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A Conceptual draft of a dynamic hydro-biological model for Lake Mead

L. G. Everett
University of Arizona
Bureau of Reclamation

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A CONCEPTUAL DRAFT
OF A
DYNAMIC HYDRO-BIOLOGICAL MODEL FOR LAKE MEAD

Submitted to the Bureau of Reclamation
Region 3
Boulder City, Nevada

by

L. G. Everett - Hydrology and Water Resources
University of Arizona
Tucson, Arizona 85721
April 8, 1971
INTRODUCTION

Food and energy transformation in an aquatic system must be understood in order to develop a simulation approach. This report provides a comprehensive study of an aquatic ecosystem. A simple food web, Fig. 1, introduces the transfer relationships that occur in Lake Mead. While this diagram is hardly exhaustive it does include the major species involved. The food web diagram illustrates the disastrous effects of having the flora shift to the blue-greens. Herbivores will not consume them because cyanophyta (blue-greens) are noted to be bad tasting, bad smelling and quite toxic.

The objectives of this report are to show:

(1) the biological relationships in an aquatic system
(2) the role of nutrients in the biological cycle
(3) the role of abiotic factors in a limnetic environment
(4) the status of the art of "Eutrophication modeling".

The role of biological indicators has been well documented. Invertebrate forms are commonly used as indicators of benthic water pollution. A known succession of species can be identified downstream from a sewage disposal plant. This succession is represented in Fig. 2.

A basic ecological principle must also be observed when the quality of the water changes. Under normal conditions there are
<table>
<thead>
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<th>Species</th>
<th>Distance from Treatment Plant (miles)</th>
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<td>Ephemeroptera - Mayflies</td>
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<td>Plecoptera - Stoneflies</td>
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<td>Amphipoda</td>
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<td>Trichoptera - Caddisflies</td>
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Figure 2. Species Succession to Indicate Water Quality Recovery.
Figure 1. A Partial Food Web of Lake Mead.
many species present with few numbers in each species. However, if stress is placed on the system the number of species is reduced and the number of individuals of each species increases. This fact is illustrated in Fig. 3. Common examples of stress are limiting amounts of nutrient or dissolved oxygen.

We feel that the phytoplankton and zooplankton can be modeled in such a way as to predict the type of species that will occur given the physical and chemical inputs. An example of this would be to take 5 species of phytoplankton \(x_1, x_2, \ldots, x_5\) and trace them across Lake Mead.

Graphically this may appear:

![Graph showing species distribution across Lake Mead](image)

It is more likely that fewer species will be seen in Boulder Basin with larger numbers of individuals of the \(x_5\) species. We feel that \(x_5\) will be a large growth of blue greens.

A major parameter in a limnological system is the phytoplankton growth rate. To date, the most accurate technique available is the
Figure 3. Ecological Principles.
C14 primary productivity procedure. There is no water pollution control model available that uses this technique. Cole (1967) assigns a value of 6.4 m3-cm2-min/mo-(mg at. wt.)-g-cal-°C to his phytoplankton growth rate due to photosynthesis. He varies his growth rate factor with a single limiting nutrient, incident solar and sky radiation, and the temperature of water. Chen (1970) describes a phytoplankton growth rate dependent upon an unlimited number of contributing nutrients, assuming he knows the required Michaelis-Menton values. The Water Resources Engineers (1968) propose several approaches to obtaining a specific growth rate factor. Di Toro et al (1970) describe in detail the most complete growth expression available. However, these models have been established without any prior physical interpretation in the field. Our approach takes measured data and develops the relationships. To date, there is no published approach that is as comprehensive as this study.

The second major contribution of our approach is the distributed phytoplankton, zooplankton relationships. Prior efforts lumped the species of both plankters. However, as Provasoli of Haskins Laboratories, New York (1969), points out, the main problem is determining what discriminatory or preferential factors govern the growth of algae - beneficial (diatoms) or obnoxious (blue-greens). In a eutrophying environment, it is not the amount of algae produced but
rather the kind of algae developing that is of concern. By understanding the phytoplankton succession to the blue green stage we can determine if a reservoir is eutrophying. We can also tell the spacial extent of the problem. Once the phytoplankton have reverted to cyanophyta (blue-greens) the food chain is discontinued. Herbivorous zooplankton will not feed on filamentous blue-greens. The result is a crash in zooplankton, reduction of fish population, and uncontrolled algal bloom.

Since discretized data has been accumulated, we have the advantage of dividing our system both longitudinally and horizontally. Lake Mead will be divided longitudinally into 8 sectors as shown in Fig. 4:

1) Las Vegas Wash
2) Bureau of Reclamation Raft
3) Beacon Island
4) Overton Arm at Echo Bay
5) Overton Arm at Miner's Cove
6) Bonecilli Landing
7) Temple Bar
8) South Cove

The exact location of the station in each sector will be given on a detailed map at a later time. Since biological stratification occurs
regardless of the thermal stratification, the system must be handled in a distributed model. Previous attempts have lumped the primary productivity over a vertical profile. This model will handle biological productivity down to and past the compensation level of 1% incident solar radiation. Each sector will be vertically discretized into layers at 0, 1, 3, 5, 7, 10, 15, and 20 meters. These same depths will be analyzed at each sector. Complete mixing or homogeneity is assumed at each depth.

A conceptual hydro-biological model is presented in Fig. 5. All of the existing transfers have been included in the diagram. It is from the master flow chart that each section of the phytoplankton model will be discussed. A brief description of the pathways in the master diagram is provided. This description is not exhaustive. It is necessary to understand this aquatic ecosystem, however, out of necessity we can only model those factors of significance.
HYDRO-BIOLOGICAL DYNAMICS

1. The natural inputs include the stream flow into the reservoir plus the artificial flows that may be created by reservoir operation (dams). Also included are the nutrients as given by Hutchinson ($\text{NO}_3$, $\text{PO}_4$, $\text{S}$, $\text{K}$, $\text{Na}$, $\text{Cl}$, $\text{Ca}$, $\text{Mg}$, $\text{Fe}$, $\text{Mn}$, $\text{Zn}$, $\text{Cu}$, $\text{B}$, $\text{Co}$). The phytoplankton, zooplankton and fish are assumed to enter the system with the nutrients.

2. Bacteria of varying numbers and types enter the reservoir.

3. The sediment loaded entering the system.

4. The nutrients are taken up by the phytoplankton. Specific species will prefer certain nutrients. Certain species will be "luxury" consumers of certain nutrients. The type of nutrients available may determine the species of phytoplankton present.

5. However, if any nutrient is in limiting supply, the phytoplankton will not develop. This reduction of growth rate because of limiting nutrients can be partially compensated for by excessive amounts of other nutrients.

6. The phytoplankton are eaten (grazed) by the herbivorous zooplankton. The kind of zooplankton present is partially dependent upon the kind of phytoplankton present.

7. If an undesirable phytoplankton population (cyanophyta) is present the species, number, and grazing rate of zooplankton is...
affected. This may cut the zooplankton population.

8. The natural output to the system includes the nutrients and species, numbers, and growth rates of phytoplankton and zooplankton. It is implicit that these events are a part of the stream outflow.

9. As the phytoplankton develop they may "self shade" themselves and thereby contribute to the effect of light extinction. In the extreme case the phytoplankton form a algal mat that covers the surface of the reservoir.

10. Light extinction is directly related to the growth rate of the phytoplankton. The depth to which the phytoplankton distribute themselves is a function of the extinction coefficient. This may increase or decrease the zone of productivity.

11. Phytoplankton give off dissolved oxygen (DO) as a result of photosynthesis.

12. A sediment load will pass out of the reservoir via the stream outflow.

13. The light extinction is a function of the sediment in the reservoir.

14. The solar radiation determines the light intensity at the surface and may push the light penetration up or down in the system. However, the light extinction coefficient should not change.
15. The solar radiation is required for photosynthesis by the phytoplankton.

16. The solar radiation is responsible for heating the reservoir waters.

17. The stratification caused by temperature indirectly effects the light extinction. I. E.: The hypolimnion is much clearer than the epilimnion.

18. The temperature determines the species of fish present, their respiration rates and their grazing rate.

19. The temperature determines the respiration rate of the zooplankton, as well as determining the species of zooplanktons present. The temperature is important in determining the life cycles of zooplankton.

20. The temperature determines the respiration rate of the phytoplankton as well as determining the species of phytoplankton present.

21. The temperature is responsible for the bacterial growth rate. This implicity effects the rate of detritus decomposition. The temperature also determines the species of bacteria present.

22. The temperature determines the solubility of the various nutrients in the reservoir. The temperature is responsible for thermal stratification of the reservoir and this reduces the availability
of nutrient deposits in the benthic zone.

23. The water in the reservoir will leave at a specific temperature.

24. The solubility of dissolved oxygen (DO) is temperature dependent.

25. The DO is required for the respiration of fish. The DO also determines the species and activity of the fish.

26. The zooplankton require DO for respiration.

27. The phytoplankton require DO for respiration. The amount respired by phytoplankton over 1 day is less than \(10\%\) of the DO evolved by photosynthesis.

28. The stream flow out of the reservoir has a measurable DO.

29. The zooplankton are eaten by the fish. The fish prefer certain zooplankton species.

30. The zooplankton has a natural and toxic death rate. The natural death rate is constant, however, increasing toxicant or nutrients in excess can cause accelerated death rates.

31. The excretion of the zooplankton adds to the detritus.

32. Phytoplankton are grazed by fish. Fish prefer certain phytoplankton.

33. The phytoplankton have a natural death rate. This death rate can be accelerated by toxicants or excessive nutrients.
Excretion may be neglected.

34. The phytoplankton have a settling rate. Since they are "wanderers" they fall through gravitational forces to the detritus.

35. Man induced waste loads from domestic effluent and campgrounds in the watershed.

36. Man induced waste adds to the sediment load.

37. Man induced waste load adds to the detritus.

38. Bacteria enter the reservoir through human waste.

39. The detritus in the system adds to the sediment load.

40. A certain number of fish are caught or swim out of the reservoir.

41. The natural or induced death of fish results in a higher detritus level.

42. The excrement from fish adds to the detritus.

43. As the detritus is broke down it adds to the nutrient pool.

44. A certain level of detritus will flow out of the reservoir.

45. As the bacteria die they contribute to the detritus. Some bacterial mats are denser than algal blooms. (i.e. sphaerotilus)

46. The bacteria feed on the detritus and break it down.

47. The bacteria live in a symbiotic growth with the algae. This symbiotic relationship could be very important for phytoplankton growth.
48. A certain level of bacteria will flow out of the system.

PHYTOPLANKTON (SYSTEM) COMPONENT

The conceptual model will be developed in terms of biomass changes or source terms \((S)\) in an element layer \((j)\). Since we are vertically discretizing the system we will discuss the approach in terms of a homogenous element layer at depth \((H)\). The source term of phytoplankton will be represented as a difference between

\[
\sum_{i=1}^{n} C_{pij} - \sum_{i=1}^{n} D_{pij}
\]

where \((p)\) identifies the quantities as referring to phytoplankton, \((i)\) refers to one of the \((n)\) phytoplankton species and \((j)\) refers to the element layer of concern. The \((\sum_{i=1}^{n} C_{pij})\) term is very accurately determined by the \(C_{14}\) technique, especially where the primary productivity is very low or very high.

LIGHT AND TEMPERATURE

Light is involved in the life of an alga from the point of settling of the zygote throughout its developmental history to the stage of reformation of a zygote. It affects polarity, morphogenesis and phototropism; it is effective in tactic movements of vegetative cells and in movement of chromatophores. However, we are interested in the aspects of algal metabolism throughout photosynthesis, movement and reproduction that is phased by light. We are also concerned
PHYTOPLANKTON DYNAMICS

Temperature -> Phytoplankton
Solar Radiation -> Phytoplankton
Nutrients -> Phytoplankton
Zooplankton Grazing -> Phytoplankton
Fish Grazing -> Phytoplankton
Settling -> Phytoplankton
Natural Death -> Phytoplankton
Respiration -> Phytoplankton

Advection Transport

Sp: i
with the light influences on the distribution of algae in time and space. Temperature on the other hand is more concerned with the rates of processes but is also involved in the distribution of species in nature.

It should be understood that the shape of the photosynthesis-light curve, which reflects the previous photic history of a phytoplankton, changes rather rapidly when the phytoplankton is transferred to a new light condition. It is also well known that at low intensities the rate of photosynthesis is not affected by the temperature, while at higher light intensities the rate increases with the temperature.

If we assume optimum or saturated intensity (no shading) and all necessary nutrients are present, we obtain a maximum or saturated phytoplankton growth rate $K_1$. It was determined from experimental data that a straight line can approximate the change in the saturated growth rate with temperature: where

$$K' = K_1 T$$

and $K_1$ has the range $0.10 \pm 0.025$ day$^{-1}$ °C$^{-1}$. This approximation over a natural population is accepted.

However, it is also understood that by varying any one of a number of inputs to the phytoplankton growth rate (i.e. $C_{ai}$) will result in an infinite set of curves.
Where $Ca_1, 2, 3$ are varying concentrations of Ca

From $10^\circ C$ to $20^\circ C$ the saturated growth rate doubles. This agrees with the reported temperature dependent ratios. This relationship is not valid at higher temperatures. Light varies as a function of time, depth. Ryther (1956) gives a plot of the light dependence of the growth rate of phytoplankton:

$$G(I) = \frac{I}{I_s} \exp (- \frac{I}{I_s} + 1)$$

where $I$ is the light intensity and $I_s$ is the optimum light intensity.

The average growth rate over the volume elements is required. We must obtain the extinction coefficient $K_e$ by knowing the light intensity.
at various depths. At depth \( Z \) the light intensity \( I_z \) is related to the surface intensity \( I_o \) by the formula:

\[
I_z = I_o \exp(-K_{ez})
\]

Thus the reduction of the saturated growth rate at any depth \( Z \) due to the non-optimum light intensity present is:

\[
G(I_z) = \frac{I_o e^{-K_{ez}} \exp(-I_o e^{-K_{ez}} + 1)}{I_s}
\]

We need to average this reduction factor throughout the depth of the volume element \( V_j \). Let \( H_o \) and \( H_j \) be the depths of the surface and bottom of the element respectively. This reduction factor must be averaged with time also. Assume the incident solar radiation as a function of time over a day is:

\[
I_o(t) = \begin{cases} 
I_a & 0 < t < f \\
0 & f < t < 1 
\end{cases}
\]

where \( f \) is the daylight fraction of the day and \( I_a \) is the average incident solar radiation during the photo period. Let \( r_j \) be the reduction of the growth rate due to non-optimum light conditions in volume \( V_j \) averaged over depth and time be:

\[
r_j = \frac{1}{H_j} \int_0^{H_j} \int_0^f \frac{I_a e^{-K_{ej}z}}{I_s} \exp(-I_a e^{-K_{ej}z} + 1) dt dz
\]

Letting \( \alpha_{ij} = \frac{I_a}{I_s} e^{-K_{ej}H_j} \) and \( \alpha_{oj} = \frac{I_a}{I_s} \) the reduction in growth rate
As the concentration of phytoplankton increases in a volume the extinction coefficient, particularly in the green wavelengths, starts to increase. This mechanism is called self shading. Let:

\[ K_{e,j} = K'_{e,j} + H(P_j) \]

where \( K'_{e,j} \) is the extinction coefficient due to other causes and \( H(P_{oj}) \) has units of chlorophyll concentration.

The nutrients follow Monod growth genetics. At an adequate level of substrate concentration, the growth rate proceeds at the saturated growth rate for that light and temperature conditions. However, at low concentrations the growth rate becomes linearly proportional to the substrate concentration.

For a nutrient (\( N'_j \)) with concentration \( N_j \) in the jth segment the factor by which the growth rate is reduced in the jth segment is:

\[ r_j = \frac{N'_j}{K'_m + N'_j} \]

The constant \( K'_m \) called the Michaelis or half saturation constant, is the nutrient concentration at which the growth rate is half the saturated growth rate.
The Michaelis-Menton relationship describes a hyperbolic curve:

\[ G = \frac{20x}{4 + x} \]

Limiting Factor X

Ketchum (1939) has determined the \( K'_m \) for phosphate to be 10 g - P/l.

The known phytoplankton growth rate can be expressed as a function of several limiting factors which may be multiplied:

\[ G_{ipj} = \frac{N_{1j}}{K_{1m} + N_{1s}} \frac{N_{2j}}{K_{2m} + N_{2s}} \frac{N_{3j}}{K_{3m} + N_{3s}} \]

The growth rate for all the phytoplankton in one element is:

\[ \prod_{i=1}^{n} G_{pij} = K \frac{T}{K_{ij}^{1j}} \left( \frac{2.718f}{K_{ij}^{1j}} \right) \left( e^{-K_{ij}^{1j}} - e^{-K_{ij}^{1j}} \right) \left( \frac{N_{ij}^{1j}}{K_{ij}^{1m} + N_{ij}^{1s}} \right) \left( \frac{N_{ij}^{1j}}{K_{ij}^{1m} + N_{ij}^{1s}} \right) \]

\[ \prod_{i=1}^{n} S_{pj}^{i} = \prod_{i=1}^{n} P_{ij} \]
The settling rate of phytoplankton (sp) is subtracted from the growth rate.

**PHYTOPLANKTON DEATH RATE**

The endogenous respiration rate, considered to be a death rate, is the rate at which the phytoplankton oxidize their organic carbon to $\text{CO}_2$. The respiration rate is a function of temperature and can be approximated by a straight line: $\text{RR} = K_2T$. The natural death rate of phytoplankton (d) is a linear function of the number of individuals.

**GRAZING BY ZOOPLANKTON**

The interaction of phytoplankton and herbivorous zooplankton is very complex. The zooplankton feed by filtering the phytoplankton. Let $C_{gi}$ be the filtering rate per unit biomass of one species (i) zooplankton. The $C_{gi}$ factor is a function of the species of phytoplankton and zooplankton present. Grazing, however, can vary with temperature, species, concentration of phytoplankton, etc. Since the model will be used in anticipation of eutrophic situations developing, the shift in flora to blue-greens could have a considerable effect on the zooplankton population and species. The death rate of phytoplankton due to grazing by zooplankton is $\sum_{i=1}^{n} C_{gi}Z_{ij}$ where $Z_{ij}$ is the concentration of one species (i) herbivorous zooplankton biomass in the jth volume element. The death rate of the phytoplankton is:
The source of the zooplankton \( S_{zj} \) is:

\[
S_{zj} = \sum_{i=1}^{n} \left( G_{zij} - D_{zij} \right) Z_{ij}
\]

where \( Z_{ij} \) is the concentration of one species \( i \) of zooplankton in \( j \)th volume. Since the efficiency of zooplankton grazing fluctuates with the changing phytoplankton concentrations a term must be introduced to account for the semi-digested forms that are excreted by the zooplankton:

\[
\sum_{i=1}^{n} G_{zij} = a_{zp} \frac{K_{mp}}{K_{mp} + P_{ij}}
\]

where \( a_{zp} \) is the utilization efficiency. The growth rate of the zooplankton is also a function of the grazing rate and can be represented by:

\[
\sum_{i=1}^{n} G_{zij} = \sum_{i=1}^{n} C_{ig} \cdot P_{ij}
\]

The zooplankton growth rate takes the form:

\[
\sum_{i=1}^{n} G_{zij} = a_{zp} \sum_{i} C_{ig} K_{mp} \left( \frac{P_{ij}}{K_{mp} + P_{ij}} \right)
\]
ZOOPLANKTON DYNAMICS

Advection Transport

Phytoplankton Biomass → Zooplankton Sp: i → Natural Death
Phytoplankton Species → Respiration
Temperature → Fish Biomass
Fish species
The respiration rate of the zooplankton can be represented by the relationship: $K_3T$. The death rate of the zooplankton is represented by the equation:

$$\sum_{k} D_{zij} = K_3T + K_4$$

Where $K_4$ is imperically derived for predation by higher trophic levels such as fish.

**THE NUTRIENT SYSTEM**

A specific species of alga has a total nutritional requirement which includes numerous essential elements. It is possible to demonstrate that there will be no growth if any one of the numerous essential elements is removed and withheld from the alga. Further, the rate of growth can be controlled by judicious additions of the element in minimal or short supply. The law of the minimum is an important principle in physiology, stating that regardless of how satisfactory one or more requirements of an organism may be, it cannot survive or flourish unless all requirements are met.

Elemental requirements for the metabolism and growth of algae are related to cellular functions such as respiration, photosynthesis and nitrogen fixation. For example, Nostoc mucorum requires the following critical concentrations for nitrogen fixation: calcium, $0.3 \text{ ppm}$; boron, $0.1 \text{ ppm}$, and molybdenum, $0.01 \text{ ppm}$. A critical
concentration is defined as the minimum concentration which will produce a maximum growth. Micro-elemental requirements for photosynthesis include manganese, iron, copper, chlorine, vanadium, etc. in addition to such major elements as carbon, hydrogen, nitrogen, phosphorus, magnesium, and potassium. Iron, manganese, zinc and copper are known to have various roles in respiration.

The sink of nutrient (i) is: $S_{Ni j}$ in the $j$th element. Phytoplankton remove nutrients from the nutrient pool. Certain species of phytoplankton have a preference for certain nutrients. The amount required of one nutrient by one species is $a_{nip} C_{pi j} P_{ij}$ when $a_{nip}$ is the nutrient to phytoplankton biomass ratio. This amount must be summed over all the nutrients and all the species. Therefore, the sink of nutrients due to phytoplankton is:

$$S_{nj} = \sum_{i=1}^{n} \sum_{p=1}^{n} a_{nip} C_{pi j} P_{ij}$$

The zooplankton excrete nutrients at a rate that is dependent upon the biomass of phytoplanktons being grazed. This excretion rate has the mathematical form of the Michaelis-Menton equation. The rate of nutrient excretion is equal to the rate grazed minus the rate metabolized and can be represented by the equation:

$$S_{nj} = \sum_{i=1}^{n} \sum_{p=1}^{n} a_{nip} C_{pi j} Z_{ij} P_{ij} \frac{(1 - a_{zip} K_{mip})}{K_{mip} + P_{ij}}$$
Letting $W_{nj}$ be the rate of addition of a nutrient (i) to the jth element by man induced practices, we obtain an equation for the nutrient system. From this equation however, we must subtract the loss of nutrients due to adsorption to the sediment particles: $A_{ni}$, where $A$ is the coefficient of adsorption.

The final nutrient pool equation includes a man induced waste term, phytoplankton uptake term, excretion rate term, phytoplankton and zooplankton death rate term, and a sediment loss term:

$$S_{nj} = \sum_{i=1}^{n} \sum_{n=1}^{N} W_{ni} - \sum_{i=1}^{n} \sum_{n=1}^{N} a_{nip} C_{pij} P_{ij} + \sum_{i=1}^{n} \sum_{n=1}^{N} a_{nip} C_{igj} Z_{ij} P_{ij} \left( 1 - a_{zip} K_{mip} \right) + \sum_{i=1}^{n} \sum_{n=1}^{N} a_{nip} K_{2} T_{pij} + \sum_{i=1}^{n} \sum_{n=1}^{N} a_{nin} K_{3} T_{zij} - \sum_{i=1}^{n} A_{ni}$$

It quickly becomes apparent that only a computer approach could solve the required equations.
### NOMENCLATURE FOR THE CONCEPTUAL MODEL

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<thead>
<tr>
<th>Notation</th>
<th>Description</th>
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<tbody>
<tr>
<td>S</td>
<td>Source or sink</td>
</tr>
<tr>
<td>G</td>
<td>Phytoplankton or zooplankton growth</td>
</tr>
<tr>
<td>D</td>
<td>Phytoplankton or zooplankton death</td>
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<tr>
<td>T</td>
<td>Temperature</td>
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<td>$K_1$</td>
<td>Temperature coefficient</td>
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<td>K</td>
<td>Saturated growth rate of phytoplankton</td>
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<tr>
<td>$I_s$</td>
<td>Light Saturation Intensity for phytoplankton</td>
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<tr>
<td>$I_o$</td>
<td>Light intensity at a depth</td>
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<td>$K_2$</td>
<td>Endogenous respiration rate of phytoplankton</td>
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<td>$C_g$</td>
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<td>Influent phytoplankton chlorophyll concentration</td>
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<td>$a_{ZP}$</td>
<td>Zooplankton conversion efficiency</td>
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<tr>
<td>$K_{mp}$</td>
<td>Phytoplankton Michaelis constant</td>
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<tr>
<td>$D_z$</td>
<td>Zooplankton death rate</td>
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</table>
Notation

\( a_{NP} \)  Phytoplankton nutrient-carbon ratio

\( W_N \)  Direct discharge rate of nutrient

\( V \)  Segment volume
SELECTED REFERENCES


Ketchum, B. N.  The Absorption of Phosphate and Nitrate by Illuminated Cultures of Nitzschia Closterium, Am. J. Botany, June, 1939.


