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Exploring Teachers' Beliefs and Practices on Mathematical Discourse for Diverse Students in Inclusive Classrooms

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EXPLORING TEACHERS' BELIEFS AND PRACTICES ON MATHEMATICAL DISCOURSE FOR DIVERSE STUDENTS IN INCLUSIVE CLASSROOMS

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ABSTRACT

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The recent implementation of rigorous standards in mathematics education has required shifts in classroom practices. Standards-based instruction places large emphasis on students' conceptual understanding, requiring them to demonstrate high cognitive levels of mastery of the content through communication of their mathematical reasoning. Teachers and students' mathematical discursive practices in the classroom can lead to meaningful discussions that integrate students' explanation, justification, and arguments of ideas or claims and understanding of the content. Research on teachers' discursive practices has shown that (a) teacher talk tends to dominate classroom instruction and (b) classroom discourse lacks frequent opportunities for teacher-student and peer interactions. The purpose of this exploratory sequential mixed methods study was to increase understanding regarding teachers' beliefs and practices related to the planning and implementation of mathematical discourse in inclusive general education elementary mathematics settings. Specifically, this research study centered on the development of a valid and reliable instrument on teachers' beliefs and practices related to mathematical discourse that could be used by teachers and researchers interested in the implementation of equitable mathematical discursive practices in the classroom that promote students' conceptual understanding. The development of the survey occurred over a multiphase process: content development (qualitative data collection and analysis), survey development and pretesting (survey validity measures), and pilot testing (survey reliability measures). Six general and three special education teachers participated in Phase 1 and 2, and 18 teachers (i.e., 13 general and 5 special education teachers) participated in Phase 3. Data sources included individual interviews, a focus group, classroom observations, teachers' lesson plans, and the *Teachers' Beliefs and Practices on Mathematical Discourse Survey*. Qualitative and quantitative findings showed that teachers believe mathematical discourse is intuitively implemented during instruction without much planning, all students should participate in the classroom discourse, and mathematical discourse should be explicitly taught and modeled to students. Findings on teachers' perceived practices showed that teachers mainly utilize discourse to assess understanding by soliciting students' mathematical reasoning, generally use the curriculum to guide their mathematical discourse practices and implement varied grouping strategies to facilitate discourse. Findings on teachers' observed practices indicated that (a) teacher-led, authoritative discourse dominated discursive practices during mathematics instruction, (b) discursive practices were mostly focused on assessing understanding and addressing misconceptions, (c) participation and engagement generally involved all students in the classroom, and (d) planning for mathematical discourse was solely based on activities explicitly included in the curriculum. Based on these findings a 50-item *Teachers' Beliefs and Practices on Mathematical Discourse Survey* was created, pretested, and pilot tested for validity and reliability purposes. The alpha coefficient for each of the two survey constructs suggested that overall measures of validity and reliability were sufficient by showing relatively high internal consistency to support the survey use in future research and program and professional development planning.

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"It is good to have an end to journey towards; but it is the journey that matters, in the end."

Ursula K. Le Guin

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CHAPTER ONE

INTRODUCTION

During the 2018-2019 academic year, only 41% of fourth grade students in the United States (U. S.) performed at or above the proficient level on the National Assessment of Educational Progress (NAEP) in mathematics; this percentage was 34% for 8th grade students (U.S. Department of Education, 2020b). Students' educational achievement has been a cause of concern since the late 20th century (Schmidt & Houang, 2012) and has led to significant educational and policy reforms. State-level and international comparisons of mathematics and science outcomes for students on several international assessments of achievement (e.g., NAEP, Trends in International Mathematics and Science Study [TIMSS], Program for International Students Assessment [PISA]) have influenced policy regarding science, technology engineering, and mathematics (STEM) preparation in the U.S. since the 1990s (Dingman et al., 2013; Suter & Camilli, 2019). Recently, students' academic achievement concerns have been exacerbated by fears of economic competition related to globalization (Schmidt & Houang, 2012; Suter & Camilli, 2019).

To improve students' academic performance, the National Research Council conducted a review of STEM competitiveness (National Academies of Sciences, Engineering, & Medicine, 2007) and concluded that there was an urgent need to improve students' educational achievement in the U.S. (Suter & Camilli, 2019). The Council suggested that one way to improve the achievement of students was through enhanced training and education of mathematics teachers to strengthen their skills. In addition, a report by the Organization for Economic Cooperation and Development recommended the adoption of a common core of internationally benchmarked standards in mathematics and language arts to ensure students' competitiveness in a global economy (Suter & Camilli, 2019). Consequently, the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA, CCSSO, & Achieve, 2008) began an initiative focused on the improvement of education based on these recommendations. With the implementation of rigorous academic standards, education officials sought to place U.S. students in a comparable position to students in other developed countries (Schmidt & Houang, 2012).

High Rigorous Standards to Promote Mathematical Proficiency

At the beginning of the 21st century, the standards-based reform (SBR) initiative was implemented in the United States to not only increase the academic performance of all public-school students (Bacon, 2015) but also to guide teachers in implementing more challenging academic curricula (Burris et al., 2008). Related to mathematics, state variations in the role and purpose of standards led to large differences in expected mathematical learning goals for students across states (Dingman et al., 2013). To reduce inconsistencies, the National Council of Teachers of Mathematics (NCTM) released a suggested set of mathematics focal points (i.e., *Curriculum Focal Points for Pre-Kindergarten Through Grade 8th Mathematics;* Fennell, 2006) that specified major mathematical topics at each grade level from elementary to middle school (Dingman et al., 2013).

A movement to standardize rigorous content in mathematics and English language arts across states led to the implementation of the Common Core State Standards (CCSS; CCSSI, 2010; Porter et al., 2011). These standards were released in the U.S. in 2010 (Schmidt & Houang, 2012). Now, 41 states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) are integrating these standards into classroom instruction, planning, and assessments (Common Core State Standards Initiative [CCSSI], 2010). The CCSS are a progression of learning expectations in mathematics and English language arts, specifically designed to prepare K-12 students for postsecondary education (Neuman & Roskos, 2013). In mathematics, these standards are based on teaching and learning of mathematics research and curricular frameworks of high-performing states and countries (Dingman, 2013). Although CCSS set grade-specific goals, they do not specify curriculum, teaching practices, nor materials needed by teachers to support students' learning (Khaliqi, 2016; Porter et al., 2011).

Implementation of rigorous standards, such as the CCSS, has required shifts in classroom practices. In mathematics, the CCSS place large emphasis on students' conceptual understanding (Jitendra, 2013), requiring them to demonstrate high cognitive levels of content mastery through communication of their mathematical thinking, reasoning, and understanding (Porter et al., 2011). Moreover, standards-based mathematical practices accentuate social interactions between teachers and students, as well as among classroom peers to provide students with an avenue to construct and build on their conceptual understanding (Nathan & Knut, 2003) and to communicate about and through mathematics (Walshaw & Anthony, 2008).

Mathematical communication, such as mathematical discourse, has been found to engage students in developing a deeper understanding of mathematics (Kosko & Gao, 2017). To reach a deeper understanding of concepts through communication of mathematical thinking and reasoning, students must move beyond explanations of procedures and link their thinking and reasoning to underlying mathematical concepts (Stein, 2007). Rittle-Johnson and Siegler (1998) defined conceptual understanding as an "understanding of principles that govern the domain and of the interrelations between pieces of knowledge in a domain" (p.77). As mathematical content becomes more complex, communication and reasoning skills become more essential for higher-order mathematical thinking (Yilmaz & Topal, 2014). With this in mind, standards for mathematical practice specify that teachers at all grade levels should (a) seek to develop their students' ability to construct viable arguments and critique the reasoning of their peers (Conley, 2011) by providing rationales, justifications, and conjectures (Kosko & Gao, 2017) and (b) show their students not only the value of mathematical accuracy, but also the value of understanding mathematical concepts (Stein, 2007). Therefore, to promote students' conceptual understanding, shifts in classroom practices related to the time students spend interacting and talking with one another around content are needed (Hakuta et al., 2013). In fact, in alignment with CCSS, classroom environments must include content and meaningful, language-rich activities for all students (Hakuta et al., 2013).

Oral language, which includes vocabulary, syntax, pragmatics, and discourse processes (Gottlieb, 2016), has been linked to mathematical thinking and reasoning (Barwell, 2016; Sarama et al., 2012) and conceptual understanding (Schleppenbach et al., 2007). Through communication with others, students explore, offer conjectures, find patterns, and construct conceptual understanding of mathematics (Wilkinson, 2018). Therefore, to show conceptual understanding, students must communicate their thinking and reasoning through multiple representations, engage in collaborative group work with peers, and explain and demonstrate knowledge using language (Jitendra, 2013; Moschkovich, 2012a). To achieve high levels of mathematical thinking, students in the classroom are required to present, explain, and justify mathematical claims in different discourse forms (e.g., teacher-led, small group, and pairs; Hakuta et al., 2013; Schleppenbach et al., 2007). Researchers have found mathematical discourse as essential to a sustained change in students' conceptual understanding because it helps students to become aware of more conceptually advanced forms of mathematical ideas or claims (Walshaw & Anthony, 2008). Mathematical discourse plays a critical role in the classroom because it gives students the opportunity to (a) explore and communicate their own mathematical reasoning and understanding and (b) consider other students' mathematical ideas and claims (Nathan & Knuth, 2003). Thus, according to CCSS, instruction is expected to support mathematical discussions (Hakuta et al., 2013), and teachers should focus on simultaneously developing their students' mathematical understanding and the specialized language of mathematics (Wilkinson, 2018).

The General Education Mathematics Classroom

In addition to federal laws (e.g., Individuals with Disabilities Education Act [IDEA], No Child Left Behind [NCLB], Every Student Succeeds Act [ESSA]), professional organizations, such as the NCTM, require and support equal educational opportunities for all students regardless of their personal characteristics, backgrounds, or physical challenges (Hudson et al., 2006). Access and equal education opportunities mainly take place in general education classrooms (García et al., 2008; Hudson et al., 2006). In general education classrooms, students should be provided with meaningful opportunities to access the general education curriculum (Cosier et al., 2013) and are expected to adhere to high academic standards that enhance academic achievement (Daniel & King, 1997). Currently, general education mathematics classrooms accommodate students with very diverse academic and linguistic strengths and needs; therefore, it is important for teachers in general education classroom settings to be prepared to capitalize on their students' strengths as well as support their needs by (a) incorporating in their daily instruction a variety of strategies, techniques, and methods, such as mathematical discourse, proven to be effective to assist their students' learning (Prast et al., 2018), and (b) creating opportunities for students to talk mathematically (Griffin et al., 2013).

Specifically, students diagnosed with a learning disability (LD) in mathematics, students who demonstrate below grade level performance (Powell et al., 2013), and students who are emergent bilinguals (EBs) with or without a disability (also called English Learners; García et al., 2008) receive mathematics instruction in general education classrooms and require instructional and/or linguistic supports to be able to reach their full academic potential.

Although many students with LD receive additional mathematics instruction in the resource classroom, students with LD predominantly learn mathematics in general education classrooms and work individually and in small groups supported and encouraged by general and special education teachers (Truelove et al., 2007). Research has shown that students with LD in general education settings have more opportunities to justify their mathematical answers and develop their communication skills needed to effectively work with a standards-based curriculum than students with LD in special education settings (Griffin et al., 2013).

In addition to students with LD, most EBs (i.e., students who demonstrate emergent abilities in speaking, reading, writing, or understanding of English; U.S. Department of Education, 2017) are also educated in general education classrooms (De Jong et al., 2013; García et al., 2008; Klingner et al., 2014). Although there have been different educational programs within the U.S. public education system to support the academic learning of EBs (e.g., submersion, structured immersion, bilingual education; García et al., 2008), after the standards-based accountability reform many schools decided to eliminate their bilingual education programs with the belief that the general education English-medium classroom was the best place to ensure emergent bilinguals' yearly academic progress (Baker et al., 2016). Therefore, due to the heterogeneity of the student population in general education classrooms, it is important for teachers to (a) consider different instructional approaches, practices, and methods (Hudson et al., 2006), and (b) focus on shaping the development of novice mathematicians who speak the precise and generalizable language of mathematics (Walshaw & Anthony 2008).

Students with Learning Disabilities

In the United States, 33% of all students with disabilities were classified as having LD during the 2018-2019 academic year (U.S. Department of Education, 2020a); approximately 4.6% of the entire public student population in the U.S is identified as having an LD (U.S. Department of Education, 2020a). Students with LD represent the largest category of students receiving special education services. Findings from the National Longitudinal Transition Survey indicated that 74% of students with LD performed below average on math calculation, and 85% of students with LD performed below average on math applied problems (Cortiella & Horowitz, 2014).

Students' verbalizations of their mathematical thinking (i.e., communication of their mathematical thinking and reasoning) have proven effective for improving the mathematical performance of students with LD (Gersten, et al., 2009; Jayanthi et al., 2008). Although academic language development has not been widely studied in the field of LD (Silliman & Wilkinson, 2015), interventions on student verbalizations have demonstrated that verbalizations help students with LD to anchor skills and strategies both behaviorally and mathematically. Even more, verbalizations may serve to facilitate students' self-regulation during problem solving to avoid impulsiveness (Gersten et al., 2009; Jayanthi et al., 2008).

Research stresses the importance of teaching students with LD to use language to guide their mathematical thinking (Gersten et al., 2009), but many students with LD find the use of the lexical-syntactic components of mathematical language and discourse challenging for two main reasons: (a) mathematics employs a highly technical, precise, and densely structured language, and (b) many mathematical terms have many different meanings in the mathematics register

(Silliman & Wilkinson, 2015; Topping et al., 2003). Instructional supports provided by teachers could facilitate the participation of students with LD in the classroom discourse. Regrettably, students with LD are often not integrated in activities involving mathematical inquiry, problem solving, and discourse because they have not yet mastered basic computation skills (due to both their disability and insufficient instructional supports and opportunities during classroom instruction; Borgioli, 2008; Griffin et al., 2013), and teachers tend to lack knowledge and skills related to differentiating instruction (Prast et al., 2018)

Emergent Bilinguals

Emergent bilinguals (EBs), students who come to school with developed oral and/or literacy practices that enable them to communicate with their families and communities (officially classified as English learners [ELs]; Kleyn & García, 2019) comprise approximately 1 in 10 (10.1%) students in the U.S. (U.S. Department of Education, 2020a). EBs often underperform in mathematics compared to their English-speaking peers; 47% of EBs in fourth grade and 72% in eighth grade demonstrate below basic mathematical achievement (U.S. Department of Education, 2019a). Therefore, EBs might need additional instructional (e.g., extra time, modeling, scaffolding, differentiated instruction) and linguistic supports (e.g., sentence frames, realia, use of native language) to engage in a highly demanding academic curriculum and to meet the expectations of content standards that require the use of language and literacy in English (de Araujo et al., 2018; Barrow & Markman-Pithers, 2016; Bunch, 2013; Kibler et al., 2014).

Supporting EBs' participation in mathematical discourse is essential for developing their mathematical understanding. Teachers play a vital role in shaping classroom structures and interactions needed for EBs to gain access and learn from mathematical discourse (deAraujo et al., 2018). Teachers' investment in EBs' cultural and linguistic backgrounds and experiences increases the opportunities for EBs to participate in mathematical discourse (deAraujo et al., 2018). Unfortunately, research has shown that some teachers position EBs as less competent than their English-speaking peers, thereby often excluding EBs from participating in classroom discourse during mathematics instruction (deAraujo et al., 2018; Walshaw & Anthony, 2008).

Emergent Bilinguals with Learning Disabilities

Additionally, some EBs might face academic challenges for reasons that go farther than their sociocultural background, second language development, and/or educational history (Klingner et al., 2014). The systematic, accurate, and valid identification process of LD for EBs continues to be underdeveloped (García & Tyler, 2010). Researchers suggest that EBs identified as having LD demonstrate many of the same academic difficulties as their English-speaking peers with LD and that EBs will experience these academic difficulties in both languages (García & Tyler, 2010). Interventions that have only been shown to be effective for monolingual speakers are inadequate for EBs with and without disabilities (Klingner et al., 2014). Importantly, effective teaching strategies that support cognitive and linguistic skills, such as providing opportunities for oral language development and discourse, are not only beneficial for EBs with and without LD but for all students in the classroom (García & Tyler, 2010; Klingner et al., 2014).

Experts in the field of special education define students with mathematics difficulties (MD) as those who (a) receive special education services in mathematics, or (b) struggle with mathematics, but have not been identified as having a learning disability (Fuchs et al., 2004; Gersten et al., 2005). Hence, students with MD are students identified as having a LD in mathematics or students considered at-risk because they have not reached basic proficiency levels in mathematics, such as many students who are EBs (NCES, 1992). Students with MD tend to perform in the low and well below average proficiency range (Gersten et al., 2005) and continue to face academic challenges in mathematics throughout their formal schooling and postsecondary education (Powell et al., 2013).

Students with MD often require additional supports during mathematics instruction, especially because they face challenges that make accessing grade-level, highly rigorous mathematics standards more difficult (Doabler et al., 2012; Powell et al., 2013). Reasonable linguistic, and instructional supports may be needed for students with MD to be able to participate in and benefit from small group and whole classroom mathematics discussions in general education classrooms, such as different types of scaffolds (e.g., conceptual, procedural, strategic; Jitendra, 2013). Teacher instruction has a major impact on student learning (Jitendra, 2013); therefore, teachers should incorporate effective teaching methods, strategies, and interventions (e.g., verbalization strategies; Gersten et al., 2009) that could greatly improve the mathematical performance of these students. To enhance and improve mathematics instruction for students with MD in general education settings, researchers suggest that teachers should promote engagement and understanding through mathematical discourse (Doabler et al., 2012).

Mathematical Discourse

The standards-based reform movement in the U.S. highlights mathematics instruction that emphasizes classroom discourse, in which students' mathematical thinking, reasoning, and understanding play a central role (Bray, 2011; Stein, 2007). To better understand mathematical discourse, it is important to describe the relation between language and discourse. Each academic content discipline has its own linguistic and discourse repertoires (Silliman & Wilkinson, 2015). Research on language of specific disciplines, such as mathematics, describes language not only as specialized vocabulary but also syntax, grammatical patterning, organization, and register (Moschkovich, 2012b; Schleppegrell, 2007) across different language modalities (i.e., listening, reading, speaking, and writing). In mathematics, many concepts are expressed with symbols and graphic representations to convey meanings in ways that words cannot represent (e.g., symbols may carry deeper meaning that require lengthier explanations; Schleppegrell, 2007). Hence, mathematics register is not only vocabulary, but meanings (e.g., symbols, order, position, orientation), styles, and modes of arguments (e.g., precision, brevity, logical coherence; Moschkovich, 2012b; Schleppegrell, 2007). The language of mathematics is multidimensional and the core component of mathematical discourse.

According to Moschkovich (2012b), the language of mathematics is essential for the successful participation of students in mathematical discourse. Gee (1990) defines discourse as:

> A socially accepted association among ways of using language, other symbolic expressions and artifacts, of thinking, feeling, believing, valuing, and acting, as well as using various tools, technologies or props that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role, or to signal that one is filling a social niche in a distinctively recognizable fashion. (p. 161)

Thus, mathematical discourse includes not only language, but also mathematical values, points of views, beliefs, expressions, and objects (Moschkovich, 2012b).

Mathematical discourse helps students clarify and connect their ideas (Schleppenbach et al., 2007). Moreover, it encourages students to question and challenge each other to explain their mathematical thinking and reasoning. Research on mathematical discourse suggests teachers should encourage all students to not only present problem-solving strategies but also explain and justify these strategies to their peers to successfully achieve conceptual understanding (Schleppenbach et al., 2007).

Moschkovich (2012b) extended the term mathematical discourse to mathematical discourse practices and emphasized that discourse is embedded in sociocultural practices and is also connected to mathematical ideas that promote conceptual understanding. Thus, mathematical discourse practices not only involve language and conceptual knowledge, but also the diverse classroom environment and the sociocultural context of mathematics learning (Hall, 1993; White, 2003). Research suggests that greater involvement of participants during mathematical discourse generates higher levels of expressed mathematical thinking by students (Wood et al., 2006). Students become active participants of mathematical discourse practices when they talk about their mathematics ideas in ways that mathematically competent people do, for instance being precise and explicit and searching for certainty (Moschkovich, 2012b; Sherin, 2002; Walshaw & Anthony, 2008). In other words, students should engage in the uniquely mathematical ways of communicating during instruction (Sfard, 2000).

Teachers' Mathematical Discourse Practices

Although mathematical discourse practices can lead to (a) meaningful discussions that integrate students' explanation, justification, and argumentation of ideas or claims (Piccolo et al., 2008) and (b) understanding of the content, research on teachers' typical discourse practices has shown that teacher talk tends to dominate classroom instruction (Piccolo et al., 2008). In fact, teacher explaining (rather than eliciting student participation) is the most frequent discourse practice used during mathematics instruction and students' participation is inconsistent (Erath et al., 2018; Piccolo et al., 2008). Research suggests that students need multiple opportunities to communicate their own understanding of the content being presented (Piccolo et al., 2008).

Therefore, teachers must strive to build students' conceptual understanding of the content through intentional mathematical discourse based on a probing, guiding, interactive dialogue (Piccolo et al., 2008), as well as strategic questioning (Topping et al., 2003).

Mathematical discourse must be intentional; in other words, teachers must intentionally plan to enhance opportunities for classroom discourse (Krussel et al., 2004). Teachers become more purposeful in the implementation of mathematical discourse during instruction when they have intentionally planned for students' opportunities to engage in classroom discourse practices (e.g., explanation, justification, argumentation; Herbel-Eisenmann et al., 2013). In fact, teachers hold the important role of managing and monitoring (a) content, (b) structure (e.g., whole, and small group), and (c) temporal boundaries (e.g., available tools, time) of classroom discourse (Krussel et al., 2004) to engage students in mathematical argumentations, explanations, and justifications of their ideas or claims.

Unfortunately, teachers' mathematical practices continue to lack frequent opportunities for peer interactions (Griffin et al., 2013). In addition, many teachers find it very challenging to include classroom discourse (e.g., posing questions that elicit, engage, and challenge students' thinking and asking students to clarify and justify their ideas orally and in writing; Stein, 2007; White, 2003) as an integral component of their instruction (Walshaw & Anthony, 2008) and struggle to include all students in classroom discussions (White, 2003). Students with learning disabilities or those at risk for mathematics difficulties tend to remain passive and reluctant to participate during mathematics discussions (Baxter et al., 2002). Furthermore, teachers' feedback during discourse frequently focuses on encouragement and/or praise and does not emphasize students' cognitive development (Walshaw & Anthony, 2008). To promote mathematical competence, teachers' feedback should allow important mathematical ideas to surface and enhance connections between language and conceptual understanding (Michaels et al., 2016).

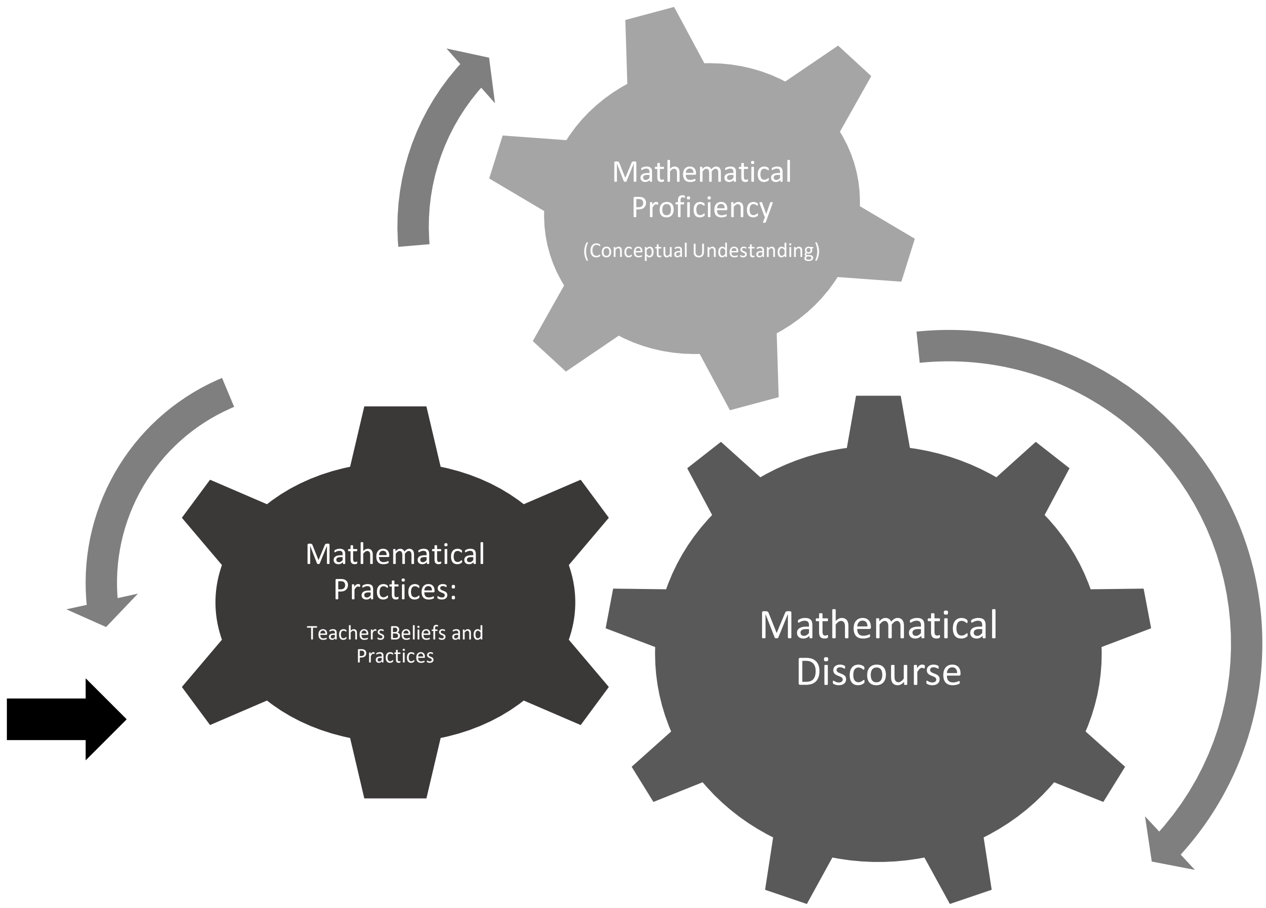
Although the language and learning of mathematics cannot be separated (Schleppegrell, 2007), many teacher, school, and district leaders continue to view mathematics as detached from language (de Araujo et al., 2018). All things considered, purposeful and intentional inquiry-based questioning promoted, modeled, and facilitated by teachers through mathematical discussions generates deeper mathematical thinking and understanding, and positive outcomes (Topping et al., 2003) for all students. Therefore, mathematical discussions become an essential component of classroom instruction in general education classrooms where teachers need to provide instructional supports and relevant learning opportunities to a linguistically and academically diverse group of students.

Conceptual Framework

The academic literacy in mathematics framework (Moschkovich, 2015) is based on a sociocultural perspective of language (Donato & MacCormick, 1994). Academic literacy in mathematics expands the concept of academic mathematical language to a complex view of mathematical proficiency as participation in rigorous practices that involve both conceptual understanding and mathematical discourse. In other words, the academic literacy in mathematics framework includes three interrelated components: (a) mathematical proficiency, (b) mathematical discourse, and (c) mathematical practices. These three components cannot be separated when considering, analyzing, and designing mathematical tasks, activities, or instruction for students at-risk for MD. The literacy in mathematics framework puts an emphasis on mathematical discourse, rather than solely mathematical language. Mathematical discourse is embedded in mathematical practices and understanding is developed through discourse participation.

Figure 1

Conceptual Framework



Note. Based on The Academic Literacy in Mathematics Framework (Moschkovich, 2015)

To understand how teachers' mathematical practices are developed, it is important to consider the relationship between teachers' beliefs and practices. Research has found opposing perspectives related to both the development of and subsequent changes to teachers' beliefs and practices (Handal, 2003). One perspective suggests that teachers' beliefs influence teachers' pedagogical decisions and classroom practices (Cross, 2009; Nathan & Knuth, 2003). According to Cross (2009), beliefs are the expression of conscious and unconscious ideas deemed to be true, as well as thoughts about the person, the world, and the person's position in the world that are developed through the participation in different social groups. Because beliefs are very personal and often reside at a level beyond the person's immediate control or knowledge, they play an influential role in the person's decision-making processes. Thus, beliefs are strong predictors of behaviors, and they tend to be highly resistant to change (Cross, 2009). In contrast, a second perspective suggests that teachers' practices influence their beliefs, thus, a change in beliefs requires engagement in new practices (Nathan & Knuth, 2003). Overall, the common denominator of both perspectives is that both teachers' practices and beliefs play an essential role during planning and instruction of the lesson to support conceptual understanding through mathematical discourse for all students in the general education classroom.

As mentioned before, general education mathematics classrooms accommodate students with very diverse academic and linguistic strengths and needs. Therefore, equitable access to educational opportunities in the classroom is dependent on how teachers manage intersectional factors (e.g., race, language, ability, socioeconomic status, background) underlaying educational disparities in general education classrooms (Carey et al., 2018). Intersectionality could be a powerful lens to understand equitable and inclusive mathematical practices related to mathematical discourse. Thus, findings of this research study will be explained and discussed through an equity and intersectionality lens. Equity and intersectionality theoretical perspectives and examples of mathematics research that have employed these perspectives to explain their findings will be described in more detail in Chapter 2.

Statement of the Problem

Although students' participation in mathematical discourse is essential for the development of their conceptual understanding (Erath et al., 2018; Moschkovich, 2015), mathematics teachers continue to face challenges in incorporating mathematical discourse during classroom instruction particularly when planning for diverse classrooms that include students with LD, emergent bilinguals, and other students at-risk for mathematics difficulties (Doabler et al., 2012; Griffin et al., 2013; Nathan & Knuth, 2003). Research on discursive practices has shown that some students face more obstacles to participating in classroom discourse than their peers (de Araujo et al., 2018; Erath. et al., 2018). In addition, most teaching approaches of oral language are focused on the word and sentence level of academic language instead of the discourse level (Erath. et al., 2018).

To promote mathematical proficiency, classroom practices must include activities that support conceptual understanding through mathematical discourse; however, the implementation of teaching practices that support mathematical discourse has been very challenging for many teachers, and many current instructional practices have not yet included discourse-based mathematical activities (Nathan & Knuth, 2003). Even if teachers understand the importance of mathematical discourse during instruction, teachers' beliefs about implementing mathematical discourse are not always congruent with their actions in the classroom (Nathan & Knuth, 2003; Nisbet & Warren, 2000). More research is needed on (a) how teachers' beliefs about intentional planning and implementation of mathematical discourse during classroom instruction influence their classroom instructional practices, and (b) how teachers' beliefs regarding the implementation of equitable mathematical discourse opportunities for all students in the classroom, including those with specific academic and linguistic needs, influence their mathematical practices. Thus, this research study centers on the development of a valid and reliable instrument to measure teachers' beliefs and practices about mathematical discourse that could be used by teachers and researchers interested in the implementation of equitable mathematical discursive practices in the classroom that promote students' conceptual understanding.

Purpose of the Study and Research Questions

The purpose of this study is to increase understanding not only of the beliefs teachers have about mathematical discourse but also how these perceived beliefs are related to their mathematical practices. In other words, what are teachers' beliefs and perceived practices regarding the intentional planning and implementation of mathematical discourse during classroom instruction to support students' conceptual understanding and if these beliefs are also observed in their mathematical practices. An exploratory sequential mixed methods design (Creswell & Creswell, 2018) was selected to develop a stronger understanding of teachers' beliefs and their corresponding practices reflected in their lesson planning and classroom instruction. First, qualitative data was collected and analyzed, then a survey was developed based on the results of the initial data set (Creswell & Creswell, 2018), and at the end, measures of reliability and validity were conducted to ensure the integrity and quality of the developed survey. Three main research questions were addressed:

1. How do teachers in general education settings describe perceived beliefs and practices related to mathematical discourse?
2. How do teachers implement mathematical discourse in general education settings as reflected in teachers' lesson plans and classroom observations?
3. Are the validity and reliability estimates of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* sufficient to support its use in research and program planning?

Significance of the Research

An increased diversity of learners in mathematics classrooms requires teachers to have a wider understanding of effective classroom discourse that supports learning and promotes positive outcomes for all students (Walshaw & Anthony, 2008). Discourse is an essential component of mathematics instruction to develop understanding, especially for EBs (deAraujo et al., 2018) and students with LD (Topping et al., 2003). Although a substantial amount of variability in student achievement gains is related to teachers, research on the effectiveness of specific instructional practices on students' learning has been more extensive than research on how teachers understand, design, and deliver instruction (Griffin et al., 2013). In addition, there has been little research related to mathematical discourse (Walshaw & Anthony, 2008) with students at-risk for MD (Silliman & Wilkinson, 2015). Therefore, more research is needed regarding how teachers' beliefs and practices can increase understanding on how they create opportunities for all students to express their mathematical thinking and reasoning through classroom discourse, which is essential to support students' conceptual understanding and equitable mathematics classrooms. There is a real need to understand the role teachers' beliefs play in how teachers engage their students in mathematical discourse practices (Bray, 2011; Walshaw & Anthony, 2008) to further advance teacher education and professional development opportunities (Griffin et al., 2013).

Limitations and Delimitations of the Study

Sample selection and procedures were limited to teachers' availability to participate in this study and to the researcher's access to recruit mathematics teachers due to *COVID19*. Teachers' mathematical practices were analyzed through classroom observations and teacher's lesson plans; however, the lesson planning content was not delineated by the study and many teachers did not create lesson plans that contained the level of detail needed to accurately analyze a qualitative data set.

This study also presented delimitations. First, only general and special educators teaching mathematics in general education classroom settings were invited to participate in this study because most students at-risk for MD take mathematics in general education settings (Griffin et al., 2013). Second, the development of a survey tool to ask teachers about their perceived beliefs and practices might bring some issues about the quality of the survey (e.g., reliable and valid); these issues were addressed by including in the study multiple measures of survey reliability and validity (i.e., internal consistency reliability, pretesting, pilot testing, response validity, and content validity; Creswell & Creswell, 2018).

Definition of Key Terms

Beliefs

Beliefs are the expression of conscious and unconscious ideas (deemed to be true) and thoughts about a person, the world, and a person's position in it, developed through participation in different social groups (Cross, 2009).

Common Core State Standards

The CCSS are a progression of learning expectations or standards in mathematics and English language arts especially designed to prepare K-12 students for a career and postsecondary education (Neuman & Roskos, 2013).

Conceptual Understanding

Understanding of principles that rule a domain and of the interrelations between pieces of knowledge in a domain (Rittle-Johnson & Siegler, 1998).

Discourse

Discourse is a "socially accepted association among ways of using language and other symbolic expressions, of thinking, feeling, believing, valuing, and acting, as well as using various tools, technologies, or props that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role, or to signal that one is filling a social niche in a distinctively recognizable fashion" (Gee, 1990).

Emergent Bilinguals

Emergent bilinguals are students demonstrating emergent abilities in speaking, reading, writing, or understanding of English (García et al., 2008), who come to school with developed oral and/or literacy practices in their home language(s) that enable them to communicate with their families and communities (Kleyn & García, 2019).

Language Register

Language register is a set of meanings that is appropriate to a particular function of language, joined with words and structures that express these meanings (Halliday, 1978).

Learning Disability

A learning disability or specific learning disability is a disorder in one or more of the basic psychological processes involved in understanding or using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations (IDEA, 2004; US Department of Education, 2019a).

Mathematical Discourse Practices

Discourse embedded in sociocultural practices to emphasize plurality of these practices and to connect discourse to mathematical ideas (Moschkovich, 2012b).

Mathematical Reasoning

Process of communication with others or with oneself that allows for inferring mathematical utterances from other mathematical utterances (Jeannotte & Kieran, 2017).

Mathematics Teaching Practices

Mathematical teaching practices are comprised of tasks, discourse, environment, and evaluation and assessment (Barkatsas & Malone, 2005).

Mathematical Thinking

Intuitive mathematical ideas (e.g., more or less concept, shape, size, location, pattern, position) that are developed in a social environment and are an essential component of a child's cognitive development (Ginsburg et al., 2006).

Mathematical Understanding

Continuous connection making of mathematical ideas that results from the integration of concepts and procedures (Cai & Ding, 2015).

Students at-Risk for Mathematics Difficulties

Students who (a) receive special education services in mathematics, or (b) struggle with mathematics, but have not been identified as having a learning disability (Fuchs et al., 2004; Gersten et al., 2005).

Organization of the Study

The present research study is composed of five chapters. Chapter 1 introduces a broad description of the research problem, the purpose and research questions that guided the study, and the significance of the study. In addition, the chapter presents some limitations, delimitations, and key terms definitions that aim to provide understanding of the overall research study. Chapter 2 includes a detailed description of the literature regarding teachers' beliefs and practices about mathematics education, and more specifically about mathematical discourse. The chapter also describes in a brief manner the theoretical framework (Equity and Intersectionality) that was used to analyze and discuss the study's findings. Chapter 3 thoroughly describes the study's methodology, which included research questions and design, participants and setting, dependent measures, and data collection and analysis. Chapter 4 presents a broad description of the study's results and findings. Last, Chapter 5 discusses the study's results and findings as they relate to the literature. The chapter also includes the interpretation of the results and findings through an equity and intersectionality lens, implications of findings for future research and practice, and a more detailed explanation of the study's limitations, mainly caused by *COVID-19* circumstances.

CHAPTER TWO

REVIEW OF THE LITERATURE

As indicated, the purpose of this exploratory sequential mixed methods study is to increase understanding not only of teachers' beliefs about mathematical discourse but also how these perceived beliefs are related to their mathematical practices. The present chapter includes three main components. First, the description of the research literature regarding teachers' beliefs and practices related to mathematics education that supports diverse students' conceptual understanding. Three main sections derived from this topic: (a) Research on teachers' beliefs and practices related to the teaching of mathematics, (b) research on teachers' beliefs and practices related to the learning of mathematics, and (c) research on teachers' beliefs and practices related to the assessment of mathematics. Second, the description of the research literature on teachers' beliefs and practices regarding mathematical discourse as it relates to the development of students' conceptual understanding. Last, the description of the equity and intersectionality theoretical framework that guided the analysis and discussion of the study's findings. In addition, the section includes mathematics educational research that employed an equity and intersectionality theory in the findings to support the development of the study's theoretical framework.

Teachers' Beliefs and Practices Related to Mathematical Instruction

Previous research has indicated that there is a close relationship between teachers' beliefs, their instructional practices, and their decision-making process in the classroom (Cross, 2009; Handal, 2003; Nathan & Knuth, 2003; Nisbet & Warren, 2000). Although it is not entirely clear whether teachers' beliefs influence their practice or practice influences teachers' beliefs

(Handal, 2003; Nisbet & Warren, 2000), to better understand the relationship between teachers' beliefs and practices it must be seen as an interdependent interaction (Quigley, 2021).

Related to mathematics instruction, teachers' mathematical beliefs encompass all belief systems held by teachers connected to the nature of mathematics and the teaching and learning of mathematics (Gantt Sawyer, 2018; Handal, 2003; Voss et al., 2013). Specifically, teachers' mathematical beliefs include (a) what mathematics is, (b) how mathematics teaching and learning happens, and (c) how mathematics teaching and learning should be implemented in the classroom (Ernest, 1991; Handal, 2003). In fact, teachers' beliefs about mathematics cannot be separated from their beliefs about the teaching and learning of mathematics (Barkatsas & Malone, 2005). These belief systems, and therefore teachers' instructional practices, are influenced by teachers' culture, socio-economic status, educational history, peer interactions, and pedagogical and mathematics knowledge (Barkatsas & Malone, 2005; Handal, 2003). Importantly, these belief systems define teachers' perceptions related to the teaching and learning of mathematics and the role students play in the classroom (i.e., active-passive, dependent-autonomous, receiver-creator of knowledge; Handal, 2003; Voss et al., 2013).

Extensive research has been done on teachers' beliefs and practices in mathematics, which was prominent during the introduction of the standards-based mathematics education at the beginning of the 21st century. Research on teachers' mathematics beliefs and practices encompasses different areas: (a) The teaching of mathematics (Barlow & Cates, 2006; Brown et al., 2007; Engeln et al., 2013; Gantt Sawyer, 2018; Good et al., 1990; Marshall et al., 2009; Quigley, 2021; Stipek et al., 2001; Yates, 2006; Yurekli et al., 2020), (b) the learning of mathematics (Aljaberi & Gheith, 2018; Diamond, 2019; Russo et al., 2020), and (c) the assessment of mathematics (Delandshere & Jones, 1999; Martínez-Sierra et al., 2020; Nisbet & Warren, 2000).

Teachers' Beliefs and Practices Related to the Teaching of Mathematics

The literature related to teachers' beliefs and practices about the teaching of mathematics is extensive and very diverse. It mainly focuses on the implementation of teaching strategies that emphasize the development of students' problem-solving skills and conceptual understanding. Overall, the teaching of mathematics literature includes diverse topics, such as, traditional versus inquiry-oriented (problem-based) teaching (Barlow & Cates, 2006; Stipek et al., 2001), inquiry-oriented mathematics teaching vs science education (Engeln et al., 2013; Marshall et al., 2009), standards-based teaching and curriculum (also known as reform-based; Gantt Sawyer, 2018; Yates, 2006; Yurekli et al., 2020), and teaching supports and strategies to promote conceptual understanding (Brown et al., 2007; Good et al., 1990; Quigley, 2021).

Traditional vs Inquiry-Oriented Teaching

Some research focused on teachers' beliefs and practices regarding the teaching of mathematics is rooted in teaching and learning theoretical perspectives (i.e., social constructivist/inquiry-oriented vs. behaviorist/passive). Driven by the standards-based reform in the early 2000s, researchers in the U.S. and around the world (e.g., Barkatsas & Malone, 2005; Barlow and Cates, 2006; Handal, 2003; Stipek et al., 2001; Voss et al., 2013) have investigated teachers' beliefs and practices directly related to inquiry-oriented or problem-based mathematics instruction (also known as anchored, hands-on, project-based, student-centered, and inductive instruction; Engeln et al., 2013), where teachers embrace a dynamic view of mathematics based on the development of problem-solving skills (Stipek et al., 2001).

For example, Stipek and colleagues (2001) investigated teachers' beliefs and practices about mathematics instruction on 21 elementary school teachers through a pre and post survey and classroom observations. They later developed a measure that directly contrasted more traditional mathematics practices to inquiry-based mathematics practices. The authors implemented a correlation analysis to measure the stability of teachers' beliefs from the beginning to the end of the school year. In addition, they conducted a factor analysis to measure the coherence in teachers' beliefs. Stipek et al. (2001) found that teachers had a coherent set of beliefs which predicted their mathematics practices. Teachers who held inquiry-oriented beliefs about teaching mathematics had higher self-confidence and enjoyed teaching mathematics more than those who embraced more traditional ideas of teaching mathematics. Inquiry-oriented teachers' beliefs and practices encouraged students to explore mathematics problems and attempt multiple solutions to solve them (Stipek et al., 2001).

Similarly, Barlow and Cates (2006) investigated changes in teachers' beliefs and practices about mathematics teaching related to problem-based mathematics instruction. Specifically, the authors examined the beliefs of 61 elementary mathematics teachers through a 24-item pre and post 24-item survey regarding problem posing mathematics instruction. Barlow and Cates analyzed scores (i.e., descriptive statistics and variance) for the pre- and post-surveys and found a significant difference between teachers' beliefs about mathematics instruction before and after implementing problem-posing strategies. Teachers also answered an open-ended question about the importance of problem-based instruction. Teachers believed that problem posing instruction (a) developed higher order thinking skills in students, (b) promoted deeper understanding of mathematics, and (c) gave students a sense of ownership of the mathematics they are learning. Equally important, teachers believed this type of problem-based instruction enhanced their lessons (a) by allowing students to explore alternative solutions to the problems and (b) by promoting mathematical conversations among peers. The authors concluded that problem-based instruction (problem posing) not only has an impact on student learning, but also on teachers' beliefs about mathematics and mathematics teaching (Barlow & Cates, 2006).

Inquiry-Oriented Mathematics Teaching vs Science Education

Researchers have also investigated teachers' beliefs and practices about inquiry-oriented mathematics and science education (e.g., Englen et al., 2013; Marshall et al., 2009). For instance, Engeln and colleagues (2013) investigated teachers' beliefs on the implementation of mathematics and science inquiry-based education in 12 European countries. A 32-item teacher questionnaire was used to examine the beliefs and practices of 917 elementary and secondary teachers on inquiry-based instruction. The authors conducted an exploratory factor analysis and found that teachers reported a positive attitude toward inquiry-based instruction, but saw classroom management, system restrictions, and resources as relevant obstacles to its implementation in their mathematics and science classrooms. After conducting a one-way analysis of variance (ANOVA), the authors found that although there were significant differences across the 12 countries, classroom management was seen as the least significant problem in all countries. In addition, the authors performed a latent class analysis to examine different lesson patterns during classroom instruction. Most of the teachers showed a teacher-oriented (traditional) instructional pattern, where inquiry-based instruction was not part of their daily mathematics teaching. Consequently, students' opportunities to explain their ideas or discuss among peers were limited. Results showed that opportunities for discussions among peers in mathematics and science classrooms were less frequent than opportunities for students to explain their ideas (Engeln et al., 2013).

In a similar way, Marshall and colleagues (2009) examined teachers' beliefs and practices about inquiry-oriented instruction in mathematics and science classrooms. One thousand two hundred and twenty-two elementary and secondary teachers answered a 58-item online survey. Through a one-way analysis of variance and correlational analyses, the authors investigated the relationship between teachers' grade level and their self-reported practices about inquiry-based instruction. The authors found that teachers believed the time students should be engaged in inquiry-based activities was higher than the actual time they reported engaging in this type of instruction. In addition, the authors found that teachers reported higher inquiry-based practices in elementary science classrooms compared to elementary mathematics classrooms. The authors attributed this finding to the tendency of mathematics assessments to measure more procedural knowledge than conceptual understanding. Marshall and colleagues (2009) concluded that to transform science and mathematics instruction, teachers need to understand and explicitly integrate concept ideas into inquiry learning activities and/or problems.

Standards-Based Teaching and Curriculum

Other researchers explicitly investigated teachers' beliefs and practices in standards-based teaching and new standards-based curriculum, which is based on the development of students' conceptual understanding (e.g., Gantt Sawyer, 2018; Yates, 2006; Yurekli et al., 2020). According to Handal and Herrington (2003), teachers' beliefs play an essential role not only in facilitating the success of the standards-based reform, but also in effectively implementing a new standards-based curriculum. Teachers might be resistant to implement standards-based reform practices (Gantt Sawyer, 2018) because they may have learned mathematics with a traditional view that emphasizes the transmission of mathematical facts and procedures instead of the acquisition of a deep understanding of concepts (Yates, 2006). For this reason, it is important that teachers believe instructional practices aligned with reform initiatives will enhance student learning and promote conceptual understanding (Yurekli et al., 2020). Yates (2006) examined teachers' beliefs and practices about standards-based mathematics teaching and curriculum. Specifically, 127 elementary mathematics teachers answered a 20-item survey. Through principal components and ANOVA analyses, Yates (2006) found that (a) teachers' beliefs about mathematics and the teaching of mathematics were not related to their age, qualifications, or teacher experience and (b) repeated exposure to reform initiatives over time caused some teachers to update their practices. Yates (2006) concluded that educational change takes place slowly over time.

More recently, Gantt Sawyer (2018) investigated the factors influencing teachers' beliefs about mathematics and the standards-based reform teaching of mathematics. Differently from other researchers, Gantt Sawyer (2018) employed a qualitative case study methodology to investigate the beliefs of an experienced (13 years) first-grade elementary teacher, through a survey, interviews, and classroom observations. The author found that different factors affect teachers' beliefs on implementing new ways of teaching mathematics. Specifically, beliefs were influenced by family, past teachers, the way mathematics was learned, teacher preparation programs, and teaching experiences. Gantt Sawyer (2018) concluded that personal factors, such as teacher preparation programs or teaching experiences, could significantly support beliefs in standards-based teaching.

Yurekli and colleagues (2020) investigated 408 elementary and middle school teachers' beliefs and self-reported practices about teaching mathematics for conceptual understanding (explicit attention to mathematical connections of concepts, operations, and relations), which is a key component of mathematics standards education. After collecting survey data, the authors analyzed teachers' beliefs and self-reported practices through a series of two-level measurement models. They found that teachers have positive beliefs about making connections explicit to promote conceptual understanding. However, teachers' beliefs about the importance of connecting concepts, operations, and relations was greater than the frequency with which they reported teaching those connections during their mathematics instruction. In addition, findings showed that students' background and standardized tests were the two most significant factors that teachers reported as impediments to teaching for conceptual understanding. Importantly, Yurekli and colleagues (2020) concluded that teachers do not always report implementing those practices that they believe are important to support students' conceptual understanding (Yurekli et al., 2020).

Teaching Supports and Strategies to Promote Conceptual Understanding

Other aspects related to teachers' beliefs and practices about the teaching of mathematics have been investigated, such as the inclusion of instructional supports (Brown et al., 2007; Quigley, 2021) and grouping strategies (Good et al., 1990) to promote students' conceptual understanding. Specifically, Brown and colleagues (2007) investigated K-12 teachers' beliefs and practices about letting students use calculators as instructional supports to develop conceptual understanding. The authors used a 28-item survey, which had 20 common statements and 8 specific statements for three different mathematics levels (i.e., elementary, middle, and high school). A total of 816 mathematics teachers answered the survey, of those only 327 were elementary teachers. Brown and colleagues (2007) performed frequency, descriptive statistics, and factor analyses to understand teachers' beliefs, knowledge, and reported practices about the use of calculators. The authors found that teachers in all grade levels believed that students could learn mathematics using calculators, and that those experiences lead to better understanding of concepts. Elementary mathematics teachers' reported practices showed that teachers only allowed their students to use calculators when solving computational tasks, whereas middle and high school teachers incorporated the use of calculators on a wider range of student problem-solving experiences. The authors concluded that elementary teachers have more difficulty finding the balance between developing computational mastery in their students and integrating the use of calculators during their instruction.

Quigley (2021) investigated teachers' beliefs and practices related to the use of concrete materials (also called manipulatives) as an instructional support to develop conceptual understanding. Specifically, 49 elementary mathematics teachers answered a questionnaire and four of those teachers also participated in an interview. Different from all other reviewed studies, the author analyzed the data quantitatively (frequencies) and qualitatively (thematic analysis). Quigley (2021) found that teachers incorporated concrete materials during their instruction for different purposes: (a) conceptual understanding, (b) engagement, (c) memory, (d) social interactions, and (e) fluency and automaticity. In addition, findings showed that teachers believe that once concrete materials are given to students, teachers are no longer the focus of the lesson. Quigley (2021) concluded that teachers that incorporate concrete materials during their lesson hold a social constructivist philosophy where students oversee constructing their own knowledge.

In addition to the study of teacher beliefs and practices around instructional supports, mathematics researchers have also investigated teachers' beliefs and practices related to grouping strategies to support conceptual understanding. For example, Good et al. (1990) investigated teachers' beliefs and reported practices about varied types of grouping strategies in elementary classrooms, mainly small group instruction. Specifically, the authors examined different types, operation, organization, and purposes of groups during mathematics instruction through a questionnaire including 1509 teacher participants. Teachers reported using different group sizes, such as whole and small (when groups were less than 13 students) group instruction. In fact, teachers reported implementing small group instruction during the middle and last part of the lesson more often than implementing small groups at the beginning of the lesson. Although teachers believed that small group instruction was particularly appropriate for problem-solving, findings showed that small group instruction was more frequently used for practice purposes than to promote problem solving skills. In addition, Good and colleagues (1990) found that the content or complexity of the lesson did not influence teachers' decision on the implementation of small group instruction.

Currently, classroom practices are composed of both (a) teacher and students' explanations of concepts and ideas (Barlow & Cates, 2006; Engeln et al., 2013; Quigley, 2021) and (b) inquiry-based and problem-posing practices (Gantt Sawyer, 2018; Marshall et al., 2009; Stipek et al., 2001; Yurekli et al., 2020) that are designed to develop higher-order thinking skills and a deeper conceptual understanding of mathematics that promotes student engagement and active participation throughout the learning process (Marshall et al., 2009). The importance of investigating teachers' beliefs and practices regarding the teaching of mathematics is clearly related to the mathematical performance and achievement of all students in the classroom and the success of the standards-based reform. Specifically, this line of research is related to the implementation of standards-based mathematical practices that emphasize the development of students' conceptual understanding.

Overall, mathematics education researchers have been actively involved using different methodologies to understand the complex relationship between teachers' beliefs and practices about the teaching of mathematics after the standards-based reform initiative. Interestingly, only two studies differed from quantitatively analyzing teachers' beliefs and practices: Gantt Sawyer (2018) included a qualitative analysis (case study), and Quigley (2021) included a qualitative (thematic) and quantitative (frequency) analyses. The present research intended to fill the gap in the literature by incorporating an exploratory mixed methods analysis of teachers' beliefs and practices related to mathematics instruction, specifically the planning and implementation of mathematical discourse.

Teachers' Beliefs and Practices Related to the Learning of Mathematics

The learning of mathematics generally happens when students are actively involved in the construction of mathematical meaning through activities and discourse embedded during instruction (Nisbet & Warren, 2000). Although less extensive than the teaching of mathematics, the literature on teachers' beliefs and practices about the learning of mathematics has focused on how students learn mathematics, for example, constructivist versus traditional learning (Aljaberi & Gheith, 2018), learning through struggle (Russo et al., 2020), and transfer of knowledge learning (Diamond, 2019).

Theories of student learning (e.g., constructivist, traditional) have been widely explored in the research literature (Woolley et al., 2004). Related to these theories, more research has focused on the teaching of mathematics than on the learning of mathematics. Nevertheless, research on teachers' beliefs and practices about the learning of mathematics has been investigated within multiple topics. For example, in combination with teachers' beliefs and practices regarding the teaching of mathematics (Aljaberi & Gheith, 2018), through students' productive struggle (Russo et al., 2020), and learning through transfer of knowledge (Diamond (2019).

Aljaberi and Gheith (2018) investigated the beliefs and practices of 111 elementary and middle school mathematics teachers about the teaching, learning, and nature of mathematics. The authors used two different scales (i.e., math beliefs and math teaching practices) to quantitatively (i.e., descriptive statistics and analysis of variance) analyze the data. In relation to the learning of mathematics, results showed that most teachers held constructivist beliefs (students learn by actively constructing knowledge), and these beliefs were consistent with both elementary and middle school teachers. Aljaberi and Gheith (2018) found a positive relationship between teachers' beliefs and their practices and concluded that constructivist beliefs about the learning of mathematics were also reflected in teachers' constructivist practices.

Another topic related to the learning of mathematics is productive struggle (also known as productive failure and zone of confusion) which is connected to students' perseverance and motivation. Specifically, productive struggle learning refers to students trying to figure out mathematical concepts and relationships that were not immediately apparent (Hiebert & Grouws, 2007). This line of research emerged in the advent of standards-based math instruction. Russo and colleagues (2020) examined the beliefs and practices about the role of student struggle in the learning of mathematics of 93 elementary teachers. The authors analyzed questionnaire data through qualitative (thematic analysis) and quantitative (chi-square test of independence) analyses. Results showed that most teachers held positive beliefs about the value of struggle to learn mathematics, such as opportunities to (a) persist through challenges, (b) take risks, (c) build autonomy, and (d) develop confidence. Russo et al. (2020) found that teachers believed struggle is a key component of mathematical learning because it provides opportunities for problem solving skills development, peer tutoring, and student-led discourse. The authors concluded that teachers holding positive beliefs about productive struggle was not enough to be reflected in their instructional practices (Russo et al., 2020).

In a different way, Diamond (2019) examined teachers' beliefs and practices about how students learn mathematics by transferring knowledge acquired from one concept to another. Specifically, Diamond (2019) investigated the beliefs of eight mathematics teachers about students' transfer of learning. The author defines transfer as the generalization, expansion, or application of knowledge to a new concept or skill. Through a qualitative analysis of interviews and observations, Diamond (2019) found that most teachers believed students' dispositions, affect, and own beliefs were key components for the transfer of learning to occur. In essence, teachers believed that (a) confidence in their own abilities, (b) usefulness and relevancy of the mathematical content, and (c) students' beliefs about mathematics were essential factors for students to productively transfer their learning. In addition, findings showed that teachers' pedagogical actions in the classroom were informed by their multiple beliefs about transfer of mathematical knowledge and understanding.

Interestingly, more recent research on teachers' beliefs and practices related to student learning of mathematics (Diamond, 2019; Russo et al., 2020) included mixed methods and qualitative methodologies to analyze their data. The incorporation of multiple methodological approaches to investigate the literature shows the need to understand the complexity of creating and implementing evidence-based research (Palinkas et al., 2015).

Overall, the research literature related to teachers' beliefs and practices on the learning of mathematics is closely connected to the literature about the teaching of mathematics. In fact, many researchers (Aljaberi & Gheith, 2018; Barkatsas & Malone, 2005; Reeder et al., 2009; Voss et al., 2013) did not separate the teaching and learning of mathematics in their research findings. The reason why the teaching and learning of mathematics are generally interconnected is because how teachers believe students learn is the focus of the teaching that is happening in the classroom. In other words, teachers base their practices and actions in the classroom on the beliefs they have about the ways students make meaning, understand concepts, problem solve, and generalize mathematical knowledge (Diamond, 2019). Because mathematical discourse plays an important role on students' meaning making and development of conceptual understanding (Schleppenbach et al., 2007), teachers' beliefs and practices on discourse will be further explored in this chapter.

Teachers' Beliefs and Practices Related to the Assessment of Mathematics

Closely related to the teaching and learning of mathematics is the research about the assessment of mathematics (Delandshere & Jones, 1999). New perspectives of mathematics teaching practices, because of the standards-based reform initiative, have changed teachers' assessment practices. Namely, teachers not only depend on students' behaviors, such as paying attention and staying on task, to assess understanding of the content (Turner et al., 2009). Although less explored than the research about the teaching and learning of mathematics, research about teachers' beliefs and practices in relation to the assessment of mathematics has also been examined (Delandshere & Jones, 1999; Martínez-Sierra et al., 2020; Nisbet & Warren, 2000). Delandshere and Jones (1999) investigated three elementary mathematics teachers' beliefs and practices about assessment and its relationship to the teaching and learning of mathematics. Specifically, through a qualitative thematic analysis of teachers' interviews and classroom observations, the authors examined the factors that define teachers' beliefs about the assessment of mathematics. Three main themes emerged from their analysis: (a) teachers' beliefs are defined by their understanding of students' learning, (b) teachers' beliefs are influenced by the purpose and function of the assessment, and (c) teachers' beliefs and practices are shaped by the curriculum they are using and by state-mandated assessments. Delandshere and Jones (1999) concluded that teachers' criteria to assess student performance is generic and they do not consider complex content knowledge. The authors suggest that teacher preparation programs and professional development should focus on teaching teachers how to move away from textbook activities and assessments that lack disciplinary content.

In a similar way, Martínez-Sierra and colleagues (2020) qualitatively investigated teachers' beliefs about assessments and how these beliefs are connected to their overall mathematics beliefs. The authors implemented a thematic analysis of semi-structured interviews to examine 18 high school mathematics teachers' beliefs and practices about assessments. Findings showed that teachers had 3 main reasons to incorporate assessments during their instruction: (a) to know what students learned, (b) to inform the teacher, and (c) to make students accountable for their learning. Martínez-Sierra and colleagues (2020) concluded that many teachers missed opportunities to use assessments to improve their own teaching practices because they lacked the knowledge on how to transform their assessment practices from only assessing what students learned (summative) to assessing for feedback of their own teaching practices (more formative).

Different from other research studies (Delandshere & Jones, 1999; Martínez-Sierra et al., 2020), Nisbet and Warren (2000) investigated 398 elementary teachers' beliefs in relation to multiple mathematics topics, including the assessment of mathematics (i.e., the content of mathematics, the teaching of mathematics, the assessment of mathematics, and factors that influence their beliefs). The authors quantitatively (factor analysis) examined teachers' responses to a survey. Of a total of 15 factors found by the analysis, only 3 factors were related to teachers' beliefs about the assessment of mathematics (i.e., use of assessment to inform the teacher, use of assessment to inform the learners, and use of assessment for accountability purposes). Specifically, results indicated that teachers mostly used assessments to inform their teaching, and that this assessment purpose decreased as the grade level increased. Nisbet and Warren (2000) concluded that teachers highly value assessment data to evaluate their personal performance and their students' progress and suggested to further investigate the influence of external factors (e.g., parents and policy) on teachers' beliefs about the assessment of mathematics.

Although some researchers claim that some teachers believe they can rely on visible, behavioral observations of their students (e.g., paying attention, staying on task) to know if they have learned the content (Turner et al., 2009), findings about teachers' beliefs and practices related to the assessment of mathematics showed that teachers believe assessments are important to both acknowledge students' understanding and meaning making processes (Martínez-Sierra et al., 2020), and to inform their teaching practices (Nisbet and Warren, 2000). Notably, researchers that focused their research on investigating teachers' assessment of mathematics (Delandshere & Jones, 1999; Martínez-Sierra et al., 2020), as opposed to holistically investigating teachers' beliefs and practices about the teaching, learning and assessment of mathematics, included in their research a qualitative inductive thematic analysis to better understand this specific component of mathematics instruction. It seems plausible that researchers are moving away from only investigating teachers' beliefs and practices through quantitative methods and are also incorporating qualitative methodologies that are intended to reach depth of understanding of the research problem (Palinkas et al., 2015).

All things considered, the research related to teachers' beliefs and practices about the teaching, learning, and assessment of mathematics has been evolving along the years as new strategies and methods are developed and implemented in mathematics classrooms to promote students' conceptual understanding. With the implementation of the standards-based reform the emphasis on students' conceptual understanding has shifted the way teachers perceive, teach, and assess mathematics. Moreover, these changes have propelled researchers to better understand the role teachers' beliefs play in their instructional practices to develop their students' conceptual understanding. Mathematical discourse has been proven effective to support students' conceptual understanding (Erath et al., 2018; Moschkovich, 2015); therefore, the purpose of the present research study was to examine teachers' beliefs and practices related to the planning and implementation of mathematical discourse during instruction. The following section of the chapter will specifically focus on teachers' beliefs and practices about mathematical discourse, as it relates to students' conceptual understanding.

Teachers Beliefs' and Practices Related to Mathematical Discourse

Research on teachers' beliefs and practices related to mathematical discourse also began with the implementation of standards-based initiatives that emphasize the development of students' conceptual understanding by providing students with many opportunities to produce, validate, and communicate mathematical ideas (Bray, 2011; Hwang, 2018; Nathan & Knuth, 2003). Unfortunately, the research literature on teachers' beliefs about mathematical discourse is very limited compared to the literature on teachers' practices related to mathematical discourse. In one of the few studies examining teachers' beliefs about teaching and learning of mathematics, Nathan and Knuth (2003) investigated how a teacher's beliefs about teaching and learning of mathematics influenced her instructional practices regarding classroom discourse after the implementation of the standards-based reform. The authors observed a sixth-grade mathematics teacher once a week and interviewed the teacher twice a month, over a 2-year period, to investigate her beliefs and rationale for her actions during her mathematics lessons. Nathan and Knuth employed discourse and comparative analyses of classroom videos and interviews (a) to examine moment to moment interactions among members of the classroom (i.e., teacher and students, student and student, and whole classroom discourse) and (b) to interpret the data. Findings showed that even though the teacher believed students learn best from their peers and class participation is essential to learn mathematics, her teaching practices (during the first year of the study) had very little student to student interactions and most of the classroom discourse was between the teacher and a student. After participating in multiple interviews, the teacher realized that interactions among students were very limited. Thus, during the second year of the study, the teacher promoted more discursive opportunities among peers during her instruction. Specifically, by removing herself from playing a leading role during classroom discourse, she provided more opportunities for student-led discussions. Nathan and Knuth (2003) concluded that teachers' reflections of their teaching practices can lead to instructional changes that would support their beliefs about mathematical discourse and its relationship with students' conceptual understanding.

Hwang (2018) also examined the beliefs and classroom norms and discourse of 3 sixth-grade mathematics teachers in relation to equitable mathematics practices. Through classroom observations and interviews, the author employed a deductive thematic analysis to investigate teachers' beliefs and practices regarding classroom discourse (e.g., initiator, purpose, content). Findings indicated that teachers' beliefs about teaching mathematics and students' equitable participation during discourse influenced the mathematical rigor of the content being taught. For example, teachers who were concerned about increasing the participation of all students in the classroom paid less attention to the mathematical accuracy of their students' explanations and justifications of the mathematical content. In addition, the author found that teachers who were concerned about their students' reasoning constantly asked questions to provide multiple opportunities for students to justify their thinking. Hwang (2018) concluded that discourse helps students establish their individual identities in mathematics classrooms. These identities determine the role students' play during instruction as providers or receivers of mathematical ideas.

In a different way, Bray (2011) examined how teachers' beliefs and mathematics knowledge influence their error handling practices during mathematical discussions. A collective case study design was employed to investigate four third-grade mathematics teachers' beliefs and knowledge on how to handle students' mathematical misconceptions during classroom discourse. The author analyzed observational and interview data (at the beginning and at the end of the school year) using a cross case qualitative thematic analysis. Bray (2011) also included in her research a mathematics and pedagogy survey to establish a profile of teachers' beliefs and mathematics knowledge before and after the implementation of a standards-based curriculum. Findings showed evidence of shifts in teachers' beliefs that better aligned to standards-based mathematics instruction by the end of the school year. Specifically, teachers believed that during classroom discourse teachers should encourage students to explain as much mathematical thinking and reasoning as possible. Related to handling misconceptions, the author found that although teachers believed in the importance of teaching for conceptual understanding, they often found students' errors hard to understand and struggled to formulate content-based questions and explanations to address misconceptions. Bray (2011) concluded that mathematical discussions focused on inquiry and argumentation were harder to accomplish than discussions based on strategy explanations because teachers not only needed mathematical and pedagogical knowledge, but also discussion management skills.

Research on teachers' beliefs about mathematical discourse has been closely related to research regarding teachers' beliefs about standards-based instruction, which emphasizes the importance of students' conceptual understanding. The research literature on teachers' beliefs about mathematical discourse not only includes quantitative methods to help researchers understand the research problem, but also involves the use of qualitative methodologies to allow researchers to explore the problem in a deeper way. In addition, to explore mathematical discourse more in depth, some researchers (Adler & Ronda, 2015; Gillies & Khan, 2009; Hamm & Perry, 2002; Hufferd-Ackles et al., 2004; Hundeland et al., 2020; Kumpulainen & Kaartinen, 2003; Louie, 2020; Martin et al., 2015; McConney & Perry, 2011; Piccolo et al., 2008; Schleppenbach et al., 2007) decided to only focus on teachers' discursive practices that promote conceptual understanding and engagement. The literature on teachers and students' discursive practices has been extensively explored, mostly using qualitative and mixed methodologies.

Mathematics Discursive Practices

Research on teachers' practices related to mathematical discourse emanated from the recognition that general education classrooms are becoming very diverse, and teachers require a wider understanding of mathematical discourse that promotes the conceptual understanding of all students in the classroom (Walshaw & Anthony, 2008). To promote understanding through discourse does not necessarily mean more talk during instruction; it requires the inclusion of strategies (e.g., probing, interpreting, scaffolding, questioning, revoicing) that focus on students' explanation, justification, and argumentation of their mathematical thinking and reasoning through language (Walshaw & Anthony, 2008). Therefore, research on teachers' practices regarding mathematical discourse encompasses diverse topics such as the implementation of instructional strategies (Martin et al., 2015; McConney & Perry, 2011; Piccolo et al. 2008; Schleppenbach et al., 2007), teacher-student talk (Adler & Ronda, 2015; Hufferd-Ackles et al., 2004; Hundeland et al., 2020), collaborative learning (Gillies & Khan, 2009; Kumpulainen & Kaartinen, 2003), and authority and agency (Hamm & Perry, 2002; Louie, 2020).

Instructional Strategies

Some researchers have explored teachers' mathematical discourse practices focusing on specific instructional strategies, such as questioning, prompting, and scaffolding. Piccolo and colleagues (2008) studied the nature of classroom discourse in teachers' practices and its impact on teachers' questioning-explanation practices. Through a grounded theory qualitative analysis, the authors observed and examined the mathematics lessons (n=183) of 48 middle school mathematics teachers over a three-year period. Before videorecording mathematics lessons, the researchers conducted a training for teachers that consisted of watching classroom videos and noticing different question types (e.g., open-ended, cloze) and the discourse that was generated afterwards. Findings showed that teacher talk was dominant. Specifically related to questioning strategies, findings revealed that teachers' open-ended questions promoted students' engagement and conceptual understanding and that persistent questioning led to a discourse that included deeper and richer students' explanations of more complex mathematical content. The authors concluded that discourse focused on teachers' questioning of mathematical reasoning provided students with the opportunity to develop their conceptual understanding.

Similarly, Martin and colleagues (2015) examined teachers' use of questions, tasks, and discourse to promote students' conceptual understanding. Specifically, 48 elementary mathematics teachers attended a school-based professional development training focused on mathematics knowledge and pedagogy and a summer institute focused on unwrapping standards and differentiation of mathematical content based on students' different ability levels. Through observations of mathematics lessons (at the beginning and at the end of the school year), the researchers investigated teacher-student interactions and different types of questions asked by teachers during classroom discourse. Martin and colleagues (2011) analyzed the data quantitatively (i.e., mean differences using a *t*-test) and qualitatively (i.e., using thematic analysis). Findings revealed that teachers' questioning influenced the levels of mathematical discourse and students were able to build understanding through discourse. Although shifts in teachers' practices were not statistically significant, findings showed evidence that teachers facilitated students' participation and engagement in classroom discourse by prompting and asking questions that promoted students' explanations of their thinking and reasoning. The authors concluded that teachers' use of questioning strategies that focused on justifying mathematical strategies and connecting topics increased the mathematical content complexity of the classroom discourse.

In like manner, McConney and Perry (2011) investigated shifts in teachers' questioning practices and students' explanations during mathematical discourse after the implementation of a standards-based curriculum. Specifically, four fourth-grade teachers were observed across two years. The participants taught mathematics using a traditional curriculum the first year of the study and a standards-based curriculum the second year of the study after attending a summer professional development program between years one and two. McConney and Perry analyzed the data quantitatively (analysis of variance from year one to year two) and qualitatively (thematic analysis). Results showed evidence that there was a statistically significant difference from year one to year two on teachers' practices related to giving students the opportunity to verbally elaborate on their reasoning and problem-solving processes. Additionally, findings of qualitative data displayed a difference in the quality of teachers' questions and students' explanations (longer students' responses and more discourse turns) from year one to year two. Namely, teachers' questions were designed to elicit longer and more in-depth student responses during the second year of the study. The authors concluded that when teachers adopt a standards-based curriculum, which emphasizes students' conceptual understanding, they will undoubtedly alter their discourse practices (questioning strategies) to assess understanding and address misconceptions.

In a very different way, Schleppenbach and colleagues (2007) investigated teachers' practices related to extended discourse (follow-up questioning). In addition, the authors compared teachers' extended discourse practices in the U.S. with teachers' extended discourse practices in China. To clarify, the authors defined extended discourse as discourse that broadens the conversation, even after a correct answer has been given, to support students' conceptual understanding. The main idea behind the implementation of extended discourse is the belief that the explanation and justification of students' reasoning is as important as the correct solution of the problem. Through a mixed methods design, the authors examined the frequency and content level of extended discourse (i.e., rule recall, computation, procedures, and reasoning) during instruction to corroborate if extended discourse promoted higher levels of mathematical thinking and conceptual understanding.

Schleppenbach and colleagues (2007) observed mathematics lessons from 15 fifth grade mathematics teachers from China and 12 fourth and fifth grade mathematics teachers from the U.S. To interpret the data, the authors employed quantitative (frequency, analysis of variance, effect size, and dynamic time warping) and qualitative (thematic) analyses. Results showed that extended discourse practices in Chinese classrooms were more frequent, included more mathematical reasoning questions, and required more formal vocabulary than extended discourse practices in U.S. classrooms. In other words, extended discourse practices in U.S. classrooms were less frequent and emphasized students' computational knowledge. The authors concluded that extended discourse is the first step to engage students in conversations that develop their conceptual understanding, but evidence of observed extended discourse practices in both countries suggested that neither country reached discourse levels that were congruent with standards-based content expectations and different enough from traditional forms of discourse.

Teacher-Student Talk

Research on teachers' practices regarding mathematical discourse also explored teacher-student talk and math-talk learning communities. For example, Adler and Ronda (2015) investigated differences in a teacher's mathematical practices related to discourse over one academic year using an analytical framework called Mathematics Discourse in Instruction (MDI). MDI emphasizes language as students' main resource to communicate, negotiate, and collaborate during instruction. Thus, MDI incorporates exemplification, explanatory talk, and learner participation to make abstract mathematical concepts, such as numbers and functions, accessible to students through teacher-student discussions. One high school mathematics teacher participated in the study and provided video recordings of two mathematics lessons. The authors analyzed the data qualitatively using deductive coding. Findings showed that there were little differences in exemplification and explanatory teacher-student talk across time. Namely, the authors found more revoicing and formal vocabulary in the teacher's discursive practices at the end of the academic year. Moreover, findings showed that the teacher's task demand was low, explanatory talk was incomplete, and student participation was limited in both lessons observed. Adler and Ronda (2015) concluded that the MDI framework gives researchers the opportunity to understand and distinguish subtle changes in teachers' mathematical practices.

Following Adler and Ronda's (2015) line of research, Hundeland et al. (2020) used the MDI framework to investigate characteristics of mathematical discourse in four kindergarten mathematics classrooms. The authors mainly examined students' opportunities to talk and teachers' actions and contributions during discourse to promote students' conceptual understanding. Hundeland and colleagues (2020) employed a randomized control trial to analyze classroom discourse in (a) two kindergarten classrooms using a specific curriculum that emphasized classroom discussions and reflections during playful learning and inquiry activities (treatment group) and (b) two kindergarten classrooms using the traditional curriculum (control group). Teachers in the treatment group attended a training to be able to implement the curriculum that emphasized students' conceptual understanding with fidelity. Though video recordings of the lessons, the authors analyzed the qualities (frequency and content complexity) of mathematical discourse and compared them between treatment and control conditions. Findings showed that in both groups (treatment and control) teachers were actively guiding mathematical discussions that included mathematical and non-mathematical content and most students were actively participating. In addition, the most significant difference between mathematical discourses observed in the treatment group compared to the control group was on the level of discourse. The mathematical discourse level in the treatment group was higher than that of the control group; mathematical discourse was mostly characterized by children contributing with answers to what, how, and why questions that promoted the communication of multiple mathematical ideas during the discussion. On the contrary, mathematical discourse in the control group was mostly characterized by children contributing with one-word answers to closed questions. Hundeland and colleagues (2020) concluded that curriculum and teacher training based on inquiry and discourse may result in richer and more profound mathematical discussions of concepts and ideas in kindergarten classrooms.

In different manner, Hufferd-Ackles and colleagues (2004) investigated teacher-student talk during discourse through the creation of a math-talk learning community, where teachers and students use discourse to support the mathematical learning of all students. Through a qualitative case study design, the researchers investigated teacher-student talk and the creation of a math talk learning community in four elementary mathematics classrooms over the course of a year. All teachers in the study taught mathematics using a standards-based curriculum that included supports (i.e., language and visual representations) to promote students' communication of mathematical thinking and reasoning. Through classroom observations and interviews, researchers employed a qualitative analysis of the data and determined that the mathematical practices of one of the four teachers exhibited considerable changes towards the implementation of reform-based instruction. Specifically, the teacher started teaching in a traditional way and later adopted classroom discourse practices that supported the mathematical learning and understanding of all members of the classroom community. Therefore, this specific classroom was selected as the focus of the case study.

Findings exhibited four main factors that captured the growth of a math-talk learning community over time: (a) questioning, (b) explaining mathematical thinking, (c) sharing mathematical ideas, and (d) embracing responsibility for learning. After analyzing the selected classroom data throughout the year, researchers determined different math-talk learning community levels from traditional practices (level 0) to discursive practices that embraced meaningful and collaborative math-talk (level 3). Notably, teacher's practices transitioned from asking questions with only numerical value answers to questions that focused on students' mathematical thinking and extended descriptions of multiple student strategies. Hufferd-Ackles and colleagues (2004) concluded that the description of different math-talk learning community levels can assist other mathematics teachers trying to build effective math-talk learning communities.

Collaborative Learning

Closely related to teacher-student talk practices are collaborative learning practices during small group and dyads discourse. Some researchers (Gillies & Khan, 2009; Kumpulainen & Kaartinen, 2003) explicitly focused their research on collaborative learning practices that promote students' conceptual understanding during small group discourse and dyads to further explore the nature of the discourse during these specific grouping strategies.

Through a comparative treatment design, Gillies and Khan (2009) investigated the effectiveness of a cognitive and metacognitive questioning teacher training to challenge students' mathematical learning, problem solving, and reasoning during small group collaborative work. The study involved two different groups of elementary and middle school mathematics teachers: (a) the cooperative and questioning condition and (b) the cooperative condition. Specifically, the authors examined differences of 28 teachers' practices in language use to promote students' reasoning and problem-solving skills and the effect of those practices on students' mathematical discourse and learning when comparing the two group conditions. Observations of teachers and students' discourse were recorded and coded at the beginning and at the end of the intervention. The researchers analyzed the data through a multivariate analysis of variance (MANCOVA) to determine if there was a significant difference between the two conditions. A random intercept model of multilevel modeling was used to determine if there was a significant difference in students' reasoning and problem-solving scores across conditions. Results showed that teachers in the cooperative and questioning condition asked significantly more reflective questions (probe and clarify, confront discrepancies, and suggest strategies) that challenged students' reasoning than their counterparts in the cooperative condition. Results also showed the effects of these questioning strategies during classroom discourse on students' discursive practices in the cooperative and questioning condition compared to students' discursive practices in the cooperative condition. Specifically, students in the cooperative and questioning condition provided more elaborative answers that included reasoning and justification of mathematical ideas. Notably, there were not significant differences on students' problem-solving and reasoning skills across conditions. Gillies and Khan (2009) concluded that when teachers are taught to use different questioning strategies, they tend to challenge more their students' thinking and reasoning.

Kumpulainen & Kaartinen (2003) also investigated collaborative reasoning and learning, but during peer dyads discourse. Specifically, the authors sought to better understand the collaborative reasoning and discourse (i.e., formulation and explanations of mathematical ideas) emerging within heterogeneous peer dyads (i.e., different mathematical competence levels). The study included 12 fifth-grade students in one mathematics classroom. Video recordings and field notes of three dyad cases were randomly selected for a close, qualitative, microlevel analysis. Students' collaborative reasoning (i.e., communicative functions, modes of social activity, problem-solving strategies, and mathematical language) was coded using inductive and deductive thematic analysis. Findings highlighted the interactional elements and mechanisms that support collaborative reasoning (e.g., equal participation in social interactions, joint reasoning of problem-solving strategies, collaboration, and appreciation of each other's contributions) and the elements that pose challenges to promote collaborative reasoning (e.g., cognitive and social conflicts created asymmetric interactions and lower collaborative reasoning). Interestingly, in some instances conflict situations resulted in peer tutoring episodes that included students' argumentation and scaffolding towards a joint conceptual understanding. The authors concluded that collaborative reasoning in heterogeneous dyads provided students with multiple opportunities to elaborate on their mathematical thinking, reasoning, and understanding.

Authority and Agency

Other factors, such as students' mathematical authority (Hamm & Perry, 2002) and agency (Louie, 2020), in relation to teachers' discursive practices have also been investigated. Hamm and Perry (2002) examined how, if at all, teachers promote feelings of mathematical authority in their students during classroom discourse. Specific teaching practices (e.g., questioning, integration of students' ideas) tell students how ideas are developed and validated. Consequently, specific teacher questions and follow up responses to students' ideas communicate to students the individual with the ultimate source of mathematical knowledge and valid ideas. Through classroom observations (i.e., video recordings) of six first-grade mathematics teachers, the authors analyzed teachers' practices during classroom discourse. Hamm and Perry scored the degree to which students engaged in higher order thinking and assessed the extent to which talking was used to understand mathematics using two different rating scales. Results showed that only one out of six teachers occasionally gave her students a sense of mathematical authority and created a classroom community conducive of higher order thinking and discourse participation. During instruction, this teacher often emphasized the importance of students finding their own ways of solving mathematical problems and backing up their reasoning with evidence of their thinking processes. Unfortunately, all other teachers who participated in the study did not show evidence of granting mathematical authority to their students. Therefore, in their classrooms, students' opportunities to learn complex mathematical concepts were limited. Hamm and Perry concluded that through teachers' practices in mathematics classrooms, many children unfortunately learn at a young age that mathematics is a discipline discovered by others and thus their contributions are irrelevant.

Closely related to research on students' mathematical authority is research on students' agency. Agency refers to students' self-perceptions of being effective learners, thinkers, and problem-solvers. Louie (2020) examined how teachers use discourse to emphasize students' abilities and agency (i.e., how teachers' practices during classroom discourse grants students a sense of mathematical authority). Twenty mathematics teachers from five different schools participated in the study during a district-wide professional development initiative focused on students' agency, authority, and identity (Teaching for Robust Understanding of Mathematics [TRU] framework; Schoenfeld, 2014). The author conducted a qualitative discourse analysis using observations of teachers' collaborative conversations (i.e., self-reported practices) and field notes. Findings showed that teachers often made use of agency discourse to promote a feeling of mathematical ownership and authority in their students. For example, teachers reported they provided opportunities for their students to take ownership of their mathematical learning. Notably, in many instances when teachers tried to promote students' agency during their instruction, they at the same time also promoted students' hierarchies (e.g., only high achieving students took ownership of their learning). In addition, many teachers reported that their participation in the agency, authority, and identity professional development prompted them to make changes in their instructional practices. Louie (2020) concluded that fostering student agency often involves the concept of student hierarchies. The author recommended that teachers should pay explicit attention to students' hierarchies when fostering student agency to be able to improve the learning opportunities and participation of all students in the classroom.

To fully understand how teachers' mathematical practices regarding mathematical discourse support all students in the classroom, many researchers have investigated equitable discursive practices concerning students with diverse abilities, cultures, and languages (Banse et al., 2017; Baxter et al., 2002; Celedón-Pattichis & Turner, 2012; Dominguez, 2017; Griffin et al., 2013; Hansen-Thomas, 2009; Lewis, 2017; Musanti & Celedón-Pattichis, 2013; Wiebe Berry & Kim; 2008; Xin et al., 2020). Specifically, researchers have found that teaching for inclusion requires the creation of classroom spaces that focus on students sharing ideas to challenge and extend their own and other students' thinking (Nathan & Knuth, 2003; Walshaw & Anthony, 2008) in order to promote all students becoming apprentice mathematicians (Schoenfeld, 2014).

Students with Learning Disabilities

With the implementation of the standards-based reform, the legislation of teaching all students through high academic standards that will prepare them for their college and post-secondary success was also implemented. Research on mathematical discourse practices has also focused on the education of students with LD (Baxter et al., 2002; Griffin et al., 2013; Lewis, 2017, Wiebe Berry & Kim, 2008; Xin et al., 2020). Researchers have investigated (a) effective mathematical discourse interventions for students with disabilities or at-risk for mathematical difficulties (Lewis, 2017; Xin et al., 2020) and (b) students with LD mathematical discursive practices in inclusive general education classrooms (Baxter et al., 2002; Griffin et al., 2013; Wiebe Berry & Kim, 2008).

For instance, Xin and colleagues (2020) examined the benefits of teacher-learner discourse moves on the mathematical reasoning and problem-solving skills of student with learning disabilities. Through statistical discourse analysis, the authors investigated the effects of discourse-oriented instruction and the characteristics of the mathematical discourse of three students with LD in the fifth grade. The 8- to 10-week daily intervention consisted of engaging students with disabilities in solving multiplication problems and explaining their mathematical reasoning behind the solution to the problem. Results showed that after the intervention all three students improved their mathematical performance, but only one student demonstrated successful transfer of multiplicative reasoning to solve a range of multiplicative word problems. Related to the mathematical discourse developed during the intervention, results showed that teachers constantly prompted students for information about their assimilation of the problem, and teachers' discourse often involved academic vocabulary. In addition, teacher-student discourse consisted of the teacher tilting the essential cognitive work to the student with LD while adapting the discourse to promote students' reasoning and understanding. Xin and colleagues (2020) concluded that a constructivist-based mathematics instruction supports the engagement and successful learning of students with LD.

In like manner, Lewis (2017) investigated the effects of a discursive intervention based on a sociocultural approach to disability and focused on bridging the conversational discourse of a student with LD and the mathematical discourse needed to support the student's reasoning and conceptual understanding. First, the author identified and analyzed ways in which the discourse of a student with LD was inadequate to fully access the mathematical discourse of the classroom. Then, the author designed an intervention (i.e., re-mediation instruction) that included alternative and more accessible mediators (e.g., different words and visuals) for the student with LD.

Through a micro genetic analysis (i.e., documenting in fine-grained detail small shifts in discourse), the author analyzed the observations during the re-mediation intervention. Specifically, the author coded for student correctness, discursive patterns, word use, and use of visuals. To measure the effectiveness of the intervention, Lewis (2017) used a pre and post mathematics assessment on fractions. Findings showed that the student's with LD discourse shifted over the course of the intervention. Across sessions, the student with LD showed an increase in mathematics accuracy and use of academic words. The author concluded that mathematics instruction in inclusive classrooms should move towards models that embrace diversity and promote successful outcomes for all students.

Moving away from discursive interventions for students with LD, Griffin et al. (2013) investigated the mathematical discursive practices in inclusive general education classrooms of two teachers and six students with LD in third and fourth grade. Through classroom observations and interviews, the authors coded and analyzed the data qualitatively. Results showed that one teacher spent 70% of the observed teaching time checking students understanding and promoting discourse during small groups or dyads. In contrast, the other teacher spent 40% of the observed teaching time checking students understanding and provided few opportunities for student-to-student interactions. The authors also analyzed students' mathematical performance through academic assessments. Results showed that most students improved their mathematical performance and performed at grade level. Griffin et al. (2013) concluded that mathematics teaching in inclusive classrooms that includes directed and strategy instruction, offers students multiple opportunities to communicate their thinking and practice the content during whole and small groups, and incorporates multimodal supports (e.g., manipulatives and visual representations) may successfully support the mathematics learning of students with LD.

In a similar way, Wiebe Berry & Kim (2008) examined the nature of classroom discourse in an inclusive first grade mathematics classroom. Teachers' discourse interactions with students with LD and low mathematics achievers were the focus of the research. Specifically, four teachers (i.e., general education, special education, student teacher, and a paraprofessional) shared teaching responsibilities in the classroom and participated in the study. Through classroom observations, interviews, and field notes, the authors employed inductive and deductive coding to analyze the frequencies of collected qualitative and quantitative data. Findings showed that teachers' discursive practices mostly included questioning to elicit participation, responding to students' responses, giving instructions, and presenting and explaining the content (e.g., recalling, explaining, and repeating). In addition, the authors found that teachers used different instructional strategies (e.g., scaffolding, feedback) considered effective for students with LD, but mathematical discourse mainly addressed low-level questions and students were not required to explain their thinking and reasoning to the teacher or their peers. Wiebe Berry & Kim (2008) concluded that to promote communication in the classroom, teachers could start by learning new questioning strategies, which have been effective to support the learning of students with LD.

Baxter and colleagues (2002) focused their research on the mathematical discourse of a teacher and her 28 fourth-grade students in a general education classroom. Specifically, Baxter et al. (2002) investigated the nature of teacher discursive practices (i.e., evolution across time from teacher to student-led, differences in students' discursive practices depending on their mathematical ability level, and the impact of the participation of all students on how the teacher mediated the discourse) in a general education classroom, where the teacher intentionally worked to include all students (with and without disabilities) in the classroom discourse. Through video recordings of classroom observations, audio recordings of small-group interactions, field notes, and interviews, the authors sought to identify systematic patterns in teacher and student statements. Findings showed that over time the teacher's discourse shifted from mostly behavior management to prompting students' mathematical reasoning. Students with LD improved their level of participation during classroom discourse, however, the teacher expressed that moving the conversation back and forth among students with different ability levels sometimes interrupted the flow of the discourse and made it hard to reach a high mathematical content level. Baxter et al. (2002) concluded that it is possible to implement effective interactive discourse practices for students with LD in general education mathematics classrooms.

Emergent Bilingual Students

Research on teachers and students' discursive practices during mathematics instruction regarding students who are emergent bilinguals have also been widely investigated. This line of research mainly emerged from the implementation of standards-based mathematics instruction and the educational accountability reform in the United States. Specifically, research on mathematical discourse practices with emergent bilingual students has focused on: (a) developing and implementing curriculum and/or instruction that places an emphasis on discourse (Banse et al., 2017; Dominguez, 2017; Hansen-Thomas, 2009), and (b) analyzing teacher and students' current discursive practices (Celedón-Pattichis & Turner, 2012; Musanti & Celedón-Pattichis, 2013).

Related to developing and implementing a discourse curriculum or instruction, Dominguez (2017) studied the mathematical discourse of EBs during the implementation of specially designed instruction that promoted and facilitated discussion. Specifically, the author observed two mathematics classrooms (i.e., fourth and fifth grade) for a two-week period to understand students' discursive practices. In addition, to fully understand how emergent bilinguals communicate, the author visited all the homes of the fourth and fifth grade students. Unlike the observed classroom communication, students constantly transitioned from English to Spanish and vice versa as they talked. The author created an inventory of students' common activities and experiences to create mathematical activities and tasks within the school context. Teachers implemented these tasks during mathematics instruction, but they expressed concerns about the mathematical content complexity. Through video recordings of classroom observations, the author qualitatively (i.e., discourse analysis) analyzed the data. Results showed that bilingual students learned and discussed mathematics within two kinds of experiences (i.e., familiar and unfamiliar) and within two languages (i.e., English and Spanish). Familiar contexts allowed students to recognize their own experiences and encouraged them to take risks and participate during the classroom discourse to solve problems using both languages. Dominguez (2017) concluded that using strategies that include mathematical tasks and problems based on students' experiences and languages promotes the participation of emergent bilingual students in the mathematical discourse of the classroom and thus, the development of their conceptual understanding.

Different from Dominguez (2017), Banse et al. (2017) studied teachers' discursive practices during the implementation of a curriculum aimed to improve students' mathematical confidence and understanding through rich discussions. Through a comparative case study, the authors examined two fourth-grade teachers' discursive practices in classrooms with a high concentration of emergent bilingual students. Specifically, Banse et al. (2017) analyzed the data of lesson video recordings qualitatively (i.e., grounded theory) using inductive coding. Results showed that teachers seldom asked referential questions to their students, and mostly included recall questions during discourse. In addition, teachers generally scaffolded their students' understanding by using repetition strategies that led to some elaboration of mathematical reasoning and use of academic vocabulary. The authors concluded that (a) despite the use of a mathematics curriculum focused on discourse, classroom instruction lacked deep rich discussions of the content and (b) the extent to which these type of discussions occurred depended on each teacher's performance.

In a similar way, Hansen-Thomas (2009) investigated the discourse practices of three sixth-grade mathematics teachers using a curriculum designed to emphasize the interactions, discussions, and problem-solving skills of students who are culturally and linguistically diverse. Specifically, the author qualitatively examined teachers' discursive practices aimed to encourage and elicit discourse that contains high mathematical content using a case study design that employed an interactional sociolinguistics discourse analysis. Findings exhibited that teachers' practices included incorporating (a) modeling, eliciting, revoicing, restating, and redirecting strategies, (b) using contextualization ques, and (c) encouraging appropriate language use. Although, all three teachers engaged their students in discourse practices, one teacher exceled in her implementation of instructional practices (e.g., modeling that elicited students' discourse aimed to promote students' conceptual understanding). The author concluded that when teachers promote mathematical discourse through continuous modeling and eliciting strategies, students who are emergent bilinguals have more opportunities to engage and participate in rich discourse that supports their conceptual understanding.

Celedón-Pattichis and Turner (2012) studied a kindergarten teacher and her emergent bilingual students' interactions supportive of mathematical discourse development. Specifically, the authors examined a kindergarten problem solving activity through a sociocultural lens, focusing on emergent bilingual students' participation in mathematical discourse. The lesson included solving a word problem, which the teacher encouraged her students to solve in ways that made sense to them. After, the teacher facilitated a group discussion that promoted students' sharing their own strategies. Through 25 classroom observations across one academic year, the authors qualitatively (i.e., case study) analyzed the data focusing on three dimensions: mathematical language, visual representations used by students to communicate their thinking, and students' contributions and teacher responses. Findings showed that by the end of the school year students began to appropriate mathematics vocabulary, to communicate their reasoning leveraging visual and symbolic representations, and to follow the rules that guided the classroom problem-solving discourse. Importantly, findings showed evidence that teacher and students worked collaboratively to support the development of the mathematical discourse. The authors concluded that young emergent bilingual students actively participate during mathematics instruction in many ways, such as discussing, explaining, symbolizing, representing, justifying, and connecting mathematical ideas.

In a very similar way, Musanti and Celedón-Pattichis (2013) used a case study design to examine a bilingual kindergarten teacher's mathematical practice that used language as a learning resource to promote emergent bilinguals' understanding of the mathematical content. Through classrooms observations, video recordings, and interviews, the authors analyzed characteristics in the teacher's instructional approach to teaching mathematics using language and discourse. Findings showed that the teacher mainly implemented three instructional practices: (a) use of mathematics stories, (b) integration of multimodal representations, (c) inclusion of collective thinking and representation to promote understanding. Findings related to discourse showed that the teacher provided multiple opportunities for students to listen to peers, compare strategies, and explain their thinking. The authors concluded that through a collective construction of meaning, students participated in a shared mathematical discourse that positioned them as effective problem solvers.

As can be seen, research on teachers' beliefs about mathematical discourse is very limited. In contrast, research on teachers' practices about mathematical discourse has been extensively explored regarding multiple topics and with diverse populations of students. Interestingly, research on teachers' discursive practices regarding diverse populations (i.e., students with LD, emergent bilingual students) has mainly focused on designing, implementing, and evaluating effective curriculum, instruction, and/or interventions (Banse et al., 2017; Dominguez, 2017; Hansen-Thomas, 2009; Lewis, 2017; Xin et al., 2020) that support these students' participation in classroom mathematical discourse, development of their conceptual understanding, and enhancement of their mathematical performance. Notably, all research studies focused on mathematical discursive practices (i.e., teachers and students) employed qualitative analyses to fully understand teachers and students' actions during discourse and explain teachers' instructional strategies that promote rich and deep mathematical discussions and equitable mathematics practices in general education classrooms.

Analysis of the literature related to teachers' beliefs and practices regarding mathematics and mathematical discourse showed that researchers strived to have a deeper understanding of the research problem by incorporating different methodological approaches. With the implementation of standards-based reform initiatives, researchers focused on understanding the beliefs and practices of teachers that adopted a constructivist or student-centered approach and placed emphasis on the development of students' conceptual understanding. However, researchers mainly relied on the use of quantitative surveys to explore teachers' beliefs and practices related to the teaching, learning, and assessment of mathematics.

Conversely, to investigate mathematical discourse, researchers moved away from using quantitative surveys and purposefully investigated teachers and students' discursive practices in mathematics through qualitative analyses. Although qualitative methodologies intend to achieve depth of understanding of the research problem, mixed methods could be very powerful on utilizing qualitative findings to further explore the problem through complex quantitative analyses. The present research study not only incorporated a mixed methods design, to investigate teachers' beliefs and practices regarding mathematical discourse, but also utilized the findings from its qualitative analysis to create a valid and reliable quantitative survey.

In addition, the present research study introduced a topic that was absent in the review of the literature related to mathematical discourse: the intentional planning of mathematical discourse. Of all research studies reviewed in the analysis of the literature, the inclusion of lesson plans to analyze teachers' mathematical discourse was absent. To support teacher preparation and professional development programs on the effective planning and implementation of meaningful and rich mathematical discourse, it is critical to understand all aspects that affect teachers' beliefs and practices related to this topic, including their lesson planning.

Theoretical Framework

The theoretical framework described in this chapter was used as a lens to analyze, discuss, and interpret the study's findings and results. As mentioned before, equitable access to educational opportunities in the classroom is dependent on how teachers manage intersectional factors (e.g., race, language, ability, socioeconomic status) underlaying educational disparities in general education classrooms (Carey et al., 2018). Therefore, equity and intersectionality theoretical perspectives could be a powerful lens to understand equitable and inclusive mathematical practices related to mathematical discourse.

Equity and Intersectionality Theoretical Perspectives

The concept of intersectionality has its origins in the racialized experiences of minority women in the United States in the 1970s, and early 1980s (Atewologun, 2018; Harris & Leonardo, 2018), which is also developed and explained in the work of U.S. critical race theorist and legal scholar Kimberlé Crenshaw (Bullock, 2018; Gillborn, 2015; Harris & Leonardo, 2018). Intersection refers to the juxtaposition of two or more social categories or systems of power, in which social identities, sociodemographic characteristics, social processes, and social systems are included (Atewologun, 2018). According to Bowleg (2012):

> Intersectionality is a theoretical framework for understanding how multiple social identities such as race, gender, sexual orientation, SES [socioeconomic status], and disability intersect at the micro level of individual experience to reflect interlocking systems of privilege and oppression (i.e., racism, sexism, heterosexism, classism) at the macro social-structural level. (p. 1267)

In other words, the concept of intersectionality was developed to acknowledge those individuals who simultaneously endure and experience different modes of oppression. These forms of oppression, when considered in parallel, seem to have an additive effect, but those who experience these oppressions face multiplicative consequences (Bullock, 2018; Sibbett, 2020).

Intersectionality encompasses three core ideas: (a) social identities consistently treated as marginal, (b) the complex nature of power, and (c) no single social label is ever complete (Harris & Leonardo, 2018). The last idea derives from the notion that social categories (e.g., race, gender, sexual orientation) are multiple, interdependent, and mutually constitutive (Bowleg, 2012), and that categories are best understood in relational terms, rather than in isolation (Carey et al., 2018). Different sophisticated and nuanced understandings of social formations have resulted in increasingly complex identity terms (e.g., LatCrit [Latinx critical theory], Dis/Crit [critical disability theory], "LGBT"; Harris & Leonardo, 2018). Consequently, intersectionality has been criticized by some scholars for the uncritical (e.g., meaningless, excessive, unjustified) use of intersections that might eventually shatter any sense of coherence (Delgado, 2011; Gillborn, 2015; Harris & Leonardo, 2018).

Intersectionality has evolved from a theory of multiple marginalization to a theory of multiple identities in the second decade of the 21st century (Sibbett, 2020). As the concept of intersectionality takes on a broader meaning, an intersectional analysis has become a way to engage this theoretical perspective in critical inquiry (Bullock, 2018). An intersectional approach aims to analyze the relationships of power and inequality within a social setting, and how individual and group identities are shaped because of these relationships (Tefera et al., 2018). Even though the intersectionality framework was developed to analyze the multiple forms of marginalization experienced by women of color, this framework also offers researchers the opportunity to examine the different ways that intersecting social dynamics affect people within and across groups (Bullock, 2018; Tefera et al., 2018).

Equity and Intersectionality in Education

An intersectionality perspective offers educational researchers and practitioners theoretical explanations of the ways in which diverse members of a group might experience education differently depending on their race, ethnicity, gender, sexuality, religion, citizenship, ability, and/or age (Tefera et al., 2018). The acknowledgement of these differences might provide insight into issues of inequality within and across teaching and learning settings. Thus, the inclusion of intersectionality theories into pedagogical actions and educational research is starting to gather increased attention and become more normative (Bullock, 2018; Tefera et al., 2018). Moreover, an intersectional perspective can (a) be applied in the field of education and special education to examine educational inequities related to the intersection of ability, race, and language, among others (Tefera et al., 2018), and (b) help teachers and school leaders create more equitable school environments (Carey et al., 2018).

To create supportive learning spaces where all identities can be safely expressed, teachers must constantly challenge the status quo and strive to develop an understanding of intersectional identities and cultural competence (Powers and Duffy, 2016). An intersectionality awareness could motivate educators to disentangle and challenge power relations, which advance the norms, values, and attributes valued by macro-structural and cultural systems, that continually favor some students and restrict other students (Carey et al., 2018). Unfortunately, this intersectional approach to challenge existing power inequities operating in educational sites is seldom taken up by educators and school leaders (Carey et al., 2018).

To acknowledge an intersectionality approach, teachers need to understand the ways that race and other oppressions operate in the classroom and the diverse identities they themselves embody (Artiles, 2019; Carey et al., 2018). How their own identities (e.g., white, female, and upper-middle class) might influence their own practices to place students who reflect non-dominant identities in disadvantage (Carey, 2018). In essence, an intersectional thinking urges educators to resist stereotypes and deficit perspectives about their students' academic potential and develop different mindsets, dispositions, and practices that combat societal oppressions manifested in the school culture, curriculum, teachers' decision making, and in student and teacher interactions inside and outside the classroom (Carey, 2018). An intersectionality perspective takes a higher meaning when learning is diminished because students feel insecure, marginalized, invisible, and threatened in the classroom (Powers & Duffy, 2016).

Equity and Intersectionality in Mathematics Education Research

The application of an Intersectional analysis in K-12 mathematics education research has been studied with a limited range of methodological approaches, narrowed scope, and disjointed educational issues. In fact, the complexity of this theoretical perspective has resulted in a limited range of methodological approaches, mostly qualitative, being used to explore it (Schudde, 2018). An intersectionality perspective and the conflict model of intersectional analysis in K-12 mathematics research has been applied to understand students and teachers' perspectives about different issues, such as, students' experiences during mathematics instruction (Gholson & Martin, 2014; Zavala, 2014), collaborative group work (Esmonde et al., 2009), and teachers' biases (Riegle-Crumb and Humphries, 2012).

Students' Perspectives. Intersectional research focused on investigating students' perspectives aimed to understand experiences and relationships during mathematics instruction: (a) for different intersectionality groups (i.e., gender, race, and age [Gholson & Martin, 2014], race and language [Zavala, 2014]), and (b) during collaborative group work (Esmonde et al., 2009). Golson and Martin (2014), via a qualitative study, investigated the intersection of gender, race, and age of two third-grade African American girls' experiences during mathematics instruction to understand and acknowledge Black girlhood as a context for nurturance, ability, potential, reinforcement, and support. They analyzed the data through single-identity lenses and later combined those analyses to negotiate conflicts among them. In their findings, the authors exposed and highlighted the tensions experienced by students who belonged to that specific identity category (i.e., girl, black, and a child) in their mathematics classroom. Similarly, Zavala

(2014) investigated the intersection of race and language of Latinx high-school students' mathematics learning. Through a Latino Critical Race Theory, the author qualitatively investigated multiple constructs to highlight the experiences lived by Latinx students learning mathematics. Findings highlighted the privilege that English-speaking students have in mathematics classes related to access to curriculum materials, and ability to communicate with their teacher. Zavala (2014) suggested further research to address intersectional identities that also include immigration status and culture and how the intersection of these multiple identities affects Latinx students' mathematics learning.

Students' perspectives during mathematics instruction were also investigated through collaborative group work (Esmonde et al., 2009). Esmonde and colleagues (2009) used an intersectional approach to understand how identities affected cooperative group work in an urban secondary mathematics classroom. Through a single case design, researchers performed two phases of analysis (i.e., whole class and individual) to see how identities determined students' experiences of group work. Esmonde and colleagues (2009) found out that identities (e.g., gender, race) negatively affected the benefits of group work for minority students in different ways, including the development of decision making and leadership skills.

Teachers' Perspectives. Only one research study employing an intersectional approach to investigate mathematics education was based on teachers' perspectives (Riegle-Crumb & Humphries, 2012). Riegle-Crumb and Humphries (2012) investigated tracking and teacher bias in high school mathematics courses through a quantitative analysis of national course-taking data from high school transcripts. Although results showed that minority students were overrepresented in low mathematics courses and underrepresented in advanced mathematics courses, there were not statistically significant racial/ethnic and gender differences on teachers' perceptions. Crumb and Humphries (2012) concluded that after taking achievement differences into consideration, teachers did not perceive male and female minority students as having a lower mathematics ability compared to their white peers (Riegle-Crumb & Humphries, 2012).

Overall, qualitative and quantitative approaches to intersectionality move beyond singular dimensions to emphasize the compound impact and consequences of multiple, intersecting, and complex social identities on students' educational outcomes (Schudde, 2018). For example, quantitative methodologies, such as heterogeneous effects (HE), move beyond focusing on the effect of a single social identity to study the differential effects of multiple identities on students' learning performance (Schudde, 2018). Unfortunately, research on intersectionality in mathematics education that simultaneously incorporates the strengths of qualitative and quantitative methodologies is very limited. The present research study aims to understand equitable and inclusive mathematical teachers' practices related to mathematical discourse employing a mixed methods methodology, which includes both qualitative and quantitative methodologies. This specific research methodology could provide different perspectives to understand how teachers' discursive practices influence students' mathematics learning, conceptual understanding, and achievement (Schudde, 2018).

Summary

The present chapter aimed to provide the reader with deep understanding on teachers' beliefs and practices related to mathematics instruction supportive of students' development of conceptual understanding. First, the chapter included extensive research on teachers' beliefs and practices regarding mathematics. To describe the research regarding teachers' beliefs and practices focused on a standards-based mathematics education supportive of students' conceptual understanding, three main categories were found: (a) teachers' beliefs and practices related to the teaching of mathematics, (b) teachers' beliefs and practices related to the learning of mathematics, and (c) teachers' beliefs and practices related to the assessment of mathematics. Although some researchers investigated one of these components in isolation, most researchers incorporated in their research the inclusion of the teaching, learning, and assessment of mathematics to fully explain teachers' beliefs and actions in the classroom. Interestingly, the research field on this topic has evolved from only employing quantitative methodologies to including qualitative and mixed methods methodologies that aim to explain the research problem more in depth.

Second, the chapter displayed the limited research on teachers' beliefs about mathematical discourse, and the extensive research regarding discursive mathematics practices. Many topics have been studied to understand teachers' actions during discourse, students' opportunities to participate during discourse, and the nature of teacher-student and student-student interactions during discourse. Specially, research has focused on interactions as they relate to the development of understanding of mathematical concepts that will increase the mathematics performance and achievement of all students in the classroom.

Last, the researcher included a brief explanation of the theoretical framework that guided this research study and helped to interpret the data observed in the findings and results. Equity and intersectionality have become an essential lens to understand students' opportunities to participate in a mathematical discourse conducive of the successful development of their conceptual understanding. Research on mathematical discourse connected students' multiple opportunities to explain their thinking and teachers' provision of equitable mathematics practices that promote students' mathematical identity, authority, agency, and access. Therefore, the purpose of this research was to increase understanding of teachers' beliefs and practices regarding the intentional planning and implementation of mathematical discourse during classroom instruction to support students' conceptual understanding, especially students from diverse backgrounds, who might be at-risk for mathematics difficulties. Importantly, this study extends the research literature on teachers' beliefs and practices regarding mathematical discourse by using qualitative data to drive the development of a quantitative instrument about the planning and implementation of mathematical discourse in inclusive classrooms.

CHAPTER THREE

METHODS

This study examined teachers' beliefs and practices related to the implementation of mathematical discourse during classroom instruction in K-5 general education mathematics settings. Research on teachers' mathematical discourse practices has demonstrated that teachers' beliefs and knowledge about mathematics and the mathematics instruction that is subsequently implemented in the classroom both influence their mathematical practices (Clark et al., 2014) and the way they incorporate and manage mathematical discourse in the classroom (Walshaw & Anthony, 2008). Thus, teachers' beliefs on mathematical discourse shape their classroom practices (Nathan & Knuth, 2003). Unfortunately, little research has been done regarding (a) teachers' beliefs related to the planning and implementation of mathematical discourse during instruction and (b) how these beliefs might be related to their discursive practices in the classroom (Nisbet & Warren, 2000; Walshaw & Anthony, 2008).

The current chapter shares the methodology of the present study, designed to further understand teachers' beliefs and practices in general education settings related to mathematical discourse. The chapter describes the study's methodology, which includes: (a) research questions, (b) research design, (c) setting, (d) participants, (e) dependent measures, and (g) data collection and analysis procedures.

Research Questions

This mixed methods study was guided by the following research questions:

1. How do teachers in general education settings describe perceived beliefs and practices related to mathematical discourse? (qualitative)

2. How do teachers implement mathematical discourse in general education settings as

measured by teachers' lesson plans and classroom observations? (quantitative and qualitative)

3. Are the validity and reliability estimates of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* sufficient to support its use in research and program planning? (quantitative)

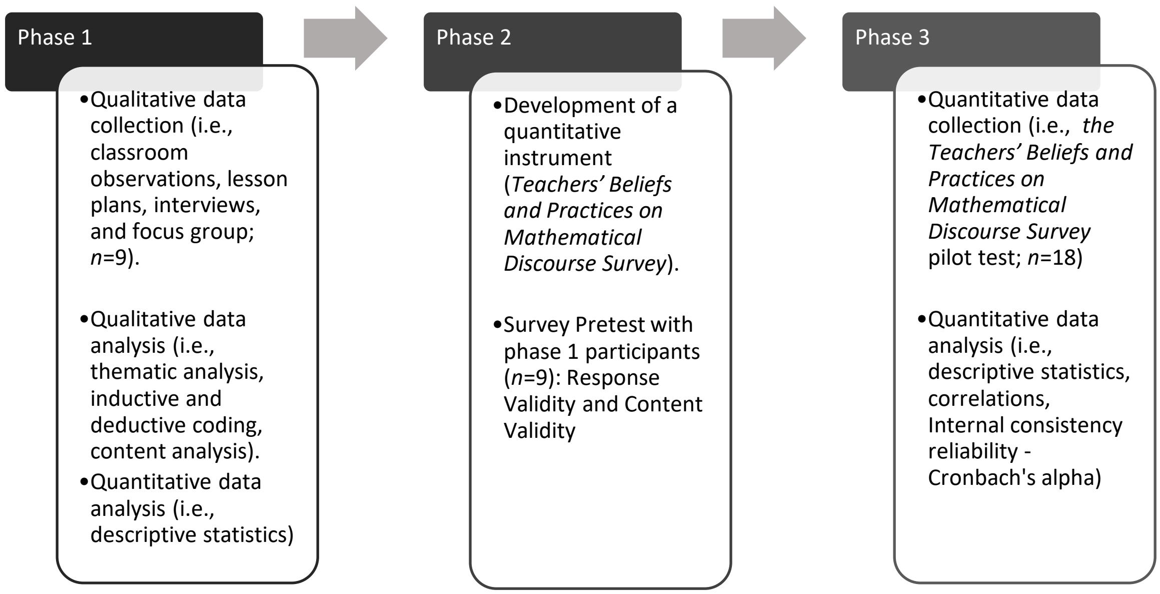
Research Design

The present research study followed an exploratory sequential mixed methods design to broadly explore and understand teachers' beliefs and practices related to mathematical discourse. Mixed methods designs are being implemented with more frequency in educational research, after many researchers have noticed that the complexity of the challenges of executing evidence-based research (e.g., innovative practices and strategies, interventions) often requires more than a single methodological approach (Palinkas et al., 2015). Contrary to qualitative and quantitative methodologies, mixed methods capitalize on the strengths of both qualitative and quantitative research (Creswell & Creswell, 2018). In fact, mixed methods research incorporates qualitative and quantitative strands of data in a single research study to address the study's research questions (Creswell & Plano Clark, 2018).

The study's exploratory sequential mixed methods design was composed of three phases. First, qualitative data were collected and analyzed (Phase 1) and identified themes were used to drive the development of a quantitative instrument (e.g., survey) to further explore the research problem. Then, the developed instrument was pretested (Phase 2; Creswell & Creswell, 2018; Creswell & Plano Clark, 2018). Last, quantitative data were collected to pilot test and validate the designed instrument (Phase 3). Specifically, two stages of analysis were conducted: (a) qualitative data to develop the content of the designed instrument, and (b) quantitative data to validate and measure the developed instrument (e.g., Berman, 2017; Dizon el at., 2011).

Figure 2

Exploratory Sequential Mixed Methods Design



Note: Adapted from Creswell and Creswell (2018) and Gehlbach and Brinkworth (2011)

Setting

Participants of the study were elementary and special education alums (last 5-7 years) in the college of education of a highly diverse American public research university; most graduates of these programs are hired to teach in a large urban school district in the southwestern United States. To be part of the study, participants were required to be teaching and/or co- teaching in an elementary general education mathematics classroom. During the 2020-2021 academic year, the local school district operated 379 schools, enrolled more than 310,000 students, and employed more than 17,900 teachers. Students' racial and ethnic distribution was 47.28% Hispanic, 22.27% White, 15.2% Black, 7.21% two or more races, 6.06% Asian, 1.64% Pacific Islander, and .34% American Indian/Alaskan Native (Nevada Accountability Portal, 2022). The district reported that 86.63% of students were eligible for Federal Free and Reduced Lunch, 15.2% were English Learners, and 12.59% had an Individual Education Program (IEP; Nevada Accountability Portal, 2022). In mathematics, proficiency scores for students in elementary and middle school (grades 3-8) are based on the Math Criterion-Referenced Test (CRT) or the Smarter Balanced Assessment Consortium test (SBAC). For students in high school (11th grade only), proficiency scores are based on the ACT mathematics (the highest possible score students can earn is 36, students who achieve a score of 22 or higher are identified as proficient). Table 1 describes student achievement scores in mathematics within the district's elementary, middle, and high schools for the year 2020-2021; due to the coronavirus pandemic, these are the most recent results available.

Table 1

Students' Achievement Scores in Mathematics for the Year 2020-2021

Test	School Grade	% Proficient
Math Criterion-Referenced Test (CRT) New NV Standards	3	24.8
Math Criterion-Referenced Test (CRT) New NV Standards	4	21.3
Math Criterion-Referenced Test (CRT) New NV Standards	5	19.3
Math Criterion-Referenced Test (CRT) New NV Standards	6	19.7
Math Criterion-Referenced Test (CRT) New NV Standards	7	23.9
Math Criterion-Referenced Test (CRT) New NV Standards	8	17.8
American College Testing (ACT)	11	21.2

(Data retrieved from Nevada Accountability Portal, 2022)

Participants

All elementary school mathematics teachers (i.e., general and special education) teaching and/or co-teaching mathematics in general education classrooms, who were alumni (last 5-7 years) of the college of education, were invited to participate in the present research study. Two different samples were included in the study. For the first sample (Phases 1 and 2 of the study), eligible participants met the following criteria: (a) currently taking or have taken graduate courses related to mathematics instruction and pedagogy in the last 5-7 years in the college of education at the researcher's university and (b) teaching and/or co-teaching in an elementary general education mathematics classroom. The researcher asked department chairs within the college of education (i.e., special education and elementary/secondary education) to distribute a recruitment email on her behalf to eligible participants. The recruitment email included contact information about the researcher and general information about the research study (e.g., purpose, procedures, inclusion/exclusion criteria). A total of 989 emails (265 graduate students in special education, 484 graduate students in elementary education, and 240 students getting professional development credits) were sent to eligible participants (K-5 general and/or special education elementary mathematics teachers teaching and/or co teaching in a general education classroom; See Appendix F). The researcher contacted potential participants (via university email) and sought to build a respectful and trusting communication.

For the second sample (Phase 3 of the study), the researcher asked the department chairs within the College of Education (i.e., special education and elementary/secondary education) to distribute a recruitment email on her behalf to potential participants. A total of 990 emails (265 graduate students in special education, 485 graduate students in teaching and learning, and 240 students getting professional development credits) were sent to eligible participants (K-5 general and/or special education elementary mathematics teachers teaching and/or co teaching in a general education classroom; see Appendix G).

During Phases 1 and 2 of the study, the researcher asked eligible participants to take part in the research study by: (a) attending interviews and a focus group, (b) letting the researcher observe their teaching practices and perform a qualitative analysis of their lesson plans, and (c) answering a quantitative survey (on Qualtrics) as well as follow up questions related to the newly developed survey (e.g., language, format, design) for pretesting purposes (i.e., survey content and response validity). During Phase 3 of the study, the researcher asked eligible participants to take part in the research study by answering a survey for pilot testing purposes.

Participant Selection Process

Sampling for Qualitative Purposes

The sample for the study's qualitative research was selected purposefully to choose specific participants that provided rich information (Palinkas et al., 2015). In purposeful sampling, researchers intentionally select individuals and sites to learn or understand the problem and research questions (Creswell & Creswell, 2018). Specifically, 37 potential participants answered the recruitment email and survey; from those, only 19 potential participants met criteria to participate in the study (i.e., 13 general education and 6 special education teachers).

The researcher identified and selected nine participants that were knowledgeable about, and experienced in, teaching mathematics in elementary general education classrooms to a diverse group of students (e.g., students with learning disabilities, emergent bilinguals; Creswell & Plano Clark, 2018). Because qualitative research is mainly intended to achieve depth of understanding, it places an emphasis on data saturation, which requires the collection of data until no new or relevant information is revealed (Creswell & Creswell, 2018; Palinkas et al, 2015). The sample size (*n*=9) was deliberately identified to be large enough to increase the richness of the data across teachers, but small enough to account for the needed time to perform an in-depth and detailed analysis (Diamond, 2019). Selected participants showed availability and willingness to take part in the present research study. Thus, for Phases 1 and 2 of the study (qualitative data collection and development of the survey), recruited participants (*n* = 9) were asked to (a) communicate experiences and opinions in a coherent, expressive, and reflective manner (Palinkas et al., 2015) during the focus group and interviews, (b) provide lesson plans and allow for classroom observations during mathematics instruction, and (c) answer a quantitative survey and follow up questions about the survey (for validity and reliability purposes). To specify, six general education and three special education teachers were selected to participate in Phases 1 and 2 of the study.

First, IRB approval was obtained from the researcher's university (see IRB participants' consent form in Appendix D). After selecting possible participants, the researcher contacted eligible participants via their university or school email. Then, the researcher met (online) with each selected participant that agreed to participate in the study to explain the purpose, logistics, procedures of the study, and ask them to sign a consent form. The consent form signed by participants included the researcher's contact information to answer any concerns and follow up questions the participants might have. In addition, the consent form also provided information about the purpose of the study, procedures, risks of participation, compensation, confidentiality, and voluntary participation.

Participants' demographic information was obtained through a brief demographic online survey (Qualtrics) sent via email. The survey included general information questions about participants (e.g., age, gender, ethnicity, teaching experience, teaching grade level, student population in their classroom, licenses and endorsements, and participation in professional development; see Appendix A). Table 2 describes Phases 1 and 2 participants' overall demographic information. In addition, Table 3 reports specific demographic information per participant, each participant was given an identifier (e.g., A, B, C) for future reference.

Table 2

Demographic Characteristics of Participants During Phases 1 and 2 of the Study

Participant Variable	Total number (%)
Gender	
Female	9 (100%)
Male	0 (0%)
Age	
< 30 years old	3 (33%)
30 - 40 years old	5 (56%)
> 40 years old	1 (11%)
Race	
White	5 (56%)
Black/African American	1 (11%)
Asian	1 (11%)
Two or More	2 (22%)
Ethnicity	
Hispanic or Latino or Spanish Origin	1 (11%)
Not Hispanic or Latino or Spanish Origin	8 (89%)
Education	
Bachelor's degree	3 (33%)
Master's degree	6 (67%)
Teaching Experience	
< 5 years	5 (56%)
5 - 10 years	2 (22%)
10 - 15 years	1 (11%)
> 15 years	1 (11%)

(Continued)

Grade level	
PK-K	1 (11%)
First grade	1 (11%)
Second grade	1 (12%)
Third grade	3 (33%)
Fourth & Fifth grade	3 (33%)
License Type	
General Education	6 (67%)
Special Education	3 (33%)
Percentage of students with LD in the classroom	
< 15%	6 (67%)
> 30%	3 (33%)
Percentage of students who are EB in the classroom	
< 15%	4 (45%)
15 - 30%	3 (33%)
> 30%	2 (22%)
Received PD related to mathematics instruction (hours)	
0 -15 hours	5 (56%)
15 - 30 hours	2 (22%)
> 30 hours	2 (22%)

Note: LD = learning disabilities; EB = emergent bilinguals; PD = professional development

Table 3

Demographic Information per Participant

Participant	Age (years old)	Race	Education	Teaching Experience (years)	Grade Level	License Type
Participant A	30 - 40	Two or more	Bachelor	< 5	4^{th} & 5^{th}	SpEd
Participant B	30 - 40	Two or more	Masters	10 - 15	3rd	GenEd
Participant C	< 30	White	Bachelor	< 5	4^{th} & 5^{th}	SpEd
Participant D	30 - 40	White	Masters	5 - 10	3rd	GenEd
Participant E	> 40	White	Masters	> 15	PK-K	GenEd
Participant F	30 - 40	Asian	Bachelor	< 5	4^{th} & 5^{th}	SpEd
Participant G	< 30	White	Master	< 5	2nd	GenEd
Participant H	30 - 40	Black/African American	Master	5 - 10	1st	GenEd
Participant I	< 30	White	Master	< 5	3rd	GenEd

Note: SpEd = special education; GenEd = general education

Sampling for Pilot Testing Purposes

Qualitative data collected during Phase 1 of the study was used to develop a survey instrument (Phase 2; e.g., Dizon et al., 2011) related to teachers' beliefs and practices about mathematical discourse planning and implementation during classroom instruction. Survey questions were built from the salient themes that emerged from the qualitative data analysis. In educational research, random selection of participants can be unfeasible and costly (Creswell & Creswell, 2018; Delice, 2010). Therefore, a convenience sample was used to implement Phase 3 (pilot testing) of the study (Creswell & Creswell, 2018). Potential participants for the pilot testing phase (Phase 3) of the study were contacted via email by the department chairs from the researcher's University College of Education. Informational emails were sent to eligible participants containing the purpose and general information of the study, and a demographics and quantitative survey (Qualtrics). By choosing to answer the survey, participants gave their consent to participate (see Appendix E). The researcher electronically sent the quantitative survey (Qualtrics) to 990 general and special education mathematics teachers teaching in general elementary education classrooms. In total, 41 potential participants accessed the survey; 16 did not meet participation criteria and were not able to answer the survey and seven did not complete the survey in its totality. Thus, 18 complete surveys were collected (i.e., 13 general and 5 special education teachers). To be able to build a representative sample for pilot testing purposes (Delice, 2010), the researcher sought to have enough participants during Phase 3 of the study by sending one reminder email per week to participants (in the span of three weeks; Saleh & Bista, 2017). Table 4 describes specific demographic information about Phase 3 participants.

Table 4

Demographic Characteristics of Participants During Phase 3 of the Study

Variable	Number (%)
Gender	
Female	16 (89%)
Male	2 (11%)
Age	
< 30 years old	5 (28%)
30 - 40 years old	4 (22%)
> 40 years old	9 (50%)
Race	
White	9 (50%)
Black/African American	1 (6%)
Asian	5 (28%)
Two or More	3 (16%)
Ethnicity	
Hispanic or Latino or Spanish Origin	3 (16%)
Not Hispanic or Latino or Spanish Origin	15 (84%)
Education	
Bachelor's degree	9 (50%)
Master's degree	8 (44%)
Higher than master's degree	1 (6%)
Teaching Experience	
< 5 years	12 (66%)
5 - 10 years	4 (22%)
10 - 15 years	1(6%)
> 15 years	1 (6%)

(Continued)

Grade level	
PK-K	2 (11%)
First grade	2 (11%)
Second grade	1 (6%)
First, second, and third grade	4 (23%)
Third grade	3 (16 %)
Fourth grade	2 (11%)
Fourth & Fifth grade	1 (6%)
Fifth grade	3 (16%)
License Type	
General Education	13 (72%)
Special Education	5 (28%)
Percentage of students with LD in the classroom	
< 15%	14 (78%)
15-30%	2 (11%)
> 30%	2 (11%)
Percentage of students who are EB in the classroom	
< 15%	6 (33%)
15 - 30%	8 (45%)
> 30%	4 (22%)
Received PD related to mathematics instruction (hours)	
0 - 15 hours	11 (61%)
15 - 30 hours	5 (28%)
> 30 hours	2 (11%)

Note: LD = learning disabilities; EB = emergent bilinguals; PD = professional development

Dependent Measures

Qualitative Data Sources (Phase 1)

Classroom Observations

Observations should provide rich information of what takes place inside a classroom for one or more content lessons (Schoenfeld et al., 2018). They provide researchers with (a) access to the process of the research problem, and (b) rich information on instructional practices of which teachers might be unaware (Scanlan et al., 2002). Specifically, classroom observations give researchers the opportunity to obtain information about everyday teachers' practices that might not have been recognized or perceived by teachers due to their busy schedule (Scanlan et al., 2002). As a reference, in public schools a typical third grade mathematics lesson lasts approximately 70 minutes (Hoyer & Sparks, 2017). For the qualitative phase of the study, participants ($n = 9$) were asked to allow researchers to observe two mathematics lessons over the span of two weeks. A total of 16 classroom observations were conducted; due to restrictions in place related to the coronavirus pandemic, one participant was not able to provide observational data (i.e., outside visitors not allowed on campus). Classroom observations allowed researchers to review and analyze teachers' discursive practices during mathematics instruction.

Lesson Plans

Writing lesson plans is considered an important component in teachers' general pedagogical knowledge, as it is closely related to classroom instruction and students' learning outcomes (Ding & Carlson, 2013). Well-thought-out and high-quality lesson plans build a solid base for classroom implementation (Ding & Carlson, 2013). Because mathematical discourse must be planned in advanced to fully support students' conceptual understanding (Krussel et al., 2004), the analysis of teachers' mathematics lesson plans (related to strategic questioning, tasks, and activities to promote mathematical discourse) in the present study was essential to fully understand teachers' practices in the classroom. According to Ferrell (1992), teachers' lesson plans could be an effective evaluation tool to supplement classroom observations. Therefore, participants during Phase 1 of the study were required to provide plans for the lessons the researchers observed. Each participant (n = 9; this included the teacher for whom observations were not conducted) submitted two lesson plans. A total of 18 lesson plans were collected and analyzed. To provide some context on the level of detail in teachers' lesson plans, Appendix I includes examples of two lesson plans (more detailed vs less detailed).

Interviews and Focus Group

Interviews provide a useful way for the researcher to learn about the research problem (Qu & Dumay, 2011). Interviews require substantial planning and preparation to yield rich data. The researcher conducted semi-structured individual (n = 9) and focus group (n = 1) virtual interviews via Zoom, a web-based platform (Creswell & Creswell, 2018). The purpose of conducting a focus group, in addition to individual interviews, was for the researcher to (a) take a less active role in guiding the discussion, (b) lower the researcher's participation in the interview process (Qu & Dumay, 2011) and (c) bring multiple ideas about the same topic into the conversation.

Questions during interviews and focus group were open-ended to allow participants to voice and communicate their experiences unconstrained by the researcher's perspectives. Individual and group interviews were implemented using the following protocol: Basic information about the research study, a brief introduction from the researcher, an opening question (ice breaker), content questions related to their beliefs and practices about mathematical discourse, and closing comments (Creswell & Creswell, 2018). All participants were asked the same predetermined questions. The researcher included probing questions to ask participants to elaborate on their answers when it was needed (e.g., tell me more, I need more detail, what do you mean?). Individual interviews and the focus group (attended by 8 participants) lasted approximately one and one and a half hours; they were recorded and transcribed verbatim for analysis.

Although interviews allow the researcher to control the line of questioning and direct the conversation towards the topics and issues related to the research problem, they also have some limitations such as lack of generalizability and the inclusion of potential bias of the researcher (Creswell & Creswell, 2018; Qu & Dumay, 2011). Therefore, the researcher sought to (a) maintain the flow of the participant's story, (b) promote a positive relationship with participants, and (c) avoid including their own bias about the topic into the conversation (Qu & Dumay, 2011). Tables 5 and 6 include planned interview and focus group questions. Interview and focus group questions include similar topics yet differ from each other to reach a broader content in teachers' explanations, experiences, and examples.

Table 5

Interview Questions

Question Number	Domain	Interview Questions
Question 1	Curriculum	Is your mathematics instruction guided by a specific curriculum? Which one?
Question 2	Teaching Strategy	What math strategies, methods, or activities can effectively support students' understanding during your classroom instruction? Why?
Question 3	Conceptual Understanding	How do you define conceptual understanding?
Question 4		Can you give me an example of a teaching strategy or activity often used in your classroom to promote conceptual understanding?
Question 5		How do students show comprehension of a mathematical concept or skill?
Question 6	Mathematical Discourse	How do you define mathematical discourse?
Question 7		Do you think mathematical discourse is an important component of mathematics instruction? Why and how?
Question 8		Do you think mathematical discourse promotes student's conceptual understanding? Why and how?
Question 9	Intentional Planning	Do you plan for the implementation or integration of mathematical discussions during your instruction? How?
Question 8		How and when do you decide you will include math discourse in your instruction?

Note: Adapted from Barkatsas & Malone (2005) and Schoenfeld (2014).

Table 6

Focus Group Questions

Question Number	Domain	Focus Group Questions
Question 1	Mathematical Discourse	Would you consider mathematical discourse and essential component of your teaching? Why?
Question 2	Grouping Strategies	What do you think about different grouping configurations during classroom instruction?
Question 3		Can grouping configuration affect mathematical discourse? How? Why?
Question 4	Mathematical Content	Do you think specific mathematical content promotes mathematical discourse during instruction? How? Why?
Question 5	Student Participation	Who does and does not participate in the mathematical discourse of the class?
Question 6		How can students participate during classroom discussions? (e.g., talking, writing, leaning in, listening hard, manipulating symbols, making diagrams, interpreting graphs, using manipulatives, connecting different strategies)
Question 7		What opportunities exist in your classroom for each student to participate in math discussions and explain their own mathematical ideas, as well as respond to each other's? What about students with LD, emergent bilinguals, or emergent bilinguals with LD?
Question 8		How can you create opportunities for more students to participate more actively during math discourse?
Question 9		How can your own interactions facilitate participation of all students?

(Continued)

	• What about students with LD? • What about emergent bilinguals? • What about emergent bilinguals with LD? How can your own interactions inhibit participation of all students? • What about students with LD? • What about emergent bilinguals? • What about emergent bilinguals with LD?
Question 10	How can you support those students that are not often involved in the classroom discourse?

Note: Adapted from Barkatsas & Malone (2005) and Schoenfeld (2014).

Quantitative Measure (Phases 2 and 3)

Teachers' Beliefs and Practices on Mathematical Discourse Survey

The quantitative survey was developed by the researcher based on the salient themes and categories that originated from the qualitative data analysis (Phase 1; See Appendix H). The researcher created the survey following specific research related to (a) the construction and validation of multiple item scales used to assess people's beliefs, values, and opinions (Spector, 1992) and (b) the design and use of research instruments to describe beliefs and practices of mathematics teachers (Swan, 2006). Before being used for pilot testing purposes (Phases 2 and 3 of the study), the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* was approved by the university IRB and pretested with Phase 1 participants (*n* = 9; Creswell & Creswell, 2018) to measure its content and response validity. Pretesting the survey helped the researcher identify statements that (a) required rephrasing, (b) needed to be removed, and (c) fit better under a different domain (Schroder et al., 2011). Last, the researcher pilot tested the survey ($n = 18$) to address the internal consistency reliability of the survey (e.g., Cronbach's alpha).

Data Collection and Analysis Procedures

Data Collection and Analysis (Phase 1)

Classroom Observations

All Participants (in Phase 1 of the study; n=9) except one were able to let researchers observe their teaching during mathematics instruction (full lesson). A total of 16 elementary mathematics lessons were observed. Classroom observations were quantitatively analyzed using Schoenfeld's Mathematical Discussions Coding (MDC) rubric (Schoenfeld, 2013; see Appendix B). The rubric describes (a) mathematics teachers' behaviors during the implementation of mathematical discourse in the classroom (i.e., richness of mathematics, teacher's mathematical integrity, soliciting student reasoning, assessing understanding, pacing discussion, opportunities for deeper mathematical conversations, and addressing/engaging misconceptions) and (b) students' behaviors during mathematical discussions (i.e., participation, risks, and student explanations). The rubric contains level descriptors (i.e., low, average, and high) for teachers and students' behavior. Although the present research study did not collect data on students, scoring students' behaviors during mathematical discourse provided a context to help the researcher better understand the actions of the teacher.

In addition to the MDC rubric (Schoenfeld, 2013), the researcher included two components (i.e., access to mathematical content; agency, authority, and identity) of the Teaching for Robust Understanding of Mathematics (TRU) rubric (Schoenfeld, 2014; See Appendix C). First, the access to mathematical content component describes teachers' support to access the content of the lesson for all students in the classroom. Second, the agency, authority, and identity component describes students' opportunities to be the source of ideas, discussions, and contributions in the classroom. Both rubrics (i.e., MDC and TRU) provided the researcher with rich information about teachers' discursive practices during instruction. Specifically, the rubrics allowed the researcher to measure different teacher and student behaviors during mathematical discourse. Importantly, the researcher had the opportunity to observe (a) students' access and participation in discussions and (b) the mathematical understanding expectations from the teacher. The use of the rubrics yielded a quantitative score that granted the researcher with the opportunity to perform a descriptive statistics analysis (i.e., mean, standard deviation, range).

The researcher also conducted a deductive thematic analysis (Nowell et al., 2017) on observations of teachers' practices during classroom discussions and observation field notes to explore patterns in teacher-student interactions and to understand how teachers promoted mathematical discourse supportive of all students in the classroom, including those at-risk of mathematics difficulties. The researcher included Mortimer and Scott's (2003) framework on the communicative approach (See Table 7) to code student-teacher discourse during mathematics instruction. The unit of analysis to code classroom observations was the lesson. This framework aims to capture the level of interactivity occurring between students and teachers during classroom discussions. Discussions can be analyzed as (a) non-interactive or interactive and (b) authoritative or dialogic (Mortimer & Scott, 2003). Interactive talk involves more than one person participating in a classroom discussion as opposed to non-interactive talk associated with the exclusion of student participation. Authoritative talk is described as a teacher-dominated discussion. Dialogic talk involves substantial co-participation between teacher and students and the consideration of multiple ideas and points of view (Silva Pimentel & McNeill, 2016). As mentioned before, coding students' participation during mathematical discourse allowed the researcher to better understand teachers' mathematical practices. To demonstrate consistent estimates of the same teacher or student behavior among multiple coders, 25% of observations of mathematics lessons ($n = 4$) were randomly selected, observed, and independently coded by two researchers. Interrater reliability (IRR) was calculated by adding the total number of agreements between researchers and dividing that number by the total number of scores contained in the observational rubrics and framework. Initial IRR was 90.6%. After comparing scores, researchers talked about their disagreements to come to an agreement. Final IRR was 100%.

The researcher incorporated the resulting data (i.e., means and standard deviations) from the quantitative analysis of observation rubrics and the qualitative data (i.e., coding patterns) from the qualitative analysis to find emerging themes from the observed data. Through a constant comparison analysis (Strauss & Corbin, 1998), the researcher found themes that arose from teachers' observed practices related to the implementation of mathematical discourse during instruction. Specifically, the researcher paid attention to commonalities observed in teachers' discursive practices through quantitative and qualitative analyses.

Table 7

Classroom Discourse Coding Framework

Topic	Type	Descriptor
Initiator	Teacher-initiated	Teacher starts the conversation
	Student-initiated	Students start the conversation
Level	Interactive	Teacher and students participate and provide ideas
	Non-interactive	Teacher is the only one that talks and gives ideas.
Type	Dialogic	Teacher assumes neutral position
	Authoritative	Teacher's authority determines direction of discourse
Answers	Brief	One-word, expressing numerical value, gesturing a value (agree/disagree)
	Extended	Multiple words and/or utterances

Note: Adapted from Mortimer & Scott (2003).

Lesson Plans

Teachers' lesson plans (n = 18) were analyzed through content analysis (Bazerman, 2006) to examine teachers' planned activities or strategies that included the explicit implementation of mathematical discourse. The unit of analysis was each instructional block (specified by teachers in their lesson plan; Avalos et al., 2021), which included activities and assessments during whole group, small group (center group), guided practice, and independent practice. The researcher implemented an inductive coding approach to analyze classroom activities included in teachers' lesson plans (Thomas, 2006), themes and categories emerged (emergent coding) from the data after the researcher completed a thorough examination (Creswell & Creswell, 2018). First, the researcher (a) read the lesson plans multiple times to describe salient categories from classroom activities and strategies planned by teachers, and (b) wrote memos about the categories to discover potential associations among them (Thomas, 2016). Namely, the researcher created a coding framework that included categories, subcategories, and codes related to the planning for mathematical discourse during the lesson. Then, the researcher used the constant comparative method (Strauss & Corbin, 1998) to be able to interpret the data. Specifically, the researcher systematically compared patterns within and across teachers' instructional blocks written in their lesson plans. As a result, main themes emerged from the observed data of teachers' lesson plans. It is important to consider that findings inevitably were shaped by the researcher's experiences and assumptions (Thomas, 2016) about classroom activities that promote mathematical discourse. Thus, triangulation from different data sources (e.g., observations, interviews, lesson plans) was essential to add validity to the findings (Creswell & Creswell, 2018). Table 8 describes the coding framework that emerged from the inductive analysis of teachers' lesson plans.

Table 8

Mathematics Lesson Plans Coding Framework

Topic	Type	Example text from lesson plans
Discourse Activities	Includes Discourse	Share and discuss solutions, have students explain their reasoning to the class, turn and talk, share with a partner (Activities that foster teacher-student and student-student interactions).
	Does not Include Discourse	Listen & look for, independently work on a package on multi-digit addition (Activities that do not require teacher and peer interactions).
Designer of Activities	Curriculum-based Activities	Solve and Share, Convince Me! Essential Question (Activities explicitly included in the curriculum).
	Teacher Created Activities	Answer questions through mystery sticks and they will discuss answers with students using sentence frames (Activities designed by the teacher).
Level of Specificity in Activities	Scripted	Can you tell how many counters there are now without counting again from 1? Where did you start? What did you do next? (Explicit description of discourse questions and scaffolds in the lesson plan)
	Not Scripted	Students will share answers, Teacher will discuss the answers as a class (No inclusion of discourse questions or scaffolds in the lesson plan).

Interviews and Focus Groups

Virtual interviews (n = 9) and focus groups (n = 1) were conducted at suitable times for the teachers. Interviews and focus groups were analyzed through a qualitative thematic analysis (Guest, MacQueen, & Namey, 2012). Interviews and focus groups were recorded and transcribed verbatim. A transcript-based analysis software (e.g., Dedoose) was used to code and analyze the data. Like the analysis of lesson plans, an inductive analysis was conducted to allow research findings to emerge from dominant themes inherent in raw data without the restraints that other frameworks and methodologies bring (Thomas, 2006). The unit of analysis included multiple conversational turns tied together by a single topic to fully capture teachers' perceptions (Bengochea & Gort, 2020; Milne & Adler, 1999). The researcher created a coding framework that included relevant data from teachers' interview and focus group responses. To explain the data, the researcher applied the constant comparative method (Strauss & Corbin, 1998), which required a continuous classification and comparison of data (similarities and differences) across categories. Specifically, the researcher coded emerging themes and categories and assigned descriptors to codes to be able to understand the data. The researcher also created memos to uncover possible connections and patterns among categories. In other words, the researcher focused on the interceptions of codes across categories to find patterns emerging from the data. The intersections of codes that were constantly observed across the data were selected to create emerging themes. Table 9 presents categories, code names, and examples of selected codes which were criteria to measure inter-coder reliability (ICR). The coding framework developed by the researcher captured the analytical significant attributes of the data (O'Connor & Joffe, 2020). As indicated, topics included in interview and focus group questions were closely related. Therefore, to code both interviews and focus group data, the researcher used the same unit of analysis and coding framework.

Table 9

Coding Framework to Analyze Topics, Activities, and Discursive Practices Addressed by Teachers in Interviews and Focus Groups

Category	Code	Examples
Activities and Strategies During Instruction	Activity	Math Freckle, review games, budget games, skip counting, would you rather math game, teacher created resources, math songs and videos.
	Instructional Strategy	Anchor charts, graphic organizers, CUBES strategy to solve word problems, think aloud, explicit instruction (I do, we do, you do), task analysis.
Challenges During Instruction	*COVID-19* Issues	Absenteeism, lower achievement, online teaching, lack of grouping strategies, slow pace.
	Time Restrictions	To include scope and sequence content, not able to finish math activities.
Curriculum	Discourse	Solve and Share activities.
	Projects	Everyday STEM projects.
	Technology	Interactive videos, workbook aligned to computer component.
	Problem Solving	Word problems, many opportunities to practice.
Discourse During Instruction	Addressing Misconceptions	Where you made your mistake? What is wrong? Immediate feedback, reteaching opportunities.
	Assessing Understanding	Do you understand? How did you get it? What did you do to solve it? Does it make sense? Can you explain it? Turn and teach to your partner.
	Assessing Procedural Knowledge	Do you know how? What comes next? (Questions about the steps to solve the problem).
	Decision Making on When to Implement Discourse	Planned before instruction, activities embedded in the curriculum, during specific "teachable moments", observing specific students' actions.
	Grouping Strategies	Collaborative learning (during small, whole group instruction, one on one, and partner

(Continued)

		share), group selection (by abilities, English language development, or gender).
	Discourse Participation	Selection of students (randomly, students who raise their hand), no participation (due to being afraid to take risks, not understanding the content, and still developing English language), confidence to speak, creating a safe space, providing multiple opportunities.
	Modeling Math Discourse	Use of sentence frames, repetition, use of examples.
	Promoting Mathematical Authority	Students share their own ways, new ideas, and strategies to solve problems.
	Soliciting Student Reasoning	Using academic vocabulary and multimodal supports (e.g., visual representations, manipulatives), prior knowledge, different types of questions.
	Dialogic Discourse	Teacher led vs student led, redirecting the conversation.
Mathematical Content	By Grade Level	Mathematics Content Standards (e.g., Common Core), complexity, facts, relationships, sequences, and patterns.
Teaching for Conceptual Understanding	Solving Problems Independently	Do you understand? Show me, Do you know how? Prove it.
	Generalization to Other Content Areas	Every day and world scenarios, project-based learning, figure out mathematics concepts attached to an idea.
Diverse populations	Language	Emergent bilinguals or English learners.
	Ability	Students with and specific learning disability (LD), below grade level, students with an individualized education program (IEP).
	Gender	Boys and girls.

To authenticate the credibility and transparency of the coding process, 21.4% (*n* = 6) of interviews, focus group, and lesson plans (*n* = 28) were independently coded by two researchers (O'Connor & Joffe, 2020; Syed & Nelson, 2015). The researcher coded all interviews, focus group, and lesson plans. To measure the percentage of agreement (intercoder reliability) between coders, the researcher randomly selected two interviews and four lesson plans. A second researcher (graduate research student) served as a reliability coder and coded the specified subset of the total data (Syed & Nelson, 2015). Initially, intercoder reliability between researchers was 78.5%, after discussing all coding differences (Campbell et al., 2013) ICR increased to 100%.

Data Collection and Analysis (Phases 2 and 3)

Teachers' Beliefs and Practices on Mathematical Discourse Survey

The development of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* occurred over a multiphase process: content development (phase 1), survey development and pretesting (phase 2), and pilot testing (phase 3). The purpose of Phase 1 of the study was to discover the content included in the survey. After performing a quantitative and qualitative analysis of teachers' perceived beliefs and practices and teachers' observed practices. Main emerging themes were used to develop a 50-item quantitative survey regarding the planning and implementation of mathematical discourse during instruction for diverse students in inclusive classrooms. Mainly, the pretest phase of the study was intended to understand how potential participants comprehended and responded to each item (e.g., survey validity). The pilot test phase of the study had the purpose to administer the survey to a larger teacher sample to test how items function within the survey (e.g., internal consistency reliability; Gelhback & Brinkworth, 2011).

To develop a good psychometric instrument that will allow the researcher to draw meaningful inferences from the survey's results, the validity and the reliability of the survey were examined (Creswell & Creswell, 2018). First, participants (Phase 1 & 2) were asked to provide specific information (i.e., scale, instrument items, instrument design) about the survey to verify its correct interpretation (Creswell & Creswell, 2018). Specifically, to examine the response validity of the survey, the researcher asked participants to read the questions and explain their thought processes in selecting their answers (Rickards et al., 2012). Feedback from participants provided the researcher with information about clarity and language complexity (Gelhback & Brinkworth, 2011). Second, for content validity purposes the researcher shared the survey with an expert to review it and provide feedback if needed. Overall, this process gave the researcher the opportunity to learn if the survey's content (e.g., themes, language, format) and construct measured what it was intended to measure (Creswell & Creswell, 2018). Last, during Phase 3 of the study (pilot testing phase, $n = 18$), reliability of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* was addressed through a measure of internal consistency reliability (i.e., Cronbach's alpha α) commonly used for questionnaires with multiple items (Bonett & Wright, 2014). Descriptive statistic (i.e., mean and standard deviation) of the pilot tested survey (Nathans et al., 2012) and correlation values were also calculated.

The analysis of the survey's internal consistency reliability was conducted using an advanced statistical analysis software (SPSS). The internal consistency reliability measure (i.e., Cronbach's alpha α) was essential to ensure that scores resulting from the survey were reliable and accurate across all items included in the survey (Creswell & Creswell, 2018). Specifically, Cronbach's alpha determined the correlation of the items in the survey to each other (Tavakol & Dennick, 2011). Because the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* has two main constructs (i.e., teachers' beliefs and teachers' practices), to avoid inflating the value of the alpha, the researcher reported it for each of the constructs rather than for the entire survey (Tavakol & Dennick, 2011).

To determine if the alpha coefficient indicated an appropriate internal consistency of the items of the survey, the researcher followed the suggestion from Tavakol and Dennik (2011) that an acceptable internal consistency coefficient might range from 0.70 to 0.95. Moreover, Gliem and Gliem (2003) suggested that an alpha of 0.80 is a reasonable goal. Specifically, the formula used to calculate the coefficient of internal consistency was = rk / [1+(k-1) r], where k is the number of items in the survey and r is the mean of all inter-item correlations (Gliem & Gliem, 2003).

Summary

The present chapter thoroughly described the methods used by the researcher to examine teachers' beliefs and practices related to the implementation of mathematical discourse during classroom instruction in K-5 general education mathematics settings. A mixed methods exploratory sequential design was implemented to investigate the research problem. Because the study centers on the development of a mathematical discourse teachers' beliefs and practices instrument, it was divided in three phases: (a) qualitative phase, (b) development of the survey and pretesting phase, and (c) pilot testing phase.

The qualitative phase of the study (Phase 1) consisted of the analysis of teachers' perceived beliefs and practices (i.e., interviews and focus group) and teachers' observed practices (i.e., observations and lesson plans). Data analysis included thematic analysis using inductive and deductive coding, content analysis, and descriptive statistics of observation rubrics. The development of the survey and pretesting phase of the study (Phase 2) included the design and construction of a 50-item survey regarding teachers' beliefs and practices about the planning and implementation of mathematical discourse during instruction. In addition, Phase 2 of the study included different measures of validity (e.g., content, response). The last phase of the study (Phase 3) was the pilot testing of the survey, which included the measure of internal consistency reliability (Cronbach's alpha) and descriptive statistics (i.e., mean and standard deviation) and correlation analyses.

To implement the study, two different teacher samples were included. The sample for Phase 1 and 2 of the study consisted of 6 general education teachers and 3 special education teachers. The sample for Phase 3 of the study included 18 teachers: 13 general education teachers and 5 special education teachers. Overall, the present chapter meticulously delineated the methodology of this mixed methods exploratory sequential study.

CHAPTER FOUR

RESULTS

The purpose of the present research study was to increase understanding not only of the beliefs teachers have about mathematical discourse but also how these perceived beliefs might be related to their mathematical practices. Specifically, the researcher sought to better understand how teachers' beliefs regarding the intentional planning and implementation of mathematical discourse during classroom instruction to support students' conceptual understanding could influence their mathematical practices in classrooms with diverse populations of students. Through an exploratory sequential mixed methods design, the researcher aimed to answer the study's following research questions.

1. How do teachers in general education settings describe perceived beliefs and practices related to mathematical discourse? (qualitative)

Analyzed through qualitative thematic analysis.

2. How do teachers implement mathematical discourse in general education settings as measured by teachers' lesson plans and classroom observations? (quantitative and qualitative)

Analyzed through qualitative thematic analysis, qualitative content analysis, and descriptive statistical (i.e., mean, and standard deviation) analysis.

3. Are the validity and reliability estimates of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* sufficient to support its use in research and program planning? (quantitative)

Analyzed through Cronbach's alpha, descriptive statistics (i.e., mean and standard deviation), and correlation analyses.

In particular, the study's exploratory sequential design was composed of a qualitative component (Phase 1), survey development and pretesting component (Phase 2), and a pilot testing component (Phase 3). Findings and results are organized and presented following the study's research design: (a) qualitative part of the research (Phase 1), which answered research questions 1 and 2, and (b) development of the mathematics discourse survey and survey validity and reliability (Phases 2 and 3), which answered research question 3.

Phase 1 (Qualitative Data Analysis)

To broadly investigate teachers' beliefs and practices about the planning and implementation of mathematical discourse during instruction, two research questions guided this specific component of the study:

1. How do teachers in general education settings describe perceived beliefs and practices related to mathematical discourse? (Interviews and focus group data analysis)
2. How do teachers implement mathematical discourse in general education settings as measured by teachers' lesson plans and classroom observations? (Lesson plans and observations data analysis)

Research Question 1: Perceived Beliefs and Practices

In reference to the first research question, findings about teachers' perceived beliefs about mathematical discourse show that teachers believe (a) mathematical discourse is intuitively implemented during instruction without much planning, (b) discourse and collaborative work among peers is less structured (i.e., includes less academic vocabulary) during small groups, but helps to promote conceptual understanding and student engagement, (c) all students should participate in classroom discourse, and (d) mathematical discourse should be explicitly taught and modeled to students. Findings about teachers' perceived practices showed that teachers (a) mainly utilize discourse to assess understanding by soliciting students' mathematical reasoning, (b) use the curriculum to guide their mathematical discourse practices, but also rely on students' behaviors and intuition to initiate discourse during instruction, (c) randomly select students to participate, and (d) implement varied grouping strategies to facilitate discourse. Table 10 provides a summary of the frequency percentage of these main codes related to mathematical discourse that emerged from interview and focus group data. Frequency was calculated by dividing the number of codes from a specific topic (e.g., addressing misconceptions) by the total number of codes coded by the researcher. The purpose of the table is to provide context relative to the main topics discussed during interviews and focus group. Although some topics (e.g., conceptual understanding, grouping strategies, student participation) were explicitly included in interviews and focus group questions, other topics (e.g., soliciting student reasoning, addressing misconceptions, mathematical authority) derived from teachers' responses during interviews and focus group.

Table 10

Code Frequency of Teachers' Perceptions About Mathematical Discourse

Code	Frequency *n* (%)
Addressing Misconceptions	26 (5%)
Assessing Understanding	78 (14%)
Decision Making on when to implement discourse	42 (8%)
Grouping Strategies	90 (16%)
Participation	86 (15%)
Modeling Math Discourse	40 (7%)
Promoting Mathematical Authority	18 (3%)
Soliciting Student Reasoning	139 (25%)
Dialogic Discourse	19 (3%)
Others	22 (4%)

Main findings regarding teachers' perceived beliefs and practices that emerged from the deductive thematic analysis of interviews and focus group data are broadly presented in Table 11, followed by a detailed explanation of emerging themes evidenced by teachers' interview and focus group excerpts and contributions.

Table 11

Main Findings Related to Teachers' Mathematical Discourse Perceived Beliefs and Practices

Theme	Category	Exemplar Excerpts
Theme 1: Assessing Conceptual Understanding Through Reasoning	Using academic vocabulary	*"Mathematical discourse has to include vocabulary; you can't understand the math concept if you don't know what the words attached to the concept are. Especially the further along that you get in math, you kind of need that."* (Interview Teacher H)
		"They need to know that equation refers to anything whether it be addition, subtraction, multiplication or division, and being able to use that content specific vocabulary." (Interview Teacher D)
	Using multimodal supports	*"If they need another visual, I have a visual video that I show them, and it breaks it down a little further with the vocabulary."* (Interview Teacher G)
		"I always provide [a] manipulative for the kids, if possible, use which you're most comfortable with. If you need blocks to count or the number line, get up, use the number line." (Focus Group Teacher F)
Theme 2: Participation of All Students in Mathematical Discourse	Using different grouping strategies	*"I guess discourse and grouping strategies work, work pretty well when there's one teacher in the classroom."* (Interview Teacher I)
		"I think discourse happens [during], small group, whole group, [and] partner share because they [students] learn from each other." (Focus Group Teacher F)
	Collaborative small groups with less structured discourse	*"I like discourse in smaller groups. I feel like [when] working with smaller groups you get to hear from less students at a time. And then you can say confidently, okay, [from] these two [students] working together, she understands it [content] and she does not. But they're working together and she's explaining it."* (Interview Teacher G)

(Continued)

		"I do a lot of small group work, I find that's also very effective, it's a lot of that peer collaboration. I think they [students] learn so much from each other." (Interview Teacher B)
Theme 3: Teaching and Modeling Discourse	Discourse planning is not needed.	*"I feel like it [discourse] just comes naturally, I just feel like I innately see it [content] through the eyes of a kid and see what they're thinking, and then I just go based on that because I know all the different potential answers that they're [students] going to get and then I just base it [discourse] on that."* (Focus Group Teacher A) *"Many times, it's on the fly [implementing discourse], just like I noticed they need to talk and so maybe I'll have them turn and talk."* (Interview Teacher B)

Theme 1: Assessing Conceptual Understanding Through Reasoning

The first theme that emerged from both qualitative sources (i.e., interviews and focus group) was that teachers implement mathematical discourse to assess understanding by soliciting students' reasoning. Specifically, teachers relied on (a) academic vocabulary and (b) multimodal supports to assess conceptual understanding when soliciting students' reasoning. Evidence of how teachers emphasized the need for mathematical discourse as a means of assessing students understanding is presented in the following paragraphs.

Teachers mostly saw mathematical discourse as an avenue to assess students' understanding. Their purpose for implementing discourse was to verify if students could explain mathematical concepts and justify their claims (found across all 9 participants). The following three excerpts show evidence of teachers' intention to assess their students' understanding of the content by soliciting explanations of their reasoning and justifications of their claims in a precise and explicit manner. Indeed, teachers' interpretation of students' conceptual understanding was listening to students' explanations and justifications of their mathematical reasoning. Even though continuous prompting of students' mathematical reasoning was the main component of the classroom discourse, teachers were convinced that students understood the concept being presented only when they were able to independently explain and justify their reasoning. For instance, Teacher F noted*, "I'm like, okay well how did you get that? Why did you get 84? What did you have to do? They understand what they need to do, but I don't know if they got the full concept."* In the excerpt below a teacher implements mathematical discourse to assess their students' understanding of the underlying mathematical concepts. In this case, teacher G stresses during her interview that students should be able to explain their reasoning without prompting or help.

Teacher G: If they [students] can explain the why and the how to me without me needing to dig for it. So, if they can…. if I say okay, how did you get that? [and the student answers] well I did this and this because of this and that and this is what I got. Okay, [I know] they understand the concepts.

The teacher's interpretation of their students' conceptual understanding is through the explanation and justification of mathematical reasoning included in students' responses. By saying "*without me needing to dig for it*", teacher G implies that students show conceptual understanding when she does not need to prompt them to correctly explain their mathematical reasoning. Similarly, the excerpt below highlights the interpretation another teacher has about their students' conceptual understanding. During the focus group, teacher H addresses the importance of students' explanations of their reasoning during discourse to assess understanding.

Teacher H: if they [students] are able to explain to you how they did it [solved the *problem], even because they might solve it [the problem] correctly, but if they don't explain their reasoning, or how they solved that [the problem], you are not certain are you? You don't feel, you don't feel like you can check that okay they understood the concept.*

Teacher H is waiting for students to correctly explain their thinking and reasoning to feel comfortable to move forward with her teaching. Teachers' continuous prompting of students' mathematical reasoning not only helps them to corroborate if students have reached an understanding of the concept, but also if students have been able to process how to solve the problem. *For instance, teacher C commented, "He knew the process, he could explain it to you. But he didn't have the number concept portion of it."* In contrast, the following interview excerpt shows how teacher E recalls a conversation with her student. After asking some questions, teacher E recognized that the student not only understood the underlying concept but used a different strategy to solve the problem.

Teacher E: And then one kid is explaining it to me [the content], and I'm looking at it, I'm listening to his explanation. And I knew what he was doing. It was definitely different than what I taught. Yeah, he was breaking it down more, so it was like 632 times five, and he's like, okay, you have to do five times 600, he's explaining it to me. I understood the concept. He understood the concept. Well, he was doing it differently. He couldn't multiply it, so he explained how to do it, although he didn't know how to do it the same way. Yeah, and that's another thing because one thing is the underlying concept of how to apply it, and then another thing is that process that they need to follow.

In this case, teacher E realized that the student understood the underlying concept of multiplication but decided to use a different strategy to multiply 632 times five because he did not know how to solve it the same way than the teacher. As can be seen, by including mathematical discourse during their instruction, teachers sought to assess their students' understanding by asking and prompting their students' thinking and reasoning. While soliciting students' reasoning, most teachers (i.e., 7 of 9 participants) required their students to use (a) academic vocabulary and/or (b) multimodal supports (e.g., manipulatives, visual representations).

Using Academic Vocabulary. Through the explanation of mathematical reasoning, academic vocabulary became an essential component of the classroom discourse. Teachers demanded the inclusion of academic vocabulary when students explained their thinking and reasoning (e.g., "*being able to use mathematical terms when explaining*"). Many teachers (i.e., 7 out of 9 participants) expressed that they required their students to use academic vocabulary when participating in classroom discourse. The following three excerpts highlight teachers' request to use academic vocabulary during discourse. In the next interview excerpt, teacher A expects her students to use academic vocabulary in their mathematics register.

Teacher A: We're talking about the problem, we're talking about the numbers, we're thinking about the strategies, right, addition, subtraction, and multiplication. How do you know what are the words that you're looking for? Right, and use your math words, what are some math words? When I say equation, what do I mean by equation?

Teacher A implies that by using specific mathematics vocabulary (e.g., equation), students know what an equation is, and therefore, by using it students have certain understanding of the content being taught. Similarly, the following interview excerpt displays teacher H's explanation of the use of mathematics vocabulary essential to convey meaning and understanding during students' explanation of their reasoning.

Teacher H: Part of place value is understanding that a group of ones can be called a 10.

So, that is a specific vocabulary term that they [students] need to know and understand. So, when they're engaging in mathematical discourse, if I asked him a question like talk to your partner about how many groups of 10 you see. When they [students] are discussing, they should use that specific vocabulary word, like oh man I see four 10s, right? They should be able to use that vocabulary, so that is part of it [the content].

The teacher suggests that by using the term tens, students have certain understanding of the mathematics content and the relationship between concepts (i.e., place value and numbers). In other words, the teacher implies that by students' saying four tens they understand the value of the number four and the place value concept. In a different way, the following excerpt from the focus group features teacher G's requisite of the use of academic vocabulary during discourse to reconcile her idea of how mathematical discourse should look like.

Teacher G: How [are] the conversations? What the conversations look like? How are we talking to each other? Are we using the vocabulary? Is it quiet, is it loud when we're doing that?

Specifically, teacher G suggests that vocabulary must be present during discourse and all students should be talking and participating (by being loud) in the classroom discourse. In addition to academic vocabulary, teachers include multimodal supports during mathematical discourse to assess conceptual understanding by soliciting mathematical reasoning.

Using Multimodal Supports. Teachers identified other discourse components in addition to academic vocabulary (i.e., 6 out of 9 participants). In fact, they emphasized the inclusion of multimodal supports (e.g., manipulatives, visual representations) during discourse to assess understanding and facilitate students' mathematical reasoning. The upcoming three excerpts accentuate how teachers leverage the use of manipulatives and visual representations to facilitate mathematical discourse. Specifically, in the following interview excerpt, teacher I suggests the manipulative plays an important role in assisting students on how to (a) conceptualize the content and (b) explain their reasoning during discourse.

Teacher I: I present a problem. And usually, I want to say 90% of the time there's a manipulative that goes along with it. I put the kids in groups, I give them the manipulative, and I give them the word problem or the equation. And I'm like, okay, solve it using these tools, that [students solving the problem] kind of gives me the opportunity to walk around and see where the kids are at before I teach it [the new content], what are they understanding, what specifically do I need to review, are they using that vocabulary?

Teacher I is suggesting that manipulatives and academic vocabulary should be present while students work collaboratively and discuss how to solve the problem with each other. In this specific case, the teacher is assessing students' prior knowledge and understanding of previously learned content before teaching new mathematical content. In her explanation, by students correctly using the manipulative and the academic vocabulary she can notice if there is some understanding of the mathematical content. In a similar way, the interview excerpt below showcases how teacher B uses a manipulative to elicit discourse that let her know if students' explanations of their reasoning show understanding of the concept or memorization of the procedural knowledge.

Teacher B: If I asked them and they're using a manipulative, I try to not give an indication of right or wrong, so I kind of just stay objective. Like, okay and [I ask] why did you do that? And then they kind of explain. And then at the end, I'll be like, can I share one of my ideas with you? And I'll show them, and then I asked them, why do you think I did it this way instead? And that kind of helps me to see if they're making the connection, or if they're *just kind of following the steps instead of understanding the deeper concept.*

In this case, teacher B tries to promote students' collaboration, discussion, and problem solving using a manipulative without her input. Once students have explained their mathematical reasoning, the teacher guides their students' thinking by solving the problem the right way. Importantly, the teacher continues to prompt her students about the reasoning behind the solution of the problem to corroborate their development of conceptual understanding, and not only a memorization of the process. Teachers also use visual representations to facilitate discourse that promotes students' explanations of their reasoning and understanding. The following excerpt from an interview showcases the use of a visual (picture of an equation) to promote mathematical discourse. The picture plays an important role during the discussion by facilitating students' mathematical thinking and reasoning, while the teacher asks questions.

Teacher D: Sometimes I just put a picture on the board, and it has usually an equation or a word problem to go with it, and I don't read it, I don't do anything. And I'm like, okay, just look at the picture, tell me what you see, what do you notice, and then find somebody who notices the same thing. As you find somebody who sees something different, talk about it [mathematical thinking]. And then there is no pressure, there's no right or wrong answer. They [students] start to find friends who see the same thing as them and [also students who see] different things, which helps them to talk about what they see.

In her answer, teacher D introduces the visual during the discourse to promote the inclusion of multiple ideas or ways to solve the problem. Students had the opportunity to justify their reasoning and hear different explanations from their peers about ways to solve the same mathematical problem. As can be seen, teachers purposefully implement discourse to assess their students' understanding. They continuously prompt their students to explain their mathematical reasoning using not only academic vocabulary, but also multimodal supports that are an essential component of the mathematics register.

Theme 2: Participation of All Students in Mathematical Discourse

The second theme that emerged from both qualitative sources (i.e., interviews and focus group) was that teachers believed all students should participate in classroom discourse (found across all participants). Specifically, teachers believed that (a) all grouping strategies (e.g., whole group, small group, partners, or one on one setting) facilitated interactive mathematical discourse and (b) small groups promoted student collaboration with a less structured discourse (using less academic vocabulary).

Teachers' perception of mathematical discourse involves interactive student participation, where students share their thoughts, ideas, and reasoning. During her interview, teacher A commented: "*I want everybody participating, right? So, like, when I think about discourse usually because my classes are only like six or 10 kids, I expect that everybody is participating. That's what discourse looks like. During our [classroom] discourse everybody is talking, everybody is following, everybody is answering. Even if the question is wrong right in my class, I feel like that's what discourse looks like, because we're in a small group setting."* As can be seen, teacher A has a clear idea of how mathematical discourse should look, which includes the participation of all students by answering and talking to each other. In the following two excerpts, teachers expressed the same belief about the participation of all students during discourse, and how they made it possible (e.g., using a check list, random selection). During the focus group, teacher E expressed how she makes sure all her students are participating during the lesson.

Teacher E: So, there's a lot of times that I kind of do a little checklist on. For instance, if I *ask a student a question, they give me an answer. Then I elaborate on it, and I, I kind of try to get them to go deeper into it, and I have just like a little checklist on who I did that with so that I don't forget students so, but sometimes it happens.*

In this specific example, teacher E tries to assess a deeper level of understanding, which requires asking multiple questions to the same student. She expresses her intention to assess all students' understanding on a deeper level by creating a small student checklist. In other cases, teachers randomly select their students to include both students that like to talk and those who are quiet in the classroom discourse. For instance, during the focus group teacher H mentioned, "*I am definitely that teacher who cold calls every single student, every single time.*" Similarly, in her interview teacher B commented:

Teacher B: I do have those students who don't like to participate. I do random [selection]. Okay, but sometimes with random sticks, if I choose their name, and they just look at me, I will tell them that I'm going to come back to you. Let's listen to what some other friends did because you know that they're struggling.

In the excerpt above, teacher B randomly selects students to make sure all students participate during discourse. She acknowledges that some students are still in the process of understanding the content and do not feel confident to answer. Thus, she accommodates her students' needs by offering to later come back to them with a question. This action communicates to students that she expects everyone to participate, but she will give them the opportunity to do it when they are ready. Teachers envision mathematical discourse as interactive conversations (i.e., including all students in the classroom) occurring during whole, small, and partners settings.

Using Different Grouping Strategies. Teachers believed all grouping strategies (e.g., whole group, small group, pairs, one on one) facilitated interactive mathematical discourse (found across all participants). Teachers expressed that mathematical discourse could be implemented at the beginning, during, and at the end of the lesson through different grouping strategies. Specifically, teachers use multiple grouping strategies as a venue to promote discourse and student participation, collaboration, and engagement (e.g., *"talking to your friend or your buddy next to you like, how do we do this and figuring out together can be more fun and engaging"*). Teachers believe that all grouping strategies facilitate the implementation of interactive mathematical discourse. As an example, teacher E made the following comment during the focus group: *"Whole group, small group, [and] one on one. All [grouping strategies] promote discourse."* The following two excerpts showcase how teachers leverage multiple ways to group their students to promote conceptual understanding and student participation in the classroom discourse. The upcoming focus group excerpt exemplifies the way teacher H uses different grouping strategies during her instruction.

> *Teacher H: Taking what they [students] experienced [during] whole group and hearing different conversations between myself and other kids back and forth. They [students] have all that going on in their mind so when they get in their group, and they're discussing with the other learners in that group, they [students] are able to kind of either pull on some of the things that they heard during the whole group if they want to*.

Teacher H implements whole group instruction to bring multiple mathematics ideas and experiences to the conversation. She emphasizes the importance of students sharing their thinking with her and listening to their peers' explanations of their thinking and reasoning. Then, the teacher implements small group discussions to facilitate mathematical discourse among peers. In a similar way, the following interview excerpt shows the way teacher B introduces different grouping strategies into her mathematics lesson.

Teacher B: [During] Whole group, I'll ask the questions, tell them what is expected, and then they go into smaller groups. Today I selected their groups for them because they needed that strong leader kid.

In the example above, during whole group discourse, teacher B prompts her students on what to discuss with their peers, then purposefully selects groups with the intention to include a specific student that can move the mathematical discourse forward. Teachers consider different grouping strategies to promote participation of all students depending on their students' abilities or preferences. For instance, during the focus group teacher I commented *"Some [students] will be quiet during whole group, and just come to life during small group and centers. Some [students]will be super quiet during small group and centers because they just don't feel like talking, you know, but they want to shine in front of everybody."* The teacher could clearly recognize that some students prefer to participate in whole group versus small group discourse or vice versa. Although teachers believed all grouping strategies were conducive of interactive mathematical discourse, some teachers specifically expressed the advantages of implementing discourse during small groups.

Collaborative Small Groups with Less Structured Discourse. Although mathematical discourse is promoted through different grouping strategies, some teachers believe small groups discourse may be less structured (includes less academic vocabulary), but better promotes participation, collaboration, engagement, and confidence (i.e., 3 out of 9 participants). Teachers believe that during small group students talk and work with their peers in a collaborative way. For instance, teacher C commented, *"The important part to do is to collaborate, because it helps with the understanding."* The three excerpts below exhibit teachers' beliefs about the benefits of implementing mathematical discourse in small group settings. Specifically, the following excerpt displays teacher's I opinion on the value of implementing small group discourse, even if students are not continuously using academic vocabulary while discussing their reasoning and thinking. During her interview, teacher I noted:

Teacher I: With partners, although they're not using that precise language all the time and the academic vocabulary all the time because they are not always challenged by their teacher. I still do find a really big benefit to it because, for I mean for a lot of reasons, like relationship building, confidence boosting, like so many things but they really it's just, it's like a non-pressured way where it's like, there's no pressure here, the whole class isn't watching you, and a partner can just sit down with their whiteboard and be like okay this is the strategy I did, I drew a picture or whatever. How about you? Try this and it's just that like friendly comfortable way to make mistakes without 24 students watching you.

Although teacher I acknowledges the importance of using academic vocabulary during discourse, she still believes small group discourse has multiple benefits for students, such as promoting students' self-confidence and collaboration. The teacher values the student-to-student interaction during partner talk. Similarly, the focus group excerpt below highlights teacher B's belief that during small group mathematical discourse is more informal, but beneficial.

Teacher B: When you do it [discourse]during whole group, it's more formal and when they [students] talk with their partners, it's a little bit less formal, and it's kind of like you know just talking in normal way. So, I feel like both are helpful.

In this case, teacher B implies that being able to talk in a normal way (without incorporating academic vocabulary) helps students feel more comfortable, which does not happen when students are formally (using academic vocabulary) discussing in a whole group setting. Moreover, teachers (i.e., 5 out of 9 participants) believe that discourse in small group settings promotes peer collaboration and learning. The following interview excerpt illustrates teacher's D opinion on the important role small group settings play in fostering peer tutoring and collaboration during discourse.

> *Teacher D: Talking about it [mathematical reasoning] with their peers, I think they learn really well from each other, so I think it [small group setting] is a really important component because they learn really well from their peers.*

In this example, the teacher clearly values the discourse among peers because it leads to a collaboration that produces mathematics learning. The teacher emphasizes the unique way students learn from one another during small group discourse. Therefore, even if small group discourse might lack the academic vocabulary essential to assess understanding, teachers believe it is still very valuable to promote self-confidence, collaboration, student interaction, engagement, and content learning. Overall, teachers believe interactive mathematical discourse happens throughout the lesson during multiple grouping strategies.

Theme 3: Teaching and Modeling Discourse

The last theme that emerged from both qualitative sources (i.e., interviews and focus group) is that teachers believe mathematical discourse should be explicitly taught and modeled to students (e.g., using discourse rules, linguistic supports). Teachers held the belief that students do not know how to carry out and participate in rich and meaningful mathematical discourse if students have not been taught how to do it previously. However, they also believe discourse does not need to be planned beforehand because it develops naturally during their instruction. In other words, they believe that the intentional planning of mathematical discourse in their lesson plans is not necessary because discourse is an implicit component of their instruction.

Teachers not only have an idea of what mathematical discourse should look like (e.g., participation of all students in the classroom, using academic vocabulary) during their instruction, they also believe mathematical discourse needs to be explicitly taught and modeled to students (i.e., 8 out of 9 participants). For instance, during her interview teacher D expressed "*So, give them the examples, teach it [discourse] and then have the examples ready because I live for an anchor chart. So, like anchor charts [are] all over the room. Oh, and we're doing [discourse]. [I say,] hey guys, it's time for partner talk if you forgot what that [partner talk] looks like, here is an example of our rules for partner talk."* In her comment, teacher D implies that she already taught her students how to discuss with their peers, and what are the rules to effectively discuss mathematics in her classroom. In addition, the teacher communicates to her students that she is expecting them to discuss mathematics content following specific rules and gives them the opportunity to check how discourse is done. The following two excerpts exemplify how teachers model and teach their students how to discuss mathematically during their instruction. Specifically, the excerpt below shows how teacher H teaches, models, and practices discourse with her students. During the focus group teacher H commented:

> *Teacher H: You have to teach kids how to do partner talk. Like, I know that we assume that they know how to talk to each other, but they don't know how to do it in an educational format. Right, that [teaching how to discuss] should be your whole lesson on unto itself. This is what partner talk looks like, we're going to practice. Here is an example of what partner talk looks like. Let me show you. Now you're going to practice with your partner about how partner talk looks like. Oh, my friends, and that's when we're doing that redirection. That is not what partner talk looks like, oh my friend, here is a beautiful example of what partner talk is, we are sitting, we're looking at each other, we're taking*

turns.

Like the previous example, teacher H has an idea of how discourse should be done. She emphasizes specific “rules” or conditions, such as looking at each other and taking turns, that need to happen when students are discussing mathematics during partner talk. The teacher suggests that teaching students how to do mathematical discourse involves multiple opportunities that require time. By mentioning that a whole lesson should be dedicated to teaching and modeling how to do discourse, the teacher is implying that it takes time to practice and learn how to effectively talk to peers and teachers. In like manner, the following focus group excerpt demonstrates how teacher E models the way she wants students to discuss mathematics.

Teacher E: You have to model how to do it. And I do, I sit and I pick a student to be my partner and we go through it back and forth. And yes, you have to model [the] conversation, you have to teach him [student], how to have a conversation with the student. I know we work a lot with sentence frames, when we discuss, so it kind of gives them what they need to like, share that they understand the concept if they're missing the words. Like I know this because, or I got the answer because things like that, to kind of help them.

In this case, teacher E not only talks about modeling the conversation to students, but also incorporating linguistic supports (e.g., sentence frames) to facilitate discourse and communication of their understanding. Similarly, in her interview teacher D commented: *“They [students] have the ability to still have communication, but they're more comfortable, they're confident, they know what they're going to say [if] they have that sentence frame to walk around with them.”* In this case, teacher D addresses the importance of linguistic supports, such as sentence frames, to model how to do discourse. By allowing students to use sentence frames, the teacher is not only giving them a linguistic support that facilitates discourse, but also promoting students' confidence to share their mathematical thinking and reasoning with their peers.

Discourse Planning is Not Needed. Teachers believe mathematical discourse does not need to be planned because it develops naturally during instruction (i.e., 7 out of 9 participants). Although research has found that teachers must intentionally plan to enhance opportunities for classroom discourse (Krussel et al., 2004), teachers believe the implementation of classroom discourse is a natural and spontaneous process that does not need to be planned before teaching the lesson. The following excerpts reflect teachers' beliefs about not planning for mathematical discourse due to multiple reasons, such as teaching experience, personality, preferences, and students and teachers' behaviors. For example, during the focus group teacher I commented: *"[In] my lesson plans, [I] don't have all that [discourse planning] in there. Like it's just because I've been teaching for X amount of years, that that [Discourse] just comes by flow."* The teacher attributes not planning for discourse in her lesson plans to her teaching experience. Suggesting that the more experience teachers have, the less they need to plan for mathematical discourse in their lesson plans. In her interview, Teacher H expressed the same teacher sentiment "*I honestly, I'm to the point in my career [that] I do not have time for that [planning for discourse]."*

Other teachers attributed not planning for mathematical discourse to their teaching preferences and personality. In this case, teachers feel that discourse comes naturally without planning. For example, in the focus group teacher D commented: *"Discourse with them [students], that just sort of comes naturally, only because, well, not only because, because I'm a talker, I never know how I feel that day, I never know what kind of questions I want to ask, kind of until I get there."* Specifically, the teacher implies that the decision to implement discourse and the type of discourse she implements during her instruction is decided on the spur of the moment. Likewise, the following excerpt highlights the same belief shared by teacher A during the focus group.

Teacher A: I don't necessarily write it [discourse planning] down because I feel a little bit like I kind of live in the moment of what I'm doing and when I'm doing it. I have a general idea of the questions that I'm going to ask, you kind of have a big picture sort of [the] situation, but I feel like it's just discoursing until it feels comfortable.

Teacher A believes that by planning for specific content and activities around the content, there is no need to plan for mathematical discourse. In other words, discourse comes along with the activities that teachers include in their lesson plans. Moreover, the teacher suggests that the duration of the discourse is determined by feeling comfortable to move on with her teaching. Other teachers implement mathematical discourse when they notice specific students or teachers' behaviors (i.e., 4 out of 9 participants), such as students not raising their hands or teachers talking too much. For example, in the interview excerpt below teacher H decides to implement discourse after noticing her students' behaviors.

Teacher H: When I realized that the problem is kind of difficult and I don't see a lot of people raising hands like, oh, I'll call them sometimes and it's like they don't know the answer. [I say], Okay, it looks like you guys need to turn and talk or something like that.

The teacher decides to implement discourse among peers when she feels students are not raising their hands, and therefore, not understanding the concept. This action implies that the teacher believes discourse will promote some understanding of the content, especially when the content is complex. In other cases, teachers decide to implement discourse when they feel they have been the only ones talking, for example, teacher I expressed in her interview *"I do discourse partner talk. That's something I turned to anytime I start to notice like I'm talking too much."* By saying I am talking too much, the teacher indicates that students also need to be talking during her instruction, and that she should not be the only one talking and explaining the reasoning behind the mathematics content.

Overall, teachers' perceived beliefs and practices related to mathematical discourse indicate that mathematical discourse is mainly implemented to assess their students' understanding. The way teachers assess understanding during discourse is by soliciting students' mathematical reasoning, which requires students to use academic vocabulary and multimodal supports to explain and justify their claims and ideas. In addition, teachers envision mathematical discourse as a natural and interactive process (e.g., happening in different group settings and including all students in the classroom) that requires teaching and modeling, but not necessarily planning prior to instruction.

Research Question 2: Implemented Mathematical Discourse Practices and Planning

In reference to the study's second research question, about how teachers implement mathematical discourse in general education settings as measured by classroom observations and teachers' lesson plans, findings showed that (a) teacher-led, authoritative discourse dominates discursive practices during mathematics instruction, (b) discursive practices are mostly focused on assessing understanding and addressing misconceptions, (c) participation and engagement generally involves all students in the classroom, and (d) planning for mathematical discourse is solely based on activities explicitly included in the curriculum. The researcher coded and scored 16 classroom observations (including field notes) using a discourse coding framework (Mortimer & Scott, 2003) and two mathematical discourse rubrics (i.e., Schoenfeld, 2013; Schoenfeld, 2014). Table 12 highlights descriptive statistic information (i.e., mean, and standard deviation) of teachers' practices scored with two mathematical discourse rubrics (i.e., *Mathematical Discussions Coding* rubric and the *Teaching for Robust Understanding of Mathematics* rubric).

Table 12

Classroom Observations Descriptive Statistics

Item	M (Low=1, Average=3, and High=5)	SD
Richness of Mathematics (engagement of underlying mathematics concepts	3.75	1.44
Teacher's Mathematical Integrity (Mathematics is generally correct and targets key ideas)	4	1.26
Soliciting Student Reasoning	3.125	1.15
Assessing Understanding	3.5	1.37
Pacing of Discussion (engaging and accessible for students)	3.25	1.77
Opportunities for Deeper Mathematical Conversations	3.125	1.36
Addressing/Engaging Misconceptions	3.875	1.26
Student Participation	4.25	1.44
Student Risk Taking	3.5	1.55
Student Explanations (include rationale of their thinking)	3.375	1.50
Access to Mathematical Content (for all students)	4	1.26
Agency, Authority, and Identity (Students are the source of ideas, which are explained and explored)	2.625	1.31

Main themes regarding teachers' observed practices that emerged from the quantitative and qualitative analysis of observations and lesson plans data are presented below. Themes include a detailed explanation of observed results and findings evidenced by rubrics scores, classroom discourse and field notes coding, and teachers' lesson plans analysis. As indicated in the previous chapter, in-person classroom observations were implemented with 8 participants as one participant was not able to provide classroom observational data due to *COVID-19* restrictions.

Theme 1: Teacher-Led and Guided Discourse

The first theme that emerged from the quantitative and qualitative analysis of classroom observations was that teachers Lead and Guided the Direction of the Discourse (observed across all classroom observations). During mathematics instruction, all observed whole group discussions were initiated and led by teachers. Teachers held the responsibility to control and redirect the discourse. Thus, teachers frequently played an authoritative role, which determined the direction of the mathematical discourse (TRU Agency, Authority, and Identity m = 2.625, SD = 1.31). They constantly prompted their students and asked questions that required specific students' answers. In other words, teachers continuously asked questions until certain mathematical content was included in their students' answers. Moreover, teachers kept asking questions to guide their students' mathematical reasoning and thinking (MDC Soliciting Students' Reasoning m = 3.125, SD = 1.15). For example, during a third-grade general education classroom observation, while students were sharing their ideas to the whole classroom on how they solved a multiplication word problem, the teacher kept asking her students to explain their problem-solving decision-making processes *"Why did you choose three times four? Many of you used the multiplicative strategy, what is a different strategy that you could use? Explain this to me, how did you solve it?"*

In addition, it was observed across most classroom observations that students had multiple opportunities to explain their thinking to their peers during dyads or small group discourse (MDC Student Explanations m = 3.375, SD = 1.50). Teachers frequently asked their students to talk to their partner to explain their thinking before asking students to share their ideas to the whole class observed across most classrooms' observations. Teachers asked their students to solve problems together and to choose someone from their group to share their reasoning with the whole class. It was during group or partner activities that some opportunities for student-led discourse were observed (observed across 3 out of 8 participants).

Although students had many opportunities to explain their thinking and reasoning, it was observed that teachers frequently used mathematical discourse to address students' misconceptions (observed across all participants; MDC Addressing/Engaging Misconceptions *m* = 3.875, SD = 1.26). Questions like *"What else are you missing? Or are you sure?"* were used by some teachers (observed across 3 out of 8 participants) to prompt their students to check their thinking. Teachers did not explicitly say to their students that their answer was incorrect; some teachers (observed across 3 out of 8 participants) instead used metacognitive strategies to scaffold their students thinking. The following excerpt includes teacher G's use of metacognitive strategies during her second-grade classroom instruction on arrays (multiplication):

> *Teacher G: What am I going to do first? This is what you should be thinking. Did I do that? Did I understand? Did I miss that step?*

In this case, it is evident that the teacher wants to guide their students' reasoning to correctly solve the problem, but in a way that gives her students the opportunity to do the mathematical thinking, reasoning, and problem solving. In a similar way, teacher D said to her third-grade students solving division problems *"You should be thinking, why did I do it this way? Is this a strategy I could use? How do I know that?"* By assessing understanding (MDC Assessing Understanding *m* = 3.5, SD = 1.37) and addressing misconceptions (MDC Addressing/Engaging Misconceptions *m* = 3.875, SD = 1.26), teachers continued to control and guide the mathematical discourse throughout the lesson.

Theme 2: Discourse Includes All Students

The second theme that emerged from the quantitative and qualitative analysis of classroom observations was that mathematical discourse involved all students in the classroom (found across most classroom observations). Teachers strived to include the participation of all students in the classroom during discursive activities (MDC Student Participation $m = 4.25$, SD = 1.44; TRU Access to Mathematical Content $m = 4$, SD = 1.26) by (a) asking different problem strategies and representations (MDC Richness of Mathematics $m = 3.75$, SD = 1.44), (b) validating students' contributions, (c) controlling the pace of the discussion (MDC Pacing of Discussion m = 3.25, SD = 1.77), and (d) accommodating their students' needs. They were constantly prompting their students to share their thoughts with their peers (observed across 6 out of 8 participants). For instance, during a third-grade classroom observation, teacher B said to her student *"It looks like you are not part of the conversation"*. It was observed that teachers encouraged broad and rich mathematical discussions (observed across 6 out of 8 participants; MDC Richness of Mathematics $m = 3.75$, SD = 1.44) by using multiple ways to promote students' participation (e.g., grouping strategies, ways to select students, classroom activities). Generally, teachers promoted participation of students by asking them the strategy they used to solve the problems. For example, one third grade teacher said to her students during whole group discourse *"This is my strategy, you all can use another strategy to solve the problem."* Standards-based mathematics teaching emphasizes the inclusion of multiple ways (e.g., different strategies) of representing and solving mathematical problems to promote the development of students' conceptual understanding. Teachers frequently prompted their students to contribute to the conversation with different ways to represent the problem (observed across all observations). For example, a kindergarten teacher teaching sorting skills to her students asked:

Teacher E: "Do you have to do it by shapes? What is a different strategy that you could use? What do you think we should do?

Students: (did not answer)

Teacher: Maybe we can do it by color or by size? What do you think?

This conversation highlights teacher's E intention to promote students' mathematical authority by letting them decide their classification criteria. Moreover, the teacher is communicating the underlying idea that there are multiple correct and valid ways to solve the problem. Teachers' discursive practices had repercussions in their students' discursive practices. Observations of students' discursive practices showed that most students took risks on sharing their ideas and explaining their thinking and reasoning (observed in most classroom observations). Even more, students' contributions were validated by the teacher and other students (observed across 5 out of 8 teachers; MDC Student Risk Taking $m = 3.5$, SD = 1.55). For example, during a first-grade lesson one student said, "I used the exact same strategy!"

Although teachers tried to foster students' mathematical identity and authority, observation data showed that in multiple occasions students' ideas were not explored or built upon during the classroom discourse (TRU Agency, Authority, and Identity $m = 2.625$, SD = 1.31). It was also observed (across most classroom observations) that teachers tried to make the content accessible for most students (TRU Access to Mathematical Content $m = 4$, SD = 1.26) by controlling the pace of the discussion and paying attention to their students' specific needs. Comments like *"Are you ready to explain your thinking? Thumbs up if you are ready to move on"* were often observed during the lesson (across most classroom observations). Teachers' accommodations of their students' specific needs were also observed (across most classroom observations) on how teachers assessed their students' understanding. Often, the flow of the discourse was modified based on the assessments teachers made to check their students' understanding. In the following excerpt a fourth-grade teacher said to her student:

Teacher: Explain this to me, how did you solve it? Do you think it's going to work for all?

Student: (thinking)

Teacher: Yeah, in this case, that strategy did not work.

In this case, after the teacher noticed the lack of understanding of her student by applying the wrong strategy to solve the problem, she redirected the discourse to provide clarification of the mathematical content. Overall, students' thinking and reasoning co-constructed mathematical knowledge during discourse. Teachers' efforts to include all students in the classroom increased the complexity and richness of the observed mathematical discourses during instruction.

Theme 3: Discourse Planning and Implementation Based on Curriculum

The last theme that emerged from the qualitative analysis of lesson plans and classroom observations was that teachers' discourse planning and implementation was mainly based on curriculum activities (e.g., mathematical tasks, instructional designs, and representations embedded in mathematics curriculum materials; Remillard & Kim, 2017). The only evidence of teachers' planning for mathematical discourse observed in their lesson plans was the inclusion of mathematics activities embedded in the curriculum. The following two excerpts show examples of instructional blocks included in teachers' mathematics lesson plans. Specifically, the following excerpt displays an instructional block (whole group work) in a third-grade teacher's lesson plan:

> *Topic 5: Review What You Know: SW [students will] try their best to complete the math problems.*
>
> *Lesson Vocabulary: column, equation, even, fact family, odd, row.*
>
> *Whole Group (I do):*

Solve and Share: TW [teacher will] read out the math problem and pick students randomly to help solve.

Look Back: SW [students will] help answer the "look back" question.

Essential Question: How can you explain patterns in the multiplication chart?

Convince Me! TW [Teacher will] model with students the "Convince Me" word problem and check for understanding.

In this example, teacher I's lesson plan includes some activities designed to promote students' conceptual understanding through mathematical discourse during whole group instruction. Although teachers included curriculum-based activities that promoted mathematical discourse (e.g., *solve and share, look back and essential question*) in their lesson plans, that was the only evidence observed of teachers' planning for mathematical discourse before teaching the lesson. Activities embedded in the curriculum, *such as solve and share and convince me*, were also observed in lesson plans of other teachers that used the same curriculum (observed across 6 out of 9 participants). Some teachers (observed across 5 out of 9 participants) also included in their lesson plans the academic vocabulary (e.g., fact families, even, odd) they were planning to teach their students to use during mathematical discourse (as part of their students' mathematical register). In the next paragraph is another example of an instructional block (e.g., small group instruction) included in a kindergarten teacher's lesson plan that also incorporated curriculum-based activities to promote mathematical discourse.

Center Group Teach

Pose the solve and share problem (page 231)

There are 7 fish in a bowl. Emily puts 1 more fish in the bowl.

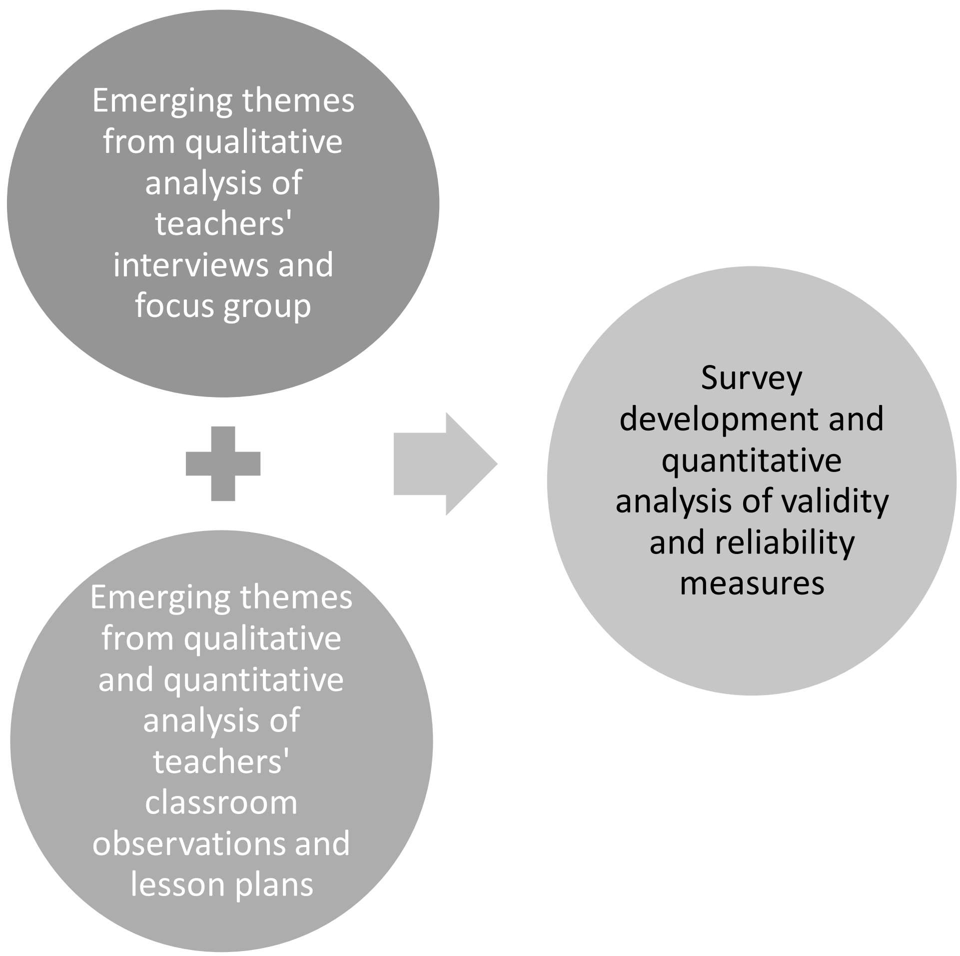
How many fish are in the bowl now? How can you solve this problem? Does something repeat in the problem? How can the solution help me solve another problem? What are *you being asked to find? What tools do you have to solve the problem? What should you do with your counters? How many counters will you need? Do you have to count all the fish shown? After you add 1 more counter do you have to count the fish again? Why not? Share and discuss solutions.*

The example includes a curriculum-based activity (e.g., *solve and share problem*) that targets students' communication of their mathematical reasoning through teacher's questioning and scaffolding. In her lesson plan, teacher E explicitly included the questions she planned to ask her students during small group instruction. It is evident that the planned discourse is intended to assess students' conceptual understanding through a set of brief and open-ended questions that prompts students to elaborate on their thinking, reasoning, and problem-solving processes. Teacher I also planned to promote discourse among her students by including a "*share and discuss*" activity during small group work. Most teachers' lesson plans (observed across 7 out of 9 participants) did not include the level of detail shown in the previous example. Lesson plans including explicit questions to promote mathematical discourse were rarely observed. Classroom observations (observed across all observations) confirmed teachers' implementation of curriculum-based activities to promote mathematical discourse during their instruction as stated in their lesson plans. For instance, third-grade teacher D said to her students *"It's time for solve and share, turn and talk to your partner"*. Students were aware of this activity, thus the transition to talk with their peers was done naturally without many instructions from the teacher.

Overall, the discourse observed during mathematics instruction in K-5 classrooms was initiated, guided, and redirected by teachers. Student-led discourse was rare and only observed during small group activities. Although teachers constantly promoted the participation of all students in the classroom to share their reasoning and thinking, students' explanations often consisted of what they did, but not why they did it. Thus, it was observed (across most observations) that students' mathematical authority and identity was seldom nourished and developed during instruction. Emerging themes were used to develop a quantitative survey related to teachers' beliefs and practices regarding the planning and implementation of mathematical discourse for diverse students in inclusive classrooms. Figure 3 highlights the survey development process through the integration of qualitative and quantitative data.

Figure 3

Data Integration for the Development of the Quantitative Survey



Phases 2 and 3 (Survey Development, Validity and Reliability Analyses)

To investigate the validity and reliability of the developed survey *Teachers' Beliefs and Practices on Mathematical Discourse*, the following research question guided Phases 2 and 3 of the present research study:

3. Are the validity and reliability estimates of the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* sufficient to support its use in research and program planning? (quantitative)

Research Question 3

The researcher developed a 50-item survey related to teachers' beliefs and practices about mathematical discourse from findings and results obtained from the qualitative and quantitative analyses of interviews, focus group, observations, and lesson plans. In other words, statements on the survey were based on the emerging themes found from qualitative and quantitative analyses during Phase 1 of the study. The survey is composed of two 25-item constructs: (a) teachers' perceived beliefs and (b) teachers' perceived practices (see Appendix H). Table 13 displays survey statements grouped by constructs, which in turn are divided in five different categories (i.e., implementation of mathematical discourse, discourse through different grouping strategies, students' participation, explicit teaching and modeling of discourse, and discourse for different purposes). In addition, both constructs include a 6-point Likert scale, which require teachers to indicate (a) the degree to which they agree or disagree with each of the surveys' beliefs statements (i.e., strongly disagree, disagree, slightly disagree, slightly agree, agree, and strongly agree) and (b) the frequency to which they engage in each of the surveys' instructional practice statements (i.e., never, almost never, rarely, occasionally, frequently, and always).

Table 13

Teachers' Beliefs and Practices on Mathematical Discourse Survey

Construct	Categories		Statement
Teachers' Beliefs	Implementation of mathematical discourse	1	There are enough discourse-related activities included in provided mathematics curriculum to integrate discourse into instruction.
		2	Specific student behaviors prompt me to implement discourse during my instruction.
		3	Mathematical discourse does not need to be planned; it comes naturally during my instruction.
		4	The teacher should play a central role during mathematical discourse.
		5	The teacher has a responsibility to maintain control of the classroom discourse.
	Discourse through different grouping strategies	6	The teacher should be part of the discussion regardless of any grouping strategy.
		7	Mathematical discourse is more effective during whole group instruction.
		8	Mathematical discourse and student collaboration are more effective during small group instruction.
		9	All grouping strategies facilitate mathematical discourse.
	Students' participation	10	The teacher should initiate mathematical discourse during instruction.
		11	All students should participate in classroom discourse even if they feel uncomfortable.

(Continued)

		12	Students do not want to participate in classroom discourse because they are still developing their language.
		13	Students do not want to participate in classroom discourse because they are afraid of taking risks in front of their peers.
		14	Students do not participate in the classroom discourse because they do not understand the content.
		15	Students only raise their hand when they understand the concepts being taught.
	Explicit teaching and modeling of discourse	16	Students need to be explicitly taught how to participate in mathematical discourse.
	Discourse for different purposes	17	Mathematical discourse is essential to address my students' understanding.
		18	Mathematical discourse promotes conceptual understanding.
		19	Students learn through discussing their ideas and thinking processes.
		20	Mathematical discourse provides opportunities for students to develop mathematical authority by sharing their own ideas.
		21	The teacher has the responsibility to lead and guide the classroom discourse.
		22	Is easier to learn mathematics if students collaborate and discuss with their peers than if they work by themselves.
		23	Mathematical discourse is essential to address students' misconceptions.

(Continued)

		24	Students should use academic vocabulary when explaining their thinking and reasoning.
		25	An essential way for students to share their reasoning is through discourse.
Teachers' Practices	Implementation of Mathematical Discourse	1	I plan for mathematical discourse in my lesson plans.
		2	I rely on curriculum activities to promote discourse during my instruction.
		3	I decide to implement mathematical discourse by observing my students' behaviors.
		4	I follow my intuition as a teacher when implementing mathematical discourse.
		5	I adapt the way I discuss mathematics with my students depending on their specific needs.
	Discourse through different grouping strategies	6	I promote mathematical discourse during my whole group instruction.
		7	I try to provide opportunities for discourse during small group and independent practice.
		8	I plan small groups having the specific needs of each student in mind.
		9	I let my students choose their own groups or partners during classroom discourse activities.
	Students' participation	10	I encourage my students to participate in the classroom discourse at least once.
		11	I randomly select my students to participate during classroom discourse.

(Continued)

		12	I allow students to ask for help from their peers when they do not know how to explain their reasoning or thinking.
		13	I choose students that raised their hand to participate during classroom discourse.
		14	I encourage my students to elaborate when explaining their thinking.
		15	I ask open-ended questions to my students.
		16	I provide multiple opportunities for students to explain and justify solutions to the problems.
	Explicit teaching and modeling of discourse	17	I explicitly teach and model how to participate in mathematical discourse to my students.
	Discourse for different purposes	18	I require my students to use academic vocabulary during their participation in classroom discourse.
		19	I promote mathematical discourse to assess students' understanding.
		20	I promote mathematical discourse to solicit students' reasoning.
		21	I provide multiple opportunities for students to share their ideas and reasoning through discourse.
		22	I promote the use of non-linguistic supports, like visual representations, during classroom discourse.
		23	I promote my students' development of mathematical authority by sharing their own ideas with the entire class.
		24	I promote mathematical discourse to address misconceptions.

(Continued)

25	I encourage students to make and discuss mistakes.

To address the validity and reliability of the developed survey (*Teachers' Beliefs and Practices on Mathematical Discourse Survey)*, the researcher both pretested and pilot tested the survey with two different participants' samples. As mentioned before, the pretesting phase of the study (Phase 2), which included Phase 1 participants (n = 9), had the purpose of analyzing the survey's content and response validity. The pilot testing phase of the study (Phase 3), which included a convenience sample (n=18), had the purpose of analyzing descriptive statistics (i.e., mean, and standard deviation), correlations, and the survey's internal consistency reliability (Cronbach's alpha).

To measure the survey's content validity, the researcher sent the survey to an expert in the field of special education for feedback. Feedback received was related to (a) clear and precise communication, (b) consistent structure and language, and (c) redundancy of survey statements. Three main edits were made to the survey. First, the researcher reworded the first question on teachers' beliefs to make it comprehensible for the reader. Second, the researcher restructured question number 8 to keep it consistent with question number 7. Last, the researcher revised question number 6 due to its similarity with another question in the survey.

To measure the survey's response validity, the researcher sent the survey to participants from Phase 1 of the study (n = 9) to pretest it (via Qualtrics). In addition, the researcher asked participants (a) to provide information about their understanding of the survey, (b) to communicate if they found any misinterpretations of questions, directions, or procedures, (c) to give an approximate of the time spent answering the survey, and (d) to give suggestions on how to improve the survey. Participants specified that survey statements were clear and easy to understand, survey questions were presented in a concise and brief manner, and survey time completion was on average 15 minutes.

During Phase 3 of the study, the researcher pilot tested the survey and conducted descriptive statistics, correlation, and internal consistency reliability (Cronbach's alpha) analyses from results obtained from Phase 3 participants' surveys. Table 14 presents the means and standard deviations for the developed survey (separated by construct) from the pilot testing phase of the study (Phase 3; $n = 18$). In addition, Table 15 displays correlational data among the five beliefs and practices categories included in the survey. Correlation coefficients (i.e., Pearson's R) were performed to indicate how strong were the relationships between teachers' beliefs and practices. Teachers' perceived practices about the implementation of mathematical discourse were significantly correlated ($p < .05$) with teachers' perceived beliefs about student participation in mathematical discourse ($r = .580$) and teachers' perceived beliefs about explicit teaching and modeling of mathematical discourse ($r = .495$). In addition, teachers' perceived practices about student participation in mathematical discourse were significantly correlated ($p < .01$) with teachers' perceived beliefs about student participation in mathematical discourse ($r = .598$) and with teachers' perceived practices about the implementation of mathematical discourse ($r = .978$). Teachers' perceived practices about explicit teaching and modeling for mathematical discourse were significantly correlated ($p < .05$) with teachers' perceived beliefs about explicit teaching and modeling of mathematical discourse ($r = .514$), with teachers' perceived practices about discourse through different grouping strategies ($r = .477$), and with teachers' perceived practices about student participation in mathematical discourse ($r = .489$). Lastly, teachers' perceived practices about the implementation of mathematical discourse for different purposes were significantly correlated ($p < .05$) with teachers' perceived beliefs about the explicit teaching and modeling of mathematical discourse ($r = .489$) and with teachers' perceived practices about the explicit teaching and modeling of mathematical discourse ($r = .563$).

Table 16 contains the survey's internal consistency reliability coefficients by construct (i.e., beliefs and practices). Cronbach's alpha normally ranges between 0 and 1, greater internal consistency is achieved when the coefficient is closer to 1. Results showed that the alpha coefficient for the 25-item teachers' beliefs construct of the survey was 0.74. Further analysis of the scale by item showed that by removing question 9 from the beliefs' construct, the alpha coefficient increased to 0.76. Results also displayed that the alpha coefficient for the 25-item teachers' practices construct of the survey was 0.76. Further analysis of the scale by item showed that by removing question 9 from the practices' construct, the alpha coefficient increased to 0.80. Results suggest that items in both constructs of the survey have relatively good internal consistency.

Table 14

Teachers' Beliefs and Practices on Mathematical Discourse Survey Descriptive Statistics

Construct	Item	M	SD
Beliefs	There are enough discourse-related activities included in provided mathematics curriculum to integrate discourse into instruction.	4.17	1.10
	Specific student behaviors prompt me to implement discourse during my instruction.	4.39	1.04
	Mathematical discourse does not need to be planned; it comes naturally during my instruction.	3.89	1.60
	The teacher should play a central role during mathematical discourse.	4.78	1.26
	The teacher has a responsibility to maintain control of the classroom discourse.	5.11	1.13
	The teacher should be part of the discussion regardless of any grouping strategy.	4	1.57
	Mathematical discourse is more effective during whole group instruction.	3.33	1.46
	Mathematical discourse and student collaboration are more effective during small group instruction.	4.67	1.19
	All grouping strategies facilitate mathematical discourse.	4.28	1.27
	The teacher should initiate mathematical discourse during instruction.	4.72	1.32
	All students should participate in classroom discourse even if they feel uncomfortable.	3.83	1.38
	Students do not want to participate in classroom discourse because they are still developing their language.	3.28	1.49
	Students do not want to participate in classroom discourse because they are afraid of taking risks in front of their peers.	3.94	1.51

(Continued)

	Students do not participate in the classroom discourse because they do not understand the content.	4.39	1.58
	Students only raise their hand when they understand the concepts being taught.	2.89	1.37
	Students need to be explicitly taught how to participate in mathematical discourse.	4.61	1.24
	Mathematical discourse is essential to address my students' understanding.	4.89	1.28
	Mathematical discourse promotes conceptual understanding.	5.11	1.23
	Students learn through discussing their ideas and thinking processes.	5.17	1.10
	Mathematical discourse provides opportunities for students to develop mathematical authority by sharing their own ideas.	5.06	1.35
	The teacher has the responsibility to lead and guide the classroom discourse.	5	.91
	Is easier to learn mathematics if students collaborate and discuss with their peers than if they work by themselves.	4.67	1.37
	Mathematical discourse is essential to address students' misconceptions.	4.94	1.30
	Students should use academic vocabulary when explaining their thinking and reasoning.	4.94	1.39
	An essential way for students to share their reasoning is through discourse.	4.50	1.58
Practices	I plan for mathematical discourse in my lesson plans.	4.50	1.25
	I rely on curriculum activities to promote discourse during my instruction.	4.56	1.04
	I decide to implement mathematical discourse by observing my students' behaviors.	4.89	.76
	I follow my intuition as a teacher when implementing mathematical discourse.	5.22	.81

(Continued)

I adapt the way I discuss mathematics with my students depending on their specific needs.	5.50	.62
I promote mathematical discourse during my whole group instruction.	4.89	1.13
I try to provide opportunities for discourse during small group and independent practice.	5	.59
I plan small groups having the specific needs of each student in mind.	5.17	1.25
I let my students choose their own groups or partners during classroom discourse activities.	3.50	1.04
I encourage my students to participate in the classroom discourse at least once.	5.39	.61
I randomly select my students to participate during classroom discourse.	4.83	1.10
I allow students to ask for help from their peers when they do not know how to explain their reasoning or thinking.	5.67	.49
I choose students that raised their hand to participate during classroom discourse.	4.44	.86
I encourage my students to elaborate when explaining their thinking.	5.61	.50
I ask open-ended questions to my students.	5.39	.85
I provide multiple opportunities for students to explain and justify solutions to the problems.	5.39	.50
I explicitly teach and model how to participate in mathematical discourse to my students.	5.11	.90
I require my students to use academic vocabulary during their participation in classroom discourse.	4.67	.77
I promote mathematical discourse to assess students' understanding.	5.11	.76
I promote mathematical discourse to solicit students' reasoning.	4.83	.71

(Continued)

I provide multiple opportunities for students to share their ideas and reasoning through discourse.	5.06	.73
I promote the use of non-linguistic supports, like visual representations, during classroom discourse.	5.33	.97
I promote my students' development of mathematical authority by sharing their own ideas with the entire class.	5.17	.86
I promote mathematical discourse to address misconceptions.	5	1.08
I encourage students to make and discuss mistakes.	5.17	.86

Table 15

Correlations among Teachers' Perceived Beliefs and Practices about Mathematical Discourse

Variable	1	2	3	4	5	6	7	8	9	10
1. Beliefs about Implementation of MD	--									
2. Beliefs about Discourse Through Different Grouping Strategies	.372	--								
3. Beliefs about Student Participation in MD	-.018	.075	--							
4. Beliefs about Explicit Teaching and Modeling of Discourse	.046	-.016	.013	--						
5. Beliefs about Discourse for Different Purposes	-.067	-.150	.341	-.207	--					
6. Practices about Implementation of MD	.444	.440	.580*	.495*	.397	--				
7. Practices about Discourse Through Different Grouping Strategies	-.302	-.193	.077	-.176	.262	-.119	--			
8. Practices about Student Participation in MD	.382	.401	.598**	.459	.453	.978**	.091	--		
9. Practices about Explicit Teaching and Modeling of Discourse	.216	.214	.050	.514*	-.165	.387	.477*	.489*	--	
10. Practices about Discourse for Different Purposes	.051	-.172	-.003	.489*	.191	.321	.235	.371	.563*	--

Note: MD = mathematical discourse; n = 18; ** = significant at $p < 0.01$; * = significant at $p<0.05$.

Table 16

Internal Consistency Reliability Coefficients

Survey Construct	*α/ρ*	Items
Teachers' Beliefs	.74	25
Teachers' Practices	.76	25

Summary

The present chapter introduced the findings and results from the three research questions that guided the study. Regarding teachers' perceived beliefs and practices on mathematical discourse, three main themes emerged from the qualitative data analysis of interviews and focus group: (a) teachers' perceived practices related to assessing conceptual understanding through reasoning using academic vocabulary and multimodal supports, such as visual representations and manipulatives, (b) teachers' perceived beliefs on the participation of all students during discourse, which was done using different grouping strategies and implementing collaborative small groups with less structured discourse, and (c) teachers' perceived beliefs on teaching and modeling discourse, which was further explained by the belief that discourse planning is not needed.

The qualitative and quantitative analysis of classroom observations and teachers lesson plans revealed three main themes related to teachers' observed practices: (a) teachers led and guided the direction of the discourse, (b) teachers included all students in the classroom discourse, and (c) teachers mainly based their mathematical discourse planning and implementation on activities included in the curriculum. From the analysis of teachers' perceived and observed beliefs and practices, the researcher developed, pretested, and pilot tested a 50-item survey. Specifically, construct and content validity were addressed during the pretesting phase of the study. After editing the survey according to generated feedback, the researcher pilot tested the survey to measure the internal consistency reliability. Overall, measures of validity and reliability were sufficient to support the *Teachers' Beliefs and Practices on Mathematical Discourse Survey* use in future research and program and professional development planning.

CHAPTER FIVE

DISCUSSION

Mathematical discourse plays an essential role during classroom instruction to support and develop students' conceptual understanding (Jitendra, 2013; Kosko & Gao, 2017). It gives students the opportunity to communicate their own thinking and reasoning and notice multiple, and sometimes more complex, mathematical ideas or claims shared by their teacher or peers (Nathan & Knuth, 2003; Walshaw & Anthony, 2008). With the implementation of the standards-based reform, shifts in teachers' practices are required to incorporate meaningful and rich classroom discourse that will generate multiple opportunities for students to develop their understanding of mathematical content and practice the specialized language of mathematics (Wilkinson, 2018). Even though there is a close relationship between teachers' beliefs, their instructional practices, and their decision-making process in the classroom (Cross, 2009; Handal, 2003; Nathan & Knuth, 2003; Nisbet & Warren, 2000), teachers' beliefs about implementing mathematical discourse are not always congruent with their actions in the classroom (Nathan & Knuth, 2003; Nisbet & Warren, 2000). Therefore, the purpose of this research study was to deepen our understanding regarding teachers' beliefs and practices related to the planning and implementation of mathematical discourse in inclusive general education elementary mathematics settings.

The Academic Literacy in Mathematics conceptual framework (Moschkovich, 2015) served as a guide to analyze the study's data. Specifically, findings and results focused on the inclusion and integration of the frameworks' three main components: Mathematical discourse, mathematical proficiency (conceptual understanding), and mathematical practices (and beliefs). Findings of this mixed methods study encompassed (a) teachers' perceived beliefs and practices,

(b) teachers' observed practices related to mathematical discourse, and (c) the development of a valid and reliable quantitative instrument related to teachers' mathematical discourse beliefs and practices based on the findings and results from the qualitative and quantitative analysis of teachers' perceived beliefs and practices and teachers' observed practices regarding the planning and implementation of mathematical discourse .

Related to teachers' perceived practices, findings showed that teachers mainly implement mathematical discourse to assess students understanding by soliciting students' reasoning; this includes the use of academic vocabulary and multimodal supports (e.g., manipulatives and visual representations). Like this finding, previous research on teachers' beliefs and practices about mathematical discourse have found that teachers believed they should constantly promote and encourage students' explanations of their reasoning and thinking to promote students' conceptual understanding (Bray, 2011; Hufferd-Ackles et al., 2004; Martin et al., 2015). Moreover, research has found that teachers and students' discursive practices included multimodal supports (Quigley, 2021) and academic vocabulary during their instruction (Celedón-Pattichis & Turner, 2012). Teachers' insistence on assessing their students understanding through reasoning clearly aligns with new standards-based educational expectations. By requiring the inclusion of academic vocabulary and multimodal supports during discourse, teachers are developing in their students the mathematical register needed to engage and communicate in uniquely mathematical ways like mathematically competent people do. In contrast, some researchers found evidence on teachers' discursive practices that deviated from standards-based instruction. Their findings showed that teachers often provided limited opportunities for students to engage in mathematical discourse (Engeln et al., 2013), frequently focused on encouragement and not on cognitive development (Walshaw & Anthony, 2008; Wiebe Berry & Kim, 2008), and relied on students' behaviors to assess understanding (Turner et al., 2009). It is important to note that this contrasting evidence was found in research conducted at the beginning of the implementation of the standards-based reform. Findings of more recent research align with this study's findings (Bray, 2011; Martin et al., 2015; Quigley, 2021).

Teachers' beliefs about mathematics and the teaching and learning of mathematics include their perceptions on the role students play in the classroom as active creators of knowledge or passive receivers of knowledge (Handal, 2003; Voss et al., 2013). There has been divergent evidence in relation to students' participation during mathematical discourse. Some researchers (Adler and Ronda, 2015; Erath et al., 2018; Piccolo et al., 2008) found in their research findings that the most frequent discourse practice was teachers' explanation of the content and student participation was very limited. In contrast, findings of this research study showed that teachers believe all students should participate in the classroom discourse and multiple grouping strategies facilitate their participation. This belief was congruent with their discursive practices. Data from classroom observations showed that mathematical discourse involved most students in the classroom during whole and small group instruction throughout the lesson. Teachers frequently tried to foster meaningful mathematical participation and engagement. Similar findings were found by other researchers investigating teachers' discursive practices (Good et al., 1990; Hundeland and colleagues, 2020; Wood et al., 2006). Particularly, findings of this research study displayed teachers believed small group discourse was less structured (e.g., included less academic vocabulary), but it promoted collaboration, engagement, and understanding. This belief is congruent with Kumpulainen & Kaartinen (2003) research that highlighted small group collective reasoning as a venue for collaboration and appreciation of others' contributions.

Research on mathematical discourse has extensively focused on developing and testing mathematics curriculum, instruction, and interventions that emphasize the inclusion of mathematical discourse in classroom activities to promote students' explanations, justifications and argumentations of their reasoning and claims (Banse et al., 2017; Dominguez, 2017; Hundeland et al., 2020; Lewis, 2017; McConney and Perry, 2011; Xin et al., 2020). Specifically, researchers have found that standards-based mathematics curriculum, which emphasizes students' understanding, will unquestionably modify teachers' discursive practices (McConney and Perry, 2011) and foster richer and more profound mathematical discussions of concepts and ideas (Hundeland et al., 2020). Extending previous findings, this research investigated teachers' mathematical discourse planning and implementation through the analysis of teachers' lesson plans and classroom observations. Findings showed that teachers' mainly plan and implement mathematical discourse based on curriculum activities. Moreover, findings displayed that teachers believe mathematical discourse does not need to be planned prior to instruction. In other words, teachers believe they have enough teaching experience to implement mathematical discourse without planning for it beforehand. Although research has found that teachers become more purposeful in the implementation of mathematical discourse when they have intentionally planned for it (Herbel-Eisenmann et al., 2013), little research has been done on what teachers believe and do about planning for mathematical discourse. To further our understanding, findings of this research study contributed to the literature by developing a survey on teachers' beliefs and practices about mathematical discourse planning before instruction and the role the curriculum plays in the planning and implementation of mathematical discourse.

The development of a quantitative survey based on the study's findings and results had the intention to further explore and understand teachers' perceptions and practices about mathematical discourse planning and implementation. Because mathematical discourse plays such an important role to support students' conceptual understanding, the development and use of a quantitative survey specifically intended to understand teachers' discursive practices for diverse students in inclusive mathematics settings could have a substantial impact in the field of teacher education. This survey could support teachers on increasing students' mathematical performance and achievement, specifically those who are at risk of mathematics difficulties.

Unfortunately, another finding from this study is the lack of discussion regarding the differentiation of mathematical discourse for diverse students. The purpose of this study was to understand teachers' beliefs and practices about mathematical discourse for diverse students in inclusive settings. Alarmingly, findings and results from qualitative and quantitative analyses of teachers perceived and observed beliefs and practices uncovered the absence of differentiation during mathematical discourse for students at risk for mathematical difficulties. Although researchers have found that some students might require instructional and/or linguistic supports to be able to fully participate in the classroom discourse due to the lexical-syntactic complexity of mathematical language and discourse (Silliman & Wilkinson, 2015; Topping et al., 2003), findings of this study evidenced the lack of mathematical discourse differentiation for diverse populations of students in general education classrooms.

The mathematics curriculum undoubtedly plays an essential role on teachers' mathematical discursive practices. As seen in these research findings, teachers mostly rely on curricular activities to initiate and promote mathematical discourse. It is essential to place emphasis on the curriculum teachers use during their instruction and supplemental activities teachers might choose to strengthen their instruction, specially to support students who are or might be at risk of mathematics difficulties. Findings on specific curriculum or instruction to support the mathematical discursive practices of students with LD and emergent bilinguals with and without disabilities have shown that curriculum and interventions based on a sociocultural approach are effective at embracing student diversity and promoting successful outcomes for all students in inclusive classrooms (Banse et al., 2017; Lewis, 2017; Xin et al., 2020). Specially designed instruction that both promotes students' familiar contexts and allows them to recognize their own experiences and encourages them to take risks and participate during discourse to solve problems is needed to successfully promote and develop students' conceptual understanding (Dominguez, 2017).

Consistent with a broad body of the literature (Hamm and Perry, 2002; Hundeland et al., 2020; Nathan and Knuth, 2003; Piccolo and colleagues, 2008), findings of observed teachers' practices related to mathematical discourse showed that teachers led and guided the direction of the mathematical discourse. Although research suggests teachers should manage and monitor the classroom discourse (Krussel et al., 2004), findings of this research study showed that teachers played an authoritative role (expert) during classroom discourse by not assuming a neutral position and influenced their students' discursive negotiations (Langer-Osuna, 2016). In other words, teachers had the control of the direction, structure, and content of the mathematical discourse and students shared their reasoning and ideas following their teachers' lead. Research has found that when teachers remove themselves from playing a leading role during classroom discourse, they provide more opportunities for student-led discussions (Nathan and Knuth, 2003). It is important for teachers to find a balance between monitoring and fostering discourse and controlling and leading discourse. The role teachers play during their discursive practices not only affects students' understanding of the content, but also their mathematical identity and authority as mathematical thinkers and problem solvers.

Mathematical identity and authority, which refer to students positioning themselves as credible and reliable sources of mathematical knowledge (Langer-Osuna, 2017), are closely related to equitable mathematics practices that allow students to have multiple opportunities to author, debate, argue, and collaborate to co-construct mathematical solutions to complex mathematical problems and participate in higher order thinking discourse (Hamm and Perry, 2002; Hwang, 2018; Langer-Osuna, 2016; Louie, 2020). The use of equity and intersectionality theories to analyze teachers' mathematics discursive practices gives the researcher the opportunity to understand mathematics access faced by diverse student populations (e.g., students with LD, emergent bilinguals, emergent bilinguals with LD) in inclusive settings.

Previous research findings on this issue showed that teachers believe students' background impedes them from teaching with the goal of conceptual understanding (Yurekli et al., 2020). This finding is troublesome because by law all students (no matter their background) should be taught using standards-based instruction, which emphasizes conceptual understanding. Thus, teachers require a better understanding of how to implement mathematical discourse that promotes conceptual understanding of all students in the classroom (Walshaw & Anthony, 2008). Importantly, equitable access to mathematics opportunities highly depends on teachers' management of intersectional factors, such as students' backgrounds, language, race, and academic and social status during instruction (Carey et al., 2018; Langer-Osuna, 2016).

Although teachers can ameliorate academic and social status and power issues during mathematical discourse by seeking opportunities to publicly acknowledge the mathematical authority of students who often struggle with mathematics (Langer-Osuna, 2017), some researchers have found that teachers rarely give their students the opportunity to develop their mathematical authority, limiting their opportunities to establish their individual identities as problem solvers and creators of mathematical knowledge (Hamm and Perry, 2002; Hwang, 2018). In addition, research has found that teachers' beliefs about students' equitable discourse participation might affect the mathematical rigor and complexity level of the discourse during their instruction (Baxter et al., 2002; Hwang, 2018). Findings of this research study showed that teachers' beliefs and practices about mathematical discourse promoted the participation of all students in the classroom. This is not enough to provide equitable mathematical practices to all students in the classroom, teachers must strive to develop in their students a sense of mathematical ownership and authority that will make them see themselves as effective mathematical thinkers and problem solvers (Louie, 2020; Musanti & Celedón-Pattichis, 2013). Nevertheless, equitable mathematics access in inclusive elementary mathematics classrooms can be achieved. Mathematics teaching that (a) incorporates strategy instruction, (b) offers multiple opportunities for students to communicate their thinking during whole and small group work, (c) requires the use of academic vocabulary, (d) includes multimodal supports, and (e) employs an standards-based curriculum that includes a wide range of relatable contexts can successfully support the mathematics learning, identity, and authority of all students in the classroom, especially those who come from diverse backgrounds and face multiple intersectional factors (Celedón-Pattichis & Turner, 2012; Dominguez, 2017; Griffin et al., 2013).

Conclusions

Recently, mathematical discourse has gained attention and prominence in mathematics research and practice due to the role it plays not only to support students' understanding, but also to foster equitable mathematics spaces through the development of students' mathematical authority and identity. Since the standards-based reform movement, research on teachers' beliefs and practices in mathematics has been broadly explored to understand the strong yet complicated relationship between them and to promote shifts in teachers' practices that will enhance and improve mathematics instruction for all students in inclusive classrooms. Research on teachers' beliefs about mathematical discourse has been limited compared to research on teachers' discursive practices. The findings of this study are congruent with more recent research findings related to mathematical discursive practices. Specifically, observed practices showed that teachers continue to lead and guide the direction of the mathematical discourse. Although findings also showed that teachers believed and practiced the inclusion of most students in the classroom discourse, that is not enough to truly foster equitable mathematics practices that will develop the mathematical identity and authority of all students no matter their background, language, race, academic and social status, or ability level. Teachers often need to assume a neutral role during the classroom discourse to encourage student-led discussions that will provide opportunities for students to author, debate, argue, justify, negotiate, and collaborate to co-construct mathematical knowledge and solutions to complex mathematical problems. That would be the first step to creating equitable mathematics spaces that will not give privileges to some students while making other students feel insecure and invisible.

Another critical topic that emerged from this study's findings is the role the curriculum plays in teachers' lesson planning and instruction. Most teachers do not have input on the selection of the curriculum they are using to teach mathematics; that decision mostly relies on district leaders that may or may not know specific characteristics of equitable mathematics activities that promote conceptual understanding. Therefore, teachers must be critical users of the curriculum and supplement it with activities that include familiar contexts that give all students the opportunity to recognize their own experiences and encourage them to see themselves as producers of knowledge, problem solvers, and critical thinkers.

The use of the survey regarding teachers' beliefs and practices related to mathematical discourse is the first step for teachers to reflect on their own teaching practices and to start making instructional changes towards more equitable practices that not only foster student participation, but also incorporate mathematical discourse differentiation and develop student mathematical authority and identity. The development of the survey will also help researchers to deep their understanding on how to support teachers on the lesson planning and implementation of mathematical discourse process that will effectively generate rich and meaningful discussions and will adapt to the social, academic, and language needs of all students in the classroom. As a final note, it is important to have in mind that conclusions should be viewed through the lens of the limitations further mentioned in this chapter.

Implications for Future Research

There are multiple implications for future research emanating from this study:

1. More research is needed on teachers' beliefs and practices regarding discourse planning and implementation. The literature on mathematical discourse has mainly focused on teachers and student discursive practices. Planning for mathematical discourse and its implementation should be investigated more in depth. Due to the bidirectional relationship between beliefs and practices, research on this matter should be further explored with diverse methodologies and analysis approaches.
2. This study contributed to the literature on teachers' beliefs and practices by developing, pretesting, and pilot testing a quantitative survey on elementary mathematics teachers' beliefs and practices regarding the planning and implementation of mathematical discourse. Future research should address other measures of validity and reliability, such as factor analysis to determine the dimensionality of the survey. In addition, future

research should incorporate a large sample of elementary mathematics teachers to investigate teachers' beliefs and practices on mathematical discourse planning and implementation using this survey to be able generalize its findings and draw meaningful conclusions.

3. Little research has been done regarding the relationship between teachers' perceived beliefs and practices and important teacher and classroom variables, such as student population in the classroom, complexity of mathematical content, teaching experience, and previous participation in professional development related to the implementation of mathematical discourse (Bray, 2011; Nisbet & Warren, 2000; Walshaw & Anthony, 2008). Therefore, future research should focus on investigating if specific teacher and classroom variables are related to and predict teachers' perceived beliefs and practices on the planning and implementation of mathematical discourse.
4. More research is needed on teachers' beliefs about equitable mathematical discursive practices that foster and develop students' mathematical ownership, identity, and authority. Due to the strong relationship between beliefs and practices, it essential to understand what teachers consider as equitable discursive practices for students who are or might be at-risk of mathematics difficulties in inclusive mathematics settings.
5. The role that the curriculum plays on the planning and implementation of mathematical discourse cannot be ignored. Moreover, most teachers do not have any input on the curriculum they must use to teach mathematics. Thus, teachers' beliefs and practices regarding the use, rationale, and implementation of supplemental mathematical discourse activities to promote students' conceptual understanding should be further investigated.

Implications for Future Practice

This study also presents some implications for future practice:

1. Findings of this research could support and guide teacher preparation and professional development programs on the effective and successful planning and implementation of rich and meaningful mathematics discourse that promotes students' conceptual understanding and fosters equitable mathematics practices focused on students' mathematical authority, identity, and ownership. Specifically, there is a need to support teachers on how to find a balance between leading and monitoring the direction of the discourse. Therefore, teacher preparation and professional development programs could support teachers on how to play a neutral role during mathematical discourse to give their students the opportunity to develop their mathematical authority and identity in the classroom.
2. Teachers' reflection of their own teaching practices could cause shifts in their mathematics instruction (Nathan and Knuth, 2003; Louie, 2020). Therefore, findings of this research could guide teacher preparation and professional development programs on the implementation of noticing and reflecting strategies regarding teachers' discursive practices to encourage shifts towards more equitable and rigorous mathematics discourse.
3. Research suggests that teacher preparation and professional development programs should focus on teaching teachers how to move away from curriculum activities and assessments that do not challenge their students' mathematical reasoning and understanding (Delandshere and Jones, 1999). Thus, findings from this research could support teacher preparation and professional development programs on teaching teachers how to choose and implement discourse-based activities and instruction to supplement any mathematics curriculum that lacks access to equitable activities and materials

(Zavala, 2014)

4. Training related to questioning, modeling, and eliciting strategies for pre-service and in-service teachers is essential to support teachers on how to provide all students with multiple opportunities for engagement and participation in rich discourse supportive of their conceptual understanding and mathematical identity and authority (Hansen-Thomas, 2009).

Limitations

This mixed-methods study had some limitations related to the methods of the study (e.g., sample size, participants recruitment, and data collection).

1. The qualitative research phase was mainly intended to achieve depth of understanding of the research problem. By including a purposeful selection of participants, the researcher intended to ensure that the data emerging from the qualitative analysis was comprehensive, complete, and saturated to fit all scenarios that could be identified in the larger population (Morse, 1999) of elementary mathematics teachers teaching in inclusive settings. Due to *COVID-19* restrictions, participant recruitment was limited and only three out of nine participants that met the study's participation criteria were special education teachers.
2. Due to *COVID-19*, district and school restrictions did not allow the researcher to collect video recordings of classroom observations. To deal with this limitation, the researcher performed in classroom observations of teachers' mathematical practices. Specifically, the researcher scored rubrics, coded the classroom discourse framework, and took field notes of teachers and students' discursive practices and behaviors during in-person observations.

3. Not all participants of the qualitative phase of the study were able to provide observational data. To mitigate the absence of the participant's in-person classroom observations due to school *COVID-19* restrictions, the researcher made the decision to increase the sample size from eight to nine participants.
4. Due to present circumstances, the sample size for the quantitative phase of the study was too small (n=18) to be able to find statistical significance of more complex analyses, such as multiple regression and correlations. Therefore, for the pilot test phase of the study, the researcher conducted a descriptive statistical analysis and measured the survey's internal consistency reliability (Cronbach's alpha) to make sure the survey was valid and reliable to be used with larger samples.

Summary

The present chapter included a thorough discussion of the study's findings and how they were related to a broad body of literature regarding teachers' beliefs and practices on mathematics instruction, especially on mathematical discourse. The researcher discussed the study's findings through an equity and intersectionality lens to broaden findings in the literature about equitable mathematics teachers' discursive practices that provide all students multiple opportunities to participate in high content level, meaningful, and rich mathematical discourse. In addition, the chapter included relevant implications for research and practice related to the findings discovered in the study.

At the end, the researcher presented some limitations related to the methods that might have affected the study's findings. Despite the study's limitations, findings of this research study contribute to the dialogue about teachers' beliefs and practices regarding the planning and implementation of mathematical discourse to improve the design and implementation of teachers' education and professional development programs in mathematics. Importantly, equitable mathematics spaces are needed to give all students in the classroom the opportunity to thrive and reach their full academic potential.

APPENDICES

Appendix A: Participants' Demographic Information Survey

Questions
What gender do you identify as?
Female
Male
Non-binary/other
Prefer not to answer
Which category below includes your age?
21-30 years old
31-40 years old
41-50 years old
51-60 years old
61+ years old
Could you specify your race?
White
Black/African American
Asian
American Indian/Alaska Native
Native Hawaiian or Other Pacific Islander
Two or more
Could you specify your ethnicity?
Hispanic or Latino or Spanish Origin
Not Hispanic or Latino or Spanish Origin
What is the highest level of education you have completed?
Bachelor's Degree
Some Graduate Education
Master's Degree
Ph.D. Degree
Which category below includes your teaching experience?
Less than 5 years
5-10 years
11-15 years
More than 15 years
What mathematics grade level do you teach?
PK/K
First Grade
Second Grade

(Continued)

Third Grade
Fourth Grade
Fifth Grade
What category best describes the percentage of students with Learning disabilities in your classroom?
0-15%
15-30%
More than 30%
What category best describes the percentage of Emergent Bilingual Students in your classroom?
0-15%
15-30%
More than 30%
What type of teaching license do you hold? (Select all that apply)
Elementary School Teacher
Elementary School Teacher, Special Education
Which endorsement and certifications do you hold? (Select all that apply)
None
Bilingual Endorsement
English Language Acquisition and Development (ELAD)
English as a Second Language
Other (Specify)
Which professional development have you participated in? (Select all that apply)
None
General Mathematics Education
Mathematical Discourse
How many hours of professional development related to mathematics instruction have you received?
0 - 15 hours
16 - 30 hours
> 30 hours

Appendix B: Mathematical Discussions Coding Rubric

Mathematical Discussions (MD)		Level of Emphasis		
	Description	Low: 1	Average: 3	High: 5
Teacher Behavior				
1	Richness of Mathematics	If underlying mathematics concepts are engaged, the engagement is superficial.	Underlying mathematics concepts are engaged, but not in ways that make connections to other mathematical ideas.	Underlying concepts are central to the discussion. The emphasis is on understanding why and making connections between mathematical ideas.
2	Teacher's Mathematical Integrity	Teacher's mathematics contains significant errors.	Teacher's mathematics is generally correct but does not help students focus on key ideas.	Teacher's mathematics is generally correct and helps students focus on key ideas.
3	Soliciting Student Reasoning	Teacher does not solicit student ideas, or only asks for answers, not reasoning or justification.	Teacher asks students to provide some reasoning and explanation about mathematical ideas, but student participation is mostly limited to student-teacher interactions.	Teacher presses students for reasoning and justification of ideas/solutions, building the discussion using student ideas, and pressing students to question/analyze each other's reasoning.
4	Assessing Understanding	Teacher does not assess student understanding or only does so superficially.	Teacher makes some attempt to check whether students are following key ideas of the discussion but fails to productively use that information.	Teacher makes sure students are following the discussion and assesses their understanding of important mathematical ideas (by using student work and asking questions). The flow of the lesson/discussion is modified as appropriate based on these assessments.
5	Pacing of Discussion	Teacher provides an excessive amount of time or an insufficient amount of time for students to engage with questions/ concepts (e.g., teacher answers own questions or always calls on firsthand).	The pace of the discussion is engaging/accessible for most students, but the teacher spends too little time on some important topics or too much time on less important topics.	The pace of the discussion is engaging/accessible for most students.
6	Opportunities for deeper mathematical Conversations	Teacher misses opportunities for deeper mathematical conversations.	Teacher leverages opportunities for deeper, conceptual conversations, but often resolves the mathematics for students.	Teacher opens deeper, conceptual conversations, and persists in having students' resolve mathematical questions as much as possible.
7	Addressing/ Engaging Misconceptions	Teacher leaves misconceptions unaddressed except when they are treated as "wrong answers" and corrected.	Teacher addresses some misconceptions but either (a) major misconceptions are left unaddressed or (b) the "fixes" are somewhat superficial.	Teacher engages misconceptions, probing for misunderstandings and building on partial understandings.

Student Behavior				
1	Participations	There is little student participation.	Participation is limited to a subgroup of students.	Many students participate.
2	Risks	Students don't share ideas.	Students share ideas when they are mostly certain they are correct.	Students take risks in sharing their ideas
3	Student Explanations	Students don't explain their ideas or solution processes.	Students' explanations consist of what they did/think but not why.	Students explain why their solutions or ideas work, as appropriate.

(Schoenfeld, 2013)

Appendix C: The Teaching for Robust Understanding of Mathematics Rubric

	The Mathematics	**Cognitive Demand**	**Access to Mathematical Content**	**Agency, Authority, and Identity**	**Uses of Assessment**
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills- oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
3	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem-solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation, but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
5	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; OR what appear to be established participation structures result in such engagement	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, AND/OR students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

(Schoenfeld, 2014)

Appendix D: Teacher Consent Form

Teacher Consent Form

UNLV

INFORMED CONSENT

Department of Early Childhood, Multilingual, and Special Education

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TITLE OF STUDY: Investigating Teachers' Beliefs and Practices on Mathematical Discourse for Diverse Students in Inclusive Classrooms
INVESTIGATOR(S): Gloria Carcoba Falomir and Dr. Joseph Morgan

For questions or concerns about the study, you may contact Gloria Carcoba or Dr. Joseph Morgan at 702-895-3329. For questions regarding the rights of research subjects, any complaints, or comments regarding the manner in which the study is being conducted you may contact **the UNLV Office of Research Integrity – Human Subjects at 702-895-2794, toll free at 888-581-2794, or via email at IRB@unlv.edu**

Purpose of the Study
You are invited to participate in a research study. The purpose of this study is to better understand teachers' beliefs about mathematical discourse, and how these beliefs are related to their mathematical practices.

Participants
You are being asked to participate in the study because you fit the following criteria: Graduate students or alums (last 5-7 years) from the UNLV College of Education who have experience teaching and/or co teaching in an elementary general education mathematics classroom.

Procedures
If you volunteer to participate in this study, you will be asked to do the following: (1) complete a teacher demographics survey, (2) be interviewed (individually and in a group) by the researcher on your ideas, beliefs, opinions and experiences teaching mathematics and implementing mathematical discourse, (3) provide two lesson plans of any daily mathematics instruction, (4) allow for two observations of your mathematics teaching (related to the lesson plans) and (5) take a quantitative survey of teachers' beliefs and practices related to mathematical discourse during instruction. The group interview will include the participation of 12 teachers. The individual interview and the focus group will last approximately 60-90 minutes each and will be video recorded.

Benefits of Participation

There may not be direct benefits to you as a participant in this study. However, we hope that this study will inform teacher training programs related to the design and implementation of teachers' professional development in mathematics related to the implementation of mathematical discourse during classroom instruction.

Risks of Participation

There are risks involved in all research studies. This study may include only minimal risks. You may feel uncomfortable answering questions about your teaching to the researcher or in a group setting. The study also includes the potential risk of loss of confidentiality due to focus group participation. Participants are asked to respect the privacy of other focus group members by not disclosing any content discussed during the study. You may feel uncomfortable having to record your own teaching practices. To minimize risks of transmission of *COVID-19*, the researcher will follow UNLV guidelines (Clark County and CDC guidelines).

Cost /Compensation

There will not be a financial cost to you to participate in this study. The study will take approximately 300 minutes of your time: (1) interview (90 minutes), (2) focus group (90 minutes), (3) demographic survey (5 minutes), (4) quantitative survey (40 minutes), (5) questions about understanding of survey (45 minutes), and (6) set up and upload (15 minutes).
You will be compensated for your time with $100 ($40 interview, $40 focus group, and $20 survey). You may be asked to provide your name, email, mailing address, and date of birth for compensation purposes, these identifiers will not be linked to the study's data.

Confidentiality

All information gathered in this study will be kept completely confidential. No reference will be made in written or oral materials that could link you to this study. All records will be stored in a locked facility at UNLV for 5 years after the completion of the study. After the storage time expires, the information gathered will be permanently deleted or shredded and destroyed.

Voluntary Participation

Your participation in this study is voluntary. You may refuse to participate in this study or in any part of this study. You may withdraw at any time without prejudice to your relations with the university. You are encouraged to ask questions about this study at the beginning or any time during the research study.

Participant Consent:

I have read the above information and agree to participate in this study. I have been able to ask questions about the research study. I am at least 18 years of age. A copy of this form has been given to me.

______________________________ ____________________________
Signature of Participant Date

____
Participant Name (Please Print)

Video Taping

I agree to be videotaped during interviews for the purpose of this research study.

____ ___________________________
Signature of Participant Date

____
Participant Name (Please Print)

Appendix E: Teacher Consent Form

Teacher Consent Form

UNLV

INFORMED CONSENT

Department of Early Childhood, Multilingual, and Special Education

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TITLE OF STUDY: Investigating Teachers' Beliefs and Practices On Mathematical Discourse for Diverse Students in Inclusive Classrooms
INVESTIGATOR(S): Gloria Carcoba Falomir and Dr. Joseph Morgan

For questions or concerns about the study, you may contact Gloria Carcoba or Dr. Joseph Morgan at 702-895-3329. For questions regarding the rights of research subjects, any complaints, or comments regarding the manner in which the study is being conducted you may contact **the UNLV Office of Research Integrity – Human Subjects at 702-895-2794, toll free at 888-581-2794, or via email at IRB@unlv.edu**

Purpose of the Study
You are invited to participate in a research study. The purpose of this study is to better understand teachers' beliefs about mathematical discourse, and how these beliefs are related to their mathematical practices.

Participants
You are being asked to participate in the study because you fit the following criteria: K-5 general and/or special education elementary mathematics teacher teaching and/or co teaching in a general education classroom.

Procedures
If you volunteer to participate in this study, you will be asked to do the following: (1) complete a teacher demographics survey, and (2) take a quantitative survey of teachers' beliefs and practices related to mathematical discourse during instruction.

Benefits of Participation
There may not be direct benefits to you as a participant in this study. However, we hope that this study will inform teacher training programs related to the design and implementation of teachers' professional development in mathematics related to the implementation of mathematical discourse during classroom instruction.

Risks of Participation

There are risks involved in all research studies. This study may include only minimal risks. You may feel uncomfortable answering questions about your teaching practices.

Cost /Compensation

There will not be a financial cost to you to participate in this study. The study will take approximately 20 minutes of your time. You will not be compensated for your time.

Confidentiality

All information gathered in this study will be kept completely confidential. No reference will be made in written or oral materials that could link you to this study. All records will be stored in a locked facility at UNLV for 5 years after the completion of the study. After the storage time expires, the information gathered will be permanently deleted or shredded and destroyed.

Voluntary Participation

Your participation in this study is voluntary. You may refuse to participate in this study or in any part of this study. You may withdraw at any time without prejudice to your relations with the university. You are encouraged to ask questions about this study at the beginning or any time during the research study.

Participant Consent:

I have read the above information and agree to participate in this study. I have been able to ask questions about the research study. I am at least 18 years of age. A copy of this form has been given to me.

Appendix F: Phases 1 & 2 Participants' Recruitment Email

Hello,

Background

We are researchers from the University of Nevada, Las Vegas in the Department of Early Childhood, Multilingual and Special Education. We are conducting a research study designed to better understand how teachers' beliefs regarding the intentional planning and implementation of mathematical discourse during classroom instruction might influence their mathematical practices. We hope that this study will inform teacher training programs related to the design and implementation of teachers' professional development in mathematics related to the inclusion of mathematical discourse during classroom instruction.

Participants

We are contacting you to participate in this research study because you are a Graduate student or alum (last 5-7 years) from the UNLV College of Education. We are seeking elementary and special education teachers who have had experience teaching mathematics in an inclusive elementary mathematics classroom, either as a teacher or co-teacher.

Study Activities

Your participation in this study is voluntary. If you decide to participate, you will be asked to do the following:

(1) complete a teacher demographics survey,
(2) be interviewed (individually and in a group) by the researcher on your ideas, beliefs, opinions, and experiences teaching mathematics and implementing mathematical discourse,
(3) provide two lesson plans of any daily mathematics instruction,
(4) allow for two classroom observations of your mathematics teaching (related to the lesson plans), and
(5) take a quantitative survey of teachers' beliefs and practices related to mathematical discourse during instruction. The individual interview and the focus group will be video recorded.

All information gathered in this study will be kept completely confidential, only the research team will have access to it. The study will take approximately 300 minutes of your time, and you will be compensated for your time with $100 ($40 interview, $40 focus group, and $20 survey). We hope that you will consider participating in this research study, as your input is essential to not only increase understanding of how teachers' beliefs regarding mathematical discourse might influence their mathematical practices but also to inform teacher training programs related to the design and implementation of teachers' professional development related to mathematical discourse.

Demographics Screening Form

If you decide to participate, could you please answer the brief survey below? This survey will gather contact information from you as well as demographic information to verify your eligibility to participate in the study.

https://unlv.co1.qualtrics.com/jfe/form/SV_8jeBxL7x4HZGIrY

Sincerely,

Gloria A. Carcoba Falomir, M.Ed.
gloria.carcobafalomir@unlv.edu

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Appendix G: Phase 3 Participants' Recruitment Email

Greetings - we are writing to invite you to participate in a 20-minute math survey if you work in elementary or special education and have had experience teaching mathematics in an inclusive environment. The link to the survey is below and more information about the study is at the end of this email.

https://unlv.co1.qualtrics.com/jfe/form/SV_cMAaUOFXSoCjlxc

THANKS FOR YOUR CONSIDERATION.

INFORMATION REGARDING THE STUDY

Hello,

Background

We are researchers from the University of Nevada, Las Vegas in the Department of Early Childhood, Multilingual and Special Education. We are conducting a research study designed to better understand how teachers' beliefs regarding the intentional planning and implementation of mathematical discourse during classroom instruction might influence their mathematical practices. We hope that this study will inform teacher training programs related to the design and implementation of teachers' professional development in mathematics related to the inclusion of mathematical discourse during classroom instruction.

Participants

We are contacting you to participate in this research study because you are a Graduate student or alum (last 5-7 years) from the UNLV College of Education. We are seeking elementary and special education teachers who have had experience teaching mathematics in an inclusive elementary mathematics classroom, either as a teacher or co-teacher.

Study Activities

Your participation in this study is voluntary. If you decide to participate, you will be asked to do the following:

(1) complete a survey of teachers' beliefs and practices related to mathematical discourse during instruction.

All information gathered in this study will be kept completely confidential, only the research team will have access to it. The study will take approximately 20 minutes of your time, and you will not be compensated for your time. We hope that you will consider participating in this research study, as your input is essential to not only increase understanding of how teachers' beliefs regarding mathematical discourse might influence their mathematical practices but also to inform teacher training programs related to the design and implementation of teachers' professional development related to mathematical discourse.

Demographics and Mathematical Discourse Survey

If you decide to participate, could you please answer the survey below? This survey will gather your demographic information as well as your perceived beliefs and practices related to the implementation of mathematical discourse during instruction.

https://unlv.co1.qualtrics.com/jfe/form/SV_cMAaUOFXSoCjlxc

Sincerely,

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Appendix H: Teachers' Beliefs and Practices on Mathematical Discourse Survey

Directions:
Listed below are some statements about your mathematics instruction. Please read each statement carefully and indicate the degree to which you agree or disagree with each statement.
1= **Strongly Disagree**
2= **Disagree**
3= **Slightly Disagree**
4= **Slightly Agree**
5= **Agree**
6= **Strongly Agree**

		1	2	3	4	5	6
1	There are enough discourse-related activities included in provided mathematics curriculum to integrate discourse into instruction.						
2	Specific student behaviors prompt me to implement discourse during my instruction.						
3	Mathematical discourse does not need to be planned; it comes naturally during my instruction.						
4	The teacher should play a central role during mathematical discourse.						
5	The teacher has a responsibility to maintain control of the classroom discourse.						
6	The teacher should be part of the discussion regardless of any grouping strategy.						
7	Mathematical discourse is more effective during whole group instruction.						
8	Mathematical discourse and student collaboration are more effective during small group instruction.						
9	All grouping strategies facilitate mathematical discourse.						
10	The teacher should initiate mathematical discourse during instruction.						
11	All students should participate in classroom discourse even if they feel uncomfortable.						
12	Students do not want to participate in classroom discourse because they are still developing their language.						
13	Students do not want to participate in classroom discourse because they are afraid of taking risks in front of their peers.						

14	Students do not participate in the classroom discourse because they do not understand the content.						
15	Students only raise their hand when they understand the concepts being taught.						
16	Students need to be explicitly taught how to participate in mathematical discourse.						
17	Mathematical discourse is essential to address my students' understanding.						
18	Mathematical discourse promotes conceptual understanding.						
19	Students learn through discussing their ideas and thinking processes.						
20	Mathematical discourse provides opportunities for students to develop mathematical authority by sharing their own ideas.						
21	The teacher has the responsibility to lead and guide the classroom discourse.						
22	Is easier to learn mathematics if students collaborate and discuss with their peers than if they work by themselves.						
23	Mathematical discourse is essential to address students' misconceptions.						
24	Students should use academic vocabulary when explaining their thinking and reasoning.						
25	An essential way for students to share their reasoning is through discourse.						
Directions: Listed below are some statements about your mathematics instruction. Please read each statement carefully and indicate the frequency to which you engage in each statement. 1= **Never** (Never happens, I never plan for it) 2= **Almost Never** (It may happen, but I do not intentionally plan for it) 3= **Rarely** (It may happen some of the time and if it happens, I do not deliberately plan for it) 4= **Occasionally** (It happens some of the time, when it happens, I deliberately plan for it) 5= **Frequently** (It happens with regularity and intentionality) 6= **Always** (It constantly happens with regularity and intentionality)							
26	I plan for mathematical discourse in my lesson plans.						
27	I rely on curriculum activities to promote discourse during my instruction.						
28	I decide to implement mathematical discourse by observing my students' behaviors.						
29	I follow my intuition as a teacher when implementing mathematical discourse.						
30	I adapt the way I discuss mathematics with my students depending on their specific needs.						

31	I promote mathematical discourse during my whole group instruction.						
32	I try to provide opportunities for discourse during small group and independent practice.						
33	I plan small groups having the specific needs of each student in mind.						
34	I let my students choose their own groups or partners during classroom discourse activities.						
35	I encourage my students to participate in the classroom discourse at least once.						
36	I randomly select my students to participate during classroom discourse.						
37	I allow students to ask for help from their peers when they do not know how to explain their reasoning or thinking.						
38	I choose students that raised their hand to participate during classroom discourse.						
39	I encourage my students to elaborate when explaining their thinking.						
40	I ask open-ended questions to my students.						
41	I provide multiple opportunities for students to explain and justify solutions to the problems.						
42	I explicitly teach and model how to participate in mathematical discourse to my students.						
43	I require my students to use academic vocabulary during their participation in classroom discourse.						
44	I promote mathematical discourse to assess students' understanding.						
45	I promote mathematical discourse to solicit students' reasoning.						
46	I provide multiple opportunities for students to share their ideas and reasoning through discourse.						
47	I promote the use of non-linguistic supports, like visual representations, during classroom discourse.						
48	I promote my students' development of mathematical authority by sharing their own ideas with the entire class.						
49	I promote mathematical discourse to address misconceptions.						
50	I encourage students to make and discuss mistakes.						

Appendix I: Teacher's Lesson Plan (Example One)

Kindergarten Math Topic 5
Lesson: 5-2: Count the number of objects in each category Date:

Aim	How can we count the number of objects in each category?
Essential understanding and CCSS / MP	Essential Understanding: Objects can be classified into two categories, based on whether they have or do not have a particular attribute. Each group can then be counted. CCSS: NY.K.MD.3 - NY-K.CC.5a MP: MP.2, MP.6, MP.7, MP.8
Vocabulary	Chart, tally mark, recording objects, classify, categories, data, attribute, sort, alike, different, in common, group
Materials	Two-colored counters, cubes, markers, yarn, chart paper, sticky notes
Math Warm UP	Math Songs - Daily Review sheet (optional)
Visual Learning Whole Group Teach	Boys and girls yesterday we learned how to classify objects into categories. Today we will learn how to count the number of objects into each category. Teacher will watch the Visual Learning Video that goes with page 256 in the student workbook. Picture 1: - *What do you see?* - *Which creature has 6 legs?* - *Which has 8 legs?* - *This creature has a lot of legs.* - *Does it have more than 6 legs?* - *You can classify the creatures by those that have 6 legs and those that do NOT have 6 legs.* Picture 2: - *Which category does this show?* - *Which category doe this show? (with X)* - *This is a chart. A chart is a way of showing what you found or how many you counted.* - *One tally mark represents each creature you count. This is a good way to make sure you count each one. Count the tally marks in each category.* - *BOOK DOES NOT SHOW TALLY CROSSING AT 5! SHOW STUDENTS HOW TO CROSS THE TALLY AT 5.* Picture 3:

	- *How is this chart different from the one before?* - *Do the numbers match the number of tally marks we counted?* - *Using a number is another way of recording your count.* - *What does 6 mean?* - *What does 7 mean?*
Solve and Share Do you Understand? Show me Center Group Teach	Pose the solve and share problem (page 255) *Carlos goes outside and sees some creatures. How many creatures does he see on the ground? How many does he see that are NOT on the ground? Tell how you know you counted all of the creatures.* - *What are you asked to do?* - *What tools do you have to help you?* - *How could you use counters to help you?* - *Where would you find the creatures that are NOT on the ground?* - *How can you know that you have counted all of the creatures?* Share and discuss solutions. Do you understand? Show me! If a worm was in Box 1, what category would it be in? how many creatures would there be in that category now?
Guided Practice / Independent Practice Quick Check Assessment	Guided Practice: 256. *Have students draw lines in the chart as they count the animals that are in the pond and animals that are NOT in the pond, and then write the numbers to tell how many in another chart.* Independent Practice: 257- 258 - *What are the categories?* - *How can you find out how many are in each category?* - *What will you use to help you count all the creatures?* - *How many tally marks or lines will you draw for each creature?* - *What are you classifying? What two categories are you using to classify these birds?* - *How many lines have you drawn for the in trees category?* - *How many lines for NOT in the trees?* - *So, what numbers will you write to show how many are in each category?* - *When do you draw a line or tally make in the chart?>. how you know where on the chart to make a line?* - *Carlos writes the same number for each category. Is he correct?* - *What are the different ways these mice could have been classified?* Quick Check: 4, 5 and 6
Differentiation	● Intervention: Tally O! Teachers Guide 259A

	• Enrichment: add 2 more little creatures to the picture. Count again. How many creatures are on the ground? How many creatures are NOT on the ground? • Enrichment / Reteach pages for the lesson
EVALUATE Center Group Reflect/Share	Reflect on the aim – Turn and talk / Share with partner Essential question reflection: How can you find the number of creatures that belong to each category or group?
Assessment	Use Quick Check and checklists to track students who have met the objective, exceeded the objective, or required intervention.
Centers	- Attribute block match up - Button sorting - Geo-block match up - Sorting mats

Appendix I: Teacher's Lesson Plan (Example Two)

<table>
<tr><td colspan="2">Math Plans</td></tr>
<tr><td>Class logistics</td><td>Class Schedule:

7:45 - 9:40 Reading
9:40 - 10:30 Specials
10:30 - 10:45 Break
10:45 - 11:30 Science/ Health/ Writing
11:30 - 12:00 Lunch
12:00 - 12:30 RTI
12:30 - 12:45 Social Studies
12:45 - 1:05 Recess
1:05 - 2:10 Math
2:15 Dismissal</td></tr>
<tr><td>Goals and Objectives (Standards Included)</td><td>SWBAT … use squares to count the area of a shape (3.MD.C.5)</td></tr>
<tr><td>Learning Tasks</td><td>Focus Tasks: use grid paper to solve area problems.

Aspect of rigor targeted in this lesson: use appropriate tools and attend to precision.

Student engagement: Students will be called on to answer questions and will work with partners using sentence frames</td></tr>
</table>

Checks for Understanding	I will call on students during guided instruction to see if they understand the math problems (use table tents)
Student Language Considerations	Speaking: Students will use math sentence frames to speak. I solved the problem by ______. The strategy I used was _________. Writing: Students will have to write out their answers when they are constructing arguments and problem solving. Reading: Students will read math word problems Listening: Students will have to use active listening skills while listening to the teacher teach.
Exit Ticket and Closure	6-1 quick check

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Yurekli, B., Stein, M. K., Correnti, R., & Kisa, Z. (2020). Teaching mathematics for conceptual understanding: Teachers' beliefs and practices and the role of constraints. *Journal for Research in Mathematics Education, 51*(2), 234–247. https://doi.org/10.5951/jresematheduc-2020-0021

Zavala, M. del R. (2014). Latina/o youth's perspectives on race, language, and learning mathematics. *Journal of Urban Mathematics Education, 7*(1), 55–87. https://doi.org/10.21423/jume-v7i1a188

CURRICULUM VITAE

GLORIA A. CARCOBA FALOMIR, M.Ed.

University of Nevada, Las Vegas
College of Education
Department of Early Childhood, Multilingual, and Special Education
4505 S. Maryland Parkway
Las Vegas, NV 89154
alecarcoba@hotmail.com

CERTIFICATION

I have read the following and certify that this curriculum vitae is a current and accurate statement of my professional record.

Signature: Gloria A. Carcoba Falomir

EDUCATION

Ph.D.	University of Nevada, Las Vegas Special Education	2022 (anticipated)
M.Ed.	University of Nevada, Las Vegas Special Education Generalist	2016
M.B.A.	Keller Graduate School of Business Management, Henderson, Nevada Human Resources	2014
B.B.A.	Instituto Tecnológico Autónomo de México, Mexico City, Mexico Operations Management	1998

RESEARCH EXPERIENCE

Project Director 2021-Present
Reinvent Schools Las Vegas Community Schools Initiative (Joseph Morgan, PI; Alain Bengochea, Co-PI). U.S. Department of Education, Office of Innovation, and Improvement Full-Service Community Schools Program. ($2,491,435)
University of Nevada, Las Vegas
Supervised by Dr. Joseph J. Morgan, University of Nevada, Las Vegas

Project Coordinator, Las Vegas 2021-Present
ROOTS Replication: A Systematic Replication of a Tier 2 Kindergarten Mathematics Intervention (Ben Clarke, PI; Joseph Morgan, Co-PI; Derek Kosty, Co-PI; Keith Smolkowski, Co-PI; Christian Doabler, Co-PI; Jessica Turtura, Co-

PI). U.S. Department of Education; Mathematics and Science Education: Special Education Research, Efficacy and Replication. ($3,600,000)
University of Oregon Center for Teaching and Learning
Supervised by Dr. Joseph J. Morgan, University of Nevada, Las Vegas and Dr. Jessica Turtura, University of Oregon.

Graduate Research Assistant 2021
UNLV-Reinvent Schools Las Vegas Community Schools Initiative
Implementation of summer learning program, Project Coordinator.
Supervised by Dr. Joseph J. Morgan and Dr. Tracy Spies.

Graduate Research Assistant 2017-2021
Project E3: Enhancing, Engaging, and Empowering Teachers for the Next Generation of English Learners (Tracy Spies, PI; Sharolyn Pollard-Durodola, Co-PI; Alain Bengochea, Co-Investigator). U.S. Department of Education. ($1,812,195)
Project Coordinator, University of Nevada, Las Vegas

Early Childhood Program Evaluation Assessor 2020-2021
Acelero Early Learning Company & Annenberg Institute for School Reform at Brown University
Supervised by Leiah Groom-Thomas, M.A. (Research Project Manager)

Graduate Research Assistant 2020
UNLV-Reinvent Schools Las Vegas Community Schools Initiative
Implementation of summer learning program
Supervised by Dr. Joseph J. Morgan

Graduate Research Assistant 2016
Project SPEN:TT (Severe and Persistent Educational Needs: Teacher Training) (Monica Brown, PI; Joseph Morgan, Co-PI). U.S. Department of Education.
Supervised by Dr. Joseph J. Morgan
University of Nevada, Las Vegas

Graduate Research Assistant 2015-2016
University of Nevada, Las Vegas
Supporting Project FOCUS with translation and observation activities, data collection on several school campuses for intervention research, 9 weeks science interventionist for a Doctoral Dissertation (Impact of an evidence-based science curriculum on the vocabulary for students with intellectual disability learning English)

Data Collector 2016-2017
A Multi-Site Randomized Controlled Trial to Assess the Efficacy of the NumberShire Level 1 Gaming Intervention for Improving Math Outcomes for Students with or At Risk for Math Learning Disabilities. (Hank Fien, PI).

Institute of Education Sciences; U.S. Department of Education; National Center for Special Education Research; Technology for Special Education, Efficacy and Replication. ($3,499,086)
University of Oregon Center for Teaching and Learning
Supervised by Dr. Joseph J. Morgan, University of Nevada, Las Vegas

PROFESSIONAL EXPERIENCE

University Teaching Experience

Instructor, Math Methods for Students with Disabilities (EDSP 462) Fall 2021
Face to Face
University of Nevada, Las Vegas

Instructor, Assessment and Evaluation of English Language Learners (EDRL 475) Remote Spring 2021
University of Nevada, Las Vegas

Instructor, Math Methods for Students with Disabilities (EDSP 462) Fall 2020
Face to Face
University of Nevada, Las Vegas

Instructor, Math Methods for Students with Disabilities (EDSP 462) Fall 2019
Face to Face
University of Nevada, Las Vegas

Instructor, Methods for English Language Learners (TESL 474) Spring 2018
Face to Face
University of Nevada, Las Vegas

K-12 Teaching Experience

Kindergarten School Teacher 2008-2009
Instituto Alpes Aguascalientes, Aguascalientes, Mexico

Assistant Principal 2002-2003
Centro Educativo TOTS, Mexico City, Mexico

Kindergarten School Teacher 2001-2002
Centro Educativo TOTS, Mexico City, Mexico

COURSE DESIGN

EDRL 474- Methods and Curriculum for Teaching English Language 2020

EDRL 475- Assessment and Evaluation of English Language Learners 2020

PROFESSIONAL AND COMMUNITY SERVICE

Editorial Assistant 2021-Present
Intervention in School and Clinic Journal
Published by the Hamill Institute on Disabilities and SAGE
A publication for the Council on Learning Disabilities

Master's representative (k-12 Special Education) 2016-2017
Executive Board for the Student Council for Exceptional Children

PROFESSIONAL AFFILIATIONS

American Educational Research Association 2018-Present
Division of Teaching and Teacher Education
Council for Exceptional Children 2017-Present
Division for Learning Disabilities
Council for Learning Disabilities 2018-Present

PUBLICATIONS

Manuscripts Accepted for Publication

Spies, T. G., Xu, Y., Deniz, F., **Carcoba Falomir, G. A**., & Sarisahin, S. (2021). English as an additional language doctoral students' ongoing socialization into scholarly writing: How do writing feedback groups contribute? *Journal of Response to Writing, 7*(1), 128–158. https://scholarsarchive.byu.edu/journalrw/vol7/iss1/5

Spies, T. G., Pollard-Durodola, S. D., Bengochea, A., **Carcoba Falomir, G. A.,** & Xu. Y. (2021). Teacher leadership in systemic reform: Opportunities for graduate education programs. *The CATESOL Journal, 32*(1), 172–187.

Carcoba Falomir, G. A. (2019). Diagramming and algebraic word problem solving for secondary students with learning disabilities. *Intervention in School and Clinic, 54*(4), 212–218. https://doi.org/10.1177/1053451218782422.

Manuscripts Undergoing Peer Review

Spies, T. G., **Carcoba Falomir, G. A**., Sarisahin, S., Deniz, F. K., & Xu, Y. (submitted 4_3_22). Dialogue in the feedback process: Doctoral English as an additional language students' path toward scholarly writing. Submitted to *Learning, Culture, and Social Interaction* Journal.

Manuscripts In Preparation

Carcoba Falomir, G. A., Spies, T.G., Bengochea, A., Xu, Y., & Pollard-Durodola, S. D. (in preparation). Teachers' mathematical practices including visual representations and discourse. To be submitted in *Educational Studies in Mathematics* Journal.

Carcoba Falomir, G. A. (in preparation). The field of special education in Mexico: What teachers in the U.S. should know. To be submitted in *Intervention in School and Clinic* Journal.

CONFERENCE PRESENTATIONS

- **Total: 14**

Carcoba Falomir, G. A. (2021, October). What teachers need to know about Mexican-origin students with learning disabilities. Poster accepted to present at CLD 43rd International Conference on Learning Disabilities in Las Vegas, Nevada.

Carcoba Falomir, G. A. (2021, March). Dialogic mathematical practices for students with mathematics difficulties. Poster presented at CEC 2021 Conference: Learning Interactive Virtual Event (L.I.V.E.).

Spies, T. G., Sarisahin, S., Xu, Y., **Carcoba Falomir, G. A.**, & Deniz, F. K. (2020, April). An academic writing feedback group in the socialization of international doctoral students into the academy. Poster accepted to present at 2020 American Educational Research Association Annual Meeting in San Francisco California. (Conference cancelled due to *COVID-19*)

Brown, M., Cooper, A., **Carcoba Falomir, G. A,** Deniz, F. & Sarisahin, S. (2019, November). In their own voices: Unmasking the truth about international doctoral students' experiences. Paper presented at TED's 42nd Annual Conference in New Orleans, Louisiana.

Carcoba Falomir, G. A. (2019, November). Supporting students with learning disabilities' mathematical understanding and discourse through visual representations. Conversation session presented at TED's 42nd Annual Conference in New Orleans, Louisiana.

Spies, T. & **Carcoba Falomir, G. A.** (2019, October). Supporting science conversations: Students with disabilities who are learning English. Structured poster presented at the CLD 41st International Conference on Learning Disabilities in San Antonio, Texas.

Spies, T. & **Carcoba Falomir, G. A.** (2019, October). STEM practices and students with disabilities who are learning English. Structured poster presented at the CLD 41st International Conference on Learning Disabilities in San Antonio, Texas.

Carcoba Falomir, G. A. (2019, October). Supporting students with learning disabilities mathematical understanding via visual representations. Poster presented at the CLD 41st International Conference on Learning Disabilities in San Antonio, Texas.

Carcoba Falomir, G. A. (2019, September). Supporting ELs Mathematical Proficiency and Discourse. Paper presented at NVTESOL First annual conference in Las, Vegas, Nevada.

Pollard-Durodola, S., Spies, T., Bengochea, A., Joseph, N., **Carcoba Falomir, G. A.,** & Xu, Y. (2019, August). Teacher empowerment: The role of reflectivity and video observations in re-constructing knowledge-building math talk**.** Paper presented at the Annual Conference of the European Early Childhood Education Research Association in Thessaloniki, Greece.

Carcoba Falomir, G. A., Bengochea, A., Xu, Y., Spies, T., & Pollard-Durodola, S. D. (2019, April). Use of visual representations to support English learners' math talk. Round table presented at the 2019 American Educational Research Association Annual Meeting in Toronto, Canada.

Spies, T., Bengochea, A., Xu, Y., **Carcoba Falomir, G. A.,** & Pollard-Durodola, S. D. (2019, April). Academic language and literacy instruction for English learners in rural communities: A convergent mixed-methods study. Round table presented at the 2019 American Educational Research Association Annual Meeting in Toronto, Canada.

Xu, Y., Bengochea, A., **Carcoba Falomir, G. A.,** Spies, T., & Pollard-Durodola, S. D. (2019, April). Examining teachers' perceptions, interpretations, and action plans of English learners needs. Round table presented at the 2019 American Educational Research Association Annual Meeting in Toronto, Canada.

Carcoba Falomir, G. A. (2018, October). Using diagrams to help secondary students with learning disabilities master algebra word problems. Poster presented at the CLD 40th International Conference on Learning Disabilities in Portland, Oregon.

PRESENTATIONS & PROFESSIONAL DEVELOPMENT

Guest Lecturer. *The field of special education in Mexico: What teachers in the U.S. should know.* (EDRL 471) Theory and Practice for Academic English Language Development course. University of Nevada, Las Vegas.	Fall 2021
Council for Learning Disabilities (CLD). Leadership Institute (Virtual Format)	2020
Scholarly Writing Feedback Group	2018-Present

HONORS AND AWARDS

Rodman Doctoral Special Education Professional Development Award University of Nevada, Las Vegas. Awarded $800	2022
University of Nevada, Las Vegas Research Graduate Assistantship	2015-2021
Satisfactory Academic Progress Scholarship Awarded $2000 per year.	2015-2021
UNLV Southwest Travel Award Awarded $500	2018
University of Nevada, Las Vegas Academic Excellence Certificate	2016
University of Nevada, Las Vegas Outstanding Contribution Certificate	2016

CERTIFICATIONS

Classroom Assessment Scoring System (CLASS) Pre-K Observer	2017-2019
Minnesota Executive Function Scale (MEFS) Assessor	2020-2021

LANGUAGES

Spanish	Native Language
English	Proficient