Optical network-on-chip architectures and designs

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OPTICAL NETWORK-ON-CHIP

ARCHITECTURES AND

DESIGNS

by

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ABSTRACT

Optical Network-on-Chip
Architectures and Designs

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As indicated in the latest version of ITRS roadmap, optical wiring is a viable interconnection technology for future SoC/SiC/SiP designs that can provide broad band data transfer rates unmatchable by the existing metal/low-\(k\) dielectric interconnects. In this dissertation study, a set of different optical interconnection architectures are presented for future on-chip optical micro-networks.

Three Optical Network-on-Chip (ONoC) architectures, i.e., Wavelength Routing Optical Network-on-Chip (WRON), Redundant Wavelength Routed Optical Network (RDWRON) and Recursive Wavelength Routed Optical Network (RCWRON) are proposed. They are fully connected networks designed based on passive switching Microring Resonator (MRR) optical switches. Given enough different routing optical wavelengths, between any two nodes in the system a bi-directional communication channel can be built. WRON, RDWRON and RCWRON share the similar network structure with different specialties that fit to different applications.

A new topology of packet switching NoC architecture, i.e., Quartered Recursive Diagonal Torus (QRDT) is proposed. It is designed by overlaying diagonal torus. Due to its small diameter and rich routing recourses, QRDT leads to highly scalable NoCs.
By combining WRON’s interconnection property and QRDT’s network topology, a group of 2D-Torus based Packet Switching ONoC (TON) architectures is proposed. The TON is further refined to a generalized open-topology ONoC architecture, called Generalized 2D-Torus-based Optical Network-on-Chip (GTON). The communication protocol in TON is packet switching. The advantages of GTON stem from Wavelength Division Multiplexing (WDM), Direct Optical Channel (DOC) and MRR passive switching. As result, GTON architecture is highly scalable, has an ultra-high bandwidth, consumes a low power, and supports fault-tolerant routing. The work includes other issues such as channel design, analyses of the transmission power loss and the buffer.
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CHAPTER 1
INTRODUCTION

Transistor integration is a technology trend. It is estimated that by 2015, 100 billion transistors can be integrated on a 300 mm$^2$ die [1]. This enables a large number of processing/IP cores integration into a multiprocessor system-on-chip (MPSoC) or chip multiprocessor (CMP) design. The International Technology Roadmap for Semiconductors (ITRS) has projected a Tera-scale computing 3D SiP by 2015, in which the target number of cores integrated on a chip is 1,000 [1]. The interconnection and associated communication infrastructures play a central role for the performance of MPSoCs [2]. For MPSoCs, a vital challenge is to realize a scalable on-chip communication infrastructure that meets the large bandwidth capacities and stringent latency in a power-efficient fashion [3]. Networks-on-Chip (NoC) can improve the on-chip communication bandwidth of MPSoCs and has been widely adopted as an alternative to the traditional bus-based on-chip communication. However, with the continuously shrinking size of features and increasing complexity of NoCs, traditional metallic interconnected NoCs cannot satisfy the bandwidth and latency requirements within the on-chip power budget due to its limited bandwidth, long delay, and relatively high power consumption [4].

Two important issues come out. 1) Material and signal front: optics (optical signal) to replace metal (electrical signal) based interconnection (signal). Looking further into the future, optical wiring could significantly raise the performance limits hindered by metal/dielectric interconnects [5]. Optical fibers are capable of carrying encoded optical data in terabits per second while maintaining near speed-of-light limited transit latencies.
Moreover, the power consumed by optical interconnect is almost independent of the interconnect length [7], and is much less compared with electrical interconnect (around 1/10 in general) [8]. 2) Architecture front: it is inevitable that Network on Chip (NoC) will replace the traditional SoC architecture. A NoC system is composed of a large number of processing units communicating to other units through an interconnection network. This interconnection network plays an important role in achieving high performance, scalability, power efficiency, and fault tolerance.

These two issues when combined lead to Optical Network on Chip (ONoC) (Figure 1 [9]). With the potential of delivering performance-per-watt scaling, ONoC has been considered as a promising candidate to overcome the limitations of electronic NoCs, which enables high bandwidth and low contention routing of data [6] using Wavelength Division Multiplexing (WDM)-enabled optical waveguides [10]. Here optical switch [11] and waveguides [12] are used in ONoC to realize the same function as a conventional electrical router but with routing based on wavelength and with no need for an arbiter [13].

Basically optical interconnection can bring the following advantages:

- Ultra-high bandwidth (Tera-scale);
- Extremely low energy consumption/transmission latency;
- Negligible electromagnetic interference (EMI);
- Unlimited chip size.
For example, optical data routing in a 64 cores chip could have 40 times the power efficiency of the traditional wire connections in estimation [14].

Conceptual 3D Integration [15, 16] (3DI) of future ONoC CMOS ICs is shown in Figure 2 [5, 17].
The main advantage of optical interconnection relies on the bit-rate-transparent property of photonic medium [18]. This property facilitates the transmission of a very high bandwidth in optical networks while avoiding the high power cost typically associated with them in traditional electronic networks [19]. At a chip-scale, the power dissipated on a photonic link is independent of transmission distance because of the ultra-low loss in optical waveguides. Energy dissipation remains essentially the same whether a message travels between two cores that are 2mm or 2cm apart [20]. Furthermore, multiple optical signals of different wavelengths can be transmitted within the same waveguide without confliction with wavelength-division multiplexing (WDM) technology [21, 22] (Figure 3), and that can extremely increase the optical communication bandwidth.

The dissertation study presented in this paper targets to the architecture of optical interconnected network-on-chip architectures. A set of different interconnect architectures are proposed for future on-chip optical micro-networks.

The first Optical Network-on-Chip (ONoC) architecture proposed is the Wavelength Routing Optical Network-on-Chip which is referred as WRON. Both its construction scheme and routing algorithm are generalized. WRON is a fully connected network. Given enough different routing optical wavelengths, between any two nodes in the system a bi-directional communication channel can be built. And all these channels are non-conflicting to each other.

With WRON as the primitive platform, a new redundant architecture referred as the Redundant Wavelength Routed Optical Network (RDWRON). The routing algorithm of RDWRON is generalized and presented. RDWRON is also a fully connected network too.
Given enough different routing optical wavelengths, between any two nodes in the system multiple bi-directional communication channels can be built. And all these channels, each between two nodes or among any pairs of nodes are all non-conflicting to each other. This property can be applied to build micro-networks to interconnect multicore clusters.

Based on RDWRON architecture, a new recursive architecture is proposed which is referred as the Recursive Wavelength Routed Optical Network (RCWRON). The routing algorithm of RDWRON is generalized and presented. RCWRON is a fully connected network as well. As recursive architecture RCWRON possesses better scalability and fault isolating capability.

WRON, RDWRON and RCWRON are all fully connected networks. Each different pair of two nodes in the network communicates via light in a different optical wavelength. And obvious the total number of different wavelength can be integrated on a single chip is very restricted, normally no higher than the level of 10s. Hence the scalability of fully connected ONoC structures is limited, which means such kind of architectures cannot be applied to manycore systems.

On the other side, a new NoC architecture is propose, referred as the Quartered Recursive Diagonal Torus (QRDT), which is constructed by overlaying diagonal torus. Due to its small diameter and rich routing recourses, QRDT is determined to be well suitable to construct highly scalable NoCs. In QRDT, data packets can be routed through a proposed minimal routing algorithm based on the Johnson codes that have traditionally been used in finite state machine designs. It has been shown that this proposed routing algorithm with minor modifications is capable of handling the single link/node failure.
To solve the limitation of fully connected ONoC architecture, a new group of ONoC architectures is proposed, based on the combination of WRON and QRDT by which the former one is adopted as the optical interconnection device and the later one’s network topology is applied. The first is a family referred as 2D-Torus based Packet Switching ONoC, or TON architectures. As it is named, data communication in TON is implemented via packet switching. By employing a limited number of different routing wavelengths in Wavelength Division Multiplexing (WDM), each node in the TON network has the same number of optical channels. Then the communication between a pair of remote nodes can be achieved by relaying data packets among different optical channels of intermediary nodes between this pair of remote nodes. Moreover, a new concept called Directed Optical Channel (DOC) is introduced on the first time in TON architecture. The DOC is a realization of the channel with a light path between two nodes that consists of one or more physical links and optical interconnection units. By utilizing a predetermined routing wavelength and with properly designed optical interconnection units light can travel through the DOC without any relay or arbitration. DOCs can be built between any two nodes. Packet Switching, WDM and DOC, this combination enables the TON architecture unlimited scalability, ultra-high bandwidth, low power consumption (compare with optical tuning architectures) and practical feasibility. And the transmission latency is also maintained in an acceptable level.

Totally three different TON architectures are presented, denoted as TON-I, TON-II and TON-III, with 4, 8 and 10 different DOCs in each node in the network architecture, respectively. And the routing algorithms for each architecture are addressed in detail, with the network performance evaluation presented with simulation results.
The topologies of TON architectures are fixed. But for different purposes networks in different topologies may be required. From this view, an improved version of TON architecture is proposed, denoted as Generalized 2D-Torus-based Optical Network-on-Chip (GTON), which is a generalized packet switching optical NoC. Given sufficient routing wavelengths, any network topology can be implemented in GTON. A systematic approach of the design of network interconnections and routers is presented in detail. The use of passive MRR switching, Wavelength Division Multiplexing (WDM) and Direct Optical Channels (DOC) allows for attaining high bandwidth and makes GTON fault tolerant. The number of different routing wavelengths required in GTON architecture is limited. Simulation results presented has shown the performance of the architecture. Related issues such as channel design, transmission power loss and the buffer analysis of GTON are also provided explicitly.

![Integrated Transmitter Die](image1.png)

![Integrated Receiver Die](image2.png)

**Figure 3**  Wavelength Devison Multiplexing (WDM) [22]

The dissertation is organized as follows. 0 introduces the concepts of ONoC and
contributions of the dissertation study. Chapter 2 presents a survey of latest breakthroughs in developing on-chip optical components/devices, and a summary of most ONoC architectures proposed so far. From 0 to 0, three different fully connected ONoC architectures WRON, RDWRON and RCWRON are presented explicitly with their routing algorithms generalized in detail. 0 presents an improved network topology QRDT which is the foundation of the later work. Based on the QRDT architecture, the packet switching ONoC architect family TON is presented in 0 with three typical cases TON-I, II and III. And their routing algorithms and related network properties are also addressed in this chapter. Furthermore, in 0 the TON architecture is refined and a generalized open-topology packet switching ONoC architecture GTON is presented with detail design scheme. And routing algorithm plus network properties analyses are presented in this chapter. Finally Chapter 9 concluded the dissertation study with the schedule of future exploration.
CHAPTER 2

OPTICAL NETWORK-ON-CHIP OVERVIEW

2.1 Optical Interconnection Components

In the past several years, the field of silicon photonics attracts attention of industrial and academic research groups [23]. A great number of essential silicon photonic components have been created such as on-chip laser sources, optical modulators[24, 25], waveguides [26] and photodetectors. Intel has announced a Tera-scale laser modulator in 2007 [24]. The first Ge-on-Si laser has been invented by MIT in March 2010 [27], while IBM invented a germanium avalanche photodetector used on-chip around the same time [28, 29]. These demonstrated silicon photonic devices and technologies make optical network-on-chip (ONoC) viable.

2.1.1 On-Chip Lasers

2.1.1.1 Raman Laser

A major attraction of Raman lasers is their ability to use light from an optical “pump” to generate coherent light emission in wavelength regions that are hard to reach with other conventional types of lasers. In addition, Raman lasers can be made from materials such as silicon that do not possess suitable energy band structures to produce laser light by stimulated emission.

The silicon Raman laser was constructed from a low loss silicon-on-insulator (SOI) rib waveguide in a ring cavity configuration, as shown in Figure 4 [22, 30]. The racetrack-shaped ring laser cavity was connected to a straight bus waveguide by means of a directional coupler that couples both pump and Raman lasing signals into and out of the cavity. The pump coupled into the laser cavity generates optical gain by stimulated
Raman scattering inside the silicon waveguide at the first-order Stokes wavelength, which is 15.6 THz redshifted from the pump. The gain increases with pump power, and the lasing threshold is reached when the gain equals the total cavity loss. The first-order Raman lasing then starts, and the laser output increases with increasing pump power [30-32].

![Schematic of Raman laser](image)

**Figure 4**  
Schematic of Raman laser [30]

### 2.1.1.2 Vertical Cavity Surface Emitting Lasers (VCSEL)

VCSELs are semiconductor devices with light emission perpendicular to the chip surface [33-35], as shown in Figure 5 [36]. It offers several advantages compared to conventional edge-emitting (in-plane) laser diodes, such as:

- Low electric power consumption;
- Capability of on-wafer testing;
- Simplified fiber coupling and packaging;
- Longitudinal single-mode emission spectrum;
- Suitability for 2d-array integration;
- Continuous-wave mode operation at room temperature.

**Figure 5** Schematic of the vertical cavity surface emitting laser [36]

### 2.1.1.3 Mode-Lock Evanescent Lasers (MLLs)

MLLs are capable of generating stable short pulses which have a corresponding wide optical spectrum of phase correlated modes. MLLs can have high extinction ratios, low jitter, and low chirp, making them excellent transmitters when combined with a data-encoding modulator [37, 38].

The mode locked laser consists of a long gain section and a short absorbing section, which are separated by electrical isolation regions. The absorber pad was designed to match a coplanar strip line probe to enable efficient injection of RF signals for hybrid mode locking. P metal pads connect to p metal above the optical waveguide, and n metal pads connect to n metal on both sides of the mesa via a metal bridge. The absorber p
contact appears to overlap the gain section’s n contact, but it is isolated by a layer of SiO2 between the two metals, as presented in Figure 6.

Figure 6  Schematic of the mode locked silicon evanescent laser [37].

2.1.1.4 Ge-on-Si Laser

On Feb. 2010, MIT made the first germanium-on-silicon laser as shown in Figure 7, which has the following properties [27].

- Can produce wavelengths of light useful for communication in multi-core system;
- The first germanium laser can operate at room temperature;
- Much less cost;
- Germanium is easy to incorporate into silicon chips – no longer “external lasers”.
2.1.2 Optical Modulators

The traditionally large device size of active optical components, particularly, the size of silicon optical modulators [39-42] presents a critical obstacle to the large-scale high-density optoelectronic integration on silicon using the well-established very-large-scale integration (VLSI) technology. This problem originates from the intrinsically weak refractive-index tuning ability possessed by silicon material [43]. Besides the size issue, comparatively low device speed is another serious concern associated with silicon optical modulators. Although extensively theoretical work has been performed since the late 1980’s to exploit the high-speed switching capability of silicon modulator in the gigahertz (GHz) domain [44-47], the experimental breakthrough came in 2004 when Liu et al. first experimentally demonstrated a gigahertz all-silicon optical modulator [42, 48].

2.1.2.1 Mach-Zehnder Modulator (MZM)

In the MZM Optical Modulator, the high-speed modulation was obtained in a metal-oxide-semiconductor (MOS) capacitor based Mach-Zehnder interferometer (MZI)
structure through the plasma dispersion effect (or free carrier dispersion effect), which is the most effective means of electrically modifying the refractive index of silicon [43].

The Mach-Zehnder interferometer silicon modulator contains two reverse-biased p-n diode phase shifters (a). The splitters are multimode interference (MMI) couplers. The radio-frequency (RF) signal is coupled to the traveling-wave electrode from the optical input side, and termination load is added to the output side. The cross-sectional view in (b) shows a p-n junction waveguide phase shifter on silicon on insulator. The refractive index modulation is based on the depletion width variation in response to the reverse bias voltage caused by the free-carrier plasma dispersion effect in silicon. The coplanar waveguide electrode was designed to match the electrical and optical velocities, as shown in Figure 8 [49].

![Figure 8 MZM optical modulator [22, 49]](image)

2.1.2.2 Cascaded Silicon Micro-ring Modulator

A cascaded modulator is fabricated on a SOI substrate. The device structure is based on the micro-ring modulator. It consists of ring resonators embedded with PIN junctions
used to inject and extract free carriers, which in turn modify the refractive index of the silicon and the resonant wavelength of the ring resonator using the mechanism of the plasma dispersion effect. The waveguides and rings are formed by silicon strips with the height of 200nm and the width of 450nm on top of a 50nm-thick slab layer. The radii of the four ring resonators are designed to be 4.98μm, 5.00μm, 5.02μm, and 5.04μm, respectively. The difference in the radii corresponds to a channel spacing of 3.6nm. The modulator operates at 4 Gbit/s (updated to 12.5 Gbit/s), as shown in Figure 9 [50].

![Figure 9](image.png)

**Figure 9** Schematics of a WDM optical interconnection system [50].

2.1.3 Photodetectors

2.1.3.1 PIN Photodetector

Figure 10 presents a photodiode (PD) structure with deep n-well (DNW) fabricated in an epitaxial substrate complementary metal–oxide–semiconductor (epi-CMOS) process is presented. The DNW buried inside the epitaxial layer intensifies the electric field deep inside the epi-layer significantly, and helps the electrons generated inside the epi-layer to drift faster to the cathode. Therefore, this new structure reduces the carrier transit time
and enhances the PD bandwidth. A PD with an area of $70 \times 70 \mu m^2$ fabricated in a 0.18 $\mu m$ epi-CMOS achieves 3-dB bandwidth of 3.1 GHz in the small signal and 2.6 GHz in the large signal, both with a 15-V bias voltage and 850-nm optical illumination. The responsivity is measured 0.14 A/W, corresponding to a quantum efficiency of 20%, at low bias. The responsivity increases to 0.4 A/W or 58% quantum efficiency at 16.2-V bias in the avalanche mode [51].

![Cross section of a PIN PD with DNW in an epi-CMOS process](image)

**Figure 10** Cross section of a PIN PD with DNW in an epi-CMOS process [51]

### 2.1.3.2 Germanium Avalanche Photodetector

Figure 11 shows a schematic of a waveguide-integrated Ge APD, with modeling and scanning electron microscope (SEM) cross-sections. The thicknesses and widths of both the Ge and Si layers were optimized to ensure the highest responsivity within the smallest possible footprint. The resulting thicknesses of 140 nm and 100 nm, and widths of 750 nm and 550 nm for the Ge and Si layers, respectively, provide propagation of at most only two optical modes in the combined layer stack for the transverse electric field polarization at both the 1.3 $\mu$m and 1.5 $\mu$m wavelengths [52]. This optical design allows
efficient coupling of light from the rooting silicon waveguide up into the fundamental mode in the Ge layer, which is further facilitated by a short Ge taper, as shown in Figure 11 (a). The resulting optical power resides almost completely (confinement factor of over 77%) in the top Ge layer. Besides ensuring a very short absorption length of the order of 10 μm even for a wavelength of 1.5 μm, this design allows us to minimize the APD capacitance to as low as 10 fF [29, 51, 52].

![Figure 11](image_url)  
Figure 11  
Schematic of Germanium avalanche photodetector [29]

2.1.4 Micro Ring Resonator (MRR) Optical Switch

An MRR optical switch is a resonating structure, and is most commonly used in “add-drop” filters [10, 53] (named so because of their capacity to add or subtract a signal from a waveguide based on its wavelength).

A basic MRR structure, which is commonly used in “add-drop” filters, is shown in Figure 12(a). The MMR is associated with a resonant wavelength, e.g. \( w_i \), which is determined by the geometric and structural parameters of it MRR, as indicated in the
following [54].

\[ mw_i = n_{eff}L \]  \hspace{1cm} (1)

where \( m \) is an integer, \( n_{eff} \) is the effective index of the optical mode, and \( L \) is the length of the resonating cavity. The operation of the MMR filter can be viewed as a wavelength-controlled switch. The operation of the switch depends on the wavelength of the signal entering the input of the structure, \( w_p \). If \( w_p = w_i \), the signal passes through the resonator, otherwise, it goes straight along the input direction.

As shown in Figure 12, one switch is composed of one or more identical microrings evanescently side-coupled to signal waveguides [55]. The electromagnetic field is propagated within the structure only for modes corresponding to specific wavelengths, where these resonant wavelength values are determined by geometric and structural parameters (substrate and microring material index, thickness and radius of microring) [56].

(a) Micronring Resontor (MRR)
The basic function of an optical switch can be viewed as a wavelength-controlled switch. The operation of the switch depends on the wavelength of the signal entering at one of the inputs of the bidirectional add-drop filter, \( w_p \). Each filter is associated with a resonant wavelength, e.g., \( w_i \) for the switch shown in Figure 12 and its real action photos.
are shown in Figure 14 [22]. For any input signal from \( w_p \), the signal will propagate to both filters. If \( w_p = w_i \) (tolerance is of the order of a few nanometer, depending on the coupling strength between the disk and the waveguide), \( w_p \) passes through the switch on the same direction as the input signal (referred as the “straight” function); if \( w_p \neq w_i \), the signal will pass through the switch on the cross direction (referred as the “across” function), as shown in Figure 13. Noticeably, the input and output of the optical switch are reversible. However, to avoid conflicts inside the optical switch (caused by the signals sent in opposite directions), it is not allowed to let inputs at opposite directions come to an optical switch simultaneously.

![Figure 13](image.png)

(a) Go straight.  
(b) Go across.

**Figure 13**  
Basic functions of the optical switch.

The advantages of such structures lie in the possibility of building highly complex, dense and passive on-chip switching networks. One application of this device is in optical crossbar networks. More elaborate \( N \times N \) switching networks have been reported in [56],
although their functionalities are subject to be verified experimentally. The optical switch shown in Figure 12 can be used to build highly complex, dense and passive on-chip switching networks, as exemplified in can be applied to build 4×4 ONoC [10]. However, across the literature, there has been no general discussion of the network properties of the ONoC. In light of this special case of ONoC structure shown in [10], here we attempt to develop a generalized $N \times N$ (where $N$ represents the number of input/output nodes) optical interconnection network suitable for ONoC. Following the same naming convention as adopted in [10], we shall name this network structure as Wavelength Routed Optical Network (WRON).

![MRR optical switch functions](image)

(a) MRR switch off  
(b) MRR switch on

Figure 14  
MRR optical switch functions

2.2  Current Works on ONoC Architectures

So far many different architectures have been proposed for future optical interconnection multi-core systems. These architectures can be divided into two groups: 1) ONoCs without MRR [57], and 2) ONoCs with MRR [5, 8, 14, 25, 54, 58-68]. The second type of ONoCs is based on the CMOS fabrication technologies, and thus has
many advantages over the former type in accuracy, reliability, control complexity and fabrication. MRR resonantly tunnels light through it when the light frequency is within its pass band width and rejects it otherwise.

The pass band width of an MRR is shiftable with thermal-optical (TO) or electrical-optical (EO) effect tuning of the inter-resonator coupling strengths [69]. Hence the second group can be classified into two classes: i) active switching networks with TO or EO tuning [8, 14, 58, 67, 70], and ii) passive routing networks using fixed wavelength assignment [54, 60-64].

For active MRR tuning ONoC architectures, Kirman et al. [5] have proposed a clustered electro-optical multicore system. However, this bus-based design has scalability limits. Shacham et al. [8] have proposed a circuit-switched on-chip optical network that uses an optical network for large packet transmission and an electronic network for both the control data and small packets transfer. Pan et al. [62] proposed a hybrid hierarchical architecture in which intra-cluster communication is based on electrical signaling and inter-cluster communication is carried on multiple optical crossbars. To avoid global switch arbitration, the crossbar is partitioned into multiple, smaller crossbars and the arbitration is localized. Cianchetti et al. [61] proposed another switch-based hybrid on-chip optical network that uses source-based routing and reconfigurable optical switches. Batten et al. [60] proposed an optical NoC based on mesh and global crossbar, where optical interconnect is used for the high throughput traffic and metallic interconnect for local and fast switching. Gu et al. [67] proposed a fat tree-based circuit-switched optical NoC.

As tuning (either TO or EO) is in general adopted to shift the passband of MRRs,
these active optical NoCs are wavelength saving networks. However, an extra layer for tuning and controlling is required to be integrated with optical signal transmissions, which is difficult to realize and also increases the complexity and costs. Moreover, the wavelength tuning range of EO tuning is limited (about 2nm [14]). Although TO tuning can provide about 20nm shift [25], it requires an extra time in scale of a few μs for direct heating, which causes extra power consumption.

In contrast, passive optical NoCs are based on wavelength routing and thus no extra circuits are needed for switching. Moreover MRR routing architectures support WDM technology and perform routing based on their wavelengths. Ultra-high-bandwidth signal transmission can be obtained in such networks but this requires as many light sources as distinct paths in the network [14]. For passive optical NoCs, Zhang et al. [54] proposed a generic wavelength routed optical architecture, namely WRON that uses cascaded MRR-based 2×2 optical switches. It is a WDM-supported non-blocking routing passive optical NoC. Similarly, Briere et al. [65] proposed a wavelength-routed multi-stage passive optical routing structure that uses multiple 2×2 switching elements called λ router. Both Zhang’s and Briere’s designs require large arrays of fixed-wavelength sources with fast wavelength-selection switches. Kirman et al. [66] proposed an all-optical network that combines wavelength-based oblivious routing, passive optical routers, and connection-based operation.

Passive architectures currently generally use a big number of wavelengths and MRRs. As every node has a separate port to all other nodes’ data channels, it requires O(N^2) modulators/transmitters, even though only O(N) of them are active at a time. For example, the Corona [64] has 256 wavelengths and 1056k MRRs for its 256-core network. It is a
high-bandwidth, low-latency optical crossbar that uses token-based optical arbitration to serialize data transmissions to each node.

Practically, according to Intel’s announcement in July 2010, there are only four lasers integrated on a chip in their latest photonic prototype [71]. Given the current technology constraints one of possible architectural solutions is to limit the number of direct interconnections and use the packet switching. Under the technology limitation packet switching with a limited number of direct interconnections is a resolving approach.

The rest of the paper is organized as follows. Section II introduces the operating mechanism of basic optical switches. Section III presents the basic structure of the WRON, and Section IV details the routing scheme of WRON. In Section V, we present the RDWRON as the basic building block to construct the RCWRON. Section VI presents the structure of RCWRON, followed by its routing scheme shown in Section VII. Section VIII concludes the paper with suggestions for future exploration.
CHAPTER 3

WAVELENGTH ROUTED OPTICAL NETWORK-ON-CHIP ARCHITECTURES

In this chapter a generalized wavelength routing ONoC architecture denoted as WRON is presented.

3.1 Basic Structures of WRON

The generalized WRON is composed of input/output nodes and multiple stages of optical switches. In WRON, the number of stages is found equal to the number of input/output nodes, except for the case when only 2 input/output nodes are present. At any stage, all the optical switches within it share the same resonating wavelength.

The structure of an $N$-input/output WRON, hereafter denoted as $N$-WRON, is dependent on the value of $N$. Basically there are two types of WRON.

3.1.1 WRON Type I

WRON type I has the following properties.

- When $N$ is an odd number (i.e., there are odd numbered input/output nodes), there are $(N-1)/2$ switches in each of $N$ stages.

- When $N$ is an even number, there are $N/2$ switches in each of the odd-numbered stages, and $(N/2)-1$ switches in each of the even-numbered stages.

Lemma 1. The number of optical switches in an $N$-WRON is

\[
\frac{N \times (N-1)}{2}
\]

Proof:

When $N$ is even, the number of optical switches is

\[
\frac{N}{2} \times \frac{N}{2} + \left( \frac{N}{2} - 1 \right) \times \frac{N}{2} = \frac{N \times (N-1)}{2}.
\]
When $N$ is odd, the number of optical switches is

$$\frac{N - 1}{2} \times N = \frac{N \times (N - 1)}{2}.$$  

Hence, Lemma 1 holds. \hfill \blacksquare

As an example, the structure of type I 4-WRON and 5-WRON are shown in Figure 15(a) and (b), respectively.

![Type I WRON](image)
In a type I WRON, all ports (nodes) in the network are labeled as follows.

- Denote the $p^{th}$ source node of an $N$-WRON as $S_p$, and the $q^{th}$ destination node as $D_q$.
- When $N$ is an odd number, label the first and the second output ports (input ports) of the $m^{th}$ switch at the $n^{th}$ stage as $O(2m-1,n)$ and $O(2m,n)$ ($I(2m-1,n)$ and $I(2m,n)$), respectively.
- When $N$ is an even number, label the first and the second output ports (input ports) of the $m^{th}$ switch at the $n^{th}$ stage as $O(2m,n)$ and $O(2m+1,n)$ ($I(2m-1,n)$ and $I(2m,n)$), respectively.

The connection of all optical switches of an $N$-WRON can be clearly described by an $N \times (N+1)$ connection matrix. In the connection matrix, only the ports (nodes) of the prior stage connected to the current input ports of the switches or the destination nodes needed to be considered.

Except the entries in the last column, any of the remaining entries in the connection matrix, denoted as $C(i,j)$, is the index of the output port (or source node) that the $i^{th}$ input port at the $j^{th}$ stage connects to. The $k^{th}$ entry in the $(N+1)^{th}$ column in the connection matrix specifies the output port which connects to destination node $D_k$. When there is no port connection, $C(i,j)$ is set to zero. This zero value also indicates a logical link that will bypass the $j^{th}$ stage’s switches (i.e., a link that crosses two stages).

The connection matrix can be constructed as follows:

**Case 1.** (When $N$ is an even number)
\[ C(i, j) = \begin{cases} 
S_i & \text{when } j = 1 \\
0 & \text{when } j = 2p & 1 < j \leq N & i = 1 \\
0 & \text{when } j = 2p & 1 < j \leq N & i = N \\
O(i, j - 2) & \text{when } j = 2p + 1 & 1 < j \leq N + 1 & i = 1 \\
O(i, j - 2) & \text{when } j = 2p + 1 & 1 < j \leq N + 1 & i = N \\
O(i, j - 1) & \text{when } j > 1 & 1 < i < N 
\end{cases} \]

Case 2. (When \( N \) is an odd number)

\[ C(i, j) = \begin{cases} 
S_i & \text{when } j = 1 & i < N \\
S_N & \text{when } j = 2 & i = N \\
0 & \text{when } j = 2p & 1 < j < N & i = 1 \\
O(i, j - 2) & \text{when } j = 2p & 2 < j \leq N + 1 & i = N \\
O(i, j - 2) & \text{when } j = 2p + 1 & 1 < j < N & i = 1 \\
0 & \text{when } j = 2p + 1 & 1 \leq j \leq N & i = N \\
O(i, j - 1) & \text{when } j = N + 1 & i = 1 \\
O(i, j - 1) & \text{when } j > 1 & 1 < i < N 
\end{cases} \]

As an example, the connection matrix of type I 4-WRON shown in Figure 15(a) is given as:

\[
\begin{bmatrix}
S_1 & 0 & O(1,1) & 0 & O(1,3) \\
S_2 & O(2,1) & O(2,2) & O(2,3) & O(2,4) \\
S_3 & O(3,1) & O(3,2) & O(3,3) & O(3,4) \\
S_4 & 0 & O(4,1) & 0 & O(4,3)
\end{bmatrix}
\]

The connection matrix of the type I 5-WRON shown in Figure 15(b) is given as:

\[
\begin{bmatrix}
S_1 & 0 & O(1,1) & 0 & O(1,3) & O(1,5) \\
S_2 & O(2,1) & O(2,2) & O(2,3) & O(2,4) & O(2,5) \\
S_3 & O(3,1) & O(3,2) & O(3,3) & O(3,4) & O(3,5) \\
S_4 & O(4,1) & O(4,2) & O(4,3) & O(4,4) & O(4,5) \\
0 & S_5 & 0 & O(5,2) & 0 & O(5,4)
\end{bmatrix}
\]

3.1.2 WRON Type II

WRON type II has the following properties.

- When \( N \) is an odd number, there are \((N-1)/2\) switches in each of the \( N \) stages.
When $N$ is an even number, there are $(N/2)-1$ switches in each of the odd-numbered stages, and $N/2$ switches in each of the even-numbered stages.

As an example, the structure of type II 4-WRON and 5-WRON are shown in Figure 15(a) and (b), respectively.

Following the same notation, the connection matrix of type II WRON can be constructed as follows:

Case 1. (When $N$ is an even number)

$$C(i, j) = \begin{cases} S_i & \text{when } j = 1 & 1 < i < N \\ 0 & \text{when } j = 2p + 1 & 1 \leq j < N & i = 1 \\ 0 & \text{when } j = 2p + 1 & 1 \leq j < N & i = N \\ O(i, j - 2) & \text{when } j = 2p & 2 < j \leq N & i = N \\ O(i, j - 2) & \text{when } j = 2p & 2 < j \leq N & i = N \\ O(i, j - 1) & \text{when } 1 < j \leq N & 1 < i < N \\ O(i, N) & \text{when } j = N + 1 \end{cases}$$

Case 2. (When $N$ is an odd number)

$$C(i, j) = \begin{cases} S_i & \text{when } j = 1 & 1 < i \leq N \\ S_i & \text{when } j = 2 & i = 1 \\ 0 & \text{when } j = 2p & 1 < j < N & i = N \\ O(i, j - 2) & \text{when } j = 2p & 2 < j \leq N + 1 & i = 1 \\ O(i, j - 2) & \text{when } j = 2p + 1 & 1 < j \leq N & i = N \\ 0 & \text{when } j = 2p + 1 & 1 \leq j \leq N & i = 1 \\ O(i, j - 1) & \text{when } 1 < j \leq N & 1 < i \leq N \\ O(i, j - 1) & \text{when } j = N + 1 & 1 < i \leq N \end{cases}$$

As an example, the connection matrix of the type II 4-WRON shown in Figure 16 (a) is given as:

$$
\begin{bmatrix}
0 & S_1 & 0 & O(1,2) & O(1,4) \\
S_2 & O(2,1) & O(2,2) & O(2,3) & O(2,4) \\
S_3 & O(3,1) & O(3,2) & O(3,3) & O(3,4) \\
0 & S_4 & 0 & O(4,2) & O(4,4)
\end{bmatrix}
$$
The connection matrix of the type II 5-WRON shown in Figure 16 (b) is given as:

\[
\begin{bmatrix}
0 & S_1 & 0 & O(1,2) & 0 & O(1,4) \\
S_2 & O(2,1) & O(2,2) & O(2,3) & O(2,4) & O(2,5) \\
S_3 & O(3,1) & O(3,2) & O(3,3) & O(3,4) & O(3,5) \\
S_4 & O(4,1) & O(4,2) & O(4,3) & O(4,4) & O(4,5) \\
S_5 & 0 & O(5,1) & 0 & O(5,3) & O(5,5)
\end{bmatrix}.
\]

Figure 16 Type II WRON
Type I WRON and type II WRON are closely related. When $N$ is even, swapping the input and output nodes of a type I WRON will convert it to a type II WRON. When $N$ is odd, rearranging the input and output nodes of type I WRON in a reversed order will convert it to a type II WRON. Therefore, the structure of type I and II WRON are isomorphic to each other, and the routing problems of type II WRON can be solved using the same solution to type I WRON combined with a simple linear numeric transform. In the following, we shall focus our study on type I WRONs only.

3.2 Routing Scheme of WRON and Its System Organization

In WRON, each routing path $P_i$ is associated with a tri-tuple $<S, D, W>$, where $S$ denotes the source node address, $D$ denotes the destination node address, and $W$ is the assigned routing wavelength for the data transmission. All the wavelength assignments of a 4-WRON (Figure 15 (a)) are tabulated in Table 1. For instance, to send data from source node $S_1$ to destination node $D_3$, only wavelength $w_1$ can be used. From the same table one can see that by using four different wavelengths, $S_1$ can reach four destinations using the same wavelength; different sources can reach different destinations in a non-blocked fashion. 0 shows the wavelengths assignment for a 5-WRON (Figure 15(b)).

<table>
<thead>
<tr>
<th>$W$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$w_2$</td>
<td>$w_3$</td>
<td>$w_1$</td>
<td>$w_4$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$w_3$</td>
<td>$w_4$</td>
<td>$w_2$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$w_1$</td>
<td>$w_2$</td>
<td>$w_4$</td>
<td>$w_3$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$w_4$</td>
<td>$w_1$</td>
<td>$w_3$</td>
<td>$w_2$</td>
</tr>
</tbody>
</table>
Table 2 The wavelength assignment of 5-WRON.

<table>
<thead>
<tr>
<th></th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$w_3$</td>
<td>$w_2$</td>
<td>$w_4$</td>
<td>$w_1$</td>
<td>$w_5$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$w_4$</td>
<td>$w_3$</td>
<td>$w_5$</td>
<td>$w_2$</td>
<td>$w_1$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$w_2$</td>
<td>$w_1$</td>
<td>$w_3$</td>
<td>$w_5$</td>
<td>$w_4$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$w_5$</td>
<td>$w_4$</td>
<td>$w_1$</td>
<td>$w_3$</td>
<td>$w_2$</td>
</tr>
<tr>
<td>$S_5$</td>
<td>$w_1$</td>
<td>$w_5$</td>
<td>$w_2$</td>
<td>$w_4$</td>
<td>$w_3$</td>
</tr>
</tbody>
</table>

In general, for an $N$-WRON, given any two of the three parameters ($S$, $D$, or $W$), the routing path is uniquely determined and the last parameter can be derived from the two known parameters as follows.

**Proposition 1.** For an $N$-WRON, given the source node address $S$ and the routing wavelength $W$, the destination node address $D$ can be uniquely determined as

$$D = f_D(N, S, W) = \begin{cases} 
1 - D^* & \text{if } D^* \leq 0 \\
D^* & \text{if } 0 < D^* \leq N \\
2 \times N + 1 - D^* & \text{if } D^* > N 
\end{cases} \quad (3)$$

where $D^* = S + (N - 2W + 1) \times (-1)^S$.

**Proposition 2.** For an $N$-WRON, given the destination node address $D$ and the routing wavelength $W$, the source node address $S$ can be uniquely determined as

$$S = f_S(N, D, W) = \begin{cases} 
1 - S^* & \text{if } S^* \leq 0 \\
S^* & \text{if } 0 < S^* \leq N \\
2 \times N + 1 - S^* & \text{if } S^* > N 
\end{cases} \quad (4)$$

where $S^* = D + (N - 2W + 1) \times (-1)^{N+D}$.

**Proposition 3.** For an $N$-WRON, given the source node address $S$ and the destination node address $D$, the routing wavelength $W$ can be uniquely determined as

$$W = f_W(N, S, D) \quad (5)$$

where
\[ W = f_w(N, S, D) \]
\[= \begin{cases} \frac{N + 1 + S - D}{2} & \text{when } S = 2s & D = 2d + 1 \\ \frac{-N + S + D}{2} & \text{when } S = 2s & D = 2d & S + D > N \\ \frac{N + S + D}{2} & \text{when } S = 2s & D = 2d & S + D \leq N \\ \frac{N + 1 - S + D}{2} & \text{when } S = 2s + 1 & D = 2d \\ \frac{3N + 2 - S - D}{2} & \text{when } S = 2s + 1 & D = 2d + 1 & S + D \geq N + 2 \\ \frac{N + 2 - S - D}{2} & \text{when } S = 2s + 1 & D = 2d + 1 & S + D < N + 2 \end{cases} \]

when \( N \) is an even number, and

\[ W = f_w(N, S, D) \]
\[= \begin{cases} \frac{N + 1 + S - D}{2} & \text{when } S = 2s & D = 2d \\ \frac{-N + S + D}{2} & \text{when } S = 2s & D = 2d + 1 & S + D > N \\ \frac{N + S + D}{2} & \text{when } S = 2s & D = 2d + 1 & S + D \leq N \\ \frac{N + 1 - S + D}{2} & \text{when } S = 2s + 1 & D = 2d \\ \frac{3N + 2 - S - D}{2} & \text{when } S = 2s + 1 & D = 2d & S + D \geq N + 2 \\ \frac{N + 2 - S - D}{2} & \text{when } S = 2s + 1 & D = 2d & S + D < N + 2 \end{cases} \]

when \( N \) is an odd number.

The proofs of Proposition 1 to Proposition 3 are given in Appendix 0.

From the above propositions, one can see that in a WRON, any pair of source and destination nodes can be routed without experiencing a conflict when using a unique routing wavelength.

For example, in a 4-WRON (Table 1), source nodes \( S_1, S_2, S_3 \) and \( S_4 \) can simultaneously communicate with the same destination node \( D_1 \), provided routing wavelength \( w_2, w_3, w_4, \) and \( w_4 \), are used. The only constraint applied to the routing in a
WRON is that bidirectional communication between the same pair of nodes cannot be possibly realized. The reason is quite simple, the wavelength used by routing from a source node to a destination node and the wavelength by routing on the reverse direction will be the same. The optical signals with the same wavelength but on opposite directions will cause interference inside an optical switch. This constraint must be observed by the communication protocol applicable to ONoC.

3.3 System Organization

To illustrate the system organization of a WRON, a 4-WRON is shown in Figure 17 [54]. There are a total of 8 processing elements that are connected by a 4-WRON. Each processing element is directly connected to a transmitter block which enables the electro-optical conversion. Each transmitter core consists of laser(s) [72], drivers and a serializer, and each core has a receiver block [73, 74] which enables the opto-electronic conversion. This opto-electronic unit features a PIN photodiode (conversion of flow of photons into photocurrent), a transimpedance amplifier (TIA), a decision circuit (digital signal regeneration) and a deserializer (DES) [55].
3.4 Conclusion

In this chapter a generalized wavelength routed optical micronetwork architecture WRON is presented, and its routing scheme is generalized.

Figure 17  System organization of a 4-WRON.
CHAPTER 4

2-D REDUNDANT OPTICAL NETWORK-ON-CHIP ARCHITECTURES

As shown in 0, a WRON is capable of routing any permutation of a set of input and output nodes given enough wavelengths. However, as WRON is not a recursive structure, a large WRON cannot be built by connecting WRONs in smaller sizes. For example, a 6-WRON cannot be directly obtained from connecting multiple 3, 4 or 5 WRONs.

Based on basic WRON structure introduced above, in what follows, we propose a new recursive structure, the 2-dimensional recursive wavelength routed optical network (2-D RCWRON), and this 2-D RDWRON serves as the basic building block to build 2-D RCWRON. The construction and the routing scheme of RDWRON will be introduced in detail followed by the introduction of the RCWRON in the next section.

4.1 Basic Units in RDWRON

There are two basic units, the Inverse Connector (IC) and WRON, to construct a RDWRON.

4.1.1 Inverse Connector (IC)

The function of an IC is to switch the input signal to specialized output port according to a fixed inverse function. We denote an IC with \( N \) source/destination nodes as \( N \)-IC, and its structure is shown in Figure 18.

**Lemma 2.** For an \( N \)-IC, if the address of a source node is \( S \), the address of its destination node \( D \) is

\[
D = N + 1 - S.
\]
4.1.2 Construction of 2-D Redundant Optical Network

The 2-D redundant wavelength routed optical network (RDWRON) is the basic building block to construct a 2-D RCWRON. A RDWRON with \( N \) input/output nodes is constructed by connecting \( N \) \( N \)-WRON and \( N-1 \) \( N \)-IC alternatively as shown in Figure 19. Wavelengths in different stages in the RDWRON are preset as 1, 2 \( \ldots \) to \( N^2 \) from the first stage of the first \( N \)-WRON to the last stage of the last \( N \)-WRON.

We denote 2-D RDWRON with \( N \) input/output nodes as \( N^2 \)-RDWRON. Figure 20 shows the structures of \( 3^2 \)-RDWRON and \( 4^2 \)-RDWRON.

**Lemma 3.** The total number of switches in one 2-D \( N^2 \)-RDWRON is
\[ O_{RDWRON} = N \times \frac{N \times (N-1)}{2} = \frac{N^2 (N-1)}{2} \] (7)

(a) 3²- RDWRON.

(b) 4²-RDWRON.

Figure 20 Examples of N²-RDWRON.

4.2 Features of 2-D RDWRON

The 2-D RDWRON has the following features:

- A set of different wavelengths can be used so that of the same source and destination pair, there are multiple routing paths.

- Different source nodes can use the same set of wavelengths to reach different destination nodes. These different wavelengths can be used to all source nodes to share the same property.

For an \(N^2\)-RDWRON with \(N\) inputs/outputs and \(N^2\) different wavelengths, all these \(N^2\) wavelengths can be segmented into \(N\) subsets \(\{W_1, W_2, \ldots, W_N\}\) in which each subset \(W_i\) \((i=1,2,\ldots,N)\) has exactly \(N\) different wavelengths and \(W_i \cap W_j = \emptyset\) for all \(i \neq j\). Then,
For each source node \( S_j \), any wavelength in the same subset \( W_i \) can lead to the same destination.

For all source nodes, the partitions of \( N^2 \) wavelengths into \( N \) subsets are same (refer to Table 3). When the partition is derived, it can be applied to all source nodes in which a) will be satisfied.

For each source node, different subsets can be used to route to different destination nodes. Hence by using all \( N \) subsets, all \( N \) destination nodes can be reached from any source node.

4.3 The routing scheme of \( N^2 \)-RDWRON

The RDWRON architecture introduced in prior sections can be referred as Level 1 RDWRON.

4.3.1 Level 1 RDWRON routing scheme

The routing scheme of Level 1 \( N^2 \)-RDWRON can be solved according to the following Propositions.

**Proposition 4.** For a Level 1 \( N^2 \)-RDWRON, given the source node address \( S \) and the routing wavelength \( w \), the destination node address \( D \) can be derived as

\[
D = f_D (N, S, w_0)
\]

where \( w_0 = \text{mod}(w - 1, N) + 1 \) and \( f_D \) is defined in Eqn. 3.

**Proposition 5.** For a Level 1 \( N^2 \)-RDWRON, given the destination node address \( D \) and the routing wavelength \( w \), the source node address \( S \) can be derived as

\[
S = f_S (N, D, w_0)
\]

where \( w_0 = \text{mod}(w - 1, N) + 1 \) and \( f_S \) is defined in Eqn. 4.

**Proposition 6.** For a Level 1 \( N^2 \)-RDWRON, a set of different routing wavelengths can
be used in routing from one source node to one destination node. Denote the set of different wavelengths of the $N^2$-RDWRON as $W$, given the RDWRON size $N$, the source node address $S$ and the destination node address $D$, $W$ can be derived as

$$W = \{w, w + N, w + 2N, \ldots, w + (N - 2)N, w + (N - 1)N\}$$

(8)

$$= \{w + (k - 1)N | k = 1, 2, \ldots, N\}$$

where $w = f_w(N, S, D)$ and $f_w$ is defined in Eqn. 5.

The proofs of Propositions 4-6 are given in Appendix 0.

For example, the routing wavelength assignment of $3^2$-RDWRON is shown in Table 3.

**Table 3** The routing wavelengths assignment of 32-RCWRON.

<table>
<thead>
<tr>
<th>$W$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$w_2$</td>
<td>$w_5$</td>
<td>$w_8$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$w_3$</td>
<td>$w_6$</td>
<td>$w_9$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$w_1$</td>
<td>$w_4$</td>
<td>$w_7$</td>
</tr>
</tbody>
</table>

### 4.3.2 Level2 RDWRON routing scheme

The wavelength selection for a RDWRON is not unique. In the following, another type of RDWRON will be introduced. We name it *Level2 RDWRON* which will be used in the construction of RCWRON. Correspondingly, we denote the RDWRON introduced before as *Level1 RDWRON* in which wavelengths setting in the optical switches are in sequence.

The wavelength presetting at the $k^{th}$ stage in *level2 RDWRON* is $w_k$, where

$$w_k = i + (j - 1) \times N$$

and
\[
\begin{aligned}
  j &= \text{mod}(k-1,N)+1 \\
  i &= \left\lfloor \frac{k-1}{N} \right\rfloor 
\end{aligned}
\]

The following propositions can be derived for solving the routing scheme of Level2 RDWRON.

**Proposition 7.** In Level2 \(N^2\)-RDWRON, given the source node address \(S\) and the routing wavelength \(w\), the destination node address \(D\) can be derived as follows:

\[
D = f_D(N, S, w_0)
\]

where

\[
w_0 = \left\lfloor \frac{w-1}{N} \right\rfloor
\]

and \(f_D\) is defined in Eqn. 3.

**Proposition 8.** In the Level2 \(N^2\)-RDWRON, given the destination node address \(D\) and the routing wavelength \(w\), the source node address \(S\) can be derived as follows:

\[
S = f_S(N, D, w_0)
\]

where

\[
w_0 = \left\lfloor \frac{w-1}{N} \right\rfloor
\]

and \(f_S\) is defined in Eqn. 4.

**Proposition 9.** In the Level2 \(N^2\)-RDWRON, given the source node address \(S\) and the destination node address \(D\), the routing wavelength set \(W\) can be derived as follows:

\[
W = \{(w-1)N+1, (w-1)N+2, \ldots, (w-1)N+(N-1), wN\}
\]

(9)

where \(w = f_w(N, S, D)\) and \(f_w\) is defined in Eqn. 5.
4.4 Conclusion

In this chapter, based on the WRON architecture introduced in 0, a generalized redundant wavelength routed optical micronetwork architecture RDWRON is presented and its routing scheme is generalized.
CHAPTER 5

2-D RECURSIVE OPTICAL NETWORK-ON-CHIP ARCHITECTURES

Based on the RDWRON architecture introduced in the 0, a new recursive architecture is to be presented in this chapter, which is denoted as RCWRON.

5.1 Structure of 2-D Recursive Optical Network

5.1.1 Construction of 2-D RCWRON

A 2-D RCWRON can be constructed by connecting multiple RDWRONs. In specific, a 2-D RCWRON has two subnetworks, each composed of $N N^2$-RDWRONs. The RDWRONs in the first and second level are Level1 RDWRONs and Level2 RDWRONs, respectively. The wavelength selections for RDWRONs in different levels are different.

Each $N^2$-RDWRON in a 2-D RCWRON has $N$ inputs/outputs and $N^2$ stages. Totally a $N^2$-RCWRON has $N^2$ inputs/outputs. Hence, we denote the RCWRON with $N^2$ inputs/outputs as $N^2$-RCWRON ($N>2$). Figure 21 shows the structure of an $N^2$-RCWRON ($N>2$).
The connection principle of $N^2$-RCWRON is explained as follows. The $i^{th}$ output node of the $j^{th}$ $N^2$-RDWRON is connected to the $j^{th}$ input node of $i^{th}$ $N^2$-RDWRON in the second level.

One may notice that the structure of an $N^2$-RCWRON is not unique. The basic rule is that each Level1 RDWRON must have a connection to each Level2 RDWRON, and vice versa.

**Lemma 4.** The total number of switches in one 2-D $N^2$-RCWRON is

$$O_{RCWRON} = 2 \times N \times \frac{N^2(N-1)}{2} = \frac{2N^3(N-1)}{2}$$

(10)
5.1.2 Fault Tolerance Capability

Compared with the WRON, a distinct advantage of RCWRON is attributed to its fault tolerance capability. As shown in Figure 21, an $N^2$-RCWRON is composed of $2N$ RDWRONs, which are independent from each other. When one path fails, the faulty RDWRON can be easily identified by checking the sub-path in different levels of subnetworks. By abandoning the faulty RDWRON and the input/output nodes connected by the faulty RDWRON, the rest of the RCWRON can still operate normally.

5.2 Routing Scheme of 2-D RCWRON

The key idea of the routing scheme for the 2-D RCWRON is to decompose the routing path into two parts, each corresponding to the sub-paths in two subnetworks, respectively, and then solve the routing problem in each RDWRON.

In the following, we will first present the rules of assigning routing wavelengths before we describe the routing scheme.

5.2.1 Routing Wavelength Assignment

$N^2$ different wavelengths can be partitioned into two disjoint subsets $W^{(1)}$ and $W^{(2)}$. For all Level1 RDWRONs, their assigned routing wavelengths are exclusively from $W^{(2)}$, as, while for all Level2 RDWRONs, assign wavelength subsets in $W^{(1)}$ as their routing wavelengths.

For any routing path $P$ in RCWRON with assigned wavelength $w$, the path can be decomposed into two segments, $P_1$ and $P_2$, where $P_1$ is the routing path in the first subnetwork (i.e., Level1 RDWRONs) and $P_2$ is the routing path in the second subnetwork (i.e., Level2 RDWRONs). For $P_1$, the routing wavelength $w$ must be in one of the subsets in the Partition 2, denoted as $W^{(2)}$. And for $P_2$, the routing wavelength $w$ must be in one of
the subsets in the Partition 1, denoted as $W^{(1)}$. The following lemmas elaborate how to determine such $w$.

**Lemma 5.** Given a set $W$ with $N^2$ different elements, there exist at least 2 different ways to partition $W$ into $N$ subsets:

$$W = W^{(1)} = \bigcup_{m=1,2,\ldots,N} W_m^{(1)} \quad \text{and} \quad W = W^{(2)} = \bigcup_{n=1,2,\ldots,N} W_n^{(2)},$$

where each subset has $N$ elements such that for any two different subsets $W_m^{(1)} \subset W^{(1)}$, $W_n^{(2)} \subset W^{(2)} (m, n=1,2,\ldots,N)$, there is one and only one common element, i.e.,

$$w_{mn} = W_m^{(1)} \cap W_n^{(2)}.$$

**Lemma 6.** There are totally $N^2$ different combinations of $W_m^{(1)}$ and $W_n^{(2)}$, where $W_m^{(1)} \subset W^{(1)}$, $W_n^{(2)} \subset W^{(2)} (m, n=1,2,\ldots,N)$. All $N^2$ common elements for the $N^2$ combinations of $W_m^{(1)}$ and $W_n^{(2)}$ are different from each other and these common elements compose the $N^2$ different elements of set $W$.

Lemmas 5 and 6 can be proved as follows.

**Proof:** Assume that $W = \{w_t, t = 1,2,\ldots,N^2\}$, then an $N\times N$ matrix $M$ can be generated as

$$M(i,j) = w_t \quad \text{if} \ t = (i - 1) \times N + j, \ 1 \leq i, j \leq N,$$

where $i$ and $j$ denote the row and column number in $M$, respectively.

The two partitions of $W$ can be obtained as follows.

$$W = W^{(1)} = \bigcup_{m=1,2,\ldots,N} W_m^{(1)} = \bigcup_{m=1,2,\ldots,N} \{M(m,j) \mid j = 1,2,\ldots,N\}$$

and

$$W = W^{(2)} = \bigcup_{n=1,2,\ldots,N} W_n^{(2)} = \bigcup_{n=1,2,\ldots,N} \{M(i,n) \mid i = 1,2,\ldots,N\}.$$

It is easy to see that the subset $W_m^{(1)}$ in $W^{(1)}$ and the subset $W_n^{(2)}$ in $W^{(2)}$ correspond
to the \(m^{th}\) row and the \(n^{th}\) column in \(M\), respectively. The \(m^{th}\) row and the \(n^{th}\) column in \(M\) must intersect at the element \(M(m, n) = w_t\), where \(t = (m-1) \times N + n\), which corresponds to that subsets \(W_m^{(1)}\) and \(W_n^{(2)}\) must have one and only one common element

\[
 w_{mn} = W_m^{(1)} \cap W_n^{(2)} = w_t. 
\]

Conversely, each element in \(M, M(m, n) = w_t\), is uniquely identified by its coordinate \((m, n)\), which corresponds to the common element of the unique combination of \(\{W_m^{(1)}, W_n^{(2)}\}\).

Hence, Lemma 5 and Lemma 6 hold.

One of the simplest ways to partition set \(W = \{1, 2, \ldots, N^2\}\) into 2 different subset groups which satisfy Lemmas 5 and 6 are:

**Partition 1:**

\[
 W = \bigcup_{m=1,2,\ldots,N^2} W_m^{(1)} = \bigcup_{m=1,2,\ldots,N} \{(m-1) \times N + i | i = 1, 2, \ldots, N\} 
\]

**Partition 2:**

\[
 W = \bigcup_{n=1,2,\ldots,N^2} W_n^{(2)} = \bigcup_{n=1,2,\ldots,N} \{n + (j-1) \times N | j = 1, 2, \ldots, N\} 
\]

It is easy to verify that Partitions 1 and 2 are just the redundant routing wavelength subsets of Level2 and Level1 RDWRONs, respectively. According to Lemma 5, \(W_m^{(1)}\) and \(W_n^{(2)}\) has one and only one common element \(w = W_m^{(1)} \cap W_n^{(2)}\). Then \(w\) is the only wavelength which can be used to route for path \(P\).

5.2.2 Routing Scheme for RCWRON

The following notations are used in our discussion.

- \(S\ (1 \leq S \leq N^2)\): the source node address of an \(N^2\)-RCWRON.
- \(D\ (1 \leq D \leq N^2)\): the destination node address of an \(N^2\)-RCWRON.
- $S_1 \ (1 \leq S_1 \leq N)$: the source node address of Level1 RDWRON.
- $D_1 \ (1 \leq D_1 \leq N)$: the destination node address of Level1 RDWRON.
- $S_2 \ (1 \leq S_2 \leq N)$: the source node address of Level2 RDWRON.
- $D_2 \ (1 \leq D_2 \leq N)$: the destination node address of Level2 RDWRON.
- $D_M \ (1 \leq D_M \leq N^2)$: the destination node address in the sub-network composed of all Level1 RDWRONs.
- $S_M \ (1 \leq S_M \leq N^2)$: the source node address in the sub-network composed of all Level2 RDWRONs.
- $w \ (w \in W)$: the routing wavelength for the whole $N^2$-RCWRON for a given $S$ and $D$.
- $w_1 \ (w_1 \in W)$: the minimum routing wavelength of Level1 RDWRON for a given $S_1$ and $D_1$.
- $w_2 \ (w_2 \in W)$: the minimum routing wavelength of Level2 RDWRON for a given $S_2$ and $D_2$.

According to the structure of RCWRON, we can obtain following equations.

\[
\begin{align*}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \text{mod}(S_M - 1, N) + 1 \\
D_1 &= \text{mod}(D_M - 1, N) + 1 \\
D_2 &= \text{mod}(D - 1, N) + 1 \\
D_M &= \left\lfloor \frac{S - 1}{N} \right\rfloor \times N + D_1 \\
S_M &= D_1 \times N + \left\lfloor \frac{S - 1}{N} \right\rfloor
\end{align*}
\]  

(11)

According to Proposition 3 and Proposition 6, and Lemma 5 and Lemma 6 we can obtain the following equation.

\[
w = w_1 + (w_2 - 1) \times N
\]

(12)

Based on equations above we can derive the following equations.
\begin{align}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \left\lfloor \frac{S - 1}{N} \right\rfloor \\
D_1 &= \text{mod}(D - 1, N) + 1 \\
D_2 &= \left\lfloor \frac{D - 1}{N} \right\rfloor \\
w_1 &= \text{mod}(w - 1, N) + 1 \\
w_2 &= \left\lfloor \frac{w - 1}{N} \right\rfloor
\end{align}
(13)

and
\begin{align}
D_1 &= f_D(N, S_1, w_1) \\
D_2 &= f_D(N, S_2, w_2) \\
S_1 &= f_S(N, D_1, w_1) \\
S_2 &= f_S(N, D_2, w_2) \\
w_1 &= f_w(N, S_1, D_1) \\
w_2 &= f_w(N, S_2, D_2)
\end{align}
(14)

Then we have the following results for determining the routes in $N^2$-RCWRON.

**Proposition 10.** For an $N^2$-RCWRON, given the source node address $S$ and routing wavelength $w$, the destination node address $D$ can be derived as
\begin{equation}
D = D_2 + (D_1 - 1) \times N
\end{equation}
(15)

where
\begin{align}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \left\lfloor \frac{S - 1}{N} \right\rfloor \\
w_1 &= \text{mod}(w - 1, N) + 1 \\
w_2 &= \left\lfloor \frac{w - 1}{N} \right\rfloor \\
D_1 &= f_D(N, S_1, w_1) \\
D_2 &= f_D(N, S_2, w_2)
\end{align}
(16)

The function $f_D$ in Proposition 10 is given by Eqn. 3.
Proposition 11. For an $N^2$-RCWRON, given the source node address $S$ and the routing wavelength $w$, the destination node address $D$ can be derived as

$$S = S_1 + (S_2 - 1) \times N$$

where

$$D_2 = \text{mod}(D-1,N)+1$$

$$D_1 = \left\lfloor \frac{D-1}{N} \right\rfloor$$

$$w_1 = \text{mod}(w-1,N)+1$$

$$w_2 = \left\lfloor \frac{w-1}{N} \right\rfloor$$

$$S_1 = f_S(N,D_1,w_1)$$

$$S_2 = f_S(N,D_2,w_2)$$

The function $f_S$ in Proposition 11 is given by Eqn. 4.

Based on the discussion in the previous subsection, the routing wavelength for $N^2$-RCWRON can be derived as follows.

Proposition 12. For an $N^2$-RCWRON, given the source node address $S$ and the destination node address $D$, the routing wavelength $w$ can be derived as

$$w = W_1 \cap W_2 = w_1 + (w_2 - 1) \times N$$

where

$$\begin{cases} w_1 = f_n(N,S_1,D_1) \\ w_2 = f_n(N,S_2,D_2) \\ W_1 = \{w_1, w_1 + N, w_1 + 2N, \cdots, w_1 + (N-2)N, w_1 + (N-1)N\} \\ W_2 = \{(w_2-1)N + 1, (w_2-1)N + 2, \cdots, (w_2-1)N + (N-1)N, w_2N\} \end{cases}$$

and
\[
\begin{align*}
S_1 &= \text{mod}(S - 1, N) + 1 \\
S_2 &= \left\lfloor \frac{S - 1}{N} \right\rfloor \\
D_2 &= \text{mod}(D - 1, N) + 1 \\
D_1 &= \left\lfloor \frac{D - 1}{N} \right\rfloor
\end{align*}
\]  
(21)

The function \( f_w \) in Proposition 12 is given by Eqn. 5.

5.2.3 One Routing Example

Here we use \( 4^2 \)-RCWRON as an example to illustrate the routing scheme. Figure 22 shows the structure of Level 1 \( 4^2 \)-RDWRON and Level 2 \( 4^2 \)-RDWRON. The structure of the \( 4^2 \)-RCWRON is shown in Figure 23.

Figure 22 Structures of \( 4^2 \)-RDWRON.
The routing wavelength assignment for Level1 and Level2 RDWRON are shown in Table 4 and Table 5, respectively. The routing wavelength assignment for the whole $4^2$-RCWRON is given in Table 6.
Table 4  Routing wavelengths for Level1 $4^2$-RDWRON.

<table>
<thead>
<tr>
<th>$w$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$w_1$</td>
<td>$w_4$</td>
<td>$w_7$</td>
<td>$w_{10}$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$w_2$</td>
<td>$w_5$</td>
<td>$w_{11}$</td>
<td>$w_{14}$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$w_3$</td>
<td>$w_6$</td>
<td>$w_{12}$</td>
<td>$w_{15}$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$w_4$</td>
<td>$w_9$</td>
<td>$w_{13}$</td>
<td>$w_{16}$</td>
</tr>
</tbody>
</table>

Table 5  Routing wavelengths for Level2 $4^2$-RDWRON.

<table>
<thead>
<tr>
<th>$w$</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$w_{13}$</td>
<td>$w_{14}$</td>
<td>$w_{15}$</td>
<td>$w_{16}$</td>
</tr>
<tr>
<td>$S_2$</td>
<td>$w_{13}$</td>
<td>$w_{14}$</td>
<td>$w_{15}$</td>
<td>$w_{16}$</td>
</tr>
<tr>
<td>$S_3$</td>
<td>$w_{13}$</td>
<td>$w_{14}$</td>
<td>$w_{15}$</td>
<td>$w_{16}$</td>
</tr>
<tr>
<td>$S_4$</td>
<td>$w_{13}$</td>
<td>$w_{14}$</td>
<td>$w_{15}$</td>
<td>$w_{16}$</td>
</tr>
</tbody>
</table>

Table 6  Routing wavelengths of $4^2 \times 4^2$ RCWRON

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Destination</th>
<th>LEVEL2 Unit1</th>
<th>LEVEL2 Unit2</th>
<th>LEVEL2 Unit3</th>
<th>LEVEL2 Unit4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>$w$</td>
<td>$D_1$</td>
<td>$D_2$</td>
<td>$D_3$</td>
<td>$D_4$</td>
</tr>
<tr>
<td>LEVEL1 Unit1</td>
<td>$S_1$</td>
<td>$w_1$</td>
<td>$w_{13}$</td>
<td>$w_5$</td>
<td>$w_9$</td>
</tr>
<tr>
<td>LEVEL1 Unit2</td>
<td>$S_2$</td>
<td>$w_4$</td>
<td>$w_{14}$</td>
<td>$w_6$</td>
<td>$w_{12}$</td>
</tr>
<tr>
<td>LEVEL1 Unit3</td>
<td>$S_3$</td>
<td>$w_3$</td>
<td>$w_{15}$</td>
<td>$w_7$</td>
<td>$w_{11}$</td>
</tr>
<tr>
<td>LEVEL1 Unit4</td>
<td>$S_4$</td>
<td>$w_9$</td>
<td>$w_{11}$</td>
<td>$w_{13}$</td>
<td>$w_{16}$</td>
</tr>
</tbody>
</table>

5.3 Conclusion

Based on WRON, a new 2-D Recursive Wavelength Routed Optical Network is
presented in this chapter which is suitable for ONoC. The 2-D RCWRON is constructed by using 2-D RDWRONs. The routing scheme for 2-D RCWRON is derived based on the routing scheme for 2-D RDWRON.

The major advantage of the proposed 2-D RCWRON over the WRON is its fault-tolerance capability at a relatively high construction cost.
CHAPTER 6
QUARTERED RECURSIVE DIAGONAL TORUS NETWORK-ON-CHIP
ARCHITECTURES

Network-on-a-chip (NoC) is an effective approach to connect and manage the communication between the variety of design elements and intellectual property blocks required in large and complex system-on-chips. In this chapter a new NoC architecture is propose, referred as the Quartered Recursive Diagonal Torus (QRDT), which is constructed by overlaying diagonal torus. Due to its small diameter and rich routing recourses, QRDT is determined to be well suitable to construct highly scalable NoCs. In QRDT, data packets can be routed through a proposed minimal routing algorithm based on the Johnson codes that have traditionally been used in finite state machine designs. It has been shown that this proposed routing algorithm with minor modifications is capable of handling the single link/node failure.

The advance ONoC architectures going to be introduced in the next two chapters are based on the WRON introduced in Chapter 1 and QRDT to be introduced in this chapter.

6.1 Introduction

The development of VLSI technology has continuously driven the increase of on-chip capacity. The International Technology Roadmap for Semiconductors (ITRS) predicts that System-on-Chips (SoCs) will grow to multi-billion transistors in next a few years [1]. One of the major challenges in designing such highly integrated SoCs will be to find an effective way to integrate multiple pre-designed Intellectual Property (IP) cores while addressing pressing power and performance concerns [75]. With a communication centric design style, Networks-on-Chips (NoCs) have emerged as a new SoC paradigm [75] to
overcome the limitations of bus-based communication infrastructure.

In general, a packet-based NoC consists of routers, the network interface between the router and the IP, and the interconnection network. The major challenges imposed upon NoC designs include scalability, energy efficiency, and reconfigurability [76]. Numerous interconnection network topologies have been considered for NoCs, including mesh [77], torus [78], fat tree [79], honeycomb, and quite a few others [76]. However, these interconnection topologies either scale poorly or have no support for reconfigurability. To address these two problems, a class of topologies named Recursive Diagonal Torus (RDT) has made its way to NoC designs [80].

An RDT structure is constructed by recursively overlaying 2-D diagonal torus [77], and it has the following features: recursive structure with constant node degree, smaller diameter and average distance, and embedded mesh/torus topology. Consider a torus network composed of $N \times N$ nodes, where $N = nk$ and both $n$ and $k$ are natural numbers. The rank-0 torus is formed by creating a link between node $(x, y)$ and each of its neighboring four nodes: $(\text{mod}(x \pm 1, N), y)$ and $(x, \text{mod}(y \pm 1, N))$. On top of rank-0 torus, rank-1 torus is formed by adding one link between node $(x, y)$ and each of the four nodes $(\text{mod}(x \pm n, N), \text{mod}(y \pm n, N))$ it needs to connect to. The direction of the rank-1 torus is at an angle of 45 degrees to the rank-0 torus. On rank-1 torus, rank-2 torus can be formed by adding 4 links to a node in the same manner. In a more general sense, a rank-$(r+1)$ torus is formed upon rank-$r$ torus.

In this chapter, a special type of RDT structure, Quartered RDT (QRDT) will be presented. The QRDT architecture exhibits a number of desirable networking properties that make it particularly appreciated in building large on-chip micro-networks. By
indexing the network nodes with the Johnson codes, a new minimal routing algorithm, named Johnson Coded Vector Routing (JCVR), is proposed for QRDT. The JCVR algorithm with minor modifications is capable of routing data packets when there is a single link/node failure in a QRDT-based NoC.

6.2 QRDT Structure and Its Network Properties

As a special type of RDT, the QRDT network is also constructed by overlaying diagonal torus. An $N$-QRDT has $N \times N$ nodes where $N=4n$ and $n$ is a positive integer. A QRDT is composed of nodes, rank-0 links, and rank-1 links. Each node $(x, y)$, where $0 \leq x, y \leq N-1$, in the QRDT has 4 rank-0 links connecting to its four neighbors $(\text{mod}(x \pm 1, N), y)$ and $(x, \text{mod}(y \pm 1, N))$ on rank-0 torus, and 4 rank-1 links connecting to its another four neighbors $(\text{mod}(x \pm n, N), \text{mod}(y \pm n, N))$ on rank-1 torus. In total, an $N$-QRDT has $N^2$ nodes and $4N^2$ links. Figure 24 shows the structure of 4-QRDT.

Figure 25 (a) shows the directions of the 8 channels, and each channel is labeled by the dimension, rank number, and direction. Figure 25 (b) shows the coordinates of all eight nodes that are connected to node $(x, y)$ through respective channels shown in Figure 25 (a).
Figure 24  Structure of 4-QRDT.

(a) Directions of 8 channels.
Lemma 7. The diameter of the $N$-QRDT network is $n+1$, where $n=N/4$.

Lemma 8. The average distance of the $N$-QRDT is

$$
\frac{32n^3}{3} + 20n^2 - \frac{32}{3}n + 2
\over
16n^2 - 1
$$

where $n=N/4$.

The complete proof of Lemma 7 and Lemma 8, based on the routing algorithm to be presented in Section 6.4, are provided in the Appendix 0.

Table 7 lists the diameters and the average distances of QRDT, mesh, torus, and hypercube with different network sizes. One can see that QRDT has smaller diameter and average distance than the other three structures for most network sizes.
Table 7  Different networks’ diameter (R) and average distance (AD)

<table>
<thead>
<tr>
<th>Size</th>
<th>QRDT</th>
<th>Mesh</th>
<th>Torus</th>
<th>Hypercube</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>AD</td>
<td>R</td>
<td>AD</td>
</tr>
<tr>
<td>4×4</td>
<td>2</td>
<td>1.47</td>
<td>6</td>
<td>2.67</td>
</tr>
<tr>
<td>8×8</td>
<td>3</td>
<td>2.31</td>
<td>14</td>
<td>5.33</td>
</tr>
<tr>
<td>16×16</td>
<td>5</td>
<td>3.77</td>
<td>30</td>
<td>10.67</td>
</tr>
<tr>
<td>32×32</td>
<td>9</td>
<td>6.51</td>
<td>62</td>
<td>21.33</td>
</tr>
</tbody>
</table>

As QRDT is a special type of RDT structure, the Vector Routing (VR) algorithm proposed for RDT can be readily applied to QRDT. The goal of the vector routing algorithm is to represent the routing vector with an expression that combines all unit vectors in rank-0 and rank-1 torus and the hop counts. However, the vector routing algorithm cannot guarantee it is minimal routing (i.e., the number of hops on the routing path may not be minimum). A minimal routing algorithm, named Johnson coded vector routing algorithm is to be explained in the following. The Johnson codes are introduced first before the JCVR algorithm is described.

6.3    Johnson Codes and Functions to Manipulate the Codes

6.3.1    Johnson Codes

The $m$-bit Johnson code has $M$ ($M = 2m$) code words, and it can be denoted as

$$\{C_0, C_1, C_2, \cdots, C_i, C_{i+1}, \cdots, C_{M-2}, C_{M-1}\}$$

where code word $C_i$ is given as

$$C_i = b_0^{(i)} b_1^{(i)} \cdots b_j^{(i)} b_{j+1}^{(i)} \cdots b_{m-2}^{(i)} b_{m-1}^{(i)}$$

and

$$b_j^{(i)} = \begin{cases} 0 & i + j < m \text{ or } i + j \geq 2m \\ 1 & m \leq i + j < 2m \end{cases}$$
For example, the eight 4-bit Johnson code words are 0000, 0001, 0011, 0111, 1111, 1110, 1100 and 1000.

On the other hand, given an $m$-bit Johnson code word $C_i$, $i$ can be determined by the $f_a$ function as

$$i = f_a(C_i) = \begin{cases} 
\sum_{j=0}^{m-1} b_j^{(i)} & \text{if } b_0^{(i)} = 0 \\
 m + \sum_{j=0}^{m-1} b_j^{(i)} & \text{if } b_0^{(i)} = 1 
\end{cases}$$

6.3.2 Basic Functions of Johnson Codes

Two basic functions of Johnson codes are defined: the distance function and the direction function. These functions will be used in the JCVR algorithm.

The distance function is used to determine the Hamming distance between two Johnson-codes. Given two $m$ bits Johnson-codes: $A = a_0a_1a_2\ldots a_{m-1}$ and $B = b_0b_1b_2\ldots b_{m-1}$, the distance $p$ between them is

$$p = \sum_{i=0}^{m-1} (a_i \oplus b_i)$$

The direction function is used to determine the output direction. The direction $c$ is calculated as:

- **Case 1:** $p = 0$, then $c = 0$
- **Case 2:** $a_0 = 0$ and $b_0 = 0$, then
  
  $$c = \begin{cases} 
  1 & \text{if } \sum_{i=0}^{m-1} b_i > \sum_{i=0}^{m-1} a_i \\
  -1 & \text{if } \sum_{i=0}^{m-1} b_i < \sum_{i=0}^{m-1} a_i 
\end{cases}$$
- **Case 3:** $a_0 = 1$ and $b_0 = 1$, then
\[ c = \begin{cases} 
1 & \text{if } \sum_{i=0}^{m-1} b_i < \sum_{i=0}^{m-1} a_i \\
-1 & \text{if } \sum_{i=0}^{m-1} b_i > \sum_{i=0}^{m-1} a_i 
\end{cases} \]  
(26)

- Case 4: \( a_0 = 0 \) and \( b_0 = 1 \), then

\[ c = \begin{cases} 
1 & \text{if } \sum_{i=0}^{m-1} b_i \leq \sum_{i=0}^{m-1} a_i \\
-1 & \text{if } \sum_{i=0}^{m-1} b_i > \sum_{i=0}^{m-1} a_i 
\end{cases} \]  
(27)

- Case 5: \( a_0 = 1 \) and \( b_0 = 0 \), then

\[ c = \begin{cases} 
1 & \text{if } \sum_{i=0}^{m-1} b_i \geq \sum_{i=0}^{m-1} a_i \\
-1 & \text{if } \sum_{i=0}^{m-1} b_i < \sum_{i=0}^{m-1} a_i 
\end{cases} \]  
(28)

For simplicity, the calculation of the distance and direction can be denoted as one function \( f_r \) as

\[ (p, c) = f_r(A, B) \]  
(29)

6.4 Routing Algorithm for QDRT

6.4.1 JCVR Algorithm

The routing steps from a source node to its destination node can be derived as

\[ \tilde{A} = vecX_1X_1 + vecY_1Y_1 + vecX_0X_0 + vecY_0Y_0, \]  
where \( X_1 \), \( Y_1 \), \( X_0 \), and \( Y_0 \) are the unit vectors in rank-1 torus and rank-0 torus (with their directions shown in Figure 25 (a)) and their respective coefficients, \( vecX_1 \), \( vecY_1 \), \( vecX_0 \), \( vecY_0 \), give the number of hops on corresponding dimensions.

In the JCVR algorithm, the coefficients for the unit vectors are calculated at the source node. For \( N \)-QRDT, given the source and the destination addresses \( S=(S_x, S_y) \), \( D=(D_x, D_y) \), where \( S_x \) and \( S_y \) (\( D_x \) and \( D_y \)) represent the X and Y coordinates of the source
(destination) node in $m$-bit ($m=N/2$) Johnson code, respectively. The direction and distance values of the two coordinates, $(p_x, c_x)$ and $(p_y, c_y)$, can be determined by calculating $f_i$ as discussed in Section 3.2.

The address of each intermediate node $M = (M_x, M_y)$ on the routing path from $S$ to $D$ is given as,

\[
M_x = f_i(S_x, c_x, N) = \begin{cases} 
\text{mod}_N(S_x + n) & \text{when } c_x \geq 0 \\
\text{mod}_N(S_x - n) & \text{when } c_x < 0 
\end{cases}
\]

\[
M_y = f_i(S_y, c_y, N) = \begin{cases} 
\text{mod}_N(S_y + n) & \text{when } c_y \geq 0 \\
\text{mod}_N(S_y - n) & \text{when } c_y < 0 
\end{cases}
\] (30)

The JCVR algorithm for QRDT is presented in Figure 26.

Vector Routing Algorithm for QRDT:

\[
\begin{align*}
&\text{begin} \\
&\quad \text{initialize vec}X_0, \text{vec}Y_0, \text{vec}X_1, \text{vec}Y_1 \text{ to zero} \\
&\quad \text{set } M_x=S_x, M_y=S_y \\
&\quad \text{while } (D_x \neq M_x \text{ and } D_y \neq M_y) \text{ do} \\
&\quad\quad \text{calculate } (p_x, c_x) \text{ and } (p_y, c_y) \text{ using } f_i \text{ function} \\
&\quad\quad \text{if } (c_x=0 \text{ and } c_y=0) \text{ then set vec}X_0, \text{vec}Y_0, \text{vec}X_1, \text{vec}Y_1 \text{ to zero} \\
&\quad\quad \text{elseif } (p_x+p_y\leq n) \text{ then} \\
&\quad\quad\quad \text{if } (c_x\geq 0) \text{ then vec}X_0=\text{vec}X_0 + p_x \\
&\quad\quad\quad \text{elseif } (c_x<0) \text{ then vec}X_0=\text{vec}X_0-p_x \\
&\quad\quad\quad \text{if } (c_y\geq 0) \text{ then vec}Y_0=\text{vec}Y_0+p_y \\
&\quad\quad\quad \text{elseif } (c_y<0) \text{ then vec}Y_0=\text{vec}Y_0-p_y \\
&\quad\quad\quad \text{go to end} \\
&\quad\quad \text{elseif } (p_x+p_y>n) \text{ then} \\
&\quad\quad\quad \text{if } (c_x\geq 0 \text{ and } c_y\geq 0) \text{ then vec}X_1=\text{vec}X_1+1 \\
&\quad\quad\quad \text{if } (c_x\geq 0 \text{ and } c_y<0) \text{ then vec}Y_1=\text{vec}Y_1-1 \\
&\quad\quad\quad \text{if } (c_x<0 \text{ and } c_y\geq 0) \text{ then vec}Y_1=\text{vec}Y_1+1 \\
&\quad\quad\quad \text{if } (c_x<0 \text{ and } c_y<0) \text{ then vec}X_1=\text{vec}X_1-1 \\
&\quad\quad\quad \text{calculate } M_x \text{ and } M_y \text{ using the } f_i \text{ function} \\
&\quad\quad \text{endif} \\
&\quad \text{endwhile} \\
&\text{end}
\end{align*}
\]

Figure 26 Vector routing algorithm for QRDT.
For an $N$-QRDT, the while loop will be executed no more than $m = N/4$ times to complete the calculation of the four coefficients ($vecX_1$, $vecY_1$, $vecX_0$, $vecY_0$).

At the source node, the four coefficients are computed and encapsulated into the packet. At the source node and each intermediate node, the coefficients for the unit vectors will be checked in a default order of $vecX_1$, $vecY_1$, $vecX_0$, $vecY_0$. The packet will be routed to the direction as determined by the first unit vector with non-zero coefficient (hop count) and this vector’s coefficient, depending on the routing direction (+/-), will be decreased/increased before the data packets are actually forwarded to the next hop towards the destination.

Compared to the VR algorithm, the advantage of the JCVR algorithm is that it can achieve minimal routing (as proved in Lemma 7), which is not guaranteed in VR.

6.4.2 Fault Tolerance Routing for QRDT under Single Link/Node Failure

It has been well recognized that fault-tolerance capability is vital for a NoC system [81], since one faulty link/processor may isolate a large fraction of IP cores from being reachable by other cores. Next we show that with minor modification, the JCVR algorithm is capable of handling single link/node failure.

The following assumptions are made first:

- Any link or node in the network can fail, and the faulty components are unusable; that is, data will not be transmitted over a faulty link or routed through a faulty node.
- The fault model is static; that is, no new faults occur during a routing process.
- Both source and destination nodes (on rank-0 or rank-1 torus) are fault-free.
- The faults occur independently.
- If a node fails, all the eight links associated with the node on rank-0/1 tori are
considered unusable.

During the routing process, when the next routing link/node is found to be faulty in a QRDT network, fault-tolerance routing can be realized dynamically as described below. The faulty link encountered must be on one of the four dimensions (i.e., $\bar{X}_1$, $\bar{Y}_1$, $\bar{X}_0$, and $\bar{Y}_0$). There are two cases to consider:

- **Case 1**: if there exists one or more other dimensions with non-zero coefficient, the routing steps on such dimension(s) will be completed first followed by the routing steps on the dimension with faulty link/node.

- **Case 2**: otherwise, one step on the dimension on the same rank and orthogonal to the dimension with faulty link/node will be completed first followed by the routing steps on the dimension with faulty link/node and one opposite routing step on the orthogonal dimension.

Figure 27 (a) shows an example of Case 1 under single link failure. The detoured routing path is $S-M_0-M_1-D$, which does not introduce extra hops compared to the original routing path $S-M_0-M_2-D$. Similarly, as shown in Figure 28 (a), no extra hop is introduced on the detoured routing path for such a case under single node failure.
Figure 27  Two cases under single link fault.

Figure 27 (b) shows an example of Case 2 with a single link failure, where the detoured routing path has two extra hops than the original routing path. As shown in Figure 28 (b), the number of extra hops on the detoured routing path is also two for such a case under single node failure.
Hence the following lemma can be derived.

**Lemma 9.** When a single faulty link/node exists in the QRDT network, the extra number of hops on the detoured route generated by the fault-tolerance JCVR routing algorithm, as compared to the number of hops on the original route for a fault-free QRDT, is no more than 2.

### 6.5 Conclusion

In this chapter the Quartered Recursive Diagonal Torus (QRDT) constructed by overlaying diagonal torus, was introduced as a novel topology for NoCs. The QRDT structure exhibits a number of desirable networking properties (including small diameter and average distance, constant node degree), which make it particularly appreciated in building large on-chip micro-networks. A minimal routing algorithm, the JCVR algorithm, was proposed for QRDT structure. It was shown that fault tolerance routing under single
link/node failure can be handled by the JCVR algorithm with minor modifications. Compared with the VR algorithm, the JCVR algorithm achieves minimal routing. The tradeoff is the moderately higher implementation cost of the JCVR-based router than the VR-based router.
CHAPTER 7

PACKET SWITCHING OPTICAL NETWORK-ON-CHIP ARCHITECTURES

In this chapter three packet switching optical Network-on-Chip architectures, i.e., TON-I, TON-II and TON-III that are built on the topology of 2D-torus are presented. Micro-ring resonator (MRR)-based optical switches are adopted for wavelength-based routing in TONs. The implementation of a packet switching optical NoC with a limited number of wavelengths, the design of routers and a schema for wavelength assignment are illustrated in detail. The proposed TON architectures explore passive MRR switching, Wavelength Division Multiplexing (WDM) and Direct Optical Channels (DOC) yield a high bandwidth, a low latency and a low power consumption. The architectures are also potentially fault-tolerant. The average frame transmission time and the buffer utilization by are evaluated with simulation. The power analysis and the transmission power losses are provided. Simulation and analysis results show that the proposed architectures can be considered as a viable solution for future NoCs [82, 83].

7.1 Introduction

In the three different 2D torus-based passive MRR routing packet-switching optical NoC architectures TON-I, TON-II and TON-III, the physical interconnections of these TONs are as in 2D-torus. However, a new concept, direct optical channels (DOC), is proposed to build the direct light path between two nodes. A DOC consists of a set of interconnected physical links (waveguides) and routers. With a predefined routing wavelength, light can travel in a DOC without relay. Optical data packets can be transmitted in the TON by relay of different DOCs.
7.2 TON Architectures

7.2.1 Interconnections in TON network

The TON is an $N \times N$, where $N=2n$, $n \in \mathbb{N}$, 2D-torus-based network. A $6 \times 6$ TON example is presented in Figure 29. Each node in the TON is denoted by a pair $(x,y)$ where $x,y=\{0,1,\ldots,N-1\}$. Each node has a router and a processor (core), and it is physically connected to four adjacent nodes. Internally, each router has a wavelength routed Optical Interconnection Unit (OIU) and an Optical Electrical Router (OER).

The TON nodes are classified into two classes: I-Node associated with I-router; and II-node with II-router. I-router has an I-OIU and an I-OER, and II-router has an II-IOU.
and an II-OER. Given coordinates \((x, y)\), a node is an I-Node if \(x + y = 2n \ (n \in \mathbb{N})\), otherwise it is a II-Node.

OIU is an MRR matrix device designed by properly connecting MRR switches with the waveguide. The function of OIU is the port-to-port routing of the optical signal \(e\) based on its wavelength. OIU has six ports, of which four are external ports (S1, S2, D1, D2), and two, i.e., (LS, LD) are internal ports to connect to the OER.

The interconnection of an \(8 \times 8\) TON is shown in Figure 30. In \((x, y)\) node, its S1 port of OIU is connected to D2 port of the OIU in \((x-1, y)\), its S2 port of OIU is connected to D1 port of the OIU in \((x, y-1)\); D1 port of the OIU is connected to S1 of \((x, y+1)\) node, and D2 is connected to S1 of the OIU of \((x+1, y)\) node.

7.2.2 Optical Interconnection Unit (OIU)

OIU structure proposed in the paper is developed from WRON structure [54]. Unlike to many other MRR structures such as in [14], WRON is 100% non-blocking. It is guaranteed in WRON that lights in same wavelength will not be inducted into same waveguide. Hence no interference will happen during the wavelength multiplexing transmission. And much less medium access control needed, which massively simplified the design.

7.2.3 Optical-Electrical Router (OER)

The function of an OER is to receive input optical signal, convert it to electrical data, read its header and convert it to an optical signal in a proper wavelength and then transmit it to the next node through a proper output channel.

An OER connects to OIU’s LS and LD ports by two waveguides. Inside the OER, these two waveguides connect to two couplers which mixed input/output optical signals.
Each coupler is connected to an optical wavelength multiplexer and a demultiplexer. If each node has \( n \) channels, then each multiplexer converges \( n/2 \) output optical signals from \( n/2 \) lasers to a waveguide, and each demultiplexer diverges \( n/2 \) input optical signals from a waveguide to \( n/2 \) photo-detectors. Each laser/photo-detector is connected to a crossbar switch via an output port/input buffer. And there is a local output port/input buffer which is also connected to the crossbar switch for handling data to/from the core.

7.2.4 Directed Optical Channel (DOC)

Data transmission in the TON is implemented via a DOC. The DOC is a realization of the channel with a light path between two nodes that consists of one or more physical links and OIUs. By utilizing a predetermined routing wavelength and with properly designed OIUs light can travel through the DOC without any relay or arbitration. DOCs can be placed between any two nodes. Examples of one-hop and two-hop DOCs in the 8×8 TON are shown in Figure 30.

Two nodes which do not have a DOC will exchange data packets by relaying in intermediary nodes. This would require electrical/optical and optical/electrical (E/O & O/E) conversion. Data transmission latency is directly related to the number of such conversions.
7.3 TON-I Architecture

7.3.1 TON-I Topology

Each node in the network is directly connected to four other nodes via four DOCs.

Given a node (x,y), its channel map is shown in Figure 31. Its channels are:
E1: (x,y) ↔ (x,y+1); W1: (x,y) ↔ (x,y-1); S1(x,y) ↔ (x+1,y); N1: (x,y) ↔ (x-1,y);

Figure 31  TON-I network topology

7.3.2 Channel Realization

As mentioned before, channels in TON network are realized with DOC. Four channels shown in Figure 31 can be realized with DOC as explained in Table 8.

<table>
<thead>
<tr>
<th>C.</th>
<th>W.</th>
<th>S.</th>
<th>D.</th>
<th>P.</th>
<th>C.</th>
<th>DOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>w1</td>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>LS</td>
<td>E</td>
<td>(x,y)LS→(x,y)D1→(x,y+1)S2→(x,y+1)LD*</td>
</tr>
<tr>
<td>S1</td>
<td>w2</td>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>LS</td>
<td>S</td>
<td>(x,y)LS→(x,y)D2→(x+1,y)S1→(x+1,y)LD</td>
</tr>
<tr>
<td>W1</td>
<td>w1</td>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>LD</td>
<td>W</td>
<td>(x,y)LD→(x,y)S2→(x,y-1)D1→(x,y-1)LS</td>
</tr>
<tr>
<td>N1</td>
<td>w2</td>
<td>(x,y)</td>
<td>(x-1,y)</td>
<td>LD</td>
<td>N</td>
<td>(x,y)LD→(x,y)S1→(x-1,y)D2→(x-1,y)LS</td>
</tr>
</tbody>
</table>

* ‘→’ represents the routing with optical resonator switching inside OIU, ‘⇒’ represents the routing with waveguide links between OIUs.
7.3.3 Routing Table of OIUs in TON-I

The routing truths of I-OIU and II-OIU in TON-I are identical, as shown in Table 9.

<table>
<thead>
<tr>
<th>Port</th>
<th>w₁</th>
<th>w₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>D₁</td>
<td>D₂</td>
</tr>
<tr>
<td>LD</td>
<td>S₂</td>
<td>S₁</td>
</tr>
</tbody>
</table>

Table 9 Routing truth table of OIUs in TON-I

7.3.4 OIUs in TON-I

The structure of I-OIU and II-OIU in TON-I are identical. It can be constructed according to Table 9, as shown in Figure 32.

![Figure 32 OIU structure of TON-I](image)

7.3.5 OERs in TON-I

Based on the routing pattern shown in Table 8, the OER in TON-I can be designed and that is displayed in Figure 33.
7.4  TON-II Architecture

7.4.1  TON-II Topology

The TON-II is an extension of the basic TON-I. By adding four more channels at each node in TON-I the total number of channels is eight, connecting the node to two neighbors in each of the four directions. Given a node (x,y), its channel map is shown in Figure 34.

The eight channels of TON-II can be classified into two groups: Level1 channels and Level2 channels as where:

Level1 channels: E1: (x,y) ↔ (x,y+1); W1: (x,y) ↔ (x,y-1); S1(x,y) ↔ (x+1,y); N1: (x,y) ↔ (x-1,y);

Level2 channels: E2: (x,y) ↔ (x,y+2); W2: (x,y) ↔ (x,y-2); S2(x,y) ↔ (x+2,y); N2: (x,y) ↔ (x-2,y).
7.4.2 Channel Realization

The eight channels shown in Figure 34 can be realized with DOC as shown in Table 10 and Table 11 for I-Node and II-Node, respectively.
Table 10  Channel Realization for I-Node in TON-II

<table>
<thead>
<tr>
<th>C.</th>
<th>W.</th>
<th>S.</th>
<th>D.</th>
<th>P.</th>
<th>Routing Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>w₁</td>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>LS</td>
<td>(x,y)LS→(x,y)D₁→(x,y+1)S₂→(x,y+1)LD</td>
</tr>
<tr>
<td>E2</td>
<td>w₂</td>
<td>(x,y)</td>
<td>(x,y+2)</td>
<td>LS</td>
<td>(x,y)LS→(x,y)D₁→(x,y+1)S₂→(x,y+1)D₁⇒(x,y+2)S₂→(x,y+2)LD</td>
</tr>
<tr>
<td>S1</td>
<td>w₄</td>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>LS</td>
<td>(x,y)LS→(x,y)D₂⇒(x+1,y)S₁→(x+1,y)D₂⇒(x+2,y)S₁→(x+2,y)LD</td>
</tr>
<tr>
<td>W1</td>
<td>w₁</td>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₂⇒(x,y-1)D₁→(x,y-1)S₁⇒(x,y-2)D₁→(x,y-2)LD</td>
</tr>
<tr>
<td>W2</td>
<td>w₂</td>
<td>(x,y)</td>
<td>(x,y-2)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₂⇒(x,y-1)D₁→(x,y-1)S₂⇒(x,y-2)D₁→(x,y-2)LD</td>
</tr>
<tr>
<td>N2</td>
<td>w₃</td>
<td>(x,y)</td>
<td>(x-2,y)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₁⇒(x-1,y)D₂→(x-1,y)S₁⇒(x-2,y)D₂→(x-2,y)LD</td>
</tr>
<tr>
<td>N1</td>
<td>w₄</td>
<td>(x,y)</td>
<td>(x-1,y)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₁⇒(x-1,y)D₂→(x-1,y)S₂⇒(x,y-2)D₁→(x,y-2)LD</td>
</tr>
</tbody>
</table>

Table 11  Channel Realization for II-Node in TON-II

<table>
<thead>
<tr>
<th>C.</th>
<th>W.</th>
<th>S.</th>
<th>D.</th>
<th>P.</th>
<th>Routing Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>w₁</td>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>LS</td>
<td>(x,y)LS→(x,y)D₁→(x,y+1)S₂→(x,y+1)LD</td>
</tr>
<tr>
<td>S1</td>
<td>w₄</td>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>LS</td>
<td>(x,y)LS→(x,y)D₂⇒(x+1,y)S₁→(x+1,y)D₂⇒(x+2,y)S₁→(x+2,y)LD</td>
</tr>
<tr>
<td>E1</td>
<td>w₁</td>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₂⇒(x,y-1)D₁→(x,y-1)S₁⇒(x,y-2)D₁→(x,y-2)LD</td>
</tr>
<tr>
<td>N1</td>
<td>w₄</td>
<td>(x,y)</td>
<td>(x-1,y)</td>
<td>LD</td>
<td>(x,y)LD→(x,y)S₁⇒(x-1,y)D₂→(x-1,y)S₂⇒(x,y-2)D₁→(x,y-2)LD</td>
</tr>
</tbody>
</table>

7.4.3  Routing Table of OIUs in TON-II

According to the DOC realization shown in Table 10 and Table 11, the routing truths of I-OIU and II-OIU in TON-II can be derived as in Table 12 and Table 13.
Table 12  Routing truth table of I-OIU in TON-II

<table>
<thead>
<tr>
<th></th>
<th>(w_1)</th>
<th>(w_2)</th>
<th>(w_3)</th>
<th>(w_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>(D_1)</td>
<td>(D_1)</td>
<td>(D_2)</td>
<td>(D_2)</td>
</tr>
<tr>
<td>(S_1)</td>
<td>(D_2)</td>
<td>(D_2)</td>
<td>(LD)</td>
<td>(LD)</td>
</tr>
<tr>
<td>(S_2)</td>
<td>(LD)</td>
<td>(LD)</td>
<td>(D_1)</td>
<td>(D_1)</td>
</tr>
</tbody>
</table>

Table 13  Routing truth table of II-OIU in TON-II

<table>
<thead>
<tr>
<th></th>
<th>(w_1)</th>
<th>(w_2)</th>
<th>(w_3)</th>
<th>(w_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>(D_1)</td>
<td>(D_2)</td>
<td>(D_1)</td>
<td>(D_2)</td>
</tr>
<tr>
<td>(S_1)</td>
<td>(D_2)</td>
<td>(LD)</td>
<td>(D_2)</td>
<td>(LD)</td>
</tr>
<tr>
<td>(S_2)</td>
<td>(LD)</td>
<td>(D_1)</td>
<td>(LD)</td>
<td>(D_1)</td>
</tr>
</tbody>
</table>

7.4.4 OIUs in TON-II

The structure of the I-OIU and II-OIU in TON-II can be constructed according to Table 12 and Table 13, presented in Figure 35.
7.4.5 OERs in TON-II

Based on the routing pattern presented in Table 10 and Table 11, I-OER and II-OER in TON-II can be built. This is shown in Figure 36.

(a) I-OER in TON-II
The TON-III is obtained from TON-II by adding two diagonal channels at each core. Each node in TON-III has 10 channels connected to its two neighbors in each of the four directions, plus its northwest and southeast neighbor nodes. The channel map of \((x,y)\) node is shown in Figure 37.

The ten channels of each node in TON-III can be classified into Level1 channels,
Level2 channels and Level3 channels as below:

- **Level1 channels:** E1: \((x,y) \leftrightarrow (x,y+1)\); W1: \((x,y) \leftrightarrow (x,y-1)\); S1\(x,y) \leftrightarrow (x+1,y)\); N1: \((x,y) \leftrightarrow (x-1,y)\);

- **Level2 channels:** E2: \((x,y) \leftrightarrow (x,y+2)\); W2: \((x,y) \leftrightarrow (x,y-2)\); S2\(x,y) \leftrightarrow (x+2,y)\); N2: \((x,y) \leftrightarrow (x-2,y)\).

- **Level3 channels:** SE: \((x,y) \leftrightarrow (x+2,y+2)\); NW: \((x,y) \leftrightarrow (x-2,y-2)\).

![Figure 37](image)

**Figure 37**  
TON-III network topology

### 7.5.2 Channel Realization

The ten channels shown in Figure 37 can be realized with DOC as described in Table 14 and 0 for I-Node and II-Node, respectively.
Table 14  Routing paths in TON-III

<table>
<thead>
<tr>
<th>C.</th>
<th>W.</th>
<th>S.</th>
<th>D.</th>
<th>P.</th>
<th>Routing Path</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>w1</td>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>LS</td>
<td>((x,y)\text{LS} \rightarrow (x,y)D_1 \Rightarrow (x,y+1)S_2 \rightarrow (x,y+1)LD)</td>
</tr>
<tr>
<td>S2</td>
<td>w3</td>
<td>(x,y)</td>
<td>(x+2,y)</td>
<td>LS</td>
<td>((x,y)\text{LS} \rightarrow (x,y)D_2 \Rightarrow (x+1,y)S_1 \rightarrow (x+1,y)D_2 \Rightarrow (x+2,y)S_1 \rightarrow (x+2,y)LD)</td>
</tr>
<tr>
<td>SE</td>
<td>w4</td>
<td>(x,y)</td>
<td>(x+2,y+2)</td>
<td>LS</td>
<td>((x,y)\text{LS} \rightarrow (x,y)D_2 \Rightarrow (x+1,y)S_1 \rightarrow (x+1,y)D_1 \Rightarrow (x+1,y+1)S_2 \rightarrow (x+1,y+2)S_2 \rightarrow (x+2,y+2)S_1 \rightarrow (x+2,y+2)LD)</td>
</tr>
<tr>
<td>E2</td>
<td>w5</td>
<td>(x,y)</td>
<td>(x,y+2)</td>
<td>LS</td>
<td>((x,y)\text{LS} \rightarrow (x,y)D_1 \Rightarrow (x,y+1)S_2 \rightarrow (x,y+1)D_1 \Rightarrow (x,y+2)S_2 \rightarrow (x,y+2)LD)</td>
</tr>
<tr>
<td>S1</td>
<td>w6</td>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>LD</td>
<td>((x,y)\text{LS} \rightarrow (x,y)D_2 \Rightarrow (x+1,y)S_1 \rightarrow (x+1,y)LD)</td>
</tr>
<tr>
<td>W1</td>
<td>w1</td>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>LD</td>
<td>((x,y)\text{LD} \rightarrow (x,y)S_2 \Rightarrow (x,y-1)D_1 \rightarrow (x,y-1)LSD)</td>
</tr>
<tr>
<td>N2</td>
<td>w3</td>
<td>(x,y)</td>
<td>(x-2,y)</td>
<td>LD</td>
<td>((x,y)\text{LD} \rightarrow (x,y)S_1 \Rightarrow (x-1,y)D_2 \rightarrow (x-1,y)S_1 \Rightarrow (x-2,y)D_2 \rightarrow (x-2,y)LD)</td>
</tr>
<tr>
<td>NW</td>
<td>w4</td>
<td>(x-2,y-2)</td>
<td>LD</td>
<td>((x,y)\text{LD} \rightarrow (x,y)S_1 \Rightarrow (x-1,y)D_2 \rightarrow (x-1,y)S_2 \Rightarrow (x-1,y-1)D_1 \rightarrow (x-1,y-2)S_2 \Rightarrow (x-2,y-2)S_1 \Rightarrow (x-2,y-2)LSD)</td>
<td></td>
</tr>
<tr>
<td>W2</td>
<td>w5</td>
<td>(x,y)</td>
<td>(x,y-2)</td>
<td>LD</td>
<td>((x,y)\text{LD} \rightarrow (x,y)S_2 \Rightarrow (x,y-1)D_1 \rightarrow (x,y-1)S_2 \Rightarrow (x,y-2)D_1 \rightarrow (x,y-2)LSD)</td>
</tr>
<tr>
<td>N1</td>
<td>w6</td>
<td>(x-1,y)</td>
<td>LD</td>
<td>((x,y)\text{LD} \rightarrow (x,y)S_1 \Rightarrow (x-1,y)D_2 \rightarrow (x-1,y)LD)</td>
<td></td>
</tr>
</tbody>
</table>

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Table 15  Routing paths in TON-III

<table>
<thead>
<tr>
<th>C.</th>
<th>D.</th>
<th>S.</th>
<th>P.</th>
<th>Routing Path</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w₁</td>
<td>(x,y)</td>
<td>P₁</td>
<td></td>
</tr>
<tr>
<td>E1</td>
<td>w₂</td>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>LS (x,y)LS→(x,y)D₁⇒(x,y+1)S₁→(x,y+1)LD</td>
</tr>
<tr>
<td>SE</td>
<td>w₂</td>
<td>(x,y)</td>
<td>(x+2,y+2)</td>
<td>LS (x,y)LS→(x,y)D₁⇒(x,y+1)S₁→(x,y+1)D₂⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x+1,y+1)S₁→(x+1,y+1)D₂⇒(x+2,y+1)S₁→</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x+2,y+1)D₁⇒(x+2,y+2)S₁→(x+2,y+2)LD</td>
</tr>
<tr>
<td>E2</td>
<td>w₃</td>
<td>(x,y)</td>
<td>(x,y+2)</td>
<td>LS (x,y)LS→(x,y)D₁⇒(x,y+1)S₁→(x,y+1)LD⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x,y+2)S₁→(x,y+2)LD</td>
</tr>
<tr>
<td>S2</td>
<td>w₅</td>
<td>(x,y)</td>
<td>(x+2,y)</td>
<td>LS (x,y)LS→(x,y)D₂⇒(x+1,y)S₁→(x+1,y)D₂⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x+2,y)S₁→(x+2,y)LD</td>
</tr>
<tr>
<td>S1</td>
<td>w₆</td>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>LD (x,y)LS→(x,y)D₂⇒(x+1,y)S₁→(x+1,y)LD⇒</td>
</tr>
<tr>
<td>W1</td>
<td>w₁</td>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>LD (x,y)LD→(x,y)S₁→(x,y-1)D₁→(x,y-1)LS</td>
</tr>
<tr>
<td>NW</td>
<td>w₂</td>
<td>(x,y)</td>
<td>(x-2,y-2)</td>
<td>LD (x,y)LD→(x,y)S₁→(x,y-1)D₁→(x,y-1)S₁⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-1,y-1)D₁→(x-1,y-1)S₁⇒(x-2,y-1)D₂→</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-2,y-1)S₁→(x-2,y-2)D₁→(x-2,y-2)LS</td>
</tr>
<tr>
<td>W2</td>
<td>w₃</td>
<td>(x,y)</td>
<td>(x,y-2)</td>
<td>LD (x,y)LD→(x,y)S₁→(x,y-1)D₁→(x,y-1)S₁⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x,y-2)D₁→(x,y-2)LS</td>
</tr>
<tr>
<td>N2</td>
<td>w₅</td>
<td>(x,y)</td>
<td>(x-2,y)</td>
<td>LD (x,y)LD→(x,y)S₁→(x-1,y)D₂→(x-1,y)S₁⇒</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-2,y)D₂→(x-2,y)LS</td>
</tr>
<tr>
<td>N1</td>
<td>w₆</td>
<td>(x,y)</td>
<td>(x-1,y)</td>
<td>LD (x,y)LD→(x,y)S₁→(x-1,y)D₂→(x-1,y)LS</td>
</tr>
</tbody>
</table>

7.5.3 Routing Table of OIUs in TON-III

According to the DOC realization shown in Table 14 and 0, the routing truths of the I-OIU and II-OIU in TON-III are obtained in Table 16 and 0.

Table 16  Routing truth table for I-OIU of TON-II

<table>
<thead>
<tr>
<th></th>
<th>w₁</th>
<th>w₂</th>
<th>w₃</th>
<th>w₄</th>
<th>w₅</th>
<th>w₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>D₁</td>
<td>LD</td>
<td>D₂</td>
<td>D₂</td>
<td>D₁</td>
<td>D₂</td>
</tr>
<tr>
<td>S₁</td>
<td>D₂</td>
<td>D₁</td>
<td>LD</td>
<td>LD</td>
<td>D₂</td>
<td>LD</td>
</tr>
<tr>
<td>S₂</td>
<td>LD</td>
<td>D₂</td>
<td>D₁</td>
<td>D₁</td>
<td>LD</td>
<td>D₁</td>
</tr>
</tbody>
</table>
Table 17  Routing truth table for II-OIU of TON-II

<table>
<thead>
<tr>
<th></th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>$D_1$</td>
<td>$D_1$</td>
<td>$D_1$</td>
<td>LD</td>
<td>$D_2$</td>
<td>$D_2$</td>
</tr>
<tr>
<td>$S_1$</td>
<td>$D_2$</td>
<td>$D_2$</td>
<td>$D_2$</td>
<td>$D_1$</td>
<td>LD</td>
<td>LD</td>
</tr>
<tr>
<td>$S_2$</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>$D_2$</td>
<td>$D_1$</td>
<td>$D_1$</td>
</tr>
</tbody>
</table>

7.5.4  OIU{s}s in TON-III

The structure of I-OIU and II-OIU in TON-III can be constructed according to Table 16 and 0, as shown in the Figure 38.

![OIU in TON-III](image)

(a) I-OIU

(b) II-OIU

Figure 38  OIU in TON-III

7.5.5  OER in TON-III

Based on the routing pattern of Table 14 and 0, I-OER and II-OER in the TON-III are designed in Figure 39.
(a) I-OER in TON-III
Figure 39 OERs in TON-III

7.6 TON Architecture Properties

7.6.1 Topological Properties of TON

Table 18 contains network properties of TON architectures and a few other popular networks. For an optical packet switching networks such as TON, ‘Average Distance
‘AD’ and ‘Diameter (D)’ denotes the number of hops, or the number of OE and EO conversions needed for data transmission between two nodes.

Table 18  Topology Properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Topologies</th>
<th>8×8</th>
<th>16×16</th>
<th>32×32</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>TON-I</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>TON-II</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>TON-III</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Mesh</td>
<td>14</td>
<td>30</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>2D-Torus</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Hypercube</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>AD</td>
<td>TON-I</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>TON-II</td>
<td>2.5</td>
<td>4.5</td>
<td>8.5</td>
</tr>
<tr>
<td></td>
<td>TON-III</td>
<td>2.31</td>
<td>3.97</td>
<td>7.3</td>
</tr>
<tr>
<td></td>
<td>Mesh</td>
<td>5.33</td>
<td>10.67</td>
<td>21.33</td>
</tr>
<tr>
<td></td>
<td>2D-Torus</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Hypercube</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

7.6.2 Architecture Properties of TON

Table 19 summarizes the architectural properties of the TON-I, TON-II and TON-III.
Table 19 Architecture Properties of TON

<table>
<thead>
<tr>
<th>Architecture Properties</th>
<th>TON-I</th>
<th>TON-II</th>
<th>TON-III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels per node</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Lasers per node</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Photodetectors per node</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Routing wavelengths per node</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Total routing wavelengths in the network</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>MRRs per node</td>
<td>4</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>MRR optical switches per node</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Electrical input queues per node</td>
<td>4</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Electrical output queues per node</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Optical buffers per node</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Optical couplers per node</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Electrical crossbar switches per node</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Wavelength multiplexers per node</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Wavelength demultiplexers per node</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>TO/EO tuning devices in the network</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

7.7 Routing Algorithms of TON Architectures

The Johnson code is used in TON for node addressing and data routing.

7.7.1 Johnson Code Addressing for TON

To implement the data routing in the network the Johnson Code explained in Section 6.3 is applied for node addresses in TON.

7.7.1.1 Addressing Nodes in TON

The address of a node \((A_h, A_v)\) in the \(N \times N\) TON can be defined by means of two \(n\)-bit Johnson Codes \((C_h, C_v)\) where \(N=2n\).

For example, \((3, 2)\) in a 6×6 GTON is addressed as 011001.

7.7.1.2 Nodes Coordinates and Address Association

For a node address \(C = C_h|C_v\), \(C\) is of \(N\) bits and \(N = 4n\), \(C_h\) and \(C_v\) are both \(2n\) bits.
where

$$C_h = b_1^{(h)}b_2^{(h)} \cdots b_{j_1}^{(h)}b_{j_1+1}^{(h)} \cdots b_{2n}^{(h)}$$  \hspace{1cm} (31)$$

$$C_v = b_1^{(v)}b_2^{(v)} \cdots b_{j_1}^{(v)}b_{j_1+1}^{(v)} \cdots b_{2n}^{(v)}$$  \hspace{1cm} (32)$$

Then the node’s coordinates \((A_h, A_v)\) can be derived from \(A_h = f_a(C_h)\) and \(A_v = f_a(C_v)\) where

$$f_a(C) = \begin{cases} 
\sum_{i=1}^{2n} b_i & \text{if } b_i = 0 \\
2n + \sum_{i=1}^{2n} b_i & \text{if } b_i = 1 
\end{cases}$$  \hspace{1cm} (33)$$

7.7.2 Routing Functions in TON

7.7.2.1 Routing Direction Function \(f_r\)

In the routing algorithm, the most important function is \(f_r\) which can be used to determine the direction of routing, in another word, to choose the next routing channel. Actually, \(f_r\) is the function to determine the position relationship between two shift-codes.

Given two \(m\) bits Johnson-codes: \(S\) and \(D\) where

\(S = s_1s_2s_3\ldots s_m\)

\(D = d_1d_2d_3\ldots d_m\)

The function \(f_r\) is \((p, c) = f_r(S, D)\) where

$$p = c \times \sum_{i=1}^{m} (s_i \oplus d_i)$$  \hspace{1cm} (34)$$

And \(c\) is calculated as following:

If \(s_1 = d_1 = 0\), then

$$c = \begin{cases} 
1 & \text{if } \sum_{i=1}^{m} d_i > \sum_{i=1}^{m} s_i \\
-1 & \text{if } \sum_{i=1}^{m} d_i < \sum_{i=1}^{m} s_i 
\end{cases}$$  \hspace{1cm} (35)$$
elseif \( s_1 = d_1 = 1 \), then

\[
c = \begin{cases} 
1 & \text{if } \sum_{i=1}^{m} d_i < \sum_{i=1}^{m} s_i \\
-1 & \text{if } \sum_{i=1}^{m} d_i > \sum_{i=1}^{m} s_i
\end{cases}
\] (36)

elseif \( s_1 = 0, d_1 = 1 \), then

\[
c = \begin{cases} 
1 & \text{if } \sum_{i=1}^{m} d_i <= \sum_{i=1}^{m} s_i \\
-1 & \text{if } \sum_{i=1}^{m} d_i > \sum_{i=1}^{m} s_i
\end{cases}
\] (37)

elseif \( s_1 = 1, d_1 = 0 \), then

\[
c = \begin{cases} 
1 & \text{if } \sum_{i=1}^{m} d_i >= \sum_{i=1}^{m} s_i \\
-1 & \text{if } \sum_{i=1}^{m} d_i < \sum_{i=1}^{m} s_i
\end{cases}
\] (38)

7.7.2.2 Routing Address Function \( f_d \)

The Routing Address Function \( f_d \) is to calculate the node address from the current node address with an offset value.

Given the shift code \( C_c \) and offset value \( O \), where

\[
C_c = b_1^{(c)} b_2^{(c)} \ldots b_{j-1}^{(c)} b_j b_{j+1}^{(c)} \ldots b_n^{(c)}
\]

The new shift code \( C_i \) can be derived by

\[
C_i = f_d (C_c, O) = b_1^{(i)} b_2^{(i)} \ldots b_{j-1}^{(i)} b_j b_{j+1}^{(i)} \ldots b_n^{(i)}
\] (39)

where

\[
b_j^{(i)} = \begin{cases} 
0 & i + j \leq n \\
1 & n + 1 \leq i + j \leq 2n \\
0 & i + j \geq 2n + 1
\end{cases}
\] (40)

and

\[
i = A + O
\] (41)
\[ A = f_a(C_c) = \begin{cases} 
\frac{2n}{\sum_{k=1}^{2n} b_k^{(c)}} & \text{if } b_1^{(c)} = 0 \\
2n + \sum_{k=1}^{2n} b_k^{(c)} & \text{if } b_1^{(c)} = 1 
\end{cases} \quad (42) \]

7.7.2.3 Routing channel function \( f_1, f_{II}, \text{and } f_{III} \)

The routing channel vector function is to derive output channels going to use in data routing.

- **I-TON**

  For I-TON, \( (v_e, v_w, v_s, v_n) = f_I(s_h, d_h, s_v, d_v) \) \quad (43)

  where

  \[ (p_h, c_h) = f_r(s_h, d_h) \]

  \[ (p_v, c_v) = f_r(s_v, d_v) \]

  \[ v_e = \begin{cases} 
1 & \text{if } p_h > 0 \\
0 & \text{if } p_h \leq 0 
\end{cases} \]

  \[ v_w = \begin{cases} 
0 & \text{if } p_h > 0 \\
1 & \text{if } p_h \leq 0 
\end{cases} \]

  \[ v_s = \begin{cases} 
1 & \text{if } p_v > 0 \\
0 & \text{if } p_v \leq 0 
\end{cases} \]

  \[ v_n = \begin{cases} 
0 & \text{if } p_v > 0 \\
1 & \text{if } p_v \leq 0 
\end{cases} \] \quad (46)

- **II-TON**

  For II-TON,

  \[ (v_{e1}, v_{e2}, v_{w1}, v_{w2}, v_{s1}, v_{s2}, v_{n1}, v_{n2}) = f_{II}(s_h, d_h, s_v, d_v) \] \quad (47)

  where
\[ m_{v2} = \begin{cases} \frac{p_h}{2} & \text{if } p_h > 0 \\ 0 & \text{if } p_h \leq 0 \end{cases} \]
\[ m_{e2} = \begin{cases} p_h - 2m_{c2} & \text{if } p_h > 0 \\ 0 & \text{if } p_h \leq 0 \end{cases} \]
\[ m_{w2} = \begin{cases} -\frac{p_h}{2} & \text{if } p_h < 0 \\ 0 & \text{if } p_h \geq 0 \end{cases} \]
\[ m_{m2} = \begin{cases} -p_h - 2m_{w2} & \text{if } p_h < 0 \\ 0 & \text{if } p_h \geq 0 \end{cases} \]
\[ m_{s2} = \begin{cases} \frac{p_v}{2} & \text{if } p_v > 0 \\ 0 & \text{if } p_v \leq 0 \end{cases} \]
\[ m_{s1} = \begin{cases} p_v - 2m_{s2} & \text{if } p_v > 0 \\ 0 & \text{if } p_v \leq 0 \end{cases} \]
\[ m_{n2} = \begin{cases} -\frac{p_v}{2} & \text{if } p_v < 0 \\ 0 & \text{if } p_v \geq 0 \end{cases} \]
\[ m_{n1} = \begin{cases} -p_v - 2m_{n2} & \text{if } p_v < 0 \\ 0 & \text{if } p_v \geq 0 \end{cases} \]

\[ v_{c1} = \begin{cases} 1 & \text{if } m_{c1} > 0 \\ 0 & \text{if } m_{c1} = 0 \\ 1 & \text{if } m_{c1} > 0 \end{cases} \]
\[ v_{c2} = \begin{cases} 1 & \text{if } m_{c2} > 0 \\ 0 & \text{if } m_{c2} = 0 \\ 1 & \text{if } m_{c2} > 0 \end{cases} \]
\[ v_{w1} = \begin{cases} 1 & \text{if } m_{w1} > 0 \\ 0 & \text{if } m_{w1} = 0 \\ 1 & \text{if } m_{w1} > 0 \end{cases} \]
\[ v_{w2} = \begin{cases} 1 & \text{if } m_{w2} > 0 \\ 0 & \text{if } m_{w2} = 0 \\ 1 & \text{if } m_{w2} > 0 \end{cases} \]
\[ v_{s1} = \begin{cases} 1 & \text{if } m_{s1} > 0 \\ 0 & \text{if } m_{s1} = 0 \\ 1 & \text{if } m_{s1} > 0 \end{cases} \]
\[ v_{s2} = \begin{cases} 1 & \text{if } m_{s2} > 0 \\ 0 & \text{if } m_{s2} = 0 \\ 1 & \text{if } m_{s2} > 0 \end{cases} \]
\[ v_{n1} = \begin{cases} 1 & \text{if } m_{n1} > 0 \\ 0 & \text{if } m_{n1} = 0 \\ 1 & \text{if } m_{n1} > 0 \end{cases} \]
\[ v_{n2} = \begin{cases} 1 & \text{if } m_{n2} > 0 \\ 0 & \text{if } m_{n2} = 0 \\ 1 & \text{if } m_{n2} > 0 \end{cases} \]

and


III-TON

For III-TON,

\[
(W, W^*) = f_{III}(s_h, d_h, s_v, d_v)\\
W = (v_{c1}, v_{c2}, v_{w1}, v_{w2}, v_{s1}, v_{s2}, v_{n1}, v_{n2}, v_{se}, v_{sw})\\
W^* = (v_{c1}^*, v_{c2}^*, v_{w1}^*, v_{w2}^*, v_{s1}^*, v_{s2}^*, v_{n1}^*, v_{n2}^*)
\]
\[
\begin{align*}
    m_{c2} &= \begin{cases} 
    \frac{p_h}{2} & \text{if } p_h > 0 \\
    0 & \text{if } p_h \leq 0 
    \end{cases} \\
    m_{e1} &= \begin{cases} 
    p_h - 2m_{c2} & \text{if } p_h > 0 \\
    0 & \text{if } p_h \leq 0 
    \end{cases} \\
    m_{w2} &= \begin{cases} 
    -\frac{p_h}{2} & \text{if } p_h < 0 \\
    0 & \text{if } p_h \geq 0 
    \end{cases} \\
    m_{w1} &= \begin{cases} 
    -p_h - 2m_{w2} & \text{if } p_h < 0 \\
    0 & \text{if } p_h \geq 0 
    \end{cases} \\
    m_{s2} &= \begin{cases} 
    \frac{p_v}{2} & \text{if } p_v > 0 \\
    0 & \text{if } p_v \leq 0 
    \end{cases} \\
    m_{s1} &= \begin{cases} 
    p_v - 2m_{s2} & \text{if } p_v > 0 \\
    0 & \text{if } p_v \leq 0 
    \end{cases} \\
    m_{n2} &= \begin{cases} 
    -\frac{p_v}{2} & \text{if } p_v < 0 \\
    0 & \text{if } p_v \geq 0 
    \end{cases} \\
    m_{n1} &= \begin{cases} 
    -p_v - 2m_{n2} & \text{if } p_v < 0 \\
    0 & \text{if } p_v \geq 0 
    \end{cases}
\end{align*}
\]

(51)

\[
\begin{align*}
    v^*_{c1} &= \begin{cases} 
    1 & \text{if } m_{c1} > 0 \\
    0 & \text{if } m_{c1} = 0 
    \end{cases} \\
    v^*_{c2} &= \begin{cases} 
    1 & \text{if } m_{c2} > 0 \\
    0 & \text{if } m_{c2} = 0 
    \end{cases} \\
    v^*_{w1} &= \begin{cases} 
    1 & \text{if } m_{w1} > 0 \\
    0 & \text{if } m_{w1} = 0 
    \end{cases} \\
    v^*_{w2} &= \begin{cases} 
    1 & \text{if } m_{w2} > 0 \\
    0 & \text{if } m_{w2} = 0 
    \end{cases} \\
    v^*_{s1} &= \begin{cases} 
    1 & \text{if } m_{s1} > 0 \\
    0 & \text{if } m_{s1} = 0 
    \end{cases} \\
    v^*_{s2} &= \begin{cases} 
    1 & \text{if } m_{s2} > 0 \\
    0 & \text{if } m_{s2} = 0 
    \end{cases} \\
    v^*_{n1} &= \begin{cases} 
    1 & \text{if } m_{n1} > 0 \\
    0 & \text{if } m_{n1} = 0 
    \end{cases} \\
    v^*_{n2} &= \begin{cases} 
    1 & \text{if } m_{n2} > 0 \\
    0 & \text{if } m_{n2} = 0 
    \end{cases}
\end{align*}
\]

(52)

Update \{m\} with the algorithm as shown in Figure 40.
Algorithm to update \{m\}.

\[
\begin{align*}
\text{Start:} \\
m_{nw} &= 0, m_{se} = 0 \\
\text{While } m_{w1} > 1 \& m_{n2} > 1 \\
& \\
m_{w1} &= m_{w1} - 1, m_{n2} = m_{n2} - 1 \\
m_{nw} &= m_{nw} + 1, m_{e1} = m_{e1} + 1 \\
\text{End} \\
\text{While } m_{n1} > 1 \& m_{w2} > 1 \\
& \\
m_{n1} &= m_{n1} - 1, m_{w2} = m_{w2} - 1 \\
m_{nw} &= m_{nw} + 1, m_{n1} = m_{n1} + 1 \\
\text{End} \\
\text{While } m_{s1} > 1 \& m_{e2} > 1 \\
& \\
m_{s1} &= m_{s1} - 1, m_{e2} = m_{e2} - 1 \\
m_{se} &= m_{se} + 1, m_{n1} = m_{n1} + 1 \\
\text{End} \\
\text{While } m_{e1} > 1 \& m_{s2} > 1 \\
& \\
M_{e1} &= m_{e1} - 1, m_{s2} = m_{s2} - 1 \\
m_{se} &= m_{se} + 1, m_{w1} = m_{w1} + 1 \\
\text{End} \\
\text{End}
\end{align*}
\]

\begin{align*}
& \text{Then} \\
& V_{el} = \begin{cases} 
1 & \text{if } m_{e1} > 0 \\
0 & \text{if } m_{e1} = 0 
\end{cases} \\
& V_{w1} = \begin{cases} 
1 & \text{if } m_{w1} > 0 \\
0 & \text{if } m_{w1} = 0 
\end{cases} \\
& V_{s1} = \begin{cases} 
1 & \text{if } m_{s1} > 0 \\
0 & \text{if } m_{s1} = 0 
\end{cases} \\
& V_{n1} = \begin{cases} 
1 & \text{if } m_{n1} > 0 \\
0 & \text{if } m_{n1} = 0 
\end{cases} \\
& V_{se} = \begin{cases} 
1 & \text{if } m_{se} > 0 \\
0 & \text{if } m_{se} = 0 
\end{cases} \\
& V_{w2} = \begin{cases} 
1 & \text{if } m_{w2} > 0 \\
0 & \text{if } m_{w2} = 0 
\end{cases} \\
& V_{w2} = \begin{cases} 
1 & \text{if } m_{w2} > 0 \\
0 & \text{if } m_{w2} = 0 
\end{cases} \\
& V_{s2} = \begin{cases} 
1 & \text{if } m_{s2} > 0 \\
0 & \text{if } m_{s2} = 0 
\end{cases} \\
& V_{e2} = \begin{cases} 
1 & \text{if } m_{e2} > 0 \\
0 & \text{if } m_{e2} = 0 
\end{cases}
\end{align*}

\[ \text{(53)} \]

7.7.2.4 Routing vector function \( f_{v1}, f_{v2}, \text{and } f_{v3} \)

- **I-TON**

For I-TON, \( V = f_{v1}(s_h, d_h, s_v, d_v) \) where the routing vector \( V \) is generate with the
algorithm as shown in Figure 41.

```
Start:
    Initialize $V = \phi$
    While $v_e > 0$
        $V = V + E$, $v_e = v_e - 1$
    End
    While $v_w > 0$
        $V = V + W$, $v_w = v_w - 1$
    End
    While $v_s > 0$
        $V = V + S$, $v_s = v_s - 1$
    End
    While $v_n > 0$
        $V = V + N$, $v_n = v_n - 1$
    End
End
```

Figure 41 Algorithm to generate the routing vector $V$ for TON-I.

- II-TON

For II-TON, $V = f_{v2}(s_h, d_h, s_v, d_v)$ where the routing vector $V$ is generate with the algorithm as shown in Figure 42.
III-TON

For III-TON, \( V = f_{v3}(s_h, d_h, s_v, d_v) \) where the routing vector \( V \) is generate with the algorithm as shown in Figure 43.
Start:
\((v_e, v_w, v_s, v_n) = f(r_3(s_h, s_v, d_h, d_v))\)
Initialize \(V = \emptyset\)
While \(v_e > 0\)
    \(V = V + E2, v_e = v_e - 1\)
End
While \(v_e > 0\)
    \(V = V + E1, v_e = v_e - 1\)
End
While \(v_w > 0\)
    \(V = V + W2, v_w = v_w - 1\)
End
While \(v_w > 0\)
    \(V = V + W1, v_w = v_w - 1\)
End
While \(v_s > 0\)
    \(V = V + S2, v_s = v_s - 1\)
End
While \(v_s > 0\)
    \(V = V + S1, v_s = v_s - 1\)
End
While \(v_n > 0\)
    \(V = V + N2, v_n = v_n - 1\)
End
While \(v_n > 0\)
    \(V = V + N1, v_n = v_n - 1\)
End
End

Figure 43 Algorithm to generate the routing vector \(V\) for TON-III.

7.7.3 Relationship between Nodes Coordinates and Address

Given a node address in the TON is \(C = C_h | C_v\) (\(C\) is \(N\) bits where \(N = 4n\), \(C_h\) and \(C_v\) are both 2n bits) where

\[
C_h = b_1^{(h)}b_2^{(h)} \ldots b_{j-1}^{(h)}b_j^{(h)} \ldots b_{2n}^{(h)}
\]  \hspace{1cm} (54)

\[
C_v = b_1^{(v)}b_2^{(v)} \ldots b_{j-1}^{(v)}b_j^{(v)} \ldots b_{2n}^{(v)}
\]  \hspace{1cm} (55)
Then the node’s coordinates \((A_h, A_v)\) can be derived by the function \(A_h = f_h(C_h)\) and \(A_v = f_v(C_v)\) where

\[
f_a(C) = \begin{cases} 
\sum_{i=1}^{2n} b_i & \text{if } b_1 = 0 \\
2n + \sum_{i=1}^{2n} b_i & \text{if } b_1 = 1 
\end{cases}
\]  

(56)

### 7.7.4 Deterministic Routing Algorithm for TONs

We classified all channels in TONs to three classes: X-channel, Y-channel and Z-channel. In I-TON, channel E and W are X-channels, channel N and S are Y-channels. In II-TON and III-TON, channel E1, E2, W1 and W2 are X-channels, channel N1, N2, S1 and S2 are Y-channels. And channel SE and NW are Z-channels.

In deterministic routing, all data transmission should follow X-Y sequence in I-TON and II-TON, and Z-X-Y sequence in III-TON.

There are two types of deterministic routing algorithms for each TON, dynamic routing and predetermined routing. In dynamic routing,

#### 7.7.4.1 Dynamic routing

In dynamic routing, the routing is implemented in each mid node. Before the data package arrive to its destination node, in each intermediary node, the router will read its header to find its destination address, and then based on the destination node address \(D\) and the current node address \(S\), one channel will be selected and the data package will be transmitted along this channel.

- **Dynamic routing algorithm I-TON**

  The dynamic routing algorithm of I-TON is shown in Figure 44.
Algorithm Routing $(S \rightarrow D)$:

Begin:

- Compare $D$ and the Current Address $S$
  - $(p_h,c_h) = f_r(s_h,d_h)$
  - $(p_v,c_v) = f_r(s_v,d_v)$

- If $c_h = 0$ & $c_v = 0$
  - channel = internal
- Elself $c_h > 0$
  - channel = E
- Elself $c_h < 0$
  - channel = W
- Elself $c_h = 0$ & $c_v > 0$
  - channel = S
- Elself $c_h = 0$ & $c_v < 0$
  - channel = N

End

End Routing

Figure 44 Dynamic routing algorithm of I-TON.

- Dynamic routing algorithm for II-TON

The dynamic routing algorithm of II-TON is shown in Figure 45.
Dynamic routing algorithm for III-TON

In routing within III-TON, we have following equal routing channel combinations.

\[
\begin{align*}
W1 + N2 &= E1 + NW \\
W2 + N1 &= S1 + NW \\
E2 + S1 &= N1 + SE \\
E1 + S2 &= W1 + SE
\end{align*}
\]  

(57)

To maximize the utilization of all channels and network throughput, Z-channels has the highest priorities in channel assignment.

The dynamic routing algorithm of III-TON is shown in Figure 46.
7.7.4.2 Predetermined routing

In predetermined routing, before a data package is sent from its source node, all its
routing path is determined and stored as a routing channel vector $V$ in the header. The routing vector is composed of a sequence of channels going to use in the routing. In each intermediary node, the router will read the header, assign the output channel according to the first element in the routing vector, then update $V$ to remove the used element and replace $V$ into the header.

- **Predetermined routing algorithm I-TON**

  The predetermined routing algorithm of I-TON is shown in Figure 47.

  ![Algorithm Routing (S→D):](image)

  **Figure 47** Predetermined routing algorithm of I-TON.

- **Predetermined routing algorithm II-TON**

  The predetermined routing algorithm of II-TON is shown in Figure 48.
7.7.5 Basic Adaptive Routing Algorithm for TONs

In basic adaptive routing, the routing paths from source node to destination node are vary depends on the traffic condition but all their lengths are maintained to be same to the minimum.

Here a new function $T_c$ is introduced to describe the traffic condition in a node which will be used in output channel decision-making in adaptive routing.
Assume amount of packages in output FIFO of channel $x$ is $n_x$, then

$$t_x = B - n_x$$  \hspace{1cm} (58)$$

where $B$ is the maximum capacity of the FIFO.

For I-TON, $T_c = (t_e, t_w, t_s, t_n)$.  \hspace{1cm} (59)$$

For II-TON, $T_c = (t_{c1}, t_{c2}, t_{w1}, t_{w2}, t_{s1}, t_{s2}, t_{n1}, t_{n2})$.  \hspace{1cm} (60)$$

For III-TON, $T_c = (t_{c1}, t_{c2}, t_{w1}, t_{w2}, t_{s1}, t_{s2}, t_{n1}, t_{n2}, t_{se}, t_{sw})$.  \hspace{1cm} (61)$$

There are also two types of adaptive routing algorithms corresponding to two deterministic routing algorithms, adaptive dynamic routing and adaptive predetermined routing.

### 7.7.5.1 Adaptive dynamic routing

In adaptive dynamic routing, before the data package arrive to its destination, in each intermediary node the router will read its header, find its destination address, and then compare it with current address and take the traffic condition into account to assign a proper output channel to it.

- **Adaptive dynamic routing algorithm I-TON**

  The adaptive dynamic routing algorithm of I-TON is shown in Figure 50.
Algorithm Routing (S→D):
Begin:
\[ A_c = f_j(s_b, d_h, s_c, d_e) \]
\[ c = \max \{ c_i \mid c_i \in (A_c \cdot T_c) \} \]
channel = c
End

* The operator ‘•’ in the algorithm represents the element-by-element product of the two arrays.

Figure 50 Adaptive dynamic routing algorithm of I-TON.

- Adaptive dynamic routing algorithm II-TON

The adaptive dynamic routing algorithm of II-TON is shown in Figure 50.

Algorithm Routing (S→D):
Begin:
\[ A_c = f_j(s_b, d_h, s_c, d_e) \]
\[ c = \max \{ c_i \mid c_i \in (A_c \cdot T_c) \} \]
channel = c
End

Figure 51 Adaptive dynamic routing algorithm of II-TON.

- Adaptive dynamic routing algorithm III-TON

The adaptive dynamic routing algorithm of III-TON is shown in Figure 52.
7.7.5.2 Adaptive predetermined routing

In adaptive predetermined routing, the sequence of routing channels in the routing vector \( V \) should be modified according to the real-time traffic condition to minimize the latency in each intermediate node.

- Adaptive predetermined routing algorithm I-TON

The adaptive predetermined routing algorithm of I-TON is shown in Figure 53.

```
Algorithm Routing (S→D):
Begin:
\[
(f_{III}(s_h, d_h, s_v, d_v), c = \max \{ i \mid c_i \in (A_v \cdot T_c) \cup (A^*_c \cdot T_c) \})
\]
channel = c
End
```

Figure 53 Adaptive dynamic routing algorithm of I-TON.

- Adaptive predetermined routing algorithm for II-TON

The adaptive predetermined routing algorithm of II-TON is shown in Figure 54.
The adaptive predetermined routing algorithm of III-TON is shown in Figure 55.

![Figure 55](image)

Algorithm Routing $(S\rightarrow D)$:
Generate the routing channels:
\[ W_{i'} = (v_{s1}, v_{s2}, v_{s3}, v_{s4}, v_{s5}) = f_{III}(s_h, d_h, s_v, d_v) \]
Routing:
While $W_{i'} \neq 0$
\[ S_{i'} = \{s_i|s_i = \text{sign}(v_i), v_i \in W_{i'}\} \]
\[ v = \max\{v_i|v_i \in (S_{i'} \cdot T_e)\} \]
channel = $v$
Update $W_{i'}$ by reducing 1 from the channel of $v$ from $W_{i'}$
End

Figure 55  Adaptive dynamic routing algorithm of III-TON.

- Adaptive predetermined routing Algorithm for III-TON

The adaptive predetermined routing algorithm of III-TON is shown in Figure 55.

It can be seen that the algorithm is not optimal because it does not include channel substitute between horizontal/vertical channels and diagonal channels, to match the fastest output channel. The reason is that such substitute will need the update of routing vector $W_{i'}$ in each intermediate node, which is almost same to adaptive dynamic routing.
algorithm.

7.8 Simulation and Analysis

7.8.1 Simulation Setup

We test the maximum traffic load the network can handle without blocking. The network is set to be homogeneous; and all nodes have the same frame generating rate. Simulation parameters are listed in Table 20.

<table>
<thead>
<tr>
<th>Table 20</th>
<th>Simulation Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Frequency</td>
<td>40G</td>
</tr>
<tr>
<td>Electrical Frequency</td>
<td>4G</td>
</tr>
<tr>
<td>Frame Length</td>
<td>256 bits</td>
</tr>
<tr>
<td>Electrical Bus Width</td>
<td>128 bits</td>
</tr>
<tr>
<td>Frame Generating Rate</td>
<td>$2.4 \times 10^{-3} \sim 0.08$ frame/ nanosecond</td>
</tr>
<tr>
<td>Frame Generation</td>
<td>Uniform Distribution</td>
</tr>
<tr>
<td>Network Size</td>
<td>$6 \times 6 \sim 16 \times 16$</td>
</tr>
<tr>
<td>Routing Algorithm</td>
<td>Deterministic Routing</td>
</tr>
<tr>
<td>Buffer service</td>
<td>FIFO</td>
</tr>
</tbody>
</table>

At least 10,000 frames are generated by each node per simulation run and transmitted to other nodes with the same probability. Frames generated by a core are switched to an output channel via a crossbar. Data are modulated to a sequence of serial optical data bits and transmitted to the next node through a waveguide. On the receiving side, the optical bit sequence is received by a photodetector and converted to a parallel electrical data frame. This process repeats until the frame arrives to its destination core. The total time of the process is the frame transmission time.

We consider first a deterministic routing algorithm, in which data are transmitted according to the following sequence: Level3 channels first, Level2 channels second and
Level1 channels last. In Level1 channels x-y routing is applied to avoid the deadlock.

7.8.2 Simulation Results for TON-I

An average frame transmission time in TON-I networks of different sizes and with different frame generating rates are shown in Figure 56. It can be observed that before a point when the network reaches the blocking condition the waiting time is negligible.

As it was mentioned before, each channel has an input buffer. An average and a maximum utilization of these buffers are shown in Figure 57 and Figure 58.
Figure 57  Average input buffer utilization in TON-I

Figure 58  Maximum input buffer utilization in TON-I
7.8.3 Simulation Results for TON-II

An average frame transmission time in TON-II of different sizes and with different frame generating rates are shown in Figure 59. It can be observed that before a point when the network reaches the blocking condition the waiting time is negligible.

![Average Transmission Time](image)

**Figure 59** Average frame transmission time in TON-II

The average and maximum utilization of input buffers are shown in Figure 60 and Figure 61.
Figure 60  Average input buffer utilization in TON-II

Figure 61  Maximum input buffer utilization in TON-II
7.8.4 Simulation Results for TON-III

An average frame transmission time in networks of different sizes and with different frame generating rates are shown in Figure 62. It can be observed that before a point when the network reaches the blocking condition the waiting time is negligible.

As it was mentioned before, each channel is equipped with an input buffer. An average and a maximum utilization of these buffers are shown in Figure 63 and Figure 64.

From simulation results shown in Figure 56 to Figure 64, it can be seen that with a number of DOCs increasing from 4 to 8 and 10, as in TON-I, II and III, respectively, the average single frame transmission time from the source node to the destination node and an average/maximum buffer usage are dropped remarkably; the buffer usage becomes aligned.

Figure 62 Average frame transmission time in TON-III
Figure 63  Average input buffer utilization in TON-III

Figure 64  Maximum input buffer utilization in TON-III
TON architectures are highly scalable due to packet-switching. The frame transmission time in TON architectures mainly depends on the number of hops or EO/OE conversion and the transmission time in each channel. The architectures utilize passive MRRs for signal switching and wavelength-multiplexing can be used in each optical channel to provide multiple simultaneous communications. Hence overall network throughput is expected to be extremely high. Direct optical channels (DOCs) offer various shortcuts and their combination significantly reduce distances between remote node pairs in the network. Due to the availability of multiple routing paths between node pairs, the communication in TON networks is highly fault tolerant.

7.9 Power Analysis

7.9.1 Electrical Back-end Components

Table 21 shows the estimated energy costs of the electrical back-end for the optical link (drivers, receivers, and clocking) using a predictive technology model for the 22nm node [62]. The dominant source of energy consumption is the modulator driver, followed by the optical receiver and clocking circuits. Based on these parameters Tab.15 can be derived which listed energy consumed for transmitting a single bit via single DOC in TON-I, II and III, respectively. Then the average energy consumption for transmitting a single bit between any two cores in TON-I, II and III are shown in Table 22 respectively.
Table 21  Estimated energy of optical components

<table>
<thead>
<tr>
<th>Component</th>
<th>Energy (fJ/b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serializer</td>
<td>1.5</td>
</tr>
<tr>
<td>Pre- Driver</td>
<td>19.0</td>
</tr>
<tr>
<td>Push-Pull Modulator</td>
<td>70.0</td>
</tr>
<tr>
<td>Analog Receiver Front End</td>
<td>40.0</td>
</tr>
<tr>
<td>Flip-Flop Sampling &amp; Monitoring</td>
<td>12.0</td>
</tr>
<tr>
<td>Deserializer</td>
<td>1.5</td>
</tr>
<tr>
<td>Optical Clocking Source</td>
<td>2.0</td>
</tr>
<tr>
<td>Clock Phase Control</td>
<td>12.0</td>
</tr>
<tr>
<td>Laser</td>
<td>300.0</td>
</tr>
<tr>
<td>Total</td>
<td>458</td>
</tr>
</tbody>
</table>

Table 22  Estimated energy consumption of single channel in TON-I, II, and III

<table>
<thead>
<tr>
<th>Architecture</th>
<th>TON-I</th>
<th>TON-II</th>
<th>TON-III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Energy (fJ/b) per Channel</td>
<td>5.715</td>
<td>7.695</td>
<td>8.880</td>
</tr>
<tr>
<td>Optical Energy (fJ/b) per Channel</td>
<td>458</td>
<td>458</td>
<td>458</td>
</tr>
<tr>
<td>Total Energy (fJ/b) per Channel</td>
<td>463.715</td>
<td>465.695</td>
<td>466.88</td>
</tr>
</tbody>
</table>

Table 23  Estimated energy consumption of single channel in TON-I, II and III

<table>
<thead>
<tr>
<th>Energy</th>
<th>Topologies</th>
<th>8×8</th>
<th>16×16</th>
<th>32×32</th>
</tr>
</thead>
<tbody>
<tr>
<td>AD</td>
<td>TON-I</td>
<td>1854.86</td>
<td>3709.72</td>
<td>7419.44</td>
</tr>
<tr>
<td></td>
<td>TON-II</td>
<td>1164.24</td>
<td>2095.63</td>
<td>3958.41</td>
</tr>
<tr>
<td></td>
<td>TON-III</td>
<td>1078.49</td>
<td>1853.51</td>
<td>3408.22</td>
</tr>
</tbody>
</table>

7.9.2 Transmission Power Loss

The optical power loss of the MRR switch matrix mainly is due to off-state, on-state losses, waveguide crossing loss and waveguide propagation loss [23].

In our analysis, the waveguide propagation loss is neglected. Other loss values are adopted according to [60], as shown in Table 24. Transmission power losses in each channel in TON-I, II and III are listed in Table 25, Table 26 and 0, respectively.
Table 24  Optical Power Budget

<table>
<thead>
<tr>
<th>Component</th>
<th>Loss(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRR off-state</td>
<td>0.06</td>
</tr>
<tr>
<td>MRR on-state</td>
<td>1.5</td>
</tr>
<tr>
<td>Waveguide Crossing</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 25  Transmission Power Loss in TON-I

<table>
<thead>
<tr>
<th>Port</th>
<th>Wavelength</th>
<th>Channel</th>
<th>Power Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>w₁</td>
<td>E₁</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>w₂</td>
<td>S₁</td>
<td>1.67</td>
</tr>
<tr>
<td>LD</td>
<td>w₁</td>
<td>W₁</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>w₂</td>
<td>N₁</td>
<td>1.67</td>
</tr>
</tbody>
</table>

Table 26  Transmission Power Loss in TON-II

<table>
<thead>
<tr>
<th>Node</th>
<th>Channel</th>
<th>Wavelength</th>
<th>Port</th>
<th>Power Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-Node</td>
<td>E₁</td>
<td>w₁</td>
<td>LS</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>E₂</td>
<td>w₂</td>
<td></td>
<td>2.22</td>
</tr>
<tr>
<td></td>
<td>S₂</td>
<td>w₃</td>
<td></td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>S₁</td>
<td>w₄</td>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>W₁</td>
<td>w₁</td>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>W₂</td>
<td>w₂</td>
<td>LD</td>
<td>2.22</td>
</tr>
<tr>
<td></td>
<td>N₂</td>
<td>w₃</td>
<td></td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>N₁</td>
<td>w₄</td>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td>II-Node</td>
<td>E₁</td>
<td>w₁</td>
<td>LS</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>S₂</td>
<td>w₂</td>
<td></td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>E₂</td>
<td>w₃</td>
<td></td>
<td>3.77</td>
</tr>
<tr>
<td></td>
<td>S₁</td>
<td>w₄</td>
<td></td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>W₁</td>
<td>w₁</td>
<td>LD</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>N₂</td>
<td>w₂</td>
<td></td>
<td>3.56</td>
</tr>
<tr>
<td></td>
<td>W₂</td>
<td>w₃</td>
<td></td>
<td>3.77</td>
</tr>
<tr>
<td></td>
<td>N₁</td>
<td>w₄</td>
<td></td>
<td>2.00</td>
</tr>
</tbody>
</table>
It can be seen from Table 25 to 0, transmission losses in TON-I, II and III is around 1.67dB, 2.64dB and 3.38dB, with maximum 1.67dB, 3.77dB and 5.78dB, respectively.

The channel transmission loss is linear to the channel length, or the number of OIUs in the channel. Losses in each OIU are related to the number of MRR switches in it, which is linear to the number of DOCs in each node.

In most ONoC architectures which use MRR tuning, optical receivers have to deal with a wide power range because the transmission power loss depends on the specific path. Wide power range receivers are difficult to implement. On the contrary, in TONs, the transmission power loss per DOC is a constant, and thus the receiver is less complex.
7.10 Conclusion

In this chapter three different 2D-torus based packet switching optical NoC architectures TON-I, TON-II and TON-III are proposed. These packet-switching TONs architectures are highly scalable. The use of passive MRR switching, WDM and direct optical channels (DOCs) offers an extremely high bandwidth, low latency, low cost, low power consumption, and fault tolerance property. The performances of the TON networks are evaluated by simulation and the results demonstrate that the TONs have high throughputs. TON-I has a lesser cost and hence a relatively lower network performance, whereas TON-III has a highest performance but requires complex hardware and control. TON-II represents a good tradeoff between these three architectures. Related issues such as communication energy and transmission power losses are also addressed. Many outstanding features of the designed TON architectures make it to be a compelling solutions for future optical NoCs.
CHAPTER 8
GENERALIZED PACKET SWITCHING OPTICAL NETWORK-ON-CHIP ARCHITECTURES

In this chapter a Generalized 2D-Torus-based Optical Network-on-Chip (GTON) is proposed. GTON is a generalized packet switching optical NoC architecture based on micro-ring resonator (MRR) optical switches. Given sufficient routing wavelengths, any network topology can be implemented in GTON. A systematic approach of the design of network interconnections and routers in GTON is presented explicitly. The use of passive MRR switching, Wavelength Division Multiplexing (WDM) and Direct Optical Channels (DOC) allows for attaining high bandwidth and makes GTON fault tolerant. The GTON architecture requires a small number of routing wavelengths and MRRs and demonstrates low cost, latency and power consumption. The performance of the network is evaluated by simulation. Related issues such as channel design, transmission power loss and the buffer analysis are provided.

8.1 Introduction

The physical interconnection in GTON is same to 2D-torus. The new concept of direct optical channel (DOC) is adopted in GTON to identify a direct light path between two nodes. A DOC is a set of interconnected physical links (waveguides). By using a predetermined routing wavelength, light travels in a DOC without relay. Optical data packets are switched between DOCs in the GTON. Theoretically, the DOC can be introduced to connect any two nodes in the network and thus any existing network topology can be built on the GTON [84].
8.2 GTON Architecture Overview

The GTON is an $N \times N$, where $N=2n$, $n \in \mathbb{N}$, 2D-torus-based network. A $6 \times 6$ GTON example is presented in Figure 65. Each node in the GTON is presented by a pair $(x,y)$ where $x,y=\{0,1,\ldots,N-1\}$. Each node has a router and a processor (core), and it is physically connected to four adjacent nodes. Internally, each router has a wavelength routed Optical Interconnection Unit (OIU) and an Optical Electrical Router (OER). OIU is a MRR matrix device with six ports, of which four are external ports (S1, S2, D1, D2), and two, i.e., (LS, LD) are internal ports to connect to the OER.

The GTON nodes are classified into two classes: I-Node associated with I-router; and II-node with II-router. I-router has an I-OIU and an I-OER, and II-router has an II-OIU and an II-OER. Given coordinates $(x, y)$, a node is an I-Node if $x + y = 2n$ ($n \in \mathbb{N}$), otherwise it is a II-Node.

![Figure 65 6x6 GTON network topology](image)
The interconnection of the 6×6 GTON is shown in Figure 65. It can be seen that for the I-Node at \((x, y)\), its S1 port is connected to the S2 port of the II-node at \((x, y-1)\), S2 port is connected to the D2 of the II-node at \((x-1, y)\), D1 is connected to the S1 of the II-node at \((x+1, y)\), and D2 is connected to the D1 of the II-node at \((x, y+1)\). And for the II-Node at \((x, y)\), its S1 port is connected to the D1 port of the I-node at \((x-1, y)\), S2 is connected to the S1 of the I-node at \((x, y+1)\), D1 is connected to the D2 of the I-node at \((x, y-1)\), and D2 is connected to the S2 of the I-node at \((x+1, y)\).

![Diagram](image-url)

Figure 66 A 6×6 GTON architecture and examples of DOCs.
Unlike other optical architectures, data transmission in GTON is via a DOC. The DOC is a light path between two nodes that consists of one or more physical links and OIUs. By utilizing a predetermined routing wavelength and with properly designed OIUs, light can travel through the DOC without any relay or arbitration. DOCs can be placed between any two nodes. Examples of a one-hop DOC and a two-hop DOC in the 6×6 GTON are shown in Figure 65.

Two nodes which do not have a DOC, data packets are transmitted by relaying in intermediary nodes. This would require electrical/optical and optical/electrical (E/O & O/E) conversion. Data transmission latency is directly related to the number of such conversions.

8.3 GTON Design Schema

In this section we introduce the design of a GTON architecture given a target topology. As it has been mentioned earlier, the GTON is a generalized architecture. Any topology which uses 2D-coordinate addressing can be mapped into the GTON. A key issue here is to design the router, i.e. OIU and OER for each node to realize DOCs required by the network topology. The design procedure includes the following steps:
1. Convert the target network topology into channel map.
2. Map channels into DOCs.
3. Derive wavelength routing table per OIU, and assign wavelengths to DOCs.
4. Design OIUs based on wavelength routing table.
5. Design OERs based on the wavelength table and the OIUs design.

The design of a special network architecture called GTON-XII is illustrated in the next subsection.
8.3.1 GTON-XII Topology

GTON-XII topology is 2D-torus based. Each node in the network is directly connected to 12 other nodes via 12 DOCs. Given a node \((x, y)\), its channel map is shown in Figure 67. The twelve DOCs are:

E1: \((x, y) \leftrightarrow (x+1, y)\); W1: \((x, y) \leftrightarrow (x-1, y)\); S1: \((x, y) \leftrightarrow (x, y+1)\); N1: \((x, y) \leftrightarrow (x, y-1)\); E2: \((x, y) \leftrightarrow (x+2, y)\); W2: \((x, y) \leftrightarrow (x-2, y)\); S2: \((x, y) \leftrightarrow (x, y+2)\); N2: \((x, y) \leftrightarrow (x, y-2)\); SE: \((x, y) \leftrightarrow (x+2, y+2)\); NW: \((x, y) \leftrightarrow (x-2, y-2)\); NE: \((x, y) \leftrightarrow (x+2, y-2)\); and SW: \((x, y) \leftrightarrow (x-2, y+2)\).

These DOC are grouped into three levels: Level 1 with channels E1, W1, S1 and N1, which are one-hop connections in X or Y direction; Level 2 with channels E2, W2, S2 and N2, which are two-hop connections in X or Y direction; and Level 3 with channels NE, NW, SE and SW, which are two-hop diagonal connections.

The minimum size of GTON-XII is 6×6 to avoid multiple DOCs between the same pair of nodes.
8.3.2 Network Properties of GTON-XII

The GTON-XII network has the following properties.

**Lemma 10.** The diameter $D$ of an $N\times N$ GTON-XII is

$$D = \left\lceil \frac{N}{4} \right\rceil + 1$$

(62)

**Lemma 11.** The average distance $AD$ between any pair of nodes in the GTON-XII can be derived from the following equations.

If $N = 4m$, then

$$AD = \frac{32m^3 + 36m^2 + m}{48m^2 - 3}$$

(63)

Else when $N+2 = 4m$, then

$$AD = \frac{32m^3 - 12m^2 - 11m + 3}{48m^2 - 48m + 9}$$

(64)

Table 28 compares the network properties of GTON and a few other popular
networks. ‘Average Distance’ denotes the number of hops, or the number of OE and EO conversions needed for data transmission between two nodes.

Table 28 Diameter (D) and Average Distance (AD) for Different Networks.

<table>
<thead>
<tr>
<th>Network</th>
<th>GTON-XII</th>
<th>Mesh</th>
<th>2D-Torus</th>
<th>Hypercube</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>D</td>
<td>AD</td>
<td>D</td>
<td>AD</td>
</tr>
<tr>
<td>8x8</td>
<td>3</td>
<td>2.13</td>
<td>14</td>
<td>5.33</td>
</tr>
<tr>
<td>16x16</td>
<td>5</td>
<td>3.44</td>
<td>30</td>
<td>10.67</td>
</tr>
<tr>
<td>32x32</td>
<td>9</td>
<td>6.09</td>
<td>62</td>
<td>21.33</td>
</tr>
</tbody>
</table>

8.3.3 Channel Mapping in GTON-XII

Mapping of the 12 channels into DOCs means selection of a direct light path for each channel. Since there are always more than one solutions available for each channel, we enforce the following rules in such selection to minimize the cost of the resulted DOC:

1. For each DOC, the internal routing path for the same type routers must be the same, which means that if one I-router in the DOC uses the internal routing path from port S1 to D1 (S1D1), then all other I-routers in this DOC must also use the internal routing path as S1D1.

2. Among different DOCs, using as less as possible different internal paths for the router of a same type such that the number of required wavelengths is minimal.

Based on these rules one solution of mapping of 12 channels in GTON-XII is presented in Table 30 - 0. Table 29 - 0 are for I-node and Table 32 - 0 are for II-node. In these tables, each line represents a DOC. The channeling column shows physical path of the optical signal.

For example, in the first row in Table 30, ‘(x,y)LS(x,y)D1(x+1,y)S1(x+1,y)LD’
indicates routing the signal from LS to D1 in node (x, y), which is an internal path; then transmission from D1 port of node (x,y) to S1 port of node (x+1,y), which is an external path. In (x+1, y) nodes it is routed from S1 to LD, which is an internal path again. A unique wavelength is assigned per each DOC.

Mapping of node (2, 2) (Figure 68 to Figure 70) and (2, 3) (Figure 71 to Figure 73) in the 6×6 GTON-XII are presented to illustrate the channel mapping of I-node and II-node, respectively. Since GTON-XII has a symmetric topology, the mappings of all nodes (either type I or type II) are identical.

<table>
<thead>
<tr>
<th>Table 29</th>
<th>I-Node Level1 Channel Mapping in GTON-XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.</td>
<td>D.</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x+1,y)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y+1)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-1,y)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y-1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 30</th>
<th>I-Node Level2 Channel Mapping in GTON-XII</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.</td>
<td>D.</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x+2,y)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y+2)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y)</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y-2)</td>
</tr>
</tbody>
</table>
### Table 31  
I-Node Level3 Channel Mapping in GTON-XII

<table>
<thead>
<tr>
<th>S.</th>
<th>D.</th>
<th>Channeling</th>
<th>W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x,y)</td>
<td>(x+2,y-2)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_1 \Rightarrow (x+1,y)S_1 \rightarrow (x+1,y)D_1 \Rightarrow (x+1,y-1)S_1 \Rightarrow (x+1,y-2)D_2 \Rightarrow (x+2,y-2)L_D$</td>
<td>w9</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x+2,y+2)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_2 \Rightarrow (x,y+1)D_1 \rightarrow (x,y+1)S_2 \Rightarrow (x,y+2)D_1 \Rightarrow (x,y+2)S_1 \rightarrow (x+1,y+2)D_2 \Rightarrow (x+2,y+2)L_D$</td>
<td>w10</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y+2)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_2 \Rightarrow (x-1,y)D_2 \rightarrow (x-1,y)S_2 \Rightarrow (x-1,y+1)S_1 \rightarrow (x-1,y+1)D_2 \Rightarrow (x-1,y+2)D_1 \rightarrow (x,y+2)L_S$</td>
<td>w11</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y-2)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_1 \Rightarrow (x,y-1)S_2 \rightarrow (x,y-1)D_1 \Rightarrow (x,y-2)D_2 \rightarrow (x,y-2)S_2 \Rightarrow (x-2,y-2)D_1 \rightarrow (x-2,y-2)L_S$</td>
<td>w12</td>
</tr>
</tbody>
</table>

### Table 32  
II-Node Level1 Channel Mapping in GTON-XII

<table>
<thead>
<tr>
<th>S.</th>
<th>D.</th>
<th>Channeling</th>
<th>W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x,y)</td>
<td>(x+1,y)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_2 \Rightarrow (x+1,y)S_2 \rightarrow (x+1,y)L_D$</td>
<td>w13</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y-1)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_1 \Rightarrow (x,y-1)D_2 \rightarrow (x,y-1)L_S$</td>
<td>w14</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-1,y)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_1 \Rightarrow (x,y-1)D_1 \rightarrow (x,y-1)L_S$</td>
<td>w15</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y+1)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_2 \Rightarrow (x,y-1)S_1 \rightarrow (x,y-1)L_D$</td>
<td>w16</td>
</tr>
</tbody>
</table>

### Table 33  
II-Node Level2 Channel Mapping in GTON-XII

<table>
<thead>
<tr>
<th>S.</th>
<th>D.</th>
<th>Channeling</th>
<th>W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x,y)</td>
<td>(x+2,y)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_2 \Rightarrow (x+1,y)S_2 \rightarrow (x+1,y)D_1 \Rightarrow (x+2,y)S_1 \rightarrow (x+2,y)L_D$</td>
<td>w17</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y-2)</td>
<td>$(x,y)L_S \rightarrow (x,y)D_1 \Rightarrow (x,y-1)D_2 \rightarrow (x,y-1)S_1 \rightarrow (x,y-2)S_2 \rightarrow (x,y-2)L_D$</td>
<td>w18</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_1 \Rightarrow (x-1,y)D_1 \rightarrow (x-1,y)S_2 \Rightarrow (x-2,y)D_2 \rightarrow (x-2,y)L_S$</td>
<td>w19</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x,y+2)</td>
<td>$(x,y)L_D \rightarrow (x,y)S_2 \Rightarrow (x,y+1)S_1 \rightarrow (x,y+1)D_1 \rightarrow (x,y+2)L_S$</td>
<td>w20</td>
</tr>
</tbody>
</table>
Table 34  II-Node Level3 Channel Mapping in GTON-XII

<table>
<thead>
<tr>
<th>S.</th>
<th>D.</th>
<th>Channeling</th>
<th>W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x,y)</td>
<td>(x+2,y-2)</td>
<td>(x,y)LD \rightarrow (x,y)D_1 \rightarrow (x,y-1)D_2 \rightarrow (x,y-2)S_2 \rightarrow (x,y-2)D_2 \rightarrow (x+1,y-2)S_1 \rightarrow (x+2,y-2)LD</td>
<td>W_{21}</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x+2,y+2)</td>
<td>(x,y)LD \rightarrow (x,y)D_2 \rightarrow (x+1,y)S_2 \rightarrow (x+1,y+1)D_1 \rightarrow (x+1,y+1)S_2 \rightarrow (x+1,y+2)S_1 \rightarrow (x+2,y+2)LD</td>
<td>W_{22}</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y+2)</td>
<td>(x,y)LD \rightarrow (x,y)S_2 \rightarrow (x,y+1)S_1 \rightarrow (x+y+1)D_1 \rightarrow (x,y+2)S_1 \rightarrow (x-1,y+2)S_2 \rightarrow (x-2,y+2)D_2 \rightarrow (x+2,y-2)LS</td>
<td>W_{23}</td>
</tr>
<tr>
<td>(x,y)</td>
<td>(x-2,y-2)</td>
<td>(x,y)LD \rightarrow (x,y)S_1 \rightarrow (x-1,y)D_1 \rightarrow (x-1,y)S_1 \rightarrow (x-1,y-1)D_1 \rightarrow (x-1,y-1)S_2 \rightarrow (x-2,y-2)D_2 \rightarrow (x-2,y-2)LS</td>
<td>W_{24}</td>
</tr>
</tbody>
</table>

Figure 68  Mapping I-Node Level 1 channels in 6x6 GTON-XII
Figure 69  Mapping I-Node Level2 channels in 6×6 GTON-XII
Figure 70  Mapping I-Node Level3 channels in 6x6 GTON-XII
Figure 71  Mapping II-Node Level1 channels in 6x6 GTON-XII
Figure 72  Mapping of II-Nodes Level 2 channels in 6×6 GTON-XII
8.3.4 Routing Wavelength Assignment

The routing table of each OIU is constrained by the channel mapping. In channel mapping, each DOC has been assigned a unique routing wavelength for a light path that goes through a set of OIUs. Based on that, the internal routing of these OIUs is obtained. The routing wavelength table for I-node shown in Table 35 can be derived from Table 30 - 0 and routing wavelength table for II-node shown in Table 36 can be derived from Table 32 - 0. For example, the first row in Table 30 shows that the light path of a DOC between
nodes \((x,y)\) and \((x+1,y)\) with wavelength \(w_1\) is

\[(x,y)_{LS} \rightarrow (x,y)_{D1} \Rightarrow (x+1,y)_{S1} \rightarrow (x+1,y)_{LD}.\]

It is given that \((x,y)\) is I-Node with I-OIU and \((x+1,y)\) is II-Node with II-OIU. Hence for this DOC the routing truth is \(LS \leftrightarrow D_1\) for I-OIU and \(S_1 \leftrightarrow LD\) for II-OIU by the use of routing wavelength of \(w_1\), as listed in the first column of Table 35. In the same manner, Table 30 - 0 can be converted to Table 35 and Table 36. The blank cells in these tables indicate that there is no constrain and thus can be filled with any value.

**Table 35**  
Routing Table for I-Nodes

<table>
<thead>
<tr>
<th>Wavelength(w)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-OIU</td>
<td>LS</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D1</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>LD</td>
<td>D2</td>
<td>D2</td>
<td>LD</td>
<td>D2</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>D2</td>
<td>D2</td>
</tr>
<tr>
<td>II-OIU</td>
<td>LS</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D1</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
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<td>D2</td>
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<td>D1</td>
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<td>LD</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
<td>LD</td>
<td>D1</td>
</tr>
</tbody>
</table>

**Table 36**  
Routing Table for II-Nodes

<table>
<thead>
<tr>
<th>Wavelength(w)</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-OIU</td>
<td>LS</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D1</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
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<td>D2</td>
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<td>D1</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>LD</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
</tr>
<tr>
<td>II-OIU</td>
<td>LS</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>LD</td>
<td>D1</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
<td>LD</td>
</tr>
</tbody>
</table>

A complete routing wavelength table can be acquired for both I-OIU and II-OIU by combining Table 35 and Table 36. However, there are some redundant wavelengths among 24 wavelengths which are to be removed. To minimize the number of require
wavelengths the following rules are applied.

- **Rule 1.** A routing wavelength can be assigned only once for a pair of ports.

- **Rule 2.** The total number of different routing patterns for all channels should be minimized.

A new routing table for all DOCs is obtained by assigning same routing wavelength to identical routing patterns, as presented in Table 37. It retains only 10 different wavelengths.

By simplifying Table 37, we obtain 0 which is the routing truth of I-OIU and II-OIU. 0 can be converted to Table 39, which can be directly used for the design of OIU and OER.

<table>
<thead>
<tr>
<th>Table 37</th>
<th>Remapped Routing Table for I-Nodes and II-Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.</td>
<td>2,3,2,2,3,2,2,3,2,2,2,3,2,2,2,2,2,2,2,2,3,2,2,2,2,2,3</td>
</tr>
<tr>
<td>D.</td>
<td>4,1,0,0,5,4,4,5,0,1,4,0,0,4,2,1,2,5,0,2,3,2,1,3,1,2,3,3,2,2,2,1,2,4,2,4,2,0,3,0,3</td>
</tr>
<tr>
<td>Original W.</td>
<td>21 12 23 10 22 24 9 11 5 18 20 7 1 15 3 13 2 14 4 16 6 8 17 19</td>
</tr>
<tr>
<td>New W.</td>
<td>1 2 3 4 5 5 6 6 7 7 7 7 8 8 8 9 9 9 9 10 10 10 10</td>
</tr>
<tr>
<td>I-OIU</td>
<td>LS LD D1 LD D2 LD D1 D1 LD D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
<tr>
<td></td>
<td>S1 D2 D2 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
<tr>
<td></td>
<td>S2 D1 D2 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
<tr>
<td>II-OIU</td>
<td>LS D1 LD D2 LD D2 LD D2 LD D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
<tr>
<td></td>
<td>S1 LD D2 D1 D2 LD D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
<tr>
<td></td>
<td>S2 D2 D1 LD D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1 D1</td>
</tr>
</tbody>
</table>
Table 38  Routing Table for OIU in GTON-XII

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-OIU</td>
<td>LS</td>
<td>LD</td>
<td>D1</td>
<td>LD</td>
<td>D2</td>
<td>LD</td>
<td>D1</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>D2</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
<td>D2</td>
<td>LD</td>
<td>LD</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
<td>LD</td>
<td>D2</td>
<td>LD</td>
<td>LD</td>
<td>D1</td>
<td>D1</td>
</tr>
<tr>
<td>II-OIU</td>
<td>LS</td>
<td>D1</td>
<td>LD</td>
<td>D2</td>
<td>LD</td>
<td>D2</td>
<td>LD</td>
<td>D1</td>
<td>D2</td>
<td>D1</td>
</tr>
<tr>
<td></td>
<td>S1</td>
<td>LD</td>
<td>D2</td>
<td>D1</td>
<td>LD</td>
<td>D1</td>
<td>D2</td>
<td>LD</td>
<td>D2</td>
<td>LD</td>
</tr>
<tr>
<td></td>
<td>S2</td>
<td>D2</td>
<td>D1</td>
<td>LD</td>
<td>D1</td>
<td>D1</td>
<td>D2</td>
<td>LD</td>
<td>D1</td>
<td>D1</td>
</tr>
</tbody>
</table>

Table 39  Routing Table for OIU in GTON-XII

<table>
<thead>
<tr>
<th></th>
<th>LD</th>
<th>D1</th>
<th>D2</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-OIU</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>S1</td>
<td>2</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>S2</td>
<td>4</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>II-OIU</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>S1</td>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>S2</td>
<td>3</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

8.3.5 OIU Design

OIU which implements routing is a MRR-based optical-switch matrix. Based on the Table 39, both I-OIU and II-OIU can be built with a series of optical switches as shown in Figure 12.(c). The OIU is designed as follows:

1. Given the routing wavelengths in Table 39, add one or two MRR optical switch(es) per wavelength into the basic OIU structure. The basic OIU structure has the port connections as LS↔LD, S1↔D1 and S2↔D2, shown in Figure 74(a).

2. Each optical switch is added into the structure along with a cross-bridge as shown in Figure 75 to make the optical switch transparent to other wavelengths.

3. We define the following port pairs as the original routing pattern: LS↔LD, S1↔D1 and S2↔D2. Given the routing pattern of a wavelength, if it is same to the original
pattern (for example, $w_5$ to I-OIU in Table 39), it can be disregarded. If its routing pattern contains only one of the three pairs (for example, $w_1$ to I-OIU in Table 39), one MRR switch is to be added into the structure. If its routing pattern contains no ports pair of the original pattern (for example, $w_6$ and $w_7$ to I-OIU in Table 39), two MRR switches are to be added.

As an example, for wavelengths $w_1$ and $w_2$ in I-OIU, their routing patterns are given in Table 40 and Table 41.

<table>
<thead>
<tr>
<th>Table 40</th>
<th>Routing Table of Wavelength $w_1$ in I-OIU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>LD</td>
</tr>
<tr>
<td>I-OIU LS</td>
<td>1</td>
</tr>
<tr>
<td>I-OIU S1</td>
<td>1</td>
</tr>
<tr>
<td>I-OIU S2</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 41</th>
<th>Routing Table of Wavelength $w_2$ in I-OIU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>LD</td>
</tr>
<tr>
<td>I-OIU LS</td>
<td>2</td>
</tr>
<tr>
<td>I-OIU S1</td>
<td>2</td>
</tr>
<tr>
<td>I-OIU S2</td>
<td>2</td>
</tr>
</tbody>
</table>

MRR switches associated with wavelength $w_1$ and $w_2$ can be integrated into I-OIU as shown in Figure 74.
(a) Direct source-destination ports connections in I-OIU.

(b) Integrating routing wavelength $w_1$ into the I-OIU

(c) Integrating routing wavelength $w_2$ into the I-OIU

Figure 74 Integrating single wavelength switches into OIU
Based on the routing in Table 39, I-OIU and II-OIU are designed as shown in Figure 76.

8.3.6 OER Design

The function of OER is to receive input optical signal and to convert it to electrical data, then read and analyze the data header to make routing decision. If relay is necessary, electrical to optical re-conversion is performed with a proper wavelength to transmit to
the next node. The OER in GTON-XII has 12 input and 12 output ports plus one input port and one output port to the local core. Demultiplexers on both ends of the OER separate multiple input optical signals and propagate each of them to the corresponding input port. A photodetector in each channel receives input optical signal and converts it to electrical data to the input buffer. The controller of the OER reads the header of a data packet and determines the output channel the packet should go through. Packets generated by a local core are stored in the input queue too.

In every OER, in each clock cycle, if an output channel is available, a packet will be switched to the output channel through the crossbar, and then it is converted to the optical signal and transmitted to next node. Packets destined to the local core will also be switched similarly yet no EO conversion is required.

According to Table 39, the routing wavelengths assignment for all channels of I-Node and II-Node is derived and shown in Figure 77.
Based on Figure 77, I-OER and II-OER of GTON-XII can be designed as shown in Figure 78.
(a) I-OER in GTON-XII
8.3.7 Architecture Properties of GTON-XII

Table 42 summarizes the architectural properties of the GTON-XII.
Table 42  Architecture Properties of GTON-XII

<table>
<thead>
<tr>
<th>Architecture Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels of each node</td>
<td>12</td>
</tr>
<tr>
<td>Lasers in each node</td>
<td>12</td>
</tr>
<tr>
<td>Photodetectors in each node</td>
<td>12</td>
</tr>
<tr>
<td>Routing wavelengths in each node</td>
<td>7</td>
</tr>
<tr>
<td>Total routing wavelengths in the network</td>
<td>10</td>
</tr>
<tr>
<td>MRRs in each node</td>
<td>28</td>
</tr>
<tr>
<td>MRR optical switches in each node</td>
<td>14</td>
</tr>
<tr>
<td>Electrical input queues in each node</td>
<td>12</td>
</tr>
<tr>
<td>Electrical output queues in each node</td>
<td>0</td>
</tr>
<tr>
<td>Optical buffers in each node</td>
<td>0</td>
</tr>
<tr>
<td>Optical couplers in each node</td>
<td>2</td>
</tr>
<tr>
<td>Electrical crossbar switches in each node</td>
<td>1</td>
</tr>
<tr>
<td>Optical wavelength multiplexers in each node</td>
<td>2</td>
</tr>
<tr>
<td>Optical wavelength demultiplexers in each node</td>
<td>2</td>
</tr>
<tr>
<td>TO/EO tuning devices in the network</td>
<td>0</td>
</tr>
</tbody>
</table>

8.4  Routing Algorithm of GTON-XII

The Johnson code is used in GTON-XII for node addressing and data routing.

8.4.1  Johnson Code Addressing for GTON-XII

To implement the data routing in the network the Johnson Code explained in Section 6.3 is applied for node addresses in GTON-XII.

8.4.1.1  Addressing Nodes in GTON-XII

The address of a node \((A_h, A_v)\) in the \(N \times N\) GTON-XII can be defined by means of two \(n\)-bit Johnson Codes \((C_h, C_v)\) where \(N=2n\). For example, \((3, 2)\) in a \(6 \times 6\) GTON is addressed as 011001. It is same as presented in Section 7.7.1.1.

8.4.1.2  Nodes Coordinates and Address Association

For a node address \(C = C_h|C_v\), \(C\) is of \(N\) bits and \(N = 4n\), \(C_h\) and \(C_v\) are both \(2n\) bits. It
is same as presented in section 7.7.1.2.

8.4.2 Routing functions for GTON-XII

8.4.2.1 Routing Direction Function \( f_r \)

In the routing algorithm, \( f_r \) can be used to determine the routing direction, that is to choose the next routing channel. It is same as the function \( f_r \) presented in Section 7.7.2.1.

8.4.2.2 Routing Address Function \( f_d \)

The Routing Address Function \( f_d \) is to calculate the node address from the current node address with an offset value. It is same as the function \( f_d \) presented in Section 7.7.2.2.

8.4.2.3 Routing channel vector \( V \)

The routing channel vector \( V \) is used in routing a data packet from a source node to its destinations. The format of vector \( V \) is as:

\[
V = \left( n_3, c_3, n_2, c_2, c_x, c_y \right)
\]

(65)

where \( n_3 \) is the number of Level3 channel \( c_3 \), \( n_2 \) is the number of Level3 channel \( c_2 \), and \( c_x, c_y \) are Level1 channels.

\[
\begin{align*}
  c_x & \in \{E1, W1\} \\
  c_y & \in \{S1, N1\} \\
  c_2 & \in \{E2, W2, S2, N2\} \\
  c_3 & \in \{NE, NW, SE, SW\}
\end{align*}
\]

(66)

Assume the source node address is \((s_h, s_v)\) and the destination address is \((d_h, d_v)\), \( f_c \) allows to derive the routing channel vector \( V = f_c \left( s_h, d_h, s_v, d_v \right) \) based on the followings:

\[
\begin{align*}
  (p_h, c_h) &= f_r \left( s_h, d_h \right) \\
  (p_v, c_v) &= f_r \left( s_v, d_v \right)
\end{align*}
\]

(67)
From the topology view can be seen there are equal channel combination transforms in GTON-XII as the following:
\[ E2 + N1 = NE + S1 \]
\[ E2 + S1 = SE + N1 \]
\[ W2 + N1 = NW + S1 \]
\[ W2 + S1 = SW + N1 \]
\[ N2 + E1 = NE + W1 \]
\[ N2 + W1 = NW + E1 \]
\[ S2 + E1 = SE + W1 \]
\[ S2 + W1 = SW + E1 \]

and

\[ NE + SE = E2 + E2 \]
\[ SE + SW = S2 + S2 \]
\[ SW + NW = W2 + W2 \]
\[ NW + NE = N2 + N2 \]

In general, given a routing vector, many isomeric routing vectors can be generated by applying the equal channel combination transforms. The routing distances by these routing vectors are same. This property will be used further in the adaptive routing algorithm.

In addition, there are also numerous unequal channel combination transforms. It can be seen that 12 channels in GTON-XII: i.e. 6 pairs, for example, channel E and W. By adding any coupled two channels into any position of a given routing vector, its destination will not change, though the routing distance increases. This property will also be utilized in a later part of the paper.

8.4.3 Deterministic Routing Algorithm for GTON-XII

In deterministic routing, all transmissions should follow the sequence of Level3 channels first, Level2 channels second and Level1 channels last.

Also, before a data packet is sent from its source node, a routing path is determined and stored as a routing channel vector \( V \) in the packet header. The routing vector is
composed of a sequence of channels. In each intermediary node, the router will read the header, assign the output channel according to the routing vector $V$, and then update $V$ to remove the used channel and substitute the new $V$ in the header.

The deterministic routing algorithm for GTON-XII can be formulated as shown in Figure 79.

```
Algorithm Routing (S→D):
Generate the routing vector $V$:
$V = (n_3, c_3, n_2, c_2, c_x, c_y) = f_s(s_h, d_h, s_v, d_v)$
Routing:
if $V=0$
    channel = internal
else
    if $c_3 \neq 0$
        channel = $c_3$, $n_3 = n_3 - 1$
    else
        if $c_2 \neq 0$
            channel = $c_2$, $n_2 = n_2 - 1$
        else
            if $c_x \neq 0$
                channel = $c_x$, $c_x = 0$
            else
                if $c_y \neq 0$
                    channel = $c_y$, $c_y = 0$
                end
            end
        end
    end
end
End
```

Figure 79  Deterministic routing algorithm for GTON-XII.

8.4.4  Adaptive Routing Algorithm for GTON-XII

In the adaptive routing, the routing path between a source and a destination is chosen dynamically based on the buffer condition on each intermediary node. But the lengths of
all different paths are maintained equal to the minimum distance between two nodes.

Here a new parameter $T_c$ is introduced to describe the traffic condition in a node which will be used for making a decision in the output channel.

Assume the number of data packets in the output buffer of channel $x$ is $n_x$, then

$$t_x = B - n_x$$  \hspace{1cm} (73)

where $B$ is the maximum capacity of the buffer.

$$T_c = (t_{c1}, t_{c2}, t_{w1}, t_{w2}, t_{s1}, t_{s2}, t_{m1}, t_{m2}, t_{sw}, t_{ne}, t_{mw})$$  \hspace{1cm} (74)

In the basic adaptive dynamic routing, before the data packet arrives to its destination, in each intermediary node the router reads the latest routing channel vector $V$, compares the buffer conditions of channels in $V$ to choose a best one, and updates the routing vector $V$, as shown in Figure 80.

```
Algorithm Routing (S→D):
Generate the routing vector $V$:
$V = (n_1, c_1, n_2, c_2, c_3, c_4) = f_s(s_h, d_h, s_v, d_v)$
Routing:
While $V \neq 0$
In each intermediary node, check the traffic condition of all available channels in the routing vector $V$, then choose the best channel for data routing.
Update $V$ by removing the channel used from the routing vector $V$.
End
```

Figure 80 Adaptive routing algorithm for GTON-XII.

8.4.5 Fault Tolerant Routing Algorithm for GTON-XII

GTON-XII is featured by the fault tolerant ability because there are many alternative routing paths between most of node pairs. This is due to 12 different channels and much
equal channel combination transforms. For example, between \((x, y)\) and \((x+3, y+1)\) there are 12 different routing paths, i.e.:

\[
\begin{align*}
E1 & \rightarrow E2 \rightarrow S1 \\
E1 & \rightarrow S1 \rightarrow E2 \\
E2 & \rightarrow E1 \rightarrow S1 \\
E2 & \rightarrow S1 \rightarrow E1 \\
S1 & \rightarrow E1 \rightarrow E2 \\
S1 & \rightarrow E2 \rightarrow E1
\end{align*}
\]

(75)

and

\[
\begin{align*}
E1 & \rightarrow N1 \rightarrow SE \\
E1 & \rightarrow SE \rightarrow N1 \\
SE & \rightarrow E1 \rightarrow N1 \\
SE & \rightarrow N1 \rightarrow E1 \\
N1 & \rightarrow E1 \rightarrow SE \\
N1 & \rightarrow SE \rightarrow E1
\end{align*}
\]

(76)

In this basic fault tolerant routing we have all routing distances equal to a shortest distance between source and destination node. When a shortest path(s) between source and destination nodes is blocked because of node/link failure, there are still many choices for a path avoiding the failed spot. In the GTON-XII, only if all 12 neighbors of the source or the destination are failed, the transmission fails.

The direct fault tolerant routing algorithm can be formulated as shown in Figure 81.
Figure 81 Direct fault tolerant routing algorithm for GTON-XII.

8.5 Simulation and Analysis

8.5.1 Simulation Setup

We test the maximum traffic load the network can handle without blocking. The network is set to be homogeneous; and all nodes have the same packet generating rate. The parameter setting of the simulation is shown in the Table 43.
Table 43  Simulation Setting

<table>
<thead>
<tr>
<th>Network Pattern</th>
<th>Homogeneous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Frequency</td>
<td>40G</td>
</tr>
<tr>
<td>Electrical Frequency</td>
<td>4G</td>
</tr>
<tr>
<td>Packet Length</td>
<td>256 bits</td>
</tr>
<tr>
<td>Electrical Bus Width</td>
<td>64</td>
</tr>
<tr>
<td>Packet Generating Rate</td>
<td>$2.4 \times 10^{-3} \sim 0.08$ packet/nanosecond</td>
</tr>
<tr>
<td>Packet Generation Pattern</td>
<td>Uniform Distribution</td>
</tr>
<tr>
<td>Network Size</td>
<td>$6 \times 6 \sim 16 \times 16$</td>
</tr>
<tr>
<td>Routing Algorithm</td>
<td>Deterministic Routing</td>
</tr>
<tr>
<td>Buffer service</td>
<td>FIFO</td>
</tr>
</tbody>
</table>

In the simulation, for each setting at least 10,000 packets are generated by each node and transmitted to all other nodes with the same probability. Packets generated from a core are switched to an output channel via a crossbar. Data are modulated to a sequence of serial optical data bits and transmitted to the next node through a waveguide. On the receiving side, the optical bit sequence will be received by a photodetector and converted to a parallel electrical data packet. This process is repetitive until the packet arrives to its destination core. The total time of the process is the packet transmission time.

We consider first a deterministic routing algorithm, in which data are transmitted according to the following sequence: Level3 channels first, Level2 channels second and Level1 channels last. In Level1 channels routing the sequence of x-y is applied to minimize deadlock.

8.5.2 Simulation Results

The average packet transmission time in networks of different sizes and with different packet generating rates are shown in Figure 82. It can be observed that before a point when the networks reaches the blocking condition the waiting time is negligible.
As it was mentioned before, each channel is equipped with an input buffer. The average and maximum utilization of these buffers are shown in Figure 83 and Figure 84.
8.6 Network Properties of GTON Architectures

8.6.1 Buffer Analysis

The buffer usage is related to the channel usage. Level3 DOCs are used much more frequently in a 12x12 GTON-XII than in a 6x6 GTON-XII when deterministic routing is applied. In the 6x6 GTON-XII most of the traffic can be handled with Level1 & 2 DOCs.

Based on deterministic routing we have obtained the channel utilization in GTON-XII of different sizes shown in Figure 85. It can be seen, when the network is smaller than 40x40, channel utilization percentiles are quite different. For high packet arrival rate larger input buffer needed. On the other hand, routing algorithm can also affect the buffer utilization.

Figure 84 Maximum input buffer utilization in GTON-XII
8.6.2 Analysis of Channel Availability

Although the GTON supports wavelength multiplexing the optical interference occurs if two signals of the same wavelength are transmitted from opposite ends of the DOC. That can be avoided by first checking the waveguide serving condition before transmission. When a channel is receiving it cannot be used to transmit simultaneously. Hence the OER controller should monitor each channel and start transmission with an idle channel only. A proper design in the MAC protocol would take care about issues such as transmission at an even clock cycle from one side of a channel leaving the other side an odd clock cycle. Here is a reasonable premise required that the network is globally-synchronized.
8.6.3 Maximum DOC Length

As shown in Section 8.4, every DOC in GTON is built with a specific routing wavelength and corresponding OIU and OER structures. In any DOC the internal routing path of each OIU is different; otherwise a dead loop will occur. Consequently, the maximum number of hops of a DOC is limited. For example, the OIU in GTON-XII has 6 ports, in which the 2 ports LS and LD are connected to OER. Other 4 ports have 2 combinations, i.e., \((S_1 \leftrightarrow D_1, S_2 \leftrightarrow D_2)\) and \((S_1 \leftrightarrow D_2, S_2 \leftrightarrow D_1)\) for both I-OIU and II-OIU. Then totally four port combinations available for DOC routing, that indicates the maximum length of a DOC is five hops. If OIU has 8 ports, the maximum distance is seven hops. A DOC can be designed to connect any pair of nodes within this maximum distance.

The performance of a GTON network is related to properties of the network topology, such as diameter and average distance (number of hops, or number of O/E & E/O conversions). And the GTON network topology is determined by how DOCS are designed for each node in the network. Adding DOCS would lessen the transmission delay, the energy consumption and improve fault tolerance; however would increase the architecture/control complexity and the fabrication cost.

8.6.4 Topological Diversity of GTON

As explained in Sect.4, because DOC can be added between any two nodes, GTONs can realize any topology. Although the proposed GTON architectures are 2D-Torus based, the concept of DOC and OIU is not limited and can be extended to high-dimensional topologies such as 3D-torus. Thus, GTON can be made high-dimensional and consequently higher dimension topologies can be realized.
8.6.5 Transmission Power Loss

The optical power loss of the MRR switch matrix mainly is due to off-state, on-state losses, waveguide crossing loss and waveguide propagation loss [23].

In our analysis, the waveguide propagation loss is neglected. Other loss values are adopted according to [60], as shown in the Table 44. Transmission power losses in each channel are listed in Table 45.

### Table 44 Optical Power Budget

<table>
<thead>
<tr>
<th>Component</th>
<th>Loss(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRR off-state</td>
<td>0.06</td>
</tr>
<tr>
<td>MRR on-state</td>
<td>1.5</td>
</tr>
<tr>
<td>Waveguide Crossing</td>
<td>0.05</td>
</tr>
<tr>
<td>Photodetector</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### Table 45 Transmission Power Loss in DOCs

<table>
<thead>
<tr>
<th>DOC</th>
<th>I-Node</th>
<th>II-Node</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W.</td>
<td>Power Loss (dB)</td>
</tr>
<tr>
<td>E1</td>
<td>8</td>
<td>5.72</td>
</tr>
<tr>
<td>S1</td>
<td>9</td>
<td>7.15</td>
</tr>
<tr>
<td>W1</td>
<td>8</td>
<td>5.93</td>
</tr>
<tr>
<td>N1</td>
<td>9</td>
<td>4.63</td>
</tr>
<tr>
<td>E2</td>
<td>7</td>
<td>7.72</td>
</tr>
<tr>
<td>S2</td>
<td>10</td>
<td>7.84</td>
</tr>
<tr>
<td>W2</td>
<td>7</td>
<td>7.72</td>
</tr>
<tr>
<td>N2</td>
<td>10</td>
<td>5.06</td>
</tr>
<tr>
<td>NE</td>
<td>6</td>
<td>9.35</td>
</tr>
<tr>
<td>SE</td>
<td>4</td>
<td>7.81</td>
</tr>
<tr>
<td>SW</td>
<td>6</td>
<td>9.35</td>
</tr>
<tr>
<td>NW</td>
<td>2</td>
<td>7.81</td>
</tr>
<tr>
<td>Ave.</td>
<td>7.17</td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Table 45, transmission losses of Level1, 2 & 3 channels are
around 5.33dB, 7.43dB and 8.39dB, with maximum 9.35 dB and minimum 3.91 dB.

The channel transmission loss is linear to the distance of the channel, or OIUs in it. The loss in each OIU relates to the number of MRR switches in it, which is linear to the number of DOCs in each node. For example, in the GTON-XII each node has 12 DOCs, consequently each OIU has 14 MRR switches. If the DOC number reduces to 8 by removing all Level3 DOCs, there are only 8 MRR switches required in each OIU. Then the power loss in each OIU is around 2.5 dB, and average power loss of all channels is around 7.5dB. This will be a more applicable architecture with better cost/performance tradeoff.

It can be seen from Figure 76 that many MRR switches in OIU are connected in parallel. It is very suitable for applying the comb switch [85, 86], which will substantially reduce the number of MRRs and consequently lesser the transmission loss.

In most of ONoC architectures which use MRR tuning, optical receivers have to deal with a wide power range because the transmission power loss depends on the specific path. Wide power range receivers are hard for practical implementation. In the GTON, on contrast, the transmission power loss per DOC is constant, and thus the receiver is less complex.

8.7 Conclusion

In this chapter a generalized 2D-torus based packet switching optical NoC architecture called GTON is proposed with the use of micro-ring resonator (MRR) optical switches. The complete schema to design a target network topology based on the GTON is presented. The GTON is highly scalable because of packet switching. The use of passive MRR switching, WDM and Direct Optical Channels offers an extremely high
bandwidth, low latency/ cost/power consumption, and makes the GTON fault tolerant architecture. The performance of the network is evaluated by simulation. Related issues such as analyses of channel utilization, transmission power loss and the buffer size are also addressed. The GTON has shown many outstanding features which make it a compelling solution for future optical NoCs.
In this dissertation study, a set of different ONoC architecture are proposed for different purposes with the adoption of wavelength division multiplexing and MRR passive-switching routing. The first a group is with WRON, RDWRON and RCWRON which are all fully connected network architectures.

To explore the topology advantage of different NoC architectures, an improved version of RDT architecture, named QRDT is proposed for NoC applications. Compare with traditional NoC topologies such as mesh, torus or hypercube, QRDT has better flexibility, higher scalability and shorter network diameter and average distance, which is suitable for ONoC applications.

By combining concepts from QRDT topology and WRON structure, a new ONoC architecture family, 2D-torus based packet switching optical NoC architecture, or TON is proposed. Three typical cases, TON-I, II, and III are presented and explained in detail. And diverse routing algorithms are provided for each of them. Network performance of each of them is presented with simulation results.

Based on the architecture family TON, a new generalized ONoC architecture, generalized 2D-torus based packet switching optical NoC architecture, or GTON is developed. Different to all existed architectures ever proposed, GTON is with high scalability and infinite topology flexibility. Since GTON is of packet-switching, given a limited number of different routing wavelengths, GTON can extend to any networks size with the compensation of transmission latency. Followed with the design scheme provided in 0, with the designated set up of proper DOCs and related OIU's, one or more
direct light path can be created between any two nodes in the network, hence GTON architecture can be built into any given network topology.

In conclusion, with the use of passive switching MRR routing, WDM and Direct Optical Channels, GTON architectures offered an extremely high bandwidth, low latency/ cost/power consumption and robust fault tolerance. The performance of the network is evaluated by simulation. Additional issues such as analyses of channel utilization, transmission power loss and the buffer size are addressed as well. The GTON has shown many outstanding features which make it a compelling solution for future optical NoCs.

This study on ONoC architecture is still on-going. Now the study is concentrating majorly on three topics. The first is the development of non-blocking WRON structure based optical on-chip passive router. And some results are already submitted/accepted to publish [87, 88]. The second is to apply the new MRR optical router to GTON architecture to achieve a performance leap. Then the third is to further improve the GTON performance by applying advanced routing algorithms and exploring more network topologies. Shortly, working on such an exciting and promising field, more complete works are expected to publish from time to time.
APPENDIX A

Proof of Propositions 1-3

For the convenience of proof, we represent the WRON using a diagonal grid structure, and expand it to a Tri-network structure by adding two virtual WRONs at both sides of the original WRON. We denote the original WRON as the Real WRON, the virtual network close to the first (last) source node of the Real WRON as the Negative WRON (Positive WRON). The Tri-network structure of N=4 is shown in Figure 86.

Each optical switch in the Tri-network is indicated by the coordinate \((C,R)\) according to Rule 1.

- **Rule 1.** In the Real WRON, when \(j\) is odd, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i-1)\); when \(j\) is even, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i)\).

  In the Negative WRON, when \(j\) is odd, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i-1-N)\); when \(j\) is even, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i-N)\).

  In the Positive WRON, when \(j\) is odd, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i-1+N)\); when \(j\) is even, the coordinate of the \(i^{th}\) switch in the \(j^{th}\) stage is \((j,2*i+N)\).

- **Rule 2.** At the up and bottom boundaries of the Real WRON, there are many Peak Nodes as marked in Figure 87.

  In the Real WRON, the coordinates of the Peak Node is indicated as \((C, R)\), where \(C\) is the stage number of the Peak Node and \(R\) equals to 0 (\(N\)) when the Peak Node is connected to the first (last) switch in the stage.

- **Rule 3.** When the routing path reaches the Peak Nodes in the network, if the horizontal coordinate \(C\) of the Peak Nodes is same as the wavelength assigned to the
path, the path will change its routing direction and return to the Real WRON. Otherwise, the routing path will keep its direction and move forward into one of the virtual WRONs.

According to Rule 3, in solving the routing scheme, we shall try to avoid the trouble arose by the veer of the routing path at some of the Peak Nodes.
Figure 86 Structure of the Tri-network for a 4×4 WRON.

All source and destination node addresses in the Tri-network are numbered following Rule 4.

- Rule 4. Every destination node address indicated in the Tri-network is the virtual
destination address $D^*$. When the routing follows Rule 1, a routing path will reach the virtual destination node $D^*$. The relationship between the virtual addresses $D^*$ and its corresponding real address $D$ is shown below.

$$D = \begin{cases} 
1 - D^* & \text{when } D^* \leq 0 \\
D^* & \text{when } 0 < D^* \leq N \\
2N + 1 - D^* & \text{when } D^* > N 
\end{cases} \quad (77)$$

Similar to the source node, the relationship between the virtual addresses of source node $S^*$ in the Tri-network and its corresponding real source address $S$ is shown below.

$$S = \begin{cases} 
1 - S^* & \text{when } S^* \leq 0 \\
S^* & \text{when } 0 < S^* \leq N \\
2N + 1 - S^* & \text{when } S^* > N 
\end{cases} \quad (78)$$

For the ease of understanding, we have the following definitions.

- **Definition 1 (Start Node):** The start node is the first node on the routing path following the source node. It can be a switch node or a peak node.

  Assume the source address for a routing path is $S$, then:

  The coordinate of the start node is $(1, S-1)$ if $S$ is even, or $(1, S)$ if $S$ is odd.

- **Definition 2 (Reflection Node):** The reflection node is the specified node in a routing path whose horizontal coordinate is same as the wavelength assigned to the path. The routing path will change its direction in the reflection node. There are two kinds of reflection nodes: reflection peak node and reflection switch node. In any routing path there is one and only one reflection node.

- **Definition 3 (Inherent Slope):** The inherent slope is the slope of the routing path starting from the source node to the reflection node.

  Assume the source address for a routing path is $S$, then:
The inherent slope is -1 if \( S \) is even, or 1 if \( S \) is odd.

- **Definition 4 (Acquired Slope)**: The acquired slope is the slope of the routing path from the reflection node to the destination node. Obviously it is the opposite value of the inherent slope.

Assume the source address for a routing path is \( S \), then:

The acquired slope is 1 if \( S \) is even, or -1 if \( S \) is odd.

- **Definition 5 (End Node)**: The end node is the last node on the routing path before the destination node. It can be a switch node or a peak node.

Given an end node with the coordinate of \((N, R)\), if the acquired slope is 1, the destination node address is \( R+1 \); if the acquired slope is -1, the destination node address is \( R \).

In the following proof, we denote the coordinate of a node as \((c, r)\) where \( r \) denote the vertical coordinate and \( c \) denote the horizontal coordinate.

A.1. **Proof of Proposition 1**

In an \( N \)-WRON, given the source node address \( S \) and the routing wavelength \( W \), the destination address \( D \) can be derived by the following procedure.

- **When \( N \) is Even**

  Case 1. When \( S \) is even

  The coordinate of the *start node* is \((1, S-1)\). The *inherent slope* is -1. The function of the routing path before the *reflection node* is \( (r-(S-1))+(c-1)=0 \), i.e., \( r=S-c \).

  Given the routing wavelength \( W \), let \( c=W \), then \( r=c-W \). Hence the coordinate of the *reflection node* is \((S-W, W)\).

  Hence the function of the routing path after the *reflection node* is \( (r-(S-W))-(c-W)=0 \),
i.e., \( r = S + c - 2 \times W \)

The vertical coordinate of the *end node* is \( N \), hence the horizontal coordinate of the *end node* is \( r = S + N - 2 \times W \).

Then the virtual address \( D^* \) of the destination node is

\[
D^* = S + (N - 2W + 1).
\]

Case 2. When \( S \) is odd

By the procedure similar to case 1, when \( S \) is odd, the virtual address of the destination node can be derived as

\[
D^* = S - (N - 2W + 1).
\]

In summary, \( D^* = S + (N - 2W + 1) \times (-1)^S \).

- When \( N \) is odd

By the same way as in Case 1, when \( N \) is odd, the virtual address \( D^* \) of the destination node can be derived as

\[
D^* = S + (N - 2W + 1) \times (-1)^S.
\]

Hence Proposition 1 holds.

A.2. Proof of Proposition 2

- When \( N \) is even:

  Case 1. When \( D \) is even

  Similar to the cases in Appendix I, the virtual address of the source node can be derived as

  \[
  S^* = D + (N - 2W + 1).
  \]

  Case 2. When \( D \) is odd

  The virtual address of the source node can be derived as
\[ S^* = D - (N - 2W + 1). \]

In summary, \( S^* = D - (N - 2W + 1) \times (-1)^{N+D}. \)

- When \( N \) is odd:

  The virtual address of the destination node can be derived as
  \[ S^* = D - (N - 2W + 1) \times (-1)^{N+D}. \]

  Hence Proposition 2 holds.

A.3. Proof of Proposition 3

For an \( N \)-WRON, given the source node address \( S \) and the destination node address \( D \), the routing wavelength \( W \) can be derived based on \( S \) as follows. The major problem in deriving \( W \) is that sometimes we should not use the real address \( D \) but the virtual address \( D^* \) in computing the correct wavelength \( W \).

- When \( N \) is even

  When \( S \) and \( D \) have different parities, the destination node of the routing path is in the \( Real \ WRON \), i.e., \( D^*=D \).

  Case 1. When \( S \) is even

    When \( D \) is odd, \( D^*=D \). The coordinate of the start node is \((1, S-1)\), the inherent slope is -1. The function of the path before the reflection node is \( r - S + c = 0 \).

    The coordinate of the end node is \((N,D-1)\). The acquired slope is 1. The function of the routing path after the reflection node is \( r - (D-1) = c - N \), i.e., \( r - D - 1 + N + c = 0 \).

    Assume the reflection node is in \((c_0,r_0)\), then

    \[
    \begin{cases}
    r_0 + c_0 - S = 0 \\
    r_0 - c_0 - D + N + 1 = 0
    \end{cases} \Rightarrow c_0 = \frac{N + S - D + 1}{2} \Rightarrow W = \frac{N + 1 + S - D}{2}
    \]

    When \( D \) is even, the destination node is in the \( Virtual \ WRON \), i.e., \( D^* \neq D \). There have two possibilities: \( D^* > N \) or \( D^* \leq 0 \). Hence the routing wavelength \( W \) for the path from
source node $S$ to destination node $D$ should be:

$$W = \frac{N + 1 + S - D^*}{2},$$

where

$$D^* = \begin{cases} 2 \times N + 1 - D & \text{when } D^* > 0 \\ 1 - D & \text{when } D^* \leq 0 \end{cases}$$

Then we have

$$W_1 = \frac{N + 1 + S - \left(2 \times N + 1 - D\right)}{2} = \frac{S + D - N}{2}$$

$$W_2 = \frac{N + 1 + S - (1 - D)}{2} = \frac{S + D + N}{2}$$

The wavelength $W$ should be greater than 0 and less than $N$, hence we have the following expressions:

$$W_1 = \frac{S + D - N}{2} \Rightarrow N < (S + D) \leq 3N \Rightarrow (S + D) > N$$

$$0 < W_1 \leq N$$

$$W_2 = \frac{S + D + N}{2} \Rightarrow -N < (S + D) \leq N \Rightarrow (S + D) \leq N$$

$$0 < W_2 \leq N$$

Then $W = W_1$ when $S + D > N$ and $W = W_2$ when $S + D \leq N$.

Case 2. When $S$ is odd

It can be derived similarly to Case 1, when $D$ is even

$$W = \frac{N + 1 - S + D}{2}$$

When $D$ is odd,
When $N$ is Odd

When $S$ and $D$ have same parities, the destination node of the routing path is in the 

Real WRON, i.e., $D^* = D$.

Case 1. When $S$ is even

Similar to the last case, when $D$ is even,

$$W = \frac{N + 1 + S - D}{2}$$

When $D$ is odd,

$$\begin{cases} 
W = \frac{S + D - N}{2} & \text{when } S + D > N \\
W = \frac{S + D + N}{2} & \text{when } S + D \leq N
\end{cases}$$

Case 2. When $S$ is odd

Similar to the case 1, when $D$ is odd,

$$W = \frac{N + 1 - S + D}{2}.$$ 

When $D$ is even,

$$\begin{cases} 
W = \frac{-S - D + 3 \times N + 2}{2} & \text{when } S + D \geq N + 2 \\
W = \frac{-S - D + N + 2}{2} & \text{when } S + D < N + 2
\end{cases}$$

Hence Proposition 3 holds. \qed
APPENDIX B

Proof of Propositions 4-6

All rules and definitions used here are same as in Appendix 0

B.1. Step I

Consider the topology structure of the WRON as shown in Figure 87. Each switch is indicated by its coordinate \((c,r)\) uniquely in the. When the routing wavelength \(w\) assigned to the routing path is different to all the wavelengths present in the WRON, we refer this situation as the “The N-WRON is irrelative to the wavelength \(w\)” . When the N-WRON is irrelative to the wavelength, the relationship between the address of the source node \(S\) and the address of the destination node address \(D\) can be derived as \(D = N \cdot S + 1\).
B.2. Step II

When an $N$-WRON is connected with an Inverse Connector (IC), there are two types of connections between IC and $N$-WRON as shown in Figure 88.

Figure 87 Structure of 8-WRON.
Given the size of IC $N$, the address of source node $S$, when the $N$-WRON is irrelative to the wavelength, the destination node address is derived as: $D = N + 1 - S$.

The routing truth of the subnet shown in Figure 89 is given by:

$$\begin{cases} 
  m = N + 1 - s \\
  d = N + 1 - m
\end{cases} \Rightarrow \quad d = s$$

Hence both sub-networks shown in Figure 89 can be substituted as the Straight Connection (SC) block (Figure 90).
Note that the integration of any number of $N$-SCs is equal to one $N$-SC and the integration of $N$-SC to other network (WRON) is equal to that network (WRON).

B.3. Step III

Given any $N^2$-RDWRON and a wavelength $w$ assigned to a special routing path, we can easily transform the RDWRON to a WRON by the following way.

In the $N^2$-RDWRON, there are $N$ $N$-WRON and $N$-1 IC blocks. Assume that

$$k = \left\lfloor \frac{w - 1}{N} \right\rfloor, \quad w_0 = \text{mod}(w - 1, N) + 1.$$  

Then the wavelength $w$ exists only in the $(k + 1)^{\text{th}} N$-WRON, i.e., all other $k$-1 $N$-WRONs are irrelative to the wavelength $w$.

For the $i^{\text{th}} N$-WRON, $0 < i < k$, in the $N^2$-RDWRON, it can be integrated with the $i^{\text{th}}$ IC to an $N$-SC; for the $j^{\text{th}} N$-WRON, $k < j \leq N$, in the $N^2$-RDWRON, it can be integrated with the $(j - 1)^{\text{th}}$ IC to an $N$-SC too. Then the $N^2$-RDWRON is composed only of SCs and WRONs which can be treated as WRON merely. This process is shown in Figure 90.
Figure 90 Transform $N^2$-RDWRON to $N$-WRON.

Hence, the routing scheme of $N^2$-RDWRON is almost same as that of $N$-WRON. The derivations of the destination address and the source address for $N^2$-RDWRON are same
as those for $N$-WRON. In deriving the routing wavelength, we can treat the $N^2$-RDWRON as $N$ different $N$-WRON and calculate them separately.

We summarize the routing scheme of RDWRON as follows.

For an $N^2$-RDWRON, given the source node address $S$ and the routing wavelength $w$, the destination node address $D$ can be derived as follows.

$$ D = f_D (N, S, w_0) $$

where $w_0 = \text{mod} (w-1, N) + 1$, and $f_D$ is the function defined in Eqn. (3).

For an $N^2$-RDWRON, given the destination node address $D$ and the routing wavelength $w$, the source node address $S$ can be derived as following.

$$ S = f_S (N, D, w_0) $$

where $w_0 = \text{mod} (w-1, N) + 1$, and $f_D$ is the function defined in Eqn. (4).

In an $N^2$-RDWRON, a set of different routing wavelengths can be used in routing from one source node to one destination node. Denote the set of different wavelengths of the $N^2$-RDWRON as $W$, given the RDWRON size $N$, the source node address $S$ and the destination node address $D$, $W$ can be derived as:

$$ W = \{ w, w+N, w+2N, \ldots, w+(N-2)N, w+(N-1)N \} $$
$$ = \{ w+(k-1)N | k = 1,2,\ldots, N \} $$

where $w = f_w (N, S, D)$, and $f_D$ is the function defined in Eqn. (5).

Hence, Proposition 4 to Proposition 6 hold. ■
APPENDIX C

Proof of Lemma 7 and Lemma 8

C.1. Proof of Lemma 7

To prove the lemma, we first introduce the Quad-Star Network (QSN). An $N$-QSN is constructed based on a $(2N+1) \times (2N+1)$ mesh by removing all nodes with their minimum distance (in number of hops) to the center node (indexed as $(N, N)$) greater than $N$ and their related links. Figure 91 shows the structure of a 3-QSN, with the center node $(C)$ and an edge node $(E)$ marked. The minimum distance between $C$ and $E$ is 3 hops.

In an $n$-QSN, the total number of nodes in the network is given by

$$U(n) = 4 \times \left[ n + (n-1) + \cdots + 2 + 1 \right] + 1$$

$$= 4 \times \left[ \frac{n(n+1)}{2} \right] + 1 = 2n^2 - 2n + 1$$

(79)

Figure 91 Structure of 3-QSN.

And the sum of lengths (in number of hops) of all shortest paths between all nodes in
the network to $C$ is

$$D(n) = 4 \times \left[ n^2 + (n - 1)^2 + \cdots + 2^2 + 1^2 \right]$$

$$= \frac{4n(n + 1)(2n + 1)}{6}$$

$$= \frac{2n(n + 1)(2n + 1)}{3} \quad (80)$$

Figure 92 shows that the 12-QRDT is partitioned into 8 QSNs in different sizes (shown in different colored regions) based on their respective location to the original node $O$ (the node at the left-up corner). Some of the QSNs are overlapped. In general, for an $N$-QRDT, it can be partitioned to one $n$-QSN ($B$ in Figure 92), four $n$-QSN ($F_1$, $F_2$, $F_3$, and $F_4$ in Figure 92) and three $(n-1)$-QSN ($S_1$, $S_2$, and $S_3$ in Figure 92), where $n=N/4$. 

180
Consider the \( n \)-QSN region \( B \) in Figure 92. Then the rank-0 path between any node in \( B \) and the original node \( O \) is less than or equal to \( n \) hops. According to the JCVR algorithm, the routing path between any node in \( B \) and the original node \( O \) is less than \( n \).

Then consider the \( n \)-QSN regions \( F_1, F_2, F_3, \) and \( F_4 \). The rank-0 path between any node in these regions and their center node is less than or equal to \( n \) hops. According to the JCVR algorithm, the routing path between any node in \( F_1, F_2, F_3 \) and \( F_4 \) to the original node \( O \) is less than \( n+1 \).

\( S_1, S_2, \) and \( S_3 \) are \((n-1)\)-QSNs. All rank-0 paths between nodes in these regions to their
original nodes are less than or equal to \( n-1 \) hops. According to JCVR algorithm, the routing path between any node in \( S_1, S_2, \) and \( S_3 \) to the original node \( O \) is less than \( n+1 \).

Note that the combination of all these regions is equivalent to the whole QRDT network. Because the QRDT is symmetric to any of its nodes, such partition can be applied to all nodes. Then we can have the first conclusion:

The diameter of the \( N \)-QRDT network is less than or equal to \( n+1 \), where \( n = N/4 \).

The routing steps from a source node to a destination node can be derived as

\[
\vec{A} = \text{vec}X_1 \vec{X}_1 + \text{vec}Y_1 \vec{Y}_1 + \text{vec}X_0 \vec{X}_0 + \text{vec}Y_0 \vec{Y}_0.
\]

The length of a routing path is determined by the summation of the four coefficients and will not change with the sequence of routing steps. From Figure 92 it can be seen that any routing path which involves more than 2 rank-1 links is not minimal. That is, any shortest routing path can have at most 2 rank-1 links.

Consider the shortest routing path between the original node \( O \) and node \( X \) as marked in Figure 92. One of the shortest routing paths from \( O \) to \( X \) with 2 rank-1 links is \( O \rightarrow P \rightarrow R \) (or \( O \rightarrow P \rightarrow T \)) plus \( n \) rank-0 links from \( R \) to \( X \) (or \( T \) to \( X \)), whose length is \( n+2 \).

And one of the shortest routing paths from \( O \) to \( X \) with 1 rank-1 link is \( O \rightarrow P \) plus \( n \) rank-0 links from \( P \) to \( X \), whose length is \( n+1 \). And there are many shortest routing paths from \( O \) to \( X \) with only rank-0 links with lengths \( 3n \). Then we can have the second conclusion:

The diameter of the \( N \)-QRDT network is greater than or equal to \( n+1 \), where \( n = N/4 \)

Combing the above two conclusions, Lemma 7 holds. 

C.2. Proof of Lemma 8

The calculation of the average distance in QRDT is performed by partitioning the network into a set of the QSNs.
According to the proof of Lemma 1, by the JCVR algorithm, the total lengths of all shortest paths between one selected node and all other nodes are

\[
\text{Sum} = D(n) + 4 \times [D(n) - n[4 + (n - 1)] + U(n) - [4 + (n - 1)]] + 3[D(n - 1) + 2U(n - 1)] \\
= \frac{32n^3}{3} + 20n^2 - \frac{32}{3}n + 2
\] (81)

And the number of all such paths is \( N^2 - 1 = 16n^2 - 1 \).

Then the average distance between one selected node and all other nodes is

\[
d = \frac{D(n) + 4 \times [D(n) - n[4 + (n - 1)] + U(n) - [4 + (n - 1)]] + 3[D(n - 1) + 2U(n - 1)]}{16n^2 - 1} \\
= \frac{\frac{32n^3}{3} + 20n^2 - \frac{32}{3}n + 2}{16n^2 - 1}
\] (82)

As the \( N \)-QRDT is a symmetric network, the average distance of \( N \)-QRDT is also \( d \).

Hence Lemma 8 holds. ■
REFERENCES


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