Estimating Kelly Fraction
Gambling & Risk Taking
Las Vegas

William Chin  Marc Ingenoso

DePaul University
Conger Asset Management, Chicago

June 9, 2016
Diffusion Model

Estimating $F$

Estimating $\sigma^2_F$

Warren Buffett

Further Work
Geometric Brownian Motion

$F$ times Kelly betting is modeled by

$$dB = \left( F \frac{\mu}{\sigma^2} \right) \mu Bdt + \left( F \frac{\mu}{\sigma^2} \right) \sigma BdW$$
Geometric Brownian Motion

F times Kelly betting is modeled by

\[ dB = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW \]

- \( F \) is the fraction of the full Kelly bet and
F times Kelly betting is modeled by

\[ dB = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW \]

- \( F \) is the fraction of the full Kelly bet and
- \( \mu \) is the “edge“ and \( F \frac{\mu}{\sigma^2} B \) is the “bet size“
Geometric Brownian Motion

F times Kelly betting is modeled by

\[ dB = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW \]

- \( F \) is the fraction of the full Kelly bet and
- \( \mu \) is the “edge“ and \( F \frac{\mu}{\sigma^2} B \) is the “bet size“
- \( W \) is standard Wiener Process
Geometric Brownian Motion

F times Kelly betting is modeled by

\[ dB = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW \]

- \( F \) is the fraction of the full Kelly bet and
- \( \mu \) is the “edge“ and \( F \frac{\mu}{\sigma^2} B \) is the “bet size“
- \( W \) is standard Wiener Process
- \( F \) corresponds to a utility function
Geometric Brownian Motion

F times Kelly betting is modeled by

\[ d\bar{B} = \left( F \frac{\mu}{\sigma^2} \right) \mu B dt + \left( F \frac{\mu}{\sigma^2} \right) \sigma B dW \]

- \( F \) is the fraction of the full Kelly bet and
- \( \mu \) is the “edge“ and \( F \frac{\mu}{\sigma^2} B \) is the “bet size“
- \( W \) is standard Wiener Process
- \( F \) corresponds to a utility function

Letting \( \theta = \frac{\mu}{\sigma} \) we have

\[ d\bar{B} = (F \theta^2) B dt + (F \theta) B dW \]
Itoh’s Lemma:

\[ \frac{dB}{B} = F \theta^2 dt + F \theta dW \]

yields

\[ d \ln B = (F - \frac{1}{2} F^2) \theta^2 dt + F \theta dW \]
Diffusion Model

Estimating $F$

Estimating $\sigma_{\hat{F}}^2$

Warren Buffett

Further Work
Estimating $F$
Estimating $F$

- Set $\rho = (F - \frac{1}{2}F^2)\theta^2$ and
Estimating F

- Set $\rho = (F - \frac{1}{2}F^2)\theta^2$ and
- $\Sigma = F\theta$
Estimating $F$

- Set $\rho = (F - \frac{1}{2}F^2)\theta^2$ and
- $\Sigma = F\theta$
- solving for $F$ yields

$$F = \frac{2\Sigma^2}{2\rho + \Sigma^2}$$
Bankroll

In order for our estimate to be useful in practice we need some idea of its statistical variability. Assume that for times $t_0, t_1, \cdots, t_N$ we know the bankrolls $B_0, B_1, \cdots, B_N$. Define $\Delta B_k = B_k - B_{k-1}$ and $\Delta t_k = t_k - t_{k-1}$. 
We use standard estimators for the parameters $\rho$ and $\Sigma^2 = \text{Var}(\ln(B))$:

$$R = \frac{\ln(B_N) - \ln(B_0)}{t_N - t_0}$$

$$v = \frac{\sum_{k=1}^{N} [\ln(B_k) - \ln(B_{k-1}) - \Delta t_k R]^2}{t_N - t_0}$$
Our estimator for the Kelly fraction $F$ is

$$\hat{F} = \frac{2\nu}{2R + \nu}$$

We estimate its standard deviation next.
Diffusion Model

Estimating $F$

Estimating $\sigma_F^2$

Warren Buffett

Further Work
Estimating $\hat{\sigma}_F^2$

Taylor approximation of $\hat{\sigma}_F^2$

$$\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma_R^2 + \left| \frac{\partial F}{\partial v} \right|^2 \sigma_v^2$$
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$

\[
\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v
\]

\[
= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right)
\]
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$

$$\approx \left| \frac{\partial F}{\partial R} \right|^2 \sigma^2_R + \left| \frac{\partial F}{\partial v} \right|^2 \sigma^2_v$$

$$= \left| \frac{-v}{(2R + v)^2} \right|^2 \left( \frac{v}{t_N - t_0} \right)$$

$$+ \left| \frac{R}{(2R + v)^2} \right|^2 \left( \frac{2v^2}{(t_N - t_0)^2} \sum_{k=1}^{N} (\Delta t_k)^2 \right)$$
Estimating $\sigma^2_{\hat{F}}$

Taylor approximation of $\sigma^2_{\hat{F}}$ with $N$ equal time intervals
Estimating $\sigma_{\hat{F}}^2$

Taylor approximation of $\sigma_{\hat{F}}^2$ with $N$ equal time intervals

$$\frac{\nu^3 + 2R^2\nu^2}{N(R + \frac{1}{2}\nu)^4}$$
Diffusion Model

Estimating $F$

Estimating $\sigma^2_{\hat{F}}$

Warren Buffett

Further Work
E. Thorp (2006) and W. Ziemba (2003) say that Warren Buffett seems to be a Full Kelly bettor. We find a Kelly Fraction of $F = 0.26 \pm 0.09$ ($\alpha = 0.05$) for closing prices 1980-present.
Diffusion Model

Estimating $F$

Estimating $\sigma_F^2$

Warren Buffett

Further Work
Methods in discrete time