Optimizing the mix of games and their locations on the casino floor

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Introduction

- Current:
  - CEO - nQube Data Science
  - Associate Professor of Astrophysics - University of Manitoba
  - Board member - Wild Rose Hedging

- Research Interests:
  - Large-scale non-linear optimization problems using large data sets
  - Mathematical modelling of physical systems
  - Evolutionary computing and artificial intelligence based optimization algorithms

Outline

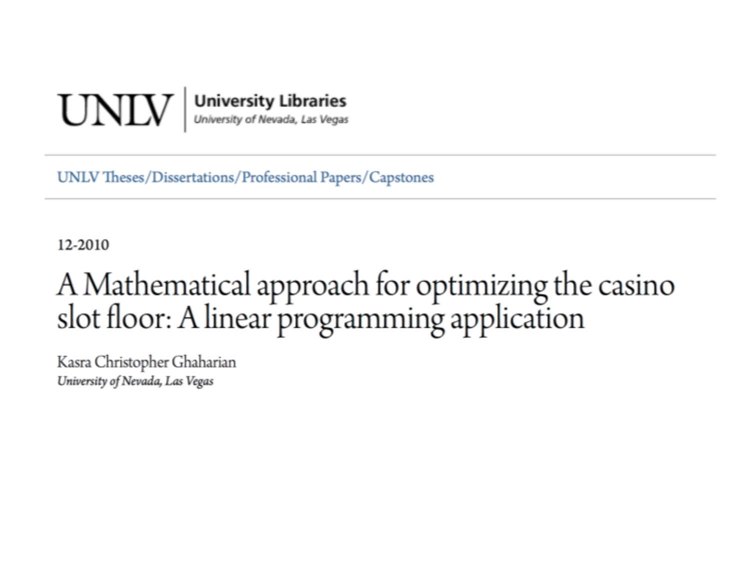
- Previous work in slot floor mix optimization
- Linear vs. non-linear models
- Two-step non-linear model for slot floor optimization:
  - Step 1 (the **inverse** problem): Find the model parameters from data
  - Step 2 (the **forward** problem): Optimize the model, given the now-known parameters
- Casino optimal model explorer – visualization tool
- Time dependent optimization of a casino floor

* Artificial data study to understand the mathematical structure of the problem and modelling framework

Previous work

- Ghaharian, Kasra Christopher. "A mathematical approach for optimizing the casino slot floor: A linear programming application." (2010).
- Uses linear programming to optimize slot floor machine mix for 2612 machines across a 6 month period.
- Linear model must be constrained:
  - No more than 10% change in mix of machines from starting configuration
  - Max/min bounds necessary for linear optimization problems, but somewhat artificial

- Optimized model outperformed the original configuration by 3.91% on coin-in and 5.65% on win.
- **No consideration of machine placement**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A Mathematical approach for optimizing the casino slot floor: A linear programming application

Kasra Christopher Ghaharian
University of Nevada, Las Vegas

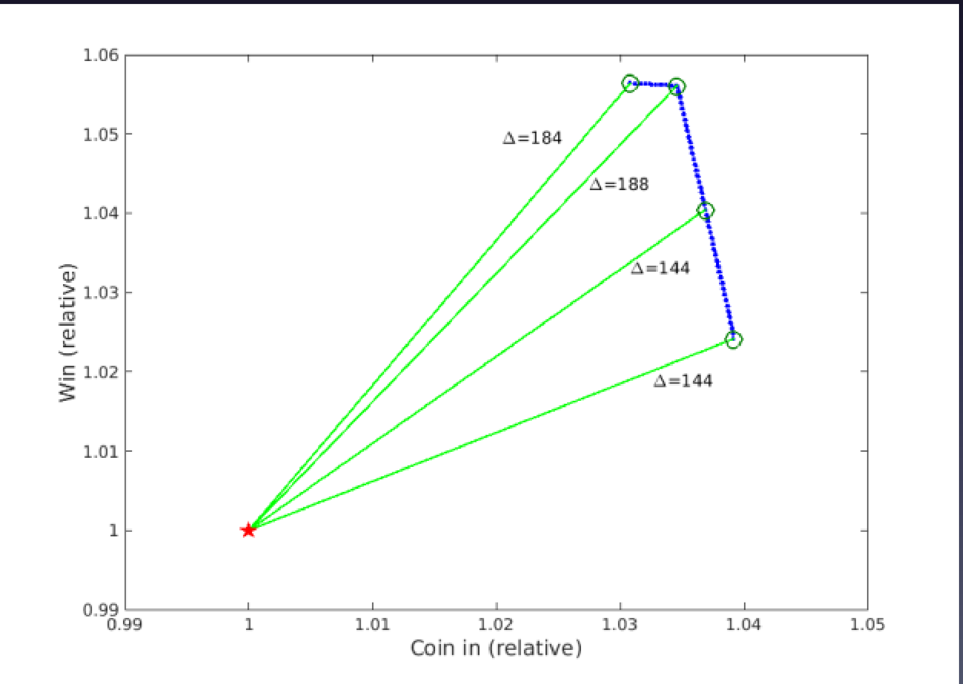
Other literature

- Bayus, Barry L., and Shiv K. Gupta. "Analyzing floor configurations for casino slot machines." Omega 13.6 (1985): 561-567.
  - location effects vs. profit
  - devising a way to predict the profit for different arrangements of slot machines on the casino floor, and (2) choosing the 'best' alternative out of all the possible arrangements

- Lucas, Anthony F., and William T. Dunn. "Estimating the effects of micro-location variables and game characteristics on slot machine volume: A performance-potential model." Journal of Hospitality & Tourism Research 29.2 (2005): 170-193.
  - Micro-location variables affecting performance:
  - Ceiling Height (CEILG), Slant-Tops (SLANT), End-units (END), Signs, Maximum Wager (MAX), Platform (PFORM), and Program (PGRM),Top Award (AWD), Max-Coin Par (PARMC), Aisle Units (AISLE), Standard Deviation, Coin-in (CI)
  - Multiple-Regression model used

nQ A multi-objective re-analysis of linear casino slot floor optimization

- Ghaharian optimized coin-in and win objectives, but *separately.*
- Multi-objective optimization methods allow us to optimize both coin-in and win *simultaneously*, and explore the *trade-offs* between them.
  - Within the set of optimal models, win cannot be improved without sacrificing coin-in, and vice-versa.

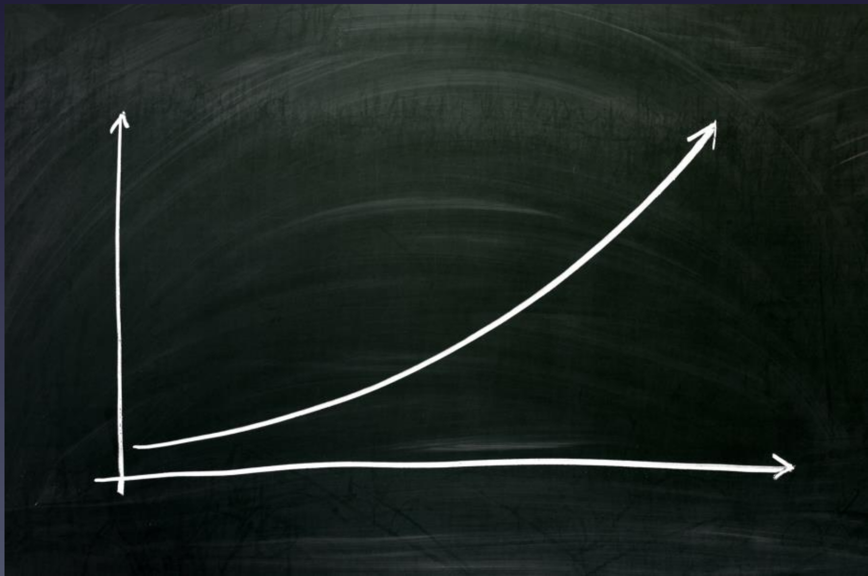


- Blue dots: trade-off curve of optimal solutions. Each point represents a model for the mix of machines on the slot floor
- Red star: casino's current slot mix
- Green circles: selection of optimal solutions
- Δ's are the total number of machines that need to be changed to achieve an optimal solution. Minimum $\Delta = 144$.

You can't (really) use a linear model to model reality, because reality isn't linear

- Most real world phenomena are not fundamentally linear.
- Non-linearity is usually the origin of complex behavior
  - especially in "messy" systems with many interacting parts: ex. a casino floor

- A linear approach can approximate systems locally (not too far from an initial state, as in Ghaharian), but **cannot** capture the full dynamics.



A non-linear model of slot floor performance

- Step 1: The Inverse Problem
  - Fitting a non-linear parameterized model to machine-level data
- Step 2: The Forward Problem
  - Optimizing the mix of machines, given the parameters from Step 1.

The Efficient Slot Floor (highly idealized)

Players:

- are thoroughly mixed throughout the slot floor;
- can easily find the their preferred machine type;
- have no spatial preference, and no preference for either crowded or isolated regions;
- are indifferent to the detailed placement of a machine within a bank of machines.

These are strong, idealized conditions, which are relaxed later…

The Efficient Slot Floor

- There exists a normalized vector of probabilities

$$\vec{\psi} = [\psi_1, \psi_2, ...\psi_M] \qquad \sum_{i=1}^{M} \psi_i = 1$$

  which specifies the odds that a given player will choose a slot machine type *i*, on a floor with *M* machine types.

- There are N_0 machines in total on the floor, and N_i of each type *i*.
- Probability that a given player chooses a *particular* machine *n* of type *i* = *MT*(*n*) is:

$$p_n^{EF} := \frac{\psi_i}{N_i}$$

$_n$Q Efficient floor: duty cycle and coin-in

- Define the "busyness" of the slot floor as

$$Q := \frac{P_0}{N_0}$$

- where there are P_0 players and N_0 machines available in total. The duty cycle of a given machine n is

$$f_{n,q} = \min(1, p_n Q N_0)$$

No differences between machines n of the same type i-MT(n)

- For an efficient floor, this becomes

$$f_n(Q) = \min\left(1, \frac{\psi_i Q}{\nu_i}\right) \text{ where } \nu_i := \frac{N_i}{N_0}$$

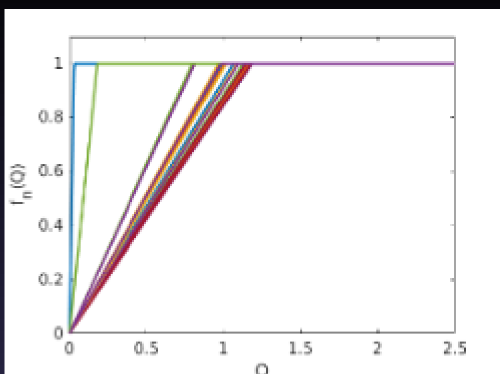
- And $CI_{n,q} := CI_i^0 * f_{n,q}$

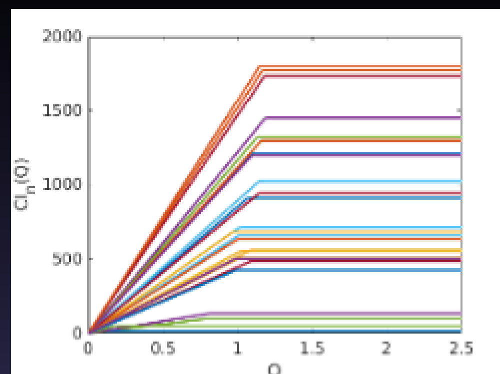
Fitting parameters to the efficient floor model

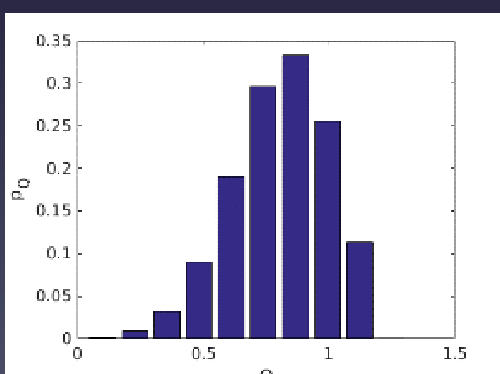
$$f_n(Q) = \min\left(1, \frac{\psi_i Q}{\nu_i}\right) \qquad CI_{n,q} := CI_i^0 * f_{n,q}$$

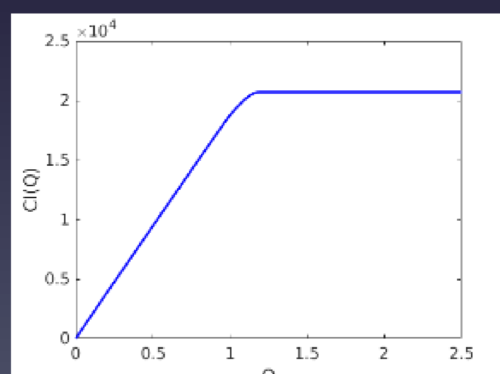
- Duty cycle and coin in are measurable for each machine on the floor. Assumed known for various values of busyness Q.
- (Step 1: Inverse problem) These data can be used to determine unknown player preferences $\psi = [\psi_1, \psi_1, \ldots, \psi_M]$.
- (Step 2: Forward problem) With ψ known, one can vary $\nu = [\nu_1, \nu_1, \ldots, \nu_M]$ to determine the optimal mix of machines to maximize coin-in, win, or both.

Efficient floor: Optimization of 25 machine types









- Efficient floor model naturally results in non-linear saturation behaviour because any given machine stops producing additional coin-in once the duty cycle $f_n(Q) = 1$, at sufficiently high busyness Q.
- The optimal coin-in solution (for the distribution of Q shown) contains a complex and non-obvious mix of machines.

Spatial and clustering effects: beyond the efficient slot floor

- The true probability p_n is modified (from the efficient probability p^{EF}) by secondary effects, such as:
  - Δ^R: **Region** that the machine resides in.
  - Δ^B: Relative position within a **bank** of machines
  - Δ^S: **Spatial** effects due to distance from hallways, walls, table games, etc.
  - Δ^C: **Clustering** effects due to proximity of other players
- Combined effect:

$$\Delta_{n,q} = \Delta^R_{n,q} + \Delta^B_{n,q} + \Delta^S_{n,q} + \Delta^C_{n,q}$$

Spatial and clustering effects

- Modified (non-efficient) normalized probability:

$$p_n = \frac{p_n^{EF}(1+\Delta'_n)}{\sum_{n=1}^{N0} p_n^{EF}(1+\Delta'_n)}$$

- But there is a problem: Δ_n may be positive or negative, but p_n must always be positive. Fix this:

$$p_n = \frac{p_n^{EF}(1+T(\Delta_n))}{\sum_{n=1}^{N0} p_n^{EF}(1+T(\Delta_n)}$$

- Where $T(\cdot)$ is a non-linear transformation that maps $(-\infty,\infty) \rightarrow [-1,\infty)$. For example:

$$T(\Delta_n) = max(-1,\Delta_n)$$

Non-negativity of probability implies additional non-linearity!

Components of Δ_n modeling spatial and clustering preferences

$$\Delta_{n,q} = \Delta^R_{n,q} + \Delta^B_{n,q} + \Delta^S_{n,q} + \Delta^C_{n,q}$$

- Regional preferences
  - Some rooms, regions, alcoves, etc. may be higher performing than others

$$\Delta^R_n = \delta^R_r$$

- Bank preferences
  - End positions on linear banks, outer positions of curves, etc may outperform other positions

$$\Delta^B_q = \delta^B_b$$

- Spatial preferences
  - Distance to entrances, walls, walkways, table games, washrooms, etc. may influence performance

$$\Delta^S_n = \sum_{m=1}^{N_A} \delta^S_{i,j} d_{n,m}^{-\gamma^S}$$

- Cluster preferences
  - Some players like the excitement of crowds, while others prefer seclusion.

$$\Delta^C_{n,q} = \delta^C_i \sum_{m=1}^{N_0} f_{m,q} d_{n,m}^{-\gamma^C}$$

nQ

“Gaming is a passive activity…A gaming room has no dynamic value.”
– Steve Wynn, opening plenary talk

- Modeling *players* is not passive though:
  - machine duty cycle is a proxy for where the *players* are.
  - Source of non-linear clustering effects
  - Gaming floor design can influence where the players are.

$$f_{n,q} = \min(1, p_n Q N_0)$$

$$\Delta^C_{n,q} = \delta^C_i \sum_{m=1}^{N_0} f_{m,q}\, d_{n,m}^{-\gamma^C}$$

$$p_n = \frac{p_n^{EF}(1+\Delta'_n)}{\sum_{n=1}^{N0} p_n^{EF}(1+\Delta'_n)}$$

Self-consistent non-linear solution required. System contains N_0 simultaneous equations, where $N_0 \sim 10^3$ machines on the floor.

Objective Function

- The total set of parameters is:
  - Ψ M parameters; number of machine types
  - Δ^R N_R parameters; number of regions
  - Δ^B N_B parameters; number of identified bank positions
  - Δ^S $M*N_S$ parameters; number of machine types * number of other spatial attributes (entrances, table games, walls, walkways, etc.)
  - Δ^C M parameters; number of machine types

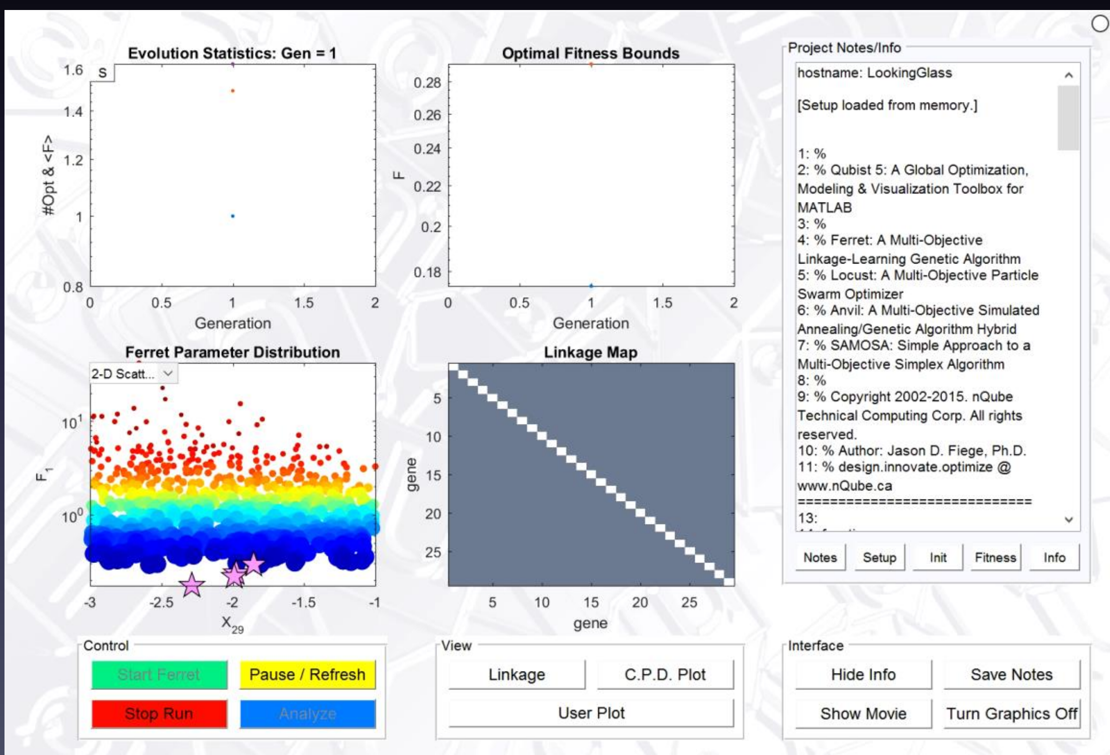
Minimize this to determine P:

$$F = \frac{\left[f_{n,q}^{model}(\mathbf{P}) - f_{n,q}^{model}\right]^2}{\langle f_{n,q}^{model}\rangle^2} + \frac{\left[CI_{n,q}^{model}(\mathbf{P}) - CI_{n,q}^{model}\right]^2}{\langle CI_{n,q}^{model}\rangle^2}$$

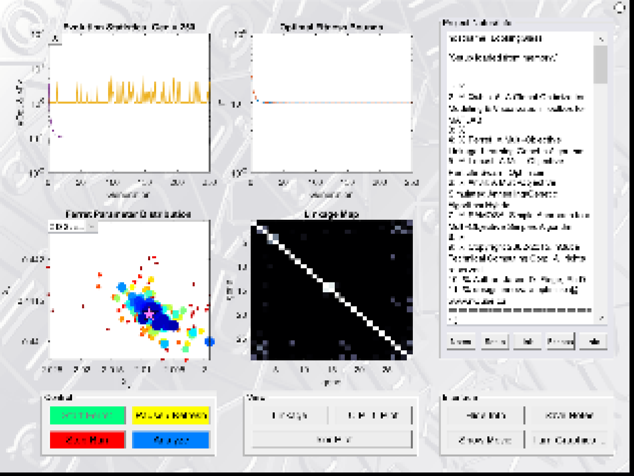
- We assume the duty cycle (f_{nq}) and coin-in (CI_{nq}) are known from machine-level data.
- <…> denotes average.

Ferret Evolutionary Optimizer (From Qubist Global optimization toolbox) www.nqube.com

Exact model inversion test: Paris casino map

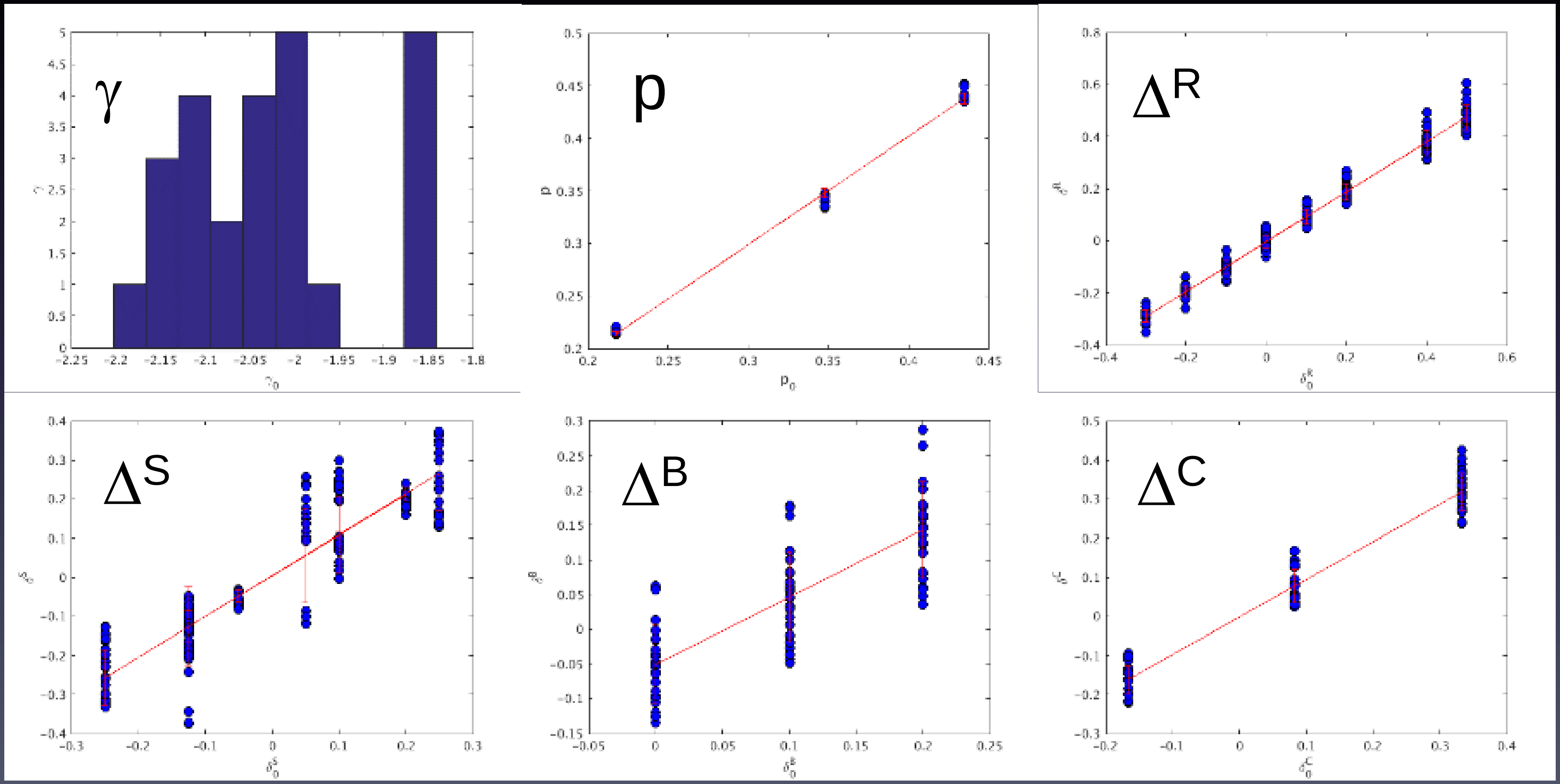


- 3 machine types
- Data is machine-level duty cycle and coin-in for a model with known parameters. Floor partially optimized for coin-in using a simulated annealing method
- Goal is to accurately determine 29 model parameters from the “data” → F = 0.

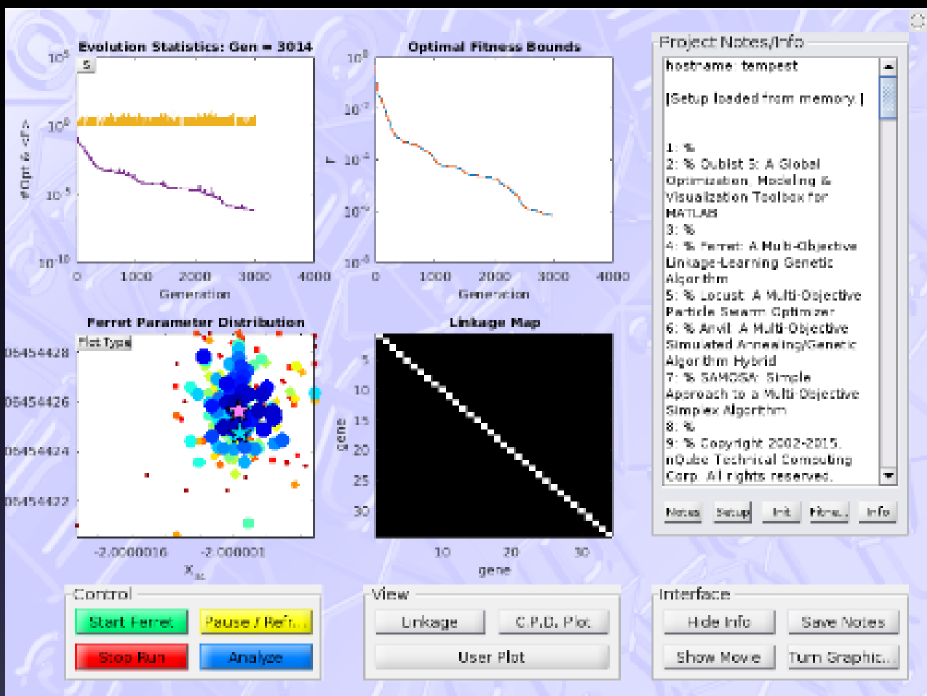


Noise robustness

- Parameter recovery experiment with 10% noise added to duty cycle (f_{nq}) and coin-in (CI_{nq})
- Power law γ recovered to ~5-10%. All parameters show good correlation with data parameters



10 Machine types exact inversion



- “Curse of Dimensionality”: optimization problems become more difficult as the number of parameters increases
- Inverse problem becomes more difficult as the number of machine types increases
- Exact inversion an be done if $\Delta^S = 0$, which is probably OK. (34 parameters)
- **Dimensionality does not affect forward problem.**

- Larger number of machines: no problem in the presence of noise – exact inversion without noise is much more difficult.

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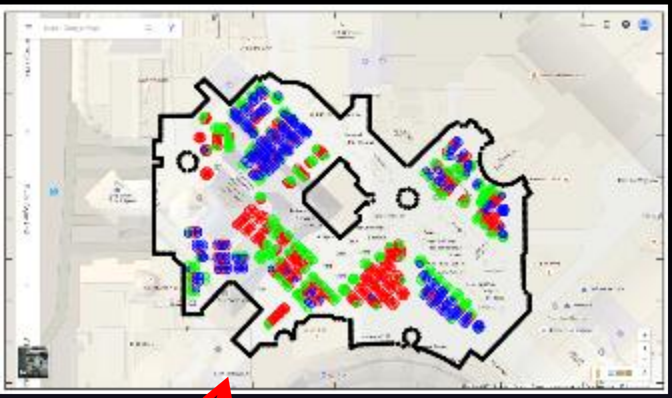
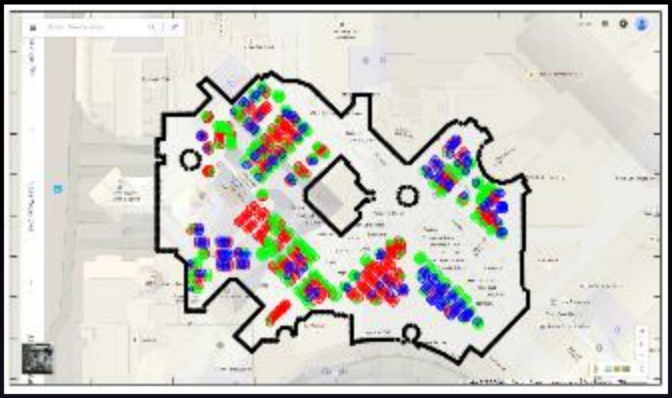
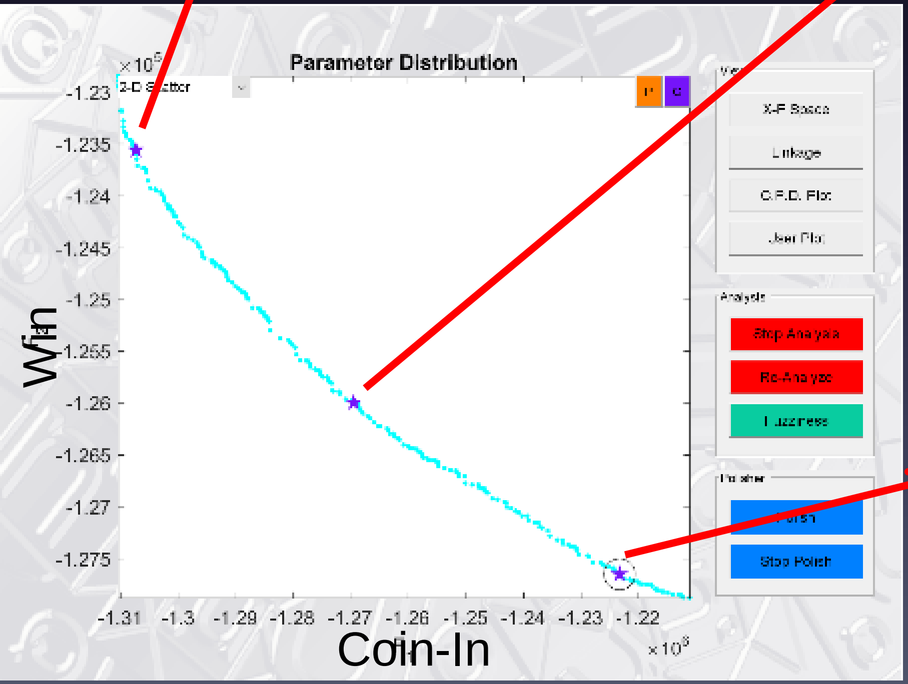
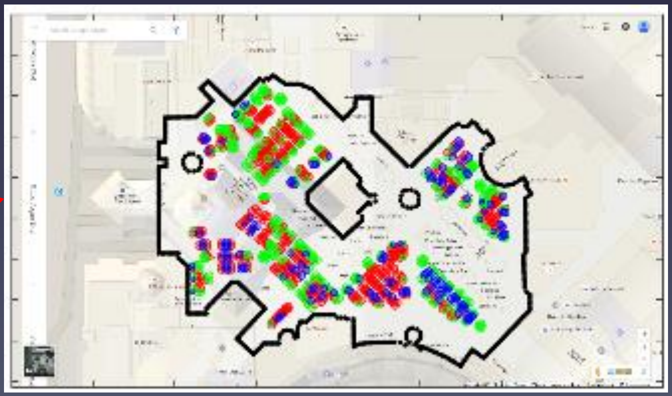
Forward models







Every point on the trade-off curve is an alternative optimized model



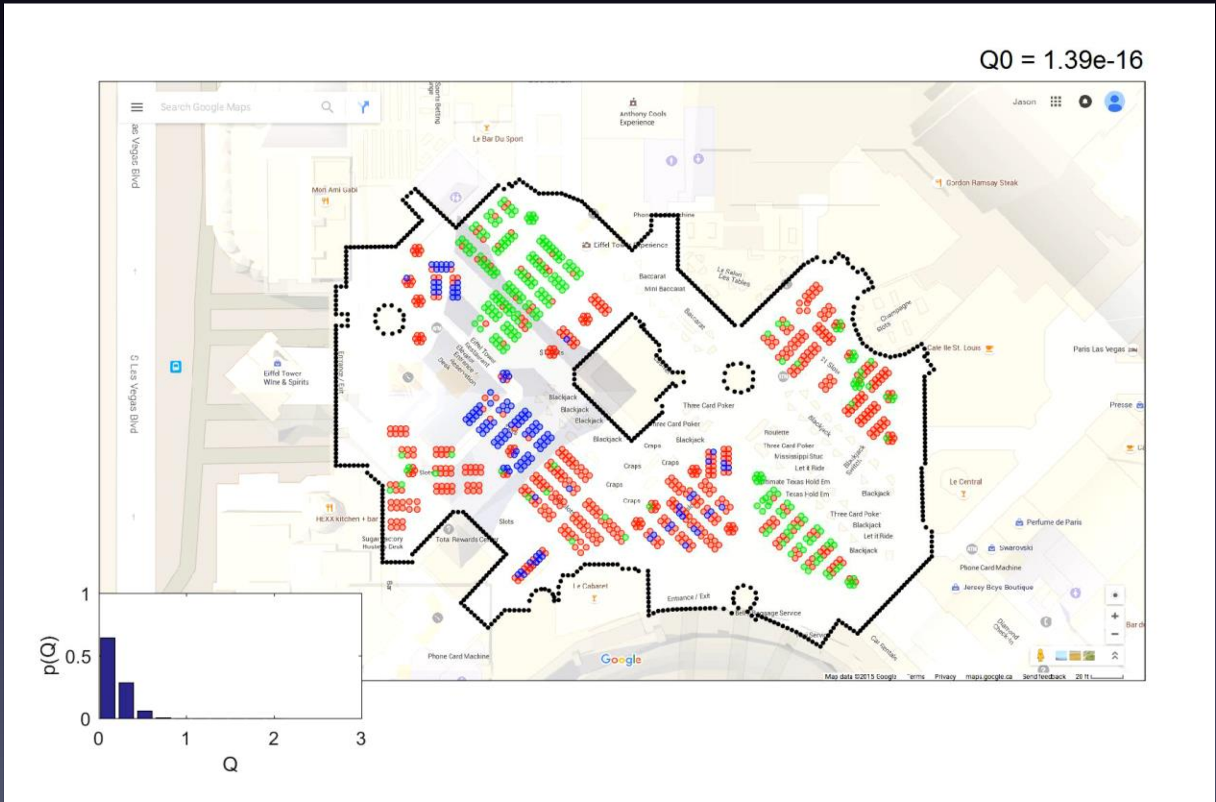
Time-dependent models via simulated annealing tracking

- Pick a point on the tradeoff curve
- The curve is convex → can assign effective weight w to coin-in relative to win objectives:

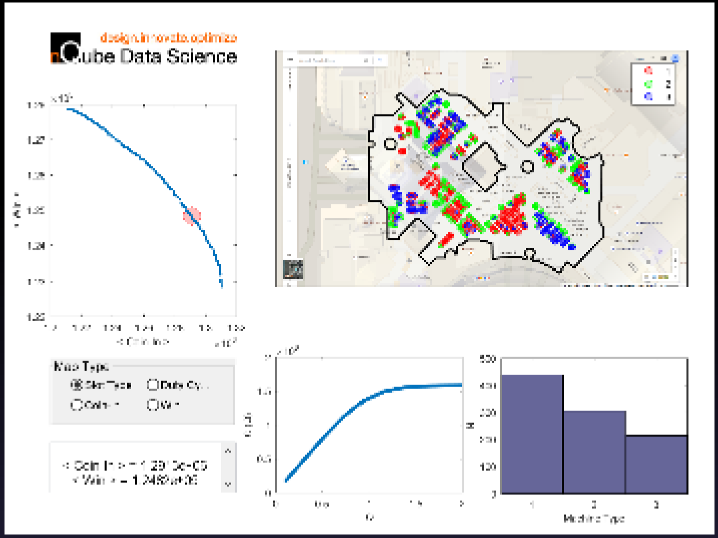
$$F = wCI + (1-w)Win$$

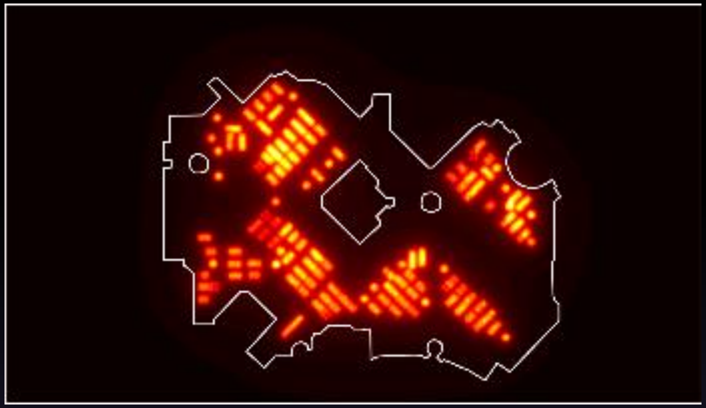
- Average change = 204 machines (out of 958) – superior to other methods investigated.

- Starting at the initial floor configuration, continually maximize *f* as the *Q* distribution changes in time.

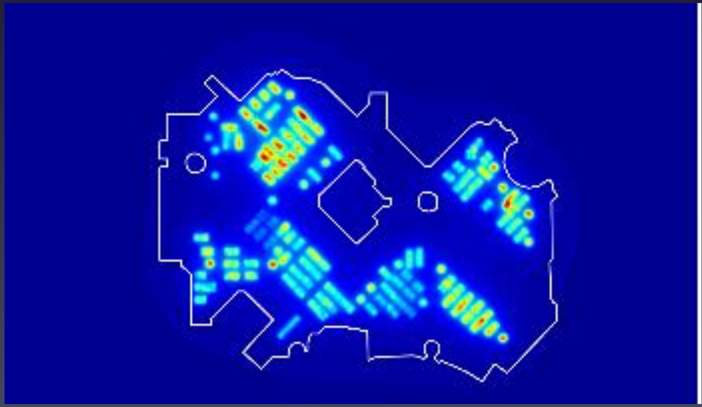


Visualization

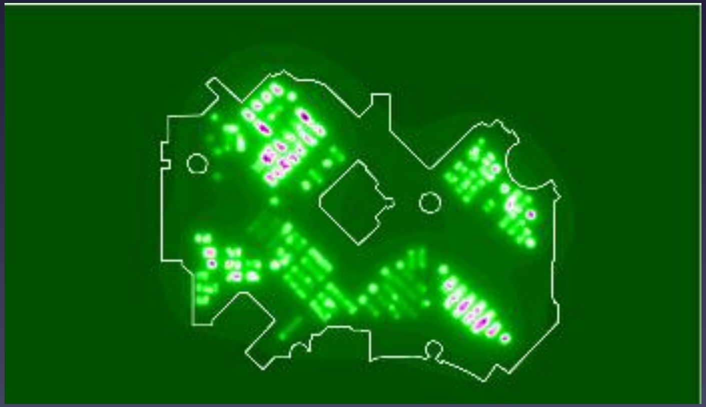




Duty cycle: where the players are



Coin-in: where the money is



Win: where the profits are

nQ

Next Steps

- Time dependent model tracking. See poster (at www.nqube.com)
  - Stationary and Time-Dependent Optimization of the Casino Floor Slot Machine Mix, Anastasia Baran, nQube Data Science.
- Further investigation of model for realistic numbers of machine types (~20)
- Inclusion of accurate pay tables to calculate win.
- Include marketing and player incentive parameters in the model.
- Find academic collaborations and industry partners to apply models to real fine grain data.

The End