Estimating Kelly Fraction

Gambling & Risk Taking
Las Vegas

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Geometric Brownian Motion

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Letting $\theta = \frac{\mu}{\sigma}$ we have

$$dB = (F\theta^2)B dt + (F\theta)B dW$$

Itoh's Lemma:

$$\frac{dB}{B} = F\theta^2 dt + F\theta dW$$

yields

$$d \ln B = (F - \frac{1}{2}F^2)\theta^2 dt + F\theta dW$$

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- solving for F yields
$$F = \frac{2\Sigma^2}{2\rho + \Sigma^2}$$

Bankroll

In order for our estimate to be useful in practice we need some idea of its statistical variability. . Assume that for times $t_0, t_1, \cdots, t_N$ we know the bankrolls $B_0, B_1, \cdots, B_N$. Define $\Delta B_k = B_k - B_{k-1}$ and $\Delta t_k = t_k - t_{k-1}$.

Estimating Drift and SD

We use standard estimators for the parameters ρ and $\Sigma^2 = \mathrm{Var}(\ln(B))$:

$$R = \frac{\ln(B_N) - \ln(B_0)}{t_N - t_0}$$

$$v = \frac{\sum_{k=1}^{N} \left[\ln(B_k) - \ln(B_{k-1}) - \Delta t_k R\right]^2}{t_N - t_0}$$

Estimating F

Our estimator for the Kelly fraction F is

$$\hat{F} = \frac{2v}{2R + v}$$

We estimate its standard deviation next

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Taylor approximation of $\sigma^2_{\hat{F}}$

$$\approx \left|\frac{\partial F}{\partial R}\right|^2 \sigma_R^2 + \left|\frac{\partial F}{\partial v}\right|^2 \sigma_v^2$$

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$$= \left|\frac{-v}{(2R+v)^2}\right|^2 \left(\frac{v}{t_N - t_0}\right)$$

$$+ \left|\frac{R}{(2R+v)^2}\right|^2 \left(\frac{2v^2}{(t_N - t_0)^2} \sum_{k=1}^{N} (\Delta t_k)^2\right)$$

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Taylor approximation of $\sigma^2_{\hat{F}}$ with N equal time intervals

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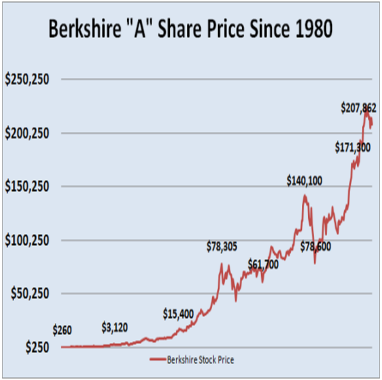
$$\frac{v^3 + 2R^2v^2}{N(R + \frac{1}{2}v)^4}$$

Berkshire Hathaway



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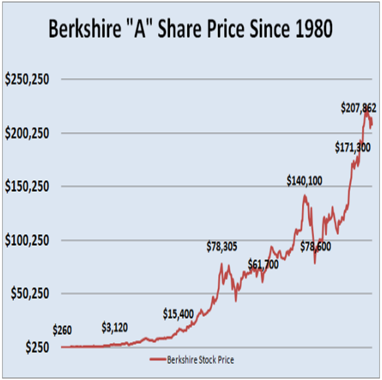

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Methods in discrete time