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Channel Estimation and ICI Cancelation in Vehicular Channels of OFDM Wireless Communication Systems

Vahid Vahidi
vahidi@unlv.nevada.edu

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CHANNEL ESTIMATION AND ICI CANCELATION IN VEHICULAR CHANNELS OF OFDM WIRELESS COMMUNICATION SYSTEMS

By

Vahid Vahidi

Bachelor of Electrical and Computer Engineering
Shiraz University
2008

Master of Electrical and Computer Engineering
Shiraz University
2011

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Howard R. Hughes College of Engineering
The Graduate College

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Vahid Vahidi

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Doctor of Philosophy-Electrical Engineering
Department of Electrical and Computer Engineering

Ebrahim Saberinia, Ph.D.  
Examination Committee Chair

Kathryn Hausbeck Korgan, Ph.D.  
Graduate College Interim Dean

Pushkin Kachroo, Ph.D.  
Examination Committee Member

Emma Regentova, Ph.D.  
Examination Committee Member

Shahram Latifi, Ph.D.  
Examination Committee Member

Alexander Paz, Ph.D.  
Graduate College Faculty Representative
Abstract

Orthogonal frequency division multiplexing (OFDM) scheme increases bandwidth efficiency (BE) of data transmission and eliminates inter symbol interference (ISI). As a result, it has been widely used for wideband communication systems that have been developed during the past two decades and it can be a good candidate for the emerging communication systems such as fifth generation (5G) cellular networks with high carrier frequency and communication systems of high speed vehicles such as high speed trains (HSTs) and supersonic unmanned aircraft vehicles (UAVs). However, the employment of OFDM for those upcoming systems is challenging because of high Doppler shifts. High Doppler shift makes the wideband communication channel to be both frequency selective and time selective, doubly selective (DS), causes inter carrier interference (ICI) and destroys the orthogonality between the subcarriers of OFDM signal. In order to demodulate the signal in OFDM systems and mitigate ICIs, channel state information (CSI) is required. In this work, we deal with channel estimation (CE) and ICI cancellation in DS vehicular channels. The digitized model of the DS channels can be short and dense, or long and sparse. CE methods that perform well for short and dense channels are highly inefficient for long and sparse channels. As a result, for the latter type of channels, we proposed the employment of compressed sensing (CS) based schemes for estimating the channel. In addition, we extended our CE methods for multiple input multiple output (MIMO) scenarios. We evaluated the CE accuracy and data demodulation fidelity, along with the BE and computational complexity of our methods and compared the results with the previous CE procedures in different environments. The simulation results indicate that our proposed CE methods perform considerably better than the conventional CE schemes.
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<tbody>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BE</td>
<td>Bandwidth Efficiency</td>
</tr>
<tr>
<td>BEM</td>
<td>Basic Expansion Model</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>CE</td>
<td>Channel Estimation</td>
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<tr>
<td>CoSaMP</td>
<td>Compressive Sampling Matching Pursuit</td>
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<td>Delay Doppler Sparsity</td>
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<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DS</td>
<td>Doubly Selective</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter Carrier Interference</td>
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<tr>
<td>LS</td>
<td>Least Square</td>
</tr>
<tr>
<td>LMMSE</td>
<td>Linear Minimum Mean Square Error</td>
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<tr>
<td>MIMO</td>
<td>Multiple Input Multiple Output</td>
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<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>MDDS</td>
<td>Modified Delay Doppler Sparsity</td>
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<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>OMP</td>
<td>Orthogonal Matching Pursuit</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal To Noise Ratio</td>
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<tr>
<td>SUI</td>
<td>Stanford University Interim</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aircraft Vehicle</td>
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Chapter 1: Introduction

Since our daily life is becoming more dependent on vehicles, vehicular wideband communications is an evolving technology which has attracted wide considerations in recent years. The goal is to develop a reliable and efficient real time communication systems. Because of high bandwidth efficiency (BE) and robustness to multipath effect, orthogonal frequency division multiplexing (OFDM) has been accustomed widely for various wideband wireless data communication systems [1-3]. However, high Doppler shift of the emerging vehicular communication systems makes the use of OFDM challenging since it makes the communication channel time selective and causes significant inter-carrier interference (ICI). High Doppler shift can occur in two different conditions; high speed vehicles, and high center frequency communication systems. The examples of high speed vehicles are fixed-wings unmanned aircraft vehicles (UAVs) and high speed trains (HSTs). For instance, a low cost supersonic 1.4-Mach UAV is made in Colorado University [4]. Because of its light weight and small engine, it can fly at low altitudes and it can be used for civilian applications. For the example of high carrier frequency communication systems, 5G standard can be considered. This emerging cellular communication systems is projected to work at the center frequency between 27.5-71 GHz in the United States [5-6]. Thus, even at regular highway car speeds, the Doppler shift of several kilohertz affects the received signal.

We studied the performance of OFDM in doubly selective (DS) UAV to ground station (GS) channels in [7]. The simulation results in that paper indicate that without proper ICI mitigation, the performance of the OFDM system degrades extensively especially when the number of subcarriers becomes larger or the Doppler shift increases. In order to remove ICI, the communications channel should be estimated. Data-aided channel estimation (CE) methods can be performed either in the frequency domain or time domain. Frequency domain CE techniques have been the subject of several studies [8-12]. Implementing Least Squares (LS)
and Linear Minimum Mean Square Errors (LMMSE) estimators based on one OFDM block as a pilot in the frequency domain was proposed in [8]. Using adjacent subcarriers of a frequency domain symbol to mitigate its ICI was proposed in [9]. MinHai et al. [10], assumed that the phase changes in a linear manner with Doppler; therefore, they estimated the Doppler by considering two consecutive channel-transfer functions. Guangxi et al. [11] estimated the channel by zero padding the pilot. They implemented DFT and a high-precision interpolation technique in the frequency domain. Iterative processing of CE in the frequency domain was performed in [12]. Several other authors proposed time domain CE methods [13-18]. By assuming that the channel impulse response (CIR) varies linearly with time during a block period, Han et al. [13] proposed a time-domain equalization technique. Gupta [14] demonstrated that their proposed time-domain ICI-mitigation technique based on DFT estimation of the channel had better performance compared to the LS and LMMSE estimators. Ahmed et al. [15] assumed that Doppler shift causes attenuation coefficients to be time-varying; therefore, they transmitted and cross-correlated a known OFDM reference block with the local known OFDM block. By comparing the difference in pick location, they were able to calculate the Doppler shift. Gupta [16] proposed an iterative time domain LMMSE (TD-LMMSE) that tracked channel variations in the time domain using an LMMSE estimator. Aggarwal et al. [17] inserted a pseudo random code in the time domain during the guard interval. Some studies combined time domain and frequency domain techniques for CE. Werner et al. [18] were able to conduct a CE by estimating the channel response in the frequency domain. Afterwards, for the sake of the interpolation error reduction, a refining step in the time domain was performed. In [19], the Doppler spread was estimated in the frequency domain and complex amplitudes were estimated in the time domain.

Most of the schemes that were reviewed ignore the non-diagonal elements of the frequency domain channel matrix to reduce the computational complexity of channel equalization [8-19].
However, others take the non-diagonal elements into consideration [20-21]. Ng and Dubey [20] obtained the other elements of the frequency domain channel matrix with the assumption that the ICI of a subcarrier was just due to its adjacent subcarriers. Nakamura et al. [21] minimized the mean square error between the received pilots and the transmitted pilots, and estimated the Doppler shift and complex amplitudes of all the paths autoregressively. They assumed each time that the received signal just routes through one path; therefore, they neglected the ICI of the other paths.

On the other hand, based on the experimental results [22-23], a number of DS vehicular communication channels exhibit a long delay and sparse structure in the time and Doppler domain. Because of the long delay of those channels, all the CE methods that are proposed for dense environment are inefficient since tremendous number of pilots would be needed for estimating the channel. Besides the inefficiency, those methods are impractical for fast time varying channels since when the channel is estimated, it would not be valid for the upcoming data if the coherence time of the channel would be smaller than the duration of the CE reference block and the data length. To utilize the inherent channel sparsity, the ground breaking compressed sensing (CS) procedure can be employed. It is indicated in [24] that how a sparse signal can be reconstructed by a few number of measurements. Several works have applied CS for sparse CE in DS channels [25-33]. Most of these studies, employ frequency domain pilots for CE [25-31] while a few number of them apply time domain training sequence [32-33].

The papers that employ frequency domain pilots can be divided into two major groups. The first group of these papers only utilize the sparsity of the channel in the delay domain [25-26]. In these papers, a sparse vector is made from the sparse pseudo-circular channel matrix in the time domain and they utilize an LS or an LMMSE approach to estimate only the diagonal elements of the frequency domain channel matrix. The second group of papers that utilize frequency domain pilots, benefit from the sparsity of both delay and Doppler shift [27-31]. In
[27-29], basic expansion model (BEM) is applied to express the DS channels. In these papers, basis vectors are defined in the frequency domain and the sparse coefficient vectors of the channel are estimated by the transmission of frequency domain pilots. As it is indicated in [27], the BEM based CE methods make large modeling error because of the truncation of the FFT that occurs while basis determination; therefore, they need a refining step. Some papers that utilize both the delay and Doppler shift sparsity, state the CE problem as finding the non-zero elements of a sparse matrix [30-31]. Each element of the matrix defines a specific delay and Doppler shift for a particular path. For instance, the authors of [30] have applied this method for proposing two off line pilot design algorithms while the utilization of the train position for deleting most of the elements of the delay Doppler matrix is performed in [31]. The disadvantageous of the methods that construct a sparse matrix of delay and Doppler shifts is that in high Doppler shift conditions, the size of the matrix increases which results in the performance degradation of the CS methods. Some papers applied time domain training sequence for CS based CE methods for time domain synchronous OFDM (TDS-OFDM) systems [32-33]. In TDS-OFDM, the cyclic prefix is replaced by a pseudorandom noise (PN) sequence which is also used for CE. However, those papers considered that the channel does not change during the transmission time of one OFDM symbol and therefore, their method is not practical for high Doppler shift conditions.

On the other hand, as we explained in the first paragraph, one of the conditions that requires CE is the next communication systems such as 5G. According to [5], it is highly probable that 5G communication systems benefit from massive multiple input multiple output (MIMO) technology to increase the channel capacity and enhance data fidelity. Since MIMO requires the full knowledge of CSI for estimating the transmitted sequences of each transmitter from the received sequences of all the receiver antennas, CE is essential in MIMO-OFDM systems. As the number of transmitters and receivers increases, more channels should be estimated at
the receiver; therefore, applying the conventional CE methods such as LS and LMMSE would be bandwidth inefficient and this results in the overall system throughput degradation. However, because of the high signal bandwidth of the 5G systems which is expected to be more than 100 MHz [5], the time resolution decreases and the tapped delay line channel model would be a sparse one. As a result, CS methods can be implemented for estimating the communications channel. The effectiveness of CS methods for sparse CE in MIMO-OFDM systems is described in [34-35]. In addition, most MIMO systems with colocation antennas have a communal channel support because of the analogous path arrival times between each pair of transmitter and receiver antenna [36]. Several papers employed that innate common sparsity to improve the CE and they applied block orthogonal matching pursuit (BOMP) in order to find non-zero channel taps [37-40]. Those papers studied the effect of designing pilot structure as an offline procedure that effects the accuracy of CE. In order to prevent interferences between the received pilots at the receiver antennas, orthogonal pilot structures are considered in [37-38]. Since the orthogonal pilot structures add overhead to the signal transmission system and reduce the effective bandwidth, several papers proposed the employment of the same locations of subcarriers for pilot transmission [39-40]. Those papers proposed the utilization of pseudorandom Bernoulli pilots in order to decrease the correlation between the received symbols.

CS based CE methods for DS channels in MIMO-OFDM systems which utilize scattered pilots in the frequency domain can be categorized into three major groups. The methods of the first group only exploit delay sparsity of the channel to estimate the diagonal elements of the frequency domain channel matrix [41-43]. Since these CE methods do not separately estimate the Doppler shifts of the channel taps in the time domain, they cannot construct the whole frequency domain channel matrix for ICI cancellation. As a result, they are not suitable for high Doppler shift scenarios. The methods of the second group consider the sparsity of both
In these papers, a sparse matrix is expressed which its elements should be found. Any non-zero element of that matrix represents the delay and Doppler shift of a path. The disadvantage of these methods is that the size of the sparse matrix increases in high Doppler shift conditions which results in the performance degradation of any CS method. The CE schemes of the third group consider the sparsity in the time domain directly and the sparsity in the Doppler domain indirectly by defining the bases that are created in the frequency domain [48-49]. Those bases are defined by BEM and are exploited to estimate the sparse vectors which are expressed in the delay domain. This type of schemes can exploit grouped CS (GCS) methods. These methods require large guard intervals between pilots and data which results in the degradation of spectral efficiency. They also require a refining step that increases the computational complexity of the CE.

In general, the CE methods that were reviewed are not appropriate for very high Doppler shift channels. In this current work, we have proposed several CE schemes which are suitable for high Doppler shift conditions. Some of those CE methods are suitable for short and dense channels while the other methods are applicable for long and sparse channels. On the other hand, some of our proposed methods utilize time domain training sequences and the others are the modified version of the previous methods that apply frequency domain scattered pilots.

The remainder of this article is organized as follows. Chapter 2 presents the background which involves the study of the transmission and demodulation of OFDM data through the DS channels, and several CE methods that their approaches are utilized in our work. Our proposed CE methods for short and dense channels, long and sparse channels, and MIMO conditions are described in Chapter 3, 4 and 5 respectively. Finally, Chapter 6 discusses the conclusion.
2.1. OFDM transmission and demodulation in doubly selective channels

- Single input single output (SISO) condition

Each time domain OFDM sample that is transmitted to the channel is obtained by calculating the IFFT from the OFDM symbols and is represented as:

\[
x_t(n) = \begin{cases} 
  \sum_{m=0}^{N-1} x_f(m) e^{j2\pi(n+G-1)m/N} & 0 \leq n \leq G - 1 \\
  \sum_{m=0}^{N-1} x_f(m) e^{j2\pi(n-G)m/N} & G \leq n \leq N - 1 + G
\end{cases}
\]

where \( x_f(m) \) is the frequency domain transmitted symbol in the \( m \)th subcarrier, \( N \) is the number of subcarriers, \( n \) is the index of the transmitted samples in the time domain, and \( G \) is the length of the cyclic prefix. The time varying channel model between the transmitter and receiver is modeled as [50]:

\[
a_t(t) = \sum_{l=1}^{L} a_l e^{j\theta_l} e^{j2\pi f_l kT_s} g_{total}(lT_s - \tau_l)
\]

where \( L \) is the number of the propagation paths, \( a_l \) is the amplitude of the \( l \)th path having a Rayleigh distribution, \( \theta_l \) is the random delay of the \( l \)th path, \( f_l \) is the Doppler of the \( l \)th path, \( \tau_l \) is the delay of the \( l \)th path, \( T_s \) is the channel resolution for resolvable paths, and \( g_{total} \) is the total impulse response of the transmitter and receiver filters.

When the signal is passed through the tapped delay line channel in (2), the received signal is expressed as [50]:
\[ y_t(n) = \sum_{l=1}^{L} x_t(n-l) a_l e^{j2\pi f_l n T_s} + w_t(n), \]

where \( y_t(n) \) is the received signal after passing through the channel and \( w_t(n) \) is the additive white Gaussian noise (AWGN) that is added to the \( n^{th} \) signal in the time domain. The other parameters were defined earlier. The received symbols in the frequency domain are calculated by calculating DFT of the received time domain samples as:

\[ y_f(m) = \sum_{m=1}^{N} x_f(m) H(k, m) + z_f(m), \]

where \( y_f(m) \) is the received symbol in \( m^{th} \) subcarrier, \( H(k, m) \) is the \((k, m)^{th}\) element of the frequency domain channel matrix, \( H \), and \( z_f(m) \) is the Fourier transform of AWGN which is added to the \( m^{th} \) received symbol. By considering (1)-(4), \( H(k, m) \) is obtained as [21]:

\[ H(k, m) = \frac{1}{N} \sum_{l=1}^{L} a_l e^{j2\pi f_l l T_s} e^{-j2\pi l k} \frac{1 - e^{jN(2\pi f_l T_s + \frac{2\pi}{N}(m-k))}}{1 - e^{j(2\pi f_l T_s + \frac{2\pi}{N}(m-k))}}. \]

When there is zero Doppler shift, \( H(k, m) = 0 \) for \( k \neq m \) and \( H \) becomes diagonal. Therefore, \( y_f(m) \) would only be a function of \( x_f(m) \) and the \( m^{th} \) diagonal element of the \( H \). However, in the presence of Doppler shift, \( H \) is not diagonal and \( y_f(m) \) would be dependent on all the transmitted subcarriers.

When channel is estimated, it can be used for detection of the transmitted symbols at the receiver. Papers have used two different approaches for symbol detection. Some studies have considered the received OFDM symbols as:

\[ y_f = H_a x_f + H_{icl} x_f + z_f \]
where $x_f$, $y_f$ and $z_f$ are the $N \times 1$ frequency domain vector of the transmitted symbols, received symbols and the additive noise respectively, $H_d$ is the diagonal channel matrix in the frequency domain and its elements are obtained from (5) by assigning $k = m$, and $H_{ICI}$ is the matrix that its diagonal elements are zero and its non-diagonal elements are presented by (5) for $k \neq m$. By considering $H_{ICI}X + Z$ as the additive noise, the transmitted symbols by using the one tap equalizer are estimated as:

$$\tilde{x}_f = \frac{y_f}{H_d}$$ (7)

where $\tilde{x}_f$ is the vector of the estimated received symbols. While this approach performs well in low Doppler shift conditions, our simulation results in the next section indicate that the one tap equalizer is not accurate for high Doppler shift scenarios. As a result, complete channel matrix of (5) should be used for data demodulation. This matrix can be used by LS method as:

$$\tilde{x}_f = inv(H) y_f$$ (8)

More accuracy would be obtained if MMSE is used instead of LS. The MMSE of the received symbols is obtained as:

$$\tilde{x}_f = \hat{H}^* \times [\hat{H} \times \hat{H}^* + \sigma^2 I]^{-1} y_f$$ (9)

where $I$ is the $N \times N$ identity matrix and $\sigma^2$ is the noise variance.

- **Multiple input multiple output (MIMO) condition**

Consider a MIMO-OFDM system with $N_T$ transmitters and $N_R$ receivers. The tap delay line channel model between the $i$th transmitter and $j$th receiver at time $n$ is expressed as:

$$h_{ij}(n) = \sum_{l=1}^{L} \alpha_{ij} e^{j2\pi f_{D_{ij}} nT_S} \delta((n - l)T_s)$$ (10)
where $h_{ij}$, $a_{ij}$ and $f_{D_{ij}}$ determine the discrete equivalent of the channel, $i^{th}$ tap complex amplitude and $i^{th}$ tap Doppler shift between the $i^{th}$ transmitter and $j^{th}$ receiver respectively, and $T_s$ is the sampling time of the system. The total number of taps is indicated by $L$ which is obtained by dividing the maximum delay spread of the channel by $T_s$.

At each receiver, an FFT is applied to the block of $N$ received data. The output of the FFT in the $j^{th}$ receiver is obtained as:

$$y_{f_j} = \sum_{i=1}^{N_T} H_{ij} x_{f_i} + z_{f_j}, \quad j = 1, 2, ..., N_R \quad (11)$$

where $x_{f_i}, i = 1, 2, ..., N_T$, is the $N \times 1$ vector of the frequency domain transmitted symbols from the $i^{th}$ transmitter and $H_{ij}$ is a $N \times N$ frequency domain channel matrix between the $i^{th}$ transmitter and $j^{th}$ receiver and its elements are obtained by calculating the $N$ point FFT of the $h_{ij}$ channel in equation (1). Vector $z_{f_j}$ is a $N \times 1$ vector and indicates the additive noise in frequency domain.

In order to obtain the transmitted signals from the received signals, the received symbols can be put in a single vector of size $N \cdot N_R \times 1$ as $y_f = [y_{f_1}^T, y_{f_2}^T, ..., y_{f_{N_R}}^T]^T$ and its relationship to the transmitted signals, $x_f = [x_{f_1}^T, x_{f_2}^T, ..., x_{f_{N_T}}^T]^T$, is defined as:

$$y_f = H x_f + z_f \quad (12)$$

where $H$ is a $N \cdot N_R \times N \cdot N_T$ matrix which is expressed as:

$$H = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1N_T} \\ H_{21} & \ddots & \cdots & H_{2N_T} \\ \vdots & \ddots & \ddots & \vdots \\ H_{N_R1} & \cdots & \cdots & H_{N_RN_T} \end{bmatrix} \quad (13)$$
and \( \mathbf{z}_f = [\mathbf{z}_{f_1}^T, \mathbf{z}_{f_2}^T, \ldots, \mathbf{z}_{f_{N_R}}^T]^T \) is the \( N \cdot N_R \times 1 \) additive noise vector. The \((k, m)\)th element of \( \mathbf{H}_{ij} \) is obtained similar to (5).

When there is no Doppler shift, \((H_{k,m})_{i,j} = 0\) for \( k \neq m \) and \( \mathbf{H}_{ij} \) becomes a diagonal matrix. However, in the presence of Doppler shift, \( \mathbf{H}_{ij} \) is not diagonal and \( \mathbf{y}_{f_j} \) would depend on all the transmitted subcarriers. When all the \( \mathbf{H}_{ij} \)'s are estimated, the transmitted data can be extracted by the employment of minimum mean square error (MMSE) estimate:

\[
\hat{\mathbf{x}} = \hat{\mathbf{H}}^* \times [\hat{\mathbf{H}} \times \hat{\mathbf{H}}^* + \sigma^2 \mathbf{I}_{N \cdot N_R}]^{-1} \mathbf{y},
\]

where \( \hat{\mathbf{H}} \) and \( \hat{\mathbf{x}} \) define the estimated channel matrix and the transmitted symbols respectively, and \( \sigma^2 \) is the noise variance.

2.2. Channel estimation methods for short and dense channels

- Least square (LS) and linear minimum mean square error (LMMSE) methods:

In small Doppler shift situations, the channel matrix \( \mathbf{H} \) is nearly diagonal and there is no ICI. The diagonal elements of the channel still should be estimated to recover transmitted symbol in each sub-carrier using what is commonly called one-tap equalization. In order to obtain the LS estimate of channel, a pilot with the length of \( N \) in the frequency domain should be transmitted. The LS estimate of the diagonal elements of the channel is obtained as [8]:

\[
\hat{\mathbf{h}}_{\text{LS}} = \begin{bmatrix} y_{f_1} & y_{f_2} & \ldots & y_{f_N} \\ x_{f_1} & x_{f_2} & \ldots & x_{f_N} \end{bmatrix},
\]

where \( x_{f_i} \) and \( y_{f_i} \) are the \( i^{th} \) transmitted pilot and received pilot respectively. By utilizing the LS estimator, the LMMSE of the channel is obtained as [8]:

\[
\hat{\mathbf{h}}_{\text{LMMSE}} = \mathbf{R}_{HH} \left[ \mathbf{R}_{HH} + \frac{\beta}{\sigma^2} \mathbf{I}_N \right]^{-1} \hat{\mathbf{h}}_{\text{LS}},
\]
where $\beta$ is defined as $\frac{E(x_{f,k}^2)}{E(x_{f,k})}$ when $x_{f,k}$ is the transmitted symbol in the frequency domain, $\sigma^2$ represents the variance of AWGN and $R_{HH}$ is the autocorrelation matrix of the frequency domain channel and is defined as [8]:

$$R_{HH}(u,v) = \sum_{l=0}^{L-1} a_t^2 e^{-j2\pi(u-v)k}.$$ (17)

2.3. Channel estimation methods for long and sparse channels

• CS based LS and LMMSE methods

While LS and LMMSE estimates of the channel matrix work for both dense and sparse wireless channels, using CS algorithms for the sparse case can decrease the size of the pilot signal needed and the number of calculations for CE. When a pilot signal is transmitted and we would like to estimate the time domain channel from the received signal $y_f$, one can rewrite the received signal presented in equation (6) as [51]:

$$y_f = X_d F h + H_{IC} x_f + z_f.$$ (18)

where $X_d$ is the $N \times N$ diagonal matrix with diagonal elements equal to pilot data $x_f$. The time domain channel vector $h$ is assumed to be sparse with only $S$ non-zero elements where $S \ll L$. The location of non-zero elements of $h$ and their values are to be estimated with CS algorithms. It is indicated in [52] and [53] that a good reconstruction of an $S$ sparse data can be obtained by using only a pilot size between $3S$ and $4S$ instead of a whole OFDM symbol of size $N$. If $P$ indicates the number of pilots used, we can separate transmitted and received pilot signals and arrange them in their vectors as [54]:

$$y_f^{CS} = X_d^{CS} F^{CS} h + z_f^{CS},$$ (19)
where $X_d^{CS}$ is a $P \times P$ diagonal matrix of pilots, $F^{CS}$ is a $P \times N$ matrix that its rows are chosen from the rows of $F$ (the rows that corresponds to the pilot places), $z_f^{CS}$ is $P \times 1$ AWGN vector and $y_f^{CS}$ is $P \times 1$ receive data corresponding to pilot subcarriers. The estimated channel $\hat{h}$ would be obtained by solving $l_0$ minimization problem as [54]:

$$\min \|h\|_0 \text{ s.t. } \|y_f^{CS} - \phi h\|_2 \leq \sigma,$$  (20)

where $\phi = X_d^{CS} F^{CS}$ is the measurement matrix and $\|h\|_0$ defines the number of non-zero elements of $h$ and $\sigma$ is the variance of AWGN. Since this problem is a NP hard problem, it is indicated in [54] that it can be replaced by a convex optimization problem as follows:

$$\min \|h\|_1 \text{ s.t. } \|y_f^{CS} - \phi h\|_2 \leq \sigma,$$  (21)

where $\|h\|_1$ is the norm 1 of the channel that is obtained by the summation of the absolute values of the channel taps.

While there are several ways to reconstruct $h$ from $y_f^{CS}$, orthogonal matching pursuit (OMP) is a common greedy algorithm for obtaining $h$. In OMP, we reconstruct $y_f^{CS}$ using selected columns of $\phi$. The column of $\phi$ that has the largest correlation with $y_f^{CS}$ is chosen to initialize the new reconstruction matrix $M$. Then, we subtract the portion of $y_f^{CS}$ that is covered by the new column of $M$, to calculate a residue which in turn is correlated with columns of $\phi$. In each step, a new column is added to matrix $M$ and the channel of ith iteration is calculated as:

$$\hat{h}_{\text{OMP}} = (M_i^H M_i)^{-1} M_i^H y_f^{CS},$$  (22)
and a new residue is calculated:

\[ r_i = y_f^{cs} - \varphi \hat{h}_{OMP_i}, \quad (23) \]

where \( M_i \) is the \( M \) matrix at \( i^{th} \) iteration. The iteration stops when \( \frac{||r_i||_2}{||r_{i-1}||_2} < \text{thresold} \). The time domain channel, \( \hat{h}_{omp} \), is obtained through the last iteration of (22).

Using \( \hat{h}_{omp} \), one can calculate \( \hat{h}_{ls} \) as:

\[ \hat{h}_{ls} = \text{diag}(F \hat{h}_{omp}), \quad (24) \]

and \( \hat{h}_{LMMSE} \) using (11).

- Delay Doppler sparsity method (DDS) [30]:
  
  By the assumption that \( f_i \in [-f_{\max} + \alpha, f_{\max} - \alpha] \), the Doppler spread can be quantized into \( 2Q + 1 \) levels. \( (Q = \alpha \left\lceil \frac{f_{\max}}{\alpha} \right\rceil) \). \( \alpha \) defines the precision of the Doppler shift quantization which is considered to be 1 in [30], and \( \left\lceil \cdot \right\rceil \) defines the floor operator. The diagonal elements of the frequency domain channel matrix is defined as [30]:

\[ H(k) = \sum_{l=0}^{L-1} \sum_{q=-Q}^{Q} a_l e^{j2\pi \alpha q L T_s} e^{-j2\pi k \frac{q}{N}} \quad (25) \]

By defining the two vectors as \( \mathbf{v}_k = [1, e^{-j2\pi \frac{k}{N}}, \ldots, e^{-j2\pi (L-1)\frac{k}{N}}] \) and \( \mathbf{v}_q = [e^{-j2\pi Q L T_s}, e^{j2\pi (1-Q) L T_s}, \ldots, e^{j2\pi Q L T_s}] \), (25) can be written as:

\[ H(k) = \mathbf{v}_k \mathbf{C} \mathbf{v}_q^T = (\mathbf{v}_q \otimes \mathbf{v}_k) \mathbf{c} \quad (26) \]

\( \mathbf{C} \) is a \( L \times (2Q + 1) \) matrix, \( \mathbf{c} \) is the \( L(2Q + 1) \) vector form of \( \mathbf{C} \) and \( \otimes \) defines the kronecker products.
By the insertion of (21) in (13), the following equation is obtained:

\[ y_f = X_d V c + N_T \]  

(27)

where \( V = [v_q \otimes v_{k_1}, v_q \otimes v_{k_2}, \ldots, v_q \otimes v_{k_p}]^T \) is a \( P \times L(2Q + 1) \) matrix and \( N_T = H_{I_{CI}} x_f + z_f \).

If only \( S \) paths out of \( L \) paths of the channel would be non-zero while \( S \ll L \) and the \( c \) vector would be a sparse one. The measurement matrix for this CS problem is \( \varphi = X_d V \). A CS method like OMP, can be applied for resolving this problem.

- Basic expansion method (BEM) [27]

In this method, the complex amplitudes and the phase variation of Doppler shifts are combined to each other and are expressed as one variable. The \( l^{th} \) channel tap is defined as:

\[
h_l = [b_0, b_1, \ldots, b_{D-1}] \begin{bmatrix} c(0, l) \\ c(1, l) \\ \vdots \\ c(D - 1, l) \end{bmatrix} + e_l,
\]  

(28)

where \( h_l = [h_l(G), h_l(G + 1), \ldots, h_l(G + N)]^T \), \( D \) is the BEM order, \( e_l \) is the BEM modeling error and \( c(d, l)_{i,j} \) is the coefficient of the \( d^{th} \) BEM base, \( b_d \), which is defined as:

\[
b_d = \begin{bmatrix} 1, \ldots, e^{j2\pi n(d-\frac{D-1}{2})}, \ldots, e^{j2\pi n(N-1)(d-\frac{D-1}{2})} \end{bmatrix}^T.
\]  

(29)

The received symbols in term of the BEM are expressed as:

\[
y_f = \sum_{d=0}^{D-1} B_d C_d x_f + w_f,
\]  

(30)

where \( B_d = F_N \text{diag}(b_d) F_N^H \), \( C_{d,i,j} = \text{diag}(\sqrt{N}F_N(C_d^T, 0_{1 \times N-L})^T) \). \( F_N \) is Fourier transform matrix, and \( w_f \) defines the BEM error in the frequency domain. By the assumption that \( D \) is an odd number and the guard interval before and after of each pilot is \( D - 1 \), the received signals are expressed as:
\[ y_{fd} = \left[ \hat{P}_1 F_{ND-1} \frac{1}{2}, \hat{P}_2 F_{ND-1} \frac{1}{2}, \ldots, \hat{P}_{N_f} F_{ND-1} \frac{1}{2} \right] \alpha_d \hat{e}_d + w_f, \]  
\hspace{1cm} (31)

where \( \alpha_d = diag(1, e^{\frac{i2\pi (d-1)}{N}}, \ldots, e^{\frac{i2\pi (d-1) \cdot N_f}{N}}) \), \( \hat{P}_i \) is the diagonal matrix of the \( i^{th} \) transmitter, and \( F_{ND-1} \) is the matrix that its rows are chosen from those rows of \( F_N \) which correspond to the position of pilots. By this formulation, the bases \( (d_s) \) will have the same sparsity. As a result, GCS methods can be applied for sparse signal estimation. At the last step, linear smoothing procedure was proposed in [27] in order to reduce the BEM modeling error and Doppler Effect on the estimation of complex amplitudes. For data demodulation, the estimated coefficients are inserted in (30).

2.4. Compressed based Channel estimation methods for MIMO-OFDM

- **LS method for static channels**

In order to estimate overall matrix \( H \) and use it in either (14) for data demodulation, we first estimate \( \mathbf{h}_{ij} \) using a pilot assisted system. Each \( \mathbf{h}_{ij} \) is a diagonal matrix whose elements are N-point FFT of \( h_{ij} \). By adding \( N - L \) zeros to the end of \( \mathbf{h}_{ij} \), we use \( N \times 1 \) vector \( \mathbf{g}_{ij} \) to represent time domain channel from the \( i^{th} \) transmitter antenna to the \( j^{th} \) receiver antenna. The equation (11) can be rewritten as:

\[ y_j = \sum_{i=1}^{N_t} \mathbf{X}_{d_i} F \mathbf{g}_{ij} + z_j, \quad j = 1, 2, \ldots, N_R \]  
\hspace{1cm} (32)

where \( \mathbf{X}_{d_i} \) is the \( N \times N \) diagonal matrix that its diagonal elements are the elements of \( \mathbf{x}_{f_i} \), i.e., \( \mathbf{X}_{d_i} = \text{diag}(\mathbf{x}_{f_i}) \), and \( F \) is the \( N \times N \) Fourier transform matrix. If we put all the channel vectors coming to the received antenna in a single vector as \( \mathbf{g}_j = [\mathbf{g}_{1j}^T, \mathbf{g}_{2j}^T, \ldots, \mathbf{g}_{N_{Tj}}^T]^T \), the equation (32) can be rewritten as:

\[ y_{j} = \mathbf{D} \mathbf{g}_j + z_j, j = 1, 2, \ldots, N_R \]  
\hspace{1cm} (33)
where \( D \) is a \( N \times N.T \) matrix which is defined as:

\[
D = [X_{d1}F, X_{d2}F, \ldots, X_{dN_T}F].
\] (34)

When only \( S \) out of \( N \) elements of \( g_{ij} \) are non-zero and \( S \ll N \), the channel is considered to be sparse. As a result, \( g_j \) would be a sparse vector. Therefore, CS algorithms can be applied in order to estimate \( g_j \) from equation (33) using small number of pilots. By setting the number of pilots to be \( P \), (33) can be written for scattered pilots as:

\[
y_{fj}^{cs} = D^{cs}g_j + z_{fj}^{cs},
\] (35)

where \( y_{fj}^{cs} \) is the \( P \times 1 \) vector of the received pilots at the \( j \)th receiver, \( z_{fj}^{cs} \) is the \( P \times 1 \) additive noise vector in frequency domain, and \( D^{cs} \) is a \( P \times N.T \) matrix which is defined as:

\[
D^{cs} = [X_{d1}^{cs}F^{cs}, X_{d2}^{cs}F^{cs}, \ldots, X_{dN_T}^{cs}F^{cs}],
\] (36)

where \( X_{di}^{cs} \) is a \( P \times P \) diagonal matrix of pilots and \( F^{cs} \) is a \( P \times N \) matrix that its rows are selected from the rows of \( F \) (the rows that corresponds to the pilot locations). Afterwards, since all the \( g_{ij}s \) indicate a common sparsity, (35) can be written as:

\[
y_{fj}^{cs} = \theta^{cs} \vartheta_j + w_{fj}^{cs},
\] (37)

where \( \vartheta_j \) is obtained from \( h_j \) by extracting the same taps and rearranging them in a group, \( \vartheta_j = [\vartheta_{j_1}^T, \vartheta_{j_2}^T, \ldots, \vartheta_{j_L}^T]^T \), where \( \vartheta_{j_i} = [\alpha_{1j}, \alpha_{1j}, \ldots, \alpha_{1N_Tj}] \) and \( \theta^{cs} \) is obtained from \( D^{cs} \) by rearranging its columns, \( \theta^{cs} = [\theta^{cs}_1, \theta^{cs}_2, \ldots, \theta^{cs}_L] \) where \( \theta^{cs}_i = [D^{cs}_{i1}, D^{cs}_{i+L}, \ldots, D^{cs}_{i+(N_T-1)L}] \). Since different CIRs have common sparse support, the whole elements of the \( \vartheta_{j_i} \) would be zero or non-zero which indicates the block sparsity. As a result, block-structured CS methods such as BOMP, can be applied for solving (37) and \( \theta^{cs} \) is the measurement matrix for that CS procedure.
As it is described in [55], the BOMP algorithm is initiated by the residue, \( r_0 = y_f^{cs} \). At the \( k \)th iteration, the block that has the most correlation to \( r_{k-1} \) is chosen:

\[
i_k = \arg \max \max_i (\theta_i^{cs} H \theta_k^{cs}) \tag{38}
\]

Those columns of \( \theta_i^{cs} \) are chosen to initialize the reconstruction matrix \( M \). Afterwards, the portion of \( y_f^{cs} \) that is covered by the new columns of \( M \) is subtracted and a new residue is calculated. In each step a new block of columns are added to matrix \( M \) at the end of each step, and the \( \theta_j \) of the \( k \)th iteration is obtained as:

\[
\overline{\theta}_j^{k} = (M_k^H M_k)^{-1} M_k^H y_f^{cs}, \tag{39}
\]

and a new residue is calculated:

\[
r_k = y_f^{cs} - \theta^{cs} \overline{\theta}_k^{k}. \tag{40}
\]

The iteration stops when \( \|r_k\|_2 < \text{threshold} \).

We should run BOMP scheme \( N_R \) times in order to estimate all the \( \theta_j, j = 1, 2, \ldots, N_R \).

Afterwards, each \( \overline{\theta}_j \) can be obtained after rearranging the elements of \( \overline{\theta}_j \).

It is indicated in [55] that the mutual block coherence of \( \theta^{cs} \) is obtained by:

\[
\mu_B(\theta^{cs}) = \max_{r \neq l} \frac{1}{N_T} \rho(\theta_r^{cs} H \theta_l^{cs}). \tag{41}
\]

It is proved in [56] that if the average mutual coherence is minimized instead of the maximum mutual coherence, more accurate estimate of the sparse vector would be obtained. As a result, in this paper we consider the average of the mutual coherence:

\[
\mu_B(\theta^{cs}) = \frac{1}{N_T N} \sum_{\theta \neq \theta, \theta \leq N_T} \theta_r^{cs} H \theta_l^{cs}. \tag{42}
\]

- MIMO-DDS method [57]
This method is based on the DDS method which was described in the previous section. Similar to the equation (27), the following formula is achieved for the MIMO condition at each receiver antenna:

\[ Y_j = \varphi c_{Tj} + Z_j \]  \hspace{1cm} (43)

where \( \varphi = P \left( v_q \otimes v_k \otimes 1_{N_T \times 1} \right) \) and \( P \) is the \( p \times p \) diagonal matrix of the transmitted pilots.

- MIMO-BEM [49]

This method is based on the BEM method that was described in the previous section. The received symbols at the \( j^{th} \) receiver are expressed as:

\[ y_{fj} = \sum_{i=1}^{N_T} \sum_{d=0}^{D-1} B_d C_{d,i,j} x_{f_i} + W_j, \]  \hspace{1cm} (44)

where all the parameters were defined earlier. By the assumption that \( D \) is an odd number and the guard interval before and after of each pilot is \( D - 1 \), the received signals are expressed as:

\[ y_{f,j,d} = \left[ \hat{P}_1 F_{N_D - 1} \frac{1}{2}, \hat{P}_2 F_{N_D - 1} \frac{1}{2}, \ldots, \hat{P}_{N_T} F_{N_D - 1} \frac{1}{2} \right] \alpha_d \left[ \hat{c}_{d1}, \hat{c}_{d2}, \ldots, \hat{c}_{d_{N_T}} \right]^T + w_d, \]  \hspace{1cm} (45)

where \( \alpha_d = diag(1, e^{j2\pi(d-1) \frac{1}{N}}, \ldots, e^{j2\pi(d-1) \frac{1}{N}}) \otimes I_{N_T} \); therefore, \( \alpha_d \left[ \hat{c}_{d1}, \hat{c}_{d2}, \ldots, \hat{c}_{d_{N_T}} \right]^T = \left[ c_{d1}^T, c_{d2}^T, \ldots, c_{d_{N_T}}^T \right]^T \).
Chapter 3. Proposed channel estimation methods for short and dense channels

3.1. Methods

- Autoregressive scheme

We proposed this method in [58]. The Autoregressive scheme utilizes time domain training sequence that its structure is designed such that it would be able to estimate the channel amplitude and Doppler shift for each tap sequentially. According to (3) the amplitude of the taps remain unchanged during an OFDM block while the phases are changed linearly by time with the rate of the Doppler frequency of that tap. To estimate these values, we send a block of ones in time domain as pilot. The length of the block ($L$) should be equal or larger than the delay spread of the channel. The block of ones is then continued with a block of zeros of length $L$, so we can collect all the data from convolution of the channel from block of one without interference. The received pilots have the following formula:

$$
\begin{align*}
    y_t(1) &= a_1 e^{j2\pi f_1 T_s} + w_t(1) \\
    y_t(2) &= a_1 e^{j2\pi f_1 T_s} + a_2 e^{j2\pi f_2 T_s} + w_t(2) \\
    &\vdots \\
    y_t(L) &= a_1 e^{j2\pi L f_1 T_s} + a_2 e^{j2\pi L f_2 T_s} + \cdots + a_L e^{j2\pi L f_L T_s} + w_t(L)
\end{align*}
$$

(46)

Now, if we repeat the same pilot structure immediately, meaning, if we continue the first block of ones and zeroes with another set, the received pilots will have the following formula:

$$
\begin{align*}
    y_t(2L + 1) &= a_1 e^{j2\pi (2L+1) f_1 T_s} + w_t(2L + 1) \\
    y_t(2L + 2) &= a_1 e^{j2\pi (2L+2) f_1 T_s} + a_2 e^{j2\pi (2L+2) f_2 T_s} + w_t(2L + 2) \\
    &\vdots \\
    y_t(2L + L) &= a_1 e^{j2\pi (2L+L) f_1 T_s} + a_2 e^{j2\pi (2L+L) f_2 T_s} + \cdots + a_L e^{j2\pi (2L+L) f_L T_s} + w_t(2L + L)
\end{align*}
$$

(47)

Comparing (46) and (47), the Doppler frequency of the first path can be estimated as:
\[ f_1 = \frac{1}{2\pi 2LT_s} \tan^{-1} \left( \text{imag} \left( \frac{y_1(2L+1)}{y_1(1)} \right) \right) \quad \text{(48)} \]

Once \( f_1 \) is obtained, \( a_1 \) can be estimated as:

\[ \hat{a}_1 = \frac{y_1(1)}{e^{j2\pi f_1 T_s}}. \quad \text{(49)} \]

Similarly, we can estimate the Doppler frequency and amplitude of the second path from \( y_2 \) and \( y_{2L+2} \) after removing the effects of the first path using \( \hat{a}_1 \) and \( \hat{f}_1 \) as follows:

\[ \hat{f}_2 = \frac{1}{2\pi 2LT_s} \tan^{-1} \left( \text{imag} \left( \frac{y_1(2L+2) - \hat{a}_1 e^{j2\pi f_1(2L+2) T_s}}{y_1(1) - \hat{a}_1 e^{j2\pi f_1(2) T_s}} \right) \right) \quad \text{(50)} \]

\[ \hat{a}_2 = \frac{y_2(2) - a_1 e^{j2\pi f_1 T_s}}{e^{j2\pi f_1 T_s}} \]

This sequential procedure leads to estimation of the Doppler and the amplitude of all the paths, and thus to the estimation of the channel matrix of (5). The structure of the transmitted pilot is presented in Figure 1. If the length of single OFDM symbol (\( N \)) is larger than 4\( L \), we can put two or more of the pilot sequences in Figure 1 in a single OFDM pilot and average the estimates from each pilot set to reduce the effect of additive white Gaussian noise.

**Figure 1. Transmitted pilot sequence**

\[
\begin{array}{cccccccccc}
1 & 1 & \ldots & 1 & 0 & 0 & \ldots & 0 & 1 & 1 & \ldots & 1 & 0 & 0 & \ldots & 0 \\
\hline
l & l & \ldots & l & l & l & \ldots & l & l & l & \ldots & l & l & l & \ldots & l \\
4L
\end{array}
\]

As it is indicated in (50), for the calculation of the amplitude or the Doppler shift of each path, a summation with the complexity of \( O(L) \) over the previous paths is needed. Besides that, the algorithm contains two loops and the complexity of each loop is \( O(L) \). Therefore, the computational complexity of this scheme is \( O(L^3) \). Since the summation over \( \frac{N}{4L} \) sets of blocks...
is applied, the final complexity for the Autoregressive algorithm is $O\left(\frac{NL^2}{4}\right)$. If the channel delay spread is large, the Autoregressive method can get computationally challenging. Therefore, it is essential to design a CE method that evaluates all the elements of the frequency domain channel matrix with low complexity.

- Linearizing scheme

We proposed this method in [59]. In the Linearizing method, the time domain training sequence is different than Autoregressive method and is designed such that the channel amplitudes and Doppler shifts for all the paths can be estimated simultaneously with two sets of linear equations.

The structure of the time domain training sequence for this method is similar to Fig. 1, except that instead of block of 1s, a block of pseudorandom noise (PN) codes are sent as a pilot. Approximating $a_i e^{j2\pi f_i n T_s}$ in (3) with $a_i (1 + j2\pi f_i n T_s)$ to linearize the equations, the received pilots are expressed as:

$$
\begin{align*}
    y_t(1) &= P_1 a_1 (1 + j2\pi f_1 T_s) + w_t(1) \\
    y_t(2) &= P_2 a_1 (1 + j2\pi 2f_1 T_s) + P_1 a_2 (1 + j2\pi 2f_2 T_s) + w_t(2) \\
    &\vdots \\
    y_t(L) &= P_L a_1 (1 + j2\pi Lf_1 T_s) + P_{L-1} a_2 (1 + j2\pi Lf_2 T_s) + \cdots + P_1 a_L (1 + j2\pi Lf_L T_s) + w_t(L)
\end{align*}
$$

(51)

where $P_i$ is the $i^{th}$ transmitted pilot and the other parameters were defined previously. Since the identical pilot block is repeated after a zero-padding block, the corresponding received pilots have the succeeding formula:
\[
\begin{align*}
\{y_t(2L+1) &= P_1 a_1 (1 + j2\pi(2L+1)f_1 T_s) + w_t(2L+1) \\
y_t(2L+2) &= P_2 a_1 (1 + j2\pi(2L+2)f_1 T_s) + P_1 a_2 (1 + j2\pi(2L+2)f_2 T_s) + w_t(2L+2) \\
&\quad \vdots \\
y_t(2L+L) &= P_L a_1 (1 + j2\pi(2L+L)f_1 T_s) + P_{L-1} a_2 (1 + j2\pi(2L+L)f_2 T_s) + \cdots + \\
&\quad P_1 a_L (1 + j2\pi(2L+L)f_L T_s) + w_t(2L+L)
\end{align*}
\]

(52)

Comparing (51) and (52), \(a_i\) can be calculated from the scaled difference of \(y_t(i)\) and \(y_t(2L+i)\). The linear equation is attained as:

\[
y_{tda} = \left(\frac{2L+i}{2L}\right) y_t(i) - \frac{i}{2L} y_t(i+L)
\]

(53)

and the set of linear equations are presented in matrix form as:

\[
y_{tda} = T_1 \times a + \mathbf{w}_{diff},
\]

(54)

where \(y_{tda}\) is a \(L \times 1\) matrix, \(y_{tda}^T = [y_{tda1}, y_{tda2}, \cdots, y_{tdaL}]\), \(a\) is \(L \times 1\) matrix of complex amplitudes \(a^T = [a_1, a_2, \cdots, a_L]\), and \(T_1\) is a \(L \times L\) lower triangular matrix of the training symbols:

\[
T_1 = \begin{bmatrix}
P_1 & 0 & \cdots & 0 \\
P_2 & P_1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
P_L & P_{L-1} & \cdots & P_1
\end{bmatrix}
\]

(55)

\(w_{diff}\) is a \(L \times 1\) matrix, \(w_{diff}^T = [w_{diff1}, w_{diff2}, \cdots, w_{diffL}]\) where \(w_{diffi} = \left(\frac{2L+i}{2L}\right) w_t(i) - \frac{i}{2L} w_t(i+2L)\).

Since (54) is a linear equation, \(a\) is estimated as:

\[
\hat{a} = (T_1)^{-1} y_{tda}
\]

(56)
When complex amplitudes are estimated, the Doppler frequency for each path is calculated based on a similar procedure. Considering (51) and (52), \( f_i \) can be calculated from the difference of \( y_t(i) \) and \( y_t(2L + i) \). The linear equation is attained as:

\[
y_{t,df} = y_t(2L + i) - y_t(i) = \bar{a}_1 P_i (j2\pi2Lf_1T_s) + \bar{a}_2 P_{i-1} (j2\pi2Lf_2T_s) + \cdots + \bar{a}_i P_1 (j2\pi2Lf_iT_s) + w_t(2L + i) - w_t(i)
\]

where \( \bar{a}_i \)s are the estimated complex amplitudes in (54). The set of linear equations in a matrix form is presented as:

\[
y_{t,df} = U_1 \times (d, j2\pi2LT_s) + w_{diff2}
\]

where \( y_{t,df} \) is a \( L \times 1 \) matrix, \( y_{t,df}^T = [y_{t,df 1}, y_{t,df 2}, \cdots, y_{t,df L}] \), \( d \) is a \( L \times 1 \) vector of Doppler frequencies, \( d^T = [f_1, f_2, \cdots, f_L] \) and \( U_1 \) is a \( L \times L \) lower triangular matrix of training symbols that is presented as:

\[
U_1 = \begin{bmatrix}
\bar{a}_1 P_1 & 0 & \cdots & 0 \\
\bar{a}_1 P_2 & \bar{a}_2 P_1 & 0 & \cdots \\
\vdots & \ddots & \ddots & \vdots \\
\bar{a}_1 P_L & \bar{a}_2 P_{L-1} & \cdots & \bar{a}_L P_1
\end{bmatrix}
\]

and \( w_{diff2} \) is a \( L \times 1 \) matrix, \( w_{diff2}^T = [w_{diff2 1}, w_{diff2 2}, \cdots, w_{diff2 L}] \) where \( w_{diff2 i} = w_t(2L + i) - w_t(i) \).

Since (58) is a linear equation, \( d \) is estimated as:

\[
\hat{d} = Real\{\frac{1}{j2\pi2LT_s} (U_1)^{-1} y_{t,df}\}
\]

In the case of \( \frac{N}{4L} \) pilot sequences, \( T_1 \) in (55) and \( U_1 \) in (59) are replaced with \( T \) and \( U \) respectively which are defined as:
\[ T^T = [T_1^T, T_2^T, \ldots, T_N^T] \],
\[ U^T = [U_1^T, U_2^T, \ldots, U_N^T] \],

where \( T_2, \ldots, T_N \) and \( U_2, \ldots, U_N \) are described similar to \( T_1 \) and \( U_1 \), respectively, but with different pseudorandom codes. Therefore, \( T \) and \( U \) are \( \frac{N}{4L} \times L \) matrixes, which consist of \( \frac{N}{4L} \) lower triangular matrixes. Then, (56) is converted to:

\[ \alpha = (T^T T)^{-1} T^T y_{tda} \] (63)

and (60) is converted to:

\[ \hat{d} = \text{Real} \left\{ \frac{1}{j2\pi 2LT} (U^T U)^{-1} U^T y_{tdf} \right\} \] (64)

It was indicated in [60] that the minimum estimation error would be obtained if \( T^T T \) and \( U^T U \) are diagonal matrixes. This is approximately achieved by the proposed training sequence. Besides that, it is proofed in [60] that the \( T^T T \) and \( U^T U \) would be invertible if and only if the columns of \( T \) and \( U \) are linearly independent. Since both \( T \) and \( U \) are upper triangular matrixes with non-zero diagonal elements, their columns are independent from each other.

By employing the pseudorandom sequences for \( P_l \)s, the non-diagonal elements of \( T^T T \) would be close to zero and the diagonal elements are calculated as:

\[ r_{a_i} = \sum_{q=1}^{\frac{N}{4L}} \sum_{w=1}^{L-i+1} (P_w)_q^2 \] (65)

where \( r_{a_i} \) is the \( i \)th diagonal element and \( (P_w)_q \) defines the \( w \)th pilot of the \( q \)th PN sequence. The relation between the estimated complex amplitudes and the actual complex amplitudes is obtained as:
\[ \hat{a}_i = a_i + \frac{1}{r_i} \sum_{q=1}^{N/4L} \sum_{w=1}^{L-1} b \left( a_i (P_{w+L-s}) q \cdot (P_w)_q + (P_{w+L-s}) q w^{diff}_i \right) \]  

(66)

where \( \hat{a}_i \) is the estimated complex amplitude and \( b \) is a parameter that could be 0 or 1. The other parameters are described earlier in the paper. Since training symbols are pseudorandom and noise samples are independent and identically distributed (iid), this approach is very efficient in noise elimination. As a result, most of the estimation error would be because of a linearization error.

Similarly, \( U^T U \) can be approximated as a diagonal matrix, and the diagonal elements are calculated as:

\[ r_{f_i} = \sum_{q=1}^{N} \sum_{w=1}^{L-i+1} \hat{a}_i^2 (P_w)_q^2 \]  

(67)

where \( r_{f_i} \) is the \( i \)th diagonal element. By the assumption that the linearization does not add any error to the estimation, the estimated Doppler shifts are obtained as:

\[ \hat{f}_i = f_i + \frac{1}{j2\pi2LT_s r_{f_i}} \sum_{q=1}^{N/4L} \sum_{w=1}^{L-1} b \hat{a}_{w+L-s} \hat{a}_w \left( a_i (P_{w+s}) q \cdot (P_w)_q + (P_{w+s}) q w^{diff}_i \right) \]  

(68)

This procedure contains four steps. Two subtraction, (53) and (57), and two LS estimation, (56) and (60). Consequently, the complexity is \( O\left(2 \left( \frac{N}{2} + \frac{N}{4L} L^2 \right) \right) = O\left( N(1 + \frac{L}{2}) \right) \) which is linearly proportional to \( L \).

3.2. Simulation results

In this section, the Q1 and L2 channel models which were defined in [7], are used to evaluate the performance of discussed CE methods. Monte Carlo simulation is employed in order to evaluate the performance of the LS, LMMSE, Autoregressive and Linearizing CE methods. A
10-MHz total bandwidth is assumed for the OFDM signal; therefore, the time resolution $T_s$ is equal to 100 ns. In order to have a fixed pilot structure, the cyclic prefix is set to 32. The number of subcarriers is chosen to be 1024. Therefore, a $\frac{1024}{4 \times 32} = 8$ pilot structure, of the form presented in Fig. 1 can be accommodated in a single OFDM pilot symbol. In the Linearizing method, the training sequence is constructed using an 8-bipolar Gold sequence with the length of 31 [61], with added bit of “1” to the end. According to [62], the number of Golden sequences with length $2^m - 1$ is $2^m + 1$ and the cross-correlation between each pair obtains three possible values, $\{-1, -t(m), t(m) - 1\}$, where $t(m) = 2^{\left\lfloor \frac{m}{2} \right\rfloor + 1} + 1$. The simulation results indicate that choosing any set of 8 out of 33 Golden sequences which have -1 cross-correlation, leads to the same and the best outcome in comparison to choosing Golden sequences with larger cross-correlations. The performance of the ICI cancellation is simulated by using two maximum Doppler shift ($f_{D_{\text{max}}} = 800$ Hz and $f_{D_{\text{max}}} = 8$ KHz) for the UAV. After the transmission of each training sequence and CE, a set of OFDM data is transmitted through the channel in order to be demodulated. The modulation in each subcarrier is assumed to be binary phase-shift keying (BPSK). Bit error rate (BER) vs. signal to noise ratio (SNR) curves for LS, LMMSE, Autoregressive and Linearizing methods for Q1 and L2 are shown in Fig. 2 and Fig.3 respectively.

As it is indicated in Fig. 2, acceptable performance is achieved at low Doppler shift of $f_{D_{\text{max}}} = 800$ Hz for all methods. However, for the high Doppler shift of $f_{D_{\text{max}}} = 8$ KHz, the performance of LS and LMMSE methods are not reliable, even at 30 dB SNR. The proposed method for the L2 channel model does not perform as well for other channel models. This degradation in the performance was expected because the length of the cyclic prefix (32) was less than the length of the channel (200). However, the performance of the proposed method still is better than the performance of the LS and LMMSE methods for the L2 channel model.
While both the Autoregressive and Linearizing methods are successful in mitigating ICI at high Doppler shift, the latter method performs channel estimation with much lower complexity in comparison to the former one. Besides that, the performance of the Linearizing method outperforms the performance of the Autoregressive method slightly since the employment of the Golden sequences as pilots makes $T^T T$ and $U^T U$ near diagonal and therefore, the LS estimation of the channel becomes robust to the additive noise. Based on our investigation by applying the Monte Carlo simulation, the performance of the LS procedure would be similar to
the averaging technique, which is applied for Autoregressive method, if all the pilots are set to 1 instead of Golden sequences. In addition, applying the truncated Taylor expansion for the channel phase, since $f_{d_{\text{max}}}T_s = 8 \times 10^{-4} \ll 1$, does not add considerable error for channel estimation.

The performance of these CE methods were evaluated by transmission of hyperspectral images. The Pavia Centre hyperspectral dataset [63] was used to analyze data quality and the accuracy of the CE methods. Fig. 4 depicts how ICI mitigation affects the quality of reconstruction from the estimated data received from the communication channel.

Figure 4. (a): original data, (b): reconstructed image with ICI and noise, (c): ICI mitigated reconstructed image, (d): ICI mitigated reconstructed image after denoising. (b, c, d: SNR =10)
Table 1 shows all of the classification accuracy for both the spectral and spectral-spatial features with the two UAV speeds (50 m/s and 500 m/s); thus, four SNR values are reported.

<table>
<thead>
<tr>
<th>Method</th>
<th>Spectral-Spatial Model</th>
<th>Spectral Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Speed (m/s)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>500</td>
</tr>
<tr>
<td>SNR (dB)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>LS</td>
<td>10.92%</td>
<td>77.94%</td>
</tr>
<tr>
<td>MMSE</td>
<td>78.88%</td>
<td>90.24%</td>
</tr>
<tr>
<td>Autoregressive</td>
<td>80.37%</td>
<td>91.63%</td>
</tr>
</tbody>
</table>

Table 1. Hyperspectral classification accuracies
Chapter 4. Channel estimation approaches for long and sparse channels

Our proposed methods in this section are categorized into two groups. The methods of the first group estimate the channel without using any information regarding the position of the non-zero channel taps. On the other hand, the methods of the second group, first estimate the positions of the non-zero taps; afterwards, calculate the complex amplitudes and Doppler shifts of those non-zero channel taps by utilizing those priori estimated positions (PEP).

4.1. Methods without PEP

- **LMMSE-OMP**

  We proposed this method in [64]. We found out that we can enhance the performance of the CE by keeping the same number of pilots but enhancing the SNR by running OMP with noiseless approximate of the received data. In this new method, we first find an approximate of the channel using OMP but instead of using that estimate to demodulate the data subcarrier we create an estimate of the received pilot subcarrier values without noise:

  \[
  y_{\text{LMMSE}CS} = \hat{h}_{\text{LMMSE}} \cdot x_d \tag{69}
  \]

  We use \( y_{\text{LMMSE}CS} \) to run another round of OMP and estimate the channel. Simulation results in the next section shows that this enhances the performance of the CE significantly.

  While we can continue doing a new set of OMP after each estimation, the improvement on performance will decrease while adding to the complexity of CE. It seems that performing OMP twice (one with original noisy received pilot signals to get and initial estimate of the channel and the other using an enhanced version of the received pilot data) gives the most improvement in performance.

  The purposed method will have twice the complexity of traditional OMP. If that is a concern we can replace the second run with a lower complexity method such as compressive
sampling matching pursuit (CoSaMP) [65]. CoSaMP is an alternative CS greedy algorithm that has lower complexity compared to OMP. Unlike OMP it requires the sparsity of the channel to be known. In CoSaMP algorithm, the matrix $M$ is initiated with $2S$ columns of the measurement matrix $\boldsymbol{\varphi}$ with the highest correlation with received pilot data. At each iteration, $M$ is updated by adding new columns that describe the reminder. The algorithm runs a fixed number of iteration between $4S$ and $5S$ and produces reasonable CE. In our method, if we first run OMP to find a better approximation of the received data, we can also have an estimate of the sparsity of the channel and use it to run CoSaMP in second round. According to [65] the complexity of OMP is of $O(S.N.P)$ and the complexity of CoSaMP is of $O(N.P)$. Therefore, the complexity of OMP-OMP for enhanced method is of $O(2S.N.P)$ and the complexity of OMP-CoSaMP is of $O((S+1).N.P)$.

Since the OMP-CoSaMP method has lower performance than OMP-OMP method, we made a recovery to the CoSaMP round of our CE method. Instead of implementing LS for CE, MMSE method is applied for reconstruction of the sparse channel from matrix $M$ at each iteration of CoSaMP:

$$\hat{\mathbf{r}}_{\text{CoSaMP}i} = \left( \mathbf{M}_i^H \mathbf{M}_i + \frac{1}{\text{SNR}} \mathbf{I} \right)^{-1} \times \mathbf{M}_i^H \mathbf{Y}^{CS}$$  \hspace{1cm} (70)

- Modified delay Doppler sparsity (MDDS)

Although the original method results in an accurate CE in moderate Doppler shift [30], our simulation results indicate that some improvements can enhance the performance of the CE greatly in high Doppler shift scenario of UAS. In the following, these improvements are discussed [66].

1. Enhanced measurement matrix

By considering (5), the diagonal elements ($m = k$) of frequency domain channel matrix are obtained as:
As a result, the precise term for $\mathbf{v}_q$ vector, which is previously described for (25), is obtained as:

$\mathbf{v}_q = \left[ e^{-j2\pi QLT_s} \frac{1 - e^{-jN(2\pi qT_s)}}{1 - e^{-jN(2\pi qT_s)}}, e^{j2\pi(1-Q)LT_s} \frac{1 - e^{jN(2\pi(1-Q)T_s)}}{1 - e^{jN(2\pi(1-Q)T_s)}}, \ldots, e^{j2\pi QT_s} \frac{1-e^{jN(2\pi qT_s)}}{1-e^{jN(2\pi qT_s)}} \right]$  

Therefore, more accurate measurement matrix, $\varphi$, would be obtained. In the low Doppler shift condition, the truncated Taylor expansion for $e^x \approx 1 + x$ can be used and the term would be simplified to $1$ and the measurement matrix would become the same as the one that is proposed in [30]. However, for high Doppler shift scenarios of UAS, this simplification causes considerable error.

2. Non-uniform Doppler spread quantization

The next issue that should be considered is the selection of $\alpha$ in (47). In a low Doppler shift scenario, $\alpha$ is set to be 1 as it is performed in [30]. However, in a high Doppler shift scenario, setting $\alpha$ to 1 results in a very large measurement matrix and large $\mathbf{c}$ vector. The complexity of any CS method is directly proportional to the size of the measurement matrix. In addition, when the size of the measurement matrix is large, the performance of the CS method degrades specifically in low SNR. On the other hand, because of the quantization error, choosing a large $\alpha$ results in an inaccurate Doppler shift even if the $\mathbf{c}$ vector is calculated accurately. However, the probability distribution function (PDF) of the Doppler shift of UAS has Jake distribution [50]. Since, the probability of a specific Doppler frequency increases sharply by the increment in the Doppler shift, if the higher $\alpha$ would be considered for low Doppler shifts and the lower
$\alpha$ is considered for higher Doppler shifts, the mentioned disadvantages of choosing low or high $\alpha$ would be resolved.

3. Guard interval insertion between pilot and data

Since only $P$ out of $N$ symbols of an OFDM packet are pilots, the location of these pilots should be chosen wisely in order to reduce the error of CE. In this paper, the modified version of the Algorithm 2 of paper [30] is used for defining the pilot pattern. In that algorithm, an iterative procedure is implemented in order to find the pilot pattern which results in the lowest coherence. In a low Doppler spread, the interference between any pilot symbol and its adjacent data symbol is negligible. However, in a high Doppler spread of the UAS communication system, the interference between the data and pilots degrades the CE accuracy greatly. As a result, in the MDDS method, we have added an extra condition in each iteration of the pilot estimation. The condition is that two adjacent places cannot be considered for pilot placement because during an OFDM transmission, a guard interval of one symbol between any pilot to its adjacent pilot or data should be considered. In the guard interval, no signal should be transmitted.

- **DS-LMMSE-OMP**

This method was proposed in [66] and is the modified version of the LMMSE-OMP method for doubly selective channels. In this method, first the MDDS scheme is applied in order to obtain the priori estimation of the complex amplitudes and Doppler shifts. In order to utilize those estimated values, we define $\mathbf{R}_{h,Y}$ and $\mathbf{R}_Y$ as:

$$R_{h,Y}(i,j) = E[h(i)y_p^*(j)] = E[h(i)(y(j)h(j)X_p(j) + Z)^*]$$
$$= \gamma(j)^*R_h(|i - j|)X_p(j),$$

(73)
\[ R_Y(i,j) = E[Y_p(i)Y_p^*(j)] \]
\[ = E[(\gamma(i)h(i)X_p(i) + Z(i))(\gamma(j)h(j)X_p(j) + Z(j))^\ast] \]
\[ = \gamma(i)\gamma(j)^\ast R_h(|i-j|)E[X_p(i)X_p(j)^\ast] + \sigma^2, \]  

where \( X_p \) and \( Y_p \) are the transmitted pilots and received pilots in the frequency domain respectively and \( \gamma \) is defined as:

\[ \gamma(k,m) = \frac{1}{N} \sum_{l \in [0,1,2,...,N]} e^{j2\pi f_l T_s} e^{-j2\pi k} \frac{1 - e^{jN(2\pi f_l T_s + 2\pi (k-m))}}{1 - e^{j(2\pi f_l T_s + 2\pi (k-m))}}. \]  

By defining \( \hat{h} \) as the estimated channel after the first step of our CE procedure, the LMMSE estimation of the channel is obtained as:

\[ \hat{h}_{LMMSE} = R_{R_Y} R_Y^\dagger \gamma \hat{h}, \]  

By substituting (73) and (74) into (76), we get:

\[ \hat{h}_{LMMSE} = (R_{HH})_{DS} \left[ (R_{HH})_{DS} + \frac{\beta}{\sigma^2} I_N \right]^{-1} \gamma \hat{h}, \]  

where \( (R_{HH})_{DS} \) is the DS autocorrelation channel matrix defined as:

\[ (R_{HH})_{DS}(u,v) = \frac{1}{N^2} \sum_{l=0}^{L-1} \hat{a}_l^2 \sum_{\rho=1}^{N} \frac{1 - e^{jN(2\pi f_l T_s + 2\pi (u-\rho))}}{1 - e^{j(2\pi f_l T_s + 2\pi (u-\rho))}} e^{-j2\pi (u-v)l} \]  

where \( \hat{a}_l \)s and \( f_l \)s are the estimated complex amplitudes and Doppler frequencies from the first step respectively.

- **CS-Linearizing**

We proposed this method in [68] and it is the evolved version of the Linearizing method.
for spars channels. In this method, we utilize a time domain training sequence for CE. The training sequence consists of two identical PN sequences that their length is larger than the length of the channel, \( L < G \).

By considering the following equations, it would be clarified how two same training sequences can be applied to estimate the Doppler frequency shifts from linear phase changes.

The first \( G \) received time domain samples can be expressed as:

\[
\begin{align*}
\gamma_1 &= P_1 a_1 (1 + j2\pi f_1 T_s) + w_1 + IB_1 \\
\gamma_2 &= P_2 a_1 (1 + j2\pi 2f_1 T_s) + P_1 a_2 (1 + j2\pi 2f_2 T_s) + w_2 + IB_2 \\
\vdots \\
\gamma_L &= P_L a_1 (1 + j2\pi LF_1 T_s) + P_{L-1} a_2 (1 + j2\pi LF_2 T_s) + \cdots \\
&\quad + P_L a_2 (1 + j2\pi LF_2 T_s) + w_L + IB_L \\
\gamma_{L+1} &= P_{L+1} a_1 (1 + j2\pi (L+1)f_1 T_s) + P_L a_2 (1 + j2\pi (L+1)f_2 T_s) \\
&\quad + \cdots + P_L a_2 (1 + j2\pi (L+1)f_2 T_s) + w_{L+1} \\
\vdots \\
\gamma_G &= P_G a_1 (1 + j2\pi GF_1 T_s) + P_{G-1} a_2 (1 + j2\pi GF_2 T_s) + \cdots \\
&\quad + P_{G-L+1} a_2 (1 + j2\pi GF_2 T_s) + w_G
\end{align*}
\]

(79)

where \( P_i \) and \( IB_i \) are the \( i^{th} \) transmitted sample and the \( i^{th} \) additive interference of the previous OFDM data block respectively; the other parameters were defined earlier. Now, since the same training sequence is repeated immediately, the corresponding second \( G \) received samples are expressed as:
By considering the last \( G - L \) equations of (50) and (51), a \( (G - L) \times 1 \) vector from the scaled difference of \( y_i \) and \( y_{G+i} \) is obtained. Each element of this vector is defined as:

\[
y_{t,da} = \left(1 + \frac{j2\pi(G + 1)f_1 T_s}{G} \right) \cdot y_i - \frac{i}{G} \cdot y_{i+G} = P_i a_1 + P_{i-1} a_2 + \ldots + P_L a_1 + P_{G-L} a_2 + \ldots + P_{G-L+1} a_L \tag{81}
\]

\[
y_{t,da} = \varphi \times a + w_{diff}, \tag{82}
\]

where \( y_{t,da} \) is a \( (G - L) \times 1 \) matrix, \( y_{t,da}^T = [y_{t,da}^1, y_{t,da}^2, \ldots, y_{t,da}^{G-L}] \), \( a \) is \( L \times 1 \) matrix of complex amplitudes \( a^T = [a_1, a_2, \ldots, a_L] \), and \( \varphi \) is a \( (G - L) \times L \) fat matrix \((G - L) \ll L\) of the training sequence which is defined as:

\[
\varphi = \begin{bmatrix}
P_L & P_{L-1} & \cdots & P_1 \\
P_L+1 & P_L & \cdots & P_2 \\
\vdots & \vdots & \ddots & \vdots \\
P_G & P_{G-1} & \cdots & P_{G-L+1}
\end{bmatrix} \tag{83}
\]
and \( \mathbf{w}_{diff} \) is a \((G - L) \times 1\) matrix, \( \mathbf{w}_{diff}^T = [\mathbf{w}_{diff_1}, \mathbf{w}_{diff_2}, \ldots, \mathbf{w}_{diff_{G-L}}] \) where \( \mathbf{w}_{diff_i} = \left( \frac{G+i}{G} \right) \cdot \mathbf{w}_i - \frac{i}{G} \cdot \mathbf{w}_{i+G} \).

Since the number of unknowns, \( L \), is larger than the number of equations, \( G - L \), LS method cannot be used for solving (82). However, since \( \mathbf{a} \) is a sparse vector and only contains \( S \) non-zero elements, a CS method such as OMP can be applied for estimating \( \mathbf{a} \) and \( \mathbf{\varphi} \) would be the measurement matrix of that CS method. Since \( \mathbf{P}_s \) have PN distribution, the coherence of \( \mathbf{\mu}[\mathbf{\varphi}] \) would be near zero.

When \( \mathbf{a} \) is calculated by a CS method, \( S \) non-zero complex amplitudes and their locations are defined. As a result, only \( S \) Doppler frequencies which correspond to the location of non-zero amplitudes should be estimated. The set of non-zero channel tap positions, complex amplitudes, and Doppler shifts can be defined by \( \{r_1, r_2, \ldots, r_K\} \), \( \{a_{r_1}, a_{r_2}, \ldots, a_{r_k}\} \), and \( \{f_{r_1}, f_{r_2}, \ldots, f_{r_k}\} \) respectively. Looking at (79) and (80), the difference of \( y_i \) and \( y_{G+i} \) results in:

\[
y_{tdf_i} = y_{G+i} - y_i = \tilde{\alpha}_{r_1} \mathbf{P}_{i-r_1} (j2\pi G f_{r_1} T_s) + \tilde{\alpha}_{r_2} \mathbf{P}_{i-r_2} (j2\pi G f_{r_2} T_s) + \cdots + \tilde{\alpha}_{r_k} \mathbf{P}_{i-r_k} (j2\pi G f_{r_k} T_s) + \mathbf{w}_{G+i} - \mathbf{w}_i \quad i = L + 1, \ldots, G
\]

(84)

where \( \tilde{\alpha}_r \)'s are the estimated complex amplitudes in (76). A set of linear equations can be written in a matrix form as:

\[
\mathbf{y}_{tdf} = \mathbf{Q} \times (\mathbf{d} \cdot j2\pi GT_s) + \mathbf{w}_{diff2} \quad .
\]

(85)

where \( \mathbf{y}_{tdf} \) is a \((G - L) \times 1\) vector, \( \mathbf{y}_{tdf}^T = [y_{tdf_1}, y_{tdf_2}, \ldots, y_{tdf_{G-L}}] \), \( \mathbf{d} \) is \( K \times 1 \) vector of Doppler frequencies, \( \mathbf{d}^T = [f_{r_1}, f_{r_2}, \ldots, f_{r_k}] \) and \( \mathbf{Q} \) is a \((G - L) \times K\) matrix which is obtained as:
\[ Q = \begin{bmatrix}
    \tilde{a}_{r_1}P_{L+1-r_1} & \tilde{a}_{r_2}P_{L+1-r_2} & \cdots & \tilde{a}_{r_k}P_{L+1-r_k} \\
    \tilde{a}_{r_1}P_{L+2-r_1} & \tilde{a}_{r_2}P_{L+2-r_2} & \cdots & \tilde{a}_{r_k}P_{L+2-r_k} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{a}_{r_1}P_{G-r_1} & \tilde{a}_{r_2}P_{G-r_2} & \cdots & \tilde{a}_{r_k}P_{G-r_k}
\end{bmatrix}, \quad (86) \]

and \( \mathbf{w}_{\text{diff}2} \) is a \((G - L) \times 1\) matrix,
\[
\mathbf{w}_{\text{diff}2}^T = [w_{\text{diff}2,1}, w_{\text{diff}2,2}, \ldots, w_{\text{diff}2,G-L}] \quad \text{where}
\]
\[
w_{\text{diff}2,i} = w_{G+i} - w_i
\]

Since (86) is a thin matrix, \( K \ll (G - L) \), the LS estimate of \( \mathbf{d} \) is obtained as:
\[
\hat{\mathbf{d}} = \text{Real} \left\{ \frac{1}{j2\pi \gamma_\text{df}} (\mathbf{Q})^\dagger \mathbf{y}_{t,\text{df}} \right\} . \quad (87)
\]

4.2. PEP based Methods

It is indicated in [69] that the division of changing rate of the non-zero complex amplitudes of the channel taps to the changing rate of the positions of the non-zero channel taps is equal to the division of center frequency of the channel to the bandwidth of the channel. Since the center frequency of the cellular communication systems is considerably larger than their bandwidth, the estimated channel tap position can be considered fixed during the transmission of several OFDM data blocks. The performance of the CS based CE schemes can be enhanced if the positions of the non-zero channel taps are known. For instance, [70] considered the non-zero channel tap positions as long term parameters that should be tracked by the outer loop of the decision feedback process while the complex amplitudes were considered as short term parameters that should be tracked with the inner loop of the decision feedback method.

As a result, in our proposed method of this section, first the non-zero tap positions are estimated and would be utilized for the second step that estimates the complex amplitudes and Doppler shifts. In this section, \( r = [r_1, r_2, \ldots, r_S] \) indicate the set of channel tap positions.

- Autoregressive-PEP

At the first step of this method which we proposed in [70], the MDDS method is applied. However, the estimated complex amplitudes and Doppler shifts of the first step would not be
utilized and only the estimated positions would be used. At the second step, a training sequence with the length, \( G \geq L \) all equal to “1” in the time domain is transmitted. The corresponding received signals in the time domain can be written as:

\[
\begin{align*}
\mathbf{y}_r &= a_1 e^{j2\pi f_1 r_1 T_s} + w_{r_1} \\
\mathbf{y}_{r_1+1} &= a_1 e^{j2\pi f_1 (r_1+1) T_s} + w_{r_1+1} \\
& \vdots \\
\mathbf{y}_{r_2-1} &= a_1 e^{j2\pi f_1 (r_2-1) T_s} + w_{r_2-1} \\
\mathbf{y}_{r_2} &= a_1 e^{j2\pi f_1 r_2 T_s} + a_2 e^{j2\pi f_2 r_2 T_s} + w_{r_2} \\
& \vdots \\
\mathbf{y}_{r_k} &= a_1 e^{j2\pi f_1 r_k T_s} + a_2 e^{j2\pi f_2 r_k T_s} + \ldots + a_k e^{j2\pi f_k r_k T_s} + w_{r_k} \\
\mathbf{y}_G &= a_1 e^{j2\pi f_1 G T_s} + a_2 e^{j2\pi f_2 G T_s} + \ldots + a_k e^{j2\pi f_k G T_s} + w_G
\end{align*}
\] (88)

The received samples in the \( y_{r_1}, \ldots, y_{r_2-1} \) only depend on \( a_1 \) and \( f_1 \). By averaging those as:

\[
y_{ave_1} = \frac{1}{2^{\left[\frac{r_2 - r_1}{2}\right]}} \sum_{i=0}^{2^{\left[\frac{r_2 - r_1}{2}\right]}-1} \frac{y_{r_1+i+1}}{y_{r_1+i}}.
\] (89)

The amplitude and Doppler frequency of the first path can be estimated as:

\[
\hat{a}_{r_1} = \frac{y_{ave_1}}{e^{j2\pi f_1 r_1 T_s}} \text{ and } \hat{f}_{r_1} = \frac{1}{2\pi T_s} \tan^{-1} \left( \frac{\text{imag}(y_{ave_1})}{\text{real}(y_{ave_1})} \right).
\] (90)

By removing the effect of the first path from the received samples that are located between \( r_2 \) and \( r_3 \), and averaging of those samples, the following term is obtained to estimate \( \hat{a}_{r_2} \) and \( \hat{f}_{r_2} \):

\[
y_{ave_2} = \frac{1}{2^{\left[\frac{r_3 - r_2}{2}\right]}} \sum_{i=0}^{2^{\left[\frac{r_3 - r_2}{2}\right]}-1} \frac{y_{r_2+i+1} - a_1 e^{j2\pi f_1 (i+1) T_s}}{y_{r_2+i} - a_1 e^{j2\pi f_1 i T_s}}
\] (91)
This procedure is continued until all the complex amplitudes and Doppler shifts are found. Through the implementation of this process, not only the autoregressive steps reduce from \( L \) to \( S \), but also the CE method become more robust to additive noise because of the averaging which is applied to the received samples.

- **CS-Linearizing-PEP1**

In this method that we proposed in [70], a training sequence of length, \( G \geq L \) is created by repeating a PN sequence with length of \( M \). The same sequence is repeated \( R = G/M \) times. By approximating \( a_i e^{j2\pi f_i n T_s} \) in (4) with \( a_i (1 + j2\pi f_i n T_s) \), the relations between the transmitted time domain samples and the received time domain samples would be defined linearly. If \( 2M \leq G - L \), the last \( 2M \) received samples are obtained as:

\[
\begin{align*}
\frac{y_{(R-2)\times M+1}}{y_{(R-1)\times M+1}} &= P_{r_1} ar_1 (1 + j2\pi ((R - 2) \times M + 1)f_{r_1}T_s) + \\
&\quad P_{r_2} ar_2 (1 + j2\pi ((R - 2) \times M + 2)f_{r_2}T_s) \\
&\quad \vdots \\
&\quad P_{r_k} ar_k (1 + j2\pi ((R - 2) \times M + 1)f_{r_k}T_s) + w_{(R-2)\times M+1} \\
\frac{y_{(R-2)\times M+2}}{y_{(R-1)\times M+2}} &= P_{r_1+1} ar_1 (1 + j2\pi ((R - 2) \times M + 2)f_{r_1}T_s) + \\
&\quad P_{r_2+1} ar_2 (1 + j2\pi ((R - 2) \times M + 1)f_{r_2}T_s) \\
&\quad \vdots \\
&\quad P_{r_k+1} ar_k (1 + j2\pi ((R - 2) \times M + 2)f_{r_k}T_s) + w_{(R-2)\times M+2} \\
\frac{y_{(R-1)\times M}}{y_{(R-1)\times M}} &= P_{r_1} ar_1 (1 + j2\pi ((R - 1) \times M + 1)f_{r_1}T_s) + \\
&\quad P_{r_2} ar_2 (1 + j2\pi ((R - 1) \times M + 2)f_{r_2}T_s) \\
&\quad \vdots \\
&\quad P_{r_k} ar_k (1 + j2\pi ((R - 1) \times M + 1)f_{r_k}T_s) + w_{(R-1)\times M} \\
\frac{y_{(R-1)\times M+1}}{y_{(R-1)\times M+2}} &= P_{r_1+1} ar_1 (1 + j2\pi ((R - 1) \times M + 2)f_{r_1}T_s) + \\
&\quad P_{r_2+1} ar_2 (1 + j2\pi ((R - 1) \times M + 1)f_{r_2}T_s) \\
&\quad \vdots \\
&\quad P_{r_k+1} ar_k (1 + j2\pi ((R - 1) \times M + 2)f_{r_k}T_s) + w_{(R-1)\times M+2} \\
&\quad \vdots \\
\frac{y_{R\times M}}{y_{R\times M}} &= P_{r_1+M-1} ar_1 (1 + j2\pi (R \times M)f_{r_1}T_s) + \\
&\quad P_{r_2+M-1} ar_2 (1 + j2\pi (R \times M)f_{r_2}T_s) \\
&\quad \vdots \\
&\quad P_{r_k+M-1} ar_k (1 + j2\pi (R \times M)f_{r_k}T_s) + w_{(R-1)\times M+2} \\
\end{align*}
\]

(92)

A \( M \times 1 \) vector from the scaled difference of \( y_i \) and \( y_{M+i} \) is calculated. Each element of this vector is defined as:
\[ y_{t,da} = \left( \frac{M + i}{M} \right) y_i - \frac{i}{M} y_{i+M} = P_{r_1+i} a_{r_1} + P_{r_2+i} a_{r_2} + \cdots + P_{r_{k+i}} a_{r_k} + \left( \frac{M + i}{M} \right) w_i - \frac{i}{M} w_{i+M} \]  

for \( i = (R-2) \times M + 1, (R-2) \times M + 2, \ldots, (R-1) \times M \).

A set of linear equations can be written in matrix form as:

\[ y_{t,da} = \varphi_M \times a + w_{\text{diff}}, \]  

where \( y_{t,da} \) is a \( M \times 1 \) matrix, \( y_{t,da}^T = [y_{t,da \ 1}, y_{t,da \ 2}, \ldots, y_{t,da \ M}] \), \( A_M \) is \( S \times 1 \) vector of the complex amplitudes \( a^T = [a_{t_1}, a_{t_2}, \ldots, a_{t_2}] \), and \( \varphi \) is a \( M \times S \) thin matrix of training sequence that is presented as:

\[ \varphi = \begin{bmatrix} P_{r_1} & P_{r_2} & \cdots & P_{r_S} \\ P_{r_1+1} & P_{r_2+1} & \cdots & P_{r_{S+1}} \\ \vdots & \vdots & & \vdots \\ P_{r_{1+M-1}} & P_{r_{2+M-1}} & \cdots & P_{r_{S+M-1}} \end{bmatrix}, \]  

and \( w_{\text{diff}} \) is a \( M \times 1 \) matrix, \( w_{\text{diff}}^T = [w_{\text{diff \ 1}}, w_{\text{diff \ 2}}, \ldots, w_{\text{diff \ M}}] \) where \( w_{\text{diff}} = \left( \frac{M+i}{M} \right) w_i - \frac{i}{M} w_{i+M} \). LS procedure can be applied to estimate for \( a \) as:

\[ \hat{a} = (\varphi^T \varphi)^{-1} \varphi^T y_{t,da}, \]  

Larger \( M \) results in more equations for LS under the \( 2M \leq G - L \) condition. For instance, for the channel of size \( L = 200 \) and \( G = 256 \), we choose \( M = 16 \).

After the calculation of \( \hat{a} \), \( S \) non-zero complex amplitudes and their locations are defined. As a result, only \( K \) Doppler frequencies should be estimated. Looking at (92), the difference of \( y_i \) and \( y_{M+i} \) results in:
\[ y_{tdf} = y_{M+i} - y_i = \tilde{a}_{r_1} P_{r_1+i}(f2\pi M f_r T_s) + \tilde{a}_{r_2} P_{r_2+i}(f2\pi G f_r T_s) + \cdots + \tilde{a}_{r_k} P_{r_k+i}(f2\pi G f_r T_s) + w_{M+i} - w_i \quad \text{for } i = 0,1,\ldots,M \]

(97)

where \( \tilde{a}_i \)'s are the estimated complex amplitudes in (96). A set of linear equations can be written in a matrix form as:

\[ y_{tdf} = U \times (d, j2\pi MT_s) + w_{diff2} \quad \text{.} \]

(98)

where \( y_{tdf} \) is a \( M \times 1 \) vector, \( y_{tdf}^T = [y_{tdf_1}, y_{tdf_2}, \ldots, y_{tdf_M}] \), \( d \) is \( K \times 1 \) vector of Doppler frequencies, and \( U \) is a \( M \times S \) matrix which is obtained as:

\[
U = \begin{bmatrix}
\tilde{a}_{r_1} P_{r_1} & \tilde{a}_{r_2} P_{r_2} & \cdots & \tilde{a}_{r_S} P_{r_S} \\
\tilde{a}_{r_1} P_{r_1+1} & \tilde{a}_{r_2} P_{r_2+1} & \cdots & \tilde{a}_{r_S} P_{r_S+1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{r_1} P_{r_1+M-1} & \tilde{a}_{r_2} P_{r_2+M-1} & \cdots & \tilde{a}_{r_S} P_{r_S+M-1}
\end{bmatrix},
\]

(99)

and \( w_{diff2} \) is a \( M \times 1 \) matrix, \( w_{diff2}^T = [w_{diff2_1}, w_{diff2_2}, \ldots, w_{diff2_M}] \) where \( w_{diff2_i} = w_{M+i} - w_i \).

The LS estimate of \( d \) is obtained as:

\[ \hat{d} = \text{Real}\left\{ \frac{1}{j2\pi G T_2} (U)^\dagger y_{tdf} \right\} \quad . \]

(100)

- **CS-Linearizing-PEP2**

We proposed this method in [68] and it is the modified version of the CS-Linearizing-PEP1. Our proposed procedure of PEP estimation is based on the fact that if the equal power time domain samples are transmitted through a static and noise free channel, the received samples would be the same between \( r_1 \) and \( r_2 \), \( r_2 \) and \( r_3 \), and so on. However, in the presence of additive white Gaussian noise (AWGN), the received symbols would be changed between any two received symbols. Motivated by the approach in [71], which was proposed for background extraction in a video with the application of the moving object tracking, we propose our channel
tap position estimation method in order to deal with the additive noise. In this method, it is assumed that two consecutive channel taps cannot be non-zero which is a valid assumption for all the sparse channels. For the training sequence, a time domain sequence with the length $G$ that all of its symbols are 1 is transmitted. A time domain guard interval of zeros with the length $G$ is also assumed before the transmission of that training sequence.

For the initialization of PEP estimation method, the mean and the variance of the first two received samples’ power are estimated and a Gaussian distribution is constructed by those mean $(\mu_{r_1})_1$ and variance $(\sigma_{r_1})_1$. If the distance of the third received symbol power to $(\mu_{r_1})_1$ is more than $2.5(\sigma_{r_1})_1$, then $r_2 = 3$; otherwise, the mean $(\mu_{r_1})_2$ and variance $(\sigma_{r_1})_2$ of the first three received samples’ power would be estimated to construct a new Gaussian distribution and the same procedure would be applied to the fourth received sample. When $r_2$ is appraised, the same process would be continued until all the non-zero channel tap positions are estimated. The simulation results indicate that this PEP estimation method is more accurate compared to the method that was used for CS-Linearizing-PEP1.

In this approach, a time training sequence with the length $\tilde{G}$ where $\tilde{G} \leq G$, is transmitted which consists of a periodic PN sequence with the period of $Pr$ where $Pr \geq S$ and we have assumed that the maximum possible sparsity of a channel in a typical environment is known priori. The number of periods “$N_{pr}$” is chosen such that $(N_{pr} - 3)S < L \leq (N_{pr} - 2)S$ and the length of the training sequence is equal to $\tilde{G} = N_{pr}S$. Simulation results indicate that considering the period to be $S$ instead of $M$ in the CS-Linearizing-PEP1, enhances the performance and bandwidth efficiency considerably. The $(N_{pr} - 1)^{th}$ set of the received samples is obtained as:
\[ y_t(i + (N_{pr} - 1)S) = \sum_{s=1}^{s} p(i + (N_{pr} - 1)K - r_k + 1) a_{rs} \left( 1 + j2\pi(i + (N_{pr} - 1)K) f_{rs} T_s \right) + w(i + (N_{pr} - 1)) \]
\[ i = 1, 2, \ldots, S \]

Subsequently, the \( N_{pr} \)th received set of the received samples is attained as:

\[ y_t(i + N_{pr}S) = \sum_{s=1}^{s} p(i + N_{pr}K - r_k + 1) a_{rs} \left( 1 + j2\pi(i + N_{pr}S) f_{rs} T_s \right) + w(i + N_{pr}) \]
\[ i = 1, 2, \ldots, S \]

Since the pilot structure is periodic, \( p(i + (r-1)K - r_k + 1) = p(i + rK - r_k + 1) \) and we obtain:

\[ y_{da2}(i) = \left( \frac{i + N_{pr}S}{S} \right) y(i) - \frac{i}{S} y(i + (N_{pr} - 1)S) = \sum_{s=1}^{s} p(i + (N_{pr} - 1)K - r_k + 1) a_{rs} + \left( \frac{i + N_{pr}S}{S} \right) w(i + (N_{pr} - 1)S) - \frac{i}{S} w(i + N_{pr}S). \]

The equations can be written in the matrix form as:

\[ y_{da} = \varphi_r \times a_r + w_{diff} \]

where \( y_{da} \) is a \( S \times 1 \) vector, \( a_r \) is \( S \times 1 \) vector of the complex amplitudes, \( w_{diff} \) is noise vector, and \( \varphi_r \) is a \( S \times S \) matrix of the pilots. Afterwards, the Doppler frequencies are obtained similar to (97)-(100).

- MDDS-PEP

We proposed this method in [68] and it is the modified version of the MDDS scheme. By
knowing the non-zero tap positions, the rows of the $\xi$ sparse matrixes in (27) reduces from $L$ to $S$, and the $\psi_k$ vector is shortened to:

$$\psi_{\text{new}}^{k} = [e^{-j2\pi\frac{k}{N}r_1}, e^{-j2\pi\frac{k}{N}r_2}, \ldots, e^{-j2\pi\frac{k}{N}r_K}]$$ (105)

As a result, the size of the measurement matrix and the sparse unknowns vectors are reduced considerably which enhances the accuracy of CS method according to [68]. On the other hand, since the sparsity of the channel is obtained, CoSaMP method can be utilized instead of OMP and therefore, the computational complexity of the CE would be reduced.

- **BEM-PEP**

We proposed this method in [73], and it is the modified version of the BEM method. If the location of non-zero elements is previously estimated, the scattered frequency domain pilots can be used within each OFDM symbol to estimate complex amplitudes and Doppler frequencies. If PEP is available, the equation (31) is written as:

$$y_{f_d} = \left[ \tilde{P}_1 \tilde{F}_{N\theta-1}^{\frac{1}{2}}, \tilde{P}_2 \tilde{F}_{N\theta-1}^{\frac{1}{2}}, \ldots, \tilde{P}_N \tilde{F}_{N\theta-1}^{\frac{1}{2}} \right] \bar{\alpha}_d \tilde{c}_d + w_d. \quad (106)$$

where $\tilde{P}_{N\theta-1}^{\frac{1}{2}}$ is obtained from $\tilde{F}_{N\theta-1}^{\frac{1}{2}}$ by choosing its $S$ columns that correspond to the non-zero tap positions, $\bar{\alpha}_d = \text{diag}(e^{\frac{j2\pi d}{N}(r_1 - \frac{1}{2})}, \ldots, e^{\frac{j2\pi d}{N}(r_K - \frac{1}{2})})$, and $\tilde{c}_d$ is obtained from $\tilde{c}_d$ by just keeping its non-zero locations. In conclusion, by the assumption that the number of pilots are larger than $S$, the number of equations would be larger than the number of unknowns and LS scheme can be applied for solving (106). Unlike original BEM, no CS procedure is needed.

4.3. Simulation results

- **LMMSE-OMP**

At the first step, we appraise the performance of running CS more than one time in zero Doppler shift conditions. As an example of a sparse channel, we use ITU/Vehicular Type B
channel model presented in [77]. The power delay profile (PDP) of the channel is indicated in Table 2.

<table>
<thead>
<tr>
<th>Channel model</th>
<th>Power profile (dB)</th>
<th>Delay profile (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITU Vehicular type B</td>
<td>-2.5, 0, -12.8, -10, -25.2, -16</td>
<td>0, 0.3, 8.9, 12.9, 17.1, 20</td>
</tr>
</tbody>
</table>

The delay spread of the channel is 20 µsec and assuming OFDM signal of 10 MHz bandwidth with \( N=256 \) sub-carriers, the digital equivalent channel has \( L=200 \) taps. Only \( S=6 \) of those taps have a non-zero amplitude. Total number of pilots used for channel estimation is equal to \( 4S=24 \) pilots.

Using Monte Carlo simulation, we have calculated the performance of conventional CS channel estimator using OMP and CoSaMP and our purposed method of double running the CS algorithms named OMP-OMP and OMP-COSAMP. We have also evaluated the enhancement when we run the algorithm more than twice. Figure 5 shows the plot for the normalized mean square error (NMSE) vs. signal to noise ratio (SNR) for these channel estimators. NMSE of the estimated channel is defined as \( \frac{||\hat{h} - h||_2^2}{||h||_2^2} \). This results show that we can get considerable improvement in channel estimation with the same number of pilots if we process the modified received data for pilots. While it is better to run both algorithms as OMP, if we are concerned about added complexity we can use CoSaMP at the second run and still get a very good result. Looking at the results for rerunning CS algorithm more than two times shows that the improvement is very marginal and may not worth the added complexity. Besides that, just by using MMSE instead of LS, the performance of OMP-CoSaMP would be better than OMP-OMP. This method is indicated by OMP-CoSaMP2 in the simulation results.

In order to show the effect of enhancing channel estimation performance on the demodulating OFDM signal, we have also simulated a binary phase shift keying (BPSK) OFDM signaling in
this channel and demodulated it using various channel estimators described before. Figure 6 shows the plot of bit error rate (BER) vs SNR in different channel estimation scenarios.

**Figure 5. The comparison between NMSE performances of different methods**

![NMSE vs SNR graph](image)

**Figure 6. The comparison between BER performances of different methods**

![BER vs SNR graph](image)

- **MDDS**

  We have considered both low Doppler shift and high Doppler shift scenarios to show why these modifications are helpful for UAS communications. As an example of the sparse UAS channel, we use UAS channel model L2 that is presented in [7]. For this channel, only $S=6$ out

48
of 200 channel taps are non-zero. An OFDM signal of 10 MHz bandwidth with \(N=512\) subcarriers is used. Total number of pilots used for channel estimation is equal to \(4S=24\).

Figure 7 (a) presents the plot of the NMSE vs. SNR for different channel estimation methods in low Doppler shift scenario \((f_{max} = 1 \text{ KHz})\) and Figure 7 (b) presents the same for high Doppler shift scenario \((f_{max} = 10 \text{ KHz})\). NMSE of the estimated channel is defined as 
\[
\frac{\|\hat{h}-h\|^2}{\|h\|^2}
\] and is considered a good measure of estimator performance. In each plot, we show the result for CS estimation without modification (indexed as original), with only first modification (indexed as precise measurement matrix), with only the second modification (indexed as statistical based quantization), with only the third modification (indexed as guard interval), and finally, with applying all three modifications (indexed as proposed). For the second modification, where we use probability based Doppler shift quantization, the Doppler shift spectrum is divided into 10 bins. For the 1 KHz scenario, \(\alpha\) decreases linearly from 10 to 1 with the step size of 1 when the Doppler shift increases from 0 to 1 KHz and for 10 KHz condition, \(\alpha\) decreases linearly from 100 to 10 with the step size of 10, when the Doppler shift increases from 0 to 10 KHz. For our proposed pilot placement procedure, a guard interval of one symbol is considered before and after of each pilot.

As it indicated in Figure 7 (a), the proposed modifications offer negligible improvement on the original channel estimation accuracy at 1 KHz Doppler shift. However, the effect of the proposed modifications are obvious for the 10 KHz Doppler shift. The implementation of the precise measurement matrix enhance the performance considerably. However, the more improvement comes from statistical based Doppler shift quantization and the guard interval insertion in the pilot placement. Since considering the precise measurement matrix and statistical based Doppler shift quantization are performed offline, they do not add complexity to the real time channel estimation. While adding the guard interval between the pilots is an offline procedure too, it reduces the efficiency of the OFDM transmission (in our simulation, 72
positions are considered for the pilots instead of the 24 positions). However, this reduction in the efficiency (from 488/512 to 440/512) is negligible in comparison to the achieved performance enhancement.

Figure 7. NMSE of channel estimation vs. SNR a) f_max=1 KHz, b) f_max=10 KHz
In order to demonstrate the effect of improving channel estimation performance on the conduct of the whole system, we have simulated a BPSK OFDM signal passing through the channel. Figure 8 presents the plot of BER vs. SNR for different Doppler shift scenarios. For
the clarity of the curves in the picture, the curves for each of the individual modifications is omitted.

As it is observed in Figure 8, the performance of the proposed method is considerably better than the original method in high Doppler shift scenario. In addition, the simulation results indicate that the BER for considering only the diagonal element or the whole channel matrix results in almost the same accuracy for 1 KHz Doppler shift. On the other hand, for 10 KHz Doppler shift, the BER performance is considerably better when the whole channel matrix is considered in comparison to the condition that only the diagonal elements are considered. In both of the two Doppler shift scenarios, MMSE indicate better performance than the two other reconstruction methods specifically in the low SNR condition.

- DS-LMMSE-OMP

The computational complexities of OMP and CoSaMP are \( O(S^2 \tau_r \tau_c) \) and \( O(S \tau_r \tau_c) \), respectively, where \( \tau_r \) and \( \tau_c \) are the number of rows and columns of the measurement matrix, consecutively. If the fast LMMSE channel estimation approach is employed, the computational complexity of the \( \hat{h}_{LMMSE} \) estimation becomes \( O(\tau_r Slog_2^S) \), which is considerably lower than the conventional LMMSE estimation which is \( O(\tau_r^3) \). As a result, the complexity of OMP-OMP is \( O(\tau_r Slog_2^S + 2S^2 \tau_r \tau_c) \), and the complexity of OMP-CoSaMP is \( O(\tau_r Slog_2^S + S(S + 1)\tau_r \tau_c) \). As it will be shown by the simulation results, the performance improvement is negligible if the CS process is added after step 2, but \( \tau_r Slog_2^S + S\tau_r \tau_c \) complexity is added. To summarize, the best tradeoff between the accuracy and the computational complexity can be attained by the OMP-CoSaMP. Furthermore, according to [65], the complexity of the CoSaMP method is defined by its proxy step of forming signal. As a result, applying MMSE instead of LS would not increase the complexity of the whole system.

- Configuration of the communications system
The channel models which are utilized for data transmission are Stanford University Interim (SUI) channel models. These models are proposed for three different terrain types (A, B, C) which are typical for the continental United States [74]. The height of the transmitter while measuring the channels was considered to be 30 m in [74]; therefore, the measured channel models are suitable for UAVs flying at low altitudes. In order to simulate the beam forming scheme, we utilize those SUI channel models that considered beam forming both at the transmitter and the receiver. These channel models are summarized in Table 3. The bandwidth of 100 MHz is considered for defining the length of the digitized channel.

<table>
<thead>
<tr>
<th>Channel model</th>
<th>Length of digitized channel ($L$)</th>
<th>K factor</th>
<th>Tap 1</th>
<th>Tap 2</th>
<th>Tap 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Delay (µs)</td>
<td>Power (dB)</td>
<td>Delay (µs)</td>
</tr>
<tr>
<td>SUI 1</td>
<td>90</td>
<td>14</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>SUI 2</td>
<td>110</td>
<td>6.9</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>SUI 3</td>
<td>90</td>
<td>2.2</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
</tr>
</tbody>
</table>

The parameters of the communication system are summarized in Table 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>Sampling time</td>
<td>$10^8$</td>
</tr>
<tr>
<td>$G$</td>
<td>Length of the cyclic prefix</td>
<td>128</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
<td>70 GHz</td>
</tr>
<tr>
<td>$f_{d_{\text{max}}}$</td>
<td>Maximum Doppler shift</td>
<td>23.33 kHz</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of subcarriers</td>
<td>1024</td>
</tr>
<tr>
<td>$\text{HPBW}_{tr}$</td>
<td>Transmitter antenna beam width</td>
<td>$120^\circ$</td>
</tr>
<tr>
<td>$\text{HPBW}_{re}$</td>
<td>Receiver antenna beam width</td>
<td>$30^\circ$</td>
</tr>
</tbody>
</table>

The maximum Doppler shift is calculated by considering the carrier frequency and setting the speed of the UAV to 100 m/sec which is a typical speed that is considered for UAVs in communication networks [75].
The duration of one OFDM symbol is 11.52 μs that is smaller than the coherence time of the channel which is obtained as $T_c = \frac{0.423}{f_{\text{max}}} = 18.13$ μs [76]. As a result, the assumption of the fixed complex amplitudes and Doppler shifts during the transmission of one OFDM symbol is valid. The number of pilots for the DDS and MDDS methods is set to 102, which results in 10% overhead. The actual number of pilots for MDDS method is 34; however, because of the guard interval, the overhead would be 102. Those pilots are scattered among the data. By considering the transmitter antenna beam width, $u \in [0.5U, U]$ span and the size of the measurement matrix for the DDS method would be $L \times 0.5U$. For MDDS and DS-LMMSE-OMP, similar to the DDS method, $u_{\text{min}} = 0.5U$ and $u_{\text{max}} = U$. The $0.5U$ span is divided into 10 bins, and $\varphi$ is decreased from 100 to 10 by the step of 10 when the Doppler shift increases from $0.5f_{\text{max}}$ to $f_{\text{max}}$. As a result, the $0.5U$ span is quantized into $0.03U$ and the size of the measurement matrix becomes $L \times 0.03U$. For these parameters, the computational complexity for various CE methods is summarized in Table 3.

<table>
<thead>
<tr>
<th>Method</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>DDS</td>
<td>$O(1.3440e+7)$</td>
</tr>
<tr>
<td>MDDS</td>
<td>$O(8.0639e+5)$</td>
</tr>
<tr>
<td>DS-LMMSE-OMP</td>
<td>$O(1.0754e+6)$</td>
</tr>
<tr>
<td>MDDS+OMP</td>
<td>$O(1.6130e+6)$</td>
</tr>
<tr>
<td>DS-LMMSE OMP+CoSaMP</td>
<td>$O(1.8820e+6)$</td>
</tr>
</tbody>
</table>

As it follows from Table 5, the computational complexity of DS-LMMSE-OMP is 1.33 times higher than that for the MDDS method; however, the complexity of MDDS+OMP is 2 times more than the MDDS method which indicates the effectiveness of utilizing CoSaMP instead of OMP for the second round of CS.

- Experiment data

The AVIRIS Indian Pines hyperspectral data set [77] is used for the experiment. Figure 9 a) shows a composite of spectral bands in false colors and b) depicts sixteen major classes of the land cover.
Figure 9. AVIRIS Indian Pines data set. (a) False color composition. (b) Ground truth as a collection of mutually exclusive classes

Each element of 220 frequency band matrices of the datacube is represented by 14 bits.

-Evaluation criteria

Monte Carlo simulation process is applied for evaluating the performance of the CE and data demodulation methods. The first phase of simulation involves generating the random BPSK data for measuring the BER versus SNR for different CE schemes. At each simulation round, the communication channel is selected randomly from one in Table 3.

The second simulation phase involves the evaluation of those CE methods for the transmission of the hyperspectral data that are to be analyzed at the ground station. We employ a hyperspectral data classification method designed in [78]. The method uses subspace-based multinomial logistic regression (MLR) process to learn the posterior probabilities and a pixel-based probabilistic support vector machine (SVM) classifier to define the number of mixed components per pixels. It is robust for mixed pixel characterization and under presence of additive noise. At the final stage, the Markov random field (MRF) based regularizer is applied to increase the accuracy of the classification. For the evaluation of the hyperspectral data fidelity, overall accuracy (OA) of classification is used [79]:
\[ OA = \frac{1}{N_{\text{test}}} \sum_{nc=1}^{N_{\text{class}}} m_{nc,nc}, \]  

(107)

where \( N_{\text{test}} \) is the total number of test samples, \( N_{\text{class}} \) is the number of classification classes, and \( m_{nc,nc} \) indicates the number of pixels that were correctly assigned to class \( nc \).

-Results

Figure 10 presents the plot for the BER vs. SNR for different CE methods. In this figure, all the curves except the DDS and MDDS utilize more than one round of CS. The TD-LMMSE employs the time domain autocorrelation matrix based on Jake distribution. Other implementations that apply more than one round of CS use our proposed DS-autocorrelation matrix.

Figure 10. BER vs. SNR for different CE schemes

As it is depicted in Figure 10, the methods that utilize the enhanced received pilots for another round of CS perform considerably better than the conventional methods that involve the CS just once. On the other hand, the results indicate a higher performance of our proposed DS-autocorrelation matrix for calculating the LMMSE estimate of the received pilots in comparison to the TD-LMMSE method. The simulation results show that the CoSaMP vs. OMP degrades
accuracy of the CE, however the use of the MMSE instead of the LS in the CoSaMP method improves the performance. Finally, the results clarify that more than two CSs are not necessary because the achieved enhancement is negligible compared to the added complexity (see, Table 3).

For different CE methods, the effect of the ICI and the additive noise on the classified hyperspectral image at SNR=10 dB is presented in Figure 11.

The classification OA vs. the number of training samples for CE methods for SNR=10, 20 and 30 dB are presented in Figure 12. In this figure, MDDS+CoSaMP (TD-LMMSE) is reported as TD-LMMSE.

Figure 11. Classified hyperspectral image at SNR=10 dB for (a) Original, (b) MDDS, (c) MDDS+CoSaMP (TD-LMMSE), (d) DS-LMMSE-OMP

![Classification OA vs. number of training sample for different CE schemes](image)

Figure 12. Classification OA vs. number of training sample for different CE schemes
As it follows from Figure 12, the number of training samples that are required to train the classifier for achieving a higher OA rates is significantly smaller for the proposed method that speaks about higher fidelity of received data. In addition, the classification performance of the DS-LMMSE-OMP method at the high number of training samples reaches that of the original data at high number of training samples.

- CS-Linearizing, MDDS-PEP, BEM-PEP and CS-Linearizing-PEP2

- computational complexity

Each channel estimation method consists of several steps; therefore, the total complexity is evaluated by the summation of the complexity of those steps. We employed OMP or CoSaMP for the CS step. According to [65], the computational complexity of the OMP and CoSaMP methods are estimated as $O(S^2K)$ and $O(KS)$ respectively where $K$ is the number of the elements of the measurement matrix.

LS:

This method only consists of two steps. First, the received $P \times 1$ vector of the pilots is divided to the $P \times 1$ vector of the transmitted pilots. Afterwards, the OMP is applied for CE. As a result, the computational complexity is obtained as $O(P + S^2NP)$.

BEM:
This is a two steps method. A block-OMP (BOMP) and a smoothing procedure. The computational complexity of the BOMP is $O(S^2 P ND_r)$ where $D_r$ is the BEM order. On the other hand, the computational complexity of the smoothing step is $O(NS)$ [27]. In conclusion, the computational complexity for all the receivers is obtained as $O(S^2 P ND_r + NS)$.

MDDS:

This method contains an OMP procedure. The $R$ span would be divided in 10 bins, and $\alpha$ would be decreased from 10 to 1 by the step of 1 when the Doppler shift increases from 0 to $f_{max}$. As a result, the $[-R, R]$ span would be quantized into $1.1R$ and the total complexity for all the receivers is calculated as $O(1.1S^2 LR)$.

CS-Linearizing:

This method contains 4 steps. The subtraction of two vectors for amplitude estimation, OMP with the measurement matrix size of $(G - L) \times L$, subtraction of two vectors for Doppler frequency estimation, and LS of the matrix with $(G - L) \times S$ size. As a result, the computational complexity is $O(2(G - L) + S^2(G - L)L + (G - L)^3)$.

PEP estimation:

This method contains the three procedures of mean estimation, variance estimation, and comparison. Therefore, the computational complexity is obtained as $O(G + 2G^2 + 3G^3)$. Since the estimated positions would be utilized for $N_r$ OFDM symbols, the whole complexity should be divided by $N_r$.

MDDS-PEP:

Since the number of rows of the measurement matrix reduces from $L$ to $S$, and CoSaMP is utilized instead of OMP, the complexity is obtained as $O(1.1S^2 R + \frac{1}{N_r}(PEP))$.

BEM-PEP:

Instead of BOMP, this method requires LS. As a result, the computational complexity is
obtained as $O\left((SP)^3 + NS + \frac{1}{N_r}(\text{Linearizing})\right)$.

CS-Linearizing-PEP2:

This method contains 6 different steps. For estimating complex amplitudes or phases, the subtraction of the equations should be applied. Then the received samples should be averaged, and finally a linear algebraic equation is solved. In conclusion. The total computational complexity is obtained as, $O\left(2(2(K + 1) + S^3) + \frac{1}{N_r}(PEP)\right)$.

-Simulation input Dataset

An actual drone image dataset was extracted from a video of a drone monitoring traffic from [80]. One hundred images were selected to represent a wide variety of traffic scenarios including highway, arterial, and parking lot. In addition, images were selected to have a variety of drone altitude and camera perspectives with respect to the road. The images were converted into 8-bit gray scale to make the final dataset. Ten images in the dataset are illustrated in Fig. 13.

Figure 13. Utilized frames from [80] for simulation

-Data Transmission

Each image is vectorized into a binary stream and sent to the OFDM modulation system using QPSK modulation in each subcarrier. The parameters of the communication system are summarized in Table 6.
Table 6. Parameters of the Communication System

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s$</td>
<td>Sampling time</td>
<td>$10^{-3}$ sec</td>
</tr>
<tr>
<td>$G$</td>
<td>Length of the cyclic prefix</td>
<td>256</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
<td>2.35 GHz</td>
</tr>
<tr>
<td>$d_{\text{max}}$</td>
<td>Maximum Doppler shift</td>
<td>652.75 Hz</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of subcarriers</td>
<td>4096</td>
</tr>
</tbody>
</table>

The maximum Doppler shift is calculated by setting the speed of the drone to 100 m/s which is the maximum speed of the drones in a communication network.

-Channel Model

The channel model is chosen from the family of channel models, SUI. Channel model SUI 6 is utilized in the results since this channel variant is defined for dense environments and high Doppler shift conditions. In addition, since the upcoming communication systems most likely will utilize beamforming both at the transmitter and receiver, the model of SUI 6 that is measured with directional antennas is suitable for our simulations. The PDP of SUI 6 channel model is summarized in Table 7.

Table 7. Employed SUI Channel Model

<table>
<thead>
<tr>
<th>Channel model</th>
<th>Power profile (dB)</th>
<th>Delay profile (µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUI 6</td>
<td>0, -16, -26</td>
<td>0, 14, 20</td>
</tr>
</tbody>
</table>

The bandwidth of the communication system is set to 10 MHz and therefore, the length of the tap delay line channel model is 200. We have also run our simulation using ITU Vehicular type B channel and the results are similar to what we report here for SUI 6 channel model.

-Channel Estimation and Data Recovery

The parameters for each CE method is chosen such that all of them have same bandwidth efficiency. For CS-Linearizing-PEP2, PN sequence length is set to 256 and it is constructed by randomly assigning -1 or +1 to each of the 256 samples. The duration of the OFDM block for the CS-Linearizing method is $(3 \times G + N) \times 100 \text{ ns} = 486.4 \mu s$. Since the coherence time of the channel, $T_c = \frac{0.423}{d_{\text{max}}} = 648.03 \mu s$, is larger than the OFDM block duration, considering the fixed complex amplitudes and Doppler shifts during the transmission of one OFDM block is
valid. For OMP part of the CS-Linearizing method, the threshold that is needed for converging the iterations is set to $10^{-5}$.

In order to obtain the same bandwidth efficiency similar to the CS-Linearizing method, the number of scattered locations in an OFDM block for the MDDS method is set to 432. Because of the one subcarrier guard interval before and after of each pilot, the actual number of non-zero pilots for the MDDS method is $\frac{432}{3} = 144$. Similarly for the BEM method, the BEM order ($D_f$) is set to 3 and the number of pilots is set to 144 to obtain the same bandwidth efficiency as CS-linearizing and MDDS. Finally, the number of pilots for the LS method is set to 144 with one subcarrier guard interval.

We have also run our simulation using CS-Linearizing-PEP2, MDDS-PEP and BEM-PEP schemes for CE. Since the duration of fixed channel tap positions is approximately equal to $f_c T_s$ times coherence time, we have chosen $N_r = 235$ in our simulations and since $L$ is equal to 200, the $(N_{pr}, \tilde{G})$ pair is equal to (69,207).

Table 8 summarizes the calculated computational complexity and the bandwidth efficiency based on these parameters. Computational complexity is evaluated by the ratio of number of steps that required for CE in each method to number of steps in LS method and bandwidth efficiency is set to $(BE = \frac{\text{assigned subcarriers for data transmission}}{\text{assigned subcarriers for data and pilot transmission}})$.

<table>
<thead>
<tr>
<th>Method</th>
<th>Steps of Run</th>
<th>Bandwidth Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>1</td>
<td>84.19</td>
</tr>
<tr>
<td>BEM</td>
<td>3.0022</td>
<td>84.19</td>
</tr>
<tr>
<td>MDDS</td>
<td>0.2436</td>
<td>84.19</td>
</tr>
<tr>
<td>CS-Linearizing</td>
<td>0.0449</td>
<td>84.21</td>
</tr>
<tr>
<td>MDDS-PEP</td>
<td>0.0325</td>
<td>84.19</td>
</tr>
<tr>
<td>BEM-PEP</td>
<td>$7.7739\times10^3$</td>
<td>84.19</td>
</tr>
<tr>
<td>CS-Linearizing-PEP2</td>
<td>0.0317</td>
<td>89.80</td>
</tr>
</tbody>
</table>

As it is indicated in Table 8, the computational complexity of the MDDS is more than 10 times lower than the BEM method while the computational complexity of the CS-Linearizing is approximately 6 times lower than the MDDS method. Besides that, utilizing PEP reduces
the computational complexity of the CE considerably. The achieved bandwidth efficiency improvement for the CS-Linearizing-PEP method is negligible since \( N=4096 \); however, in higher Doppler shift conditions the coherence time of the channel is decreased and therefore, the number of subcarriers should be decreased, and the bandwidth efficiency improvement of the CS-Linearizing-PEP would become more considerable. Higher Doppler shift occurs when drones communicate with the moving vehicles.

Image Processing and Car detection

Finally, the output of the communication system is processed by the traffic surveillance unit. In this section, we focus on finding how the performance of a car detection algorithm changes based on defend results from each CE scheme. For that purpose, the Faster R-CNN [81] method is utilized for car detection from the reconstructed images. Faster R-CNN is a deep learning-based detector which uses a shallow region proposal network to find candidate objects followed by the Fast R-CNN classifier for high performance detection and recognition.

- Simulation results

Several masers have been used to compare the performance of the CE schemes. In each run of the simulation, we calculate and plot the normalized mean square error (NMSE) for the channel estimation and bit error rate (BER) of data transmission vs. signal to noise ratio (SNR). For the evaluation of the car detection performance, the precision-recall criterions are considered. The precision \( \frac{t_p}{t_p+f_p} \) and recall \( \frac{t_p}{t_p+f_n} \) are obtained from \( t_p, f_p \) and \( f_n \) which are the number true positive, false positive and false negative detections respectively. When the score threshold is reduced, more potential vehicles are returned by the detector resulting in fewer \( f_n \) and improved recall but at the cost of more \( f_p \) and lower precision. As a result, higher precision and higher recall indicate better detection performance.

- NMSE Results

In order to measure the Doppler shift effect on CE performance, besides the \( d_{max}=652.75 \) Hz
that is considered for the application layer, $d_{\text{max}}=3$ KHz scenario is also used for the evaluation of CE performance. The results for $d_{\text{max}}=652.75$ Hz and $d_{\text{max}}=3$ KHz are indicated in Figure 14 and Figure 15 respectively. In these figures, the curves for the methods that utilize PEP by the assumption that ideal PEP is available, are presented.

Figure 14. NMSE vs. SNR for CE methods for $f_{\text{max}}=652.75$ Hz

![Figure 14. NMSE vs. SNR for CE methods for $f_{\text{max}}=652.75$ Hz](image)

Figure 15. NMSE vs. SNR for CE methods for $f_{\text{max}}=3$ KHz

![Figure 15. NMSE vs. SNR for CE methods for $f_{\text{max}}=3$ KHz](image)

As it is indicated in Figures 14 and 15, the CS-Linearizing performs better than LS, BEM, and MDDS methods. In addition, the exploitation of PEP enhances the accuracy of CE and data demodulation.

On the other hand, by comparison of the PEP based CE methods in the conditions that they
use estimated PEP or ideal PEP, it is understood that in the lower Doppler shift condition, PEP is estimated more accurately while in the higher Doppler shift scenario, the degradation in the performance is mostly because of the estimation error in PEP. The degradation in the performance is more obvious in higher SNR while in lower SNR the performance is mostly affected by the noise. Nevertheless, the performance of the CE methods that utilizes PEP is better than the other methods even in the high Doppler shift scenario.

BER Results

Figure 16 and Figure 17 present the plots of BER vs. SNR for $d_{\text{max}}=652.75$ Hz and $d_{\text{max}}=3$ KHz respectively. It is observed from the simulation results that the BER curves follow the same pattern as the NMSE curves.

![Figure 16. BER vs. SNR for CE methods for $f_{\text{max}}=652.75$ Hz](image)

![Figure 17. BER vs. SNR for CE methods for $f_{\text{max}}=3$ KHz](image)
Visual Detection Results

Qualitative assessment of detector performance for the various CE methods is shown for “frame 5” in Figure 13. It shows both the quality of reconstructed image (visual noise) and overlays the Faster R-CNN detection results in yellow boxes at SNR=10 dB. Note the CS-Linearizing reconstruction (Figure 18(d)) has less noise resulting in fewer spurious detections (hallucinated vehicles) in comparison to the LS, BEM, and MDDS method (see palm tree in center median in Figure 18(a)). Additionally, performance is further enhanced when the PEP is utilized (Figure 18(e)-18(f)). Table 9 shows the precision-recall values for “frame 5” when the score threshold value is 0.5 confirming visual performance assessment.

Figure 18. Car detection in frame 5 at SNR=10 dB for a) LS, b)BEM, c)MDDS, d) CS-Linearizing, e) MDDS-PEP, f) CS-Linearizing-PEP2
### Table 9. Precision-Recall Values for Frame 5

<table>
<thead>
<tr>
<th>Method</th>
<th>Precision</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>0.59</td>
<td>0.64</td>
</tr>
<tr>
<td>BEM</td>
<td>0.73</td>
<td>0.75</td>
</tr>
<tr>
<td>MDDS</td>
<td>0.71</td>
<td>0.75</td>
</tr>
<tr>
<td>CS-Linearizing</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>MDDS-PEP</td>
<td>0.78</td>
<td>0.81</td>
</tr>
<tr>
<td>BEM-PEP</td>
<td>0.80</td>
<td>0.84</td>
</tr>
<tr>
<td>CS-Linearizing-PEP2</td>
<td>0.81</td>
<td>0.84</td>
</tr>
</tbody>
</table>

The full quantitative performance assessment is performed over the 100 frame image set. The precision-recall curves for SNR=10 (solid lines) and 30 dB (dashed lines) are presented in Figure 19. The curves show that CS-Linearizing always outperforms LS, BEM, and MDDS and that PEP significantly improves performance. MDDS-PEP is in fact superior to CS-Linearizing alone without PEP.

**Figure 19. Precision-Recall curves for CE methods**
The performance of each CE method is summarized by the area under curve (AUC) in Table 10. The AUC, which is the integral of the precision-recall curve, provides a single metric to compare classifiers over a range of operating conditions. In comparison to the MDDS method, CS-Linearizing is 5% better, MDDS-PEP is 7% better and BEM-PEP and CS-Linearizing-PEP is 10% better. Even with a state-of-the-art deep-learning method, detection performance is still dependent on the quality of video data modulation. Although, it might be possible to improve the detector performance by training on examples with communication noise, this type of data is not readily available and data collection at deep learning scale is expensive. Therefore, more accurate channel estimation techniques are desired for upcoming communication systems and applications to facilitate reuse of existing analysis modules.

- **Autoregressive-PEP, CS-Linearizing-PEP1**

### Table 10. AUC for Precision-Recall curves

<table>
<thead>
<tr>
<th>Method</th>
<th>SNR=10 dB</th>
<th>SNR= 30 dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>0.5993</td>
<td>0.7013</td>
</tr>
<tr>
<td>BEM</td>
<td>0.6302</td>
<td>0.7307</td>
</tr>
<tr>
<td>MDDS</td>
<td>0.6297</td>
<td>0.7419</td>
</tr>
<tr>
<td>CS-Linearizing</td>
<td>0.6681</td>
<td>0.7748</td>
</tr>
<tr>
<td>MDDS-PEP</td>
<td>0.6704</td>
<td>0.7906</td>
</tr>
<tr>
<td>BEM-PEP</td>
<td>0.6963</td>
<td>0.8012</td>
</tr>
<tr>
<td>CS-Linearizing-PEP</td>
<td>0.6977</td>
<td>0.8254</td>
</tr>
</tbody>
</table>
Computational complexity

Autoregressive-PEP

There are two loops in this method. The inner function of the loop is the summation over $r_2 - r_1, r_3 - r_2, \ldots, r_S - r_{S-1}$. As a result, the complexity is obtained as $O(S^2(r_2 - r_1 + r_3 - r_2 + \ldots + r_S - r_{S-1}))$ and it can be simplified to $O(S^2 N)$.

CS-Linearizing-PEP1

This procedure contains four steps. Two subtraction and two LS estimation. Consequently, the complexity is $O(2(M + S^2 M))$.

Three different scenarios are considered for simulation: low Doppler shift, medium Doppler shift, and high Doppler shift. The bandwidth of the system is chosen to be 10 MHz. The channel model that is exploited for simulation is vehicular type B channel. The length of the guard interval and training sequences are designated to be 256. Since the coherence time of the channel, $T_c$, is proportional to the maximum Doppler frequency, the number of subcarriers for data transmission should be chosen in a way that the assumption of non-varying channel gains and Doppler frequencies during the training sequence and data transmission would be valid. The other parameters for the three scenarios are illustrated in Table 11.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$T_c$ (µs)</th>
<th>Number of subcarriers</th>
</tr>
</thead>
<tbody>
<tr>
<td>low Doppler shift ($f_{dmax} = 275$ Hz)</td>
<td>1538</td>
<td>4096</td>
</tr>
<tr>
<td>Medium Doppler shift ($f_{dmax} = 2.75$ KHz)</td>
<td>153.8</td>
<td>1024</td>
</tr>
<tr>
<td>high Doppler shift ($f_{dmax} = 5.5$ KHz)</td>
<td>76.91</td>
<td>256</td>
</tr>
</tbody>
</table>

The number of scattered pilots for obtaining the channel tap positions is set to 60. For CS-linearizing-PEP method, a 32 length training sequence -which is constructed using bipolar Gold sequence ([82]) with the length 31 with added bit of “1” to the end- is repeated 8 times in order to make a 256 length time domain training sequence. For frequency domain methods, the pilots are chosen based on the Algorithm 2 in [31]. For the first round of MDDS that performs coarse Doppler shift estimation, the span of $[-f_{dmax}, f_{dmax}]$ is divided into 200 equal size bins.
while for the fine step, \( \tau \) is chosen to be 1.

By applying these arrangements and the computational complexity of the CE methods in the previous section, the bandwidth efficiency of each method and the number of steps (NoS) for running each method are indicated in Table 12. In this table, \( N_r \) is considered to be 240 since the carrier frequency for current cellular communication systems is more than 2.4 GHz while the bandwidth of 10 MHz is considered for our simulations. The numbers for SE are in percentage.

<table>
<thead>
<tr>
<th>Method</th>
<th>( f_{D\max} = 275 \text{ Hz} )</th>
<th>( f_{D\max} = 2.75 \text{ KHz} )</th>
<th>( f_{D\max} = 5.5 \text{ KHz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SE</td>
<td>NoS</td>
<td>SE</td>
</tr>
<tr>
<td>Autoregressive-PEP</td>
<td>88.87</td>
<td>8.34x10^5</td>
<td>66.63</td>
</tr>
<tr>
<td>CS-Linearizing-PEP</td>
<td>88.87</td>
<td>6.87x10^5</td>
<td>66.63</td>
</tr>
<tr>
<td>DDS (Original)</td>
<td>88.89</td>
<td>1.30x10^9</td>
<td>66.67</td>
</tr>
<tr>
<td>MDDS</td>
<td>88.89</td>
<td>7.05x10^8</td>
<td>66.67</td>
</tr>
<tr>
<td>MDDS-PEP</td>
<td>88.87</td>
<td>5.75x10^7</td>
<td>66.63</td>
</tr>
</tbody>
</table>

As it can be observed in Table 12, exploiting the channel tap positions as the priori information reduces the computational complexity of CE without decreasing the spectral efficiency. In addition, it is understood from that table that the computational complexity of the time domain CE methods are considerably lower than the computational complexity of the frequency domain CE approaches.

Monte Carlo process is applied in order to calculate the performance of the CE methods. In each round of the simulation, while the power of channel taps were chosen from vehicular type B channel model, their locations were randomly chosen in [1,200] range in order to compare the CE methods in different channel models. Figure 20 indicates the plot for the NMSE vs. SNR for different CE schemes.

Figure 20. NMSE of channel estimation vs. SNR a) \( f_{D\max}=275 \text{ Hz} \), b) \( f_{D\max}=2.75 \text{ KHz} \) c) \( f_{D\max}=5.5 \text{ KHz} \)
As it is indicated in Figure 20 (a), in low Doppler shift scenario, the most important factor that defines the difference between the performances of the CE methods is to use or not to use the estimated channel tap positions as the priori information. It can be understood from the
results that the performance of frequency domain methods are more affected by increasing the Doppler shift in comparison to the performance of time domain methods. CS-Linearizing-PEP1 method has the best performance among the other methods in low SNR because of the effect of the specific structure of training sequence on LS calculation. However, in high SNR and high Doppler shift- that the performance degradation is affected by Doppler shift more than to be affected by noise- the Autoregressive-PEP method indicates the best accuracy since it models the Doppler shift accurately.

In order to measure the performance of CE methods on signal demodulation, we used a BPSK. Figure 21 indicates the plot of BER vs. SNR for different Doppler shift scenarios.

Figure 21. BER of channel estimation vs. SNR a) $f_{D_{\text{max}}}=275$ Hz, b) $f_{D_{\text{max}}}=2.75$ KHz c) $f_{D_{\text{max}}}=5.5$ KHz
As it is indicated in Figure 21, the BER performance of the different CE methods obey the same pattern as the NMSE curves. It is noticeable that even in low Doppler shift condition, a considerable difference between the performances of the various methods is observed. This difference is because of the high number of subcarriers that are considered for the low Doppler shift condition.
Chapter 5. Channel estimation approaches for MIMO-OFDM

5.1. Methods

- LMMSE-BOMP

We proposed this method in [83]. The main contributions of this method are a new method for pilot placement and exploiting the group sparsity for channel estimation.

-Pilot design structure

In this method, we proposed an optimized pilot pattern that operates based on the minimization of the average block coherence by considering both the placement of the pilots and their values simultaneously. The method is dubbed as joint placement and amplitude optimization (JPAO) method and it is summarized in Figure 22. In this paper, we utilized Golden sequences as pilots. According to [82], the number of Golden sequences with length $2^{m-1}$ is $2^{m+1}$ and the cross-correlation between each pair obtains three possible values, $\{-1, -t(m), t(m) - 1\}$ where $t(m) = 2^{[\frac{m}{2}]} + 1$. In JPAO algorithm, a set of $N_T$ Golden sequences which have the lowest cross-correlation are chosen from the $2^{m+1}$ Golden sequences. This set is indicated as $\beta = \{\beta_1, \beta_2, ..., \beta_{N_T}\}$. The remaining $Q = 2^m + 1 - N_T$ Golden sequences are stacked in a set $\alpha = [\alpha_1, \alpha_2, ..., \alpha_Q]$. Besides that, $R$ different pilot placements, $\gamma = [\gamma_1, \gamma_2, ..., \gamma_R]$, that are created by randomly choosing $P$ pilots out of $N$ subcarriers are produced. At the first step, the pilot placement that leads to the minimum average mutual block coherence is found, $\gamma_{ca}^{opt(0)}$; afterwards, the selected pilot placement is modified. In order to optimize the selected pilot placement, while the position of the $P$-1 pilots are fixed during each iteration, the position of the remaining one pilot is chosen from all the $N-P$ possible placements in order to obtain the minimum average mutual block coherence. This procedure continues for all the pilots of the selected pilot placement. At the second stage, the pilot placement is not going to change; instead, one out of $N_T$ Golden sequences are changed with another Golden sequence.
of $\alpha$ at each iteration in order to obtain the minimum average mutual block coherence. The computational complexity of the JPAO algorithm is much higher than choosing the pilots from Bernoulli distribution; however, since the pilot arrangement is an offline procedure, its computational complexity is not important for real time systems.

**Figure 22. JPAO method for pilot placement and pilot sequence selection**

<table>
<thead>
<tr>
<th>Input: $G$ different set of Golden sequences, $\beta = [\beta_1, \beta_2, ..., \beta_G]$, $\beta_{ca}$ contains $N_T$ different Golden sequences, $Q$ different Golden sequences, $\alpha = [\alpha_1, \alpha_2, ..., \alpha_Q]$, $M$ different set of pilot placements, $\gamma = [\gamma_1, \gamma_2, ..., \gamma_M]$, $\gamma_{cb}$ is generated by choosing $P$ out of $N$ positions randomly.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output:</strong> $\gamma_{cb}^{opt(\text{final})}$, and $\beta_{opt(\text{final})}$</td>
</tr>
<tr>
<td>1. For $Ca=1,2, ..., G$</td>
</tr>
<tr>
<td>2. $\gamma_{ca}^{opt(0)} = \gamma_1$</td>
</tr>
<tr>
<td>3. For $Cb=2, ..., M$</td>
</tr>
<tr>
<td>4. If $(\mu_B(\theta^{CS})<em>{ca,cb}) &lt; (\mu_B(\theta^{CS})</em>{ca,cb-1})$</td>
</tr>
<tr>
<td>5. $\gamma_{ca}^{opt(0)} = \gamma_{cb}$</td>
</tr>
<tr>
<td>6. End if</td>
</tr>
<tr>
<td>7. End for $Cb$</td>
</tr>
<tr>
<td>8. For $Ce=1,2, ..., G$</td>
</tr>
<tr>
<td>9. For $Cd={\text{Position}_1, \text{Position}<em>2, ..., \text{Position}</em>{N-P}}$</td>
</tr>
<tr>
<td>10. If $(\mu_B(\theta^{CS})<em>{ca,cd}) &lt; (\mu_B(\theta^{CS})</em>{ca,cd-1})$</td>
</tr>
<tr>
<td>11. $\gamma_{ca}^{opt(\text{final})} = \gamma_{cd}^{opt(\text{final})}$</td>
</tr>
<tr>
<td>12. End if</td>
</tr>
<tr>
<td>13. End for $Cd$</td>
</tr>
<tr>
<td>14. End for $Cc$</td>
</tr>
<tr>
<td>15. For $Ce=1,2, ..., G$</td>
</tr>
<tr>
<td>16. If $(\mu_B(\theta^{CS})<em>{ce,\gamma</em>{opt(\text{final})}}) &lt; (\mu_B(\theta^{CS})<em>{ce-1,\gamma</em>{ce-1,\gamma_{opt(\text{final})}}})$</td>
</tr>
<tr>
<td>17. $\beta_{opt(0)} = \beta_{ce}$</td>
</tr>
<tr>
<td>18. End if</td>
</tr>
<tr>
<td>19. End for $Ce$</td>
</tr>
<tr>
<td>20. For $Cg=1,2, ..., N_T$</td>
</tr>
<tr>
<td>21. For $Cf=1,2, ..., G$</td>
</tr>
<tr>
<td>22. For $Cg=[G_sequence_1, G_sequence_2, ..., G_sequence_q]$</td>
</tr>
<tr>
<td>23. If $(\mu_B(\theta^{CS})<em>{cf,g,sequence</em>{cg}}) &lt; (\mu_B(\theta^{CS})<em>{cf,g,sequence</em>{cg-1}})$</td>
</tr>
<tr>
<td>24. $\beta_{opt(\text{final})} = \beta_{opt(\text{sequence}_{cg})}$</td>
</tr>
<tr>
<td>25. End if</td>
</tr>
</tbody>
</table>

-LMMSE-BOMP
The performance of the CE procedure can be enhanced more without the requirement of extra information, in price of adding a little more complexity to the data analysis process. The new method is nominated as LMMSE-BOMP and it is described as follows.

At the first step, all the $\hat{g}_j$ s are estimated. However, they will not be utilized for data demodulation. Instead, they will be utilized for obtaining a better estimate of the received pilots. By taking the FFT of each $\hat{h}_{ij}$, $\hat{h}_{fij}$ would be estimated. Afterwards, the LMMSE of each channel is estimated as:

$$\hat{h}_{LMMSE_{ij}} = R_{HH_{ij}} \left[ R_{HH_{ij}} + \frac{B}{\sigma^2 I_N} \right]^{-1} \hat{h}_{fij},$$

where $R_{HH_{ij}}$ is the autocorrelation matrix of the frequency domain channel and it is constructed by the estimated channel parameters at the first step, the number $B$ is defined as $\frac{E(X_k^2)}{E(X_k)}$ when $X_k$ is the transmitted symbol in the frequency domain and $\sigma^2$ represents the variance of AWGN. Matrix $I_N$ is the $N \times N$ identity matrix. Afterwards, a more accurate estimate of the received pilots is obtained as:

$$y^{CS}_{LMMSE_{ij}} = \hat{h}^{CS}_{LMMSE_{ij}} x^{CS}_f,$$

where $y^{CS}_{LMMSE_{ij}}$ is the $P \cdot N_T \times 1$ vector and it is the accurate estimate of the received pilots at the $j^{th}$ receiver, $x^{CS}_f$ is the $P \cdot N_T \times 1$ vector of the transmitted pilots, and $\hat{h}^{CS}_{LMMSE_{ij}}$ is obtained by selecting the corresponding –to the pilot positions- rows of all $\hat{h}_{LMMSE_{ij}}$. The estimated $y^{CS}_{LMMSE_{ij}}$ can be applied to another round of BOMP in order to estimate $\hat{d}_{LMMSE_{ij}}$.

However, the computational complexity of the $\hat{d}_{LMMSE_{ij}}$ estimation is twice the estimation of $\hat{d}_j$ since it needs two rounds of BOMP. It is indicated that the compressive sampling matching
pursuit (CoSaMP) method has almost the same accuracy as OMP while its computational complexity is \( S \) times less than the complexity of the latter method. However, CoSaMP needs the sparsity level of the vector as a priori information. Since at the first run of \( \hat{\mathbf{f}} \) estimation the sparsity of the channel is determined, it can be exploited for the second run of CS method which was called block-CoSaMP (BCoSaMP) [84]. In BCoSaMP method, the reconstruction matrix \( \mathbf{M} \), is initiated with \( 2SN_T \) columns of the measurement matrix \( \mathbf{\theta}^{cs} \) with the maximum correlation with the received pilots. At each iteration, \( \mathbf{M} \) is updated by adding \( N_T \) columns that define the reminder. The process runs a predetermined number of iteration between 4\( S \) and 5\( S \), and yield an accurate \( \hat{\mathbf{f}}_{LMMSE} \) estimation.

While we can continue doing a new set of OMP after each estimation, the improvement on performance will decrease while adding to the complexity of channel estimation. It seems that performing OMP twice (one with original noisy received pilot signals to get and initial estimate of the channel and the other using an enhanced version of the received pilot data) gives the most improvement in performance.

Since the performance of BOMP-BCoSaMP method is lower in comparison to the performance of BOMP-BOMP method, we proposed a modification to the BCoSaMP phase of our CE scheme. Instead of employing LS for \( \mathbf{f} \) estimation, MMSE method is utilized for the reconstruction of the \( \mathbf{f} \) vector from matrix \( \mathbf{M} \) at each iteration of BCoSaMP:

\[
\hat{\mathbf{f}}_{j,k} = \left( \mathbf{M}_k^H \mathbf{M}_k + \frac{1}{\sigma^2} \mathbf{I} \right)^{-1} \times \mathbf{M}_k^H \mathbf{y}_j^{cs},
\]

We nominate this method as LMMSE-BOMP in the rest of this paper.

The complexity of OMP is of \( O(SNP) \) and the complexity of CoSaMP is \( O(NP) \) [30]. As a result, the computational complexity of BOMP is \( O(SNP N_T) \) and the computational complexity of BCoSaMP is \( O(NP N_T) \). By utilizing the fast LMMSE channel estimation method that is
proposed in [32], the computational complexity of the $\hat{h}_{LMMSE_{ij}}$ estimation for $N_T$ transmitter is $O(N_T NS\log_2 S)$. Therefore, the complexity of BOMP-BOMP for modified method is $O(N_T NS\log_2 S + 2SNP N_T)$ and the computational complexity of BOMP-BCoSaMP is $O(N_T NS\log_2 S + (S+1)NP N_T)$. According to the simulation results, by adding each round of CS method after the second round, a negligible enhancement is obtained while $N_T NS\log_2 S + NP N_T$ would be added to the computational complexity. As a result, the best tradeoff between the accuracy and the computational complexity is to perform BOMP-BCoSaMP.

- **MIMO-MDDS**

  We proposed this method in [57] and it is the modified version of our proposed MDDS method which is described in the previous chapter by applying all the modifications that were proposed for the MDDS method. The received symbols are obtained as:

  $Y_j = \varphi c_{T_j} + Z_j$ \hspace{1cm} \text{(111)}

  where $\varphi = P \left( (v_q \otimes v_k) \otimes 1_{N_T \times 1} \right)$ and $P$ is the $p \times p$ diagonal matrix of the transmitted pilots.

- **MIMO-CS-Linearizing**

  We proposed this method in [57]. If the duration of a single OFDM block is less than the coherence time of the channel, one can assume that the complex amplitudes and Doppler shifts do not change during the transmission of one OFDM block. However, the phases of the exponential terms are changed proportional to the Doppler shifts. By applying the truncated Taylor expansion, $\alpha_{t_{ij}} e^{j2\pi f_{D_{t_{ij}}} nT_{sample}}$ term can be approximated by $\alpha_{t_{ij}}(1 + j2\pi f_{D_{t_{ij}}} nT_{sample})$.

  For estimating the channel, we proposed the utilization of two identical PN sequences in time domain in order to reduce the mutual coherence of the measurement matrix as it is discussed later in this section. The length of each PN sequence is $Q$, where $Q \geq G > L$. Those
PN sequences are transmitted before the cyclic prefix and the OFDM data block. As a result, the first $Q$ received time domain samples can be defined as:

$$y_{tj}(n) = \sum_{n_c=1}^{\max\{L,n\}} \sum_{i=1}^{N_T} p_{n-n_c+1} \alpha_{n_cij} \left( 1 + j2\pi f_{p_{n_cij}} T_s \right) + BL_D(n) + w_j(n),$$

$$n = 1, 2, \ldots, Q$$

(112)

where $y_{tj}(n)$ is the received time domain sample at the $j^{th}$ receiver and time $n$, $p_{n-n_c+1}$ is the transmitted pilot from the $i^{th}$ transmitter at the time $n-n_c+1$, $w_j(n)$ is the additive noise to the $n^{th}$ received sample at the $j^{th}$ receiver. The block interference (BI) of the previous OFDM data block is indicated by $BL_D(n)$ where $BL_D(n) = 0$ for $L < n$ since for $L < n$, the received samples only face with inter block interference (IBI). As the similar PN sequence is transmitted after the first one, the corresponding second $Q$ received samples are obtained as:

$$y_{tj}(n) = \sum_{n_c=1}^{\max\{L,n\}} \sum_{i=1}^{N_T} p_{n-n_c+1} \alpha_{n_cij} \left( 1 + j2\pi (n + Q) f_{p_{n_cij}} T_s \right) + BL_{PN}(n)$$

$$+ w_j(n), n = 1, 2, \ldots, Q$$

(113)

where $BL_{PN}(n)$ is the BI of the first PN sequence and $BL_{PN}(n) = 0$ for $L < n$. By considering the last $Q - L$ equations of (112) and (113) which are free of BI, a $(Q - L) \times 1$ vector of the scaled difference of (112) and (113) is obtained where each element is expressed as:

$$y_{t_{\text{diff}} j}(n) = \left( \frac{Q + n}{Q} \right) y_j(n) - \frac{n}{Q} y_j(n + Q) =$$

$$\sum_{i=1}^{L} \sum_{l=1}^{N_T} p_{n-l+1} \alpha_{lij} + \left( \frac{Q + n}{Q} \right) w_j(n) - \frac{n}{Q} w_j(n + Q),$$

(114)

and all the elements are written in a matrix form as:
\[ y_{t_{da}j} = \varphi \times \alpha + w_{j_{diff1}}, \]  

where \( y_{t_{da}j} \) is a \((Q - L) \times 1\) vector, \( y_{t_{da}j}^T = [y_{t_{da}j 1}, y_{t_{da}j 2}, \ldots, y_{t_{da}j L - 1}] \), \( \alpha \) is a \( \sqrt{N_T}L \times 1 \) vector of complex amplitudes, \( \alpha^T = [\alpha_{11j}, \alpha_{12j}, \ldots, \alpha_{1N_Tj}, \ldots, \alpha_{L1j}, \alpha_{L2j}, \ldots, \alpha_{LN_Tj}] \), \( w_{j_{diff1}} \) is noise vector, and \( \varphi \) is a \((Q - L) \times \sqrt{N_T}L\) matrix of the training sequence that its elements are obtained as:

\[
\varphi = \begin{bmatrix}
  p_{L_1} & p_{L_2} & \cdots & p_{L_N_T} & p_{1_1} & p_{1_2} & \cdots & p_{1_N_T} \\
  p_{L+1_1} & p_{L+1_2} & \cdots & p_{L+1_N_T} & p_{2_1} & p_{2_2} & \cdots & p_{2_N_T} \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
  p_{G_1} & p_{G_2} & \cdots & p_{G_N_T} & p_{G-L+1_1} & p_{G-L+1_2} & \cdots & p_{G-L+1_N_T} 
\end{bmatrix},
\]

Since the number of columns of \( \varphi \) is larger than its rows, the equation (115) cannot be solved linearly; however, by considering a sparse tapped delay line channel model, \( \alpha \) would be a sparse vector. As a results, a CS method can be applied for solving (115).

When \( \alpha \) is estimated, \( KN_T \) complex amplitudes and their positions are obtained. Afterwards, \( KN_T \) Doppler shifts of those non-zero complex amplitudes should be calculated. By defining the set of estimated non-zero complex amplitudes as \( \{ \bar{\alpha}_{r_{11j}}, \bar{\alpha}_{r_{12j}}, \ldots, \bar{\alpha}_{r_{N_Tj}}, \ldots, \bar{\alpha}_{r_{K1j}}, \bar{\alpha}_{r_{K2j}}, \ldots, \bar{\alpha}_{r_{KN_Tj}} \} \) and the set of their corresponding frequencies as \( D^T = \{ f_{D_{r_{11j}}}, f_{D_{r_{12j}}}, \ldots, f_{D_{r_{1N_Tj}}}, \ldots, f_{D_{r_{K1j}}}, f_{D_{r_{K2j}}}, \ldots, f_{D_{r_{KN_Tj}}} \} \) where \( r = \{ r_1, r_2, \ldots, r_K \} \) is the set of non-zero positions, the subtraction of (112) and (113) results in:

\[
y_{t_{df1}} j(n) = y_{t_j}(n + Q) - y_{t_j}(n) = \sum_{\tau=1}^{K} \sum_{i=1}^{N_T} p_{n-r_\tau i} \bar{\alpha}_{r_{\tau ij}} (j2\pi Q f_{D_{r_{\tau ij}}} T_s) + w_j(n + Q) - w_j(n) .
\]

The upper equation can be written in matrix form as:

\[
y_{t_{df1}i} = \Omega \times (d, j2\pi QT_s) + w_{j_{diff2}},
\]
where $\mathbf{y}_{tdf_1^j}$ is a $(Q - L) \times 1$ vector, $\mathbf{y}_{tdf_1^j}^T = [y_{tdf_1^j, 1}, y_{tdf_1^j, 2}, \ldots, y_{tdf_1^j, Q-L}]$, and $\Omega$ is a $(Q - L) \times KN_T$ matrix that its elements are obtained as:

\[
\begin{bmatrix}
\hat{\mathbf{a}}_{1,1} p_{L+1-r_1} & \hat{\mathbf{a}}_{1,2} p_{L+1-r_2} & \cdots & \hat{\mathbf{a}}_{1,r} p_{L+1-r_N} \\
\hat{\mathbf{a}}_{2,1} p_{L+2-r_1} & \hat{\mathbf{a}}_{2,2} p_{L+2-r_2} & \cdots & \hat{\mathbf{a}}_{2,r} p_{L+2-r_N} \\
\vdots & \vdots & \ddots & \vdots \\
\hat{\mathbf{a}}_{r,1} p_{L+T-r_1} & \hat{\mathbf{a}}_{r,2} p_{L+T-r_2} & \cdots & \hat{\mathbf{a}}_{r,r} p_{L+T-r_N}
\end{bmatrix},
\tag{119}
\]

By the assumption that the number of rows of $\Omega$ is larger than the number of its columns, LS can be applied for solving (118) as:

\[
\hat{\mathbf{a}} = \text{Real} \left\{ \frac{1}{j2\pi Q T_s} (\Omega)^\dagger \mathbf{y}_{tdf_1^j} \right\}.
\tag{120}
\]

- **MIMO-MDDS-PEP**

We proposed this method in [57]. By knowing the non-zero tap positions, the rows of the delay-Doppler sparse matrixes in reduces from $L$ to $S$, and the $\mathbf{v}_k$ vector is shortened to:

\[
\mathbf{v}_k^{\text{new}} = [e^{-j2\pi k r_1}, e^{-j2\pi k r_2}, \ldots, e^{-j2\pi k r_S}]
\tag{121}
\]

As a result, the size of the measurement matrix and the sparse unknowns vectors are reduced considerably which enhances the accuracy of CS method; however, still MIMO-MDDS-PEP requires a CS method in contrast to the MDDS-PEP scheme.

- **MIMO-BEM-PEP**

We proposed this method in [57]. If the location of non-zero elements is previously estimated, the scattered frequency domain pilots can be used within each OFDM symbol to estimate complex amplitudes and Doppler frequencies. If PEP is available, we have:

\[
\mathbf{Y}_d = \left[ \hat{P}_1 F_{ND-1}^{-1}, \hat{P}_2 F_{ND-1}^{-1}, \ldots, \hat{P}_{N_T} F_{ND-1}^{-1} \right] \hat{\mathbf{a}}
\tag{122}
\]

\[
\begin{bmatrix}
\hat{c}^T_{d1}, \hat{c}^T_{d2}, \ldots, \hat{c}^T_{dN_T}
\end{bmatrix}^T + \mathbf{W}_d,
\]

where $\hat{P}_{ND-1}^{\frac{1}{2}}$ is obtained from $F_{ND-1}^{\frac{1}{2}}$ by choosing its $S$ columns that correspond to the non-
zero tap positions, \( \bar{\mathbf{c}}_d \) = \( \text{diag}(e^{j2\pi (d-\frac{D-1}{2})r_1^T}, \ldots, e^{j2\pi (d-\frac{D-1}{2})r_S^T} \otimes \mathbf{I}_{N_T} \), and \( \hat{\mathbf{c}}_{d_i}^T \) is obtained from \( \hat{\mathbf{c}}_{d_i}^T \) by just keeping its non-zero locations. In conclusion, by the assumption that the number of pilots are larger than \( KN_T \), the number of equations would be larger than the number of unknowns and LS scheme can be applied for solving (122).

- MIMO-CS-Linearizing-PEP

In this scheme that we proposed in [57], we use a time domain pilot structure before each OFDM symbol to estimate the amplitude and Doppler frequencies of the known non-zero paths. A periodic PN sequence, which is nominated as \( \hat{\mathbf{P}} \mathbf{N} \), with period of \( SN_T \) is chosen. The number of periods “\( r \)” is chosen such that \( (r-3)SN_T < L \leq (r-2)SN_T \) and the length of the training sequence is equal to \( Q_{LP} = rSN_T \). The \((r-1)^{th}\) set of the received samples is defined as:

\[
y_{t_j}(n + (r - 1)SN_T) = \sum_{s=1}^{S} \sum_{i=1}^{N_T} \hat{p}_{(n+(r-1)SN_T-r_s+1)_i} \alpha_{rs_{ij}} \\
\left( 1 + j2\pi(n + (r - 1)SN_T)f_{D_{njij}} T_s \right) + w_j(n + (r + 1)N_T). \tag{123}
\]

\( n = 1, 2, \ldots, SN_T \)

Afterwards, the \( r^{th} \) received set of the received samples is obtained as:

\[
y_{t_j}(n + rSN_T) = \sum_{s=1}^{S} \sum_{i=1}^{N_T} \hat{p}_{(n+rSN_T-r_k+1)_i} \alpha_{rs_{ij}} \\
\left( 1 + j2\pi(n + rSN_T)f_{D_{njij}} T_s \right) + w_j(n). \tag{124}
\]

\( n = 1, 2, \ldots, SN_T \)

Because of the periodic structure, \( \hat{p}_{(n+(r-1)SN_T-r_s+1)_i} = \hat{p}_{(n+rSN_T-r_s+1)_i} \), we obtain:
\[ y_{t_{da_2} j}(n) = \left( \frac{n + rKNT}{KNT} \right) \cdot y_{t_j}(n) - \frac{n}{KNT} \cdot y_{t_j}(n + (r - 1)KNT) \]

\[ = \sum_{k=1}^{K} \sum_{i=1}^{N_T} \tilde{p}_{(n+(r-1)KNT-r_k+1)_i} \alpha_{r_{kij}} + \]

\[ \left( \frac{n + rKNT}{KNT} \right) \cdot w_j(n + (r - 1)KNT) - \frac{n}{KNT} \cdot w_j(n + rKNT) . \]

The equations can be written in the matrix form as:

\[ y_{t_{da} r} = \xi_r \times \alpha_r + w_{j_{diff_{1r}}} , \quad (126) \]

where \( y_{t_{da} r} \) is a \( SN_T \times 1 \) matrix, \( y_{t_{da} r}^T = [ y_{t_{da} 1}, y_{t_{da} 2}, \ldots, y_{t_{da} j \times KNT} ] \), \( \alpha_r \) is \( SN_T \times 1 \) vector of complex amplitudes, \( \alpha_r^T = [ \alpha_{r_{1j}}, \alpha_{r_{2j}}, \ldots, \alpha_{r_{N_{T}j}}, \ldots, \alpha_{r_{SN_Tj}} ] \), \( w_{j_{diff_{1r}}} \) is noise vector, and \( \xi_r \) is a \( SN_T \times SN_T \) matrix of the pilots which are defined according to (124). Since \( \xi_r \) is a square matrix, \( A_r \) can be calculated by LS procedure. The Doppler shifts can be defined similar to (117)-(120).

- **MIMO-CS-Linearizing-PEP-M**

This method is the modified version of the previous method and we proposed it in [57]. Since the computational complexity of LS is \( O(\mathbb{C}^3) \) where \( \mathbb{C} \) is the maximum dimension of the coefficient matrix, the computational complexity of the LS procedure in the MIMO-CS-Linearizing-PEP method is \( O(SN_T^3) \). If we consider the fact that almost all the energy of vehicular channel models is concentrated in the first two non-zero channel taps, the MIMO-CS-Linearizing-PEP scheme can be modified such that the channel taps would be estimated autoregressively and therefore, the computational complexity would be reduced. This assumption is valid in all the current available vehicular channel models, such as ITU vehicular type A and B, six vehicular channel models of SUI that are defined for typical US continent, D2a for moving network and B5b for all the three street to street levels which are defined in
The modified version of the MIMO-CS-Linearizing-PEP method is dubbed MIMO-CS-Linearizing-PEP-M and it is described as follows.

- Estimating the channel

In the MIMO-CS-Linearizing-PEP-M procedure, the OFDM+pilot block starts with zero block with length $G$ followed by a periodic $PN$ sequence with length $G$ as the training sequence. The zero block makes the training sequence to be free of interference. The period of the $PN$ sequence is $N_T$ and it is assumed that $r_{r+1} - r_t \geq 2N_T$ for all $\tau$s (the case $r_{r+1} - r_t < 2N_T$ will be discussed later). As a result, the total number of periodic sequences is $R_{MLP} = \left\lfloor \frac{G}{N_T} \right\rfloor$. Therefore, the differential equations can be constructed among the received samples with the time difference of $N_T$:

$$y_{tda_2, j}(n) = \left(\frac{n + N_T}{N_T}\right) \cdot y_{tj}(n) - \frac{n}{N_T} \cdot y_{tj}(n + N_T)$$

$$= \sum_{i=1}^{r} \sum_{i=1}^{N_T} p_{n-r_{i+1}} \alpha_{r_{ij}} \left(\frac{G + n}{G}\right) \cdot w_j(n) - \frac{n}{G} \cdot w_j(n + G),$$

if $n \leq r_{r+1} + N_T$ for $n = 1, 2, ..., G - N_T$.

The $n \leq r_{r+1} + N_T$ condition expresses that only the equations that are affected by the same number of channel taps should be subtracted from each other.

When $3N_T \leq r_{r+1} - r_t$, more than one equation would be obtained. By averaging those equations, the noise effect would be reduced:

$$y_{tda_r, j}(n) = \frac{1}{\left[\frac{r_{r+1} - r_t}{N_T}\right]^{-1}} \sum_{i=0}^{\left[\frac{r_{r+1} - r_t}{N_T}\right]-1} y_{tda_2, j}(n + r_t - 1 + IN_T),$$

$$n = 1, 2, ..., N_T.$$
where $r_{s+1} = G$. The complex amplitudes are obtained through an autoregressive procedure.

The first tap of all the channel pairs are calculated by solving the following equation:

$$y_{t_{da_1}} j = \xi_{t_{da_1}} a_{r_1} + w_{j_{diff_{r_1}}},$$

(129)

where $a_{r_1} = [a_{r_11j}, a_{r_12j}, \ldots, a_{r_1N_Tj}]^T$, $\xi_{t_{da_1}}$ is the matrix of corresponding pilots, and $w_{j_{diff_{r_1}}}$ is the noise vector. Afterwards, the following equation should be solved for the calculation of $a_{r_2} = [a_{r_21j}, a_{r_22j}, \ldots, a_{r_2N_Tj}]^T$:

$$y_{t_{da_2}} j - y_{t_{da_1}} j = \xi_{t_{da_2}} a_{r_2} - \xi_{t_{da_1}} a_{r_1} + w_{j_{diff_{r_2}}}. $$

(130)

This procedure should be continued until all the complex amplitudes are estimated. Afterwards, Doppler shifts can be defined similar to (117)-(120).

Up to now, it was assumed that $r_{s+1} - r_s \geq 2N_T$ for all $r_s$. This assumption might not be always valid; however in the worst case scenario, an accurate estimate of the channel taps in $r_1$ and $r_2$ locations can be obtained by considering that mean power of those two taps are considerably larger than the other taps. If $r_2 - r_1 < 2N_T$, then the remaining $G - 2N_T$ equations can be solved for obtaining $a_{r_1 s}, a_{r_2 s}, f_{D_{r_1}} s$ and $f_{D_{r_2}} s$ by neglecting the effect of the other channel taps on those equations. If $r_2 - r_1 \geq 2N_T$, first the $a_{r_1 s}$ and $f_{D_{r_1}} s$ are obtained, then $a_{r_2 s}$ and $f_{D_{r_2}} s$ would be estimated by utilizing the remaining $G - r_2$ equations. There are several scenarios that the other channel taps can be calculated based on the number of available equations between the consecutive channel taps. For instance, if $r_4 - r_3 \geq 2N_T$, then $a_{r_3 s}$ and $f_{D_{r_3}}$ can be estimated and if $r_4 - r_3 < 2N_T$ but $r_5 - r_4 \geq 4N_T$, $a_{r_3 s}$, $a_{r_4 s}$, $f_{D_{r_3}} s$ and $f_{D_{r_4}}$ can be estimated simultaneously.

-Data demodulation

Since CP is not used for MIMO-Linearizing-PEP-M scheme, data demodulation cannot be
performed conventionally since the interference of the last $L$ data subcarriers on the first $L$ data subcarriers is missing and instead, there is the interference of the training sequence on the first $L$ data subcarriers. At the first step, the estimated complex amplitudes and Doppler shifts should be employed to reduce the block interference (BI) of the training sequences from the OFDM data. The interference from the training sequence affects the received data samples in $r_1 \leq n < r_2$ span by all the channel taps except the first one while in the $r_2 \leq n < r_3$ span affects the data by all the channel taps except the first and second ones and so on so forth. As a result, BIs can be expressed by the following equation:

$$B_{lj}(n) = \sum_{l=1}^{N_r} p_{G-r_n+n+1} \hat{a}_{r_i} e^{j2\pi f_{D}\gamma (r_{n}+G)T_S} + \sum_{l=1}^{N_r} p_{G-r_n+n+1} \hat{a}_{r_{n+1}} e^{j2\pi f_{D}\gamma (r_{n}+G)T_S} + \cdots$$

(131)

where $B_{lj}(n)$ for $r_{n-1} \leq n < r_1$ expresses the BI at the jth receiver and the nth received sample, where the samples are located between the $r_{n-1}$ sample and the $r_1$ sample.

For data demodulation, the effect of the CP can be generated at the receiver and added to the beginning of the received OFDM data for regular OFDM demodulation. Since there is a zero block after each OFDM data, what is received at the duration of the zero block can be considered as the summation of the interferences of the received CPs from the transmitter antennas. In the case of non-varying channel, the receive signal at the zero block can be added directly to the beginning of data block. However, in doubly selective channels, the variation of the channel should be compensated. Since the channel has been previously estimated, it can be applied to drag the received CP of each antenna individually.

In general, the last $L$ out of $N$ symbols of the OFDM block should be estimated and their interference should be constructed by considering the time effect and added to the beginning of the data block. An autoregressive procedure can be applied to calculate the last $N - r_S$ symbols at the first regression, then the symbols between $N - r_S-1$ to $N - r_S$, and so on. The
computational complexity of this approach is very high; however, since the first two non-zero channel taps are considerably stronger than the other taps in the vehicular channels, the interferences that are caused by the channel taps by \( r_\tau \) for \( 3 \leq \tau \) can be neglected. The interferences of the second channel taps in the zero block is obtained as:

\[
CPI_j(n) = \sum_{i=1}^{N_T} y_{ri}(N - r_2 + n + 1)\hat{a}_{rij} e^{j2\pi f_{D12}(n+N)T_s} + O.t \quad n = 1, 2, \ldots, r_2
\]  

(132)

where \( CPI_j(n) \) is the \( n^{th} \) received sample at the zero block, \( y_{ri}(n) \) is the \( n^{th} \) received sample of the \( i^{th} \) transmitter, and \( O.t \) defines the effect of the other channel taps and additive noise. Equation (132) can be written in matrix form for all the receiver antennas as:

\[
CPI = \varepsilon y_r + O.t \quad n = 1, 2, \ldots, r_2
\]  

(133)

where \( CPI = [CPI_1(1), CPI_2(1), \ldots, CPI_{N_R}(1), \ldots, CPI_1(r_2), CPI_2(r_2), \ldots, CPI_{N_R}(r_2)]^T \), \( y_r = [y_{r_1}(N - r_2 + 1), y_{r_2}(N - r_2 + 1), \ldots, y_{r_{N_R}}(N - r_2 + 1), \ldots, y_{r_1}(N), y_{r_2}(N - r_2 + 1), \ldots, y_{r_{N_R}}(N - r_2 + 1)]^T \), and \( \varepsilon \) is a \( N_R r_2 \times N_T r_2 \) matrix that its elements are obtained according to (132).

By solving (133), \( y_r \) is obtained and its elements are utilized to make the interference that should be added to the first \( r_2 \) received data samples of the receivers. The variation of the channel by time should be compensated as:

\[
CPI_{new} = \varepsilon y_{r_{new}},
\]  

(134)

where \( y_{r_{new}}(n) = y_r(n)e^{j2\pi f_{D12}(n+N)T_s} \). The signal that should be added to the \( n^{th} \) received sample of the \( j^{th} \) receiver is obtained as:

\[
\bar{CPI}_j(n) = \sum_{j=1}^{N_R} CPI_{new_j}(n).
\]  

(135)

After the addition of the CP to the received data, the received data.
5.2. Simulation results

- Methods for static channel

At the first step of the system simulation, the captured data from a traffic image [24], was converted to 8-bit binary data. These binary data were sent to the OFDM transmission system and then transmitted through the sparse and wide band UAV to ground communication channel. By exploiting the estimated channel, the homographic traffic image was reconstructed from the demodulated received OFDM data at the receiver. Afterwards, the Harris edge detection was employed in order to find the edges at the picture.

For data transmission, two MIMO structure were considered, 2 × 2 and 10 × 10. The bandwidth of the transmitted OFDM signal at each channel was considered to be 10 MHz and BPSK was exploited for data transmission. ITU/Vehicular Type B channel model was chosen for the channel model since it presents an appropriate transmission channel model for the drones that are flying at low altitude in a dense urban environment. The delay spread of the channel is 20 μsec; therefore, the digital equivalent channel has \( L = 200 \) taps at 10 MHz bandwidth. Only \( S = 6 \) of those taps have a non-zero amplitude. The number of subcarriers was set to 1024, and in order to prevent inter symbol interference (ISI), the length of the cyclic prefix was set to be 256. The number of pilots was set to 31 and 127 for 2 × 2 and 10 × 10 conditions, respectively. The number of pilots were set to the minimum length that lead to an acceptable CE accuracy.

Monte Carlo simulation was applied to measure the enhancements of proposed pilot selection and CS method. Fixing the pilot selection method to the proposed one, JPAO, the performance of the conventional BOMP method was compared with the performance of BOMP-BCoSaMP, BOMP-BOMP and LMMSE-BOMP. In addition, we evaluated the improvement of the CE by running the CS method for three time, BOMP-BCoSaMP BCoSaMP. By considering the communication parameters and the computational complexity of the methods, the number
of steps for performing each CE method is evaluated and the results are illustrated in Table 1 by normalizing all the values to the BOMP method in 2×2 condition.

Table 13. Required number of steps for CE methods

<table>
<thead>
<tr>
<th>Method</th>
<th>MIMO 2 × 2 (P = 31)</th>
<th>MIMO 10 × 10 (p = 127)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOMP</td>
<td>1</td>
<td>20.4839</td>
</tr>
<tr>
<td>BOMP-BOMP</td>
<td>2.0834</td>
<td>41.3858</td>
</tr>
<tr>
<td>BOMP-BCoSaMP</td>
<td>1.2501</td>
<td>24.3148</td>
</tr>
<tr>
<td>LMMSE-BOMP</td>
<td>1.2501</td>
<td>24.3148</td>
</tr>
<tr>
<td>BOMP-BCoSaMP-BCoSaMP</td>
<td>1.5001</td>
<td>28.1471</td>
</tr>
</tbody>
</table>

It is observed in Table 13 that the required steps of BOMP-BCoSaMP and LMMSE-BOMP is considerably lower than the BOMP-BOMP in both MIMO scenarios.

Figure 23 and Figure 24 demonstrate the plot of BER vs. SNR by exploiting different channel estimation schemes for 2 × 2 and 10 × 10 respectively.

Figure 23. The comparison between BER performances of different methods, MIMO 2 × 2

Figure 24. The comparison between BER performances of different methods, MIMO 10 × 10
The simulation results indicate the considerable enhancement that is achieved by utilizing the modified received pilots for the second round of the CS method. While the performance of BOMP-BOMP method is better than the BOMP-BCoSaMP method, its double complexity in comparison to the latter method makes it impractical for real time traffic surveillance and data transmission in upcoming cellular network. Besides that, rerunning CS algorithm more than two times indicates that the enhancement would be negligible and it doesn’t worth to add more complexity to the system. Furthermore, just by the employment of MMSE instead LS, the performance of LMMSE-BOMP would be better than BOMP-BOMP. On the other hand, the difference between the performance of conventional Bernoulli pilots and our proposed JPAO method for pilot arrangement can be seen in Figure 23 and Figure 24 for the identical CS estimation methods. As it is indicated, JPAO method enhances the CE greatly, specifically in Figure 24 and LMMSE-BOMP method. In addition, by comparing Figure 23 and Figure 24, we observe that although five times more channel capacity is achieved in MIMO 10 × 10 condition in comparison to the MIMO 2 × 2, the BER is almost the same for LMMSE-BOMP method when it utilizes JPAO pilot arrangement. This enhancement is achieved in price of around 10 times more complexity for the former condition. The efficiency per channel just reduced from 0.77 to 0.7.
The criterion for discovering a car is to consider the detected corners in a specific neighborhood as one car. Figure 25 depicts how the implementation of the homography increases the emphasis on further cars in the picture. As a result, the same criterion can be applied for all the discovered corners in the picture, in order to cluster them as one car. Figure 25 (b) indicates that how the quality of the image is degraded at SNR=10 dB when the BOMP method is applied for CE and no denoising method is applied at the receiver for the reconstructed image. The effect of the denoising process is indicated in Figure 25 (c) and Figure 25 (d), show the reconstructed picture by applying LMMSE-BOMP method for CE and denoising phase for reconstructing the image.

The result of the car detection by exploiting BOMP and LMMSE-BOMP methods are indicated in Table 14. In addition, the effect of our proposed pilot arrangement is presented by comparing it with the conventional Bernoulli method while the LMMSE-BOMP is applied. The result of Table 14 are indicated for SNR=10 dB and the 10 × 10 scenario which is the concentration of this paper. The comparison between the different methods is done by
considering the probability that an existing car is not detected—probability of miss \(P_{miss}\)—and the probability that a non-existing car is distinguished—probability of false alarm \(P_{FA}\). As a result, lower \(P_{miss}\) and \(P_{FA}\) are more preferable. While missing a car happens when its edges are not detected or its edges are considered to belong to another car, the false alarm happens in the condition that an intensity variation—because of the impulsive noise—in a specific part of the picture simulates an edge.

**Table 14.** \(P_{miss}\) and \(P_{FA}\) for different channel estimation methods in \(10 \times 10\) condition

<table>
<thead>
<tr>
<th>Method</th>
<th>SNR (dB)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(P_{miss})</td>
<td>(P_{FA})</td>
<td>(P_{miss})</td>
<td>(P_{FA})</td>
</tr>
<tr>
<td>BOMP Bernoulli</td>
<td>9.8 %</td>
<td>12.7 %</td>
<td>6.8 %</td>
<td>10.1 %</td>
<td>4.2 %</td>
</tr>
<tr>
<td>LMMSE-BOMP Bernoulli</td>
<td>9.6 %</td>
<td>12.2 %</td>
<td>6.3 %</td>
<td>9.2 %</td>
<td>3.3 %</td>
</tr>
<tr>
<td>LMMSE-BOMP JPAO</td>
<td>7.2 %</td>
<td>10.4 %</td>
<td>4.6 %</td>
<td>8.4 %</td>
<td>2.1 %</td>
</tr>
</tbody>
</table>

The results of Table 14 indicate how both the proposed pilot arrangement and sparse CE methods enhances the car detection procedure.

- Methods for doubly selective channel

To compare the performance of our CE methods, we utilize ITU vehicular type B channel model. The number of subcarriers is set to 4096 and the length of the cyclic prefix is chosen as 256 to prevent ISI. The number of transmitter and receiver antennas is 12. The center frequency and maximum speed are set to 2.35 GHz and 300 km/h respectively which results in 652.75 Hz Doppler shift. \(D=3\) and \(N_p=192\) which results in 960 overhead for MIMO-BEM and MIMO-BEM-PEP schemes and for the MIMO-CS-Linearizing method, \(Q\) is set to 512. Since \(2 \times 6 < L \leq 3 \times 6 \times 12\), \(Q_{LP} = 5 \times 6 = 360\) in MIMO-CS-Linearizing-PEP method. The coherence time of the channel is obtained as \(T_c = \frac{0.423}{f_{D_{max}}} = 648.0276\) μs. The longest OFDM block belongs to the MIMO-CS-Linearizing method with the duration of \((2 \times 512 + 256 + 4096) \times 100\) ns =537.6 μs. As a result, the assumption of fixed complex amplitudes and
Doppler shifts during the transmission of one OFDM block is valid. \( N_r = f_c T_{sample} = 235 \) where \( f_c \) is the carrier frequency. By the employment of these communication parameters and the calculated computational complexity of the methods in the previous section, the steps of run (SOR) that are needed for CE methods and their spectral efficiency are compared with each other in Table 15. All the values for the SOR are normalized to the MIMO-BEM method and for the sake of brevity, the MIMO prefix is omitted from the name of the methods.

**Table 15. Computational complexity and spectral efficiency of CE methods**

<table>
<thead>
<tr>
<th>Method</th>
<th>SOR</th>
<th>Bandwidth Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEM</td>
<td>1</td>
<td>72.06</td>
</tr>
<tr>
<td>MDDS</td>
<td>0.0811</td>
<td>72.06</td>
</tr>
<tr>
<td>CS-Linearizing</td>
<td>0.0265</td>
<td>76.19</td>
</tr>
<tr>
<td>BEM-PEP</td>
<td>2.5913e+03</td>
<td>72.06</td>
</tr>
<tr>
<td>MDDS-PEP</td>
<td>0.0108</td>
<td>72.06</td>
</tr>
<tr>
<td>CS-Linearizing-PEP</td>
<td>8.4527e-04</td>
<td>86.93</td>
</tr>
<tr>
<td>CS-Linearizing-PEP-M</td>
<td>1.1369e-04</td>
<td>88.89</td>
</tr>
</tbody>
</table>

As it is indicated in Table 15, the SOR of the MIMO-CS-Linearizing method is lower than the MIMO-BEM method. On the other hand, when the employment of PEP decreases the SOR of the MIMO-CS-Linearizing method and it increases the SOR of the BEM scheme. In addition, the SOR of the MIMO-CS-Linearizing-PEP is approximately 8 times more than the MIMO-CS-Linearizing-PEP-M.

Monte Carlo procedure is employed to calculate the performance of the CE and data demodulation schemes. QPSK data are generated for data transmission. The results for NMSE estimation are presented in Figure 26. For the sake of brevity, the MIMO prefix is omitted from the name of the methods in all the following figures.

**Figure 26. NMSE of channel estimation vs. SNR for ITU vehicular type B channel model**
As it is presented in Figure 26, the Linearizing method measures the channel more accurately than the BEM method. Besides that, the employment of PEP enhances the performance of the CE procedure considerably. The simulation results also depicts that the performance of the MIMO-CS-Linearizing-PEP is even better than the Linearizing-PEP since it performs averaging among the received samples which reduces the noise effect.

Figure 27 indicates the NMSE for CE methods in a new channel model. In order to measure the performance of the CE methods in general case, a channel with the same average power and consecutive power order of vehicular type B channel is utilized; however, the delays of the second to the six taps are randomly generated between 2 to 200.

Figure 27. NMSE of channel estimation vs. SNR for random channel

The simulation results in Figure 27 indicate that the performance of the MIMO-CS-Linearizing-PEP method is better than the MIMO-BEM and MIMO-CS-Linearizing methods and almost the same as the MIMO-BEM-PEP even when the places of non-zero channel taps
are chosen randomly which results in the conditions that only two taps of the channels could be estimated.

In order to evaluate the effect of CE on the data demodulation fidelity, BER vs. SNR curves for different CE methods are presented in Figure 28.

![Figure 28. BER vs. SNR for ITU vehicular type B channel model](image)

As it is presented in Figure 28, the BER curves follow the same pattern as the NMSE curves except for the MIMO-CS-Linearizing-PEP at low SNR, which indicates worse performance than MIMO-CS-Linearizing-PEP which is because of the error propagation of CE on BI cancellation and CP construction. On the other hand, when the SNR increases, the performance degradation would be more because of the Doppler Effect compared to the noise effect and therefore, the performance of the MIMO-CS-Linearizing-PEP and MIMO-CS-Linearizing-PEP-M would become close to each other.
Chapter 6. Conclusion and future works

In this dissertation, we proposed several CE approaches for DS channels and OFDM communication systems in order to cancel the ICI more accurately and increase the fidelity of the demodulated received data. We presented our CE schemes in three categories: CE methods for short and dense channels, CE methods for long and sparse channels, and CE methods for MIMO communication systems. Our proposed methods utilize either scattered frequency domain pilots or time domain training sequences. The performance modification of our proposed CE methods that exploit scattered frequency domain pilots is because of new pilot placement algorithms. On the other hand, the performance modification of our proposed CE methods that employ time domain training sequences is because of the specific proposed training sequence structures, some approximations that alleviated the CE procedures, and exploiting the fundamental concepts of the DS channels. The mathematical derivations and simulation results indicate that our proposed CE schemes are superior to the conventional CE schemes regarding higher accuracy, higher bandwidth efficiency, and lower computational complexity.

For the future studies, the other application of MIMO communication systems, besides increasing the channel capacity that was considered in this dissertation, which is beamforming can be deliberated. Beamforming or spatial filtering is a signal processing technique used in sensor arrays for directional signal transmission or reception. This is achieved by combining elements in an antenna array in such a way that signals at particular angles experience constructive interference while others experience destructive interference. Beamforming can be used at both the transmitting and receiving ends in order to achieve spatial selectivity. The improvement compared with omnidirectional reception/transmission is known as the directivity of the array. The first phase of any beamforming method is to estimate the angle of arrival (AoA) of the signals and for AoA estimation, the number of multipath should be
estimated. As a result, beamforming is a hierarchical procedure and it is challenging in DS channels. Since beamforming in DS channels has rarely been studied in literatures, it can be considered as the next step of this research.
Appendix

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R. Yao, Y. Liu, G. Li and J. Xu, "Channel estimation for orthogonal frequency division multiplexing uplinks in time-varying channels," in *IET Communications*, vol. 9, no. 2, pp. 156-166, 1 22 2015.


[57] Mimo doubly


Curriculum Vitae

Email: vahid.vahidi.2014@gmail.com

Education

University of Nevada, Las Vegas (UNLV)
Doctor of Philosophy in Electrical and Computer Engineering
Dissertation: “Channel estimation and ICI cancelation in doubly selective channels of OFDM wireless communication systems”
Expected graduation: May 2018
GPA: 4.0

Shiraz University
Master of Science in Electrical and Computer Engineering
Aug 2008-Aug 2011
GPA: 3.28

Shiraz University
Bachelor in Electrical and Computer Engineering
Project: “Design and manufacturing of smart aquarium by using AVR microcontroller”
GPA: 3.02

Research Expertise

Topics: OFDM; channel estimation; Doppler Effect; signal processing; compressed sensing; detection and estimation; UWB technology; localization; antenna and microwave circuit design
Applications: 5G communication systems; unmanned aircraft systems (UAS); high speed train (HST); traffic surveillance; remote sensing

Publications

[J]: Journal [C]: Conference
Accepted or published:

3. [C] V. Vahidi and E. Saberinia, "A low complexity and bandwidth efficient

Submitted:

Submitted proposals

**Academic work experience**

**UNLV**
Lecturer Jan 2018 Present
Graduate Assistance Aug 2014-Dec 2017

- Teaching Communications Lab (460L), General Engineering Lab (EGG 100L) and Circuit Discussion (EE 220D)
- Teacher assistance for Communications (EE 460), Signals and Systems (EE 380), Circuit II (EE 222), Digital Signal Processing (EE 480), Wireless Communications (EE 466).

**ITI University**
Lecturer for: "Signal and System analysis", "Analog communication", "Digital communication", "Electrical circuits", "Electronic" and "Digital communication Lab".

**Pasargad University**

**Pishtazan University**
Lecturer for: "Signal and System analysis", "Digital communication”.

**Industrial work experience**

**Derag System**
Research manager Sep 2009-July 2014

As a leader, I guided a 5 member team (2 Master and 3 Bachelor) for:

- Designing algorithms in the various areas of wireless communication:
o wireless sensor network localization methods
o beam forming
o radar systems
• Designing and making hardware for wireless communication:
o Rotman lens at 6-18 GHz
o Rotman lens at 36-40 GHz
o Array of Vivaldi antennas
o New wideband and wide beam width horn antenna
o Wide band micro strip antenna for airplane wings
o Efficient wideband power divider

ITMC Sep 2006-Sep 2009
Technician
• Working in the backup and manufacturing of the communications Racks
• Writing weekly report on available wireless communication standards and networks

Honors and awards
Summer Doctoral Fellowship (summer 2017)
IEEE transaction reviewer (since Fall 2016)
First rank student in all of my classes in UNLV
Receiving Graduate Assistantship from UNLV (2014)
Waiver of the M.Sc. tuition fee because of high rank in the entrance exam (2008).
Waiver of the B.Sc. tuition fee because of high rank in the entrance exam (2003).

Computer Skills
• Languages: Matlab; C++
• Electronic Tools: OPNET (Network Simulator); ADS; microwave office; HFFS; CST;
  Orcad-Pspice.