal location on the tower layer and top layer velocity through a simple linear equation (see Fig. 3.9):

\[
U_{\text{top}} \quad (V_{\text{top}}) \quad \text{or} \quad U_{\text{top}} \quad (V_{\text{top}})
\]

\[
\text{or} \quad u_{\text{t}} (v_{\text{t}}) \quad \text{or} \quad u_{\text{t}} (v_{\text{t}})
\]

Figure 3.9 Formulation of Linear Equation
The upper layer velocity can be calculated as:

\[ u(x, y, z) = \left( U_{\text{top}}(x, y, z_{\text{top}}) - u_r(x, y, z_{\text{low}}) \right) \frac{z - z_{\text{top}}}{z_{\text{top}} - z_{\text{low}}} + U_{\text{top}}(x, y, z_{\text{top}}) \]  

\( (3.8) \)

\[ v(x, y, z) = \left( V_{\text{top}}(x, y, z_{\text{top}}) - v_r(x, y, z_{\text{low}}) \right) \frac{z - z_{\text{top}}}{z_{\text{top}} - z_{\text{low}}} + V_{\text{top}}(x, y, z_{\text{top}}) \]  

\( (3.9) \)

where \((u, v)\) is the upper layer velocity, and \((u_r, v_r)\) is the tower layer velocity.

**Divergence Reduction Of Upper Layer**

For this particular model, a slightly modified version of a simple three-point filter is used in the vertical direction to smooth the upper layer values. The new value at a given point is the average of the value at the point and the values at two adjacent points in the vertical direction. The equation for smoothing is:

\[ u(i)^{\text{new}} = \frac{1}{3} \left( u(i)^n + u(i + ng)^n + u(i - ng)^n \right) \]  

\( (3.10) \)

\[ v(i)^{\text{new}} = \frac{1}{3} \left( v(i)^n + v(i + ng)^n + v(i - ng)^n \right) \]  

\( (3.11) \)

where \(ng\) is the number of the node on the ground (there are 625 ground nodes).

Divergence is likewise determined by

\[ \varepsilon = u(i)^{\text{new}} - u(i)^n \text{ and } \varepsilon = v(i)^{\text{new}} - v(i)^n \]  

\( (3.12) \)

where \(\varepsilon \leq 10^{-4}\).

A FORTRAN code was developed to calculate the surface wind field and upper layer wind field. Figure 3.10 shows the initial FORTRAN program flow chart.
INITIAL.FOR

read initial data

calculate initial velocities on tower layer

check the convergence of tower layer velocities

- if $\varepsilon > 10^{-4}$
  - let $u_a(i) = u(i)$,
  - $v_a(i) = v(i)$
  - calculate $u, v$

- if $\varepsilon \leq 10^{-4}$
  - call upperlayer
  - calculate initial velocities of upper layer
  - check the convergence of upper layer
    - if $\varepsilon > 10^{-4}$
      - let $u_a(i) = u(i)$,
      - $v_a(i) = v(i)$
      - calculate $u, v$
    - if $\varepsilon \leq 10^{-4}$
      - call printf
      - printf: write $x, y, z, u, v, w$ to data set

stop

Figure 3.10 The Initial Fortran Program Flow Chart
Vertical Velocity Generation

The vertical wind velocity is not a commonly observed variable in meteorology, and its estimation appears as one of the most difficult problems for modeling studies. The vertical velocity is an integral component of the three dimensional structure of the atmospheric motion and encountered in many diagnostic and prognostic problems. The simplest method for the computation of the vertical motion is the integration of the mass continuity equation using the large scale horizontal wind observations and accounting for the divergence correction.

The continuity equation is:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\] (3.13)

In this thesis, a simple difference version of Eq.(3.13) is used to calculate vertical velocities, i.e.,
\[
\frac{u(i+1) - u(i-1)}{x(i+1) - x(i-1)} + \frac{v(i + ncolumn) - v(i - ncolumn)}{y(i + ncolumn) - y(i - ncolumn)} + \frac{w(i) - w(i - ng)}{z(i) - z(i - ng)} = 0
\] (3.14)

Thus, the vertical velocities are obtained as:
\[
w(i) = w(i - ng) - \frac{u(i+1) - u(i-1)}{x(i+1) - x(i-1)} + \frac{v(i + ncolumn) - v(i - ncolumn)}{y(i + ncolumn) - y(i - ncolumn)}(z(i) - z(i - ng))
\] (3.15)

where \(ng\) is the number of the node on the ground and \(ncolumn\) is the number of the column in the tower layer.

A short FORTRAN subroutine is used to calculate vertical velocities at all the nodal points based on Eq.(3.15) and horizontal velocity values, \(u\) and \(v\). At this point, the
initial 3-D wind field in NTS region is now generated.

The Adjustment Of The Windfield

Model Description

A mass-adjusted, three-dimensional wind field model was first developed and put into production use to provide wind fields for the ADPIC pollutant transport model (Lange, 1978). The theoretical basis for this model was originally developed by Sasaki (1958, 1970a,b). The general variational analysis formalism defines an integral function whose extremal solution minimizes the variance of the difference between the observed and analyzed variable values, subject to physical constraints which are satisfied exactly or approximately by the analyzed values. Subsidiary conditions that are to be satisfied exactly are known as strong constraints; conditions that are imposed approximately are weak constraints. A minimal solution exists when the number of strong constraints is less than the number of variables. For this model a function is needed to minimize the variance of the difference between the adjusted values and the original values, subject to the strong constraint that the three-dimensional analyzed wind field is nondivergent. Euler-Lagrange Methods and minimization are discussed in Appendix A.

The specific function used in this study is:

\[
E(u,v,w,\lambda) = \int \left[ \alpha_x^2 (u-u_0)^2 + \alpha_y^2 (v-v_0)^2 + \alpha_z^2 (w-w_0)^2 + \lambda \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right] dx dy dz
\]

(3.16)

where \( u, v, w \) are the adjusted velocity components in the \( x, y, z \) directions, respectively;
\( u_0, v_0, w_0 \) are the corresponding observed variables; \( \lambda(x, y, z) \) is the Lagrange multiplier, and values of \( \alpha_i \) are Gauss precision moduli taken to be: \( \sigma_i^2 = \frac{1}{2\sigma_i^2} \) (where the values of \( \sigma_i \) are observation tower errors and \( \alpha_i \) or deviations of the observed field from the desired adjusted field).

Identical Gauss precision moduli are assumed for the horizontal directions, \( x \) and \( y \). While distinctions can be large between horizontal and vertical directions, distinctions between the \( x \) and \( y \) coordinates are minimal. The associated Euler-Lagrange equations whose solutions minimize Eq. (3.16) are:

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \quad (3.20)
\end{align*}
\]

The equation for \( \lambda \) is derived by differentiating equations (3.17)-(3.19) and substituting the results into the continuity equation, giving:

\[
\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \left( \frac{\alpha_1^2}{\alpha_2^2} \right) \frac{\partial^2 \lambda}{\partial z^2} = -2\alpha_1^2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \quad (3.21)
\]

which is a Poisson equation for the multiplier, \( \lambda \).

While this technique is a stable deterministic procedure for producing a three-
dimensional nondivergent wind field over complex terrain, the appropriateness of the final wind field is dependent on the specifications of $\alpha_1$ and $\alpha_2$. Using an interpolated wind field given by random errors added to a nondivergent wind field, a series of numerical experiments were conducted for various ratios and values of $\alpha_1$ and $\alpha_2$ (Sherman, 1978). These experiments showed that the assumption of zero initial vertical velocities is reasonable if the atmospheric conditions are near neutral. The assumption is not valid when strong convective activity exists. The value of $(\alpha_1/\alpha_2)^2$ should be proportional to the magnitude of the expected $(w/u)^2$. If this ratio is large, the adjustment is predominantly in the vertical component. If it is smaller, the horizontal adjustment dominates (Sherman, 1978). In this study, the values of $\alpha_1$ and $\alpha_2$ were taken to be 0.01 and 0.1, respectively.

Boundary Conditions of The Model

Either the multiplier, $\lambda$, or the normal velocity component variation, must be zero on a boundary. Specifying both over-specifies the problem and violates the conditions for solution uniqueness. When $\lambda$ is zero on a boundary, the normal derivative of $\lambda$ is, in general, not zero. Thus, an adjustment of the observed velocities results from Eqs. (3.17)-(3.19). A non-zero adjustment of the velocity component normal to the boundary implies a change in the amount of mass entering or leaving the volume. Therefore, the boundary condition $\lambda = 0$ is appropriate for open or “flow-through” boundaries. A constant value for $\lambda$ on an open boundary also implies no adjustment is made in the non-normal velocity components, since the non-normal derivatives of $\lambda$ are zero.
If the variation of the normal velocity component is zero on the boundary, the adjusted value of the normal velocity must be the observed value. From Eqs. (3.17)-(3.19), it is apparent that setting the normal derivative of $\lambda$ equal to zero on the boundary specifies no variation of the normal velocity component at that boundary. If the observed normal velocity is zero, this boundary condition implies no transport of mass across the boundary. Therefore, the condition $\partial \lambda / \partial n = 0$ is used for closed or "no-flow-through" boundaries. Figure 3.11 shows boundary conditions of this model.

![Figure 3.11 Boundary Conditions of The Model](image)

Application of The Finite Element Method

Equation (3.21) is a Poisson equation solved for the multiplier $\lambda$. Applying the Galerkin weighted residuals technique, Eq. (3.21) becomes:

$$
\int_{\Omega} N_i \left[ -\frac{\partial^2 \lambda}{\partial x^2} - \frac{\partial^2 \lambda}{\partial y^2} - \left( \frac{\alpha_2}{\alpha_2} \right)^2 \frac{\partial^2 \lambda}{\partial x^2} - 2\alpha_1^2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right] dx dy dz = 0
$$

Equation (3.22) can be written in matrix equivalent form as $[K]\{\lambda\} = \{f\}$.
where

\[
K = \int_{\Omega} \left[ \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \left( \frac{\alpha_i}{\alpha_j} \right)^2 \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dxdydz \tag{3.23}
\]

and

\[
f = \int_{\Omega} N_i 2x_1^2 \left( \frac{\partial u_0}{\partial x} + \frac{\partial v_0}{\partial y} + \frac{\partial w_0}{\partial z} \right) dxdydz \tag{3.24}
\]

with \( \{\lambda\} = \{\phi\} \) in Eq.(2.30). Note that the relation has been "weakened" to a first order equation. The boundary condition flux for \( \lambda \) is automatically assumed to be zero in the finite element procedure, which is ideal for this problem.

A Cholesky decomposition method is used for the matrix solver. Hexahedral elements are generated requiring the use of 2x2x2 Gauss point quadrature integration.

**Check of Divergence**

Once \( \lambda \) is calculated, the velocity components are adjusted (except for the tower values and ground values) through Eq.(3.17)-(3.19). A check of divergence is again determined by:

\[
\epsilon_u(i) = u(i) - u_0(i) \tag{3.25}
\]

\[
\epsilon_v(i) = v(i) - v_0(i) \tag{3.26}
\]

\[
\epsilon_w(i) = w(i) - w_0(i) \tag{3.27}
\]

\[
\epsilon_\lambda(i) = \lambda(i) - \lambda_0(i) \tag{3.28}
\]

If any one of these \( \epsilon_{\text{max}} > 10^4 \), then \( u_0(i) = u(i), v_0(i) = v(i), w_0(i) = w(i), \lambda_0(i) = \lambda(i) \) and the solution returns to Eq. (3.22) until \( \epsilon_{\text{max}} \leq 10^4 \). A FORTRAN based, 3-D finite element code was written to solve for the \( \lambda \)'s, and the adjustment of wind field. Fig. 3.12 shows the program flow chart for the 3-D windfield adjustment.
3-DWIND.FOR

MAIN
start computation

INITIALIZE
quadrature setup
calculate element volume
calculate bandwidth

calculate Gauss points

calculate stiffness matrix

boundary conditions

time steps

calculate potential ($\lambda$) function
and backsubstitute

calculate velocity correction

check for residual in overall flow field

if $\varepsilon_{\text{max}} > 10^{-4}$, $\lambda_0 = \lambda$
uo = u, vo = v, wo = w

if $\varepsilon_{\text{max}} \leq 10^{-4}$, call print, call printf

print -
x,y,z,u,v,w,\lambda

printf-write
x,y,z,u,v,w,\lambda
to data set

stop

Figure 3.12 The Program Flow Chart for The 3-D Windfield Adjustment

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Computing Methods

All the images and calculations of the 3-D velocity field were performed on the CRAY YMP/2-216 Supercomputer located in the National Supercomputer Center for Energy and Environment (NSCEE) at UNLV. Memory required on the Cray YMP was about 100 megabytes. Tecplot and Fieldview software were used to graphically display results. This method was first used to solve 3-D air flow over a cavity, and then subsequently used to solve 3-D wind flow over the NTS region (Pepper and Brueckner, 1992).

For solving 3-D cavity problems, the INITIAL.FOR code converged within one minute before 45 time steps. The 3-DWIND.FOR code converged within 5 minutes before 500 time steps.

For solving 3-D wind flow over the NTS region, the INITIAL.FOR code converged within 4 minutes before 350 time steps. The 3-DWIND.FOR code converged about 20 minutes after 3600 time steps.
3-D Cavity Problem

In this study, a 3-D cavity model was first performed to verify and test the 3-D finite element model. 3-D air flow over a cavity was simulated as shown in Fig. 4.1 (a,b). The 3-D diagnostic wind flow model was used and results compared with a full, viscous, incompressible Navier-Stokes solution, as shown in Fig. 4.2 (a,b). The Reynolds number was \( \text{Re} = 100 \). Utilizing the full viscous results, some node values were selected as "tower values" and used for the 3-D cavity problem. A total of five tests of the 3-D cavity problem were performed.

Fig. 4.1a shows the boundary conditions for the cavity model. Fig. 4.1b shows 3-D air flow over the cavity. Fig. 4.2a shows 2-D velocity result of Navier-Stokes solution, and Fig. 4.2b shows streamlines of the Navier-Stokes solution.
Figure 4.1a Boundary Conditions of Cavity Model

Figure 4.1b 3-D Air Flow Over a Cavity
Figure 4.2a 2-D Velocity Result of Navier-Stokes Solution

Fig. 4.2b 2-D Streamlines of Navier-Stokes Solution.
Test-1 was conducted for the 3-D cavity problem just using the 3-D windfield adjustment code, and the boundary conditions. Initialization was not run to obtain initial velocities. In this test, the adjustment code was used to solve the equation for potential flow instead of solving the diagnostic Poisson equation. Figure 4.3 shows the results of the potential flow solution.

In Test-2, the flow within the 3-D cavity was first calculated by Initial FORTRAN code (see Fig. 3.10) with boundary conditions, and initial velocities generated within the cavity. The adjustment code was used to solve the diagnostic Poisson equation. Fig. 4.4 shows results of Test-2. Comparing the plots of Test-1 and Test-2 shows the distributions of the velocity streamlines were quite different. In Test-1, streamlines flowed along the wall of the cavity. there is no air circulation within the cavity (see Fig. 4.3). In Test-2, circular streamlines are formed within the cavity (see Fig. 4.4).

Based on Test-2, one tower vector was interpolated in the cavity for Test-3. Two tower vectors were interpolated in the cavity for Test-4, and three tower vectors were interpolated in the cavity for Test-5. These fixed nodal values were selected from the full viscous solution. Fig. 4.5 shows 3-D cavity results with one interpolated tower. Fig. 4.6 shows 3-D cavity results with two interpolated towers, and Fig. 4.7 shows 3-D cavity results with three interpolated towers. Comparing the plot of Test-2, Test-3, Test-4 and Test-5, the results of these tests show that the shapes of streamline distributions were changed. With increases in the number of interpolated towers, the shapes of streamlines are close to the plot of 2-D Navier-Stokes Solution (Fig. 4.2b).

From results of these tests for 3-D cavity problem, we can see that initialization is
very important in the process of solving the problem. The accuracy of the results calculated by the 3-D diagnostic method was affected by the number of towers. The results became more accurate when more towers were selected.

Figure 4.3 3-D Cavity Problem Adjusted Only With Boundary Condition (Without initialization)
3-D cavity problem adjusted with boundary condition and initialized data with no tower.

Figure 4.4 Streamlines for Test-2

3-D cavity problem adjusted with boundary condition and initialized data with one tower. $x_{tt} = 8.8889, z_{tt} = 7.7778, u_{tt} = -0.0309, w_{tt} = -0.246$

Figure 4.5 Streamlines for Test-3
3-D cavity problem adjusted with boundary condition and initialized data with two towers.

- $x_{t1} = 8.8889$, $z_{t1} = 7.7778$
- $u_{t1} = -0.0309$, $w_{t1} = -0.246$
- $x_{q1} = 2.2222$, $z_{q1} = 6.6667$
- $u_{q1} = 0.00536$, $w_{q1} = 0.0878$

Figure 4.6 Streamlines for Test-4

3-D cavity problem adjusted with boundary condition and initialized data with three towers.

- $x_{t1} = 8.8889$, $z_{t1} = 7.7778$
- $u_{t1} = -0.0309$, $w_{t1} = -0.246$
- $x_{q1} = 2.2222$, $z_{q2} = 6.6667$
- $u_{q1} = 0.00536$, $w_{q1} = 0.0878$
- $x_{q2} = 5.5556$, $z_{q2} = 4.4444$
- $u_{q2} = -0.0997$, $w_{q2} = 0.00646$

Figure 4.7 Streamlines for Test-5

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3-D Wind Flow Over NTS Region

The wind field adjustment program converged after 3600 time steps for the NTS region. Fig. 4.8 shows the 3-D wind field vectors with topographic contours. Fig. 4.9 shows the top view of the tower layer (Layer-2) wind field, Fig. 4.10 shows the top view of the 50m layer (Layer-3) wind field, and Fig. 4.11 shows the top view of the 300m layer (Layer-4) wind field. This problem also was calculated by Gayle Sugiyama and Stevens Chan at Lawrence Livermore National Laboratory (LLNL) using the LLNL finite element model (based on a structured mesh and interpolated tower location), and a terrain following finite difference method.

Appendix B shows results of the LLNL work. Comparing plots of the results, flow simulations are similar even though they basically use two different schemes.

Figure 4.8 The Result of 3-D Windfield with Topographic Contours
Figure 4.9 Top View of Result of Tower Layer (10 M) Windfield

Figure 4.10 Top View of Result of Layer-3 (50 M) Windfield
Figure 4.11 Top View of Result of Layer-4 (300 M) Windfield

From the plots of different layers, it is clear that the windfields are different on different layers. The higher the layer, the smoother the windfield. Comparing these plots with the results of LLNL (see Appendix B), interpolated windfields of tower layer are similar (see Fig. 3.8 and B-1). Adjusted windfield of tower layer are similar (see Fig. 4.9 and B-4). Fig. 4.10 is close to B-5.

Two tests were performed to check the influence of the windfield with available data. Test-A removed one tower and then checked the difference of tower locations in the calculated windfield with and without the tower. In this test, Tower No. 14 was removed (see Fig. 3.3 and Fig. 3.6). The differences in the results are shown in Table 4.1. Fig. 4.12 shows the interpolated windfield of the tower layer after the tower was removed. From Table 4.1 and Fig. 4.12, the value and direction of the flow at this tower location were
altered significantly. Comparing Fig. 4.12 with Fig. 3.10, the directions of the windfields around the Tower No. 14 are changed, and the values are quite different. Data from Tower No. 14 is very important for calculating windfields in the NTS region.

Test-B removed two towers, and the difference examined. In this test, Tower No. 14 and No. 23 were removed (see Fig. 3.3 and Fig. 3.6). The difference in the results for these two tower points is shown in Table 4.2. Fig. 4.13 shows the interpolated windfield of the tower layer after the two towers were removed. Comparing Table 4.1 and Table 4.2, we find that node Tower No. 14 is almost the same, but node Tower No. 23 has changed significantly. Fig. 4.13 looks like the same with Fig. 4.12. The windfields were not affected a lot by removing Tower No. 23. This means that the calculated windfields around Tower No. 23 are usually more smooth than around Tower No. 14. The accuracy of the interpolated windfields using the 3-d diagnostic method is affected more by some towers than others. The results generally become more accurate when more towers are selected.
Table 4.1  The Comparison Between All Towers and One Tower Removed

<table>
<thead>
<tr>
<th>Velocity of Tower No.</th>
<th>All Towers (m/s)</th>
<th>One tower Removed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u_{14}</td>
<td>-5.42</td>
<td>-0.17</td>
</tr>
<tr>
<td>v_{14}</td>
<td>2.57</td>
<td>1.85</td>
</tr>
<tr>
<td>u_{23}</td>
<td>3.22</td>
<td>3.22</td>
</tr>
<tr>
<td>v_{23}</td>
<td>3.32</td>
<td>3.32</td>
</tr>
</tbody>
</table>

Figure 4.12  Interpolated Windfield of Tower Layer (10 M) After One Tower Removed
Table 4.2 The Comparison Between All towers and Two Towers Removed

<table>
<thead>
<tr>
<th>Velocity of Tower No.</th>
<th>All Towers (m/s)</th>
<th>Two towers Removed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_{14}$</td>
<td>-5.42</td>
<td>-0.71</td>
</tr>
<tr>
<td>$v_{14}$</td>
<td>2.57</td>
<td>1.82</td>
</tr>
<tr>
<td>$u_{23}$</td>
<td>3.22</td>
<td>0.18</td>
</tr>
<tr>
<td>$v_{23}$</td>
<td>3.32</td>
<td>4.06</td>
</tr>
</tbody>
</table>

Figure 4.13 Interpolated Windfield of Tower Layer (10 M) After Two Towers Removed
CHAPTER 5

CONCLUSIONS

The finite element method is a unique method which permits a physical domain to be modeled directly using unstructured grids in physical coordinates. Using simple modifications, the finite element method is competitive with finite difference schemes in terms of computational speed and memory.

An objective analysis technique is used to first initialize a 3-D wind field, based on available meteorological data and interpolation. A surface boundary layer technique is used to perform the upper level windfields. Vertical velocities are developed from successive solutions of the continuity equation. An adjustment of the mass consistent wind field is then obtained. The goodness of the prediction of windfield using this method is verified by solving 3-D cavity problem. The influence of accuracy of the interpolated windfields by tower values is discussed. Compared to the LLNL method, the computational time of this method is faster than LLNL method. The use of unstructured grids to model 3-D flows appears to be very attractive.

It appears that viscous, incompressible flow can quickly be simulated utilizing a minimum but sufficient number of actual data points. This method not only is good for atmospheric flow but also can be used in solving ground water problems, heat conduction, stress-strain analysis, etc. The method is especially attractive for situation where minimal
data have been experimentally obtained, and there is interest in modeling flow within the entire problem domain.

In the future, adaptive techniques will be used in this model. Adaptive grids will allow the mesh to adjust for regions where steep gradients in velocity or concentration occur and to unadapt where the flow is uniform.
In ordinary calculus, minimum problems are concerned with those values of the independent variables for which a given function attains its minimum. If a differentiable function has a minimum at a point, then its derivative vanishes at that point. It turns out that this property can be generalized to the case of a minimum of a functional on a normed space.

Euler-Lagrange equation

The most remarkable classical Euler-lagrange variational problem is to determine a function \( u(x) \) on the interval \([a, b]\) satisfying the boundary conditions \( u(a) = \alpha \) and \( u(b) = \beta \), and extremizing the functional

\[
I(u) = \int_a^b F(x, u, u')\,dx \quad \left(u'(x) = \frac{du}{dx}\right) \tag{A-1}
\]

where \( u \) is a twice continuously differentiable function on \([a, b]\) \((u \in \mathcal{C}^2([a, b]))\), \( F \) is continuous in \( x, u \) and \( u' \), and has continuous partial derivatives with respect to \( u \) and \( u' \). We assume that \( I(u) \) has an extremum at some \( u \in \mathcal{C}^2([a, b]) \). Then we consider the set of all variations \( u + tv \), for an arbitrary fixed \( v \in \mathcal{C}^2([a, b]) \), such that \( v(a) = v(b) = 0 \). Then
Using the Taylor series expansion

\[
F(x, u + tv, u' + tv') = F(x, u, u') + t \left( \frac{\partial F}{\partial u} + v' \frac{\partial F}{\partial u'} \right) + \frac{t^2}{2!} \left( \frac{\partial^2 F}{\partial u^2} + v' \frac{\partial^2 F}{\partial u'^2} \right) + \cdots,
\]

it follows from (A-2) that

\[
I(u + tv) = I(u) + t I_{(u,v)} + \frac{t^2}{2!} I_{(u,v)}^{(2)} + \cdots,
\]

where the first and the second Fréchet differentials are given by

\[
dI(u,v) = \int_a^b \left( \frac{\partial F}{\partial u} + v' \frac{\partial F}{\partial u'} \right) dx,
\]

\[
d^2 I(u,v) = \int_a^b \left( \frac{\partial^2 F}{\partial u^2} + v' \frac{\partial^2 F}{\partial u'^2} \right) dx.
\]

The necessary condition for the functional \( I \) to have an extremum at \( u \) is that \( dI(u, v) = 0 \) for all \( v \in \mathcal{E}^2([a, b]) \) such that \( v(a) = v(b) = 0 \), that is,

\[
0 = dI(u,v) = \int_a^b \left( \frac{\partial F}{\partial u} + v' \frac{\partial F}{\partial u'} \right) dx.
\]

Integrating the second term in the integrand in (A-6) by parts, we obtain

\[
\int_a^b \left[ \frac{\partial F}{\partial u} - \frac{d}{dx} \left( \frac{\partial F}{\partial u'} \right) \right] v dx + \left[ v \frac{\partial F}{\partial u'} \right]_a^b = 0.
\]

Since \( v(a) = v(b) = 0 \), the boundary terms vanish and the necessary condition becomes

\[
\int_a^b \left[ \frac{\partial F}{\partial u} - \frac{d}{dx} \left( \frac{\partial F}{\partial u'} \right) \right] v dx = 0
\]

for all functions \( v \in \mathcal{E}^2([a, b]) \) vanishing at \( a \) and \( b \). This is possible only if
\[ \frac{\partial F}{\partial u} - \frac{\partial}{\partial x} \left( \frac{\partial F}{\partial u'} \right) = 0 \]  

(A-9)

This is called the Euler-Lagrange equation.

**Optimization**

We consider optimization problems with a given auxiliary condition. It is often required to determine the function \( y = y(x) \) which minimizes the functional

\[ I(y) = \int_{x_1}^{x_2} F(x, y, y') \, dx = 0 \]  

(A-10)

subject to auxiliary condition

\[ J(y) = \int_{x_1}^{x_2} G(x, y, y') \, dx = C \]  

(A-11)

It can easily be shown that this problem is equivalent to the problem already discussed earlier, namely, that of determining the function \( y = y(x) \) which minimizes the functional

\[ I_1(y) = I(y) + \lambda J(y) = \int_{x_1}^{x_2} [F(x, y, y') + \lambda G(x, y, y')] \, dx. \]  

(A-12)

The constant \( \lambda \) involved in (A-12) must be determined from the auxiliary condition (A-11).

The resulting Euler-Lagrange equation is

\[ \frac{\partial}{\partial y'} (F + \lambda G) - \frac{d}{dx} \left[ \frac{\partial}{\partial y'} (F + \lambda G) \right] = 0. \]  

(A-13)

Eq. (3.16) is formulated based on Euler-Lagrange method and minimization which were introduced above. In Eq.(3.16)

\[ F = \alpha_1^2 (u - u_o)^2 + \alpha_2^2 (v - v_o)^2 + \alpha_3^2 (w - w_o)^2 \]  

(A-14)
\[ G = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \]  

Eq. (3.17)-(3.19) were formed by using (A-13) in Eq. (3.16).
APPENDIX B

SOLUTION OF NTS MODEL BY LLNL

The Nevada Test Site simulation was performed by Gayle Sugiyama and Stevens Chan of Lawrence Livermore National Laboratory (LLNL) on August 1996. A 15-level model was generated using a combined finite element-finite difference method. Figure B-1 to Figure B-8 show the results of the NTS region calculated by Gayle Sugiyama and Stevens Chan.
Program execution date & time

- yyyyymmdd : 13960816
- hhmmss.sss : 153053.248
- time relative to UTC in hhmm : -1700

NTS -- wind field using Darrel NTS data
Computational domain :
- \( \text{umax} \) \( j_{\text{max}} \) \( k_{\text{max}} \) = 25 15 15
- number of zones = 9064
- number of mesh points = 9375
- level for printouts = 0
- weighting factor for \( u \), \( v \) = 1.000E-00
- weighting factor for \( w \) = 1.000E-00
- scaling factor for \( z \) = 1.000E-00
- terrain = yes
- map_factor_flag = on

Parameters for solver :
- Iterative solver used is iccg
- Convergence criteria = 1.000E-04
- Allowed No. of iterations = 300
- Stabilization option = 2
- Stabilization coefficient = 5.30012E-31

Name of netCDF output file is : wand_gtx.nc
Name of netCDF wind correction file is :
Name of netCDF grid file is : nts_13930101_51000.nc
Name of netCDF wind file is : nts_13930101_011000.nc
READ_GTX : netCDF file source code string :
- MAGIC 1.0

READ_GTX : netCDF grid file :
- grid/grid_nts_gtx.nc

READ_GTX : netCDF valid date and time :
- 1993-01-01 11:00:00

Min u = -5.03C137E+00 at node 909
Max u = 9.58C317E+00 at node 2702
Min v = -3.17C352E+00 at node 927
Max v = 8.438800E+00 at node 4034
Min w = 0.30C000E+00 at node 1
Max w = 0.30C000E+00 at node 1
Min map_factor = 1.000000E+00 at node 1
Max map_factor = 1.000000E+00 at node 1

Initial total kinetic energy = 1.343315E+14

Initial RMS norm of Div(U) = 1.851673E-03
Min Div (U) = -2.2E4922E-03 in element 151
Max Div (v) = 2.337547E-02 in element 170

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<td>5.3501E-01</td>
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Iterations taken: 32
||r l / i b l 1| error: 1.7243E-04
||dx / x l | error: 9.5941E-05
allowable error: 1.0000E-04

Final RMS norm of Div(U) = 2.229931E-04
Min Div (u) = -1.816992E-03 in element 563
Max Div (u) = 1.528501E-03 in element 293
Min Div (u) = -1.067748E-06 in macro element 44
Max Div (u) = 1.184608E-06 in macro element 57

Final total kinetic energy = 7.336094E+14
Min u = -5.176109E+00 at node 209
Max u = 9.616958E+00 at node 1707
Min v = -3.069009E+00 at node 127
Max v = 8.732174E+00 at node 174
Min w = -2.030811E+00 at node 126
Max w = 1.355661E+00 at node 150

Timing Information
Mesh, initial wind field [sec.] .......... 4.1872E-01
Divergence & mass matrices [sec.] .......... 2.3014E-01
Normals, New Ct, BCs, & RHS [sec.] .......... 5.3094E-01
Form & decompose FEM matrix [sec.] .......... 5.1931E-01
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$Z_{\text{top}} = 3000 \text{ m}$

$\sigma_z = \frac{z - z_q}{Z_{\text{top}} - z_q}$

Interpolated only
BIBLIOGRAPHY


Sugiyama, Gayle; Chan, Stevens *Solution of NTS Model*. LLNL, Livermore, CA, August 1996.