Effects of explicit subtraction instruction on fifth grade students with learning disabilities

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EFFECTS OF EXPLICIT SUBTRACTION INSTRUCTION ON FIFTH GRADE
STUDENTS WITH LEARNING DISABILITIES

by

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ABSTRACT

Effects of Explicit Subtraction Instruction on Fifth Graders With Learning Disabilities

by

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This study involved an investigation of the effects of strategy instruction integrated with the concrete-representational-abstract teaching sequence on students with learning disabilities. A multiple probe design across subjects with one replication was used in this study. Two sets of data were analyzed to determine effectiveness of the independent variable (intervention lessons). The first data set consisted of pre and posttest percentage scores and the second data set consisted of baseline, intervention, and maintenance probe scores that were collected throughout the study per the parameters of a multiple probe design. The probe scores were plotted in line graph format and analyzed using visual analysis related to level, trend, and variability of the data points.

A total of six fifth grade students (five males and one female) with learning disabilities participated in this study. The participants ranged in age from 10 years 10 months to 12 years 0 months. Each participant met the State of Nevada Administrative Code eligibility criteria for specific learning disabilities and failed to meet their school district’s standards related to subtraction with regrouping. The six participants were divided into two triads.

The students’ learning disability teacher staggered the introduction of the scripted intervention lessons according to the parameters of a multiple probe design. Each
intervention lesson contained pedagogically sound systematic and explicit instruction which included (a) an advance organizer, (b) a describe and model stage of instruction, (c) a guided practice stage of instruction, (d) an independent practice stage of instruction, and (e) a problem solving stage of instruction. Additionally, the lessons followed the concrete-representation-abstract teaching sequence. The principal and student investigator observed 20% of the total lessons to ensure that the learning disability teacher implemented the lessons with fidelity. The percentage of agreement between the two observers was 99% indicating a high level of implementation fidelity.

Interscorer reliability was established before analyzing the data sets. The learning disability teacher scored all pre-, post-, and maintenance tests for the participants and the student investigator scored 20% of the pre-, post-, and maintenance tests. Interscorer reliability was determined to be 100%.

A comparison of pre- and posttests revealed that participants’ performance increased on the posttests. As a group, the participants raised their total number of correct responses from an average of 6 correct answers to 14.3 correct answers out of a total of 20 computation subtraction problems that required regrouping to solve. Participants also achieved an average of 21.6 more correct digits from pretests scores to posttest scores on a fluency measure that contained computation subtraction problems that required regrouping. Participants increased the number of correct responses on average by 4.3 on word problems that required subtraction with regrouping skills to be applied. Participants maintained these new skills over time and indicated high levels of satisfaction with regard to the mathematics intervention program. Finally, implications of the current study and suggestion for future research are discussed.
With regard to the ongoing probe data, all six participants demonstrated an increase in level from baseline condition to intervention condition. This increase in level was sustained during the maintenance condition for all six participants. All six participants demonstrated a relatively stable flat trend during the intervention condition. With regard to variability, Participant 1 was the only one who demonstrated little variability during intervention condition. Participants 2, 3, 4, 5, and 6 each demonstrated notable variability during the intervention condition and had to repeat 2 to 6 sessions in which they did not attain mastery criteria on their first try.
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CHAPTER 1
INTRODUCTION

The field of mathematics education has experienced numerous changes over the 20\textsuperscript{th} and 21\textsuperscript{st} centuries. Much emphasis has been placed on trying to improve mathematics education and the subsequent mathematics performance of students within the United States. Mathematics education is viewed as one of the hallmarks of an advanced society. Thus, in an attempt to strengthen mathematics performance, many new approaches have been tried with somewhat disappointing results.

Historical Overview of Mathematics Education

\textit{1900 – 1950: The Foundation of Mathematics Education}

During the early 1900s, the scope and sequence of mathematics education expanded. Many educators began to verbalize their concern about the necessity of teaching all students every concept within the mathematics curriculum. One such teacher educator was William Heard Kilpatrick (Klein, 2003). Kilpatrick challenged traditional views regarding what content was to be taught at the high school level. His belief, “was that subjects should be taught to students based on their direct practical value, or if students independently wanted to learn those subjects” (Klein, 2003, p. 178). This complimented the pedagogical methods supported by the progressive education movement because it limited the content of what teachers were expected to cover. The benefit of using condensed content lies in the potential to allow educators to maintain an instructional pace reflective of their students’ needs. This progressive way of thinking was supported in Edward L. Thorndike’s (1901) research findings
that students become too easily confused when they are asked to transfer between mathematical skills too quickly.

In 1915, Kilpatrick was asked to chair a committee for the National Education Association’s Commission on the Reorganization of Secondary Education to further disseminate his views on limiting the content of mathematical skills taught (Klein, 2003). The result of this committee work was a report titled *The Problem of Mathematics in Secondary Education*. The Mathematical Association of America (MAA) did not accept Kilpatrick’s progressive argument. Members of this national organization believed in the theory of maintaining the amount of content being taught for the sake of promoting mental discipline. E.R. Hedrick, the first president of the MAA, responded to Kilpatrick’s progressive thinking by creating the *National Committee on Mathematical Requirements*. The report from this committee was delayed due to World War I, but was ultimately released in 1923. This comprehensive report, referred to as the *1923 Report*, reviewed the curriculum in secondary schools, investigated the professional development of preservice teachers in other countries, and “justified the study of mathematics in terms of its applications as well as its intrinsic value” (Klein, 2003, p.180). In other words, the committee members supported providing an extensive mathematics curricula because they believed the study of mathematics such as algebra, trigonometry, and geometry, provided valuable cognitive experiences, from which all students gain cognitive value. They also recognized that it was impossible to predict what exact types of mathematics would be needed post school years for specific students, so it would be a disservice to limit the content taught. The *1923 Report* maintained the standard of providing a
comprehensive mathematics curriculum and began to invoke educational reform including establishment of the College Examination Board at the teacher educator level.

Around the same time, in 1920, The National Council of Teachers of Mathematics (NCTM) was founded and was strongly supported by the MAA. This national organization maintained the importance of teaching all parts of mathematics to the masses. Additionally, C.M. Austin, the first president of the NCTM, advocated that “curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from the educational reformers” (Klein, 2003, p.180). Thus, members of NCTM believed teachers should be empowered to participate in important decisions related to the work they do.

Kilpatrick’s influence infiltrated the 1930s. Klein (2003) titled this decade the “Activity Movement”. Classroom teachers began to feel the strains of teaching too much content within their limited time. They sought out other options including those of the progressive mathematics curriculum. Initially, they began to actively promote the integration of subjects in elementary school and promoted the idea of designing instruction based on the needs of students instead of allowing the curriculum to drive instruction. Although the Activity Movement had a strong presence in elementary schools across the nation, it met some resistance in secondary education. Rightfully, secondary school teachers specialized in specific subject matter that did not lend itself to a more holistic, blending of the skills approach.

The Life Adjustment Movement began in the mid 1940’s (Klein, 2003). This movement emerged in part because of military concerns related to the lack of
mathematical skills among army recruits. This reinforced the idea that secondary teachers were trying to cover too many skills; subsequently basic math skills were not being mastered. The result was public school students who lacked (a) daily living mathematical skills (e.g., maintaining personal funds), and (b) adequate preparation for college entry programs. The proposed solution was to offer fundamental mathematics courses reflective of these needs and, to avoid negative stigma inadvertently placed on those students who were enrolled in such courses. Although there appeared to be a great need for fundamental mathematics courses, school personnel were hesitant to publicly acknowledge remedial needs among their student population. Unfortunately, this hesitancy was reinforced through parents’ pursuits to resist the proposed curricular changes. Parents were not fond of limiting their expectations for what their children might achieve if exposed to the right curriculum, and insisted on continuing to enroll their children in advanced mathematics courses with complete disregard for whether or not previously acquired skills were learned at a mastery level. At the end of the Life Adjustment Movement (late 1940’s) several scientific discoveries emerged (e.g., radar, atomic energy). These discoveries influenced school personnel to maintain their current school standards related to mathematics and further eliminated any thoughts of an adjusted math curriculum.


By 1951, the U.S. government had spent close to 14 billion dollars supporting WWII veterans who took advantage of the G.I Bill of Rights and enrolled in colleges or universities across the nation (Gutek, 1986). This translated to the government establishing vested interest in these collegiate students playing a strong role in our
nation’s pursuit within the Cold War. During the early 1950’s, there was general consensus that Americans continued to fall short of meeting academic and global expectations (Walmsley, 2003). This resulted in overwhelming challenges to educators at all levels (i.e., elementary, secondary, post-secondary). Unfortunately, it took a monolithic event to bring these underlying issues to light and demand public attention at the federal level. The event was the USSR launch of the first satellite, Sputnik, into space on October 4, 1957. The publicity that this event drew sensationalized to the American people that the U.S. was not as advanced, scientifically or mathematically, as its Cold War counterpart. The launch of the first U.S. satellite on January 31, 1958 did not pacify the American public and blame soon fell on President Eisenhower and the education system in general.

Congress responded by passing the 1958 National Defense Education Act (NDEA). The funding provided by NDEA supported the advancement of education in science, mathematics, and modern foreign languages. It provided one billion dollars to be spent over four years at the collegiate level in the form of scholarships, grants, and loans. Many organizations, made up primarily of mathematicians, began to meet and develop ideas for “new math” curricula that would support our national drive to out perform other countries’ mathematics education. The American Mathematical Society set up the School Mathematics Study Group (SMSG), which was one of the most influential groups to the New Math era (Klein, 2003; Walmsely, 2003). Its director, Edward G. Begle, headed efforts to develop “new curriculum” for primary and secondary education and produced a series of reports that were published in Random House titled *The New Mathematical Library* (Klein, 2003). The NCTM
developed the Secondary School Curriculum Committee in 1959. This group of educators tried to provide their own curriculum based on pedagogical issues and concerns they had experienced. They published the *Revolution in School Mathematics* (NCTM, 1961), which carried two main themes. First, students were able to learn more mathematics earlier than in past generations, and second, students needed to be grouped according to their present ability levels when disseminating the knowledge to ensure mastery was reached. Unfortunately, with the public only interested in the technological advances being produced, mathematicians had already established a firmer influence on the future of curriculum content.

Walmsley (2003) suggested that increased mathematics research, automation, and the introduction of computers continued to drive the need for more complex mathematical processes through the 1960s. In addition to algebra, trigonometry, and geometry, courses such as calculus were listed on the expected mathematics transcripts of high school aged students. This further exacerbated debates about mathematics instruction becoming more formal and less attention being given to basic skills or application of basic skills (Klein, 2003).

Teacher educators were not able to keep up with the growing mathematics content and the frustration of unprepared teachers increased. The National Science Foundation offered some relief when they offered summer workshops and training programs to teachers who needed to develop their mathematical foundation beyond a computational level. Another “quick fix” approach was a course called Contemporary Mathematics on the National Broadcasting Company’s (NBC) Continental Classroom. This course was offered on television from 6:00 to 6:30 a.m. in 1961.
through 1962. Each of these solutions had minimal effect. In addition to the teacher
frustration that had emerged, Schoenfeld (2004) noted that parent frustration had also
increased because they were not able to support and assist their children with this new
approach to mathematics. Increases in complex content and insufficient teacher
preparation collectively lead to the demise of The New Math Era.


Walmsley (2003) credited the “new math” reform with producing a large
population of students who were not able to effectively use basic math skills
necessary in everyday living. President Nixon published a document in March of
1970 titled Education for the 1970s: Renewal Reform. In this document, the President
took the focus off mathematics and science education. Instead, he suggested that the
way to rectify the deficits in education, including those in math education, was to
concentrate on how to provide quality education for all Americans with a focus on
literacy and the social impact equal education could have. This included a push to
increase efforts to educate the poor and find avenues that allowed any student the
opportunity to attend college. Nixon stressed that both public education and higher
education needed to be held accountable for providing equal access to quality
education for all students. In mathematics, a strong emphasis on basics was viewed as
quality education. Undoubtedly, this perspective was adopted due to the perceived
failure of the previous “New Math” movement. During the 1970s, most states
interpreted this accountability component of quality education as being addressed
through standardized tests that included minimum competency tests in basic skills
(Klein, 2003). Thus, teachers were encouraged to return to traditional math and traditional testing once again.

During the 1970s, standardized test scores were viewed as the most appropriate means of evaluating the effectiveness of the New Math Movement; unfortunately student performance on these tests was less than acceptable (Klein, 2003; Walmsley, 2003). The National Conference Board of Mathematical Science appointed a team in 1975 to evaluate the impact New Math had on both student achievement and student attitudes with regard to mathematics. This team was titled the National Advisory Committee on Mathematical Education (NACME) and they examined performance trends within standardized test given at both state and national levels.

The NACME analyzed standardized mathematics test scores in California and New York because these states had collected test score data long enough to determine valid trends (NACME, 1975). For example, the New York State Department of Education had administered standardized tests to all students in third, sixth, and ninth grades each fall beginning in 1966. For the subsequent tests given each year after that, students were compared against the bottom 23% of the students from the 1966 testing population with the expectation being student achievement would continue to substantially increase higher than the 1966 reference point. Despite the reform New Math had demanded, data from this standardized test suggested that New Math had not impacted student achievement positively. By 1973, 32% of the students in grade 6 and 34% of the students in grade 9 performed below the 1966 reference point (NACME, 1975). This was a marked decrease in mathematical performance for sixth and ninth graders from 1966 to 1973.
California’s Comprehension Tests of Basic Skills (CTBS) mathematics tests suggested a similar trend between the years of 1969 – 1973 with declining or stagnant median scores of 47 in 1969, to 38 in 1971 which is where it remained for the next two years (NACME, 1975). Again, it was concluded that the New Math Movement had been unsuccessful.

Scholastic Aptitude Test (SAT) scores were used routinely to determine whether students were admitted to colleges or universities across the nation. Even at this level, data very clearly depicted a negative trend for student achievement in mathematics. Between 1962 and 1975, the mean score on the quantitative section of the SAT had decreased as much as 30 points. Equally dramatic was the percentage of scores above 600 in mathematics declining from 20.2 to 16.4 over the same span of years (NACME, 1975).

The combination of the president’s cry for reform in education and the disheartening data related to the standardized test scores created to monitor math achievement within the United States, revealed an issue that could not be ignored. Educators quickly abandoned the belief that New Math curriculum would impact student performance positively and went back to what had been comfortable to them in the past, which was a focus on basic computational skills including algebra and arithmetic (Walmsley, 2003). As a result of this shift in focus within math education, the “Back to the Basics” term was coined to represent this decade.

1980 - 1990: Preparing for More Reform

The NCTM gave careful consideration to the data collected by the NACME between 1970 and 1980 and used it as leverage to take on a leadership role for
mathematics reform. This influence first manifested itself in a document published by
the NCTM in 1980 titled *An Agenda for Action* (Walmsley, 2003). The preface of this
document very pointedly states that the NCTM, comprised of professional educators
who investigated the mathematics performance results of the last decade, felt the need
to publish realistic recommendations for the future of mathematics education. Of the
recommendations listed and thoroughly reviewed in *An Agenda for Action*, three
greatly impacted the direction of mathematics curricula and the eventual development
of national mathematics standards. Those three recommendations were:

1. Problem solving should be the focus of school mathematics in the 1980s.
2. Basic skills in mathematics should be defined to encompass more than
   computational facility.
3. Stringent standards of both effectiveness and efficiency should be applied to
   the teaching of mathematics (NCTM, 1980).

These recommendations resulted in another movement away from basic
computational skills and back toward the skills focused on during the New Math Era.
Problem solving was encouraged to overshadow basic computation skills and taking a
hard look at teacher education programs was at the forefront of discussion once again.

Another influential report published a mere three years after *An Agenda for Action*
was titled *A Nation At Risk* (Klein, 2003; Walmsley, 2003). The National
Commission on Excellence, appointed by the U.S. Secretary of Education Terrell
Bell, wrote this report. *A Nation At Risk* was a title that was easily identifiable to the
American public and therefore overshadowed any presence *An Agenda for Action*
might have established before (Klein, 2003). Although specific weaknesses in current
math education were only some of the many educational issues addressed in this report, they were poignant in nature. For example, the report highlighted the fact that many students who graduate from high school need remedial math instruction in post secondary settings. This need translated into two things: millions of dollars being spent by business and military leaders on remedial math education and an increase of 72% in the amount of remedial mathematics courses needed within public colleges (Goldberg & Harvey, 1983; Klein, 2003). The diminutive number of high school students completing courses such as algebra and calculus, the need to address teacher shortages in math and science, and the criticism that much of the teacher workforce came from the bottom quarter of graduating high school and college students were some other examples of why *A Nation At Risk* jolted Americans to continue to demand effective reform in mathematics education.

The NCTM embraced the public’s current interest in mathematical reform and began its drive to develop standards that addressed a strong focus on basic skills while maintaining high standards. The NCTM established the Commission on Standards for School Mathematics in 1986. This commission developed the *Curriculum and Evaluation Standards for School Mathematics* in the summer of 1987, which was ultimately revised and published in 1989. This pivotal piece of literature has been commonly referred to as the *NCTM Standards*, or the *Standards* (Klein, 2003; Walmsley, 2003). These standards were broad in nature as they indicated topics that should be covered throughout a child’s math education within three bands of grades: K-4, 5-8, and 9-12. The *Standards* suggested which of the topics should receive “increased attention” and which topics should receive
“decreased attention” within these grade spans. The Standards were then distributed to school districts or administrators throughout the country (Walmsley, 2003). The nation’s math educators were ready for change that would improve student outcomes and therefore, the Standards were welcomed and adopted by most states and used to dictate curriculum.

1990 – 2009: Standards Dictating Education

The NCTM capitalized on its success as being a leader in mathematical reform and continued to publish documents that supported the ideology found within its original Standards. The Professional Standards for Teaching Mathematics was published in 1991. The intention of this document was to establish professional teaching standards that would guide reform in school mathematics. A second NCTM publication published in 1995, was the Assessment Standards for School Mathematics. Included in this publication was an analysis of current assessment practices with the goal of improving such practices.

In April of 2000 at the annual NCTM conference, a second edition of the Standards was launched (NCTM, 2000). This edition was titled the Principle and Standards for School Mathematics and is more commonly referred to as the 2000 Standards. The NCTM suggested that the need to update the 1989 standards was a result of the “New knowledge, tools, and ways of doing and communicating mathematics” (NCTM, 2000, p.5). Specifically, the increase in availability of affordable and advanced technology including calculators and ease of dissemination of information via the internet were two driving forces for the revisions.
Similar to the 1989 *Standards*, the *2000 Standards* attempted to acknowledge that differences among students exist; therefore the standards were to continue to be addressed within broad grade levels. The bands of grade levels in the current standards were divided into four groups: Pre-K – 2, grades 3 – 5, grades 6 – 8, and grades 9 – 12. Within these grade levels, there are a variety of content and process standards expected to be reviewed in connection to specific mathematical topics appropriate to that grade level. The five content standards include Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The five process standards address Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. The number of subskills found within each content and process standard for each span of grade level is vast.

Teachers were responsive to the *2000 Standards* and by 2006 had come up with new challenges surrounding the implementation of the standards. School districts had used the *2000 Standards* to develop scope and sequence charts that clarified learning expectations to be addressed at each grade level. Due to the span of grade levels used within the *2000 Standards*, many of the learning expectations at each grade level were repetitive. The result was curricular expectations within each grade level that were wide-ranging. Teachers were overwhelmed with the daunting task to effectively cover all the benchmarks expected. At the same time, a dramatic increase in educators being held accountable for ensuring student success spawned from new legislation such as the No Child Left Behind Act (2002). This law was created to increase student achievement by enforcing penalties on schools or districts where sufficient achievement was not being met per high stakes test performance. The NCTM
recognized that the 2000 Standards were too broad to offer immediate guidance (NCTM, 2006). Teachers needed a more focused coherent curriculum to work from.

In 2006, the NCTM published the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. These Focal Points provide consistency in the grade placements of mathematics topics. The Focal Points allow teachers to commit more time each year to topics needing special emphasis. They are concise and provide the direction teachers need to make their way efficiently through grade level expectations. There are no more than three Focal Points per grade level and corresponding connections are identified. These Focal Points differ from a benchmark that typically states the single outcome expected of the student. Instead, Focal Points are used as a beginning point of reference for what should be emphasized during instruction throughout a grade level. Teachers can now use the Focal Points as a new resource that encourages instruction that ends with mastery of few specific skills within each grade level versus repetitive presentation of numerous skills with no criteria for mastery being adhered to.

Finally, in April of 2006 President Bush, via the U.S. Department of Education, created the National Mathematics Advisory Panel (NMAP) as a means to ensure that the United States was prepared to defend its “peerless mathematical prowess” among other leading nations in the world (National Mathematics Advisory Panel, 2008, p. xi). This panel acknowledged that the United States science and engineering workforce was in danger of facing depleting employment rates as a result of accelerating retirements during a time of increased growth in job opportunities within this discipline. Although the nation’s strategy in the past had been to outsource for
these job positions, the ease of use of the internet has created success of foreign financial systems which limits the amount of quality personnel seeking U.S. employers (U.S Department of Education, 2008). The NMAP’s mission was to develop a report that would explain what our country needed to do in order to adapt to this change and maintain its leadership in mathematics. In March of 2008, the NMAP published *The Foundations for Success: The Final Report of National Mathematics Advisory Panel*. This document reported six elements that needed to be addressed to keep America competitive with other nations:

1. The mathematics curriculum in Grades PreK – 8 should be streamlined and should emphasize a well-defined set of the most critical topics in the early grades.

2. Use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing a) the advantages for children in having a strong start; b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts; and c) that effort, not just inherent talent, counts in mathematical achievement.

3. Our citizens and their educational leadership should recognize mathematically knowledgeable classroom teachers as having a central role in mathematics education and should encourage rigorously evaluated initiatives for attracting and appropriately preparing prospective teachers, and evaluating and retaining effective teachers.
4. Instructional practices should be informed by high-quality research, when available, and by the best professional judgment and experience of accomplished classroom teachers. High-quality research does not support the contention that instruction should be either entirely “student centered” or “teacher directed.” Research indicates that some forms of particular instructional practices can have a positive impact under specified conditions.

5. NAEP and state assessments should be improved in quality and should carry increased emphasis on the most critical knowledge and skills leading to Algebra.

6. The nation must continue to build capacity for more rigorous research in education so that it can inform policy and practice more effectively (U.S Department of Education, 2008, p. viii - xiv).

These six elements reflect many of the same constructs that have challenged American students for over a century.

Statement of the Problem

Despite intense mathematical discussion and reform, the quality and substance of the curriculum that students have presented to them is still debatable. There is a clear disconnect between what researchers suggest as good teaching of skills and what is practiced within classrooms today; and a perfect measurement to determine accountability of skills among our students has yet to be discovered. The result is below average achievement in mathematics within our nation (Stein et. al., 2006).
**Math Performance of General Population**

Math deficiencies are among serious educational problems that correlate to student drop out rates, delinquency, and lifelong underachievement (Stillington & Frank, 1993). The National Assessment of Educational Progress (NAEP) (2007) reports that only 32% of our students are at or above the proficient level in Grade 8 mathematics. An even less impressive outcome of the NAEP (2007) indicates that this rate of proficiency drops to 23% for students in Grade 12. The increase in remedial mathematics courses being offered within four-year colleges and community colleges across the nation further substantiates that American students continue to fall short of expected math achievement (National Mathematics Advisory Panel, 2008). Jordan and Hanich (2003) suggest that although students with math difficulties who are good readers outperform those students who have both math and reading difficulties on math related tasks, both subgroups demand a high degree of remedial instruction. Research interventions for students who fall within these subgroups are of national interest to ensure a workforce prepared to compete internationally (National Mathematics Advisory Panel, 2008). Although math deficiencies among American youth have reached an incidence level equal to reading deficits, they are not as effectively addressed in classrooms (Bryant & Bryant, 2008; Jordan & Hanich, 2003; Mabbott & Bisanz, 2008).

**Math Performance of Students with Learning Disabilities**

In addition to facing the challenge of improving mathematics instruction for all students, one particular subgroup, those with learning disabilities, brings unique characteristics that magnify the challenge. These characteristics include a range of
deficits in information processing, attention, verbal – auditory discrimination, and visual-spatial processing (Goldman, 1989; Maccini & Hughes, 1997; Mercer, 1997; Miles & Forcht, 1995). Kroesbergen and Van Luit (2003) and Keeler and Swanson (2001) suggest that another common attribute that students with learning disabilities habitually exhibit is a deficit within their working memory. Each child with a learning disability contains a unique combination of these deficits, which results in a need for instruction that effectively and efficiently meets these needs.

Among students with learning disabilities, two out of every three experience mathematics–related problems (Maccini & Hughes, 1997). Students with learning disabilities have a long history of poor math performance (Baroody & Hume, 1991; Ehrlich, Buckley, Midouhas, & Brodesky, 2008; Englemann, Carnine, & Steely, 1991; Hofmeister, 1993; Maccini & Hughes, 1997; McLeod & Armstrong, 1982; Mercer & Miller 1992). More recently, students with disabilities in the fourth- and eighth-grades who took the 2007 National Assessment of Educational Progress performed significantly behind their peers even when accommodations were permitted during the examination (Bryant et. al., 2008; Lee, Griggs, & Dion, 2007; NAEP, 2007). Additionally, the percentage of children with learning disabilities in math has grown from 6% of the general population to close to 10% of the general population (Badian, 1983; Bryant et. al., 2008). Of particular concern are the individual skills frequently associated with math learning disabilities. Among Bryant and Bryant’s (2008) top ranked mathematics difficulties are problem solving, multi-step problems, verifying answers, recalling number facts, and borrowing/renaming errors. Authorities agree that deficits in mathematical reasoning can have a

Although researchers and educators have attempted to address the poor math performance through a variety of interventions, most of the intervention studies to date address basic math fact recall, basic computation skills, and problem solving (i.e. word problems) (Garnett, 1992; Miller, Strawser, & Mercer, 1996; Montague & Brooks, 1993; Montague, 2008). There is a paucity of research that addresses regrouping skills and more advanced computation skills. Reviews of mathematics literature (Maccini & Ruhl, 2000; Mercer & Miller, 1992) reveal that the concrete-representational-abstract teaching sequence and strategy instruction, among other interventions, are effective for teaching initial single digit addition and subtraction skills. Despite these initial investigations, additional research is needed related to higher-level computation skills.

Purpose of the Study and Related Research Questions

The purpose of this study is to investigate the effects of strategy instruction and the concrete-representational-abstract sequence to teach subtraction with regrouping to students with learning disabilities. The following research questions have been identified to address this purpose:
1. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems?

2. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems?

3. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems?

4. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems?

5. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping?

6. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems?

7. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence report high, medium, or low levels of satisfaction related to the subtraction with regrouping intervention lessons?
Significance of the Study

A reoccurring theme within current literature both in the field of special education and general education, suggests that one effective way to minimize early difficulties in mathematics is to deliver effective instruction to all students within those early grades (Bryant et. al., 2008; Chard et. al, 2008; Clark, Baker, & Chard, 2007; Fuson, Smith, & LoCicero, 1997; Griffin, 2004; Jordan, Kaplan, Locuniak, & Ramineni, 2007). Computation is one such skill that research supports as being pivotal to master within the early grades (Bryant, Smith, Bryant, 2008; Gersten, Jordan, & Flojo, 2005). Several methodologies have emerged as being particularly effective for teaching mathematics computation to students with disabilities. Included among these are strategy instruction, explicit instruction, and instruction that follows the concrete-representational-abstract teaching sequence. Strategy instruction includes teaching a series of steps that students follow to achieve a specified goal (Carnine, 1997). Hudson and Miller (2006) define explicit instruction as being instruction that is highly structured in which the teacher thoughtfully and specifically presents new material in small steps as driven by student-measured performance. The explicit instruction sequence usually includes an advanced organizer, teacher demonstration, guided practice, independent practice, and maintenance checks (Miller & Hudson, 2007). The concrete-representational-abstract teaching sequence requires the student to show mastery on a new math skill first taught using three-dimensional manipulative devices (Hudson & Miller, 2006). Next, the student demonstrates mastery on the same math skill when applying it to two-dimensional pictures representing the math concept. Finally, the student demonstrates competency using
the math skill on a more abstract-level including number sentences without manipulative devices or pictorial supports.

Most of the research related to strategy instruction, explicit instruction, and the concrete-representational-abstract teaching sequence is devoted to early computation skills and problem solving (i.e. word problems) that involves basic math facts (Good & Grouws, 1979; Harris, Miller, & Mercer, 1995; Keeler & Swanson, 2001; Kroesbergen & Van Luit, 2003; Mercer & Miller, 1992; Montague, 2008; Morin & Miller, 1998; Rivera & Smith, 1988; Swanson & Hoskyn, 2001). Before moving into more complex mathematical skills, basic computation must be mastered. In 2001, the National Research Council (NRC) correlated a student’s inability to fluently navigate through basic computation skills as being as debilitating as a student’s inability to decode words when applying reading comprehension skills. In 2002, the NRC further stressed the importance of fluency within basic addition, subtraction, multiplication, and division facts when it listed computation as the second of five main strands in mathematics. Without a strong foundation of these basic skills, many students with mathematical learning disabilities demonstrate mathematical misconceptions (Marchand-Martella, Slocum, & Martell, 2004). These mathematical misconceptions incapacitate a student’s ability to acquire higher order mathematics skills (Calhoon, Emerson, Flores, & Houchins, 2007).

Subtraction with regrouping is one of the early math computation skills that is expected to be mastered by students at the third grade (Cawley, Parmar, Foley, Salmon, & Roy, 2001). Unfortunately it is a computation skill that many students with learning disabilities struggle to achieve. There are limited studies found within
the current literature that address how to teach subtraction with regrouping effectively and efficiently to this population of students (Riccomini, 2005). Moreover, Calhoon, et al., (2007) suggest that descriptive studies devoted to the investigation of the computational performance of students with math disabilities have not taken place for over 15 years. This study will provide new information and help address the void in research related to the effectiveness of using strategy instruction, explicit instruction, and the concrete-representational-abstract teaching sequence to help students with learning disabilities acquire and maintain subtraction with regrouping skills.

Limitations of the Study

This study has several limitations. First, the participants from this study will be selected based on a sample of convenience. They will be fifth grade students who attend a single elementary school in the Southwestern United States. Therefore, the selection of participants will not be based on randomized criteria and the results may not generalize to other school populations. Second, the participants in this study will be students with specific learning disabilities. The findings should not be generalized to dissimilar student populations in elementary schools. Finally, this study addresses pedagogy related to teaching students with learning disabilities how to subtract multi-digit problems that require regrouping. Therefore, the results of this study should not be generalized to other mathematics skills without further investigation.
Definition of Terms

The following terms and definitions were used in this study.

*Advanced Organizer.* Material introduced at the beginning of a lesson in which previously learned material is briefly reviewed, the lesson objective is explained and connected to previously learned material, and relevance is established between the objective and why the students are learning the new material (Hudson & Miller, 2006).

*Base Ten Blocks.* These are a type of mathematics manipulative device that can be used to reinforce understanding of the base-ten number system. These 3-dimensional wooden, plastic, or foam blocks consist of small cubes that represent units of one, rectangular rods equal in length to ten ones joined together (sometimes referred to as a long) that represents tens, a square block equal in size to ten rods joined together (sometimes referred to as a flat) that represents hundreds, and a larger cube equal in size to ten hundreds blocks piled on top of one another that represents thousands (Tucker, Singleton, & Weaver, 2002).

*Basic Facts.* There are a total of 390 basic facts (i.e., 100 each of addition, subtraction, and multiplication facts and 90 division facts). Basic fact equations consist of two single digit numbers (i.e., $4 + 4 = 8; 2 -1 = 1; 2 \times 3 = 6; 4 \div 2 = 2$) or two single digit numbers and one double digit number (i.e., $6 + 6 = 12; 18 - 9 = 9; 7 \times 8 = 56; 81 \div 9 = 9$) (Stein, et al., 2006).

*Concrete-Representational-Abstract Teaching Sequence.* An instructional process that sequentially introduces a math concept through the use of: (a) concrete three-dimensional manipulative devices, (b) two-dimensional representational
drawings, and finally (c) abstract representations of the math concept usually in the form of number sentences (Hudson & Miller, 2006).

*Conceptual Knowledge.* “A connected web of information in which the linking relationships are as important as the pieces of discrete information that are linked” (Goldman, S., Hasselbring, T.S., & and the Cognition and Technology Group at Vanderbilt, 1997, p. 4)

*Curriculum-based Assessment (CBA).* Refers to an assessment that is administered to students throughout various times while learning course curriculum as a means to assess their ongoing performance (Hudson & Miler, 2006).

*Declarative Knowledge.* It is one of the four instructional domains within math instruction. Declarative knowledge includes memorization of, but not limited to, math facts such as number recognition, recalling of basic facts, and telling time (Hudson & Miller, 2006).

*Difference.* The difference refers to the answer in a subtraction equation.

*Explicit Instruction.* A method of teacher-directed instruction that is highly structured and calls for the presentation of new skills in small steps at a pace that is driven by student progress. Academic concepts and skills are taught in a clear, direct manner to promote student understanding (Hudson & Miller, 2006; Miller, 2009).

*Fluency.* The rate at which a student can solve a math problem.
Focused Curriculum-based Assessment. An assessment designed to measure a narrow span of skills. It is comprised of problem types within a specified skill (Hudson & Miller, 2006).

Guided Practice. Guided practice refers to the portion of a lesson where the students are practicing the new mathematics task with teacher assistance and guidance. Teacher support is gradually withdrawn as students become more independently successful with the new mathematics task (Hudson & Miller, 2006).

Independent Practice. Independent practice refers to the portion of a lesson where the students are practicing the new mathematics task without teacher support (Hudson & Miller, 2006).

Learning Disability. Is a condition in which a student of average intelligence has dysfunction in processing information typically found in language-based activities, resulting in learning challenges (Friend & Bursuck, 2009).

Mathematical Concepts. Include a wide variety of math related skills found within the mathematical discipline (Sherman, Richardson, & Yard, 2009).

Mild-to-Moderate Disabilities. A condition in which students have some difficulty meeting the academic and social demands of general education classrooms due in large part to below average intellectual functioning (55 – 70 on an IQ test) (Friend & Bursuck, 2009).

Minuend. The number in a subtraction equation that you are subtracting from.

National Council for Teachers of Mathematics (NCTM). The National Council of Teachers of Mathematics is a national organization that supports teachers in
ensuring equitable mathematics learning of the highest quality for all students (NCTM, 2000).

*Place Value.* Place Value is “the system by which the value of a digit is determined by the position it occupies relative to the decimal point” (Stein, Silbert, & Carnine, 1997, p. 51.).

*Problem Solving.* This is one of the five NCTM Process Standards. Problem solving refers to a student’s ability to apply their mathematical knowledge to find a solution to a real-world situation (Hudson & Miller, 2006).

*Procedural Knowledge.* Procedural knowledge is the ability to follow a set of sequential steps to solve a mathematical task (Bottge, 2001; Carnine, 1997; Goldman et al., 1997).

*Regrouping.* Refers to the actions taken to solve a subtraction problem when an exchange of base groups is required. Regrouping can take place in the four operations of addition, subtraction, multiplication, and division (Sherman et al., 2009).

*Renaming.* Another term for regrouping (i.e., actions taken to solve a subtraction problem when an exchange of base groups is required).

*Strategy Instruction.* Refers to instruction that facilitates students becoming independent learners and involves teaching students how to learn and perform (Deshler, Ellis, & Lenz, 1996).

*Student-Directed Instruction.* Instruction that progresses at a rate that is individualized to each student’s needs. Students need to meet criteria standards
before being eligible to move onto next skill or next level of skill. Usually student directed instruction takes place during the guided practice stage of acquisition. *Subtraction.* “The removal of a subset from a set” (Stein et al., 2006, p. 121). *Subtrahend.* The amount to be taken away from a total quantity (Stein, et al., 2006).

**Teacher-Directed Instruction.** Instruction in which the teacher is in charge of maintaining the pace of students’ acquisition of new skills. Usually, teacher directed instruction takes place during the describe and model stages of acquisition.

**Verbal Practice.** Verbal practice takes place when students are learning to independently recall specific steps of a strategy. Students are expected to have a high level of automaticity when recalling strategy steps before they attempt to apply the steps to specific problems (Hudson & Miller, 2006).

**Summary**

The history of mathematics education has been very contentious since the early 1900s. One central discussion that continued to surface over the years was that of curriculum. Kilpatrick’s (1915) report titled *The Problem of Mathematics in Secondary Education* was one of the first public records that suggested a limited mathematic curriculum that was more individualized to students’ practical needs. The 1923 *Report*, published by the National Committee on Mathematical Requirements, opposed Kilpatrick’s proposal and argued for a more extensive math curricula. Key
professionals in the field of math education continued this debate, which ultimately manifested itself in political reform.

Although teachers in the early 1930s were supporting Kilpatrick’s progressive math curriculum standards, by the late 1940s through early 1960s new technological advances dictated the need for a reexamination of current curriculum standards. National leaders believed that educators had a responsibility to ensure that the current math curriculum would maintain a work force that was competitive. Federal legislation, like the 1958 National Defense Education Act, began to monetarily support this broadening of the mathematics curriculum. This new curriculum came to be referred to as New Math (Klein, 2003; Walmsley, 2003).

Many set backs within the New Math curriculum existed. Unprepared teachers were unable to maintain sound pedagogy when teaching these more complex math skills expected within the New Math curriculum. Parents were unable to support their childrens’ learning because there was a clear disconnect between what they had been exposed to while they were in school and what was currently expected. By the late 1970s to early 1980s a large population of students existed who were unable to effectively use basic math skills. The New Math curriculum did not support mastery learning of basic skills before moving into more complex math skills. A call for reform once again made its way on a National level.

In the 1980s a few national organizations published seminal reports that supported the development of a reasonable curriculum that would address both basic skills to a mastery level (National Commission on Excellence in Education’s A Nation At Risk, 1983) and higher order math skills that supported advancement of technology.
(NCTM’s *An Agenda for Action*, 1980). By the end of the 1980s, The NCTM had published national standards that attempted to address both sides of the incessant curriculum debate. These standards have been adopted, revised, and condensed over the last 3 decades in response to national legislation (NCTM, 1989; NCTM, 2000, NCTM, 2006). For example, the No Child Left Behind Act (2002) dictated an increase in accountability measures to ensure that all curricula are effectively taught to all students, including mathematics curricula. To support teacher accountability success, the current math standards were reviewed, condensed, and published by the NCTM in a document titled *Curriculum Focal Points for Prekindergarten through Grades 8 Mathematics: A Quest for Coherence*.

Despite the reform that has taken place over the last century, math curriculum continues to be scrutinized. Recently, the National Mathematics Advisory Panel (2008) published *The Foundations for Success: The Final Report of National Mathematics Advisory Panel*. This document dictates a need for further reform of current mathematics curricula as a result of cyber technology. It seems that no definitive answer related to the best mathematics curriculum is near.

One commonality on which all sides of the curricular debate agree is that basic computation instruction must be addressed effectively. Unfortunately, students within the United States continue to perform below acceptable expectations (NAEP, 2007; National Mathematics Advisory Panel, 2008). Of particular concern are students with learning disabilities. There has been a dramatic increase in the number of students with learning disabilities within our general population and there is consensus regarding the importance of basic computation skills for these students (Badian, 1983
Perhaps this is why there is a primary focus within the mathematics literature on providing basic math fact instruction (i.e. single digit facts) to students with disabilities (Bryant et al., 2008; Garnett, 1992; Gersten et al., 2005; Miller et al., 1996; Montague, 2008; Montague & Brooks, 1993). Unfortunately, there is limited research on other basic computation skills (e.g. multidigit subtraction) that are equally important in terms of further progress to higher order math skills (Riccomini, 2005). The intent of this study is to contribute information regarding the effectiveness of strategy instruction and the concrete-representational-abstract teaching sequence when teaching subtraction with regrouping to students with learning disabilities. The results of this study have direct and immediate practical implications for classroom teachers of mathematics.

Details related to this study are discussed in the subsequent chapters. A review of literature relevant to this study is presented in Chapter 2. Methodology used for implementation of the study is discussed in Chapter 3. The results and discussion of their implications are reported in Chapters 4 and 5.
CHAPTER 2
REVIEW OF LITERATURE

There are two purposes for this chapter. The first is to summarize and analyze existing professional literature related to mathematics strategy instruction for students with learning disabilities. The second purpose is to summarize and analyze existing professional literature related to the concrete-representational-abstract teaching sequence. Knowledge of these two literature bases is needed to understand current best practices for teaching mathematical concepts such as subtraction with regrouping to students with learning disabilities. The chapter begins with the literature review procedures and selection criteria used for experimental studies related to mathematics strategy instruction for students with learning disabilities. A review and analysis of studies related to mathematics strategy instruction for students with learning disabilities follows. Next, the literature review procedures and selection criteria used for experimental studies related to the concrete-representational-abstract teaching sequence are explained. A review and analysis of studies related to the concrete-representational-abstract teaching sequence follows. Finally, a summary and synthesis of the research about mathematics strategy instruction for students with learning disabilities and the concrete-representational-abstract teaching sequence is provided.

Literature Review Procedures Related to Mathematics Strategy Instruction for Student with Learning Disabilities

Studies included in this review were located through a comprehensive search of studies from the following databases: Academic Search Premier, Elton B. Stephens
Company (EBSCO), Education Resources Information Center (ERIC), and Digital Dissertations. The following descriptors were used: strategy instruction, learning disabilities and disabilities. Also, a manual search through selected journals, and an ancestral search through the reference lists of obtained articles were conducted.

Selection Criteria Used for Studies Related to Mathematics Strategies for Students with Learning Disabilities

Studies were included in this review of literature if: (a) the procedures and data-based results were published between 1978-2008, (b) the research examined mathematics strategies, (c) the participants were elementary or middle school students or teachers, (d) the study explored the impact of cognitive disabilities on student success in mathematics and (e) the purpose of the study was to explore the effectiveness of mathematics strategies. Studies were excluded from this review if: (a) the procedures and data-based results were published before 1978, (b) mathematics strategies were not explored, (c) the participants were not students, teachers, or in some way related to the education field, (d) the study did not investigate the impact of cognitive disabilities on student success in mathematics, and (e) data or results of the study did not provide information related to mathematics strategies.

Summary and Analysis of Studies Related to Mathematics Strategies for Students with Learning Disabilities

Historically, special educators have been seeking out instructional techniques and curricula that maximally promote independence and success in their students (Ellis,
The articles that are reviewed clearly indicate that when specific instructional procedures are followed, students with learning disabilities are able to independently apply instructional strategies and experience success within a variety of mathematical constructs. Deshler and Schumaker (1984) define strategy instruction as techniques, principles, or rules that enable a student to learn, to solve problems, and to complete tasks independently.

**Computation and Related Instruction**

Ozaki, Williams, and McLaughlin (1996) conducted a multiple baseline across behaviors design study to assess the effects of the Cover-Copy-Compare procedure on the percent of multiplication facts correctly completed by a sixth grade student with a learning disability. The student was 11 years 1 month old and the study took place in a resource room where he typically received his special education services.

The student participated in pretesting to assess how much prior declarative knowledge of multiplication facts he had. Next, the intervention phase took place in which the student participated in instruction that covered the five steps of Cover-Copy-Compare procedure. These steps were: (a) look at the first completed math fact, (b) read the problem aloud and copy the answer, (c) cover the problem, (d) read the problem aloud and write it from memory, and (e) compare the answer to the original problem. The student participated in this instruction over 18 sessions that lasted an average of 15 minutes per session 3 times a week.

Visual analysis of the substantial level increase from the student’s baseline scores to his scores after the intervention was implemented was evident. Still, this study allows for limited generalization to other populations because only one student was
studied for this research. Future research regarding this effect should include more students and an alternative means of improving declarative knowledge in order to further confirm that the Cover-Copy-Compare intervention improves student achievement.

Problem-Solving Instruction

*Cognitive Strategies – student think-alouds.* Naglieri and Johnson (2000) conducted a study to determine if an instruction designed to facilitate planning, given by teachers to their class as a group, would have differential effects on the specific Planning, Attention, Simultaneous, Successive (PASS) cognitive characteristics of each child.

Nineteen students in grades six through eighth participated in this study. Their ages ranged from 12 to 14 years and there were 16 male participants and 2 female participants. Most of the participants were students with learning disabilities although some were identified as having mild intellectual impairments. They all attended a public school in southern California that served rural and suburban communities with low to lower-middle class levels of socioeconomic status.

Researchers administered the Cognitive Assessment System (CAS) to the nineteen participants who were then placed into the experimental group or one of four comparison groups based on their ability levels related to the four fundamental processes for planning and successfully executing cognitive tasks. The intervention condition consisted of the students completing math worksheets and teachers identifying effective strategies the students used to solve math problems. Results indicate that the students who were identified as having low planning scores from the
CAS measure demonstrated the greatest gains from baseline to intervention on the math worksheets. Researchers point out that this instruction does not use teacher scripts or rigidly formatted procedures that make the intervention easily replicated. Replication studies investigating the effects of the PASS cognitive instruction are needed, especially related to other students with various types of learning challenges.

*Schema-based Instruction – visual and graphic depictions.* Jitendra and Hoff (1996) conducted a multiple probe-across-students study to assess the effects of a schema-based direct instruction strategy on word-problem-solving performance. The participants were 3 third- and fourth-grade students; there were 2 girls and 1 boy. Each of the students had a documented learning disability and ranged in age from 8 years 10 months to 10 years 10 months. All of the subjects were White, attended a northeastern private elementary school, and were from middle to upper-middle income homes. This private elementary school was specifically designed for students who had learning disabilities. A conference room, on the school campus, that was adjacent to the students’ classroom provided the setting for this study.

The study began with a probe condition where all students concurrently completed three probes that assessed all 3 problem types (i.e. change problems, group problems, and compare problems) across 3 days. Next, the students were given instruction in how to identify and represent problem schemata followed by Probe 2. Following Probe 2, the students participated in staggered schema-based direct strategy instruction that followed scripted lessons. Once the first student reached a criterion of 100% correct for 2 consecutive days, another probe was administered (Probe 3), and instruction started for the second student. The study concluded with a
maintenance probe given 2 or 3 weeks after Probe 3 was administered to each student.

Interscorer reliability checks were completed on 20% of all probes to ensure that each of the probes was being scored accurately. Likewise, fidelity of intervention implementation checks during 20% of the problem schemata and intervention training sessions took place. Visual analysis of data was used to determine intervention effectiveness.

The results of this study indicated a significant level increase from their baseline probes to the probes administered within the intervention condition for all three subjects. This gain of skills was maintained over time as all three subjects also scored well above their baseline probe average within the maintenance phase of the study. The authors point out that further investigations should be conducted to determine the extent to which students who learn schema-based instruction would be able to generalize these skills into typical math curriculum. They also suggest that conducting this study using a larger population of students would be beneficial (i.e. different populations of students with learning challenges) to better identify who else could benefit from this type of instruction. Additionally, whether the effectiveness of this instruction lies within the schema-based diagrams provided or simply that this instruction fosters conceptual understanding should be investigated. This investigation provides useful information with regard to the implementation of conceptual instruction that has the potential to benefit students at various grade levels. Further study is needed to determine the specific range of appropriate grade levels.
More recently, Jitendra, Griffin, Haria, Leh, Adams, Kaduvettoor (2007) conducted a study to assess the effects of schema-based instruction versus multiple strategy instruction. The participants were 88 students in third grade; there were 49 boys and 39 girls. Close to ten percent of the participants were identified as having learning disabilities. The students attended an elementary school in a northeastern urban district. Participants completed pretests and posttests on mathematical problem solving and computational tests. Additionally, participants were posttested on the Pennsylvania System of School Assessment Mathematics test used to measure students’ progress on current state mathematical standards. Students were put into six instructional groups. Three groups received schema-based instruction (SBI), which included schematic diagrams designed to promote mathematical problem solving. The other three instructional groups served as a comparison group and received general strategy instruction (GSI), which included use of objects, draw a diagram, write a number sentence, and use data from a graph. Both the SBI and GSI groups were taught how to solve a word problem under their respective conditions using scripted lessons for 25 minutes a day five days a week.

A one-way between subjects analysis of covariance (ANCOVA) was applied to posttest scores. The results indicated significant differences between the two instructional groups for solving mathematical word problems. The SBI group out performed their GSI counterparts on both the mathematical word problem-solving skills posttests and on the Pennsylvania System of School Assessment measure.

Jitendra et al. (2007) concluded that schema-based instruction resulted in dramatic improvement for a group of third grade students who were solving mathematical
word problems. However, this research could be extended in several ways. The sample of subjects was not reflective of current variances within a typical general education classroom. There was a small sample size of students with learning disabilities and a lack of students who represented those with specific mathematical learning disabilities. The statistical findings among this subgroup of students differed from the larger group outcomes. There were no statistical differences between the SBI group and the GSI group when looking at only the performance on the posttests for those students with learning disabilities. While, schema-based instruction might be a useful approach for students in a general education classroom setting, looking at modifying the current scripted lessons might be necessary to adequately address the needs of students with learning disabilities.

To further substantiate the effects of schema-based instruction on problem solving, Fuchs, Fuchs, Prentice, Hamlett, Finelli, and Courey (2004) conducted a study. The purpose of this study were to assess the effects of schema-based instruction in promoting mathematical problem solving while investigating schema instruction as a mechanism in the development of mathematical problem solving. They also examined the added value of guided sorting practice on schema development and problem solving skills.

Fuchs et al. (2004) split 24 third-grade female teachers from six southeastern urban schools into 3 groups. The three groups were schema-based instruction; schema-based instruction plus sorting practice; and a contrast group which included teacher-designed and implemented instruction on the four problem types. Each of the groups was comprised of an average of 122 third-grade students who were given
pretests 3 weeks prior to instruction. After 16 weeks of whole class instruction conducted within their math classes, each group was given posttests on mathematical problem solving and schema development. A two-factor mixed model analysis of variance (ANOVA) was conducted. Condition was the between-teachers variable while initial student status was the within-teacher variable.

Results from this study show that the schema-based instruction groups out performed the contrast group for both schema development and successful problem solving. This study was of particular interest because the researchers incorporated general problem solving strategies (i.e. making sure answers make sense; lining up numbers from text to perform math operations; checking computation; and labeling work with words, monetary signs, and mathematical symbols) into each of the three conditions. Therefore, they were able to isolate the effects of schema-based instruction from more general problem solving strategies.

While this study included a population of students that ranged in ability level, it didn’t have a strong showing of students with various disabilities; most of the students with disabilities included were students with learning disabilities. Further investigation could be conducted to determine if the same effects emerge for other populations such as English Language Learners and students with other types of disabilities (e.g., students with autism who struggle with problem solving). Additionally, these researchers did not find statistically significant differences between the two types of schema-based instruction (with sorting activities and without sorting activities). Future studies examine the differences of these two types of schema-based instruction among various students with learning challenges.
Summary and Analysis of Studies Related to the Concrete-Representation-Abstract Teaching Sequence for Students with Learning Disabilities

The Concrete-Representation-Abstract Teaching sequence is a graduated instructional sequence that supports learning of a variety of mathematical skills for students with learning disabilities (Ketterlin-Geller, Chard, & Fien, 2008). This unique teaching sequence begins by promoting learning through *concrete* or hands-on instruction using manipulative devices. Next, students learn through pictorial *representations* of the previously used manipulative objects. Finally, students’ learning continues through the *abstract* stage of instruction in which the mathematical concept is presented using only numbers and operational symbols (Witzel, Riccomini, & Schneider, 2008).

Peterson, Mercer, Tragash, and O’Shea (1987) investigated the effectiveness of teaching initial place value skills using two different teaching methods. The control group received instruction that presented initial place value skills on an abstract level only. The treatment group received instruction that presented the same mathematical concept in a concrete, semiconcrete (representational), abstract teaching sequence. The twenty-four subjects in this study were randomly assigned to the control group and intervention groups. The subjects ranged in age from 8 to 13 and all were identified as having learning disabilities. Each of the subjects received math instruction in special education classrooms located in Florida.

The researchers examined skill acquisition, maintenance, retention and generalization via a 2x3 mixed design with one between (treatment) and one within (performance over time) group factor. Each group received its respective instruction
and then three teacher-made research instruments were used to measure maintenance
(one week after instruction) and retention (three weeks after instruction). The
instruction delivered to both groups was similar; the lessons were scripted and
included an advance organizer, a demonstration and model stage of instruction, a
guided practice stage of instruction, and an independent practice stage of instruction.
The only difference in instruction between the two groups was that the intervention
group received three lessons using concrete manipulative devices, three lessons using
semiconcrete or pictorial representations, and three lessons that included abstract
level instruction while the control group received nine lessons all at the abstract level
of instruction.

The results of the data collected indicated that the concrete-semiconcrete-abstract
teaching sequence was more effective than abstract level instruction when teaching
initial place value skills to students with learning disabilities. Peterson et al. (1987)
concluded that the students who participated in the concrete-semiconcrete-abstract
teaching sequence not only acquired initial place value skills better than the control
group, but that this three tiered teaching sequence also had positive effects on the
students’ ability to maintain this skill over time. The researchers concluded that this
three tiered teaching sequence was necessary to effectively teach conceptual
understanding of place value and should be further investigated to determine its exact
effect on the students’ ability to generalize these skills.

Peterson, Mercer, McLeod, and Hudson (1989) further examined the effectiveness
of using the concrete-semiconcrete-abstract teaching sequence when teaching initial
place value skills by using a multiple baseline single subject design. The three
participants involved in this study were all male elementary aged students with learning disabilities and varying levels of deficits with mathematics skills. All subjects were given a pretest to qualify them for this study. This study took place on a college campus that was hosting a training program for school-aged students. The participants attended a specialized diagnostic classroom for five weeks before returning to their home schools.

The three phases utilized in this study were the baseline phase, the treatment phase, and the posttreatment phase. During the baseline phase, each student participated in one minute timed probes where students were asked to identify place value markers (i.e., ones, tens). During the treatment phase, each subject was taught initial place value skills using the concrete-semiconcrete-abstract teaching sequence. Each subject received daily 15 minute individualized instruction that followed a direct instruction model which included an advanced organizer, a demonstration or model stage of instruction, a guided practice stage of instruction, and an independent practice stage of instruction. The treatment phase varied for each student and ranged from 9 to 15 sessions depending on the rate in which each subject reached the criteria level set for each session. Finally, during the posttreatment phase of this study, the subjects were given a posttest that was similar to the pretest.

The pretest scores were compared to the posttest scores. Each subject scored substantially better on their posttests than their pretests (i.e., at least 40 percentage point increase). Additionally, each subject earned 80% on the retention measure that was given 3 weeks after instruction took place. This retention measure took place in the subjects’ home schools, a different setting than where intervention instruction
took place. Researchers concluded that the three tiered teaching sequence was, again, an effective teaching sequence that promoted skill retention and generalization. Limitations in this study include that the subjects were not of varying genders and that the instruction took place in a clinical setting not reflective of typical classroom activity.

Harris, Miller, and Mercer (1995) investigated the effectiveness of using strategy instruction and the concrete-representational-abstract sequence to teach basic multiplication facts and related word problems to students with disabilities. The researchers used a multiple baseline across classrooms design with one replication. A total of 13 students (i.e., 12 with learning disabilities and 1 with emotional disturbance) participated in the study. All instruction took place within six second-grade general education classrooms within a public elementary school and was provided from the general education teachers.

The teachers implemented 21 multiplication lessons designed to teach basic multiplication facts. The first 10 lessons incorporated the concrete-representational-abstract teaching sequence and integrated a learning strategy using the mnemonic device DRAW (i.e., Discover the sign. Read the problem. Answer, or draw and check. Write the answer.) Beginning with lesson 11, the instructional emphasis changed to solving word problems and developing fluency with the basic multiplication facts.

Rate data obtained from 1-minute timings were used to monitor student performance throughout the study. Additionally, a researcher-constructed pre- and posttest was administered to determine the effectiveness of the intervention. Twelve
of the 13 participants demonstrated more incorrect responses than correct responses on the baseline probes. During the intervention condition, this pattern changed. All 13 participants increased the number of correct responses to an average of 10.6 and decreased the number of incorrect responses to an average of 2.9. Pre- to posttest increases ranged from 25 to 85 percentage points. To further examine the effect of the multiplication lessons, pretest, posttest and learning sheet (i.e., worksheets that accompanied each lesson) scores of students with disabilities were compared to the scores of the general education students in the same six classrooms who also received the multiplication instruction. Median performance on the pretest (i.e., 15%) and 9 of the 21 learning sheets (i.e., 70%-100%) was the same for both students with and without disabilities. Median performance on the remaining 12 learning sheets (i.e., 70%-100%) revealed either a 10 or 20 percentage point lower score (i.e., one or two problem difference) for students with disabilities. The median posttest score for students with disabilities was 80%; while the median posttest score for students without disabilities was 90%. A majority of the differences between students with and without disabilities occurred during the later lessons that focused on solving word problems. These findings indicated that students with disabilities were able to learn multiplication skills at acceptable levels (i.e., at least 80% accuracy on posttest) within general education classroom settings.

Although the results of this study were positive, it should be noted that the instructional lessons were explicit, scripted, and integrated best teaching practices for students with disabilities. The results may have been different if the typical general education basal text curricula had been used thus generalization of these findings are
somewhat limited. Additionally, no maintenance measures were implemented, so it is not possible to determine whether the multiplication skills were retained over time.

Maccini and Ruhl (2000) studied the effects of the concrete-semiconcrete-abstract teaching sequence on eighth grade students’ ability to solve algebraic math word problems. Three male participants were examined in this study whose ages ranged from 14 to 15 years of age. They all met similar academic criteria including special education eligibility status for learning disabilities. Each of these participants, however, spent a majority of their academic day in general education classrooms. A multiple probe design across subjects was implemented with the instruction taking place in a conference room on the students’ home school campus.

The study consisted of three phases: the baseline phase, the intervention phase, and the generalization and maintenance phase. During baseline, students completed four probes that required the students to demonstrate accurate problem representation, problem solution, and use of the STAR strategy. The intervention phase consisted of the students learning the STAR strategy that involves the use of a mnemonic device to self guide students through solving a problem. The steps of STAR are: (a) search the word problem; (b) translate words into an equation/picture form; (c) answer the problem; and (d) review solution. The STAR strategy was taught in the three tier concrete-semiconcrete-abstract teaching process during lessons that were between 20 and 30 minutes in length. Each student had to meet 80% mastery criteria throughout each lesson to be considered ready to move to the next lesson. After 17 lessons, students moved into the generalization and maintenance phase. During this last stage,
students took a posttest probe that was reflective of what was asked of them on the pretest.

A visual inspection of prettest scores to posttest scores indicated that adolescent students with learning disabilities can learn to successfully solve algebraic word problems when taught a math strategy using the concrete-semiconcrete-abstract teaching sequence. Each students’ percent of accuracy on problem solution increased 69, 43.5, and 50.5 percentage points from baseline to the abstract stage of instruction. Six weeks after intervention instruction ceased, each student was able to score 100% accuracy on the maintenance probe given (Maccini & Ruhl, 2000). Although the results of this study continue to validate the effectiveness of the concrete-semiconcrete-abstract teaching sequence, some limitations include that the students experienced their learning in a non-classroom environment and the sample size used was small. Students were asked to come to a conference room on their school campus that lacked some of the variables of a typical classroom setting (i.e. typical student-teacher ratios, noise, interruptions). Generalization of treatment effects to a larger population is also limited due to the experimental design and small sample size.

Butler, Miller, Crehan, Babbitt, and Pierce (2003) conducted a study to compare the effects of a concrete-representational-abstract (CRA) instructional sequence to representational-abstract sequence (RA) on the learning of fraction equivalence concepts by middle school students with mild to moderate disabilities. A majority of the 50 sixth-, seventh-, and eighth-grade students who were split into the CRA group (N= 26) and RA (N= 24) group were students with learning disabilities. There were 27 male students and 23 female students whose ages ranged from 11 to 15 years old.
For comparison, 65 eighth-grade students enrolled in general education math classes took the postassessment only. This study was conducted in a public middle school located in a large urban area of the southwestern United States.

The students in the two intervention groups (CRA and RA) participated in three phases: preassessments, intervention implementation, and postassessments. The preassessments consisted of five subtests that measured various levels of student understanding of fractions. The intervention lessons were scripted and included the following components: advance organizer, teacher demonstration, guided practice, independent practice, problem-solving practice, a feedback routine, and cue cards and notes. Additionally, the scripted lessons for the CRA group had three lessons that focused on conceptual development using concrete manipulative devices, three lessons that involved the use of representational devices, and four lessons that introduced the abstract algorithm for computing equivalent fractions. The only difference in the RA lessons was that they received 6 initial lessons that involved representational drawings and no concrete manipulative devices. At the conclusion of the 10 intervention lessons, postassessments were administered.

Results of this study indicated that both treatment groups improved from pre- to posttest. Each of the achievement measures indicated that students in the CRA group had overall higher mean scores than did students in the RA group. The researchers suggest that the students in the CRA group and the RA group scored as well as the students from the general education math classes who only participated in the postassessment. Future investigations should include pretest data on the typical students. Another interesting find within this study was the results from the attitude
measure. This attitude measure revealed that the students enjoyed using the manipulative devices and did not perceive the materials to be for younger or less competent students.

The researchers noted that most of the students involved in this study were students with learning disabilities so caution should be used when generalizing the results to other populations. Future related studies should investigate the use of this teaching sequence when teaching fractions to students with disabilities who receive their math instruction within general education classrooms.

Witzel, Mercer, and Miller (2003) conducted a study to test the effectiveness of a new explicit CRA algebra model that was designed to represent more complex equations. The researchers suggest that the instructional model used in this study presents the conceptual components in concrete and pictorial representations in a manner that prepares the student to succeed in more advanced algebra concepts.

Twelve classrooms with approximately 358 sixth- and seventh-grade students and 10 teachers were involved in this study. The study took place in a southeastern United States urban county. The researcher matched thirty-four students identified as students with learning disabilities with thirty-four students with similar characteristics and placed each of the sets into two different treatment groups. One group was taught equivalent algebra lessons using the CRA model, and the other group received traditional instruction. Fidelity of treatment checks were conducted throughout the lessons to be sure that each teacher was implementing the lessons with fidelity.

The instruction in both groups included the following four things: (1) introduction of a skill, (2) the skill was modeled, (3) guided practiced was conducted, and (4)
independent practice work was given. Instruction for the traditional group was done at the abstract level only, while instruction in the treatment group included concrete, representational, and abstract lessons.

Repeated measures of analysis of variance were performed on two levels on instruction (CRA vs. abstract) and three levels of occasions (pretest, posttest, and follow up). Results of this study indicated that although both groups showed significant growth from pretest to posttest, students in the CRA group outperformed those in the traditional instruction group. Self-identified limitations of the study as noted by the researchers involved the assessment instrument used for pretest, posttest, and follow-up in this study. The researchers pointed out that the assessment instrument had not been fully evaluated and did not address all the hands-on success the students gained.

Literature Review Summary

There were two purposes for this chapter. The first purpose was to summarize and analyze existing professional literature related to mathematics strategy instruction for students with learning disabilities. The second purpose was to summarize and analyze existing professional literature related to concrete-representational-abstract teaching sequence. Knowledge of these two literature bases is needed to understand current best practice for teaching mathematical concepts to students with learning disabilities.

From this literature review, it is evident that more research is needed to determine the effectiveness of strategy instruction integrated with the concrete-representational-abstract teaching sequence for students with learning disabilities on additional
mathematics skills (e.g., advanced computation that requires regrouping). Research supports that strategy instruction and the concrete-representational-abstract teaching sequence can be effective to teach elementary and middle school students with learning disabilities a variety of mathematical concepts (i.e., word problems, place value, multiplication, fractions, algebra). No research was located that investigated the effects of strategy instruction and the concrete-representational-abstract teaching sequence for teaching advanced computation skills that require regrouping. Thus, additional research that incorporates these validated practices to teach subtraction with regrouping skills is needed to see if student success can be replicated when focusing on this particular skill.
CHAPTER 3

METHODOLOGY

The purpose of this study was to investigate the effects of strategy instruction and the concrete-representational-abstract sequence to teach subtraction with regrouping to students with learning disabilities. This chapter is designed to describe the methodology used in this study. The following ten sections will be discussed in this chapter: (a) research questions, (b) participants, (c) setting, (d) instrumentation, (e) materials and equipment, (f) design, (g) procedures, (h) interscorer reliability, (i) fidelity of treatment, (j) treatment of data.

Research Questions

The following research questions have been answered in this study:

1. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems?

2. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems?

3. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems?
4. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems?

5. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping?

6. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems?

7. Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence report high, medium, or low levels of satisfaction related to the subtraction with regrouping intervention lessons?

Participants

A total of six fifth grade students with learning disabilities participated in this study. The participants ranged in age from 10 years 10 months to 12 years 0 months. Of the six participants 5 were males and 1 was female. The following ethnicities were represented in this sample: Black, White, Hispanic, and Asian Pacific Islander. See Table 1 for a summary of demographic data related to each participant.
Table 1

*Participant Demographic Data*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Demographic Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td></td>
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<td>Gender</td>
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</tr>
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<td>Ethnicity</td>
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</tr>
<tr>
<td>Age</td>
<td>11 years 6 months</td>
</tr>
<tr>
<td>Grade</td>
<td>Five</td>
</tr>
<tr>
<td>Disability</td>
<td>Learning Disability</td>
</tr>
<tr>
<td>Intelligence</td>
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</tr>
<tr>
<td>Quotient</td>
<td>Composite SS = 95 (percentile = 37)</td>
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<tr>
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<tr>
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<tr>
<td>Grade</td>
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<tr>
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<td>Learning Disability</td>
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<td>Achievement</td>
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<td>--------------</td>
<td>--------------------------</td>
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<td>Participant 3</td>
<td>Gender: Male, Ethnicity: Hispanic, Age: 10 years 11 months, Grade: Five, Disability: Learning Disability, Intelligence Quotient: Universal Nonverbal Intelligence Test Full Scale SS = 83 (percentile = 13), Math Achievement: Kaufman Test of Educational Achievement, Math Concepts and Applications SS = 66 (percentile = 1)</td>
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<td>Participant 4</td>
<td>Gender: Male, Ethnicity: Asian Pacific Islander, Age: 11 years 3 months, Grade: Five, Disability: Learning Disability, Intelligence Quotient: (Not available, student transfer from out-of-state), Math Achievement: Weschler Individual Achievement Test, Math Composite SS = 77 (percentile = 6)</td>
</tr>
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</table>
### Table 1 (continued).

<table>
<thead>
<tr>
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<tr>
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<td>Math Composite SS = 88 (percentile = 21)</td>
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<td><strong>Participant 6</strong></td>
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<td>Math</td>
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<tr>
<td>Achievement</td>
<td>Math Composite SS = 82 (percentile = 12)</td>
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</table>
Participant Pool

The six participants were selected from a sample of convenience within one elementary school. These participants were selected from a single learning disability teacher’s caseload of 8 fifth grade students. This pool of participants consisted of students who qualify for direct special education services for more than 50% of the school day.

Participant Selection

There were three criteria that each participant met to be eligible for study participation. The participants had: (a) met the state of Nevada Administrative Code eligibility criteria for specific learning disabilities, (b) been enrolled in the fifth grade, and (c) failed to meet both the school district’s Curriculum Essential Framework’s Standards and the Power Standards for 5th Grade related to subtraction with regrouping (Curriculum Professional Development Division, 2008). Additionally, to be included in this study, parents were required to provide informed consent and participants were required to provide student assent.

Setting

This study took place in a professional development school located in a metropolitan city in the southwestern United States. This school was a public school open to any student living within a delineated zone of the fifth largest school district in the United States. The school employed 29 grade level teachers, four teacher specialists (i.e. art, music, library, physical education teachers) and nine teachers who
provided services to specific populations of students (e.g. special education students, gifted and talented students, English language learners). The population of this school consisted of approximately 608 students of which 12% were identified as students with disabilities. Further demographic data from this school included the following: (a) 54.8% of the student population was male and 45.2% of the student population was female (b) 7.1% of the student population was Asian/Pacific Islander, (c) 56.9% of the student population was Hispanic, (d) 18.9% of the student population was Black/African American, (e) 16.4% of the student population was White, and (f) 44.7% of the student population was considered to have Limited English Proficiency. The school was located in a countywide school district that covered an area of approximately 8,091 square miles and served approximately 308,554 students.

This elementary school shared its campus with a public university with a Carnegie rating of a high research institution. The school location helped facilitate the partnership that existed between university and school personnel. Personnel who worked within the professional development school worked closely with university personnel to provide a collaborative culture in which teacher candidates enhance their learning of various instructional methodologies through observations and structured experiences within the actual educational setting in which the methodologies are taking place. The professional development school personnel also gained from this collaborative partnership. The employees of the school were exposed to current teacher education programs, professional development initiatives, site-based research projects, and grant writing opportunities.
Instrumentation

Pre- and Posttests

Four curriculum-based assessments and one interview were used in this study. The first curriculum-based assessment (CBA), the *Subtraction With Regrouping Pretest*, had 20 problems that require regrouping to solve (See Appendix A). Of the 20 problems, ten had 2-digit minuends and 2-digit subtrahends. Of those ten problems, three had zeros in the ones place value within the minuend where the student was required to regroup across zeros. The other ten problems were made up of 3-digit minuends and 3-digit subtrahends. Three of those problems had zeros in the tens place value within the minuends. This CBA was designed to assess the participants’ ability to successfully solve subtraction problems with regrouping, a single skill set, and it was not timed. This CBA, therefore, was considered an untimed-focused curriculum-based assessment (Hudson & Miller, 2006).

The second CBA, the *Subtraction With Regrouping Minute*, had 16 problems that require regrouping to solve (See Appendix B). Of the 16 problems, eight had 2-digit minuends and 2-digit subtrahends. Of those eight problems, two had zeros in the ones place value within the minuend. The other eight problems were made up of 3-digit minuends and 3-digit subtrahends. Two of those problems had zeros in the tens place value within the minuends. The participants were given one minute to complete this CBA. This CBA is designed to assess the participants’ ability to successfully solve subtraction problems with regrouping, a single skill set, and it is timed. This CBA was considered a timed-focused curriculum-based assessment (Hudson & Miller, 2006).
The third CBA, the *Subtraction Review Minute*, had 16 subtraction problems (See Appendix C). Of the 16 subtraction problems, four did not require regrouping to solve. Of those four problems not requiring regrouping, two consisted of 2-digit minuends and 2-digit subtrahends while the other two problems were made up of 3-digit minuends and 3-digit subtrahends. The remaining twelve problems required subtraction with regrouping to solve. Six of those twelve problems were comprised of 2-digit minuends and 2-digit subtrahends. Of those six problems, two problems had zeros in the ones place value within the minuend. The other six problems requiring regrouping included 3-digit minuends and 3 digit subtrahends. Of those six problems, two problems had zeros in the tens place value of the minuend. The participants were given one minute to complete this CBA. This CBA is designed to assess the participants’ ability to successfully solve subtraction problems both with and without regrouping, covering a narrow span of skills, and it is timed. This CBA was considered a timed-focused curriculum-based assessment (Hudson & Miller, 2006).

The fourth CBA, the *Subtraction Word Problem Pretest*, included ten word problems that require subtraction with regrouping to solve (See Appendix D). To ensure that this assessment measured participants’ mathematic skills and not their reading ability, this assessment was read out loud to all of the participants. Six of the problems consisted of 3-digit minuends and 3-digit subtrahends. Of those six problems, one had a zero in the tens place value within the minuend to require regrouping across zeros. The other four word problems were comprised of 2-digit minuends and 2-digit subtrahends. While none of these five problems had zeros as a digit, one of these word problems did include a sentence of extraneous information.
Participants were expected to ignore the irrelevant information, including a 2-digit number, to solve that problem correctly. This CBA was designed to assess the participants’ ability to successfully solve subtraction word problems with regrouping, a single skill set, and it is not timed. This CBA, therefore, was considered an untimed-focused curriculum-based assessment (Hudson & Miller, 2006).

Last, the Subtraction Interview Pretest was administered prior to participants receiving instruction on subtraction with regrouping (See Appendix E). This interview included asking the participants to solve six subtraction with regrouping problems. The first three problems required the participant to show the interviewer how they would solve the subtraction problems using base ten blocks. The interviewer prompted the participants to explain what they were doing with the manipulative devices as they solved the problems. Two of these first three problems were comprised of 3-digit minuends and 3-digit subtrahends. The last of the first three problems had 2-digit minuends and 2-digit subtrahends. Next, the interviewer asked the participant to solve the last three problems requiring subtraction with regrouping without using any manipulative devices. The participants were prompted to explain what they were doing as they solved these problems. Again, two of the last three problems were comprised of 3-digit minuends and 3-digit subtrahends while the last problem had a 2-digit minuend and a 2-digit subtrahend.

While the participant was solving each problem on this interview, the interviewer was scoring his or her actions based on the twenty-one conditions listed on the subtraction interview pretest scoring protocol (See Appendix F). For the first three problems, the participant was asked to use manipulative devices to explain how he or
she was solving the problems. The scoring conditions were organized into four domains: (a) participant represented first number accurately, (b) participant stated need to regroup to subtract tens, (c) participant regrouped accurately, and (d) participant subtracted accurately. For the last three problems, the participant was asked to solve the problems without using manipulative devices and to explain the steps he or she used to solve the problems. The scoring conditions were organized into three domains: (a) participant stated need to regroup to subtract ones, (b) participant regrouped accurately, and (c) participant subtracted accurately. This interview was not timed.

Identical problems from the curriculum-based assessments administered prior to instruction on subtraction with regrouping were found on the posttests administered after the intervention had been implemented. The *Subtraction with Regrouping Minute* and the *Subtraction Review Minute* were the same curriculum-based assessment both as a pretest and as a posttest (See Appendices B and C). The problems from the *Subtraction With Regrouping Pretest* and the *Subtraction Word Problem Pretest* were presented in reverse order on the *Subtraction With Regrouping Posttest* (See Appendix G) and the *Subtraction Word Problem Posttest* (See Appendix H). Last, the problems from the *Subtraction Interview Pretest* were presented in a different order on the *Subtraction Interview Posttest* (See Appendix I). Differing the order of problems on the respective posttests helped reduce the likelihood of practice effect on these measures.
**On-going Monitoring Probes**

During each subtraction lesson, participants were required to complete a *Learning Sheet*. The last ten problems on these *Learning Sheets* were used as the *Learning Sheet Probes* to monitor participant progress throughout the study. See Appendix J for a sample *Learning Sheet Probe*.

**The Subtraction With Regrouping Satisfaction Questionnaire**

The Subtraction With Regrouping Satisfaction Questionnaire was a questionnaire used to evaluate the level of satisfaction of the participants. The questionnaire consisted of 10 questions designed to measure the participant’s level of satisfaction with the subtraction with regrouping intervention lessons. The questionnaire was based on a five-point likert scale with 5 being least satisfied and 1 being most satisfied (see Appendix K). Students were given verbal directions by their learning disability teacher who then read each statement aloud before the student selected a value.

**Materials and Equipment**

*Subtraction With Regrouping Lessons* (Miller & Kaffar, 2008)

The *Subtraction with Regrouping Lessons* each included a list of goals to be addressed during instruction and a list of materials needed for delivery of instruction. Additionally, each lesson was scripted to ensure the delivery of pedagogically sound systematic and explicit instruction which included: (a) an advance organizer, (b) a describe and model stage of instruction, (c) a guided practice stage of instruction, (d) an independent practice stage of instruction, and (e) a problem solving stage.

*Subtraction With Regrouping Learning Sheets* (Miller & Kaffar, 2008)
Every lesson presented during this intervention has a *Learning Sheet Probe* to support participant understanding. Each *Learning Sheet Probe* contained three sections titled *Describe and Model, Guided Practice, Independent Practice,* and *Problem-Solving Practice.* Each of these sections contained subtraction problems that were reviewed throughout the various stages of lesson instruction. The types of problems on each *Learning Sheet Probe* correlated to the skill presented in the lesson. The last ten problems of each *Learning Sheet Probe* were completed independently and then plotted on the student *Subtraction Progress Chart.*

**Base Ten Blocks**

The base ten blocks used in this study were 3-dimensional plastic blocks that consisted of small cubes that represented units of one, rectangular rods equal in length to ten ones joined together (sometimes referred to as a long) that represented tens, and square blocks equal in size to ten rods joined together (sometimes referred to as a flat) that represented hundreds.

**Place Value Mat**

The place value mat was a single sheet of construction paper measuring 8 ½ inches wide by 24 inches long. It was divided into three columns. The far left column was titled *Hundreds,* the middle column was titled *Tens* and the far right column was titled *Ones.* Participants used the Place Value Mat when working within the five initial lessons that work on developing a conceptual understanding of subtraction with regrouping using manipulative devices (see Appendix L).
Design

A multiple probe design across subjects with one replication was used in this study (Barlow & Hersen, 1984; Horner & Baer, 1978; Zirpoli, 2008). There was three design conditions: baseline, intervention, and maintenance. There were two groups of three students. Each student was in the fifth grade.

Baseline Condition

Once pretesting was complete, the multiple probe study began. All subjects received baseline Subtraction with Regrouping Probes (see Appendix M). The baseline condition involved collection of data that was reflective of the participants’ pre instructional skills related to subtraction with regrouping. These baseline data were used to help determine the efficacy of the intervention (Barlow & Hersen, 1984). Baseline probes were administered to all participants over a minimum of three sessions until stability was clear (Baer, Wolf, Risley, 1968; McNamara & MacDonough, 1972). Once baseline stability was achieved with two participants (one from each group) the intervention condition began.

Intervention Condition

Participant 1 and Participant 4 began initial instruction of intervention lessons. The scripted lessons the participants received follow explicit instruction pedagogy which included: (a) an advance organizer, (b) a describe and model stage of instruction, (c) a guided practice stage of instruction, (d) an independent practice stage of instruction, and (e) a problem solving stage of instruction. Additionally, the lessons followed the concrete-representational-abstract instructional process. Of the total intervention lessons, there were five concrete methodology lessons, three
representational methodology lessons, one lesson in which a mnemonic device was learned and mastered, and five abstract methodology lessons. The remaining 12 lessons were designed to help students build word problem and fluency skills. The delineation of the amount and types of lessons taught when using the CRA instructional sequence aligned with current best practice reported in the literature.

The concrete methodology lessons were designed to facilitate mastery related to conceptual understanding of subtraction with regrouping. Base ten blocks were used to provide hands-on experiences that correlated to the verbal descriptions of what takes place when subtracting with regrouping. Using these three-dimensional objects allowed participants to understand and develop mental images of the math concept. The representational methodology lessons shifted the learners’ use of subtraction with regrouping from a three-dimensional understanding to a two-dimensional understanding. Participants were taught how to use visual depictions of the skill to aid in solving problems in which subtraction with regrouping was necessary. Next, the mnemonics FAST, RENAME, and BBB were taught to the students. The letters F-A-S-T cued the students to: Find what you’re solving for, Ask yourself, “What are the parts of the problem?” Set up the numbers, and Tie down the sign. The letters R-E-N-A-M-E cued the students to: Read the problem, Examine the ones column: use the BBB sentence for ones, Note the ones in the ones column, Address the tens column: use the BBB sentence for tens, Mark the tens in the tens column, and Examine and note hundreds; Exit with a quick check. The letters B-B-B cued the students to recognize if the Bigger number was on the Bottom, it means you need to Break down and trade. Finally, the abstract methodology lessons removed any visual supports the
participants previously used to solve subtraction with regrouping problems. This scaffolding of instruction within the concrete-representational-abstract process overtly supported participants’ shift from a level of understanding that requires tangible objects to a more abstract understanding of this new mathematical concept (Hudson & Miller 2006).

Per the parameters of a multiple probe design, ongoing probes of participant performance took place during the Intervention Condition. Specifically, the percentage scores of Participant 1 and 4 on lesson Learning Sheet Probes were plotted on a graph to monitor their independent success with this new skill. Once Participant 1 and Participant 4 achieved 80% correct on these Learning Sheet Probes three days in a row, Participant 2, 3, 5 and 6 received an additional baseline probe prior to beginning the intervention lessons with Participant 2 and 5. Because there was stability in baseline trends, Participant 2 and Participant 5 began the Intervention Condition. Once Participant 2 and Participant 5 achieved 80% correct on these Learning Sheet Probes three days in a row, Participant 3 and Participant 6 received an additional baseline probe prior to beginning the intervention lessons. Because stability in baseline trend existed, Participant 3 and 6 began the Intervention Condition.

Maintenance Condition

Seven days after the intervention condition ended Maintenance Probes were administered. These maintenance scores were used to measure the participants’ retention of the newly acquired skill.
Procedures

There were five phases in this study. These phases were as follows: (a) preparation for study, (b) pretest and baseline, (c) implementation of mathematics intervention lessons, (d) post-assessments, and (e) maintenance.

Phase 1: Preparation for Study

Obtaining permission. Permission to implement the study was obtained from the University of Nevada, Las Vegas Office for the Protection of Research Subjects (OPRS) and from the Clark County School District Research Review Board.

Next, the approved letters of consent and assent were placed in sealed envelopes to be disseminated to the subjects by the learning disability teacher at the school site (see Appendices N and O). The potential participants took the letters home to their parents for review and then returned the consent and assent forms to the learning disability teacher. Those who returned signed forms were eligible to participate in the study.

Phase 2: Pretest and Baseline

Four curriculum-based assessments and one interview were administered. The Subtraction With Regrouping Pretest and the Subtraction Word Problem Pretest were administered to all participants of the study within the special education classroom. These were untimed-focused curriculum-based assessments so the subjects were provided as much time as needed to complete the assessment. Percentages were calculated to reflect how accurately the participants were able to answer subtraction with regrouping problems.
The *Subtraction With Regrouping Minute* and the *Subtraction Review Minute* pretests also were administered to all participants of the study within the special education classroom. These were timed-focused curriculum-based assessments in which participants were given one minute to answer as many problems as possible. The number of correct digits and incorrect digits obtained within one minute was determined to reflect how accurately the students were able to answer subtraction with regrouping problems.

The learning disability teacher also administered the *Subtraction Interview Pretest* to the participants within the special education classroom. Points were given to the student if they met the stated criteria on the *Subtraction Interview Checklist* (i.e., student stated a need to regroup, student regrouped correctly, student solved correctly)

During ongoing baseline monitoring, a subtraction with regrouping *Baseline Learning Sheet Probe* was administered. *Baseline Learning Sheet Probes* were administered to all students over a minimum of three sessions until stability was clear (Baer et al., 1968; McNamara & MacDonough, 1971). Baseline probes were administered within the students’ learning disability class.

**Phase 3: Implementation of Mathematic Intervention Lessons**

After three days of baseline probes, because stability was established, the subtraction with regrouping intervention lessons began according to the implementation schedule (See Appendix M). The scripted lessons were delivered following explicit teaching principles and the concrete-representational-abstract process.
During the advance organizer of each lesson three basic things occurred. First, a review of previously learned skills was conducted. Second, the lesson objective was presented in a way that it was overtly connected to prior knowledge. Finally, relevance for why the participants were learning the new concept or skill was provided to enhance their motivation to participate in the rest of the lesson (Hudson & Miller, 2006).

Next, the Describe and Model stage of instruction was implemented. Three things took place during this stage. First, the instructor modeled what the participants were expected to do in order to solve the problem. The instructor thought out loud while solving problems so that participants were exposed to the metacognitive process being used while solving the problem. Second, the instructor maximized participants’ engagement during the demonstration (e.g., the teacher sat the students less than 2 feet from instruction and used verbal cues to keep students engaged) to ensure student attention was maintained. Third, the instructor monitored participant understanding through questioning and the provision of feedback.

During the guided practice stage of instruction the teacher gradually allowed the participants to take on more responsibility to solve problems independently. The teacher provided various levels of support during guided practice to ensure participant success. Throughout guided practice, the teacher gradually removed assistance (i.e. prompts) so that participants were being supported while working towards independence. The teacher was simultaneously asking both factual and process type questions to help monitor participant performance with the new skill.
During the independent practice stage of instruction the participants had an opportunity to become more fluent with newly learned skill. The teacher removed all support to allow participant to solve problems independently.

Performance feedback was provided during both the guided and independent stages of instruction. Specifically, an elaborative feedback routine was provided that included (a) helping the participant plot his or her score on a progress chart, (b) providing one specific positive statement about the participant’s work, (c) identifying one area for improvement, (d) demonstrating how to compute a missed problem using think aloud methodology, (e) asking participant to complete one similar problem, and (f) closing the feedback session with a positive statement about the participant’s performance during the feedback process and stating positive expectations related to future performance on similar problems.

Phase 4: Post-Assessments

After the participants received the subtraction with regrouping intervention lessons, they were given four curriculum-based assessments. The four curriculum-based assessments were the Subtraction With Regrouping Posttest, the Subtraction With Regrouping Minute, the Subtraction Review Minute, and the Subtraction Word Problem Posttest. The Subtraction Interview also was readministered for posttest purposes.

The Subtraction With Regrouping Posttest and the Subtraction Word Problem Posttest were administered to all the participants within the special education classroom. These were untimed-focused curriculum-based assessments so the participants were provided as much time as needed to complete the assessment.
Percentage scores were calculated to reflect how accurately the participants were able to answer subtraction with regrouping problems.

Both the Subtraction With Regrouping Minute and the Subtraction Review Minute were timed-focused curriculum-based assessments. The learning disability teacher administered these assessments to the subjects within the special education classroom. The number of correct digits and incorrect digits obtained within one minute were determined to reflect how accurately the participants were able to answer subtraction with regrouping problems.

Last, the learning disability teacher administered the Subtraction Interview Posttest to individual subjects one at a time on the school campus. The student was given points on the Subtraction Interview Checklist if they met the stated criteria.

Finally, to assess social validity of the study, the participants filled out the Subtraction With Regrouping Satisfaction Questionnaire. This questionnaire addressed the likeliness of the subjects continued motivation to successfully utilize the newly learned subtraction with regrouping skill (Barlow & Hersen, 1984). The questionnaire contained nine statements related to the level of the participants’ satisfaction with various components of the Subtraction with Regrouping Intervention Lessons. The participants rated each statement on a scale from 1 to 5 with 1 being the most favorable (see Appendix K). Students were given verbal directions by their learning disability teacher who then read each statement aloud before the student selected a value.

The test results were communicated to the students by the learning disability teacher in individual meetings on the school campus. The participants were exposed
to their pretest scores in order to illustrate the progress they made after participating in the subtraction with regrouping intervention lessons.

Phase 5: Maintenance

Seven days after the Post-Assessments, the four CBA assessments (i.e., Subtraction with Regrouping, Subtraction Word Problem, Subtraction With Regrouping Minute, and Subtraction Review Minute) and the Subtraction With Regrouping Probe were administered in the students’ learning disability classroom. These maintenance probes were given to all subjects within the special education classroom. The results of these probe were shared with the participants in individual meetings with the learning disability teacher.

Interscorer Reliability

The learning disability teacher scored each student on the pre- and posttests: (a) the Subtraction With Regrouping Pre- and Posttest, the Subtraction With Regrouping Minute, the Subtraction Review Minute, and the Subtraction Word Problem Pre- and Posttest. The Subtraction Interview was scored by the learning disability teacher and the student investigator. To determine interscorer reliability, the student investigator scored 20% of each of the pre-, post-, and maintenance tests. The primary scorer was the learning disability teacher and the secondary scorer was the student investigator. An agreement was counted when both the student investigator and the learning disability teacher recorded the same score for an answer. The formula “agreements ÷ (agreements + disagreements)” was used to determine reliability levels (Barlow & Hersen, 1982).
Fidelity of Treatment

To determine interobserver agreement related to fidelity of treatment, the principal and student investigator completed a fidelity of treatment checklist while observing the learning disability teacher deliver 20% of the subtraction with regrouping intervention lessons (see Appendix P). Items on the fidelity of treatment checklist were marked with a checkmark to indicate compliance with the scripted intervention lessons. The formula “agreements ÷ (agreements + disagreements)” was used to establish the fidelity of treatment level.

Treatment of Data Related to Visual Analysis

Visual analysis of the participants’ *Subtraction with Regrouping Probes* and *Learning Sheet Probes* occurred to determine the effects of the subtraction with regrouping intervention lessons. Each participant’s performance was graphed according to the specifications of multiple probe designs (Barlow & Hersen, 1984; Horner & Baer, 1978; Zirpoli, 2008). The level, trend, and variability of performance data were visually inspected to determine the effectiveness of the intervention. Level refers to the mean performance of the dependent variable. The intervention lessons were deemed successful if the level of the dependent variable (*Learning Sheet Probes*) increased when compared to *Baseline Subtraction With Regrouping Probes*. Trend refers to a visual inspection of the data, which reveals a constant rate of behavior, either in an upward, downward, or stable manner. The intervention lessons were deemed successful if there was an increase in the line’s slope or stability at an acceptable level. Variability refers to the consistency of the data points around the
mean performance. A successful intervention shows little variability indicating consistent performance and a change in level and trend. By replicating the study results with an additional three subjects (external validity), confidence was increased that changes in subtraction with regrouping were due to the intervention lessons. Rate changes were sequentially observed in more than one subject, but only after the treatment variable had been directly applied to each, so the experimenter gained confidence in the efficacy of the procedure (Barlow & Hersen, 1984). Excel software was used to create single-subject design line graphs for this study.

Treatment of Data Related to Research Questions

Research Question 1: Do Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems? The on-going Learning Sheet Probes were used to answer this question. Baseline probes were compared to Instruction probes. Level, trend, and variability were inspected visually to determine effectiveness of the intervention. Additionally, descriptive data (i.e., participant scores from the Subtraction with Regrouping Pretest and Subtraction with Regrouping Posttest) were used to answer this question. The scores from the participants’ pretests were compared to their posttest outcomes.

Research Question 2: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems? Descriptive data from participant performance on the Subtraction Word Problem Pretest and the
Subtraction Word Problem Posttest were used to answer this question. The scores from the participants’ pretest were compared to their posttest outcomes.

Research Question 3: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems? Descriptive data from participant performance on the Subtraction With Regrouping Minute were used to answer this question. The scores from the participants’ pretests were compared to their posttest scores.

Research Question 4: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems? Descriptive data from the Subtraction Interview Pretest and the Subtraction Interview Posttest were used to answer this question. The scores from the participants’ pretests were compared to their posttest outcomes.

Research Question 5: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping? Descriptive data from the Subtraction Review Minute were used to answer this question. The scores from the participants’ pretests were compared to their posttest outcomes.

Research Question 6: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems? An additional
on-going monitoring probe was administered seven days after instruction completion and visual analysis was used to determine differences in level between the intervention condition and the maintenance condition. Additionally, descriptive data from the CBA posttest probes (i.e., Subtraction with Regrouping Posttest, Subtraction Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute) was re-administered and compared as maintenance probes.

Research Question 7: How satisfied will students with learning disabilities be with learning the subtraction with regrouping intervention lessons? The Subtraction With Regrouping Satisfaction Questionnaire was answered by the participants at the end of the study and were analyzed to determine levels of satisfaction. The questionnaire contained nine questions being rated on a level of 1 to 5 with 1 being the most favorable.
CHAPTER 4
DATA ANALYSIS

The purpose of this study was to investigate the effects of strategy instruction and the concrete-representational-abstract sequence to teach subtraction with regrouping to students with learning disabilities. Data were collected to answer seven research questions related to the participants’ ability to learn and apply the concrete-representational-abstract sequence when solving subtraction problems that require regrouping. In addition, participants’ satisfaction levels were assessed in relation to learning through the concrete-representation-abstract sequence. A summary of the collected data following the parameters of the multiple probe design are reviewed first. Next, the results related to the seven research questions are shared. Third, interscorer reliability and fidelity of treatment data are provided. The chapter concludes with a summary of the results obtained in this study.

Overview of Collected Data Using a Multiple Probe Design

According to the parameters of a multiple probe design, data collection was staggered (Horner & Baer, 1978). Six participants were arranged in two triads. Baseline and intervention data were collected for both triads simultaneously. The second triad’s performance was used to increase external validity (Barlow & Hersen, 1984). Student performance related to baseline, instruction (i.e., advanced organizer, describe and model, guided practice, independent practice, and problem solving), and maintenance are displayed in Figures 1 and 2 and discussed in greater detail related to the respective research questions in this study.
Baseline Condition

The research study began with baseline probes being administered to all six participants for three consecutive days. The baseline probes consisted of 10 two- and three-digit subtraction problems that required regrouping to solve. There were a total of eight computation problems and two word problems. As soon as stability was achieved with two students (one from each group) the intervention condition was initiated. These two students were identified as Participant 1 and Participant 4 as indicated on the Implementation Schedule (see Appendix Q). When Participant 1 and Participant 4 achieved 80% or higher on three consecutive Learning Sheet Probes, Participants 2, 3, 5 and 6 received an additional baseline probe. Combining the results of the first three baseline probes with this fourth probe, stability in baseline trends was further established for Participant 2 and Participant 5. Thus, the intervention condition was initiated with Participant 2 and 5. When Participant 2 and 5 achieved 80% or higher on three consecutive Learning Sheet Probes, Participant 3 and Participant 6 were given one more baseline probe to solidify stability in their baseline data and then the intervention condition was initiated. In summary, adhering to the criteria established for initiating instruction (i.e., 80% or higher on three consecutive probes), Participants 1 and 4 received three baseline probes, Participants 2 and 5 received four baseline probes, and Participants 3 and 6 received five baseline probes (see Figures 1 and 2).
Figure 1. Percent of Correct Responses by Participants 1, 2, and 3.

Note. BL = Baseline; IC = Intervention Condition; M = Maintenance.
Figure 2. Percent of Correct Responses by Participants 4, 5, and 6.

Note. BL = Baseline; IC = Intervention Condition; M = Maintenance.
**Intervention Condition**

Participants 1 and 4 began instruction of intervention lessons within their special education math class. The rest of the participants were not scheduled to be in any other environment where they might be exposed to math instruction because their Individualized Education Plans (IEPs) mandated that they receive math instruction from the learning disability teacher. This facilitated control of what all participants were exposed to while waiting to begin in the intervention phase of this study. Each of the scripted lessons followed explicit instruction pedagogy. Each lesson started with an advanced organizer in which a review of previously learned skills took place, the lesson objective was presented and connected to prior knowledge, and relevance was established for why the students were learning the new concept. Next, the students participated in a describe and model stage of the lesson in which the special education math teacher accomplished three things: (a) modeled what the students were expected to do in order to solve the problem including talking through the metacognitive process that takes place while solving the problem, (b) maximized student engagement during the demonstration by sitting students within close proximity to the demonstration taking place and using verbal cues to ensure student focus was maintained, and (c) monitored student understanding by using verbal questioning and providing feedback.

The third part of the scripted intervention lessons included a guided practice stage of instruction. During this stage, the special education math teacher provided supportive verbal or visual prompts that were gradually removed to help students become more independent with the newly modeled skill. Once the students
demonstrated independent success, the teacher moved into the independent practice stage of the lesson. During this stage of instruction, students were encouraged to maintain their level of independence while practicing the newly learned skill. Within both the guided practice stage of instruction and independent stage of instruction the special education math teacher followed a specific routine when performance feedback was provided to the students. The performance feedback routine included the following steps: (a) helping the student plot his or her score on a progress chart, (b) providing one specific positive statement about the student’s work, (c) identifying one area for improvement, (d) demonstrating how to compute a missed problem using think aloud methodology, (e) asking the student to complete one similar problem, and (f) closing the feedback session with a positive statement about the student’s performance during the feedback process and positive expectations related to future performance on similar problems.

The scripted lessons followed the concrete-representational-abstract instructional process. The first five instructional lessons were designed to help participants develop concrete understanding of the skill. The next three lessons were designed to help participants develop representational understanding of the skill. The next lesson (i.e., lesson nine) was designed to help participants memorize the steps in a mnemonic device related to the procedural steps involved in solving subtraction with regrouping problems. The remaining 17 lessons were designed to help participants with advanced word problem and fluency skills.
Maintenance Phase

Seven days after the intervention condition ended for each participant, the Maintenance Probes were administered. For each participant, the seven days included five typical days of attendance in school and two weekend days.

Research Questions and Related Findings

Question 1: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems?

Two data sets were used to answer this question. The first data set was obtained from the Subtraction With Regrouping Pretests and Posttests. The second data set was obtained from the Learning Sheet Probes completed at the end of each intervention lesson.

The pre- and posttest percentage scores for the Subtraction With Regrouping Pretest and Posttest were compared as evidence related to the strategy instruction integrated with the concrete-representational-abstract teaching sequence’s effectiveness to improve students’ ability to solve subtraction with regrouping problems. The Subtraction with Regrouping Pretest and Posttest consisted of 20 problems total. Ten of the 20 problems had 2-digit minuends and 2-digit subtrahends. Of those 10 problems, 3 had zeros in the ones column within the minuend. The other 10 problems were made up of 3-digit minuends and 3 digit subtrahends. Three of those problems had zeros in the tens column within the minuends. Pre- and posttest
percentage scores were determined based on the total number of correct problems divided by the total possible (i.e., 20).

The pretest and posttest scores for Participant 1 were 100% (20/20) and 95% (19/20) respectively representing a 5 percentage point decrease. Pretest and posttest scores for Participant 2 were 0% (0/20) and 85% (17/20), respectively, representing a gain of 85 percentage points. Pretest and posttest scores for Participant 3 were 0% (0/20) and 40% (8/20) respectively representing a gain of 40 percentage points. Pretest and posttest scores for Participant 4 were 80% (16/20) and 85% (17/20) respectively representing a gain of 5 percentage points. Pretest and posttest scores for Participant 5 were 0% (0/20) and 95% (19/20) respectively representing a gain of 95 percentage points. Pretest and posttest scores for Participant 6 were 0% (0/20) and 30% (6/20) respectively representing a gain of 30 percentage points. Thus, the percentage point gain from the Subtraction With Regrouping Pretest and Posttests ranged from -5 to 95 for the participants in this study. See Table 2.

In addition to these descriptive data, a paired-sample t-test was conducted to evaluate the impact of the intervention lessons on students’ scores on the Subtraction With Regrouping Tests. There was no statistically significant increase from the Subtraction With Regrouping Pretest ($M = 6.00, \text{SD} = 9.381$) to the Subtraction With Regrouping Posttest ($M=14.33, \text{SD} = 5.785$), $t(5) = (-2.493), p<.05$. Based on the guidelines in Cohen (1988), the eta squared statistic (0.55) indicated a large effect size.
Table 2

Participants’ Pre- and Posttest Percentage Scores on the Subtraction With Regrouping Pretest and Posttest

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest Subtraction With Regrouping</th>
<th>Posttest Subtraction With Regrouping</th>
<th>Percentage Point Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>100</td>
<td>95</td>
<td>-5</td>
</tr>
<tr>
<td>Participant 2</td>
<td>0</td>
<td>85</td>
<td>+85</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0</td>
<td>40</td>
<td>+40</td>
</tr>
<tr>
<td>Participant 4</td>
<td>80</td>
<td>85</td>
<td>+5</td>
</tr>
<tr>
<td>Participant 5</td>
<td>0</td>
<td>95</td>
<td>+95</td>
</tr>
<tr>
<td>Participant 6</td>
<td>0</td>
<td>30</td>
<td>+30</td>
</tr>
</tbody>
</table>

The second data set used to answer this research question was obtained from a combination of the Baseline Probes and Learning Sheet Probes. All Baseline Probes consisted of ten problems. The first 8 problems were computation subtraction problems that required regrouping to solve correctly. The last 2 problems on the Baseline Probes were word problems. The Learning Sheet Probes consisted of 10 problems from the Learning Sheets that accompanied each lesson. All six participants were able to reach mastery performance on each Learning Sheet Probe (i.e., score a minimum of 8 or more correct). Mastery performance, on the first trial, was reached
for a majority of the Learning Sheet Probes. Out of the 26 scripted intervention lessons taught to each of the 6 participants (i.e., 156 total lessons) there were only 13 times when a participant needed to complete the lesson more than once in order to reach the mastery performance criteria.

The Baseline Probe scores for Participant 1 were 90%, 70%, and 90% (M= 83%); scores on the Learning Sheet Probes were 100%, 100%, 100%, 90%, 90%, 100%, 90%, 100%, 80%, 100%, 80%, 90%, 100%, 100%, 100%, 100%, 80%, 100%, 90%, 100%, 100%, 100%, 90%, 100%, 90%, 100%, 100%, 90%, and 90% (M= 94.8%). Visual analysis of data (see Figure 1) indicates a level increase of 10 percentage points from baseline condition to intervention condition. With regard to trend, Participant 1 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 1 demonstrated little variability during the intervention condition. He only missed two problems on 3 probes, one problem on 7 probes, and no problems were missed on the remaining 16 probes. Participant 1 did not need to repeat any lessons in order to reach performance mastery on the accompanying probe.

The Baseline Probe scores for Participant 2 were 0%, 0%, 0%, and 0% (M = 0); scores on the Learning Sheet Probes were 100%, 80%, 100%, 80%, 90%, 100%, 100%, 100%, 80%, 100%, 70%, 90%, 90%, 90%, 30%, 100%, 100%, 90%, 100%, 100%, 100%, 80%, 90%, and 90% (M = 90.7%). Visual analysis of data (see Figure 1) indicates a level increase of 100 percentage points from baseline condition to intervention condition. With regard to trend, Participant 2 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 2 demonstrated notable variability during the intervention condition. He
missed seven problems on one probe, three problems on one probe, two problems on 4 probes, 1 problem on 7 probes, a and zero problems were missed on the remaining 14 probes. Participant 2 had to repeat two lessons in order to reach performance mastery on the accompanying probe.

The Baseline Probe scores for Participant 3 were 0%, 0%, 0%, 0%, and 0% (M = 0%); scores on the Learning Sheet Probes were 100%, 80%, 60%, 90%, 80%, 70%, 80%, 50%, 80%, 100%, 90%, 90%, 60%, 90%, 80%, 90%, 80%, 80%, 90%, 80%, 80%, 80%, 80% and 80% (M = 84.1%). Visual analysis of data (see Figure 1) indicates a level increase of 100 percentage points from baseline condition to intervention condition. With regard to trend, Participant 3 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 3 demonstrated notable variability during the intervention condition. He missed five problems on 1 probe, four problems on 2 probes, three problems on 1 probe, two problems on 10 probes, 1 problem on ten problems, and zero problems were missed on the remaining 5 probes. Participant 3 had to repeat four lessons in order to reach performance mastery on the accompanying probe.

The Baseline Probe scores for Participant 4 were 70%, 70%, and 70% (M = 70%); scores on the Learning Sheet Probes were 100%, 90%, 100%, 90%, 80%, 100%, 100%, 90%, 90%, 100%, 100%, 80%, 60%, 70%, 100%, 80%, 80%, 70%, 80%, 90%, 100%, 90%, 100%, 90%, 80%, 90%, 100%, and 100% (M = 89.2%). Visual analysis of data (see Figure 2) indicates a level increase of 30 percentage points from baseline condition to intervention condition. With regard to trend,
Participant 4 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 4 demonstrated notable variability during the intervention condition. He missed four problems on 1 probe, three problems on 2 probes, two problems on 6 probes, one problem on 8 probes, and zero problems were missed on the remaining 11 probes. Participant 4 had to repeat three lessons in order to reach performance mastery on the accompanying probe.

The Baseline Probe scores for Participant 5 were 0%, 0%, 0% and 0% (M = 0%); scores on the Learning Sheet Probes were 100%, 90%, 90%, 50%, 100%, 80%, 100%, 100%, 100%, 100%, 90%, 80%, 70%, 90%, 90%, 100%, 80%, 100%, 100%, 100%, 100%, 100%, 100%, 90%, and 90% (M = 92.5%). Visual analysis of data (see Figure 2) indicates a level increase of 100 percentage points from baseline condition to intervention condition. With regard to trend, Participant 5 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 5 demonstrated notable variability during the intervention condition. He missed five problems on 1 probe, three problems on 2 probes, two problems on 3 probes, one problem on 7 probes and zero problems were missed on the remaining sixteen probes. Participant 5 had to repeat two lessons in order to reach performance mastery on the accompanying probe.

The Baseline Probe scores for Participant 6 were 0%, 0%, 20%, 0% and 0% (M = 4%); scores on the Learning Sheet Probes were 100%, 80%, 100%, 80%, 90%, 80%, 50%, 80%, 90%, 80%, 80%, 100%, 100%, 80%, 80%, 90%, 90%, 90%, 80%, 30%, 90%, 70%, 90%, 70%, 80%, 80%, 70%, 80%, 70%, 80% and 80% (M = 80.9%). Visual analysis of data (see Figure 2) indicates a level increase of 100 percentage
points from baseline condition to intervention condition. With regard to trend, Participant 6 maintained a flat trend of mastery level scores once the instruction was introduced. Participant 6 demonstrated notable variability during the intervention condition. She missed seven problems on 1 probe, five problems on 5 probes, three problems on 4 probes, two problems on 14 probes, one problem on 7 probes, and zero problems were missed on the remaining four probes. Participant 6 had to repeat six lessons in order to reach performance mastery on the accompanying probes.

Question 2: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems? One data set was used to answer this question. The data set consisted of the Subtraction Word Problem Pretest and Posttest scores.

The pre- and posttest scores for the Subtraction Word Problem Pretest and Posttest were compared to determine the effectiveness of strategy instruction integrated with the concrete-representational-abstract teaching sequence for improving students’ ability to solve subtraction with regrouping word problems. The Subtraction Word Problem Pretest and Posttest consisted of 10 problems. Four of the ten problems had 2-digit minuends and 2-digit subtrahends. The other six problems had 3-digit minuends and 3-digit subtrahends. One problem included extraneous information (i.e., word and numerical information not needed to solve the problem). Pre- and posttest percentage scores were determined based on the total number of correct problems divided by the total possible (i.e., 10).
The pretest and posttest scores for Participant 1 were 90% and 100% respectively a gain of 10 percentage points. The pretest and posttest scores for Participant 2 were 0% and 70% respectively representing a gain of 70 percentage points. The pretest and posttest scores for Participant 3 were 0% and 40% respectively representing a gain of 40 percentage points. The pretest and posttest scores for Participant 4 were 70% and 80% respectively representing a gain of 10 percentage points. The pretest and posttest scores for Participant 5 were 0% and 70% respectively representing a gain of 70 percentage points. The pretest and posttest scores for Participant 6 were 0% and 70% respectively representing a gain of 70 percentage points. Thus, the gain of points from the Subtraction With Regrouping Pretest and Posttest. ranged from 10% to 70% for the participants in this study. See Table 3.

In addition to these descriptive data, a paired-sample t-test was conducted to evaluate the impact the intervention lessons had on students’ scores on the Subtraction With Regrouping Word Problem Tests. There was a statistically significant increase from the Subtraction With Regrouping Word Problem Pretest (M = 2.67, SD = 4.179) to the Subtraction With Regrouping Word Problem Posttest (M=7.17, SD = 1.941), t (5) = -3.737, p<.05. Based on the guidelines in Cohen (1988), the eta squared statistic (0.73) indicated a large effect size.

Question 3: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems?
Table 3

*Participants’ Pre- and Posttest Percentage Scores on the Subtraction Word Problem*

**Pretest and Posttest**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest Subtraction Word Problem</th>
<th>Posttest Subtraction Word Problem</th>
<th>Percentage Point Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>90</td>
<td>100</td>
<td>+10</td>
</tr>
<tr>
<td>Participant 2</td>
<td>0</td>
<td>70</td>
<td>+70</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0</td>
<td>40</td>
<td>+40</td>
</tr>
<tr>
<td>Participant 4</td>
<td>70</td>
<td>80</td>
<td>+10</td>
</tr>
<tr>
<td>Participant 5</td>
<td>0</td>
<td>70</td>
<td>+70</td>
</tr>
<tr>
<td>Participant 6</td>
<td>0</td>
<td>70</td>
<td>+70</td>
</tr>
</tbody>
</table>

One data set was used to answer this question. The data set was obtained from the *Subtraction With Regrouping Minute Pretest* and *Posttest*. The pre- and posttest raw scores for the *Subtraction With Regrouping Minute* were compared to determine the effectiveness of strategy instruction integrated with the concrete-representational-abstract teaching sequence for improving students’ ability to solve subtraction with regrouping problems fluently. The *Subtraction With Regrouping Minute* consisted of 16 problems total. Eight of the 16 problems had 2-digit minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-digit subtrahends. Pre-
and posttest raw scores are reported as total number correct digits and total number of incorrect digits obtained within one minute.

Pretest and posttest scores for Participant 1 were 3 correct digits with 10 errors and 23 correct digits with 0 errors respectively representing a gain of 20 correct digits in one minute. Pretest and posttest scores for Participant 2 were 3 correct digits with 30 errors and 31 correct digits with 3 errors respectively representing a gain of 28 correct digits in one minute. Pretest and posttest scores for Participant 3 were 0 correct digits with 6 errors and 13 correct digits with 11 errors respectively representing a gain of 13 correct digits in one minute. Pretest and posttest scores for Participant 4 were 19 correct digits with zero errors and 19 correct digits with three errors respectively representing no gain in correct digits. Pretest and posttest scores for Participant 5 were 8 correct digits with six errors and 28 correct digits with zero errors respectively representing a gain of 20 correct digits in one minute. Pretest and posttest scores for Participant 6 were 3 correct digits with zero errors and 30 correct digits with one error respectively representing a gain of 27 correct digits in one minute. Thus, the gain of points from the *Subtraction With Regrouping Minute Pretest and Posttest* ranged from 0 to 28 correct digits in one minute for the participants in this study. See Table 4.
Table 4  

*Participants’ Pre- and Posttest Raw Scores on the Subtraction With Regrouping*  

**Minute Pretest and Posttest**

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest Subtraction With Regrouping</th>
<th>Posttest Subtraction With Regrouping</th>
<th>Raw Point Improvement for Correct Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pretest Subtraction Min</td>
<td>Posttest Subtraction Min</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Digit/Errors</td>
<td>Digit/Errors</td>
<td></td>
</tr>
<tr>
<td>Participant 1</td>
<td>3C/10E</td>
<td>23C/0E</td>
<td>20C</td>
</tr>
<tr>
<td>Participant 2</td>
<td>3C/30E</td>
<td>31C/3E</td>
<td>28C</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0C/6E</td>
<td>13C/11E</td>
<td>13C</td>
</tr>
<tr>
<td>Participant 4</td>
<td>19C/0E</td>
<td>19C/3E</td>
<td>0C</td>
</tr>
<tr>
<td>Participant 5</td>
<td>8C/6E</td>
<td>28C/0E</td>
<td>20C</td>
</tr>
<tr>
<td>Participant 6</td>
<td>3C/0E</td>
<td>30C/1E</td>
<td>27C</td>
</tr>
</tbody>
</table>

**Question 4:** Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems?

One data set was used to answer this question. The data set was obtained from the *Subtraction Interview Pretest* and *Posttest*. The pre- and posttest percentage scores for the *Subtraction Interview Pretest* and *Posttest* were compared as evidence related
to strategy instruction integrated with concrete-representational-abstract teaching sequence’s effectiveness to improve students’ conceptual understanding related to solving subtraction with regrouping word problems. The *Subtraction Interview Pretest* and *Posttest* consisted of six subtraction with regrouping problems. The first three problems required the students to show the interviewer how they would solve the subtraction problems using base ten blocks. The interviewer prompted the students to explain what they were doing with the manipulative devices as they solved the problems. Two of the first three problems were comprised of 3-digit minuends and 3-digit subtrahends. The last of the first three problems had 2-digit minuends and 2-digit subtrahends. Next, the interviewer asked the students to solve the last three problems requiring subtraction with regrouping without using any manipulative devices. The students were prompted to explain what they were doing as they solved these problems. Again, two of the last three problems were comprised of 3-digit minuends and 3-digit subtrahends while the last problem had a 2-digit minuend and a 2-digit subtrahend.

While the student solved each problem included in the interview, the interviewer scored his or her actions based on the 21 conditions listed on the subtraction interview pretest scoring protocol (See Appendix F). For the first three problems, the student was asked to use manipulative devices to explain how he or she solved the problems. The scoring conditions were organized into four domains: (a) student represents first number accurately, (b) student states need to regroup to subtract tens, (c) student regroups accurately, and (d) student subtracts accurately. For the last three problems, the student was asked to solve the problems without using manipulative devices and
to explain the steps he or she used to solve the problems. For these problems, the scoring conditions were organized into three domains: (a) student states need to regroup to subtract ones, (b) student regroups accurately, and (c) student subtracts accurately. This interview was not timed.

Pretest and posttest scores for Participant 1 were 85.7% (18/21) and 100% (21/21) respectively representing a gain of 14.3 percentage points. Pretest and posttest scores for Participant 2 were 10.5% (2/21) and 71.4 % (15/21) respectively representing a gain of 60.9 percentage points. Pretest and posttest scores for Participant 3 were 0% (0/21) and 66.6% (14/21) respectively representing a gain of 66.6 percentage points. Pretest and posttest scores for Participant 4 were 38% (8/21) and 90.4 % (19/21) respectively representing a gain of 52.4 percentage points. Pretest and posttest scores for Participant 5 were 0% (0/21) and 100% (21/21) respectively representing a gain of 100 percentage points. Pretest and posttest scores for Participant 6 were 28.5 % (6/21) and 100% (21/21) respectively representing a gain of 71.5 percentage points. Thus, the gain of percentage points from the Subtraction Interview Pretest and Posttest ranged from 14.3 to 100 for the participants in this study. See Table 5.

In addition to these descriptive data, a paired-sample t-test was conducted to evaluate the impact the intervention lessons had on students’ scores on the Subtraction Interview Tests. There was a statistically significant increase from the Subtraction Interview Pretest (M = 5.67, SD = 6.861) to the Subtraction Interview Posttest (M=18.50, SD = 3.209), t (5) = -5.347, p<.05. Based on the guidelines in Cohen (1988), the eta squared statistic (0.85) indicated a large effect size.
<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest Subtraction Interview</th>
<th>Posttest Subtraction Interview</th>
<th>Percentage Point Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>85.7</td>
<td>100</td>
<td>+ 14.3</td>
</tr>
<tr>
<td>Participant 2</td>
<td>10.5</td>
<td>71.4</td>
<td>+ 60.9</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0</td>
<td>66.6</td>
<td>+ 66.6</td>
</tr>
<tr>
<td>Participant 4</td>
<td>38</td>
<td>90.4</td>
<td>+ 52.4</td>
</tr>
<tr>
<td>Participant 5</td>
<td>0</td>
<td>100</td>
<td>+ 100</td>
</tr>
<tr>
<td>Participant 6</td>
<td>28.5</td>
<td>100</td>
<td>+ 71.5</td>
</tr>
</tbody>
</table>

Question 5: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping?

One data set was used to answer this question. The data set was obtained from the *Subtraction Review Minute Pretests and Posttests*. The pre- and posttest raw scores for the *Subtraction Review Minute Pretest* and *Posttest* were compared to determine the effectiveness of strategy instruction integrated with the concrete-representational-
abstract teaching sequence for improving students’ ability to solve subtraction with and without regrouping problems fluently. The *Subtraction Review Minute* consisted of 16 problems total. Eight of the 16 problems had 2-digit minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-digit subtrahends. Pre- and posttest raw scores are reported as total number correct digits and incorrect digits obtained within one minute.

Pretest and posttest scores for Participant 1 were 12 correct digits with 0 errors and 26 correct digits with 1 error respectively representing a gain of 14 correct digits in one minute. Pretest and posttest scores for Participant 2 were 7 correct digits with 20 errors and 30 correct digits with 0 errors respectively representing a gain of 23 correct digits in one minute. Pretest and posttest scores for Participant 2 were 0 correct digits with 2 errors and 9 correct digits with 2 errors respectively representing a gain of 9 correct digits in one minute. Pretest and posttest scores for Participant 4 were 18 correct digits with 0 errors and 27 correct digits with 1 error respectively representing a gain of 9 correct digits in one minute. Pretest and posttest scores for Participant 5 were 8 correct digits with 10 errors and 15 correct digits with 2 errors respectively representing a gain of 7 correct digits in one minute. Pretest and posttest scores for Participant 6 were 9 correct digits with 0 errors and 12 correct digits with 1 error respectively representing a gain of 3 correct digits in one minute. Thus, the gain of points from the *Subtraction Review Minute Pretest and Posttest* ranged from 3 to 23 correct digits in one minute for the participants in this study. See Table 6.
Table 6

Participants’ Pre- and Posttest Raw Scores on the Subtraction Review Minute Pretest and Posttest

<table>
<thead>
<tr>
<th>Participants</th>
<th>Pretest Subtraction Review Minute Correct Digit/Errors</th>
<th>Posttest Subtraction Correct Digit/Errors</th>
<th>Raw Point Improvement for Correct Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>12C/0E</td>
<td>26C/1E</td>
<td>14C</td>
</tr>
<tr>
<td>Participant 2</td>
<td>7C/20E</td>
<td>30C/0E</td>
<td>23C</td>
</tr>
<tr>
<td>Participant 3</td>
<td>0C/2E</td>
<td>9C/2E</td>
<td>9C</td>
</tr>
<tr>
<td>Participant 4</td>
<td>18C/0E</td>
<td>27C/1E</td>
<td>9C</td>
</tr>
<tr>
<td>Participant 5</td>
<td>8C/10E</td>
<td>15C/2E</td>
<td>7C</td>
</tr>
<tr>
<td>Participant 6</td>
<td>9C/0E</td>
<td>12C/1E</td>
<td>3C</td>
</tr>
</tbody>
</table>

Question 6: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems?

A total of five maintenance measures were used to answer this question. These measures consisted of Subtraction With Regrouping Test, Subtraction with Regrouping Minute, Subtraction Review Minute, Subtraction Interview, and Subtraction with Regrouping Word Problem Test.
The maintenance scores were compared to the *Subtraction with Regrouping Posttest* scores to determine whether the participants maintained their ability to solve subtraction with regrouping problems. The *Subtraction with Regrouping Maintenance Test* was given seven days after the *Subtraction with Regrouping Posttest* was given to each participant. The *Subtraction with Regrouping Posttest* and *Maintenance Test* consisted of 20 problems. Ten of the 20 problems had 2-digit minuends and 2-digit subtrahends. Of those 10 problems, 3 had zeros in the ones place value column within the minuend. The other 10 problems were made up of 3-digit minuends and 3 digit subtrahends. Three of those problems had zeros in the tens place value column within the minuends. Posttest and maintenance scores are reported as percent correct.

Posttest and maintenance scores for Participant 1 were 95% and 95% respectively. Posttest and maintenance scores for Participant 2 were 85% and 95% respectively. Posttest and maintenance scores for Participant 3 were 40% and 90% respectively. Posttest and maintenance scores for Participant 4 were 85% and 90% respectively. Posttest and maintenance scores for Participant 5 were 95% and 90% respectively. Posttest and maintenance scores for Participant 6 were 30% and 85% respectively. Five of the six participants demonstrated maintenance scores equivalent to or higher than their posttest scores after seven days of no instruction. The remaining participant decreased five percentage points after the week of no instruction, but still performed at skill mastery level (i.e., 80% or higher). See Table 7.
Table 7

*Participants’ Posttest and Maintenance Tests Percent Scores on the Subtraction With Regrouping Posttest and Maintenance Tests*

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttest Subtraction With Regrouping</th>
<th>Maintenance Subtraction With Regrouping Test</th>
<th>Skill Mastery Maintained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>95</td>
<td>95</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 2</td>
<td>85</td>
<td>95</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 3</td>
<td>40</td>
<td>90</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 4</td>
<td>85</td>
<td>90</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 5</td>
<td>95</td>
<td>90</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 6</td>
<td>30</td>
<td>85</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Second, the *Subtraction with Regrouping Minute Maintenance and Posttest* scores were compared to determine whether the participants maintained their fluency when solving subtraction with regrouping problems. The *Subtraction with Regrouping Minute Maintenance Test* was given seven days after *Subtraction with Regrouping Minute Posttest* was given to each participant. The Subtraction With Regrouping Minute consisted of 16 problems total. Eight of the sixteen problems had 2-digit minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-
digit subtrahends. Pre- and posttest raw scores are reported as total number correct digits and total number of incorrect digits obtained within one minute.

Posttest and maintenance scores for Participant 1 were 23 correct digits with zero errors and 24 correct digits with 4 errors respectively. Posttest and maintenance scores for Participant 2 were 31 correct digits with 3 errors and 21 correct digits with 1 error respectively. Posttest and maintenance scores for Participant 3 were 13 correct digits with 11 errors and 5 correct digits with 0 errors respectively. Posttest and maintenance scores for Participant 4 were 19 correct digits with 3 errors and 20 correct digits with 0 errors respectively. Posttest and maintenance scores for Participant 5 were 28 correct digits with 0 errors and 20 correct digits with 2 errors respectively. Posttest and maintenance scores for Participant 6 were 30 correct digits with 1 error and 20 correct digits with 4 errors respectively. Two of the six participants demonstrated maintenance scores equivalent to or higher than their posttest scores after seven days of no instruction. The remaining four participants decreased an average of nine correct digits after the week of no instruction. It should be noted that the average increase of correct digit among all participants on the Subtraction With Regrouping Minute Pretest to the Subtraction With Regrouping Minute Posttest was 18 more correct digits. Even with the four participants scoring fewer correct digits on the Subtraction With Regrouping Minute Maintenance Test, none of them scored near their original score on the pretest. See Table 8.
Table 8

Participants’ Posttest and Maintenance Test Scores on the Subtraction With Regrouping Minute Posttest and Maintenance Tests

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttest Subtraction With Regrouping Minute Correct Digit/Errors</th>
<th>Maintenance Subtraction With Regrouping Minute Correct Digit/Errors</th>
<th>Skill Maintained Within 10 Correct Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>23C/0E</td>
<td>24C/4E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 2</td>
<td>31C/3E</td>
<td>21C/1E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 3</td>
<td>13C/11E</td>
<td>5C/0E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 4</td>
<td>19C/3E</td>
<td>20C/0E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 5</td>
<td>28C/0E</td>
<td>20C/2E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 6</td>
<td>30C/1E</td>
<td>20C/4E</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Third, the Subtraction Review Minute Maintenance and Posttest scores were compared to determine whether the participants maintained their ability to discriminate between subtraction problems that require regrouping and those that do not. The Subtraction Review Minute Maintenance Test was given seven days after Subtraction Review Minute Posttest was given to each participant. The Subtraction Review Minute consisted of 16 problems total. Eight of the 16 problems had 2-digit
minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-digit subtrahends. Pre- and posttest raw scores are reported as total number correct digits and total number of incorrect digits obtained within one minute.

Posttest and maintenance scores for Participant 1 were 26 correct digits with one errors and 17 correct digits with five errors respectively. Posttest and maintenance scores for Participant 2 were 30 correct digits with zero errors and 22 correct digits with zero errors respectively. Posttest and maintenance scores for Participant 3 were 9 correct digits with two errors and 15 correct digits with four errors respectively. Posttest and maintenance scores for Participant 4 were 27 correct digits with one error and 18 correct digits with 1 error respectively. Posttest and maintenance scores for Participant 5 were 15 correct digits with two errors and 22 correct digits with zero errors respectively. Posttest and maintenance scores for Participant 6 were 12 correct digits with one error and 17 correct digits with two errors respectively. Three of the six participants demonstrated maintenance scores equivalent to or higher than their posttest scores after seven days of no instruction. The remaining three participants decreased an average of 8.6 correct digits after the week of no instruction. See Table 9.

Fourth, the Subtraction Interview Maintenance and Posttest scores were compared to determine whether the participants maintained their conceptual understanding of how to solve subtraction with regrouping problems. The Subtraction Interview Maintenance Test was given seven days after Subtraction Interview Posttest was given to each participant. The Subtraction Interview Posttest and Maintenance Test consisted of six subtraction with regrouping problems. The first three problems
required the student to show the interviewer how they would solve the subtraction problems using base ten blocks. The interviewer prompted the students to explain what they were doing with the manipulative devices as they solved the problems. Two of the first three problems were comprised of 3-digit minuends and 3-digit subtrahends. The last of the first three problems had 2-digit minuends and 2-digit subtrahends. Next, the interviewer asked the student to solve the last three problems requiring subtraction with regrouping without using any manipulative devices. The

Table 9

Participants’ Posttest and Maintenance Test Scores on the Subtraction Review

Minute Posttest and Maintenance Tests

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttest Subtraction Correct Digit/Errors</th>
<th>Maintenance Review Correct Digit/Errors</th>
<th>Skill Maintained Within 10 Correct Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>26C/1E</td>
<td>17C/5E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 2</td>
<td>30C/0E</td>
<td>22C/0E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 3</td>
<td>9C/2E</td>
<td>15C/4E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 4</td>
<td>27C/1E</td>
<td>18C/1E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 5</td>
<td>15C/2E</td>
<td>22C/0E</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 6</td>
<td>12C/1E</td>
<td>17C/2E</td>
<td>Yes</td>
</tr>
</tbody>
</table>
student was prompted to explain what he/she was doing as they solved these
problems. Again, two of the last three problems were comprised of 3-digit minuends
and 3-digit subtrahends while the last problem had a 2-digit minuend and a 2-digit
subtrahend.

While the student solved each problem included in the interview, the interviewer
scored his or her actions based on the 21 conditions listed on the subtraction interview
pretest scoring protocol (See Appendix F). For the first three problems, the student
was asked to use manipulative devices to explain how he or she solved the problems.
The scoring conditions were organized into four domains: (a) student represents first
number accurately, (b) student states need to regroup to subtract tens, (c) student
regroups accurately, and (d) student subtracts accurately. For the last three problems,
the student was asked to solve the problems without using manipulative devices and
to explain the steps he or she used to solve the problems. For these problems, the
scoring conditions were organized into three domains: (a) student states need to
regroup to subtract ones, (b) student regroups accurately, and (c) student subtracts
accurately. This interview was not timed.

Posttest and maintenance scores for Participant 1 were 100% and 100%
respectively. Posttest and maintenance scores for Participant 2 were 71% and 71%
respectively. Posttest and maintenance scores for Participant 3 were 66% and 90%
respectively. Posttest and maintenance scores for Participant 4 were 90% and 100%
respectively. Posttest and maintenance scores for Participant 5 were 100% and 100%
respectively. Posttest and maintenance scores for Participant 6 were 100% and 95%
respectively. Five of the six participants demonstrated maintenance scores equivalent
to or higher than their posttest scores after seven days of no instruction. The remaining participant decreased five percentage points after the week of no instruction, but still performed at skill mastery level (i.e., 80% or higher). See Table 10.

Fifth, the Subtraction with Regrouping Word Problem Maintenance and Posttest scores were compared to determine whether the participants maintained their ability to solve subtraction with regrouping word problems. The Subtraction with Regrouping Word Problem Maintenance Test was given seven days after Subtraction with Regrouping Word Problem Posttest was given to each participant. The Subtraction with Regrouping Word Problem Posttest and Maintenance Test consisted of 10 problems. Four of the 10 problems had 2-digit minuends and 2-digit subtrahends. The other 6 problems had 3-digit minuends and 3-digit subtrahends. One problem included extraneous information (i.e., word and numerical information not needed to solve the problem). Posttest and maintenance tests percentage scores were determined based on the total number of correct problems divided by the total possible (i.e., 10).

Posttest and maintenance scores for Participant 1 were 100% and 90% respectively. Posttest and maintenance scores for Participant 2 were 70% and 70% respectively. Posttest and maintenance scores for Participant 3 were 40% and 80% respectively. Posttest and maintenance scores for Participant 4 were 80% and 100% respectively. Posttest and maintenance scores for Participant 5 were 70% and 80% respectively. Posttest and maintenance scores for Participant 6 were 70% and 90%
respectively. Five of the six participants demonstrated maintenance scores equivalent to or higher than their posttest scores after seven days of no instruction. The remaining participant decreased 10 percentage points after the week of no instruction, but still performed at skill mastery level (i.e., 80% or higher). See Table 11.
Table 11

Participants’ Posttest and Maintenance Test Percent Scores on the Subtraction With Regrouping Word Problem Posttest and Maintenance Tests

<table>
<thead>
<tr>
<th>Participants</th>
<th>Posttest Subtraction With Regrouping Word Problem</th>
<th>Maintenance Subtraction With Regrouping Word Problem Test</th>
<th>Skill Mastery Maintained</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant 1</td>
<td>100%</td>
<td>90%</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 2</td>
<td>70%</td>
<td>70%</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 3</td>
<td>40%</td>
<td>80%</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 4</td>
<td>80%</td>
<td>100%</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 5</td>
<td>70%</td>
<td>80%</td>
<td>Yes</td>
</tr>
<tr>
<td>Participant 6</td>
<td>70%</td>
<td>90%</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Question 7: Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence report high, medium, or low levels of satisfaction related to the subtraction with regrouping intervention lessons?

The researcher-developed *Subtraction With Regrouping Satisfaction Questionnaire* was used to answer this research question. The participants completed
the Subtraction With Regrouping Satisfaction Questionnaire during the posttest phase of the study. The questionnaire included nine questions designed to measure the participant’s level of satisfaction with the subtraction with regrouping intervention lessons. The questionnaire included a five-point likert scale with 5 being least satisfied (i.e. Strongly Disagree) and 1 being most satisfied (i.e. Strongly Agree) (see Appendix K).

On Statement 1, participants rated their satisfaction level related to the statement: “Base ten blocks helped me with subtraction.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 2, participants rated their satisfaction level related to the statement: “Drawing helped me with subtraction.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 3, participants rated their satisfaction level related to the statement: “The RENAME strategy helped me with subtraction.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 4, participants rated their satisfaction level related to the statement “The BBB Phrase helped me know when to regroup.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 5, participants rated their satisfaction level related to the statement: “The Subtraction Minute helped me get faster at subtraction.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 6, participants rated their satisfaction level related to the statement “The Subtraction Review Minute helped me with regrouping.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 7, participants rated their satisfaction level related to the statement: “The FAST RENAME strategy helped me
with Word Problems.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 8, participants rated their satisfaction level related to the statement: “This program helped me get better at Subtraction.” All of the participants indicated that they “Strongly Agree” with that statement. On Statement 9, participants rated their satisfaction level related to the statement: “Overall, I liked this Subtraction Program.” Five of the participants indicated that they “Strongly Agree” with that statement while the sixth participant indicated that they “Agree” for that statement. See Table 12.

Interscorer Reliability

The learning disability teacher scored each student on the pre-, post-, and maintenance tests: (a) the Subtraction With Regrouping Test, the Subtraction With Regrouping Minute, the Subtraction Review Minute, the Subtraction Interview, and the Subtraction Word Problem Test. To determine interscorer reliability, the student investigator scored 20% of each of the pre-, post-, and maintenance tests. Twenty percent of each of the pre-, post-, and maintenance tests is equivalent to 2 of 6 tests in each set. The primary scorer was the learning disability teacher and the secondary scorer was the student investigator. An agreement was considered when both the student investigator and the learning disability teacher recorded the same score for an answer. If there were a disagreement in response, the item was marked as a disagreement. The formula agreements ÷ (agreements + disagreements) was used to determine reliability levels (Barlow & Hersen, 1982).
### Table 12

**Participants’ Ratings on the Subtraction With Regrouping Satisfaction Questionnaire**

<table>
<thead>
<tr>
<th>Questionnaire Statements</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Base ten blocks helped me with subtraction.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>2. Drawings helped me with subtraction.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>3. The RENAME strategy helped me with subtraction.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>4. The BBB Phrase helped me know when to regroup.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>5. The Subtraction Minute helped me get faster at subtraction.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>6. The Subtraction Review Minute helped me with regrouping.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>7. The FAST RENAME strategy helped me with Word Problems.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>8. This program helped me get better at Subtraction.</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>9. Overall, I liked this Subtraction Program.</td>
<td>1.0</td>
<td>2.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.2</td>
</tr>
</tbody>
</table>

*Note.* P1 = Participant 1; P2 = Participant 2; P3 = Participant 3; P4 = Participant 4; P5 = Participant 5; P6 = Participant 6; M = mean score for questionnaire statement.

On the *Subtraction With Regrouping Pretest*, the learning disability teacher and student investigator agreed on 40 out of 40 items for an overall total percentage agreement of 100%. On the *Subtraction With Regrouping Posttest*, the learning disability teacher and student investigator agreed on 40 out of 40 items for an overall
total percentage agreement of 100%. On the Subtraction With Regrouping
Maintenance Test, the learning disability teacher and student investigator agreed on
40 out of 40 items for an overall total percentage agreement of 100%.

On the Subtraction With Regrouping Minute Pretest, the learning disability
teacher and student investigator agreed on 32 out of 32 items for an overall total
percentage agreement of 100%. On the Subtraction With Regrouping Minute Posttest,
the learning disability teacher and student investigator agreed on 62 out of 62 items
for an overall total percentage agreement of 100%. On the Subtraction With
Regrouping Minute Maintenance Test, the learning disability teacher and student
investigator agreed on 30 out of 30 items for an overall total percentage agreement of
100%.

On the Subtraction Review Minute Pretest, the learning disability teacher and
student investigator agreed on 30 out of 30 items for an overall total percentage
agreement of 100%. On the Subtraction Review Minute Posttest, the learning
disability teacher and student investigator agreed on 47 out of 47 items for an overall
total percentage agreement of 100%. On the Subtraction Review Minute Maintenance
Test, the learning disability teacher and student investigator agreed on 38 out of 38
items for an overall total percentage agreement of 100%.

On the Subtraction Interview Pretest, the learning disability teacher and the
student investigator agreed on 42 out of 42 items for an overall total percentage
agreement of 100%. On the Subtraction Interview Posttest, the learning disability
teacher and the student investigator agreed on 42 out of 42 items for an overall total
percentage agreement of 100%. On the Subtraction Interview Maintenance Test, the
learning disability teacher and the student investigator agreed on 42 out of 42 items for an overall total percentage agreement of 100%.

On the *Subtraction With Regrouping Word Problem Pretest*, the learning disability teacher and student investigator agreed on 20 out of 20 items for an overall total percentage agreement of 100%. On the *Subtraction With Regrouping Word Problem Posttest*, the learning disability teacher and student investigator agreed on 20 out of 20 items for an overall total percentage agreement of 100%. On the *Subtraction With Regrouping Word Problem Maintenance Test*, the learning disability teacher and student investigator agreed on 20 out of 20 items for an overall total percentage agreement of 100%. See Tables 13, 14 and 15.

**Fidelity of Treatment**

To determine interobserver agreement related to fidelity of treatment, the principal investigator and student investigator completed a Fidelity of Treatment Checklist while observing the learning disability teacher deliver 20% of the subtraction with regrouping intervention lessons. Items on the Fidelity of Treatment Checklist were marked with a checkmark to indicate compliance with the scripted intervention lessons. The formula agreements ÷ (agreements + disagreements) was used to establish the fidelity of treatment level. Twenty percent of the 78 lessons taught is equivalent to 16 lessons. On the Fidelity of Treatment Checklists, the principal investigator and student investigator agreed on 166 out of 167 opportunities for an overall total percentage agreement of 99%. See Table 16.
Table 13

*Interscorer Reliability on Pretests*

<table>
<thead>
<tr>
<th>Test</th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction With Regrouping Test</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Minute</td>
<td>32</td>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Review Minute</td>
<td>30</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Interview</td>
<td>42</td>
<td>42</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Word Problem Test</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Overall</td>
<td>164</td>
<td>164</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 14

*Interscorer Reliability on Posttests*

<table>
<thead>
<tr>
<th></th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction With Regrouping Test</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Minute</td>
<td>62</td>
<td>62</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Review Minute</td>
<td>47</td>
<td>47</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Interview</td>
<td>42</td>
<td>42</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Word Problem Test</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Overall</td>
<td>211</td>
<td>211</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 15

*Interscorer Reliability on Maintenance*

<table>
<thead>
<tr>
<th></th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction With Regrouping Test</td>
<td>40</td>
<td>40</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Minute</td>
<td>30</td>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Review Minute</td>
<td>38</td>
<td>38</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction Interview</td>
<td>42</td>
<td>42</td>
<td>100</td>
</tr>
<tr>
<td>Subtraction With Regrouping Word Problem Test</td>
<td>20</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Overall</td>
<td>170</td>
<td>170</td>
<td>100</td>
</tr>
</tbody>
</table>
Table 16

*Fidelity of Treatment*

<table>
<thead>
<tr>
<th>Session Observed</th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
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<tr>
<td>3</td>
<td>12</td>
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<td>100</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td>12</td>
<td>100</td>
</tr>
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<td>14</td>
<td>8</td>
<td>8</td>
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</table>
Summary of Results

The purpose of this study was to investigate the effects of strategy instruction and the concrete-representational-abstract sequence to teach subtraction with regrouping to students with learning disabilities. Data were collected from pre-, post-, and maintenance tests (i.e. Subtraction With Regrouping Test, Subtraction with Regrouping Minute, Subtraction Review Minute, Subtraction Interview Test, and the Subtraction With Regrouping Word Problem Test). Additionally, curriculum-based assessments (i.e. Baseline Probes and Learning Sheet Probes) were analyzed.

A multiple probe design across subjects (Horner & Baer, 1978) with one replication was used in this study. By replicating the study results with an additional triad, (external validity), confidence is increased that the changes in mathematics performance were due to the intervention. Staggered introduction of the intervention using a multiple probe design helps determine intervention effectiveness for multiple subjects (Horner, Carr, Halle, McGee, Odom, & Wolery, 2005).

Table 16 (continued).

<table>
<thead>
<tr>
<th>Session Observed</th>
<th>Total Agreements</th>
<th>Total Agreements + Disagreements</th>
<th>Percentage Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>12</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>16</td>
<td>12</td>
<td>12</td>
<td>100</td>
</tr>
<tr>
<td>Overall</td>
<td>167</td>
<td>166</td>
<td>99</td>
</tr>
</tbody>
</table>
Although prior to intervention lessons, all students in the study failed to meet both district’s Curriculum Essential Framework’s Standards and the Power Standards for 5th grade related to subtraction with regrouping, 4 of the 6 participants had very limited prior understanding of how to subtract with regrouping based on scores obtained from their Subtraction With Regrouping Pretest, Subtraction Word Problem Pretest, and Subtraction Interview Pretest. Of the two participants who showed some prior understanding of how to subtract with regrouping, one still showed very limited conceptual understanding of subtraction with regrouping based on scores from their Subtraction Interview Pretest. Comparison of pre- and posttests revealed that participants’ performance increased on the subtest.

Comparison of pre- and posttest scores on the Subtraction With Regrouping Tests revealed growth for most participants. The average score of correct answers on the Subtraction With Regrouping Pretest was 6 correct answers. The average score of correct answers on the Subtraction With Regrouping Posttest was 14.3 (out of 20 problems total). This was an average increase of 8.3 correct answers on the Subtraction With Regrouping Posttest. Separating out the data from the four participants who scored zero on their pretest, the average gain of points after participation of intervention lessons took place was 12.5. Additionally, a comparison of posttest and maintenance tests on the Subtraction With Regrouping Tests revealed that all participants were able to maintain their newly acquired skill of how to subtract with regrouping within an acceptable mastery level (i.e., 80% or higher).

Data collection reflective of student fluency improvement with subtraction problems was derived from two measures: the Subtraction with Regrouping Minute
and the Subtraction Review Minute. The Subtraction With Regrouping Minute only contained subtraction problems that required regrouping to ascertain the correct answer. Comparison of pre-and posttest scores on the Subtraction With Regrouping Minute revealed substantial growth for most of the participants (i.e. an average of 21.6 correct digit increase from pretest scores to posttest scores). Only one participant, one of the two who demonstrated some prior understanding of how to subtract with regrouping, did not improve his/her fluency after the intervention lessons took place. It should be noted that this participant maintained his fluency level of 19 correct digits.

The second fluency measure, the Subtraction Review Minute, contained equal number of problems that required regrouping as problems that did not require regrouping. Comparison of pre- and posttest scores on the Subtraction Review Minute Tests revealed significant growth for all participants. The average gain of correct digits after the participation of intervention lessons took place was 10.8. Four of the six participants were able to improve their score from their Subtraction Review Minute Posttest to their Subtraction Review Maintenance Test. The other two participants were still able to achieve 17 and 18 correct digits on their Subtraction Review Maintenance Test. Despite these scores not being reflective of an improvement, they were still reflective of the students’ ability to maintain their skills over time. All participants scored an average of 18.5 total correct digits on the Subtraction Review Maintenance Test. This was more than double the average total correct digits achieved on the Subtraction Review PreTest, which was 9 total correct digits.
A comparison of pre- and posttest scores on the *Subtraction Word Problem Tests* revealed growth for all participants. The participants averaged 2.6 correct answers on the *Subtraction Word Problem Pretest* and 7.1 correct answers on the *Subtraction Word Problem Posttest* (out of 10 problems total). This was an average increase of 4.5 correct answers on the *Subtraction Word Problem Posttest*. Separating out the data from the four participants who scored zero on their pretest, the average gain of points after participation of intervention lessons took place was 6.25. Additionally, a comparison of posttest and maintenance tests on the *Subtraction Word Problem Tests* revealed that all participants were able to maintain their newly acquired skill of how to subtract with regrouping successfully because they scored just as well if not better on their maintenance tests.

A comparison of pre- and posttest scores on the *Subtraction Interview Tests* revealed growth for all participants. The participants averaged 5.6 total points on the *Subtraction Interview Pretest* and 18.5 total points on the *Subtraction Interview Posttest* (out of 21 points total). This was an average increase of 12.8 correct answers on the *Subtraction Word Problem Posttest*. The participant who demonstrated some prior knowledge as to how to procedurally solve subtraction problems that require regrouping on their *Subtraction With Regrouping Pretest*, was unable to demonstrate any prior conceptual knowledge related to subtraction with regrouping. Although this participant showed minimal growth in their ability to compute subtraction with regrouping problems, this same student showed significant growth in their conceptual understanding of subtraction with regrouping as evidenced when comparing their *Subtraction Interview Pretest* and *Posttest* scores, which were 8 and 19 respectively.
Additionally, a comparison of posttest and maintenance tests on the *Subtraction Interview Tests* revealed that all participants were able to maintain their newly acquired skill of subtracting with regrouping successfully since they all scored just as well if not better on their maintenance tests.

Visual analysis of the participants’ *Baseline Probes, Learning Sheets*, and *Maintenance Probe* was used to determine whether the effects of strategy instruction integrated with the concrete-representational-abstract teaching sequence increase students’ ability to solve subtraction with regrouping problems. Each participant’s performance was graphed according to the specifications of a multiple probe design (Horner & Baer, 1978). The level, trend and variability of performance data were visually inspected to determine the effectiveness of the intervention lessons. Level refers to the differences in performance from one condition (e.g. baseline) to another (e.g. intervention instruction). Trend refers to the direction of the ‘best-fit’ straight line of the dependent variable data points. Variability refers to the consistency of the data points around the mean.

All six participants in this study achieved a stable baseline prior to the intervention lessons. Also, analysis of the data indicated a level increase from the Baseline Probes to initial intervention lesson for all six participants. Additionally, five of the six participants’ data indicated no level change between the intervention condition and the maintenance condition of the study. The one participant who decreased in level from the intervention condition to the maintenance condition was still performing at a mastery level (i.e., 80%).
Visual analysis of data for Participant 1 revealed a flat trend throughout the intervention lessons with little variability. The average score Participant 1 achieved on the *Learning Sheet Probes* throughout the intervention condition was 94.8%. This participant was able to achieve a percentage score of 100% on 15 of the 25 total *Learning Sheet Probes* (60% of the *Learning Sheet Probes* had a score of 100%). Additionally, 7 of the 25 *Learning Sheet Probes* had a score of 90% (28% of the *Learning Sheet Probes* had a score of 90%) and the remaining 3 *Learning Sheet Probes* had a score of 80% (12% of the *Learning Sheet Probes* had a score of 80%). Overall, 100% of the intervention data for Participant 1 fell within the mastery criteria of achieving a minimum of 80% or better on *Learning Sheet Probes*.

Visual analysis of data for Participant 2 revealed a flat trend throughout the intervention lessons with notable variability. The average score Participant 2 achieved on the *Learning Sheet Probes* throughout the intervention condition was 90.7%. This participant was able to achieve a score of 100% on 14 of the 27 total *Learning Sheet Probes* (52% of the *Learning Sheet Probes* had a score of 100%). Additionally, 7 of the 27 *Learning Sheet Probes* had a score of 90% (26% of the *Learning Sheet Probes* had a score of 90%), 4 of the 27 *Learning Sheet Probes* had a score of 80% (15% of the *Learning Sheet Probes* had a score of 80%), 1 of the 27 *Learning Sheet Probes* had a score of 70% (4% of the *Learning Sheet Probes* had a score of 70%), and 1 of the 27 *Learning Sheet Probes* had a score of 30% (4% of the *Learning Sheet Probes* had a score of 30%). The variability within this participant’s data occurred as a result of the one *Learning Sheet Probe* that fell well below the average (The *Learning Sheet Probe* that received a score of 30%). Overall, 92% of the intervention data fell within
the mastery criteria of achieving a minimum of 80% or better on Learning Sheet Probes.

Visual analysis of data for Participant 3 revealed a flat trend throughout the intervention lessons with notable variability. The average score Participant 3 achieved on the Learning Sheet Probes throughout the intervention condition was 84.1%. This participant was able to achieve a raw score of 100% on 5 of the 29 total Learning Sheet Probes (17% of the Learning Sheet Probes had a score of 10). Additionally, 10 of the 29 Learning Sheet Probes had a score of 90% (34% of the Learning Sheet Probes had a score of 90%), 10 of the 29 Learning Sheet Probes had a score of 80% (34% of the Learning Sheet Probes had a score of 80%), 1 of the 29 Learning Sheet Probes had a score of 70% (3% of the Learning Sheet Probes had a score of 70%), 2 of the 29 Learning Sheet Probes had a score of 60% (7% of the Learning Sheet Probes had a score of 60%), and 1 of the 29 Learning Sheet Probes had a score of 50% (3% of the Learning Sheet Probes had a score of 50%). The variability within this participant’s data occurred as a result of three Learning Sheet Probes that fell well below the average (the two Learning Sheet Probes that received a score of 60% and the one that received a score of 50%). Overall, 86% of the intervention data for Participant 3 fell within the mastery criteria of achieving a minimum of 80% or better on Learning Sheet Probes.

Visual analysis of data for Participant 4 revealed a flat trend throughout the intervention lesson with notable variability. The average score Participant 4 achieved on the Learning Sheet Probes throughout the intervention condition was 89.2%. This participant was able to achieve a raw score of 100% on 11 of the 28 total Learning
Sheet Probes (39% of the Learning Sheet Probes had a score of 100%). Additionally, 8 of the 28 Learning Sheet Probes had a score of 90% (29% of the Learning Sheet Probes had a score of 90%), 6 of the 28 Learning Sheet Probes had a score of 80% (21% of the Learning Sheet Probes had a score of 80%), 2 of the 28 Learning Sheet Probes had a score of 70% (7% of the Learning Sheet Probes had a score of 70%), and 1 of the 28 Learning Sheet Probes had a score of 60% (4% of the Learning Sheet Probes had a score of 60%). The variability within this participant’s data occurred as a result of three Learning Sheet Probes that fell well below the average (the two Learning Sheet Probes that received a score of 70% and the one that received a score of 60%). Overall, 89% of the intervention data for Participant 4 fell within the mastery criteria of achieving a minimum of 80% or better on Learning Sheet Probe.

Visual analysis of data for Participant 5 revealed a flat trend throughout the intervention lessons with notable variability. The average score Participant 5 achieved on the Learning Sheet Probes throughout the intervention condition was 92.5%. This participant was able to achieve a raw score of 100% on 16 of the 28 total Learning Sheet Probes (57% of the Learning Sheet Probes had a score of 100%). Additionally, 7 of the 28 Learning Sheet Probes had a score of 90% (25% of the Learning Sheet Probes had a score of 90%), 3 of the 28 Learning Sheet Probes had a score of 80% (11% of the Learning Sheet Probes had a score of 80%), 1 of the 28 Learning Sheet Probes had a score of 70% (4% of the Learning Sheet Probes had a score of 70%), and 1 of the 28 Learning Sheet Probes had a score of 50% (4% of the Learning Sheet Probes had a score of 50%). The variability within this participant’s data occurred as a result of two Learning Sheet Probes that fell well below the average (the two
Learning Sheet Probes that received a score of 70% and 50%). Overall, 93% of intervention data for Participant 5 fell within the mastery criteria of achieving a minimum of 80% or better on Learning Sheet Probe.

Visual analysis of data for Participant 6 revealed a flat trend throughout the intervention lessons with notable variability. The average score Participant 6 achieved on the Learning Sheet Probes throughout the intervention condition was 80.9%. This participant was able to achieve a raw score of 100% on 4 of the 31 total Learning Sheet Probes (12% of the Learning Sheet Probes had a score of 100%). Additionally, 7 of the 31 Learning Sheet Probes had a score of 90% (22% of the Learning Sheet Probes had a score of 90%), 14 of the 31 Learning Sheet Probes had a score of 80% (45% of the Learning Sheet Probes had a score of 80%), 4 of the 31 Learning Sheet Probes had a score of 70% (12% of the Learning Sheet Probes had a score of 70%), 1 of the 31 Learning Sheet Probes had a score of 50% (3% of the Learning Sheet Probes had a score of 50%), and 1 of the 31 Learning Sheet Probes had a score of 30% (3% of the Learning Sheet Probes had a score of 30%). The variability within this participant’s data occurred as a result of the six Learning Sheet Probes that fell well below the average (the four Learning Sheet Probes that received a score of 70% and the other two Learning Sheet Probes that received scores of 50% and 30%). Overall, 80% of the intervention data for Participant 6 fell within the mastery criteria of achieving a minimum of 80% or better on Learning Sheet Probe.

In addition to increasing their ability to solve subtraction with regrouping problems by participating in strategy instruction integrated with the concrete-representational-abstract teaching sequence, participant satisfaction related to this
intervention instruction was high. With the exception of one participant and one question, every participant ranked the statements on the *Subtraction With Regrouping Satisfaction Questionnaire* as a statement they “Strongly Agree” with. Even the exception ranked the statement “Overall, I liked this Subtraction Program.” as a statement that they “Agree” with. This indicates that students enjoyed utilizing the base ten blocks, the drawings, the RENAME strategy, the BBB phrase, the Subtraction Minute, the Subtraction review Minute, and the FAST RENAME strategy. Finally, all the participants believed this program helped them learn how to subtract with regrouping and they indicated that they liked this program.

In conclusion, all participants’ demonstrated an improvement in their ability to subtract with regrouping. All participants made performance growth as indicated by the difference between their pretest scores to their posttest scores. There was statistically significant improvement from pre to post on both the *Subtraction Word Problem* pre- and posttests and the *Subtraction Interview* pre- and posttests. Visual analysis of the multiple probe graph revealed an increase in level between baseline and intervention instruction for all participants. Trends were stable with notable variability for most of the participants. The graphed data revealed an improvement on the dependent variable (i.e. solve subtraction with regrouping problems) with the introduction of the independent variable (i.e., the intervention lessons). Finally, mastery level performance was maintained by all participants as evidenced in the graphed data in the maintenance condition.
CHAPTER 5
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The field of mathematics education has experienced numerous changes over the 20th and 21st centuries. The center of these changes focused on legislative and curricular reform with the aim of strengthening mathematic performance. Currently, these legislative and curricular reforms (i.e. the NCTM’s Principle and Standards for School Mathematics and Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence, No Child Left Behind Act, NMAP’s The Foundations for Success: The Final Report of National Mathematics Advisory Panel) have not effectively enforced high quality curriculum. A clear disconnect between what researchers suggest as good teaching of skills and what is practiced within classroom today exists as evidenced by the below average achievement in mathematics within our nation (Stein et. al., 2006).

Evidence-based math interventions for students are of national interest to ensure a workforce prepared to compete internationally (NMAP, 2008). Although math deficiencies among American youth have reached an incidence level equal to reading deficits, they are not as effectively addressed in classrooms (Bryant & Bryant, 2008; Jordan & Hanich, 2003; Mabbott & Bisanz, 2008). Specifically, students with learning disabilities have a need for specialized interventions that can neutralize the unique characteristics they share which magnify their learning challenges (Goldman, 1989; Keeler & Swanson, 2001; Kroesbergen & Van Luit, 2003; Maccini & Hughes, 1997; Mercer, 1997; Miles & Forcht, 1995). Verification of the negative impact these characteristics have on this sub group of students was reported in the 2007 National
Assessment of Educational Progress (NAEP). The 2007 NAEP reported that students with disabilities in the fourth- and eighth-grade grades performed significantly behind their peers (Bryant et al., 2008; Lee, Griggs, & Dion, 2007; NAEP, 2007).

Of particular concern are the individual skills frequently associated with math learning disabilities. Among Bryant and Bryant’s (2008) top ranked mathematics difficulties are problem solving, multi-step problems, verifying answers, recalling number facts, and borrowing/renaming errors. Authorities agree that deficits in mathematical reasoning can have a debilitating effect on an individual’s quality of life (Chard et al., 2008; Kroesbergen & Van Luit, 2003; Maccini & Hughes, 1997; McLeod & Armstrong, 1982; Riccomini, 2005; Van de Rijt & Van Luit, 1998; U.S. Department of Education, 2008).

Although researchers and educators have attempted to address the poor math performance through a variety of interventions, most of the intervention studies to date address basic math fact recall, basic computation skills, and problem solving (i.e. word problems) (Garnett, 1992; Miller, Strawser, Mercer, 1996; Montague & Brooks, 1993; Montague, 2008). There was a paucity of research that addressed regrouping skills and more advanced computation skills. Reviews of mathematics literature (Maccini & Ruhl, 2000; Mercer & Miller, 1992) reveal that the concrete-representational-abstract teaching sequence and strategy instruction, among other interventions, are effective for teaching initial single digit addition and subtraction skills. Despite the initial investigations, additional research was needed related to higher-level computation skills.
The purpose of this study was to investigate the effects of strategy instruction and the concrete-representational-abstract sequence to teach subtraction with regrouping to students with learning disabilities. Seven research questions were answered to address the purpose of this study. Seven types of assessment (i.e., curriculum-based assessments, fluency measures, ongoing performance monitoring probes, interviews, and a satisfaction questionnaire) were used to answer the seven research questions. The first two curriculum-based assessments used as pre-, post-, and maintenance measures were the *Subtraction With Regrouping Tests* and the *Subtraction With Regrouping Word Problem Tests*. The two fluency measures that were used as pre-, post-, and maintenance measures were the *Subtraction Review Minute* and the *Subtraction With Regrouping Minute*. Another type of assessment utilized was the *Baseline Probes, Learning Sheet Probes, and Maintenance Probes*. These were used to monitor student performance throughout the study. The *Subtraction Interview* pre-, post-, and maintenance assessments were used to review student’s conceptual growth. Finally, a satisfaction questionnaire was administered at the conclusion of the study to assess student satisfaction related to subtraction with regrouping intervention lessons.

This chapter includes the following six sections: (a) discussion of findings related to the research questions, (b) informal observations related to implementing the subtraction with regrouping intervention lessons, (c) conclusions and related discussion, (d) practical implications, and (f) recommendations for future research.
Discussion of Findings Related to Research Questions

The seven research questions used to guide the design and implementation of this study are presented in this section of the chapter. The findings for each question are reviewed followed by discussion.

*Question 1*

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems?

Two data sets were used to answer this question. The first set of data was obtained from the *Subtraction With Regrouping Pretest* and *Posttest*. The second set of data was obtained from the *Learning Sheet Probes* completed at the end of each intervention lesson.

The *Subtraction With Regrouping Pretest* and *Posttest* consisted of 20 subtraction with regrouping problems. Results from this first set of data reveal that five of the six participants made gains on the *Subtraction With Regrouping Test* from pre- to posttest.

Participants 1 and 4 showed the least amount of growth when comparing scores from pretest to posttest. One plausible explanation of this could be because these two students scored above a proficient level (above 80% accuracy) during the *Subtraction With Regrouping Pretest*, leaving very little room for growth to occur. Although these two participants were able to solve subtraction with regrouping accurately, they both showed no conceptual understanding as evidenced in their *Subtraction Interview Pretest*. 
Participants 2, 3, 5, and 6 all scored zero correct during pretesting. These students all made various errors ranging from their inability to identify a need for regrouping to regrouping inaccurately. Participants 2 and 5 demonstrated substantial increases on their *Subtraction With Regrouping Posttests*; whereas, Participants 3 and 6 made minimal gains on the same posttest. One plausible explanation is that these students were not experiencing a typical day the day the *Subtraction With Regrouping Posttest* was administered. Their general education teacher was preparing to move her current classroom into another classroom on the school campus. As a result, she was showing a movie in her room for the entire afternoon. Because these two participants usually go to their special education math class in the afternoon, they were very reluctant to attend to their special math class to take the *Subtraction With Regrouping Posttest* instead of watching the movie with their peers. Once in the testing setting, it seemed that they both rushed through the assessment as it took them less than half the time to complete it compared to previous days. Participants 3 and 6 scored below mastery level on their *Subtraction With Regrouping Posttest* as a result. Although their scores revealed progress from their pretest performances, these gains were not optimal. The scores for Participants 3 and 6 on the *Subtraction With Regrouping Maintenance Test*, taken seven days after the intervention condition concluded, further substantiate the previously mentioned plausible explanation. The scores on the *Subtraction With Regrouping Maintenance Test* for Participant 3 and 6 were both at mastery levels. This substantiates that these participants clearly gained more skills than was displayed on their posttests.
Descriptive statistical analysis revealed no significant improvement from the *Subtraction With Regrouping Pretest* to *Subtraction Posttest*. Perhaps this small sample size and high pretest performance of Participants 1 and 4 contributed to this finding. There was a large effect size which indicates that the magnitude of the intervention lesson’s effect in practice was great.

The second set of data was obtained from the *Learning Sheet Probes* completed at the end of each intervention lesson. A multiple probe design was used to assess the effects of strategy instruction integrated with the concrete-representational-abstract teaching sequence on students’ ability to solve subtraction with regrouping problems.

All six participants were able to reach and maintain mastery performance (80% or better on the 25 *Learning Sheet Probes*) throughout the intervention lessons. The number of trials needed to reach mastery differed across participants. Participant 1 was able to achieve mastery criteria within 25 sessions. Participant 2 was able to achieve mastery criteria within 27 sessions having to repeat two sessions. The first repeated session the participant made 3 identical errors where he did not regroup across zeros correctly. The second repeated session for Participant 2 was impacted because it occurred on a day when the student was going to be participating in a homework party taking place directly after his math class. The participant seemed very anxious during the initial delivery of the lesson because he was going to be missing the beginning of his homework party by being in his special education math class. The second time he was exposed to the same lesson, he perceived it as being under more desirable conditions resulting in his obtaining a 100%.
Participant 3 was able to achieve mastery criteria within 29 sessions having to repeat four sessions. Participant 3 had the fewest amount of academic skills when compared to the other participants as indicated on his achievement tests (see Table 1). Additionally, his disability was negatively affected by his English language learner deficits. The lessons Participant 3 had to repeat due to earning a score of less than 80% were reflective of his need, at times, to have more opportunities to process a newly learned skill. These needs align with what current researchers suggest as to how students with learning disabilities often benefit from participating in instruction where the teacher supports student learning by providing prompts throughout multiple practice opportunities until they are able to complete the task accurately and without teacher assistance (Hudson & Miller, 2006). Participant 4 and 5 were both able to achieve mastery criteria within 28 sessions. They each had to repeat three sessions. These two participants relied heavily on the use of their fingers or drawing tallies when subtracting because they did not have strong declarative knowledge of basic subtraction facts. Each of these participants could have made computational errors as a result. Participant 4 was usually more accurate than Participant 5 when drawing tallies with the exception of session 14 when he had to repeat that same lesson twice. In this particular lesson, Participant 4 seemed highly motivated to complete his Learning Sheet Probe before the other student who was in his group. On this particular day, he was unable to complete his Learning Sheet Probe before the other student in his group and this made him particularly agitated. This agitation was carried over to the next time the participant was exposed to the same lesson because he continued to feel victimized by his inability to finish the lesson before the other
student. In order to encourage the participant to regain focus, the learning disability
teacher pointed out to Participant 4 that he was not going to be able to keep up with
the other student until he reached mastery criteria of 80% or better on that particular
lesson. It was then that Participant 4 was able to let go of the agitation and complete
the *Learning Sheet Probe* under his normal mental conditions easily achieving 100%
on the third exposure to that *Learning Sheet Probe*.

Having to repeat six sessions, participant 6 was able to achieve mastery criteria
within 31 sessions having to repeat six sessions. This participant took the greatest
number of sessions to reach mastery criteria. One plausible explanation for this could
be that she was habitually absent (i.e., 6 days) during initial instruction of the lessons.
The six sessions she had repeated all fell on days after she had been absent. The
initial exposures to these lessons were, therefore, used as a review of previously
mastered skills since she was not able to practice these skills on sequential days like
the other participants of these lessons.

In conclusion, data analyses for Question 1 indicated that strategy instruction
integrated with the concrete-representational-abstract teaching sequence did improve
students with learning disabilities’ ability to solve subtraction with regrouping
problems. The number of sessions needed to achieve mastery criteria of the
intervention lessons ranged from 25 to 31 sessions. Individual variables unique to
each participant could not be controlled and seemed to impact the number of sessions
each participant needed more than the content of the lessons themselves. In spite of
these variables, all participants demonstrated substantial gains on their *Subtraction
with Regrouping Maintenance Tests* compared to their baseline performance.
The findings in this study concur with the findings of other researchers who found that strategy instruction integrated with the concrete-representational-abstract teaching sequence can improve students’ mathematical ability (Maccini & Ruhl, 2000; Mercer & Miller, 1992). Studies have involved the use of the concrete-representational-abstract teaching sequence for a variety of mathematical concepts including place value involving tens and ones, fraction equivalence, basic facts, and algebraic math word problems (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Harris, Miller, & Mercer, 1995; Hudson, Peterson, Mercer, & McLeod, 1988; Mercer & Miller, 1992; Miller, Harris, Strawser, Jones, & Mercer, 1998; Miller, Manccini & Ruhl, 2000; Peterson, Mercer, & O’Shea, 1988;). The current study extends the current literature in that strategy instruction integrated with the concrete-representational-abstract teaching sequence was examined to determine the effect it had on the students’ ability to subtract with regrouping.

**Question 2**

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems?

One data set was used to answer this question. The data set was obtained from the *Subtraction Word Problem Pretest* and *Posttest*.

The pre- and posttest raw scores for the *Subtraction Word Problem Pretest* and *Posttest* were compared. The *Subtraction Word Problem Pretest* and *Posttest* consisted of 10 problems total. Results from this data set reveal that all six of the participants made gains on the *Subtraction Word Problem Test* from pre- to posttest.
Participants 1 and 4 made the least amount of growth when comparing scores from pretest to posttest. One plausible explanation for this could be because these two students scored higher on the pretest leaving very little room for growth to occur. Again, although these two participants were able to solve subtraction with regrouping word problems relatively accurately, they both showed no conceptual understanding as evidenced on their Subtraction Interview Pretest.

Participants 2, 3, 5, and 6 all scored zero correct during pretesting. These students all made various errors ranging from their ability to identify a need for regrouping, regrouping accurately, and identifying the correct information within a word problem to accurately solve the problem. Participants 2, 3 and 5 were able to raise their scores substantially on the Subtraction Word Problem Posttest; whereas, Participant 3 made minimal gains. The plausible explanation for this concurs with the explanation stated earlier for why it also seemed like this participant made minimal gains when looking at their Subtraction With Regrouping Pretest and Posttest scores. The day the posttest was being administered was atypical because his general education teacher was showing a movie in her class in the afternoon during this student’s special education math class time. He was very reluctant to attend his math class because he did not want to miss parts of the movie. Consequently, he rushed through the posttest and failed to reach mastery. Although he still demonstrated progress from his performance on the pretest, this gain was not reflective of his true ability. The score for Participant 3 on the Subtraction Word Problem Maintenance Test, taken seven days after the intervention condition concluded, revealed skill mastery. This further substantiates that Participant 3 clearly gained more skills than evident on his posttest.
In conclusion, data analyses for Question 2 indicated that strategy instruction integrated with the concrete-representational-abstract teaching sequence did improve students with learning disabilities' ability to solve subtraction with regrouping word problems. The level of progress made from pre- to posttest varied among the participants.

Additionally, descriptive statistical analysis conducted indicated that the difference obtained in pre- and posttest scores on the *Subtraction With Regrouping Word Problem* tests was unlikely to occur by chance. Likewise, there was a large effect size which indicates that the magnitude of the intervention lesson’s effect in practice was great.

The findings in this study concur with the findings of other researchers who found that strategy instruction could improve students’ ability to solve mathematical word problems. Kroesbergen and Van Luit (2003) reviewed many of the successful practices used to teach students with learning challenges how to problem solve (e.g. procedural strategies and schema-based instruction). The current study extends the current literature in that it examined how strategy instruction integrated with the concrete-representational-abstract teaching sequence affected students with learning disabilities’ ability to solve subtraction with regrouping word problems.

*Question 3*

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems?
One data set was used to answer this question. The data were obtained from the
*Subtraction With Regrouping Minute Pretest* and *Posttest*.

The pre- and posttest raw scores for the *Subtraction With Regrouping Minute Pretest* and *Posttest* were compared. The *Subtraction With Regrouping Minute Pretest* consisted of 16 problems total. Results from this data set reveal that five of the six of the participants made substantial gains on the *Subtraction With Regrouping Minute* from pre- to posttest.

Participant 4, however, scored exactly the same amount of correct digits on the posttest as he did on the pretest. There are two plausible reasons for why this occurred. First, Participant 4 was one of the participants who consistently showed some prior procedural knowledge related to subtracting with regrouping (i.e., high scores on the *Subtraction With Regrouping Pretest* and the *Subtraction With Regrouping Word Problem Pretest*). It is, therefore, logical that Participant 4 would also score high on the *Subtraction With Regrouping Minute Pretest*. Secondly, his high initial score on the *Subtraction With Regrouping Minute Pretest* left very little room for growth. Scores on computation minute timings are somewhat limited based on how quickly the student is able to write numbers. This participant’s writing ability rate may have limited his ability to increase his written computation rate.

In conclusion, data analyses for Question 3 indicated that strategy instruction integrated with the concrete-representational-abstract teaching sequence did increase fluency related to solving subtraction with regrouping problems for five of the six participants. The remaining participant had high fluency performance for both the pre- and posttest.
The findings in this study concur with the findings of other researchers who found that strategy instruction integrated with the concrete-representational-abstract teaching sequence could increase students’ fluency related to solving subtraction with regrouping problems (Harris, Miller, & Mercer, 1995; Mercer & Miller, 1992; Miller, Harris, Strawser, Jones, & Mercer, 1998). The current study extends the current literature in that it examined how strategy instruction integrated with the concrete-representational-abstract teaching sequence affected students with learning disabilities fluency related to subtraction with regrouping.

**Question 4**

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems?

One data set was used to answer this question. The data were obtained from the *Subtraction Interview Pretest* and *Posttest*.

The pre- and posttest percentage scores for the *Subtraction Interview Pretest* and *Posttest* were compared. The *Subtraction Interview Test* consisted of 6 subtraction with regrouping problems total. The participants were asked to explain what they were doing as they went through the procedural steps of how to solve these types of problems. While the student solved each problem, the interviewer scored his or her actions based on the 21 conditions listed on the subtraction interview pretest scoring protocol (See Appendix F). Results from this data set reveal that all of the participants made gains on the *Subtraction Interview* from pre- to posttest.
Participant 1, however, made the least amount of gain, the three point errors he made on the pretest all included his inability to regroup correctly using manipulative devices. This clearly indicates that the participant was really relying on his procedural knowledge, with no conceptual understanding of what regrouping represents. Current researchers suggests that, without a strong conceptual understanding of mathematical skills, students may develop mathematical misconceptions which, in turn, negatively impact the students’ ability to acquire higher order mathematic skills (Calhoon, Emerson, Flores, & Houchins, 2007). Therefore, although Participant 1 only missed three points on the Subtraction Interview Pretest, these three points included pivotal concepts that needed remediation in order to facilitate future success with more complex mathematics. Participant 1 was able to attain a perfect score on the Subtraction Interview Posttest, which is indicative that the vital growth that was needed was attained.

The scores obtained by Participants 2 – 6 all suggested that these students didn’t have a strong conceptual understanding of the meaning of regrouping meant as indicated in their Subtraction Interview Pretest. Combined, the average Subtraction Interview Pretest score for participants 2 – 6 was 15%. The average score for Participants 2 through 6 was 70%.

In conclusion, data analyses for Question 4 indicated that strategy instruction integrated with the concrete-representational-abstract teaching sequence did increase the participants’ conceptual understanding related to solving subtraction with regrouping problems. Each of the participants was able to score at mastery levels on the Subtraction Interview Posttest. It should be noted that none of the missed points
from the participants’ posttests came from them misusing the manipulative devices. This further substantiates that each of the participants made significant conceptual gains.

Descriptive statistical analysis revealed significant improvement from the Subtraction Interview Pretest to the Subtraction Interview Posttest, which means the difference in the two sets of scores was unlikely to occur by chance. Also, there was a large effect size which indicates that the magnitude of the intervention lesson’s effect in practice was great.

The findings in this study concur with the findings of other researchers who found that strategy instruction integrated with the concrete-representational-abstract teaching sequence increased students’ conceptual understanding related to solving a variety of mathematical concepts including place value involving tens and ones, fraction equivalence, basic facts, and algebraic math word problems (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Harris, Miller, & Mercer, 1995; Hudson, Peterson, Mercer, & McLeod, 1988; Mercer & Miller, 1992; Miller, Harris, Strawser, Jones, & Mercer, 1998; Miller, Manccini & Ruhl, 2000; Peterson, Mercer, & O’Shea, 1988;). The current study extends these finding to subtraction with regrouping skills.

Question 5

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping?

One data set was used to answer this question. The data were obtained from the Subtraction Review Minute Pretest and Posttest.
The pre- and posttest raw scores for the *Subtraction Review Minute Pretest* and *Posttest* were compared. The *Subtraction Review Minute* consisted of 16 problems total. Four of the 16 problems did not require any regrouping skills to be applied. Results from this data set reveal that all of the six participants made gains on the *Subtraction Review Minute* from pretest to posttest.

Participants 1 and 2 made the greatest gain as evidenced by the scores on the *Subtraction Review Minute Posttest*. Participants 3, 4, and 5 made a reasonable gains as evidenced by their scores on the same measures. Participant 6 demonstrated the least gain on the *Subtraction Review Minute Posttest*. This performance was consistent with her other posttests completed on the same day. This evidence continues to build the case that this participant might not have been focused on the posttests due to her desire to go back to her other class for the movie.

In conclusion, data analysis for Question 5 indicated that strategy instruction integrated with the concrete-representational-abstract teaching sequence did successfully increase students’ ability to discriminate between subtraction problems that do and do not require regrouping. There is, however, the possibility of more preferred activities interfering with student performance.

The findings in this study concur with the findings of other researchers who found that students with learning challenges need to be taught how to successfully discriminate between similarities and differences that may exist between mathematical concepts (Engleman & Carnine, 1982; Kame’enui & Simmons, 1990) Hudson & Miller (2006) suggest that teaching using explicit instruction and the concrete-representational-abstract teaching sequence can increase the likelihood that
students with learning challenges will recognize the sameness among mathematical concepts. The current study extends the literature in that it involves discrimination skills specifically related to subtraction with regrouping.

*Question 6*

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems?

Five measures were used to answer this question. These measures were four curriculum-based assessments and one interview. The four curriculum-based assessments were the *Subtraction With Regrouping Posttest*, the *Subtraction With Regrouping Minute*, the *Subtraction Review Minute* and the *Subtraction Word Problem Posttest*. The interview was the *Subtraction Interview Posttest*. Maintenance scores were compared to posttest scores.

The *Subtraction With Regrouping Posttest* and *Maintenance Test* consisted of 20 subtraction with regrouping problems. The *Subtraction With Regrouping Maintenance* test was administered seven days after the intervention condition ended. Results from this data set reveal that all six participants maintained their skill growth over time.

Participants 1, 2, 4, and 5 all scored within 10 percentage points between posttest scores and maintenance tests scores. Participants 3 and 6 actually showed marked improvement between their *Subtraction With Regrouping Posttest* and *Maintenance Test*. It is assumed that these two participants’ scores on their *Subtraction With Regrouping Maintenance Tests* were more reflective of the gains they actually made.
throughout the instructional process due the underlying circumstances of the events that occurred on their posttest day (i.e. movie being played in their general education classroom).

The *Subtraction With Regrouping Minute Posttest and Maintenance* tests consisted of 16 problems total. Eight of the 16 problems had 2-digit minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-digit subtrahends. Pre- and posttest raw scores are reported as total number correct digits and total number of incorrect digits obtained within one minute.

Participants 1 and 4 each scored higher on their *Subtraction With Regrouping Minute Maintenance Test* than on their *Posttest*. Participants 2, 3, 5, and 6 demonstrated a decrease in performance, but scored well above their pretest measures. It is possible that these students needed daily fluency practice to maintain their posttest level performance. Additionally, it should, again, be noted that Participants 3 and 6 were unhappy about missing a movie in the general education class. This may have influenced their maintenance performance.

*The Subtraction Review Minute Posttest and Maintenance Test* consisted of 16 problems total. Eight of the 16 problems had 2-digit minuends and 2-digit subtrahends. The other 8 problems had 3-digit minuends and 3-digit subtrahends. Pre- and posttest raw scores are reported as total number correct digits and total number of incorrect digits obtained within one minute.

Participants 3, 5, and 6 all scored higher on the *Subtraction Review Maintenance Test* than they did on the posttest measure. Participants 1, 2, and 4 all scored lower. It is interesting to note that Participants 1 and 4 each attained more correct digits on the
Subtraction With Regrouping Minute Maintenance Test but were unable to outperform their posttest scores on the Subtraction Review Minute Maintenance Test. While both were fluency measures, it can be concluded that these two participants were able to maintain their ability to solve subtraction with regrouping problems fluently to a higher degree than they were able to maintain their ability to discriminate between subtraction problems that did and did not require regrouping.

The Subtraction Interview Posttest and Maintenance Test consisted of six subtraction with regrouping problems. The first three problems required the student to show the interviewer how they would solve the subtraction problems using base ten blocks. The interviewer prompted the students to explain what they were doing with the manipulative devices as they solved the problems. Two of the first three problems were comprised of 3-digit minuends and 3-digit subtrahends. The last of the first three problems had 2-digit minuends and 2-digit subtrahends. Next, the interviewer asked the student to solve the last three problems requiring subtraction with regrouping without using any manipulative devices. The student was prompted to explain what he/she was doing as they solved these problems. Again, two of the last three problems were comprised of 3-digit minuends and 3-digit subtrahends while the last problem had a 2-digit minuend and a 2-digit subtrahend.

While the student solved each problem included in the interview, the interviewer scored his or her actions based on the 21 conditions listed on the subtraction interview pretest scoring protocol (See Appendix F). For the first three problems, the student was asked to use manipulative devices to explain how he or she solved the problems. The scoring conditions were organized into four domains: (a) student represents first
number accurately, (b) student states need to regroup to subtract tens, (c) student regroups accurately, and (d) student subtracts accurately. For the last three problems, the student was asked to solve the problems without using manipulative devices and to explain the steps he or she used to solve the problems. For these problems, the scoring conditions were organized into three domains: (a) student states need to regroup to subtract ones, (b) student regroups accurately, and (c) student subtracts accurately. This interview was not timed.

Five of the participants were able to score equal to or higher than their posttest scores on the Subtraction Interview Maintenance Test. Even the participant who scored lower on their maintenance test than their posttest scored well above 80% (i.e. mastery criteria). These results unquestionably validate that using explicit instruction integrated with the concrete-representational-abstract teaching sequence successfully teaches students with learning disabilities how to conceptualize important mathematical skills and maintain this skill over time.

The Subtraction with Regrouping Word Problem Posttest and Maintenance Test consisted of 10 problems. Four of the ten problems had 2-digit minuends and 2-digit subtrahends. The other six problems had 3-digit minuends and 3-digit subtrahends. One problem included extraneous information (i.e., word and numerical information not needed to solve the problem). Posttest and maintenance tests percentage scores were determined based on the total number of correct problems divided by the total possible (i.e., 10).

Five of the participants were able to score equal to or higher than their posttest scores on the Subtraction With Regrouping Word Problem Maintenance Test. Even
the participant who scored 10 percentage points lower on his/her maintenance test than their posttest scored well above 80% (i.e. mastery criteria). It should also be noted that this participant made a computational error on the maintenance measure and not a conceptual or procedural error. Again, these results unquestionably validate that using explicit instruction integrated with the concrete-representational-abstract teaching sequence enables students with learning disabilities to solve subtraction with regrouping word problems and maintain this skill over time. In conclusion, data analyses for Question 6 indicated that strategy instruction integrated with the concrete-representational-abstract teaching sequence did result in skill maintenance related to students’ ability to solve subtraction with regrouping problems.

The findings in this study concur with the findings of other researchers who found that strategy instruction integrated with the concrete-representative-abstract teaching sequence could promote maintenance of a variety of mathematical skills (i.e. place value of ten and ones, addition facts 0 – 9, addition facts 10 – 18, subtraction facts 0 – 9, subtraction facts 10 – 18, multiplication facts 0 – 81, and division facts 0 – 81) when paired with effective memory devices (Mercer & Miller, 1991 – 1994). The current study extends the literature related to solving word problems to a new skill area (i.e., subtraction with regrouping problems).

**Question 7**

Do students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence report high, medium, or low levels of satisfaction related to the subtraction with regrouping intervention lessons?
The researcher developed the *Subtraction With Regrouping Satisfaction Questionnaire* to determine participant satisfaction with receiving strategy instruction integrated with the concrete-representational-abstract teaching sequence. The *Subtraction With Regrouping Questionnaire* was answered by the participants within their learning disability classroom at the conclusion of the posttests. The *Subtraction With Regrouping Satisfaction Questionnaire* contained nine statements. The learning disability teacher read the questionnaire aloud to the participants. The participants then rated the statements from 1 to 5 with 1 being ‘Strongly Agree’ to 5 being ‘Strongly Disagree’. Participants answered all questions with “Strongly Agree” with the exception of one participant answering one of the questions with “Agree”.

All participants rated questions 1 through 8 with “Strongly Agree”. This means that all participants strongly agreed that the base ten blocks, drawings, and RENAME strategy helped them with subtraction. The participants also all strongly agreed that the Subtraction Minute helped them get faster at subtraction. Additionally, they all strongly agreed that the Subtraction Review Minute helped them with regrouping and the FAST RENAME strategy helped them with word problems. Finally, all the participants strongly agreed that the program helped them get better at Subtraction.

While five of the six participants “Strongly Agreed” that overall, they liked the Subtraction Program, Participant 2 only “Agreed” with the statement. It should be noted that directly after the learning disability teacher read that statement, this participant said, “What does this mean? Is this over?” When the learning disability teacher explained that he was done learning this skill with these lessons, the participant sulked and marked “Agree”. One plausible explanation for why this
participant rated this statement one level lower from the other part is, therefore, because he was sad that it was over, not because he was less satisfied with program overall. In conclusion, all the participants picked responses on the *Subtraction With Regrouping Satisfaction Questionnaire* that indicated their level of satisfaction with these lessons was very high.

Conclusions and Related Discussions

Based on the results obtained in this study, several conclusions may be drawn. Included among these conclusions are the following:

1. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping problems.

2. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their ability to solve subtraction with regrouping word problems.

3. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their fluency related to solving subtraction with regrouping problems.

4. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence increase their conceptual understanding related to solving subtraction with regrouping problems.
5. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence successfully discriminate between subtraction problems that do and do not require regrouping.

6. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence maintain their ability to solve subtraction with regrouping problems.

7. Students with learning disabilities who receive strategy instruction integrated with the concrete-representational-abstract teaching sequence report high levels of satisfaction related to the subtraction with regrouping intervention lessons.

Practical Implications

There are many practical implications related to implementing strategy instruction integrated with the concrete-representational-abstract teaching sequence that have emerged based on the formal assessments and observations that took place during this study. These implications involve the instructional design and implementation of the intervention lessons.

**Instructional Design and Implementation of Lessons**

When providing instruction on subtraction with regrouping to students with learning disabilities, several teaching enhancements need to be in place. These teaching enhancements facilitate successful conceptual and procedural understanding of a new skill such as subtraction with regrouping. Several teaching enhancements
were already included in the scripted intervention lessons (i.e. lessons followed best practice for explicit instruction and the concrete-representational-abstract teaching sequence).

As indicated by the performance of Participant 1 and 4, when students do not receive a sufficient amount of instruction at the concrete level related to the meaning of basic subtraction with regrouping, they will not be able to use that skill on a practical level. This was evidenced by their low pretest scores on both the Subtraction Review Minute (where they were expected to discriminate between when to apply regrouping skills and when not to) and on the Subtraction Interview Pretest (where they were asked to demonstrate with manipulative devices, or explain what they were doing to solve the problems). The intervention lessons used in this study ensured sufficient amount of concrete level practice was provided to ensure that all participants were at a mastery level of conceptual understanding before moving on to representational instruction. It may be especially useful to use other teaching enhancements such as establishing a set routine for how the students handle the manipulative devices, during this stage of instruction. For example, the learning disability teacher found it necessary to establish an additional routine during the concrete stage of instruction. The routine was stated as follows:

…So when you go to switch out one of your sticks of 10 for 10 single cubes, it’s as if you are going to shatter your stick. If the stick was to really shatter it might knock other things off our place value mat, so first put the single cubes you already have on your mat in a safe place above the line. Now that you
know they are safe, you can switch out your stick of 10 for 10 single cubes using the rest of the area in the ones column.

This routine kept the participants from confusing one of the single cubes already within the ones column for one of the new single cubes needed to represent the stick of 10 being traded to facilitate the needed regrouping. Providing this routine ensured that the students were as set up for success as possible.

Similarly, during the representational stage of instruction a routine for differentiating between when a drawing is getting regrouped into another drawing (a vertical line representing a stick of 10 gets “crossed off” so that it can get regrouped into individual horizontal tally marks) versus when you are “crossing it off” because it is being subtracted away. For example, the learning disability teacher in this study set up the routine that when something was being regrouped, the participants crossed it off using a squiggly line versus when something was being subtracted, it was crossed off with a straight line.

Each of the participants easily found success during the abstract stage of instruction as a result of their successes throughout the concrete and representational stages of instruction. During this stage of instruction the learning disability teacher used the additional teaching enhancement of color coding different steps throughout the computational process on the abstract subtraction problems during the model phase of instruction. Other teachers may have to use this teaching enhancement to help support student transition into the abstract stage. The learning disability teacher used one color to write the original problem on the white board, another color to
indicate regrouping, and another color for the answer. This seemed to help the students organize the different digits into more meaningful content.

During this stage of instruction, the teacher also kept a larger version of the FAST RENAME Cue card with the BBB sentence on it displayed next to where he was modeling each problem. The teacher could then constantly refer to the correct procedural step when modeling to the students. This proved to be a useful signal to students that needed a quick reminder of the strategy steps. It also reminded students to have their cue cards available for assistance. Naturally, the students continued to keep their cue cards out; however, they began to rarely look at them as they became more proficient with the procedural steps. The amount of time each student needed to refer to their cue card before they successfully internalized the step varied. Teachers should keep this in mind and continue to encourage their students to have the cue cards available until it is clear they no longer need them. This will help prevent students from practicing the wrong steps.

Lastly, the ongoing assessments provided on the Learning Sheet Probes were an important part of this instructional sequence. It allowed the teacher to be able to immediately provide feedback to address the students’ current learning needs. Additionally, it was obvious that the students were highly motivated to plot their scores at the conclusion of each session to see if they kept their scores above the “dark line” (i.e. the line that indicated 80%). Future students may continue to need this visual reinforcement of plotting their scores too.
Recommendations for Future Research

This study represents a continued contribution to literature involving strategy instruction infused with the concrete-representational-abstract teaching sequence. Reflection on the methods used in this study, as well as the results obtained, led to the following recommendations for future study.

1. Future research related to using strategy instruction infused with the concrete-representational-abstract teaching sequence should be conducted to determine the effectiveness of this type of instruction for students with various learning challenges (e.g., students with English as a second language, students with behavioral and emotional issues, students identified to receive tier two interventions within a response to intervention system, students with language disorders, and students with more severe intellectual disabilities).

2. Future research related to using strategy instruction infused with the concrete-representational-abstract teaching sequence should include students in other grade levels (e.g., high school).

3. Future research related to using strategy instruction infused with the concrete representational-abstract teaching sequence should be conducted with a larger sample size of students with learning disabilities and/or a larger sample size of students without disabilities.

4. Future research related to using strategy instruction infused with the concrete-representational-abstract teaching sequence should be conducted to include longer periods of maintenance to be assessed.
5. Future research related to using strategy instruction infused with the concrete-representational-abstract teaching sequence should be conducted to include other multi-digit computational skills (e.g., addition with regrouping, multi-digit multiplication with regrouping, and division with remainders).

6. Future research that compares using strategy instruction infused with the concrete-representational-abstract teaching sequence to other teaching strategies should be conducted.
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Appendix A

Subtraction With Regrouping Pretest

Subtraction With Regrouping Pretest

\[
\begin{array}{cccc}
35 & 542 & 205 & 50 \\
- 16 & - 127 & - 163 & - 36 \\
\hline
944 & 42 & 64 & 314 \\
- 572 & - 17 & - 38 & - 127 \\
\hline
204 & 54 & 342 & 20 \\
- 121 & - 36 & - 118 & - 13 \\
\hline
74 & 358 & 483 & 53 \\
- 25 & - 167 & - 149 & - 26 \\
\hline
66 & 713 & 70 & 808 \\
- 47 & - 255 & - 27 & - 464 \\
\end{array}
\]
Appendix B

Subtraction With Regrouping Minute

Subtraction with Regrouping Minute

\[
\begin{array}{cccc}
64 & 437 & 40 & 319 \\
-37 & -118 & -19 & -184 \\
\end{array}
\]

\[
\begin{array}{cccc}
406 & 56 & 554 & 33 \\
-163 & -37 & -268 & -19 \\
\end{array}
\]

\[
\begin{array}{cccc}
43 & 714 & 34 & 606 \\
-26 & -365 & -19 & -341 \\
\end{array}
\]

\[
\begin{array}{cccc}
964 & 80 & 769 & 75 \\
-237 & -35 & -298 & -47 \\
\end{array}
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## Subtraction Review Minute

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Subtraction Word Problem Pretest

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<td>1. Sue had 303 dollars. She spent 122 dollars. How many dollars does Sue have now?</td>
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<tr>
<td>2. The shop had 217 coffee cups. 165 coffee cups were sold. How many coffee cups are left at the shop?</td>
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<tr>
<td>3. Sue had 558 cookies. Tim had 164 cookies. How many more cookies did Sue have than Tim?</td>
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<tr>
<td>4. There were 231 brown dogs at the pet store, but 115 were sold. How many brown dogs are left at the pet store?</td>
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<tr>
<td>5. Harry saw 317 cars. Joe saw 184 cars. How many more cars did Harry see than Joe?</td>
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</table>
6. Matt has learned to play 43 songs on the piano. Larry has learned to play 25 songs on the piano. How many more songs does Matt know than Larry?

7. Terry had 23 pencils. She gave 17 pencils to Bob. How many pencils does Terry have left?

8. Mary has 45 pretzels. Bill has 10 potato chips. Mary gave 18 pretzels to Betty. How many pretzels does Mary have left?

9. Pat had 274 pieces of candy. She lost 128. How many pieces of candy does she have now?

10. Bill had 37 baseball cards. Tom has 19 baseball cards. How many more cards does Bill have than Tom?
Appendix E

Subtraction Interview Pretest

Subtraction Interview Pretest

1. Show me how to solve the following problems using these _______ (base ten blocks or interlocking cubes). As you solve the problems, tell me what you are doing to solve the problem.

   \[
   \begin{array}{c}
   246 \\
   - 173 \\
   \end{array} \quad \begin{array}{c}
   24 \\
   - 17 \\
   \end{array} \quad \begin{array}{c}
   132 \\
   - 115 \\
   \end{array}
   \]

2. Show me how to solve the following problems without using __________ (base ten blocks or interlocking cubes). As you solve the problems, tell me what you are doing to solve the problem.

   \[
   \begin{array}{c}
   354 \\
   - 128 \\
   \end{array} \quad \begin{array}{c}
   547 \\
   - 175 \\
   \end{array} \quad \begin{array}{c}
   37 \\
   - 19 \\
   \end{array}
   \]
Appendix F

Subtraction Interview Pretest Scoring Protocol

1. Problem One: Student represents first number accurately
   Student states need to regroup to subtract tens
   Student regroups accurately
   Student subtracts accurately

2. Problem Two: Student represents first number accurately
   Student states need to regroup to subtract ones
   Student regroups accurately
   Student subtracts accurately

3. Problem Three: Student represents first number accurately
   Student states need to regroup to subtract ones
   Student regroups accurately
   Student subtracts accurately

4. Problem Four: Student states need to regroup to subtract ones
   Student regroups accurately
   Student subtracts accurately

5. Problem Five: Student states need to regroup to subtract ones
   Student regroups accurately
   Student subtracts accurately

6. Problem Six: Student states need to regroup to subtract ones
   Student regroups accurately
   Student subtracts accurately
Appendix G

Subtraction With Regrouping Posttest

**Subtraction With Regrouping Posttest**

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Subtraction Word Problem Posttest

1. Bill had 37 baseball cards. Tom has 19 baseball cards. How many more cards does Bill have than Tom?

2. Pat had 274 pieces of candy. She lost 128. How many pieces of candy does she have now?

3. Mary has 45 pretzels. Bill has 10 potato chips. Mary gave 18 pretzels to Betty. How many pretzels does Mary have left?

4. Terry had 23 pencils. She gave 17 pencils to Bob. How many pencils does Terry have left?

5. Matt has learned to play 43 songs on the piano. Larry has learned to play 25 songs on the piano. How many more songs does Matt know than Larry?
6. Harry saw 317 cars. Joe saw 184 cars. How many more cars did Harry see than Joe?

7. There were 231 brown dogs at the pet store, but 115 were sold. How many brown dogs are left at the pet store?

8. Sue had 558 cookies. Tim had 164 cookies. How many more cookies did Sue have than Tim?

9. The shop had 217 coffee cups. 165 coffee cups were sold. How many coffee cups are left at the shop?

10. Sue had 303 dollars. She spent 122 dollars. How many dollars does Sue have now?
Appendix I

Subtraction Interview Posttest

Subtraction Interview Posttest

1. Show me how to solve the following problems using these _______ (base ten blocks or interlocking cubes). As you solve the problems, tell me what you are doing to solve the problem.

   24
   - 17

   132
   - 115

   246
   - 173

2. Show me how to solve the following problems without using __________ (base ten blocks or interlocking cubes). As you solve the problems, tell me what you are doing to solve the problem.

   37
   - 19

   354
   - 128

   547
   - 175
Appendix J  
Example Learning Sheet  
**Learning Sheet 1**

### Describe and Model

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<td>25</td>
<td>+ 12</td>
</tr>
<tr>
<td>2</td>
<td>33</td>
<td>+ 24</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>+ 25</td>
</tr>
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</table>

### Guided Practice

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<td>5</td>
<td>35</td>
<td>+ 14</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>+ 36</td>
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### Independent Practice

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<tbody>
<tr>
<td>7</td>
<td>51</td>
<td>+ 34</td>
</tr>
<tr>
<td>8</td>
<td>43</td>
<td>+ 12</td>
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<td>9</td>
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<td>+ 13</td>
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<td>10</td>
<td>44</td>
<td>+ 22</td>
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<tr>
<td>11</td>
<td>23</td>
<td>+ 33</td>
</tr>
<tr>
<td>12</td>
<td>34</td>
<td>+ 11</td>
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</table>
### Problem-Solving Practice

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>13)</strong></td>
<td>Sally had 10 books. She got 11 more books from her sister. How many books does Sally have now?</td>
</tr>
<tr>
<td><strong>14)</strong></td>
<td>Tim had 16 caps. Tom had 11 caps. How many caps do Tim and Tom have together?</td>
</tr>
</tbody>
</table>
### Subtraction With Regrouping Satisfaction Questionnaire

<table>
<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Neither Agree or Disagree</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Base ten blocks helped me with subtraction.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>2. Drawings helped me with subtraction.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>3. The RENAME strategy helped me with subtraction.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>4. The BBB Phrase helped me know when to regroup.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>5. The Subtraction Minute helped me get faster at subtraction.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>6. The Subtraction Review Minute helped me with regrouping.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>7. The FAST RENAME strategy helped me with Word Problems.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<tr>
<td>8. This program helped me get better at Subtraction.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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<td><img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
</tr>
<tr>
<td>9. Overall, I liked this Subtraction Program.</td>
<td><img src="1" alt="1" /> <img src="2" alt="2" /> <img src="3" alt="3" /> <img src="4" alt="4" /> <img src="5" alt="5" /></td>
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Appendix L

Example Place Value Mat

<table>
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<td><strong>HUNDREDS</strong></td>
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## Baseline Probes

### Baseline Probe A

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<tr>
<td></td>
<td>-67</td>
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<tr>
<td>4)</td>
<td>308</td>
<td>5)</td>
</tr>
<tr>
<td></td>
<td>-29</td>
<td></td>
</tr>
<tr>
<td>7)</td>
<td>57</td>
<td>8)</td>
</tr>
<tr>
<td></td>
<td>-29</td>
<td></td>
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</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
<tr>
<td><strong>9)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sam had 73 apples. He gave 38 apples away. How many apples does Sam have now?</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bob had 82 pens. He gave 49 to his friend. How many pens does Bob have now?</td>
<td></td>
<td></td>
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</table>
## Baseline Probe B

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<table>
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<td>3) 640</td>
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<tr>
<td>- 29</td>
<td>- 46</td>
<td>- 94</td>
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<tr>
<td>4) 603</td>
<td>5) 57</td>
<td>6) 32</td>
</tr>
<tr>
<td>- 35</td>
<td>- 38</td>
<td>- 18</td>
</tr>
<tr>
<td>7) 82</td>
<td>8) 954</td>
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<tr>
<td>- 46</td>
<td>- 69</td>
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<tr>
<td>Problem-Solving Practice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td></td>
<td></td>
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<tr>
<td><strong>9)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>There 53 dogs at the park. It started to rain so 28 of the dogs left. How many dogs were left at the park?</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ann had 93 pencils. She gave 67 to her teacher. How many pencils does Ann have now?</td>
<td></td>
<td></td>
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</table>
## Baseline Probe C

<p>| | | | |</p>
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<td>- 35</td>
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<tr>
<td>3)</td>
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<td>- 37</td>
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<tr>
<td>4)</td>
<td>905</td>
<td>5)</td>
<td>91</td>
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<td>- 47</td>
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<td></td>
<td>- 36</td>
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<td>7)</td>
<td>96</td>
<td>8)</td>
<td>865</td>
</tr>
<tr>
<td></td>
<td>- 37</td>
<td></td>
<td>- 49</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>9)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>John had 83 cats. His mom made him give 49 away. How many cats does John have left?</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>10)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Patty had 74 stickers. She gave 29 of them to her friend. How many stickers does she have left?</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix N

Parent Consent Form

PARENT PERMISSION FORM

Department of Special Education

TITLE OF STUDY: Investigating the Effects of Strategy Instruction Subtraction
Performance among Elementary Students with Learning Disabilities

INVESTIGATOR(S):

CONTACT PHONE NUMBR:

Purpose of the Study
Your child is invited to participate in a research project. The purpose of this study is to explore the effectiveness of mathematics lessons designed to help students solve addition and subtraction problems that require regrouping.

Participants
Your child is being asked to participate in the study because he/she receives special education services to improve skills in mathematics and because he/she needs help with complex addition or subtraction problems.

Procedures
If you allow your child to volunteer to participate in this study, the scores your child earns on a pretest, posttest, and daily worksheets that accompany his addition or subtraction lessons will be shared with the investigator.

Benefits of Participation
There may be direct benefits to your child as a participant in this study. Allowing the investigator to analyze your child’s mathematics performance using the tests and worksheets he/she completes during his/her mathematics instruction will help inform his/her special education teacher about the effectiveness of the instruction he is providing to your child. This information will help plan future mathematics instruction to better address your child’s educational needs.

Risks of Participation
There are risks involved in all research studies. The risks associated with this study are minimal. It is possible that your child may experience minimal stress or discomfort related to the sharing of test and worksheet scores if he/she makes errors on some of the problems.

Cost/Compensation
There will not be financial cost to your child to participate in this study. There will be no compensation.

Participant Initials: __ __
TITLE OF STUDY: Investigating the Effects of Strategy Instruction on Addition and Subtraction Performance among Elementary Students with Learning Disabilities
INVESTIGATOR(S): Dr. Susan Miller

CONTACT PHONE NUMBER: 702 895-1108

Contact Information
If you or your child have any questions or concerns about the study, you may contact Dr. Susan Miller at 702-895-1108. For questions regarding the rights of research subjects, any complaints or comments regarding the manner in which the study is being conducted you may contact the UNLV Office for the Protection of Research Subjects at 702-895-2794.

Voluntary Participation
Your child's participation in this study is voluntary. Your child may refuse to participate in this study or in any part of this study. Your child may withdraw at any time without prejudice to your relations with the university or Paradise Professional Development School. You or your child are encouraged to ask questions about this study at the beginning or any time during the research study.

Confidentiality
All information gathered in this study will be kept completely confidential. No reference will be made in written or oral materials that could link your child to this study. All records will be stored in a locked facility at UNLV for three years after completion of the study. After the storage time, the information gathered will be destroyed and computer files erased.

PARENT PERMISSION:
I have read the above information and agree to allow my child to participate in this study. I am at least 18 years of age. A copy of this form has been given to me.

Signature of Parent

Date

Parent Name (Please Print)

CHILD'S NAME

Participant Note: Please do not sign this document if the Approval Stamp is missing or is expired.
Appendix O

Student Assent Form

UNLV
UNIVERSITY OF NEVADA LAS VEGAS

STUDENT FORM
ASSENT TO PARTICIPATE IN RESEARCH

Investigating the Effects of Strategy Instruction on
Subtraction Performance among Elementary Students
with Learning Disabilities

1. My name is

2. We are asking you to be in a project that will help us teach math better.

3. If you agree to be in the project, he will tell me some of your math grades. He will tell
   me the grades you get on addition and/or subtraction tests and worksheets.

4. You might not feel good about having your grades shared if you miss some of the problems,
   but I think you are going to get good grades on this work.

5. If you agree to let him tell me your math grades, we’ll be able to tell if the instruction
   helped you learn.

6. Please talk to your parents before you decide to participate in this project. We will also ask
   your parents to give their permission for you to take part in this project. But even if your
   parents say “yes” you can say “no.”

7. If you don’t want to share your grades, you don’t have to. No one will be mad if you don’t
   want to share your grades. If you say “yes” now and change your mind later, that’s OK. You
   can stop sharing your grades any time you want.

8. You can ask any questions that you have about this project. You can call me at
   If I don’t answer your questions or you do not want to talk to me about your question,
   you or your parent can call the UNLV Office for the Protection of Research Subjects at

9. Signing your name on the line below means that you agree to be in this project. You and your
   parents will be given a copy of this form after you have signed it.

______________________________
Print your name

______________________________
Date

______________________________
Sign your name
Appendix P

Fidelity of Treatment Form

Check Marks in boxes indicate the component was included in the lesson.

<table>
<thead>
<tr>
<th>Advanced Organizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe and Model Stage of Instruction</td>
</tr>
<tr>
<td>Guided Practice Stage of Instruction</td>
</tr>
<tr>
<td>Independent Practice Stage of Instruction</td>
</tr>
<tr>
<td>Problem Solving Stage of Instruction</td>
</tr>
<tr>
<td>Fluency Stage of Instruction</td>
</tr>
</tbody>
</table>
### Appendix Q

#### Implementation Schedule

<table>
<thead>
<tr>
<th>Estimated Sessions</th>
<th>Participants / Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Participant 1, 2, 3, 4, 5, &amp; 6: Administer Pretests (Interview, Subtraction Pretest, Word Problem Pretest, Subtraction with Regrouping Minute, Subtraction Review Minute)</td>
</tr>
</tbody>
</table>
| Session 2          | Participant 1: Baseline Probe  
Participant 2: Baseline Probe  
Participant 3: Baseline Probe  
Participant 4: Baseline Probe  
Participant 5: Baseline Probe  
Participant 6: Baseline Probe |
| Session 3          | Participant 1: Baseline Probe  
Participant 2: Baseline Probe  
Participant 3: Baseline Probe  
Participant 4: Baseline Probe  
Participant 5: Baseline Probe  
Participant 6: Baseline Probe |
| Session 4          | Participant 1: Baseline Probe  
Participant 2: Baseline Probe  
Participant 3: Baseline Probe  
Participant 4: Baseline Probe  
Participant 5: Baseline Probe  
Participant 6: Baseline Probe |
| Session 5          | Participant 1: Lesson 1 (assuming stable baseline)  
Participant 2:  
Participant 3:  
Participant 4: Lesson 1 (assuming stable baseline)  
Participant 5:  
Participant 6: |
| Session 6          | Participant 1: Lesson 2  
Participant 2:  
Participant 3:  
Participant 4: Lesson 2  
Participant 5:  
Participant 6: |
| Session 7 | Participant 1: Lesson 3  
| Participant 2:  
| Participant 3:  
| Participant 4: Lesson 3  
| Participant 5:  
| Participant 6:  |
| Session 8 | Participant 1: Lesson 4  
Participant 2: Baseline Probe (assuming Participant 1 met criterion of 80% on Lessons 1, 2, and 3)  
Participant 3: Baseline Probe (assuming Participant 1 met criterion of 80% on Lessons 1, 2, and 3)  
Participant 4: Lesson 4  
Participant 5: Baseline Probe (assuming Participant 4 met criterion of 80% on Lessons 1, 2, and 3)  
Participant 6: Baseline Probe (assuming Participant 4 met criterion of 80% on Lessons 1, 2, and 3) |
| Session 9 | Participant 1: Lesson 5  
Participant 2: Lesson 1  
Participant 3:  
Participant 4: Lesson 5  
Participant 5: Lesson 1  
Participant 6:  |
| Session 10 | Participant 1: Lesson 6  
Participant 2: Lesson 2  
Participant 3:  
Participant 4: Lesson 6  
Participant 5: Lesson 2  
Participant 6:  |
| Session 11 | Participant 1: Lesson 7  
Participant 2: Lesson 3  
Participant 3:  
Participant 4: Lesson 7  
Participant 5: Lesson 3  
Participant 6: |
|-------------|-----------------------------------------------|
| Session 12 | Participant 1: Lesson 8  
Participant 2: Lesson 4  
Participant 3: Baseline Probe (assuming Participant 2 met criterion of 80% on Lessons 1, 2, and 3)  
Participant 4: Lesson 8  
Participant 5: Lesson 4  
Participant 6: Baseline Probe (assuming Participant 5 met criterion of 80% on Lessons 1, 2, and 3) |
| Session 13 | Participant 1: Lesson 9  
Participant 2: Lesson 5  
Participant 3: Lesson 1  
Participant 4: Lesson 9  
Participant 5: Lesson 5  
Participant 6: Lesson 1 |
| Session 14 | Participant 1: Lesson 10  
Participant 2: Lesson 6  
Participant 3: Lesson 2  
Participant 4: Lesson 10  
Participant 5: Lesson 6  
Participant 6: Lesson 2 |
| Session 15 | Participant 1: Lesson 11  
Participant 2: Lesson 7  
Participant 3: Lesson 3  
Participant 4: Lesson 11  
Participant 5: Lesson 7  
Participant 6: Lesson 3 |
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<th>Participant 3: Lesson</th>
<th>Participant 4: Lesson</th>
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| Session 21 | Participant 1: Lesson 17  
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|           | Participant 3: Lesson 9  
|           | Participant 4: Lesson 17  
|           | Participant 5: Lesson 13  
|           | Participant 6: Lesson 9  
|           |  
| Session 22 | Participant 1: Lesson 18  
|           | Participant 2: Lesson 14  
|           | Participant 3: Lesson 10  
|           | Participant 4: Lesson 18  
|           | Participant 5: Lesson 14  
|           | Participant 6: Lesson 10  
|           |  
| Session 23 | Participant 1: Lesson 19  
|           | Participant 2: Lesson 15  
|           | Participant 3: Lesson 11  
|           | Participant 4: Lesson 19  
|           | Participant 5: Lesson 15  
|           | Participant 6: Lesson 11  
|           |  
| Session 24 | Participant 1: Lesson 20  
|           | Participant 2: Lesson 16  
|           | Participant 3: Lesson 12  
|           | Participant 4: Lesson 20  
|           | Participant 5: Lesson 16  
|           | Participant 6: Lesson 12  
|           |  
| Session 25 | Participant 1: Lesson 21  
|           | Participant 2: Lesson 17  
|           | Participant 3: Lesson 13  
|           | Participant 4: Lesson 21  
|           | Participant 5: Lesson 17  
|           | Participant 6: Lesson 13  
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| Session 26 | Participant 1: Lesson 22  
Participant 2: Lesson 18  
Participant 3: Lesson 14  
Participant 4: Lesson 22  
Participant 5: Lesson 18  
Participant 6: Lesson 14 |
|-------------|---------------------------------------------------------------|
| Session 27 | Participant 1: Lesson 23  
Participant 2: Lesson 19  
Participant 3: Lesson 15  
Participant 4: Lesson 23  
Participant 5: Lesson 19  
Participant 6: Lesson 15 |
| Session 28 | Participant 1: Lesson 24  
Participant 2: Lesson 20  
Participant 3: Lesson 16  
Participant 4: Lesson 24  
Participant 5: Lesson 20  
Participant 6: Lesson 16 |
| Session 29 | Participant 1: Lesson 25  
Participant 2: Lesson 21  
Participant 3: Lesson 17  
Participant 4: Lesson 25  
Participant 5: Lesson 21  
Participant 6: Lesson 17 |
| Session 30 | Participant 1: Lesson 26  
Participant 2: Lesson 22  
Participant 3: Lesson 18  
Participant 4: Lesson 26  
Participant 5: Lesson 22  
Participant 6: Lesson 18 |
| Session 31 | Participant 1: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
Participant 2: Lesson 23  
Participant 3: Lesson 19  
Participant 4: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
Participant 5: Lesson 23  
Participant 6: Lesson 19 |
|---|---|
| Session 32 | Participant 1:  
Participant 2: Lesson 24  
Participant 3: Lesson 20  
Participant 4:  
Participant 5: Lesson 24  
Participant 6: Lesson 20 |
| Session 33 | Participant 1:  
Participant 2: Lesson 25  
Participant 3: Lesson 21  
Participant 4:  
Participant 5: Lesson 25  
Participant 6: Lesson 21 |
| Session 34 | Participant 1:  
Participant 2: Lesson 26  
Participant 3: Lesson 22  
Participant 4:  
Participant 5: Lesson 26  
Participant 6: Lesson 22 |
| Session 35 | Participant 1:  
Participant 2: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
Participant 3: Lesson 23  
Participant 4:  
Participant 5: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
Participant 6: Lesson 23 |
<table>
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<td>Participant 2:</td>
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<th>Participant 1: Administer Maintenance Probes (Baseline-Type, Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)</th>
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<td>Participant 3: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)</td>
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<td>Participant 6: Administer Posttests (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)</td>
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| Session 41 | Participant 1:  
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| Session 42 | Participant 1:  
|           | Participant 2: Administer Maintenance Probes (Baseline-Type, Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
|           | Participant 3:  
|           | Participant 4:  
|           | Participant 5: Administer Maintenance Probes (Baseline-Type, Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
|           | Participant 6:  |
| Session 43 | Participant 1:  
|           | Participant 2:  
|           | Participant 3:  
|           | Participant 4:  
|           | Participant 5:  
|           | Participant 6:  |
| Session 44 | Participant 1:  
|           | Participant 2:  
|           | Participant 3:  
|           | Participant 4:  
|           | Participant 5:  
|           | Participant 6:  |
| Session 45 | Participant 1:  
|           | Participant 2:  
|           | Participant 3:  
|           | Participant 4:  
|           | Participant 5:  
|           | Participant 6:  |
| Session 46 | Participant 1:  
Participan 2:  
Participant 3: Administer Maintenance Probes (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute)  
Participant 4:  
Participant 5:  
Participant 6: Administer Maintenance Probes (Interview, Subtraction Posttest, Word Problem Posttest, Subtraction with Regrouping Minute, Subtraction Review Minute) |
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University of Nevada, Las Vegas

Danielle Ferreira

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Publications:


Dissertation Title: Effects of Explicit Subtraction Instruction on Fifth Grade Students with Learning Disabilities

Dissertation Examination Committee:
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Committee Member, Nancy Sileo, Ed.D
Committee Member, Pamela Campbell, Ph.D
Committee Member, Gregory Schraw, Ph. D