Introduction

Logistic growth equation was introduced in 1838 by Pierre-François Verhulst as a way to measure different resources (particularly biological) proportionally with respect to the change of time [2]. The differential equation used to represent population growth in an environment with unlimited food and space is:

$$\frac{dy}{dt} = ky(M-y)$$

The variable $k$ is a constant which defines the growth rate and allows it to be proportional to the size of the population. When there are limited resources the differential formula:

$$\frac{dy}{dt} = ky\left(M-y\right)$$

is used where $M$ is the carrying capacity the population cannot exceed without running out of its essential resources such as food and space. The variable $y$ represents the population.

Application (cont.)

If $M$ is the maximum level of performance of which the learner is capable, then the equation, (with $k$ a positive constant):

$$\frac{dP}{dt} = k(M-P) \quad \text{(2)}$$

is a reasonable model for learning since RHS of (2) is always positive, so the level of performance $P$ is increasing. As $P$ gets close to $M$, $dP/dt$ gets close to 0, that is, the performance levels off.

The solution is derived from:

$$\frac{dP}{dt} = k(M-P)$$

$$\int \frac{dP}{M-P} = kt + c$$

and is:

$$P(t) = M - M e^{-kt}$$

The figure above shows the performance of someone learning a skill as a function of the training time $t$.

Application in Biology

Biologists stocked a lake with 400 fish of one species and estimated the species’ carrying capacity in the lake to be 10,000. The number of the fish tripled in the first year. Assuming that the size of the fish population satisfies the logistic equation, we find the expression for the size of the population after $t$ years. Based on (1), with $y_0=400$, $y=1200$, $M=10000$, and $t=1$, we get:

$$k = 1.185623 \cdot 10^{-4}$$

b) How long will it take for the population to increase to 5000?

$$5000 = 400 + \left(9.6 \cdot 10^{-4}\right) e^{1.185623 \cdot 10^{-4} t}$$

$t = 2.7$ years

Application in Psychology

Psychologists are interested in learning theory study - learning curves. A learning curve is the graph of a function $P(t)$, the performance of someone learning a skill as a function of the training time $t$. The derivative $dP/dt$ represents the rate at which performance improves.

Conclusions and Future Directions

Understanding and predicting the behavior of a population is essential in numerous applications. Logistic differential equations can be used to accurately predict a rate of change of a population over a period of time, such as in biological, psychological, and economic processes. Future directions will consist of:

- Application in which there is a minimum population such that the species will become extinct if the size of the population falls below $M$.

- Application with seasonal-growth model, given by:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{K}\right)$$

where $k$, $r$, $\varphi$ are constants that describe the seasonal variations in the rate of growth.

- Application in which the model for the growth function for a limited population is given by the Gompertz function, i.e. the solution of:

$$\frac{dP}{dt} = c e^{k \left(1 - \frac{P}{K}\right)}$$

where $c$ is constant and $K$ is the carrying capacity.

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References


For further information

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