May 2018

Linear and Nonlinear Adaptive Attitude Control of Asteroid-Orbiting Spacecraft Using State Feedback and Output Feedback

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LINEAR AND NONLINEAR ADAPTIVE ATTITUDE CONTROL OF
ASTEROID-ORBITING SPACECRAFT USING STATE FEEDBACK AND OUTPUT
FEEDBACK

by

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Bachelor of Science - Electrical Engineering
University of Nevada, Las Vegas
2016

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science - Electrical Engineering

Department of Electrical and Computer Engineering
Howard Hughes College of Engineering
The Graduate College

University of Nevada, Las Vegas
May 2018
This thesis prepared by

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entitled

Linear and Nonlinear Adaptive Attitude Control of Asteroid-Orbiting Spacecraft Using State Feedback and Output Feedback

is approved in partial fulfillment of the requirements for the degree of

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ABSTRACT

LINEAR AND NONLINEAR ADAPTIVE ATTITUDE CONTROL OF ASTEROID-ORBITING SPACECRAFT USING STATE FEEDBACK AND OUTPUT FEEDBACK

by

Nicholas Moya

Dr. Sahjendra Singh, Advisory Committee Chair
Professor of Electrical and Computer Engineering
University of Nevada, Las Vegas

This thesis presents the derivation of both a linear and nonlinear adaptive control law for the attitude states of a spacecraft in orbit around rotating asteroids. The asteroid is assumed to be irregularly shaped and in an elliptical orbit around the sun. The linearized, time-varying spacecraft model will assume to include unknown parameters and will have external disturbances present. The objective is to control the roll, pitch, and yaw angle trajectories of the spacecraft such that they track desired reference trajectories. To achieve this, the control law will be composed of (1) a feedback controller, sufficiently robust to disturbance such that the system is stable, and (2) an adaptation law to estimate unknown parameters. The design of the linear control law will assume that only the angle measurements of the attitude states will be available for feedback. A high-gain observer will then be designed to estimate the higher-order states necessary to synthesis the control law. It will be shown
that the tracking error, as well as the observation error, approach zero in finite time within the closed-loop system. The controller is able to accomplish this while being bounded to a saturation limit to reflect genuine thruster constraints. When the nonlinear, time-varying model is considered, unknown parameters are again assumed to be present. To govern the nonlinearities, and uncertainties present in the model, an adaptive control law is designed using the backstepping method. The control law also incorporates an adaption law, derived from an appropriate Lyapunov function. Using state feedback, it can be seen that the tracking error approaches zero while managing the nonlinearities of the model. For the both the linear and nonlinear systems, simulation results present full attitude control of the spacecraft, despite unknown parameters within the model.
ACKNOWLEDGEMENTS

There are several people whom I wish to thank for their help during my work on this thesis. I wish to thank my parents for their support to help me complete my education as well as this thesis, I wish to thank my friends from outside the ECE department, Fidel, Mayah, and Ian, for their understanding at my absence when I would have to stay home and work instead of accompanying them on some fun adventure. Another large thanks goes to my friends in the ECE department, including Jon, Ron, and Justin, who, from undergrad to grad school, have motivated me to do more with my education by leading with their inspiring example. I would especially like to thank Justin who has shared my investment in Control Theory from day one, and has attended every Control Theory class with me from the undergrad level to the graduate level. It’s because of his efforts that I was able to take the classes that were needed to learn the material used in this thesis. I would also like to say thank you to my girlfriend, Ann, for her patience when I would have to stay home and work instead of going out. Of course I want to thank the committee members, many of whom have been my professors for several years, for their teachings. Lastly, I would like to thank my advisor Dr. Singh for his help and guidance on this thesis have been invaluable. He has believed in me even when I had lost hope, and I know I will be a successful engineer because of his teachings.
TABLE OF CONTENTS

ABSTRACT ................................................................................................................. iii

ACKNOWLEDGEMENTS ........................................................................................ v

LIST OF TABLES ......................................................................................................... vii

LIST OF FIGURES ..................................................................................................... ix

CHAPTER 1 Introduction .......................................................................................... 1

CHAPTER 2 Spacecraft Dynamics ............................................................................. 6
  State-Variable Representation .................................................................................. 9
  Error Dynamics ...................................................................................................... 14

CHAPTER 3 State Feedback ......................................................................................... 17
  Control Law .......................................................................................................... 18
  Adaption Laws ...................................................................................................... 20
  Results of State Feedback ..................................................................................... 24
  Stability Margins ................................................................................................... 25
  Closed-Loop Simulation ....................................................................................... 27
  Effects of Disturbance .......................................................................................... 46

CHAPTER 4 Output Feedback ..................................................................................... 64
  Observer Design ................................................................................................... 64
  Observer Results ................................................................................................. 66

CHAPTER 5 Nonlinear Model and Control ................................................................. 80
  Model .................................................................................................................... 80
  Backstepping ....................................................................................................... 84

CHAPTER 6 Nonlinear Simulation Results ............................................................... 92

CHAPTER 7 Conclusion ............................................................................................ 112

BIBLIOGRAPHY ....................................................................................................... 118

CURRICULUM VITAE ............................................................................................... 120
LIST OF TABLES

2.1 Asteroid and spacecraft parameters .................................................. 9
3.1 Asteroid parameter values for Eros ....................................................... 30
3.2 Asteroid parameter values for Vesta ..................................................... 34
3.3 Asteroid parameter values for Ida ....................................................... 38
3.4 Asteroid parameter values for Gaspra .................................................. 42
6.1 Asteroid parameter values for Eros ....................................................... 98
6.2 Asteroid parameter values for Vesta ..................................................... 99
6.3 Asteroid parameter values for Ida ....................................................... 107
6.4 Asteroid parameter values for Gaspra .................................................. 108
## LIST OF FIGURES

1.1 Pictures of four prominent small asteroids. ........................................... 2

2.1 Satellite in orbit around an asteroid. ..................................................... 6

3.1 Direct adaptive control structure. ............................................................ 17
3.2 Mass of the spacecraft during its orbit. .................................................. 29
3.3 Spacecraft attitude response in orbit around Eros; zero reference trajectory. .. 31
3.4 Spacecraft attitude response in orbit around Eros; constant reference trajectory. 32
3.5 Spacecraft attitude response in orbit around Eros; sinusoidal reference trajectory. 33
3.6 Spacecraft attitude response in orbit around Vesta; zero reference trajectory. .... 35
3.7 Spacecraft attitude response in orbit around Vesta; constant reference trajectory 36
3.8 Spacecraft attitude response in orbit around Vesta; sinusoidal reference trajectory 37
3.9 Spacecraft attitude response in orbit around Ida; zero reference trajectory ....... 39
3.10 Spacecraft attitude response in orbit around Ida; constant reference trajectory 40
3.11 Spacecraft attitude response in orbit around Ida; sinusoidal reference trajectory 41
3.12 Spacecraft attitude response in orbit around Gaspra; zero reference trajectory 43
3.13 Spacecraft attitude response in orbit around Gaspra; constant reference trajectory 44
3.14 Spacecraft attitude response in orbit around Gaspra; sinusoidal reference trajectory 45
3.15 Disturbances present for a spacecraft orbiting a celestial body at varying altitudes.................. 46
3.16 Compare attitude states and reference trajectories; zero reference trajectory .. 49
3.17 Compare attitude tracking error; zero reference trajectory ......................... 50
3.18 Compare attitude angular velocity tracking error; zero reference trajectory .. 51
3.19 Compare actuator effort; zero reference trajectory ................................. 52
3.20 Compare estimated parameters; zero reference trajectory ......................... 53
3.21 Compare attitude states and reference trajectories; constant reference trajectory 54
3.22 Compare attitude tracking error; constant reference trajectory ................... 55
3.23 Compare attitude angular velocity tracking error; constant reference trajectory 56
3.24 Compare actuator effort; constant reference trajectory ............................ 57
3.25 Compare estimated parameters; constant reference trajectory .................... 58
3.26 Compare attitude states and reference trajectories; sinusoidal reference trajectory 59
3.27 Compare attitude tracking error; sinusoidal reference trajectory ............... 60
3.28 Compare attitude angular velocity tracking error; sinusoidal reference trajectory .................. 61
3.29 Compare actuator effort; sinusoidal reference trajectory ............................................. 62
3.30 Compare estimated parameters; sinusoidal reference trajectory ................................. 63

4.1 Spacecraft attitude response in orbit around Eros; zero reference trajectory .................. 68
4.2 Spacecraft attitude response in orbit around Eros; constant reference trajectory ............... 69
4.3 Spacecraft attitude response in orbit around Eros; sinusoidal reference trajectory .......... 70
4.4 Spacecraft attitude response in orbit around Vesta; zero reference trajectory .................. 71
4.5 Spacecraft attitude response in orbit around Vesta; constant reference trajectory ............... 72
4.6 Spacecraft attitude response in orbit around Vesta; sinusoidal reference trajectory .......... 73
4.7 Spacecraft attitude response in orbit around Ida; zero reference trajectory ..................... 74
4.8 Spacecraft attitude response in orbit around Ida; constant reference trajectory ................. 75
4.9 Spacecraft attitude response in orbit around Ida; sinusoidal reference trajectory ............... 76
4.10 Spacecraft attitude response in orbit around Gaspra; zero reference trajectory ................. 77
4.11 Spacecraft attitude response in orbit around Gaspra; constant reference trajectory .......... 78
4.12 Spacecraft attitude response in orbit around Gaspra; sinusoidal reference trajectory .......... 79

6.1 Spacecraft attitude response in orbit around Eros; zero reference trajectory ................. 95
6.2 Spacecraft attitude response in orbit around Eros; constant reference trajectory ................. 96
6.3 Spacecraft attitude response in orbit around Eros; sinusoidal reference trajectory .......... 97
6.4 Spacecraft attitude response in orbit around Vesta; zero reference trajectory ................. 100
6.5 Spacecraft attitude response in orbit around Vesta; constant reference trajectory ............... 101
6.6 Spacecraft attitude response in orbit around Vesta; sinusoidal reference trajectory .......... 102
6.7 Spacecraft attitude response in orbit around Ida; zero reference trajectory .................... 104
6.8 Spacecraft attitude response in orbit around Ida; constant reference trajectory ................. 105
6.9 Spacecraft attitude response in orbit around Ida; sinusoidal reference trajectory ............... 106
6.10 Spacecraft attitude response in orbit around Gaspra; zero reference trajectory ................. 109
6.11 Spacecraft attitude response in orbit around Gaspra; constant reference trajectory .......... 110
6.12 Spacecraft attitude response in orbit around Gaspra; sinusoidal reference trajectory .......... 111
CHAPTER 1

INTRODUCTION

While the study of asteroids and comets has been well established in the fields of astronomy and aerospace for understanding the origins of the solar system, it is expected that more missions to asteroids will be planned as they move from an object of observation, to a possible future source of minerals and resources. For this reason, it is important to conduct orbital and attitude dynamic analyses for small bodies within our solar system. Indeed, there have already been many missions of satellites orbiting small asteroids, and in some cases, even landing on the surface. Some missions that have already been conducted include Galileo - NASA Mission to Jupiter via asteroids Gaspra and Ida (1989) and Near Earth Asteroid Rendezvous (NEAR) Shoemaker - NASA Mission to asteroid 433 Eros (1996), and another mission Dawn - NASA Mission to asteroids Ceres and Vesta (2007) [9], [15].

Indeed, many papers presented on the analysis and control of the orbit of spacecraft around asteroids have used four primary asteroids in their study; 433 Eros, 4 Vesta, 243 Ida, and 951 Gaspra. For simplicity, these asteroids will be refereed to as Eros, Vesta, Ida, and Gaspra, respectively, for the rest of this thesis. The numbers that precedes the asteroid’s names designate the order in which they were discovered. A photo of these asteroids can be seen in figure 1.1, taken during NASA missions near each asteroid. To provide some background for each asteroid, we begin with Eros. Eros is a relatively small, elongated asteroid located near the orbit of Mars. Relatively small here means it has a mean diameter
Figure 1.1: Pictures of four prominent small asteroids.

(a) Eros  
(b) Vesta  
(c) Ida  
(d) Gaspra
of 16.8 km; about half of the distance of the city limits of Las Vegas, Nevada. It was discovered in 1898 by a German astronomer and named after the Greek god of love. It became the first asteroid to be studied in orbit when the NEAR Shoemaker probe visited the asteroid in 1998, when into orbit in 1998, and landed on the surface in 2001 [24]. By contrast, Vesta is the second largest asteroid in the asteroid belt. To give some prospective, its 525 km diameter is roughly the distance from the western most side of the state of Colorado to the eastern most side. It was discovered almost a hundred years before Eros in 1807 by another German astronomer and was named after the roman goddess of home and hearth. In 2011, the NASA spacecraft Dawn entered orbit around Vesta for one year before continuing with its mission [16]. Ida, another asteroid in the asteroid belt, was discovered in 1884 by an Austrian astronomer and named after a nymph from Greek mythology. It’s mean radius is 15.7 km, making it about double the size of Eros, or roughly the size of Las Vegas. One notable feature is that Ida has its own natural satellite named Dactyl. Ida was last visited by the spacecraft Galileo in 1993 [17]. Our last asteroid, Gaspra is also found in the asteroid belt and was discovered by a Russian astronomer in 1916 and named after a coastal resort town near the Black Sea. It is a little smaller than Ida with a mean diameter of 12.2 km, and like Ida, it too was visited by the Galileo spacecraft in 1991 [18].

Previous work done in the subject of controlling the attitude states of spacecraft usually include either linear control, nonlinear control, and/or adaptive control. Some of the work dealing in linear feedback include Lou and Zhou’s paper Magnetic attitude control of bias momentum spacecraft by bounded linear feedback, which uses bounded linear feedback to control the attitude states of a spacecraft in orbit around the Earth [13]. Another paper that uses linearized dynamics but with adaptive linear feedback, is Kumar’s Attitude Dynamics
and Control of Satellites Orbiting Rotating Asteroids, which controls the attitude states of a spacecraft in orbit around asteroids. Some of the work done with nonlinear systems include Singh and Yim’s Nonlinear Adaptive Backstepping Design for Spacecraft Attitude Control Using Solar Radiation Pressure, which combines adaptive control via the backstepping technique to control the attitude states of a spacecraft in orbit.

While many papers have detailed the attitude dynamics of spacecraft in orbit around asteroids, the asteroids are usually assumed to be spherical, and their orbits circular [21]. These assumptions maybe appropriate for larger asteroids or planets, or even for orbits with extremely large orbital radii, however for orbits close to a non-uniform asteroid, these assumptions are inappropriate. The mass distribution, rotational rate, and irregular shape will significantly effect the orbit of the spacecraft and its attitude states over time. Specifically, the asteroid’s gravity coefficients $C_{20}, C_{22}$ are representative of the asteroid’s gravity field. These terms are much larger for asteroids, rather than planets, or other sufficiently large bodies, and so must be considered. Indeed, for irregular asteroid bodies, the non-uniformity and ellipticity will effect the gravitational potential because they are dependent on the mass distribution and shape of the asteroid [19]. Because of this necessity, the attitude of spacecraft in orbit around small asteroids has been an active area of research [20], [11].

This thesis builds on the attitude control of spacecraft orbiting small, irregularly shaped asteroids in elliptical orbits. Specifically, we design an adaptive controller such that the attitude states can be controlled even when the gravity coefficients are unknown. Because the controller is adaptive, the system will take into account the mass of the spacecraft, which may be unknown during the mission, and it will still be able to maintain the desired orientation. The closed-loop system is designed such that the tri-axis attitude states of the
spacecraft (pitch, roll, yaw) track desired trajectories, making the tracking error approach zero. To thoroughly test this controller, we will simulate a spacecraft in orbit around four of the most prominent small body asteroids (Eros, Vesta, Ida, Gaspra). Simulation results will then be carried out to demonstrate the effectiveness of the control law to stabilize the closed-loop system.
To begin forming the problem of controlling the pitch, roll, and yaw states of a spacecraft orbiting a rotating asteroid, we consider the model of a rigid body spacecraft, assumed to be in orbit in the equatorial plane of an irregularly shaped asteroid (Fig. 2.1). In the figure,
vertical, and the $Z_I$ axis completes the right handed frame. The orbital plane, as the name implies, is a plane that is intersected by both the asteroid and the spacecraft as they orbit around their shared barycenter; the center around which both masses orbit each other. For our purposes, we assume that the mass of the asteroid is much larger than the spacecraft, such that the barycenter is located within the center of mass of the asteroid. From figure 2.1, we see that the attitude states, or orientation, of the spacecraft is specified by three rotations; pitch ($\alpha$), roll ($\phi$), and yaw ($\gamma$). The asteroid is assumed to be irregularly shaped; meaning that it is not symmetric along any of its three axes. It is also assumed that the asteroid rotates at a constant rate about an axis that is perpendicular to its equatorial plane. The orbit of the asteroid is presumed to be planar and periodic for elliptical orbits.

A complete formation of the equations of motion for a spacecraft in equatorial orbit around an asteroid using the Lagrangian method is done by Kumar [11]. Not surprisingly, the dynamics are nonlinear, coupled, and time-varying. While these dynamics are not presented here, we will instead focus on a system of linearized equations of motion (Eqs. (2.1)-(2.3)),
again derived by Kumar [11].

\[ \alpha'' = -3k_2 \left[ 1 - \left( \frac{R_e}{R} \right)^2 \left( \frac{5}{2} C_{20} - 19C_{22}\cos(\nu\theta) \right) \right] \alpha \]
\[ -24k_2C_{22} \left( \frac{R_e}{R} \right)^2 \sin(\nu\theta) + 2e\sin(\theta) + Q_\alpha u_\alpha \tag{2.1} \]

\[ \phi'' = -(1-k_1)\gamma' - k_1 \left[ 4 - 3 \left( \frac{R_e}{R} \right)^2 (3.5C_{20} - 17C_{22}\cos(\nu\theta)) \right] \phi \]
\[ + 24k_3 \left[ C_{22} \left( \frac{R_e}{R} \right)^2 \sin(\nu\theta) \right] \gamma + \frac{Q_\phi}{K_{xx}} u_\phi \tag{2.2} \]

\[ \gamma'' = (1-k_3)\phi' - k_3 \left[ 1 - 3 \left( \frac{R_e}{R} \right)^2 (C_{20} - 2C_{22}\cos(\nu\theta)) \right] \gamma \]
\[ + 24k_3 \left[ C_{22} \left( \frac{R_e}{R} \right)^2 \sin(\nu\theta) \right] \phi + \frac{Q_\gamma}{K_{yx}} u_\gamma \tag{2.3} \]

From equations (2.1) - (2.3), we define \( k_2 = (k_1 - k_3)/(1-k_1k_3) \), \( \nu = 2(1 + (\omega/\Omega)) \), \( Q_q = (1-e^2)^2/(I_x\Omega^2(1+e\cos(\theta))^3) \), \( q \in \{ \alpha, \phi, \gamma \} \), \( \Omega = (\mu/(R_a^3)^{1/2} \) and \( R = R_a(1-e^2)/(1+e\cos(\theta)) \).

An important distinction to be made from these equations of motion, is that we use the prime symbol (') to denote the derivatives of a state instead of the dot symbol (\( \dot{} \)) because derivatives are taken with respect to true anomaly \( \theta \) and not time \( t \); \( \dot{q} = \dot{\theta}q' \). The reader is referred to the nomenclature in table 2.1, for a comprehensive definition of the terms involved.

The derivation of these equations of motion was conducted by linearizing the system about the equilibrium condition that the states, their derivatives, and the control law, is set equal to zero. Specifically, if we define the states as \( w = [\alpha, \phi, \gamma, \alpha', \phi', \gamma']^T \in \mathbb{R}^{6 \times 1} \), the set of equations can be represented as \( \dot{w} = f(w, t, u = 0) \). When the equilibrium condition \( w_e \) is applied, the system becomes \( f(w_e, t, u = 0) = 0 \) and the equilibrium state is found to be \( w_e = 0 \).
Nomenclature

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha, \phi, \gamma )</td>
<td>pitch, roll, yaw</td>
</tr>
<tr>
<td>( \theta )</td>
<td>true anomaly</td>
</tr>
<tr>
<td>( k_1, k_2, k_3 )</td>
<td>spacecraft’s mass distribution parameters</td>
</tr>
<tr>
<td>( K_{yx}, K_{zx} )</td>
<td>spacecraft’s mass distribution parameters</td>
</tr>
<tr>
<td>( I_i )</td>
<td>spacecraft’s moment of inertia about the ( i )-axis, ( i = x, y, z )</td>
</tr>
<tr>
<td>( m )</td>
<td>spacecraft’s mass</td>
</tr>
<tr>
<td>( C_{20}, C_{22} )</td>
<td>asteroids’s gravitational coefficients</td>
</tr>
<tr>
<td>( R )</td>
<td>asteroid’s orbital radius</td>
</tr>
<tr>
<td>( R_a )</td>
<td>asteroid’s orbital semi-major axis</td>
</tr>
<tr>
<td>( R_e )</td>
<td>asteroids’s characteristic length</td>
</tr>
<tr>
<td>( e )</td>
<td>asteroid’s orbital eccentricity</td>
</tr>
<tr>
<td>( \nu )</td>
<td>prograde orbit parameter</td>
</tr>
<tr>
<td>( \omega )</td>
<td>asteroid’s spin rate</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>asteroid’s mean orbital rate</td>
</tr>
<tr>
<td>( \mu )</td>
<td>asteroid’s gravitational constant parameter</td>
</tr>
<tr>
<td>( M )</td>
<td>asteroid’s mass</td>
</tr>
</tbody>
</table>

Table 2.1: Asteroid and spacecraft parameters

State-Variable Representation

With a desire for simplicity, we are motivated to form the equations of motion (Eqs. 2.1 - 2.3) into a more compact form. However we will assume that certain parameters are unknown, namely, the moment of inertia in the x-axis \( I_x \), the spacecraft mass distribution parameters \( k_1, k_2, k_3 \), and the gravity coefficients \( C_{20} \) and \( C_{22} \). The moment of inertia and mass distribution parameters are assumed to be unknown because the mass of the spacecraft will be diminishing throughout its mission with the expenditure of fuel while performing maneuvers. This fact is easier to see when we include the full definition of these parameters:

\[
k_1 = (I_x - I_y)/I_z, \quad k_2 = (I_z - I_y)/I_x, \quad k_3 = (I_x - I_z)/I_y \quad \text{and} \quad K_{yx} = (1 - k_1)/(1 - k_1 k_3), \quad K_{zx} = (1 - k_3)/(1 - k_1 k_3).\]

Thus we can not assume these terms to be constant. The gravity parameters are also assumed to be unknown, not necessarily because they are constantly changing, instead because they are usually difficult to compute, and even then, they are...
often an inaccurate simplification.

Due to the presence of these unknown terms, the equations of motion cannot be put into a simple state-space form. Instead, we will collect all the terms into two categories; known and unknown. To begin, we expand the equations of motion (Eqs. (2.1)-(2.3)), and collect the terms for each state. Note that we use the dot (\(\dot{}\)) symbol from here on to denote differentiation with respect to true anomaly \(\theta\).

\[
\ddot{\alpha} = -3\alpha k_2 + \frac{15}{2} \left(\frac{R_e}{R}\right)^2 \alpha k_2 C_{20} - 57 \left(\frac{R_e}{R}\right)^2 \alpha \cos(\nu \theta) k_2 C_{22} \\
+ 24 \left(\frac{R_e}{R}\right)^2 \sin(\nu \theta) k_2 C_{22} + 2 \epsilon \sin \theta + \frac{1}{I_x} Q_1 u_\alpha
\]

\[
= \psi_{10} + \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} \\ k_2 & k_2 C_{20} & k_2 C_{22} \end{bmatrix} + b_1 Q_1 u_1 \tag{2.4}
\]

\[
\ddot{\phi} = -\dot{\gamma} + \dot{\gamma} k_1 - 4 \phi k_1 + 10.5 \left(\frac{R_e}{R}\right)^2 \phi k_1 C_{20} - 51 \left(\frac{R_e}{R}\right)^2 \phi \cos(\nu \theta) k_1 C_{22} \\
+ 24 \gamma \left(\frac{R_e}{R}\right)^2 \sin(\nu \theta) k_2 C_{22} + \frac{1}{I_x K_{zz}} Q_2 u_\gamma 
\]

\[
= \psi_{20} + \begin{bmatrix} \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ k_1 & k_1 C_{20} & k_1 C_{22} & k_2 C_{22} \end{bmatrix} + b_2 Q_2 u_2 \tag{2.5}
\]
\[
\ddot{\gamma} = \dot{\phi} - (\dot{\phi} + \gamma) k_3 + 3 \left( \frac{R_e}{R} \right)^2 \gamma k_3 C_{20} + 6 \left( \frac{R_e}{R} \right)^2 \cos(\nu \theta) \gamma k_3 C_{22} \\
+ 24 \phi \left( \frac{R_e}{R} \right)^2 \sin(\nu \theta) k_3 C_{22} + \frac{1}{I_x K_{yx}} Q_3 u_\gamma
\]

\[
= \psi_{30} + \begin{bmatrix} \psi_{31} & \psi_{32} & \psi_{33} \end{bmatrix} \begin{bmatrix} k_3 \\ k_3 C_{20} \\ k_3 C_{22} \end{bmatrix} + b_3 Q_3 u_3 \tag{2.6}
\]

We define \( Q_i = (1 - \epsilon^2)^3/(\Omega^2(1 + \cos \theta)^3) \), \( i = 1, 2, 3 \). The definition of the \( \psi \) and \( b \) terms in Eqs. (2.4) - (2.6) can be done by inspection, and \( \psi_{10} = 0 \). Grouping Eqs. (2.4) - (2.6) into a matrix form:
\[
\begin{bmatrix}
\ddot{\alpha} \\
\ddot{\phi} \\
\ddot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
\psi_{10} \\
\psi_{20} \\
\psi_{30}
\end{bmatrix} +
\begin{bmatrix}
\psi_{11} & \psi_{12} & \psi_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} & \psi_{33}
\end{bmatrix}
\begin{bmatrix}
k_2 \\
k_2C_{20} \\
k_2C_{22} \\
k_1 \\
k_1C_{20} \\
k_1C_{22} \\
k_3 \\
k_3C_{20} \\
k_3C_{22}
\end{bmatrix}
\]

\[
\begin{bmatrix}
b_1 & 0 & 0 \\
0 & b_2 & 0 \\
0 & 0 & b_3
\end{bmatrix}
\begin{bmatrix}
Q_1 & 0 & 0 \\
0 & Q_2 & 0 \\
0 & 0 & Q_3
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\]

(2.7)

Defining vectors

\[
w_1 = [\alpha, \phi, \gamma]^T \in \mathbb{R}^{3\times1}
\]

(2.8)

\[
w_2 = [\dot{\alpha}, \dot{\phi}, \dot{\gamma}]^T \in \mathbb{R}^{3\times1}
\]

(2.9)
where $w_2 = \dot{w}_1,$

$$w = [w_1^T, w_2^T]^T \in \mathbb{R}^{6 \times 1}$$  \hspace{1cm} (2.10)

$$\Psi_0 = [\psi_{10}, \psi_{20}, \psi_{30}]^T \in \mathbb{R}^{3 \times 1}$$  \hspace{1cm} (2.11)

$$u = [u_1, u_2, u_3]^T \in \mathbb{R}^{3 \times 1}$$  \hspace{1cm} (2.12)

$$P = [k_2, k_2 C_{20}, k_2 C_{22}, k_1, k_1 C_{20}, k_1 C_{22}, k_2 C_{22}, k_3, k_3 C_{20}, k_3 C_{22}]^T \in \mathbb{R}^{10 \times 1}$$  \hspace{1cm} (2.13)

and the following matrices,

$$\Psi_1 = \begin{bmatrix}
\psi_{11} & \psi_{12} & \psi_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} & \psi_{33}
\end{bmatrix} \in \mathbb{R}^{3 \times 10}$$  \hspace{1cm} (2.14)

$$B = \text{diag}[b_1, b_2, b_3] \in \mathbb{R}^{3 \times 3}$$  \hspace{1cm} (2.15)

$$Q = \text{diag}[Q_1, Q_2, Q_3] \in \mathbb{R}^{3 \times 3}$$  \hspace{1cm} (2.16)

where ‘diag’ means diagonal matrix.

Then, a state-variable representation of the attitude dynamics from Eq (2.7) can be written in a more compact form as

$$\ddot{w}_1 = \Psi_0(w_2) + \Psi_1(w_1, w_2, \theta)P + BQu$$  \hspace{1cm} (2.17)
where $\Psi_0(w_2)$ and $\Psi_1(w_1, w_2, \theta)$ are matrices where the value of its parameters are known, while $P$ and $B$ are matrices where the value of its parameters are unknown.

**Error Dynamics**

For a given vector of a smooth, desired pitch, roll, and yaw reference trajectory, $w_{r1} = [\alpha_r, \phi_r, \gamma_r]^T$, where $w_{r2} = \dot{w}_{r1}$, the objective is to design a robust state-variable feedback control law such that the attitude states of the spacecraft, $w_1$, track the desired reference trajectories, $w_{r1}$, despite disturbances acting on the system and despite the uncertain parameters ($P, B$) present in the system. This must be done while the control law is realistically bounded to the physical constraints of the amount of torque, in newton meters, that a thruster of a spacecraft can produce. More specifically, we want the tracking error, defined as

$$\tilde{w}_1 = w_1 - w_{r1},$$

(2.18)

as well as its derivative,

$$\tilde{w}_2 = \dot{\tilde{w}}_1$$

$$= \dot{w}_1 - \dot{w}_{r1}$$

$$= w_2 - w_{r2},$$

(2.19)
to approach zero as time goes to infinity. Taking this definition of error (Eqs. 2.18 - 2.19) and applying it to our state-variable representation (Eq. 2.17), we derive our error dynamics:

\[
\dot{\tilde{w}} = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tilde{w} + \begin{bmatrix} 0_{3\times1} \\ -\ddot{w}_r + \Psi_0(w_2) + \Psi_1(w_1, w_2, \theta)P \end{bmatrix} + \begin{bmatrix} 0_{3\times3} \\ BQ \end{bmatrix} u
\]

(2.20)

\[
= A_{OL} \dot{\tilde{w}} + E_{OL}(w_1, w_2, w_r, \theta) + B_{OL} u
\]

(2.21)

where

\[
\dot{\tilde{w}} = [\tilde{w}^T_1, \tilde{w}^T_2]^T \in R^{6\times1}
\]

(2.22)

\[
A_{OL} = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \in R^{6\times6}
\]

(2.23)

\[
E_{OL} = \begin{bmatrix} 0_{3\times1} \\ -\ddot{w}_r + \Psi_0(w_2) + \Psi_1(w_1, w_2, \theta)P \end{bmatrix} \in R^{3\times1}
\]

(2.24)

\[
B_{OL} = [0_{3\times3}, BQ]^T \in R^{6\times3}
\]

(2.25)

In order to reduce the number of sensors needed during implementation, we will assume that only the angle of the spacecraft, and not its angular velocity or acceleration, are available for measurement, and thus feedback. This idea is represented by Eq. (2.26).

\[
y = [I_{3\times3}, 0_{3\times3}] \begin{bmatrix} \dot{\tilde{w}}_1 \\ \dot{\tilde{w}}_2 \end{bmatrix}
\]

(2.26)

Because the control law will require higher order attitude states, an observer needs to be
designed to estimate the states and their derivatives.
CHAPTER 3

STATE FEEDBACK

In this chapter, the design of a adaptive control law is derived to control the attitude states of the satellite. This derivation is partially based on the direct control method presented in Ioannou and Fidan’s *Adaptive Control Tutorial* [4]. In the direct control method, the values of the unknown plant parameters are estimated and substituted into the control law which is then fed back into the system. A visual representation of this process is presented in figure 3.1, where $\theta$ is the estimate of the unknown parameter $\theta^*$, and $\theta_C$ is the estimate that is fed into the controller to act on the plant. This differs from the indirect control method,

![Figure 3.1: Direct adaptive control structure.](image-url)
where the plant model is parametrized in terms of the control law parameters, which are then estimated, and fed back into the system.

The outline of this control law derivation is as follows; first, we design a control law such that the closed-loop system is asymptotically stable, second, we use an appropriate Lyapunov function to derive the adaption laws for parameter estimation, and lastly, we will simulate the results of the control law acting on our linear model. It will also be observed how well the controller can act on the plant when disturbance present in the model.

**Control Law**

Starting with our error dynamics (Eqs. 2.19), we add an estimation term \( \hat{\rho}_i, i = 1, 2, 3 \) as an estimate of \( b_i^{-1}, i = 1, 2, 3 \). We also introduce a new ‘synthetic’ control signal \( u_a \in R^{3x1} \) which will be derived at a later part in this chapter. The relationship between these terms is described in Eq. (3.1).

\[
\begin{align*}
  u_i &= Q_i^{-1} \hat{\rho}_i u_{ai} \\
      &= Q_i^{-1}(\hat{\rho}_i - b_i^{-1} + b_i^{-1})u_{ai} \\
      &= Q_i^{-1}(b_i \hat{\rho}_i + 1)u_{ai}
\end{align*}
\]  

**(3.1)**

We are ‘allowed’ to do this, because we have control over our control signal \( u \). Then, we can say

\[
\begin{align*}
  b_i u_i &= b_i Q_i^{-1} \hat{\rho}_i u_{ai} \\
         &= b_i Q_i^{-1}(\hat{\rho}_i - b_i^{-1} + b_i^{-1})u_{ai} \\
         &= Q_i^{-1}(b_i \hat{\rho}_i + 1)u_{ai}
\end{align*}
\]  

**(3.2)**
where \( \tilde{\rho} = \text{diag}[\tilde{\rho}_1, \tilde{\rho}_2, \tilde{\rho}_3] \in \mathbb{R}^{3\times 3} \) and \( \tilde{\rho}_i = \hat{\rho}_i - \hat{b}_i \) is an estimation error term. We created this synthetic control signal so that we can uncouple our control signal from the B matrix. With our control signal now freed, we can derive a control law that can act directly to stabilize our tracking error state as well as to implement an estimation law for our unknown parameters.

Substituting Eq. (3.3) into our error dynamics (Eq. 2.19), our expression becomes

\[
\dot{\tilde{w}} = \begin{bmatrix}
0_{3\times 3} & I_{3\times 3} \\
0_{3\times 3} & 0_{3\times 3}
\end{bmatrix} \tilde{w} + \begin{bmatrix}
0_{3\times 1} \\
-\tilde{w}_{r1} + \Psi_0 + \Psi_1 \hat{P}
\end{bmatrix} + \begin{bmatrix}
0_{3\times 3} \\
B \tilde{\rho}
\end{bmatrix} u_a + \begin{bmatrix}
0_{3\times 3} \\
I_{3\times 3}
\end{bmatrix} u_a
\]

You will notice that we omit the arguments of \( \Psi_1 \) and \( \Psi_0 \). This is for visual simplification only, and at no loss of generality. Given our modified error dynamics, (Eq. 3.3), we selected the following control law (Eqs. 3.4 - 3.5),

\[
u_a = -K_{f1} \tilde{w}_1 - K_{f2} \tilde{w}_2 - \Psi_0 - \Psi_1 \hat{P} + \tilde{w}_{r1} \quad (3.4)
\]

\[
u = Q^{-1} \hat{\rho} u_a \quad (3.5)
\]

where \( K_{f1} = \text{diag}[k_{f11}, k_{f12}, k_{f13}] \in \mathbb{R}^{3\times 3} \) and \( K_{f2} = \text{diag}[k_{f21}, k_{f22}, k_{f23}] \in \mathbb{R}^{3\times 3} \). \( K_{f1} \) and \( K_{f2} \) are our feedback gains and \( \hat{P} \in \mathbb{R}^{10\times 1} \) is an estimate of \( P \). The adaptation law for \( \hat{P} \) is derived from our Lyapunov function, later in this chapter. Substituting this control law (Eq. 3.4) into our error dynamics (Eq. 3.3), the resulting closed-loop system (Eq. 3.6) becomes

\[
\dot{\tilde{w}} = \begin{bmatrix}
0_{3\times 3} & I_{3\times 3} \\
-K_{f1} & -K_{f2}
\end{bmatrix} \tilde{w} + \begin{bmatrix}
0_{3\times 1} \\
-\Psi_1 \hat{P} + B \tilde{\rho} u_a
\end{bmatrix}
\]

19
It is easily seen from Eq. (3.6) that as the estimation error terms \( \hat{P} \) and \( \hat{\rho} \) go to zero, the closed-loop error dynamics become

\[ \dot{\tilde{w}} = A_{CL} \tilde{w} \]  

(3.7)

where

\[ A_{CL} = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -K_{f1} & -K_{f2} \end{bmatrix} \]  

(3.8)

The solution to this system of ODEs is

\[ \tilde{w} = \tilde{w}(0)e^{A_{CL}} \]

where \( \tilde{w}(0) \in \mathbb{R}^{6 \times 1} \) are the initial conditions of \( \tilde{w} \). If \( K_{f1} \) and \( K_{f2} \) are chosen such that \( A_{CL} \) is Hurwitz, the system will be asymptotically stable and the tracking error \( \tilde{w} \) will converge to zero as time goes to infinity. In the next section, we will use the Lyapunov function to derive the adaption laws for \( \hat{P} \) and \( \hat{\rho} \).

Adaption Laws

A powerful tool for determining stability is to use a Lyapunov function, \( V(x) \). Generally speaking, a Lyapunov function is a kind of energy function. Stability can be determined from a Lyapunov function if \( V(x) > 0 \) and \( \dot{V}(x) \leq 0 \). When this condition is satisfied, we say that \( x \) is stable in the sense of Lyapunov [23].

Another important tool for checking stability is to use the Lyapunov Theorem. The Lyapunov Theory states that a matrix \( A \) is stable if and only if for a given p.d. (positive
definite) symmetric matrix $N$, the Lyapunov equation

$$A^T M + MA = -N \quad (3.9)$$

has a unique symmetric solution $M$ which is p.d \cite{3}. For our purposes, that means that if $K_{f1}$ and $K_{f2}$ are chosen such that $A_{CL}$ is Hurwitz, there exists a positive definite matrix $H \in R^{6 \times 6}$ for any positive definite matrix $L \in R^{6 \times 6}$ such that

$$HA_{CL} + A_{CL}^T H = -L \quad (3.10)$$

Since $L$ can be any p.d. matrix, it acts as a gain that can be selected by the user. However, it has been shown that the convergence rate of the tracking error $\tilde{w}$ will be fastest for $L = I_{6 \times 6}$ \cite{23}, where $I_{n \times n}$ is an $n \times n$ identity matrix. Using this result, we will select $L$ to be $I_{6 \times 6}$. The Lyapunov Theorem will be used to derive the adaption laws for $\hat{P}$ and $\hat{\rho}$ from our Lyapunov function.

Looking at our closed-loop error dynamics Eq. (3.6) motivates us to consider the following Lyapunov function

$$V = \tilde{w}^T H \tilde{w} + \tilde{P}^T \Gamma^{-1} \tilde{P} + \sum_{i=1}^{3} \tilde{\rho}_{i}^2 \gamma_{ai}^{-1} |b_i| \quad (3.11)$$

where $\Gamma = \Gamma^T \in R^{10 \times 10}$ and $\gamma_{ai}, i = 1, 2, 3 \in R$ are positive definite adaption gains, to be tuned by the user to achieve the desired response. We also note that the sign of $B$ is always known, and in this case it will always be positive. This Lyapunov function is considered appropriate because it is p.d. and it is representative of our closed-loop system. We note that Eq. (3.11) is not unique; we could have chosen a different Lyapunov function, but the
results will verify that this function is sufficient to achieve our objective of reducing the tracking error \( \tilde{w} \) such that it approaches zero. Taking the derivative of Eq. (3.11) yields

\[
\dot{V}(\tilde{w}) = \dot{\tilde{w}}^T H \tilde{w} + \tilde{w}^T H \dot{\tilde{w}} + \dot{\tilde{P}} \Gamma^{-1} \tilde{P} + \dot{\tilde{P}}^T \Gamma^{-1} \dot{\tilde{P}} + 2 \sum_{i=1}^{3} \dot{\rho}_{i}\gamma_{ai}^{-1}|b_i|\tilde{\rho}_i
\]

\[
= (\tilde{w}^T A_{CL}^T + [0_{1 \times 3}, -\dot{\tilde{P}}^T \Psi_1^T + u_a^T \rho^T B^T]) H \tilde{w} + \tilde{w}^T H (A_{CL} \tilde{w} + \begin{bmatrix} 0_{3 \times 1} \\ -\Psi_1 \dot{\tilde{P}} + B\tilde{\rho}u_a \end{bmatrix})
\]

\[
+ 2 \dot{\tilde{P}}^T \Gamma^{-1} \dot{\tilde{P}} + 2 \sum_{i=1}^{3} \dot{\rho}_{i}\gamma_{ai}^{-1}|b_i|\tilde{\rho}_i \]

\[
= \tilde{w}^T A_{CL}^T H \tilde{w} + [0_{1 \times 3}, -\dot{\tilde{P}}^T \Psi_1^T + u_a^T \rho^T B^T] H \tilde{w} + \tilde{w}^T H A_{CL} \tilde{w} + \tilde{w}^T H \begin{bmatrix} 0_{3 \times 1} \\ -\Psi_1 \dot{\tilde{P}} + B\tilde{\rho}u_a \end{bmatrix}
\]

\[
+ 2 \dot{\tilde{P}}^T \Gamma^{-1} \dot{\tilde{P}} + 2 \sum_{i=1}^{3} \dot{\rho}_{i}\gamma_{ai}^{-1}|b_i|\tilde{\rho}_i \]

\[
= \tilde{w}^T (A_{CL}^T H + HA_{CL}) \tilde{w} + 2\tilde{w}^T H \begin{bmatrix} 0_{3 \times 1} \\ -\Psi_1 \dot{\tilde{P}} + B\tilde{\rho}u_a \end{bmatrix} + 2 \dot{\tilde{P}}^T \Gamma^{-1} \dot{\tilde{P}} + 2 \sum_{i=1}^{3} \dot{\rho}_{i}\gamma_{ai}^{-1}|b_i|\tilde{\rho}_i
\]

\[
= -\tilde{w}^T L \tilde{w} + 2\tilde{w}^T H \begin{bmatrix} 0_{3 \times 1} \\ I_{3 \times 1} \end{bmatrix} (-\Psi_1 \dot{\tilde{P}}) + 2 \tilde{w}^T H \begin{bmatrix} 0_{3 \times 1} \\ I_{3 \times 1} \end{bmatrix} (B\tilde{\rho}u_a)
\]

\[
+ 2 \dot{\tilde{P}}^T \Gamma^{-1} \dot{\tilde{P}} + 2 \sum_{i=1}^{3} \dot{\rho}_{i}\gamma_{ai}^{-1}|b_i|\tilde{\rho}_i
\]

Here, we define the following term

\[
\tilde{w}^T H \begin{bmatrix} 0_{3 \times 1} \\ I_{3 \times 1} \end{bmatrix} = [s_1, s_2, s_3] = s
\]
and use it to continue our expression of $\dot{V}$.

\[
\dot{V}(\tilde{w}) = -\tilde{w}^T L\tilde{w} + 2\tilde{P}^T \Gamma^{-1}\dot{\tilde{P}} - 2s\Psi_1 \dot{\tilde{P}} + 2sB\tilde{\rho} u_a + 2 \sum_{i=1}^{3} \dot{\tilde{\rho}}_i \gamma_{ai}^{-1} |b_i| \tilde{\rho}_i
\]

Using the fact that $b = sgn(b)|b|$, we complete our expression for $\dot{V}$.

\[
\dot{V}(\tilde{w}) = -\tilde{w}^T L\tilde{w} + 2\tilde{P}^T \Gamma^{-1}(\dot{\tilde{P}} - \Gamma \Psi_1^T s^T) + 2 \sum_{i=1}^{3} \gamma_{ai}^{-1} |b_i| (\dot{\tilde{\rho}} + \gamma_{ai}s_i sgn(b_i) u_{ai}) \tilde{\rho}_i \quad (3.13)
\]

In order to achieve stability in the sense of Lyapunov, $\dot{V}(\tilde{w}) \leq 0$. Looking at our expression for $\dot{V}(\tilde{w})$, we see that $\dot{V}(\tilde{w})$ can be $\leq 0$ if we choose $\tilde{P}$ and $\dot{\tilde{P}}$ such that they cancel their respective terms. Thus, it is from this expression of $\dot{V}(\tilde{w})$ that we derive our following adaption laws:

\[
\dot{\tilde{P}} = \dot{\tilde{P}} - \dot{\tilde{P}} = \dot{\tilde{P}} = \Gamma \Psi_1^T s^T \quad (3.14)
\]

\[
\dot{\tilde{\rho}}_i = \dot{\tilde{\rho}}_i - \dot{\tilde{\rho}}_i = \dot{\tilde{\rho}}_i = -\gamma_{ai}s_i sgn(b_i) u_{ai} \quad (3.15)
\]

where $P$ and $\rho_i$ are constants and so $\dot{P}, \dot{\rho}_i = 0$.

While these adaption laws guarantee that $\tilde{w}$ is stable in the sense of Lyapunov, it does not guarantee that it is asymptotically stable. A system is asymptotically stable if the Lyapunov function is $V(\tilde{w}) > 0$ and $\dot{V}(\tilde{w}) \leq 0$ but a system is asymptotically stable if
\( V(\tilde{w}) > 0 \) and \( \dot{V}(\tilde{w}) < 0 \). Put another way, a system is stable if \( V(\tilde{w}) \) is p.d. (positive definite) and \( \dot{V}(\tilde{w}) \) is n.s.d. (negative semidefinite), but a system is asymptotically stable if \( V(\tilde{w}) \) is p.d. and \( \dot{V}(\tilde{w}) \) is n.d. (negative definite). However, LaSalle-Yoshizawa Theorem [10] relaxes the condition that \( \dot{V}(\tilde{w}) \) must be n.d. so that \( \dot{V} \) only has to be n.s.d. Thus, because we can say that our adaption laws guarantee that \( \dot{V}(\tilde{w}) \leq 0 \), but \( \dot{V}(0) = 0 \) only at the origin, we can say that \( \tilde{w} \) is asymptotically stable and will converge to zero in as time goes to infinity. The LaSalle-Yoshizawa Theorem is a special time-varying case of the LaSalle Theorem. The LaSalle Theorem, and the LaSalle-Yoshizawa Theorem by extension, are built upon Barbalat’s Lemma, which states, if a differentiable function \( f(t) \) has a finite limit as \( t \to \infty \), and \( \dot{f}(t) \) is uniformly continuous, then \( \dot{f}(t) \to 0 \) as \( t \to \infty \). It is easily seen that this is used to establish stability in the sense of Lyapunov by using \( V(t) \) as \( f(t) \).

Results of State Feedback

With the completed design of our control law and adaption laws, we are now ready to implement our closed-loop system and simulate the results. However, before we review the simulation results, we will discuss the stability margins of our controller and we will employ a simple modification that can be made to our feedback gains \( K_{f1} \) and \( K_{f2} \). We begin by noting that our system is in fact a second order system, and as such, we can make a slight modification to the feedback gains.
Stability Margins

Reviewing our closed-loop dynamics when the estimation errors go to zero, Eq. (3.7), we can decompose our system from its matrix form into a system of ODEs.

\[
\dot{\tilde{w}} = A_{CL}\tilde{w}
\]

\[
\begin{bmatrix}
\dot{\tilde{w}}_1 \\
\dot{\tilde{w}}_2
\end{bmatrix} = \begin{bmatrix}
0_{3\times3} & I_{3\times3} \\
-K_f & -K_f
\end{bmatrix} \begin{bmatrix}
\tilde{w}_1 \\
\tilde{w}_2
\end{bmatrix}
\]

\[
\begin{align*}
\dot{\tilde{w}}_1 &= \tilde{w}_2 \\
\dot{\tilde{w}}_2 &= -K_f \tilde{w}_1 - K_f \tilde{w}_2
\end{align*}
\tag{3.16}
\]

We can rewrite Eq. (3.16) in terms of \(\tilde{w}_1\) so to arrive at the following expression

\[
\begin{align*}
\ddot{\tilde{w}}_1 &= -K_f \dot{\tilde{w}}_1 - K_f \tilde{w}_1 \\
\ddot{\tilde{w}}_1 + K_f \dot{\tilde{w}}_1 + K_f \tilde{w}_1 &= 0_{3\times1}
\end{align*}
\tag{3.17}
\]

Observing Eq. (3.17), it is easily recognized as the general form of a second order system. Acting on this conclusion, we take the Laplace transform of Eq. (3.17) and transition from
the time domain to the frequency domain such that it becomes

\[ \ddot{w}_1 s^2 + K_{f2} \dot{w}_1 s + K_{f1} \dot{w} = 0_{3 \times 1} \]  

(3.18)

\[ s^2 + K_{f2} s + K_{f1} = 0_{3 \times 3} \]  

(3.19)

where \( s = \text{diag}[s_1, s_2, s_3] \in R^{3 \times 3} \). For clarification, it should be noted that ‘\( s \)’ here is the variable of differentiation and not the previously defined term ‘\( s \)’ from our Lyapunov function. The general form of the characteristic equation for a second order system is provided below,

\[ s^2 + 2 \zeta \omega_n s + \omega_n^2 = 0_{3 \times 3} \]  

(3.20)

where \( \omega_n = \text{diag}[\omega_{n1}, \omega_{n2}, \omega_{n3}] \in R^{3 \times 3} \) and \( \zeta = \text{diag}[\zeta_1, \zeta_2, \zeta_3] \in R^{3 \times 3} \). Equating the coefficients from Eq. (3.19) and Eq. (3.20), we can relate our feedback gains to the damping ratio and natural frequency.

\[ K_{f1i} = \omega_{ni}^2, \quad i = 1, 2, 3 \]  

(3.21)

\[ K_{f2i} = 2 \zeta_i \omega_{ni}, \quad i = 1, 2, 3 \]  

(3.22)

This allows us to choose any desired value for our damping ratio and natural frequency, and the resulting feedback gains will be automatically set. To find the range of values that will yield a stable closed-loop system, we observe the solved expression for \( s_i \) in the general form
of a second order system (Eq. 3.20).

\[ s_{i,(1,2)} = -\zeta_i \omega_{ni} \pm \omega_{ni} \sqrt{\zeta_i^2 - 1}, \quad i = 1, 2, 3 \]  

(3.23)

From Eq. (3.23), it is easy to see that for closed-loop stability to be established, \( \zeta_i, \omega_{ni} > 0 \).

Of course, this is not a strict condition in that simply setting \( \zeta_i, \omega_{ni} > 0 \) may not be enough to ensure the tracking error converges to zero. Indeed, the system can still become unstable for a sufficiently large magnitude of the reference trajectory, initial tracking error, or disturbance. The system can even become unstable if \( \zeta_i \) and \( \omega_{ni} \) are too large. For these reasons, the gain parameters \( \zeta_i, \omega_{ni} \) still need to be manually tuned to find the desired response of the system.

Closed-Loop Simulation

In order to test the effectiveness of our controller, we will digitally simulate Eqs. (3.4) - (3.5) as they are applied to the models of four asteroids; Eros, Vesta, Ida, and Gaspra. The simulations will be conducted using Simulink because of its ease in modeling dynamic systems. The spacecraft will be given three desired attitude trajectories to track; a zero constant trajectory, a nonzero constant trajectory, and a sinusoidal trajectory, as provided
below.

\[
w_{r1} = [0, 0, 0]^T \text{ (deg)} \tag{3.24}
\]

\[
w_{r1} = [15, 40, 140]^T \text{ (deg)} \tag{3.25}
\]

\[
w_{r1} = [15\sin(3\omega t), 40\sin(\omega t), 140\sin(\omega t)]^T \text{ (deg)} \tag{3.26}
\]

\[
(3.27)
\]

We note that \( \omega = \frac{2\pi}{T} \), where the period, \( T = 30 \) rad. The zero trajectory is used to measure the systems ability to regulate its orientation from a nonzero initial angle to zero. A constant trajectory is meaningful in situations where the spacecraft needs to maintain a constant orientation, such as to point its solar panels toward the the sun during solar recharging sessions. It could also be used when the spacecraft needs to point to other far away bodies. Lastly, the sinusoidal trajectories would be necessary when the spacecraft needs to conduct observational scans of relatively close asteroids.

In an effort to reduce transience error, and to reduce the strain on our controller, all three desired trajectories will begin at the origin. For the constant trajectories, this means that a reference generator had to be designed. In most cases, a reference generator is a dedicated subsystem that receives the desired trajectories as input and outputs a slightly modified version such that it meets the needs of the controller. This is called input shaping. In our case, this simply means that our desired constant trajectory is modified from Eq (3.25) to

\[
w_{r1} = [15(1 - e^{-\tau t}), 40(1 - e^{-\tau t}), 140(1 - e^{-\tau t})]^T \text{ (deg)} \tag{3.28}
\]
where $\tau$ is a time constant to be selected such that it meets the desired settling time requirement. For a period of $T = 30\, rad$, we set $\tau = 0.2$.

In an effort to faithfully represent the realistic conditions of the system, we will add a modification to the controller; we will implement an upper and lower bound on the control law. This is done to represent the realistic constraint on the thrusters, in that there is a limit to how much torque they can produce at a given time. During these simulations, we will place an upper actuator bound on our controller of 1 Newton Meter ($N \cdot m$) and a lower actuator bound of -1 Newton Meters, which is a commonly assumed constraint for spacecraft already in orbit. Continuing our efforts to realistically represent the closed-loop system, we will have the mass of the spacecraft decrease during each simulation, as seen in Fig 3.2. This will also serve to check the competency of the adaption laws.

![Spacecraft Mass](image)

**Figure 3.2:** Mass of the spacecraft during its orbit.
Case A: Eros  
Beginning with Eros, we examine the performance of the controller. This test will determine how well the controller can handle an orbit with a relatively large eccentricity. The orbit parameter values used during this simulation are collected in table 3.1 which includes parameter values for both the spacecraft and the asteroid. The figures 3.3, 3.4, and 3.5 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. The maximum value of the attitude error is about -5.5 deg and also occurs when the desired reference trajectory is sinusoidal. Unsurprisingly, the control law bounds between its saturation value of $\pm 1$ during every response and its largest steady state value $\pm 1$ newton meters materializes during the constant and sinusoidal reference trajectory simulation. Finally, looking at the estimated parameter values, we see that the norm of $\hat{P}$ stays close to zero while the $\hat{\rho}$ elements converge to constant values. We note that these constant values do not correspond to their respective $b_i^{-1}$ values, and yet, the tracking error still converges to near zero.

<table>
<thead>
<tr>
<th>Eros Orbit Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>spacecraft mass</td>
</tr>
<tr>
<td>distribution parameters</td>
</tr>
<tr>
<td>gravitational coefficients</td>
</tr>
<tr>
<td>orbital radius</td>
</tr>
<tr>
<td>orbital semi-major axis</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
</tr>
<tr>
<td>orbital eccentricity</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
</tr>
<tr>
<td>asteroid mass</td>
</tr>
</tbody>
</table>

Table 3.1: Asteroid parameter values for Eros

Case B: Vesta  
Next, we observe how the controller performs when the spacecraft is
Figure 3.3: Spacecraft attitude response in orbit around Eros. Desired trajectory $w_{r1} = [0, 0, 0]^T$ (deg), initial conditions $w_1(0) = [2, 2.5, 3.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1], \omega_n = 20\text{diag}[2, 1, 1], \Gamma = 10^{-3}I_{10 \times 10}, \gamma_a = 1, 1, 1, H = I_{6 \times 6}$. 


Figure 3.4: Spacecraft attitude response in orbit around Eros. Desired trajectory $w_{r1} = [15, 40, 140]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[3, 5, 1]$, $\Gamma = 10^{-3}I_{10 \times 10}$, $\gamma_a = 1, 1, 1$, $H = I_{6 \times 6}$. 
Figure 3.5: Spacecraft attitude response in orbit around Eros. Desired trajectory $w_r = [15\sin(3\omega t), 40\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\dot{\Gamma}(0) = 0_{10 \times 1}$, $\dot{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[3, 3, 1]$, $\Gamma = 10^{-6}I_{10 \times 10}$, $\gamma_a = 1, 1, 1$, $H = I_{6 \times 6}$. 
in orbit around Vesta. This orbit will be a unique evaluation, as Vesta has the largest orbital radius, characteristic length, and mass among the four asteroids. The orbit parameter values used during this simulation are collected in table 3.2 which includes parameter values for both the spacecraft and the asteroid. The figures 3.6, 3.7, and 3.8 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Examining again the steady state response of the various measurements, we see that the maximum value of the attitude error is about -5.5 deg and also occurs when the desired reference trajectory is sinusoidal. Again, The control law bounds between its saturation value of ±1 during every response, however, its largest steady state value is also ±1 newton meters, and materializes during the during the constant reference trajectory simulation. Similar to Eros, we see that the norm of \( \hat{P} \) stays close to zero while the \( \hat{\rho} \) elements look to slowly converge to constant values. One more, these constant values do not correspond to their respective \( b_i^{-1} \) values, and yet, the tracking error still converges near zero.

<table>
<thead>
<tr>
<th>Vesta Orbit Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>spacecraft mass distribution parameters</td>
<td>( k_1 = -0.5, k_3 = 0.3 )</td>
</tr>
<tr>
<td>spacecraft mass</td>
<td>( m = 625 \text{ kg} )</td>
</tr>
<tr>
<td>gravitational coefficients</td>
<td>( C_{20} = 0.0512, C_{22} = -0.0055 )</td>
</tr>
<tr>
<td>orbital radius</td>
<td>( R = 640 \text{ km} )</td>
</tr>
<tr>
<td>orbital semi-major axis</td>
<td>( R_a = 3.5332 \times 10^8 \text{ km} )</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
<td>( R_e = 244.3 \text{ km} )</td>
</tr>
<tr>
<td>orbital eccentricity</td>
<td>( e = 0.0891526 )</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
<td>( \omega = 3.293074 \times 10^{-4} \text{ s}^{-1} )</td>
</tr>
<tr>
<td>asteroid mass</td>
<td>( M = 2.668 \times 10^{20} \text{ Kg} )</td>
</tr>
</tbody>
</table>

Table 3.2: Asteroid parameter values for Vesta
Figure 3.6: Spacecraft attitude response in orbit around Vesta. Desired trajectory $w_{r1} = [0, 0, 0]^T$ (deg), initial conditions $w_1(0) = [2, 2, 2.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\dot{\Gamma}(0) = 0_{10 \times 1}$, $\dot{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = diag[1, 1, 1], \omega_n = 20 diag[2, 1, 1], \Gamma = 10^{-6} I_{10 \times 10}, \gamma_a = 1, 1, 1, H = I_{6 \times 6}$. 
Figure 3.7: Spacecraft attitude response in orbit around Vesta. Desired trajectory \( \omega_{r1} = [15, 40, 140]^T \) (deg), initial conditions \( \omega_1(0) = [0, 0, 0]^T \) (deg), \( \omega_2(0) = [0.1, 0.1, 0.1]^T \), \( \hat{\Gamma}(0) = 0_{10 \times 1} \), \( \hat{\rho}(0) = 0_{3 \times 3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1] \), \( \omega_n = 20 \text{diag}[3, 5, 1] \), \( \Gamma = 10^{-6} I_{10 \times 10} \), \( \gamma_a = 1, 1, 1 \), \( H = I_{6 \times 6} \).
Figure 3.8: Spacecraft attitude response in orbit around Vesta. Desired trajectory \( w_{r1} = [15\sin(3\omega t), 40\sin(\omega t), 140\sin(\omega t)]^T \) (deg), initial conditions \( w_1(0) = [0, 0, 0]^T \) (deg), \( w_2(0) = [0.1, 0.1, 0.1]^T \), \( \hat{\Gamma}(0) = 0_{10 \times 1}, \hat{\rho}(0) = 0_{3 \times 3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1] \), \( \omega_n = 20\text{diag}[3, 3, 1] \), \( \Gamma = 10^{-6}I_{10 \times 10}, \gamma_a = 1, 1, 1, H = I_{6 \times 6} \).
Case C: Ida  
Our next analysis concerns the performance of the controller during an orbit around Ida. Ida separates itself from the other four asteroids for having the largest semi-major axis and the fastest revolution period. The orbit parameter values used during this simulation are collected in table 3.3, including parameter values for both the spacecraft and the asteroid. The figures 3.9, 3.10, and 3.11 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Review of the steady state response of various measurements reveal that the maximum value of the attitude error is about -5.5 deg, again similar to Vesta, and also occurs when the desired reference trajectory is sinusoidal. After bounding between its saturation limits, the control law reaches its maximum steady state value of ±1 newton meters, which materializes during the constant reference trajectory simulation. The estimated parameter values, are similar to Vesta in that the norm of \( \hat{P} \) stays close to zero while the \( \hat{\rho} \) elements slowly converge to constant values which are not the respective \( b_i^{-1} \) values.

<table>
<thead>
<tr>
<th>Ida Orbit Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>spacecraft mass distribution parameters</td>
<td>( k_1 = -0.5, k_3 = 0.3 )</td>
</tr>
<tr>
<td>spacecraft mass</td>
<td>( m = 625 \text{ kg} )</td>
</tr>
<tr>
<td>gravitational coefficients</td>
<td>( C_{20} = -0.0902, C_{22} = 0.0408 )</td>
</tr>
<tr>
<td>orbital radius</td>
<td>( R = 43.4 \text{ km} )</td>
</tr>
<tr>
<td>orbital semi-major axis</td>
<td>( R_a = 4.2845 \times 10^8 \text{ km} )</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
<td>( R_e = 15.22 \text{ km} )</td>
</tr>
<tr>
<td>orbital eccentricity</td>
<td>( e = 0.0413593 )</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
<td>( \omega = 3.7696 \times 10^{-4} \text{ s}^{-1} )</td>
</tr>
<tr>
<td>asteroid mass</td>
<td>( M = 4.121 \times 10^{16} \text{ Kg} )</td>
</tr>
</tbody>
</table>

Table 3.3: Asteroid parameter values for Ida

Case D: Gaspra  
We end our simulation results by looking at performance of the
Figure 3.9: Spacecraft attitude response in orbit around Ida. Desired trajectory $w_{r1} = [0, 0, 0]^T$ (deg), initial conditions $w_1(0) = [2.5, 2.5, 2.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1], \omega_n = 20\text{diag}[2, 1, 1], \Gamma = 10^{-6}I_{10 \times 10}, \gamma_a = 1, 1, 1, H = I_{6 \times 6}$. 
Figure 3.10: Spacecraft attitude response in orbit around Ida. Desired trajectory $w_{r1} = [15, 40, 140]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[3, 5, 1]$, $\Gamma = 10^{-6}I_{10 \times 10}$, $\gamma_a = 1, 1, 1$, $H = I_{6 \times 6}$. 
Figure 3.11: Spacecraft attitude response in orbit around Vesta. Desired trajectory
\( \mathbf{w}_r(1) = [15\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T \) (deg), initial conditions \( \mathbf{w}_1(0) = [0, 0, 0]^T \) (deg), \( \mathbf{w}_2(0) = [0.1, 0.1, 0.1]^T \), \( \hat{\Gamma}(0) = 0_{10 \times 1} \), \( \hat{\rho}(0) = 0_{3 \times 3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1] \), \( \omega_n = 20\text{diag}[3, 3, 1] \), \( \Gamma = 10^{-6}I_{10 \times 10} \), \( \gamma_a = 1, 1, 1 \), \( H = I_{6 \times 6} \).
controller during the orbit around Gaspra. Unlike the previous asteroids, Gaspra doesn’t have a maximum parameter value to act as a unique test for the controller; all of its parameters are in a middle to low range of values. The orbit parameter values used during this simulation are collected in table 3.4 which includes parameter values for both the spacecraft and the asteroid. The figures 3.12, 3.13, and 3.14 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Examining the steady state response of the various measurements, we see that the maximum values for the various measurements are very similar to Vesta and Gaspra; the maximum value of the attitude error is about -5.5 deg and also occurs when the desired reference trajectory is sinusoidal. The control law also follows the trend of Vesta and Gaspra by bounding between ±1 during every response, and having its largest steady state value of ±1 newton meters during the constant reference trajectory simulation. The parameter estimations also have results similar to the previous asteroids.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>spacecraft mass distribution parameters</td>
<td>$k_1 = -0.5, k_3 = 0.3$</td>
</tr>
<tr>
<td>spacecraft mass</td>
<td>$m = 625 \text{ kg}$</td>
</tr>
<tr>
<td>gravitational coefficients</td>
<td>$C_{20} = -0.0729, C_{22} = 0.0301$</td>
</tr>
<tr>
<td>orbital radius</td>
<td>$R = 25.5 \text{ km}$</td>
</tr>
<tr>
<td>orbital semi-major axis</td>
<td>$R_a = 3.306 \times 10^8 \text{ km}$</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
<td>$R_e = 6.793 \text{ km}$</td>
</tr>
<tr>
<td>orbital eccentricity</td>
<td>$e = 0.1734017$</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
<td>$\omega = 2.4933 \times 10^{-4} \text{ s}^{-1}$</td>
</tr>
<tr>
<td>asteroid mass</td>
<td>$M = 2.5 \times 10^{16} \text{ Kg}$</td>
</tr>
</tbody>
</table>

Table 3.4: Asteroid parameter values for Gaspra
Figure 3.12: Spacecraft attitude response in orbit around Gaspra. Desired trajectory \( \dot{w}_{r1} = [0, 0, 0]^T \) (deg), initial conditions \( w_1(0) = [2.5, 2.5, 2.5]^T \) (deg), \( w_2(0) = [0.1, 0.1, 0.1]^T \), \( \dot{\Gamma}(0) = 0_{10 \times 1}, \dot{\rho}(0) = 0_{3 \times 3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1], \omega_n = 20\text{diag}[2, 1, 1], \Gamma = 10^{-6}I_{10 \times 10}, \gamma_a = 1, 1, 1, H = I_{6 \times 6} \).
Figure 3.13: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $w_{r1} = [15, 40, 140]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{r}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20 \text{diag}[3, 5, 1]$, $\Gamma = 10^{-6} I_{10 \times 10}$, $\gamma_a = 1, 1, 1$, $H = I_{6 \times 6}$. 
Figure 3.14: Spacecraft attitude response in orbit around Gaspra. Desired trajectory \( w_{r1} = [15\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T \) (deg), initial conditions \( w_1(0) = [0, 0, 0]^T \) (deg), \( w_2(0) = [0.1, 0.1, 0.1]^T \), \( \hat{\Gamma}(0) = 0_{10\times1}, \hat{\rho}(0) = 0_{3\times3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1] \), \( \omega_n = 20\text{diag}[3, 3, 1] \), \( \Gamma = 10^{-6}I_{10\times10} \), \( \gamma_a = 1, 1, 1 \), \( H = I_{6\times6} \).
Effects of Disturbance

The last modification that will be added to the model is disturbance. Realistic spacecraft orbit missions will always have to deal with some type of disturbance present. When considering the attitude states of a spacecraft, disturbance takes the form of a gravity gradient torque, an aerodynamics torque, a magnetic torque, and a solar radiation torque [5]. The gravity gradient torque comes from the celestial body’s gravitational force acting over the asymmetric body of the spacecraft. The aerodynamics torque is caused by interaction between the upper atmosphere and the spacecraft. The magnetic torque comes from the interaction between the magnetic fields of the spacecraft and the celestial body. Lastly, the solar radiation torque is produced from solar radiation particles hitting the spacecraft’s surface. Fig 3.15 shows the intensity of these disturbance torques at varying attitudes.

![Figure 3.15: Disturbances present for a spacecraft orbiting a celestial body at varying altitudes.](image)

Figure 3.15: Disturbances present for a spacecraft orbiting a celestial body at varying altitudes.
However, in the case of a spacecraft orbing a relatively small asteroid, the gravity torque, and magnetic torque are very small, and the aerodynamic torque is nonexistent. Because the spacecraft and asteroid are usually so far from other planets, the solar radiation torque is negligibly small. For these reasons, the orbit of spacecraft around small asteroids is usually assumed to be free of disturbance. Despite this, we still wish to test the robustness of our controller, so we will add disturbance to our plant such that the plant will be modified from Eq. (2.17) to

\[
\ddot{w}_1 = \Psi_0(w_2) + \Psi_1(w_1, w_2, \theta)P + B(u + D(t)) \tag{3.29}
\]

where \(D(t) = [d_{d1}, d_{d2}, d_{d3}]^T \in \mathbb{R}^{3 \times 1}\) and

\[
\begin{align*}
    d_{d1} &= (1 - 10\sin(\theta) - 2\sin(2\theta)) \times 10^{-6} \quad N \cdot m \\
    d_{d2} &= (1 - 5\sin(\theta) - 0.5\sin(2\theta)) \times 10^{-6} \quad N \cdot m \\
    d_{d3} &= (2 + 3\sin(\theta) + 2\sin(2\theta)) \times 10^{-6} \quad N \cdot m
\end{align*}
\]

This model of disturbance comes from Lee and Singh’s paper *A Higher-Order Sliding Mode Three-Axis Solar Pressure Satellite Attitude Control System* [12].

Using the model of a spacecraft in orbit around Eros, we will again simulate the system’s response to the zero, constant, and sinusoidal trajectories, this time, with disturbance acting on the system. For each simulation, the respective desired trajectories, tuning gains, and initial conditions remain the same from figures 3.3 - 3.5. The results will be displayed side-by-side with figures 3.3 - 3.5, to highlight any disparities between the system’s response with no disturbance and a system’s response with disturbance. These comparisons can be seen in
figures 3.16 - 3.30.

**Case E: Disturbance**  
Careful examination of the results from figures 3.16-3.30 reveal that when disturbances are present, the system’s response differs quite negligibly from when there is no disturbance present. Indeed, all five responses, including attitude tracking error $\tilde{w}_1$, attitude angular velocity error $\tilde{w}_2$, actuator effort $u$, and the parameter estimates, show no meaningful difference when disturbance is present. This fact remains true for all three trajectories; zero, constant, and sinusoidal. Although it is not presented here, the effect of disturbance acting on spacecraft in orbit around the other three asteroids, Vesta, Ida, and Gaspra, were tested and found to have the same minimal consequence on the system’s response. However, these results are not surprising as we have already established that the magnitude of disturbance is quite small and is often even ignored because of this fact. Despite this, we will continue to include disturbance in our future simulations of the system to demonstrate the robustness of our controller to any disturbance that might be present.
Figure 3.16: Spacecraft attitude response in orbit around Eros during a zero reference trajectory. Top - Attitude states and reference trajectories, no disturbance acting on the system. Bottom - Attitude states and reference trajectories, disturbance present in the system.
Figure 3.17: Spacecraft attitude response in orbit around Eros during a zero reference trajectory. Top - Attitude tracking error, no disturbance acting on the system. Bottom - Attitude tracking error, disturbance present in the system.
Figure 3.18: Spacecraft attitude response in orbit around Eros during a zero reference trajectory. Top - Attitude angular velocity tracking error, no disturbance acting on the system. Bottom - Attitude angular velocity tracking error, disturbance present in the system.
Figure 3.19: Spacecraft attitude response in orbit around Eros during a zero reference trajectory. Top - Actuator effort, no disturbance acting on the system. Bottom - Actuator effort, disturbance present in the system.
Figure 3.20: Spacecraft attitude response in orbit around Eros during a zero reference trajectory. Top - Estimated parameters, no disturbance acting on the system. Bottom - Estimated parameters, disturbance present in the system.
Figure 3.21: Spacecraft attitude response in orbit around Eros during a constant reference trajectory. Top - Attitude states and reference trajectories, no disturbance acting on the system. Bottom - Attitude states and reference trajectories, disturbance present in the system.
Figure 3.22: Spacecraft attitude response in orbit around Eros during a constant reference trajectory. Top - Attitude tracking error, no disturbance acting on the system. Bottom - Attitude tracking error, disturbance present in the system.
Figure 3.23: Spacecraft attitude response in orbit around Eros during a constant reference trajectory. Top - Attitude angular velocity tracking error, no disturbance acting on the system. Bottom - Attitude angular velocity tracking error, disturbance present in the system.
Figure 3.24: Spacecraft attitude response in orbit around Eros during a constant reference trajectory. Top - Actuator effort, no disturbance acting on the system. Bottom - Actuator effort, disturbance present in the system.
Figure 3.25: Spacecraft attitude response in orbit around Eros during a constant reference trajectory. Top - Estimated parameters, no disturbance acting on the system. Bottom - Estimated parameters, disturbance present in the system.
Figure 3.26: Spacecraft attitude response in orbit around Eros during a sinusoidal reference trajectory. Top - Attitude states and reference trajectories, no disturbance acting on the system. Bottom - Attitude states and reference trajectories, disturbance present in the system.
Figure 3.27: Spacecraft attitude response in orbit around Eros during a sinusoidal reference trajectory. Top - Attitude tracking error, no disturbance acting on the system. Bottom - Attitude tracking error, disturbance present in the system.
Figure 3.28: Spacecraft attitude response in orbit around Eros during a sinusoidal reference trajectory. Top - Attitude angular velocity tracking error, no disturbance acting on the system. Bottom - Attitude angular velocity tracking error, disturbance present in the system.
Figure 3.29: Spacecraft attitude response in orbit around Eros during a sinusoidal reference trajectory. Top - Actuator effort, no disturbance acting on the system. Bottom - Actuator effort, disturbance present in the system.
Figure 3.30: Spacecraft attitude response in orbit around Eros during a sinusoidal reference trajectory. Top - Estimated parameters, no disturbance acting on the system. Bottom - Estimated parameters, disturbance present in the system.
CHAPTER 4

OUTPUT FEEDBACK

In an effort to reduce the number of sensors that need to be on board the spacecraft, an observer can be designed to estimate the higher-order states required for implementation. Specifically, for our system, we can design an observer that estimates the $\tilde{w}$ and $\dot{\tilde{w}}$ states. It is assumed that the spacecraft is at least equipped with onboard sensors that can measure the angles of its pitch, roll, and yaw. Then we can assume that the angle error $\tilde{w}$ is available for feedback. Using an observer, we will estimate the $\dot{\tilde{w}}$ state, or the angular velocity error state. This will negate the need for the spacecraft to have an onboard velocity sensor.

Observer Design

Because our system has unknown parameters present, we cannot use a simple Luenberger observer. There are adaptive observers we can use, however, we will elect to use a high-gain observer, as presented by Khalil and Praly [8]. The general form of a high-gain observer from Khalil and Praly’s paper has been designed to be appropriate for our adaptive control scheme; it is presented below in Eqs. (4.1)-(4.2).

\[
\begin{align*}
\dot{\tilde{w}}_{e1} &= \tilde{w}_{e2} + \varepsilon^{-1}d_1(\tilde{w}_1 - \tilde{w}_{e1}), \\
\dot{\tilde{w}}_{e2} &= -\tilde{w}_{r1} + \Psi_0(\tilde{w}_{e2}, w_{r2}) + \varepsilon^{-2}d_2(\tilde{w}_1 - \tilde{w}_{e1})
\end{align*}
\]
Because we assume the satellite has angle measurement state feedback, we can set \( \tilde{w}_e(0) = \tilde{w}_1(0) \), however, we can not say the same for the angular velocity states, \( \tilde{w}_e(0) \neq \tilde{w}_2(0) \). \( \tilde{w}_1 \) and \( \tilde{w}_2 \) are the states of the observer, where \( \tilde{w}_1 \) is the estimated value of \( \tilde{w}_1 \), and \( \tilde{w}_2 \) is the estimated value of \( \tilde{w}_2 \). The terms \( \varepsilon \) and \( d_1, d_2 \) are gains to be tuned. Specifically, \( \varepsilon > 0 \) should be chosen to be sufficiently small and \( d_1, d_2 \) are selected such that the roots of the polynomial

\[
\xi^2 + d_1 \xi + d_2 = 0
\]

are stable. It has been proven by Khalil and Esfandiari [7] that an appropriate choice of the observer gains will yield a fast convergence of the estimation errors to the origin. This, in turn, will keep the closed-loop system stable, including the observer. Eqs. (4.1)-(4.2) are designed in this way such that the closed loop of the observer is asymptotically stable.

Defining our estimation errors as

\[
\tilde{w}_{ee1} = \tilde{w}_1 - \tilde{w}_e1 \tag{4.3}
\]

\[
\tilde{w}_{ee2} = \tilde{w}_2 - \tilde{w}_e2 \tag{4.4}
\]

in conjunction with Eqs. (4.1 - 4.2), we can say that

\[
\dot{\tilde{w}}_{ee1} = \dot{\tilde{w}}_1 - \dot{\tilde{w}}_e1
\]

\[
= \tilde{w}_2 - \tilde{w}_e2 - \varepsilon^{-1}d_1(\tilde{w}_1 - \tilde{w}_e1)
\]

\[
= \tilde{w}_{ee2} - \varepsilon^{-1}d_1\tilde{w}_{ee1} \tag{4.5}
\]
and likewise

\[
\dot{\tilde{w}}_{ee2} = \dot{\hat{w}}_{2} - \hat{w}_{e2} \\
= -\ddot{w}_{r1} + \Psi_{0}(w_{2}) + \ddot{w}_{r1} - \Psi_{0}(\tilde{w}_{e2}, w_{r2}) \\
- \varepsilon^{-2}d_{2}(\tilde{w}_{1} - \tilde{w}_{e1}) \\
= \Psi_{0}(w_{2}) - \Psi_{0}(\tilde{w}_{e2}, w_{r2}) - \varepsilon^{-2}d_{2}\tilde{w}_{ee1} \tag{4.6}
\]

It is noted, that the $\Psi_{1}, B,$ and $u$ terms are omitted here because they contain terms that are assumed to be unknown such as $B, P,$ and $\rho$. Finally, we neatly collect Eqs. (4.5) - (4.6) into one closed-loop system Eqs. (4.7) - (4.8).

\[
\dot{\tilde{w}}_{ee1} = \tilde{w}_{ee2} - \varepsilon^{-1}d_{1}\tilde{w}_{ee1} \tag{4.7}
\]
\[
\dot{\tilde{w}}_{ee2} = \Psi_{0}(w_{2}) - \Psi_{0}(\tilde{w}_{e2}, w_{r2}) - \varepsilon^{-2}d_{2}\tilde{w}_{ee1} \tag{4.8}
\]

As stated earlier, an appropriate choice of $\varepsilon$, and $d_{1}, d_{2}$ will make the closed-loop system in Eqs. (4.7)-(4.8) stable. This validates our earlier observer design, and guarantees that the value of the estimated state $\tilde{w}_{e2}$ will converge to the value of the true state $\tilde{w}_{2}$ as time goes to infinity.

Observer Results

Here, we will test our observer by simulating the system’s response to three reference trajectories; zero, constant, and sinusoidal, in orbit around Eros, Vesta, Ida, and Gaspra. The observer will be challenged to estimate the attitude angular velocity tracking error and
substitute this estimation $\tilde{w}e_2$ in place of the actual attitude angular velocity tracking error $\tilde{w}_2$. As stated previously, we will include disturbance as an input into the system.

Examination of the estimation error from figures 4.1-4.12 demonstrates that the high-gain observer provides a fast, tight convergence of the estimated angular velocity tracking error $\tilde{w}e_2$ to the true angular velocity tracking error $\tilde{w}_2$. This can be seen as the estimation error $\tilde{w}ee_2$ converges to zero before half of the orbit is complete. This can be attributed to the ‘high-gain’ nature of the observer in that we chose it’s parameters $d_1, d_2,$ and $\varepsilon$ to be of a relatively high value; $d_1 = 20, d_2 = 100$, and $\varepsilon = 10^{-3}$. These gains were manually selected and determined to be sufficient for our purposes. Higher gains would presumably have lead to faster, tighter convergence however this increases simulation time and our current gain values already yield adequate results. We also note that the observer is robust enough to operate when disturbances are present in the system. If this were not the case, another option would be to use an extended state observer (ESO) which would estimate both the higher-order states and the disturbance and use that estimate of disturbance to cancel to actual disturbance acting on the system.
Figure 4.1: Spacecraft attitude response in orbit around Eros. Desired trajectory $w_{r1} = [0, 0, 0]^T$ (deg), initial conditions $w_1(0) = [2, 2.5, 2.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 10}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[1, 1, 1]$, $\Gamma = 10^{-3}I_{10 \times 10}$, $\gamma_a = 68$. 

- Attitude States and Desired Reference Trajectories
- Attitude Tracking Error
- Angular Velocity Attitude Tracking Error
- Attitude Velocity States and Desired Velocity Reference Trajectories
- Estimation Error of Angular Velocity Tracking Error
- Thruster Torques
- Estimated Parameters
Figure 4.2: Spacecraft attitude response in orbit around Eros. Desired trajectory $\mathbf{w}_r = [10, 40, 140]^T \text{ (deg)}$, initial conditions $\mathbf{w}_1(0) = [0, 0, 0]^T \text{ (deg)}$, $\mathbf{w}_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\mathbf{r}}(0) = 0$ $\text{deg}$, $\hat{\phi}(0) = 0$ $\text{deg}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20 \text{diag}[3, 5, 1]$, $\Gamma = 10^{-3} \Lambda_{10 \times 10}$, $\gamma_a = 69$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Attitude Velocity States and Desired Velocity Reference Trajectories

Estimation Error of Angular Velocity Tracking Error

Thruster Torques

Estimated Parameters
Figure 4.3: Spacecraft attitude response in orbit around Eros. Desired trajectory $w_1 = [10\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 70$. 
Figure 4.4: Spacecraft attitude response in orbit around Vesta. Desired trajectory $\psi_{r1} = [0, 0, 0]^T$ (deg), initial conditions $\psi_1(0) = [2.5, 2.5, 3]^T$ (deg), $\psi_2(0) = [0.1, 0.1, 0.1]^T$, $\dot{\theta}(0) = 0$, $\dot{\phi}(0) = 0$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[2, 1, 1]$, $\Gamma = 10^{-3}I_{10\times10}$, $\gamma_a = 71$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Estimation Error of Angular Velocity Tracking Error

Thruster Torques

Estimated Parameters
Figure 4.5: Spacecraft attitude response in orbit around Vesta. Desired trajectory \( w_{r1} = [10, 40, 140]^T \) (deg), initial conditions \( w_1(0) = [0, 0, 0]^T \) (deg), \( w_2(0) = [0.1, 0.1, 0.1]^T \), \( \hat{\Gamma}(0) = 0_{10 \times 1}, \hat{\rho}(0) = 0_{3 \times 3} \). Tuning gains \( \zeta = \text{diag}[1, 1, 1], \omega_n = 20 \text{diag}[3, 5, 1], \Gamma = 10^{-3}I_{10 \times 10}, \gamma_a = 72 \).
Figure 4.6: Spacecraft attitude response in orbit around Vesta. Desired trajectory $\mathbf{w}_1 = [10\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $\mathbf{w}_1(0) = [0, 0, 0]^T$ (deg), $\mathbf{w}_2(0) = [0, 0.1, 0.1, 0.1]^T$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 73$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Attitude Velocity States and Desired Velocity Reference Trajectories

Estimation Error of Angular Velocity Tracking Error

Thruster Torques

Estimated Parameters

Estimated Parameter Values
Figure 4.7: Spacecraft attitude response in orbit around Ida. Desired trajectory $w_{r1} = [0, 0, 0]^T$ (deg), initial conditions $w_1(0) = [2.5, 2.5, 2.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = [0, 0, 0]$, $\hat{\rho}(0) = 0 3\times3$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[2, 1, 1]$, $\Gamma = 10^{-3}I_{10\times10}$, $\gamma_a = 74$. 

- Attitude States and Desired Reference Trajectories
- Attitude Tracking Error
- Angular Velocity Attitude Tracking Error
- Attitude Velocity States and Desired Velocity Reference Trajectories
- Estimation Error of Angular Velocity Tracking Error
- Thruster Torques
- Estimated Parameters
Figure 4.8: Spacecraft attitude response in orbit around Ida. Desired trajectory $w_{r1} = [10, 40, 140]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\dot{\hat{r}}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[3, 5, 1]$, $\Gamma = 10^{-3}I_{10 \times 10}$, $\gamma_a = 75$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Attitude Velocity States and Desired Velocity Reference Trajectories

Estimation Error of Angular Velocity Tracking Error

Thruster Torques

Estimated Parameters

Estimated Parameter Values
Figure 4.9: Spacecraft attitude response in orbit around Ida. Desired trajectory $w_1 = [10\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 76$. 
Figure 4.10: Spacecraft attitude response in orbit around Ga spra. Desired trajectory $w_{r1} = [0,0,0]^T$ (deg), initial conditions $w_1(0) = [2.5, 2.5, 2.5]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10}$, $\hat{\rho}(0) = 0_{3}$. Tuning gains $\zeta = \text{diag}[1,1,1]$, $\omega_n = 20\text{diag}[2,1,1]$, $\Gamma = 10^{-3}I_{10}$, $\nu_e = \cdots$
Figure 4.11: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $w_{r1} = [10, 40, 140]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0.1, 0.1, 0.1]^T$, $\hat{\Gamma}(0) = 0_{10\times10}$, $\hat{\rho}(0) = 0_{3\times3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 20\text{diag}[3, 5, 1]$, $\Gamma = 10^{-3}I_{10\times10}$, $\zeta_a = 78$. 

Figure 4.12: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $w_r = [10\sin(3\omega t), 35\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $w_1(0) = [0, 0, 0]^T$ (deg), $w_2(0) = [0, 1, 0, 1]^T$, $\hat{\Gamma}(0) = 0_{10 \times 1}$, $\hat{\rho}(0) = 0_{3 \times 3}$. Tuning gains $\zeta = \text{diag}[1, 1, 1]$, $\omega_n = 79$. 

\[\begin{align*}
\text{Attitude States and Desired Reference Trajectories} \\
\text{Attitude Tracking Error} \\
\text{Angular Velocity Attitude Tracking Error} \\
\text{Attitude Velocity States and Desired Velocity Reference Trajectories} \\
\text{Estimation Error of Angular Velocity Tracking Error} \\
\text{Thruster Torques} \\
\text{Estimated Parameter Values}
\end{align*}\]
CHAPTER 5

NONLINEAR MODEL AND CONTROL

Concluding the study of the linearized model and its control, one might naturally wonder if the original nonlinear model could yield different results. Specifically, it is speculated that the nonlinear model would be a more accurate representation of the equations of motion, and thus a new controller, derived from the nonlinear model, might be a more honest portrayal of the efforts required to control a spacecraft in orbit.

Model

With this motivation in mind, we consider the equations of motion for the attitude states of a spacecraft examined by Chauvineau and Mignard [2]. During the derivation of these equations of motion, the following assumption are maintained:

1. Orbital motion of the spacecraft is periodic, closed, and planar.

2. The gravitational attraction of the asteroid is the only external force acting on the spacecraft (no disturbance)

3. The rotational rate of the asteroid is constant (circular orbit)

4. The spacecraft is rigid (continuous distribution of mass)

5. The spacecraft attitude dynamics negligibly affect its orbit
With these assumptions, Euler’s equations of motion for a rigid body are used to describe the attitude equations of motion, as provided by Misra and Panchenko [14],

\[
\begin{align*}
I_1 \dot{\omega}_1 - (I_2 - I_3) \omega_2 \omega_3 &= M_1 \\
I_2 \dot{\omega}_2 - (I_3 - I_1) \omega_3 \omega_1 &= M_2 \\
I_3 \dot{\omega}_3 - (I_1 - I_2) \omega_1 \omega_2 &= M_3 
\end{align*}
\]

where $I_1, I_2, I_3$ are the principle moments of inertia of the spacecraft, $\omega_1, \omega_2, \omega_3$ are the respective elements of angular velocity along the principle moments, and $M_1, M_2, M_3$ are the external moments. Given that the satellite is in an equatorial orbit, if the satellites attitude motion is allowed to be three-dimensional and the asteroid has a constant angular velocity
Ω (rad/s), the gravity gradient torque components can be expressed as

\[
M_1 = \frac{\mu}{R_c^3} [(3 + 5\phi)(I_3 - I_2)\cos(\theta_1)\cos^2(\theta_2)\sin(\theta_1) \\
+ 5x(-\frac{2}{5}I_1\cos(\theta_1)\sin(\theta_3) + (I_1 - I_2 + I_3)\sin(\theta_1)\cos^2(\theta_2)\cos(\theta_3))] \\
\quad \text{(5.4)}
\]

\[
M_2 = \frac{\mu}{R_c^3} [(3 + 5\phi)(I_3 - I_1)\cos(\theta_1)\cos(\theta_2)\sin(\theta_1) \\
+ 5x(-\frac{2}{5}I_2(\sin(\theta_1)\sin(\theta_2) - \cos(\theta_1)\cos(\theta_3))) \\
+ 5x((I_2 - I_1 + I_3)(\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin^2(\theta_2)\cos(\theta_1)\cos(\theta_3)) \\
+ 5x(I_2 - I_1 + I_3)\cos(\theta_1)\cos^2(\theta_2)\cos(\theta_3))] \\
\quad \text{(5.5)}
\]

\[
M_3 = \frac{\mu}{R_c^3} [(3 + 5\phi)(I_1 - I_2)\cos(\theta_2)\sin(\theta_1)\sin(\theta_2) \\
+ 5x(\frac{2}{5}I_3(\sin(\theta_1)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3))) \\
+ 5x((I_2 - I_1 + I_3)(\sin(\theta_1)\cos(\theta_1)\sin(\theta_3) - \sin^2(\theta_2)\sin(\theta_1)\cos(\theta_3)) \\
- 5x(I_1 - I_2 + I_3)\sin(\theta_1)\cos^2(\theta_2)\cos(\theta_3))] \\
\quad \text{(5.6)}
\]

where

\[
\phi = \left[-\frac{3}{2}C_{20} + 9C_{22}\cos(2\lambda_c)\right]\left(\frac{r_0}{R_c}\right) \\
\quad \text{(5.7)}
\]

\[
x = [6C_{22}\sin(2\lambda_c)]\left(\frac{r_0}{R_c}\right) \\
\quad \text{(5.8)}
\]

\[
\lambda_c = \eta + \Omega_t \\
\quad \text{(5.9)}
\]

We note that \(R_c\) refers to the orbital radius of the spacecraft orbiting the asteroid, \(r_0\) is the characteristic length of the asteroid, \(\eta\) is the true anomaly, \(\mu\) is the standard gravitational
parameter, and $C_{20}, C_{22}$ are the gravitational coefficients of the asteroid, same as the linear model.

Misra also presents a derived expression for the relationship between the angular velocities $\omega_1, \omega_2, \omega_3$ and their respective attitude states roll ($\theta_1$), pitch ($\theta_2$), and yaw ($\theta_3$), as presented below,

$$
\begin{bmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{bmatrix}
= \begin{bmatrix} 1 & 0 & -\sin(\theta_2) \\ 0 & \cos(\theta_1) & \sin(\theta_1)\cos(\theta_2) \\ 0 & -\sin(\theta_1) & \cos(\theta_1)\cos(\theta_2) \end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix}
+ \begin{bmatrix}
\cos(\theta_2)\sin(\theta_3) \\
-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3) \\
\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \sin(\theta_1)\cos(\theta_3)
\end{bmatrix} \dot{\eta} \tag{5.10}
$$

where $\dot{\eta}$ is the instantaneous orbital rate.

Continuing our desire for simplicity, we will form the equations of motion (Eqs. 5.1 - 5.3) into a more compact form, as was previously done with the linear model. Again, we will assume that the spacecraft moment of inertia parameters $I_1, I_2, I_3$ and the asteroid gravity coefficients $C_{20}, C_{22}$ are unknown. Because the dynamics (Eqs. 5.1 - 5.3 and Eq. 5.10) have unknown terms, and are nonlinear, the equations of motion cannot be put into a simple state-space form. Instead, we begin by condensing Eqs. (5.1) - (5.3) into a matrix form, as shown below,

$$
I \dot{\omega} = -\omega \times I \omega + T_g + u \tag{5.11}
$$

where $\omega = [\omega_1, \omega_2, \omega_3]^T \in \mathbb{R}^{3 \times 1}$, $I = \text{diag}[I_1, I_2, I_3] \in \mathbb{R}^{3 \times 3}$, $T_g = [M_1, M_2, M_3]^T \in \mathbb{R}^{3 \times 1}$, and
\[ u = [u_1, u_2, u_3]^T \in \mathbb{R}^{3\times1} \text{ is our control signal, provided by the torque from the spacecraft thrusters. We also note that the ‘×’ symbol in Eq. (5.11) is the cross product. We will also simplify Eq. 5.10 into a similar matrix form,} \]

\[ \omega = L(\theta)\dot{\theta} - l(\theta)\dot{\eta} \quad (5.12) \]

or

\[ \dot{\theta} = L^{-1}(\theta)\omega + L^{-1}(\theta)l(\theta)\dot{\eta} \quad (5.13) \]

where

\[
L(\theta) = \begin{bmatrix}
1 & 0 & -\sin(\theta_2) \\
0 & \cos(\theta_1) & \sin(\theta_1)\cos(\theta_2) \\
0 & -\sin(\theta_1) & \cos(\theta_1)\cos(\theta_2)
\end{bmatrix}
\]

and

\[
l(\theta) = \begin{bmatrix}
\cos(\theta_2)\sin(\theta_3) \\
\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3) \\
\cos(\theta_1)\sin(\theta_2)\sin(\theta_3) - \sin(\theta_1)\cos(\theta_3)
\end{bmatrix}
\]

Backstepping

Examining our neatly collected dynamics below,

\[ \dot{\theta} = L^{-1}(\theta)\omega + L^{-1}(\theta)l(\theta)\dot{\eta} \quad (5.14) \]

\[ I\dot{\omega} = -\omega \times I\omega + T_g + u \quad (5.15) \]
we notice an interesting problem. While it is our desire to control the attitude states, $\theta$, such that they track the desired reference trajectories, $\theta_r$, where $\theta_r$ is a given vector of a smooth, desired roll, pitch, and yaw reference trajectory, $\theta_r = [\theta_{r1}, \theta_{r2}, \theta_{r3}]^T$, we observe that the state we wish to control, $\theta$, does not have a means of control, $u$ present in the dynamics. Indeed, our control signal $u$ appears in Eq. (5.15) and not Eq. (5.14). In linear systems, this problem is resolved easily, as demonstrated by the double integrator example below.

**Double Integrator Example** It is desired to control the $x_1$ state such that it converges to zero, from the given linear system below, by careful selection of the control law $u$.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u
\end{align*}
\]

We form the dynamics into a condensed matrix form.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
u
\end{bmatrix}
\]

By selecting the control law to be $u = -k_1x_1 - k_2x_2$, all the states can be controlled to go to zero.

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-k_1 & -k_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]
However, in nonlinear systems, it is not possible to separate the states into vectors and select the control law such that the system matrix is Hurwitz. This is due to the nonlinear relationship of the states present in the dynamics. So, to control $\theta$ in our system (Eqs. 5.14 - 5.15) we will need to use a unique, nonlinear control technique, known as Backstepping. Backstepping is a more robust version of another nonlinear control scheme known as feedback linearization. Some of the strengths of feedback linearization is providing global stability with exponential tracking error and being linear in modeled domain. Its drawbacks include requiring exact knowledge and special class of the system, as well as not being robust to uncertainties. Backstepping, by comparison, can provide global asymptotic stability and can handle modeled uncertainties; the cost being that the feedback control law will become more and more complex for higher-order systems [1]. To begin, we form the error dynamics. The tracking error is defined as

$$z_1 = \theta - \theta_r,$$  \hspace{1cm} (5.16)

Taking the derivative of Eq. (5.15) yields

$$\dot{z}_1 = \dot{\theta} - \dot{\theta}_r = L^{-1}(\theta)\omega + L^{-1}(\theta)l(\theta)\ddot{\eta} - \dot{\theta}_r$$  \hspace{1cm} (5.17)

Here, we begin the first step of backstepping; inserting a synthetic control term (or backstepping term) $\omega_c$ into Eq. (5.17). This will be used to control the $\omega$ states, which will then
control the \( \theta \) states.

\[
\dot{z}_1 = L^{-1}(\theta)(\omega - \omega_c + \omega_c) + L^{-1}(\theta)l(\theta)\dot{\eta} - \dot{\theta}_r \tag{5.18}
\]

Defining our second error term (or our backstepping term error) as

\[
z_2 = \omega - \omega_c \tag{5.19}
\]

we plug Eq. 5.19 into Eq. 5.18.

\[
\dot{z}_1 = L^{-1}(\theta)(z_2 + \omega_c) + L^{-1}(\theta)l(\theta)\dot{\eta} - \dot{\theta}_r \tag{5.20}
\]

Looking at Eq. (5.14), we select \( \omega_c \) such that the closed-loop dynamics of Eq. (5.17) are stable. For this reason, we select \( \omega_c \) to be

\[
\omega_c = -l(\theta)\dot{\eta} + L(\theta)[\dot{\theta}_r - k_{f1}(\theta - \theta_r)] \tag{5.21}
\]

where \( k_{f1} = \text{diag}[k_{f11}, k_{f12}, k_{f13}] \in R^{3 \times 3} \) is a diagonal matrix feedback gain to be tuned. When Eq. (5.21) is plugged back into Eq. (5.20), the dynamics become

\[
\dot{z}_1 = L^{-1}(\theta)(z_2 - l(\theta)\dot{\eta} + L(\theta)[\dot{\theta}_r - k_{f1}(\theta - \theta_r)]) + L^{-1}(\theta)l(\theta)\dot{\theta}_r - \dot{\theta}_r + k_{f1}z_1 + L^{-1}(\theta)l(\theta)\dot{\eta} - \dot{\theta}_r
\]

\[
= -k_{f1}z_1 + L^{-1}(\theta)z_2 \tag{5.22}
\]
From Eq. (5.22), we see that \( z_1 \) will go to zero \( (z_1 \rightarrow 0) \), when \( z_2 \) goes to zero \( (z_2 \rightarrow 0) \). In the second step, we consider a Lyapunov function chosen from the dynamics of \( \dot{z}_1 \).

\[
V_1 = \frac{z_1^T z_1}{2}
\]  
(5.23)

The derivative of Eq. (5.23) becomes

\[
\dot{V}_1 = z_1^T \dot{z}_1
= z_1^T (-k_{f_1} z_1 + L^{-1}(\theta) z_2)
= -z_1^T k_{f_1} z_1 + z_1^T L^{-1}(\theta) z_2
\]  
(5.24)

We see from Eq. (5.24) that \( \dot{V}_1 \) will be n.s.d. if we can design \( u \) such that \( z_2 \rightarrow 0 \), which confirms our initial statement. To derive this control signal, we now look at the error dynamics for \( z_2 \).

\[
\dot{z}_2 = \dot{\omega} - \dot{\omega}_c
\]

\[
I \dot{z}_2 = I \dot{\omega} - I \dot{\omega}_c
= -\omega \times I \omega + T_g + u - I \Phi_0
\]  
(5.25)

where

\[
\dot{\omega}_c = -\dot{l}(\theta) \ddot{\eta} - l(\theta) \dddot{\eta} + \dot{L}(\theta)[\ddot{\theta}_r - k_{f_1}(\theta - \theta_r)] + L(\theta)[\dddot{\theta}_r - k_{f_1}(\dot{\theta} - \dot{\theta}_r)]
= \Phi_0(\theta, \omega, \dot{\eta}, \ddot{\eta}, t)
\]
Because our model has uncertainties present, we must group the terms of Eq. (5.25) into functions that contain only known terms, and functions that contain only unknown terms. This is done, as in the case of the linear model, so that the adaptation law can be derived from the Lyapunov function to estimate the unknown terms. With this in mind, we format Eq. (5.25) as such

$$I \dot{z}_2 = \Psi \gamma + u$$  \hspace{1cm} (5.26)

where $\Psi \in \mathbb{R}^{3 \times 30}$ is a function of known terms, $\gamma \in \mathbb{R}^{30 \times 1}$ is a function of unknown terms, and $u \in \mathbb{R}^{3 \times 1}$ is our control signal. Examining Eq. (5.26), we select the control law below

$$u = -\hat{\Psi} \hat{\gamma} - k_{f2} z_2 + u_a$$  \hspace{1cm} (5.27)

where $\hat{\gamma} \in \mathbb{R}^{30 \times 1}$ is an estimate of $\gamma$, $k_{f2} = \text{diag}[k_{f21}, k_{22}, k_{23}] \in \mathbb{R}^{3 \times 3}$ is a feedback gain to be tuned, and $u_a \in \mathbb{R}^{3 \times 1}$ is an additional control signal, to be selected later. One may wonder why our unknown parameter array has 30 elements when there are only 5 parameter values to estimate ($I_1, I_2, I_3, C_{20}, C_{22}$). This is because we would often have terms that had some product combination of these terms such as $I_1 C_{20}, I_2 C_{22}$, etc. This means that our parameter estimate $\gamma$ is over parameterized. While it is possible to reduce this parameterization down to 5 terms, it has been shown that over parameterization can be beneficial in certain situations, and can even yield better results than if the estimates where not over parameterized. For the sake of simplicity, we will leave our estimate $\gamma$ over parameterized. This was the third step in the backstepping method. Substituting Eq. (5.27) into Eq. (5.26) yields the closed-loop
dynamics for $z_2$.

$$I \dot{z}_2 = -\Psi \tilde{\gamma} - k_f z_2 + u_a$$  \hspace{1cm} (5.28)

where $\tilde{\gamma} = \hat{\gamma} - \gamma$. Concluding with the fourth step, we use Eq. 5.28 to consider the following Lyapunov function,

$$V = V_1 + \frac{z_T I z_2}{2} + \frac{\tilde{\gamma} T \Gamma^{-1} \tilde{\gamma}}{2}$$  \hspace{1cm} (5.29)

where $\Gamma$ is p.d. and $\Gamma = \Gamma^T \in R^{30 \times 30}$ is a adaption gain to be tuned. Taking the derivative of Eq. (5.29) yields

$$\dot{V} = \dot{V}_1 + z_T I \dot{z}_2 + \tilde{\gamma}^T \Gamma \dot{\tilde{\gamma}}$$

$$= -z_1^T k_f z_1 + z_1^T T \Gamma^{-1}(\theta) z_2 + z_2^T (\hat{\gamma} - k_f z_2 + u_a) + \tilde{\gamma}^T \Gamma^{-1} \dot{\tilde{\gamma}}$$

$$= -z_1^T k_f z_1 - z_2^T k_f z_2 - z_2^T \Psi \tilde{\gamma} + \tilde{\gamma} T \Gamma^{-1} \dot{\tilde{\gamma}} + z_1^T \Gamma^{-1}(\theta) z_2 + z_2^T u_a$$

$$= -z_1^T k_f z_1 - z_2^T k_f z_2 + \tilde{\gamma} \Gamma^{-1}(\dot{\tilde{\gamma}} - \Gamma \Psi T z_2) + z_2^T (u_a + \Gamma^{-1}(\theta) z_1)$$  \hspace{1cm} (5.30)

Observing Eq. 5.30, we see that if the additional control signal $u_a$ and the adaption law $\dot{\tilde{\gamma}}$ are chosen as

$$u_a = -(\Gamma^{-1}(\theta))^T z_1$$  \hspace{1cm} (5.31)

$$\dot{\tilde{\gamma}} = \dot{\gamma} = \Gamma \Psi^T z_2$$  \hspace{1cm} (5.32)

then Eq. (5.30) reduces to

$$\dot{V} = -z_1^T k_f z_1 - z_2^T k_f z_2$$  \hspace{1cm} (5.33)
It is easily seen that Eq. (5.33) will be n.s.d. as long as $k_{f1}$ and $k_{f2}$ are chosen to be p.d.

As already stated in the case of the linear model, LaSalle-Yoshizawa Theorem [10] relaxes the condition that $\dot{V}$ must be n.d. so that $\dot{V}$ only has to be n.s.d. Thus, because we can say that our adaption laws guarantee that $\dot{V} \leq 0$, but $\dot{V} = 0$ only at the origin, we can say that $z_1$ and $z_2$ are asymptotically stable and will approach zero in as time goes to infinity. With the derivation of the complete control law, the backstepping method is finished.
CHAPTER 6

NONLINEAR SIMULATION RESULTS

Now, we will test the effectiveness of our nonlinear control law by digitally simulating our control laws Eqs. (5.27), (5.31), and adaption law Eq. (5.32) as they are applied to the models of four asteroids; Eros, Vesta, Ida, and Gaspra. The simulations will be conducted using Simulink because of its ease in modeling dynamic systems. Again, the spacecraft will be given three desired attitude trajectories to track; a zero constant trajectory, a nonzero constant trajectory, and a sinusoidal trajectory. They are provided below for convenience.

\[
\theta_r = [0, 0, 0]^T \quad \text{(deg)} \quad (6.1)
\]

\[
\theta_r = [10, 40, 140]^T \quad \text{(deg)} \quad (6.2)
\]

\[
\theta_r = [10 \sin(3\omega t), 40 \sin(\omega t), 140 \sin(\omega t)]^T \quad \text{(deg)} \quad (6.3)
\]

We note that \( \omega = 2\pi/T \), where the period, \( T = 30 \) rad. As in the case of controlling the linear model all three desired trajectories will begin at the origin. For the constant trajectories, this means that we will continue to use the same reference generator that was used in the case of the linear model. After the reference generator, the constant trajectory command, is modified to be

\[
\theta_r = [10(1 - e^{-\tau t}), 40(1 - e^{-\tau t}), 140(1 - e^{-\tau t})]^T \quad \text{(deg)} \quad (6.4)
\]
where $\tau$ is a time constant to be selected such that it meets the desired settling time requirement. For a period of $T = 30$ rad, we set $\tau = 0.2$.

When we were concerned with controlling the linear model, a upper and lower bound was put onto the control law of $\pm 1$ newton meters. This was done to represent the realistic constraint on the thrusters, in that there is a limit to how much torque they can produce at a given time. However we will depart from this requirement for control of the nonlinear model for two reasons; first, it will be seen that the control law significantly exceeds this bound and second, it has been observed during testing that even if the control law magnitude is slightly larger than the bound, the controller will be unable to control the system and indeed the response will become unstable. Another deviation is that disturbance will not be present in the model. This is done as a simplification since the magnitude of disturbance is too small to have any meaningful effect on the control of the system, as seen from figures 3.16 - 3.30. Lastly, it should also be noted that the same spacecraft and asteroid parameters from tables 3.1 - 3.4 that were used in linear feedback, will be used here. The tables are slightly updated, as seen in tables 6.1 - 6.4, where the variable of some parameters have changed and, the largest change, all asteroids are assumed to have circular orbits. This means that the orbital eccentricity $e$ and the orbital semi-major axis $R_a$ are omitted from our simulations. This comes from our 3rd assumption in beginning of the Nonlinear Model and Control Chapter.

**Case A: Eros** For testing the performance of our nonlinear controller, we again begin with Eros. The orbit parameter values used during this simulation are collected in table 6.1 which includes parameter values for both the spacecraft and the asteroid. The figures 6.1, 6.2, and 6.3 display the simulation results for the attitude response to a zero
trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Examining the attitude tracking error, we see that it has a maximum transient error of 5 deg during the systems response to zero the reference trajectory, and a maximum steady state error of ±0.1 deg during the systems response to the sinusoidal reference trajectory. Conversely, the attitude tracking error has a minimum transience error of 0.6 deg during the sinusoidal test, and a minimum steady state tracking error of ±10⁻³ deg during the zero trajectory test. Looking at the angular velocity tracking error, we see a maximum transient error of 55 deg/s when the system regulates itself to the origin and a maximum steady state error of ±0.2 deg/s when the system is responding to the constant trajectory test. The transient, and steady state, angular velocity tracking error is only slightly less during the constant and sinusoidal reference trajectory simulations, where it is 12 deg/s and ±0.005 deg/s, respectively. Our defined backstepping term tracking error sees a maximum transient error of 60 deg/s during the zero trajectory test and a maximum steady state error of ±1 when the sinusoidal reference trajectory is applied. Conversely, the minimum transient error for the backstepping term is 50 during the sinusoidal trajectory test and the steady state equivalent is ±10⁻³ during the zero trajectory test. The torque required from the thrusters has a maximum transient value of -17 N·m during the sinusoidal test and a maximum steady state value of ±6 N·m during the sinusoidal reference trajectory. The control signal is minimum both in transient and steady state effort when the reference trajectory is zero, where the transient effort is -1 N·m and the steady state effort is ±22 N·m. As in the case of linear feedback, we note that the estimated parameters do not converge to the true parameter values, however, the system is still stable and the tracking error is within a reasonable limit for all three tests.

**Case B: Vesta**

Now we test the performance of our controller with a spacecraft
Figure 6.1: Spacecraft attitude response in orbit around Eros. Desired trajectory $\theta_r = [0, 0, 0]^T$ (deg), initial conditions $\theta(0) = [5, 5, 5]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\dot{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_{f1} = \text{diag}[1, 1, 1]$, $k_{f2} = \text{diag}[1, 1, 1]$, $\gamma = 10^{-3}I_{10 \times 10}$. 

Estimated Parameter Values
Figure 6.2: Spacecraft attitude response in orbit around Eros. Desired trajectory $\theta_r = [40, 10, 140]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_{f1} = 25 \text{diag\{2, 1, 1\}}$, $k_{f2} = 10 \text{diag\{1, 1, 1\}}$, $\gamma = 10^{-1} I_{10 \times 10}$. 
Figure 6.3: Spacecraft attitude response in orbit around Eros. Desired trajectory $\theta_r = [40\sin(3\omega t), 10\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}(0) = 0$. Tuning gains $k_{f1} = 40\text{diag}[2, 1, 1]$, $k_{f2} = 20\text{diag}[1, 1, 1]$, $\gamma = 97$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Backstepping Term Tracking Error

Thruster Torques

Estimated Parameters
in orbit about Vesta. Unlike Eros, this orbit will be a unique evaluation, because Vesta has the largest orbital radius to the spacecraft, characteristic length, and mass among the four asteroids. The orbit parameter values used during this simulation are collected in table 6.2 which includes parameter values for both the spacecraft and the asteroid. The figures 6.4, 6.5, and 6.6 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Examining the attitude tracking error, we see that it has a maximum transient error of 7 deg during the systems response to the zero reference trajectory, and a maximum steady state error of ±0.02 deg during the systems response to the constant, and sinusoidal reference trajectory. Conversely, the attitude tracking error has a minimum transience error of 0.8 deg during the sinusoidal test, and a minimum steady state tracking error of ±10⁻³ deg during the zero trajectory test. Looking at the angular velocity tracking error, we see a maximum transient error of 55 deg/s when the system regulates itself to the origin and a maximum steady state error of ±0.2 deg/s when the system is responding to the sinusoidal trajectory test. The transient, and steady state, angular velocity tracking error is only slightly less during the sinusoidal and zero reference trajectory simulations, where it is 50 deg/s and ±0.005
deg/s, respectively. Our defined backstepping term tracking error sees a maximum transient error of 60 deg/s during the zero trajectory test and a maximum steady state error of ±0.2 when the sinusoidal reference trajectory is applied. Conversely, the minimum transient error for the backstepping term is 50 during the sinusoidal trajectory test and the steady state equivalent is ±10\(^{-3}\) during the zero trajectory test. The torque required from the thrusters has a maximum transient value of -20 \(N \cdot m\) during the sinusoidal test and a maximum steady state value of ±23 \(N \cdot m\) during the sinusoidal reference trajectory. The control signal is minimum both in transient and steady state effort when the reference trajectory is zero, where the transient effort is ±5 \(N \cdot m\) and the steady state effort is ±2 \(N \cdot m\). Likewise, we note that the estimated parameters do not converge to the true parameter values, however, the system is still stable and the tracking error is within a reasonable limit for all three tests.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>spacecraft mass distribution parameters</td>
<td>(k_1 = -0.5, k_3 = 0.3)</td>
</tr>
<tr>
<td>spacecraft mass</td>
<td>(m = 625 \text{ kg})</td>
</tr>
<tr>
<td>gravitational coefficients</td>
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</tr>
<tr>
<td>orbital radius</td>
<td>(R = 640 \text{ km})</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
<td>(R_e = 244.3 \text{ km})</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
<td>(\omega = 3.293074 \times 10^{-4} \text{ s}^{-1})</td>
</tr>
<tr>
<td>asteroid mass</td>
<td>(M = 2.668 \times 10^{20} \text{ Kg})</td>
</tr>
</tbody>
</table>

Table 6.2: Asteroid parameter values for Vesta

**Case C: Ida**

Our next analysis concerns the performance of the controller during an orbit around Ida. Ida separates itself from the other four asteroids for having the fastest revolution period. The orbit parameter values used during this simulation are collected in table 6.3, including parameter values for both the spacecraft and the asteroid. The figures
Figure 6.4: Spacecraft attitude response in orbit around Vesta. Desired trajectory $\theta_r = [0, 0, 0]^T \text{(deg)}$, initial conditions $\theta(0) = [5, 5, 5]^T \text{(deg)}$, $\omega(0) = [0, 0, 0]^T$, $\gamma^T(0) = 0_{30 \times 1}$. Tuning gains $k_{f1} = \text{diag}[1, 1, 1]$,$k_{f2} = 4\text{diag}[1, 1, 1]$, $\gamma = 10^{-3}I_{10 \times 10}$.
Figure 6.5: Spacecraft attitude response in orbit around Vesta. Desired trajectory $\theta_r = [40, 10, 140]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_f^1 = 20 \text{diag}[2, 1, 1]$, $k_f^2 = 10 \text{diag}[2, 1, 2]$, $\gamma = 10^{-2}I_{30 \times 30}$. 101
Figure 6.6: Spacecraft attitude response in orbit around Vesta. Desired trajectory
\( \theta_r = [40\sin(3\omega t), 10\sin(\omega t), 140\sin(\omega t)]^T \) (deg), initial conditions \( \theta(0) = [0, 0, 0]^T \) (deg), \( \omega(0) = [0, 0, 0]^T \), \( \hat{\gamma}^T(0) = 0 \). Tuning gains \( k_{f1} = 20\text{diag}[2, 1, 1] \), \( k_{f2} = 20\text{diag}[1, 1, 1] \), \( \gamma = 102 \).
6.7, 6.8, and 6.9 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Again, we begin by observing the attitude tracking error, where we see that it has a maximum transient error of 7 deg during the system's response to zero the reference trajectory, and a maximum steady state error of ±0.04 deg during the system's response to the constant, and sinusoidal reference trajectory. Conversely, the attitude tracking error has a minimum transience error of 1.2 deg during the sinusoidal test, and a minimum steady state tracking error of ±0.05 deg during the zero trajectory test. Looking at the angular velocity tracking error, we see a maximum transient error of 60 deg/s when the system regulates itself to the origin and a maximum steady state error of ±0.5 deg/s during the same test. The transient, and steady state, angular velocity tracking error is only slightly less during the sinusoidal and zero reference trajectory simulations, where it is 50 deg/s and ±0.2 deg/s, respectively. Our defined backstepping term tracking error sees a maximum transient error of 60 deg/s during the zero trajectory test and a maximum steady state error of ±0.5 when the sinusoidal reference trajectory is applied. Conversely, the minimum transient error for the backstepping term is 50 during the sinusoidal trajectory test and the steady state equivalent is ±0.5 during the zero trajectory test. The torque required from the thrusters has a maximum transient value of -10 \( N \cdot m \) during the constant test and a maximum steady state value of ±13 \( N \cdot m \) during the sinusoidal reference trajectory. The control signal has a minimum steady state effort of ±4 \( N \cdot m \) during the zero trajectory test. Again, we note that the estimated parameters do not converge to the true parameter values, however, the system is still stable and the tracking error is within a reasonable limit for all three tests.

**Case D: Gaspra** We end our simulation results by looking at the performance of
Figure 6.7: Spacecraft attitude response in orbit around Ida. Desired trajectory $\theta_r = [0, 0, 0]^T$ (deg), initial conditions $\theta(0) = [5, 5, 5]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\dot{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_f^1 = \text{diag}[1, 1, 1], k_f^2 = 2\text{diag}[1, 1, 1], \gamma = 10^{-3}I_{10 \times 10}$. 

$\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, $\dot{\gamma}_1$, $\dot{\gamma}_2$, $\dot{\gamma}_3$.
Figure 6.8: Spacecraft attitude response in orbit around Ida. Desired trajectory $\theta_r = [40, 10, 140]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_{f1} = 20 \text{diag}[1, 1, 1], k_{f2} = 10 \text{diag}[1, 1, 1], \gamma = 10^{-1} I_{30 \times 30}$. 
Figure 6.9: Spacecraft attitude response in orbit around Ida. Desired trajectory $\theta_r = [40\sin(3\omega t), 10\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}^T(0) = 0$. Tuning gains $k_{f1} = 20\text{diag}[1, 0.2, 1]$, $k_{f2} = 10\text{diag}[1, 1, 1]$, $\gamma = 10^6$. 

106
the controller during the orbit around Gaspra. Like Eros, Gaspra doesn’t have a maximum parameter value to act as a unique test for the controller; all of its parameters are in a middle to low range of values. The orbit parameter values used during this simulation are collected in table 6.4 which includes parameter values for both the spacecraft and the asteroid. The figures 6.10, 6.11, and 6.12 display the simulation results for the attitude response to a zero trajectory, a constant trajectory, and a sinusoidal trajectory, respectively. Examining the attitude tracking error, we see that is has a maximum transient error of 6.5 deg during the systems response to zero the reference trajectory, and a maximum steady state error of ±0.05 deg during the systems response to the constant and sinusoidal reference trajectory. Conversely, the attitude tracking error has a minimum transience error of 0.8 deg during the sinusoidal test, and a minimum steady state tracking error of ±5 × 10⁻³ deg during the zero trajectory test. Looking at the angular velocity tracking error, we see a maximum transient error of 55 deg/s when the system regulates itself to the origin and a maximum steady state error of ±0.25 deg/s when the system is responding to the sinusoidal trajectory test. The transient, and steady state, angular velocity tracking error is only slightly less during the constant reference trajectory simulations, where it is 50 deg/s and ±0.2 deg/s,

<table>
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<th>Parameter</th>
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<td>spacecraft mass distribution parameters</td>
<td>$k_1 = -0.5, k_3 = 0.3$</td>
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<tr>
<td>spacecraft mass</td>
<td>$m = 625 \text{ kg}$</td>
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<td>gravitational coefficients</td>
<td>$C_{20} = -0.0902, C_{22} = 0.0408$</td>
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<td>orbital radius</td>
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<tr>
<td>asteroid mass</td>
<td>$M = 4.121 \times 10^{16} \text{ Kg}$</td>
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</table>

Table 6.3: Asteroid parameter values for Ida
respectively. Our defined backstepping term tracking error sees a maximum transient error of 60 deg/s during the zero trajectory test and a maximum steady state error of ±1 when the constant reference trajectory is applied. Conversely, the minimum transient error for the backstepping term is 50 during the constant and sinusoidal trajectory test, and the steady state equivalent is ±0.01 during the zero trajectory test. The torque required from the thrusters has a maximum steady state value of ±35 $N \cdot m$ during the sinusoidal reference trajectory. The control signal is minimum in steady state effort when the reference trajectory is zero, where the transient effort is ±4. Finally, we note that the estimated parameters do not converge to the true parameter values, however, the system is still stable and the tracking error is within a reasonable limit for all three tests.

<table>
<thead>
<tr>
<th>Gaspra Orbit Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>spacecraft mass distribution parameters</td>
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<tr>
<td>spacecraft mass</td>
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<tr>
<td>gravitational coefficients</td>
</tr>
<tr>
<td>orbital radius</td>
</tr>
<tr>
<td>characteristic length of the asteroid</td>
</tr>
<tr>
<td>rotation rate of asteroid</td>
</tr>
<tr>
<td>asteroid mass</td>
</tr>
</tbody>
</table>

Table 6.4: Asteroid parameter values for Gaspra
Figure 6.10: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $\theta_r = [0, 0, 0]^T$ (deg), initial conditions $\theta(0) = [5, 5, 5]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\dot{\gamma}^T(0) = 0_{30 \times 1}$. Tuning gains $k_{f1} = \text{diag}[1, 1, 1]$, $k_{f2} = 10\text{diag}[1, 0.5, 1]$, $\gamma = 10^{-3}I_{10 \times 10}$. 

Attitude States and Desired Reference Trajectories

Attitude Tracking Error

Angular Velocity Attitude Tracking Error

Thruster Torques

Estimated Parameters
Figure 6.11: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $\theta_r = [40, 10, 140]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\hat{\gamma}(0) = 0 30\times1$.

Tuning gains $k_{f1} = 10\text{diag}[1, 1, 1]$, $k_{f2} = 10\text{diag}[1, 1, 1]$, $\gamma = 10^{-1}I_{30\times30}$.
Figure 6.12: Spacecraft attitude response in orbit around Gaspra. Desired trajectory $\theta_r = [40\sin(3\omega t), 10\sin(\omega t), 140\sin(\omega t)]^T$ (deg), initial conditions $\theta(0) = [0, 0, 0]^T$ (deg), $\omega(0) = [0, 0, 0]^T$, $\dot{\gamma}(0) = 0$. Tuning gains $k_f = \text{diag}[1, 1, 1]$, $k_f = 20\text{diag}[1, 1, 1]$, $\gamma = 10^{-1}I_{10 \times 10}$. 
CHAPTER 7

CONCLUSION

In this thesis, an adaptive control law was derived to control the attitude states (pitch, roll, yaw) of a spacecraft in orbit around an asteroid. The four asteroids considered were not spherical and their orbits were not circular. It was assumed that there were unknown parameters present in each orbit, namely the mass of the spacecraft and the gravity coefficients of the asteroid. The robustness of the controller’s ability to maintain agency over the attitude states was tested as a sinusoidal disturbance was introduced into the system. After forming the error dynamics from the linearized motion of equations for the attitude of a spacecraft in orbit around an asteroid, a control law was derived that decoupled itself from the rest of the system. Once the control law was fed back into the plant, the closed-loop system was established. Then, using Lyapunov stability, the adaption laws were derived to estimate the unknown parameters that would be used in the control law, completing the system. To complete the linear design, a high-gain observer was designed such that the first derivative states of the system (pitch angular velocity, roll angular velocity, yaw angular velocity) are estimated for implementation, relaxing the sensor requirements of the spacecraft. During testing, the feedback gains, adaption gains, and observer gains were tuned manually to find the desired attitude response. Simulation results of the linear controller show that the attitude states of the spacecraft can be controlled even with a bounded controller and with disturbance present in the model. With the completion of our linear controller, we exam-
ined a nonlinear model and derived an adaptive, nonlinear control law via the backstepping method to control the attitude states. Digital implementation revealed that the nonlinear controller could control the attitude states as well, however, the control law could not be bounded and required a larger actuator effort than the linear controller.

**Examination of Results**

During the state-feedback chapter, the closed-loop system was tested with four different asteroids and three unique reference trajectories to track for each asteroid, making 12 simulations total. During each simulation, we examined the attitude tracking error (pitch error, roll error, and yaw error) $\tilde{w}_1$, the angular velocity attitude tracking error $\tilde{w}_2$, the actuator effort of the thrusters from the control law $u$, and lastly, the convergence of the estimated parameters, namely the norm of $\hat{P}$ and the elements of $\hat{\rho}$.

In the output-feedback chapter, all of the previous results were examined with the addition of the estimation error of $\tilde{w}_2$ from the observer. During the simulations where the desired reference trajectory was zero, the magnitude for the error signals were the lowest across the board. No surprisingly, the magnitude of the control law was also the lowest. The reason for this might be because zero is the equilibrium point of the plant, however zero is only the equilibrium point of the first state, pitch ($\alpha$), while the other two states, $\phi$ and $\gamma$, do not have equilibrium points. However, this actually appears to be a contradiction as systems that are controlled to track an equilibrium point, exert a control law that converges to zero as time goes to infinity. In fact, the simulations in this thesis show that the control laws for the roll and yaw states converge to zero during the system’s response to a zero desired trajectory, while the control law acting on the pitch state does not converge to zero but instead remains sinusoidal. Comparing the magnitude of the control law between different reference trajectories, we can see that the constant and sinusoidal reference trajectory causes
the most steady state effort required on the thrusters. A trait shared by all measurements across all reference trajectories, and asteroids, is the fact that there is always a brief period of transient response which, after about 1/3 of the first orbit, gives way to a steady state response that is well within the desired error and thruster magnitude limits. It is noted that one full orbit corresponds to $\theta = 2\pi$. In these tests, an error less than 0.1 is considered to be a reasonable desired result.

Motivated for a more accurate representation of the attitude dynamics of a spacecraft in orbit around an asteroid, we used the nonlinear, time-varying model of attitude dynamics to derive a controller. The controllers nonlinear, and adaptive, nature is derived via the backstepping method. Each asteroid, with three reference trajectories each, is used to test the controller, just like the linear case. Unlike the linear case, all asteroids are assumed to have circular orbits and the control law isn’t bounded, nor is disturbance present in the model. During each test, we were concerned with five measurements; attitude tracking error $z_1$, angular velocity tracking error $\dot{z}_1$, backstepping term tracking error $z_2$, the control signal $u$, and the parameter estimations $\hat{\gamma}$. During testing, we observed some notable differences between the orbits of different asteroids. Among all the asteroid orbits, Gaspra required the most control effort to maintain a sufficiently low tracking error. This is curious as Gaspra had no maximum or minimum value from which to attribute this unique result. Likewise, Eros required the least amount of control effort, which is almost fitting as it, like Gaspra, has no maximum or minimum parameter value either. While the attitude tracking error, angular velocity tracking error, and backstepping term tracking error remain relatively the same for each respective test across all asteroids, the estimated parameters seem to be the exception. The convergence of the estimated parameters depends on the adaption gain $\Gamma$
where a lower value tends to make the estimates converge to a constant and higher values will make the estimates take on more of a sinusoidal behavior. Not surprisingly, a larger adaption gain will improve the tracking error of the system, however, if the gains are too large, the response can be seen to ‘chatter’ around a value. This is not desirable as it puts strain on the actuators, in this case the thrusters. For this reason, the gains needed to be tuned selectively so the response is nominal without the effect of chattering.

It is also interesting to observe how varying the value of specific asteroid parameters lead to different results of the system’s response during orbit. During testing, it was discovered that asteroids with a large eccentricity cause the pitch state to be harder to control. This is understandable, as the pitch equations of motion has the only term where eccentricity \( e \) appears. A small value for the orbital radius \( R \) of the spacecraft about the asteroid makes the roll and yaw states diverge faster. Since \( R \) has an inverse relationship to the attitude states in the equations of motion, a small \( R \) would make the terms larger and thus would require a larger magnitude of the controller gains \( K_{f1}, K_{f2} \) to maintain control of the closed-loop system. Likewise, increasing the characteristic length of the asteroid \( R_e \) has the same effect since it is directly proportional to the attitude states. The system was found to be uncontrollable for attitude state initial conditions larger than 2 deg or 2.5 deg. It should also be mentioned that the inverse of these statements are also true, namely, both a small set of initial conditions, eccentricity, characteristic length of the asteroid, and a large \( R \) will make the system more stable and easier to control. The remaining parameters, including orbital semi major axis of the asteroid’s orbit \( R_a \), mass of the asteroid \( M \), and the rotation period of the asteroid \( \omega \), did not negatively, nor positively, effect the stability of the closed-loop system in any significant way when extreme values were used.
One of the more curious results is the convergence of the estimates for the unknown parameters, specifically, how they don’t appear to converge to their respective true value of the unknown parameters, and yet, the control law works and the tracking error stays near zero. Indeed, the transient response stabilizes for all other measurements at the same time as the unknown parameter estimates appear to converge. This implies that convergence of the estimation parameters is linked to the stability of the system, even though the convergence values are not their respective values of the true parameter being estimated. One possible reason for this is because the adaption laws establish that the closed-loop system will be stable, but not necessarily that the parameter estimates will converge to their true value. Despite this, the control law performs well enough to keep the tracking error within a sufficiently low degree. The last odd fact to discuss is the promise made in Chapter 3 when we designed the controller such that the tracking error would go to zero, and that the observer error would go to zero as well. Indeed, even running the simulation for hundreds of orbits, we see that these error signals approach zero, but usually converge to a constant only close to zero. One possible explanation for this phenomenon is that fact that the system is time-varying (technically the true anomaly varies, but it produces the same effect) and yet the design of the control law does not take this fact into consideration. Another possible reason could be the fact that we introduced disturbances in the system, without designing the controller to reject disturbance, such as using an extended state observer. Lastly, the fact that the estimated parameters do not converge to their respective true values might play a part in the error signals not converging to zero. In the end, this phenomenon doesn’t deny the fact that the spacecraft successfully tracks desired attitude trajectories within an adequate margin of error.
Future Work

With these modest results concluded, the final task to consider is how to improve on the design, and the future work required to make the controller ready for implementation. If the linearized equations of motion were continued to be used, it would inevitably need a redesign of a digital controller for implication onto a computer system that would read sensor data and control the torque of the thrusters. Almost all control designs ultimately must be discretized for digital implementation. If one wished to keep the continuous controller design, one improvement could be to make the system more robust to disturbances. This could be done by modifying the adaption law with a robust adaption law, such as sigma modification, or by using a extended state observer, which estimates the higher-order states of the system as well as the disturbances present. The system can then take this estimation for the disturbance and use it to cancel the real disturbance, effectively giving the controller a disturbance rejection property. Lastly, an LQR controller could provide both small tracking error and at a minimal cost to the control law. In the attitude control system, the bound on the thrusters is the most critical part of the design, where as other elements, such as settling time, can be more relaxed. An LQR designed control law would be able to satisfy both the strict constraints of the thruster and provide a reasonable attitude tracking error.
BIBLIOGRAPHY


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