8-1-2016

Complex Traffic Network Modeling & Area-wide Hierarchical Control

Saumya Gupta
University of Nevada, Las Vegas

COMPLEX TRAFFIC NETWORK MODELING & AREA-WIDE HIERARCHICAL CONTROL

by

Saumya Gupta

Bachelor of Technology - Electronics & Instrumentation Engineering
Uttar Pradesh Technical University, Lucknow, India
2009

A thesis submitted in partial fulfillment
of the requirements for the

Master of Science - Electrical and Computer Engineering

Department of Electrical and Computer Engineering
Howard R. Hughes College of Engineering
Graduate College

University of Nevada, Las Vegas
August 2016

UNLV | GRADUATE COLLEGE

Thesis Approval

The Graduate College
The University of Nevada, Las Vegas

April 8, 2016

This thesis prepared by

Saumya Gupta

entitled

Complex Traffic Network Modeling & Area-Wide Hierarchical Control

is approved in partial fulfillment of the requirements for the degree of

Master of Science - Electrical and Computer Engineering
Department of Electrical and Computer Engineering

Pushkin Kachroo, Ph.D.
Examination Committee Chair

Yingtao Jiang, Ph.D.
Examination Committee Member

Ke-Xun Sun, Ph.D.
Examination Committee Member

Amei Amei, Ph.D.
Graduate College Faculty Representative

Kathryn Hausbeck Korgan, Ph.D.
Graduate College Interim Dean

ABSTRACT

Complex Traffic Network Modeling & Area-wide Hierarchical Control

by

Saumya Gupta

⟨Dr. Pushkin Kachroo⟩, Examination Committee Chair
Professor of Electrical and Computer Engineering Department
University of Nevada, Las Vegas

In this thesis, we present a novel methodology to divide a traffic region into subregions such that in each subregion a Macroscopic Fundamental Diagram (MFD) can be utilized to determine the state of that subregion. The region division is based on the theory of complex networks. We exploit the inherent network characteristics through PageRank centrality algorithm to identify the most significant nodes in the traffic network. We use these significant nodes as the seeds for a Voronoi diagram based partitioning mechanism of the network. A network wide hierarchical control framework is then presented which controls these sub regions individually and the network as a whole. At the subregion level a feedback controller is designed based on MFD concept. At the network level we develop a dynamic toll pricing algorithm to control the inflows into the network. This dynamic toll pricing is coupled with the subregion controller and thus forming a network wide hierarchical control. We use optimal control theory to design the dynamic toll pricing. A cost function is designed

and then using Hamilton-Jacobi-Bellman equation, we derive an optimal control law that uses real-time information. The objective of the dynamic toll algorithm is to strike a balance between the toll price and optimal traffic conditions in each of the subregions. A case study is performed for the Manhattan area in New York city and results are provided through simulations.

ACKNOWLEDGEMENTS

I would like to thank my advisor Dr. Pushkin Kachroo, for his expertise, guidance, motivation and patience throughout my thesis. Dr. Kachroo was always eager to understand and help whenever I ran into a trouble spot or had a question about my thesis. I wouldn't have made this so far in my research and learning without his persistent support.

I am also highly grateful to all the thesis committee members, Dr. Yingtao Jiang, Dr. Amei Amei and Dr. Ke-Xun Sun for their guidance and feedback.

I thank my colleagues at TRC, namely Sergio, and Michael for all their support.

It is my privileged to thank my husband Dr. Shaurya Agarwal for his constant encouragement, timely suggestions and untiring support throughout my thesis.

Thankful to Almighty, who bestows me with knowledge and wisdom. Last but not the least, I want to express my profound gratitude towards my parents, Alka Gupta and Er. Chandra Bhushan for constantly showing me the importance of knowledge and supporting me in all my endeavors.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

1.1 Complex Networks

Albert-László Barabási stated that behind every complex system there is a network. If we consider any real world system like internet, human body, human society, economy, transportation, biological, they are a type of complex networks. Each complex network structure has millions of nodes and links. There are lot of studies going on network analysis which includes node significance ranking, routing optimization, hub detection, network modeling.

Real-world Networks have so many challenges as they are under random attacks and targeted attacks. Researches are working to design resilient networks that are robust against attacks and faults. The study of robustness and resilience of complex networks is important topic of network analysis. Networks must be robust to both targeted attacks and to the random attacks. Metrics, like degree centrality, eigen vector centrality, Katz centrality, Page rank centrality, Betweenness and closeness centrality, assess the robustness of a network. More specifically if we consider Internet as an example for real word network, it was concluded that the Internet exhibited this, "Robust yet Fragile," nature (Robust yet Fragile implies that the Internet might survive (robust) against random attacks but it might be susceptible to

the targeted attacks). This conclusion was based on experiments performed using structural metrics. It was showed that the Internet's robustness was due to its self-organizational properties and its fragilities were due to Internet hijackings. Although the Internet's topology indeed was scale-free, the high node degree hubs were on the periphery of the network. Therefore, attacks to highly connected vertices would only affect the network locally, the core would remain unaffected. Thus metrics which capture a network's robustness with respects to its connectivity are insufficient and generally provide misleading conclusions about a network's robustness with respect to its behavior.

Transportation systems are complex network in which roads or streets are form a network structure. There is noticeable progress in the analysis of transportation system using the knowledge of complex network theory. Figure 1.1 shows the freeway network of Las Vegas area. One way of representing this freeway network is shown in 1.2, where $O-1$ and $O-2$ represent the starting (origin) nodes and $D-1$ and $D-2$ represent the final (destination) nodes.

Figure 1.3 shows a generalized topology of road network where nodes like O, A, B, C etc, are connected by edges $1, 2, 3$. f_{in} represents the number of vehicles flowing in node O and parameters $\ell_1, v_{f_1}, \rho_{m_1}$ are the properties of Edge 1. ℓ_1 represents length, v_{f_1} represents free-flow speed whereas ρ_{m_1} represents bumper to bumper density or jam-density.

Figure 1.1: Las Vegas Freeway Network

1.2 LWR and Greenshields' Model

The Lighthill-Whitham-Richards (LWR) model is a macroscopic traffic model. It is represented mathematically by equation (1.1). This is basically the conservation law.

$$\frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}f(t,x) = 0 \tag{1.1}$$

here, f represents the flux, and ρ represents the traffic density. There are several models which establish a relationship between traffic density to speed. For example Greenshield's Model, Underwood Model etc. Greenshield's model uses a linear model

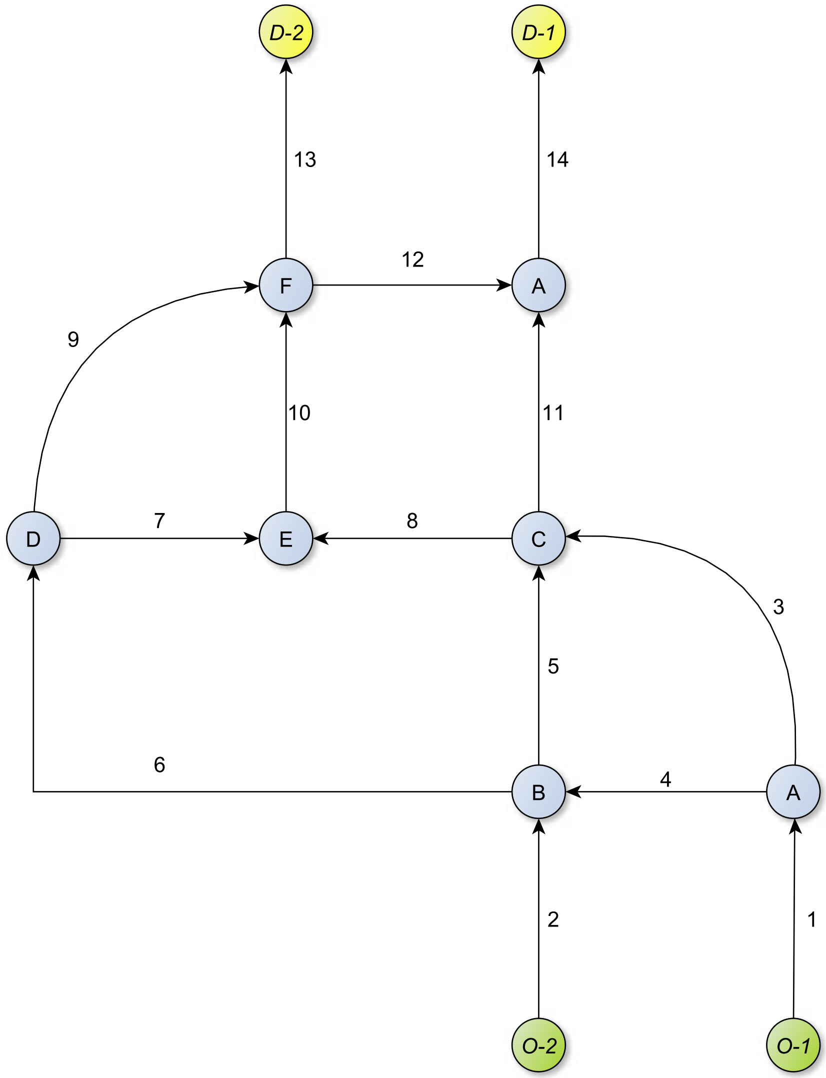


Figure 1.2: Representational Graph of Las Vegas Freeway Network

to establish the relationship between ρ and v. In the Greenshields' model [1], v is a an affine function of ρ given by

$$v(\rho) = v_f \left(1 - \frac{\rho}{\rho_m}\right) \tag{1.2}$$

Here, v_f indicates the free flow speed. ρ_m indicates the maximum possible density or

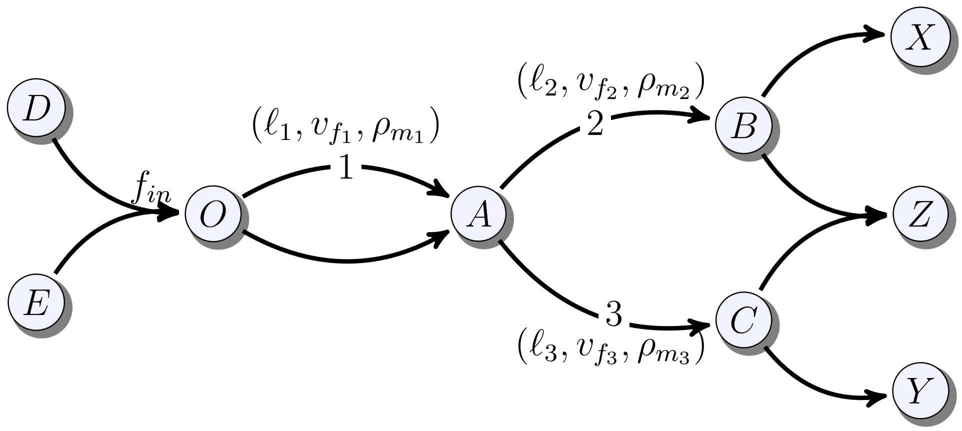


Figure 1.3: Transportation Road Network

jam density.

f is the product of speed and density, hence we get

$$f(\rho) = v_f \rho \left(1 - \frac{\rho}{\rho_m}\right) \tag{1.3}$$

These two relationships for this model are shown in Figure 1.4, which is called the fundamental diagram.

Traffic on the roads can be modeled microscopically or macroscopically. The microscopic dynamics of vehicles include models such as the car-following models. Starting from these models we can derive macroscopic dynamics of traffic (for example the LWR model) [2]. Both of these modeling techniques are important. Macroscopic traffic dynamics and modeling is extremely helpful in designing aggregated controls such as ramp metering [3], observability and controllability analysis [4] [5], financial analysis and modeling [6] etc. In addition to micro and macro models, there also exists mesoscopic models. Apart from control and management strategies, knowledge

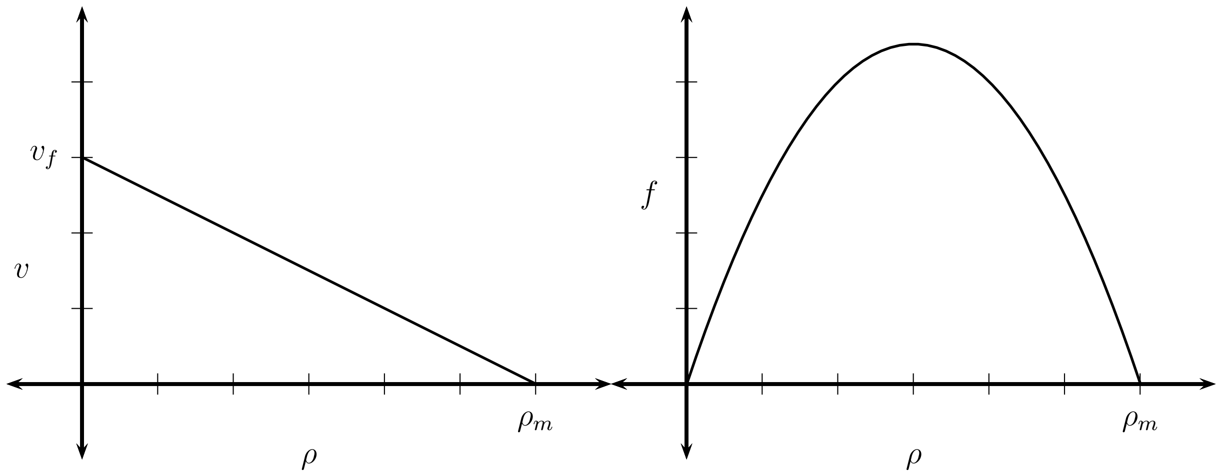


Figure 1.4: Fundamental Diagram using Greenshields' Model

of these studies are also very insightful in enhancing traffic safety [7] [8] and developing behavioral models [9].

Additionally, models that combine both microscopic and macroscopic systems (mesoscopic models) have been widely used by researchers to evaluate transportation systems for infrastructure and economic improvements [10]. Furthermore, the inbuilt interdependence of transportation systems with other Environmental, Economic, and Social systems were also studied to understand the broader concept of interconnected systems and networks [11] [12]. Also, the dynamic nature of transportation systems was analyzed for sustainability using dynamical systems, system dynamics and other modeling designs [13]. The results of such studies have helped decision makers and authorities to propose and implement suitable control mechanisms for policy making [14] [15].

1.3 Macroscopic Fundamental Diagram

Aggregation of traffic variables over a region has interested researchers for a while, as understanding of aggregated pattern of traffic behavior can lead to efficient management of traffic in bigger regions. The idea of MFD has existed for some time now, but has only recently started to gain a significant traction and attention from researchers. Although the concept of MFD is promising and has a lot of potential, its existence is not guaranteed [16]. A well defined MFD exists only under certain conditions [17]. Authors in [17] laid down the network properties required for a well defined MFD with low scatter and without hysteresis effects. Geroliminis and Boyaci further investigated the effect of signal characteristics and link lengths, on urban network characteristics and generation of an MFD [18].

Applying the MFD aggregation principle to speed aggregation in Greenshields' model we obtain

$$v\left(\rho_a\right) = \frac{1}{n}\sum_{i=1}^{n} v(\rho_i) \tag{1.4}$$

where $v(\rho)$ is given by equation (1.2). Denoting aggregated traffic density ρ_a, aggregated traffic speed, and aggregated traffic flow by

$$\begin{gathered} \rho_a = \frac{1}{n}\sum_{i=1}^{n}\rho_i, \quad v_a = \frac{1}{n}\sum_{i=1}^{n} v(\rho_i), \text{ and} \\ f_a = \frac{1}{n}\sum_{i=1}^{n} f\left(\rho_i\right) \end{gathered} \tag{1.5}$$

we see that (see equations (1.4) and (1.5)) the aggregated speed and aggregated

density have the Greenshields' relationship as well, i.e.

$$v_a = v(\rho_a) = v_f \left(1 - \frac{\rho_a}{\rho_m}\right) \tag{1.6}$$

This results is based on aggregation over sections where the parameters v_f and ρ_m are the same.

1.4 Motivation

Controlling and organizing the enormous urban traffic networks always remains a big challenge for managers and researchers. Challenges include uncertainty in user behavior, accurate estimation of OD trip matrices, and a good estimate of a network state. In such a large scale urban network its not easy to have microscopic network control strategy due to high amount of stochasticity involved in traveler behavior and network parameters as at each link and intersection. Therefore, researchers prefer macro control approach in which the idea is to perform perimeter control of a area based on aggregation of traffic conditions in a network. Therefore the concept of MFD came into existence and researchers have utilized this concept to improve urban mobility. However, due to large network size, sometimes existence of MFD is not guaranteed. Hence, the idea to subdivide the network into smaller regions for the existence of sub-MFDs in each of these subregions. These concept are very important in designing macroscopic traffic controls for large-scale urban networks.

1.5 Contribution

In our work we have performed the control techniques on the sub-MFD region of a road network and then further performed hierarchical area control by dynamically varying toll prices. We present a novel methodology to divide an urban network into sub-networks so that in each sub-network an MFD can be used to determine the state of that sub-networks. The region division is based on the theory of complex networks. We exploit the inherent network characteristics through PageRank algorithm to identify the most significant nodes in the traffic network. We then use these significant nodes as the seeds for a Voronoi diagram based partitioning mechanism of the network. A network wide hierarchical feedback controller is then presented which controls these sub regions individually. Another novelty of our work is that it present a method for hierarchical control of a whole network. This method is dynamic pricing of toll lanes which maintains inflow into the network.

1.6 Structure

This thesis is a work on control techniques which are applied on road transport complex network. Now to apply control techniques we have to perform modeling of these complex road networks. The structure of the thesis has two parts. Part One is about road network modeling. We have utilized the inherent network characteristics through PageRank centrality algorithm to identify the most significant nodes in the traffic network. Then we use these significant nodes as the seeds for a Voronoi diagram based partitioning mechanism of the network into sub refions. This algorithm then

use MFD for each sub region to develop feedback control law on these sub regions. Part Two of the thesis is about presenting a model to achieve a higher level of traffic control, such as to control an inflow into the network, for instance at a bridge to the entire region. We have designed a model based on dynamic congestion pricing.

Part I

COMPLEX TRAFFIC NETWORK MODELING

CHAPTER 2

INTRODUCTION

2.1 Summary

This part of thesis presents a novel methodology to divide a region into subregions so that in each subregion a MFD can be utilized to determine the state of that subregion. The region division is based on the theory of complex networks. We exploit the inherent network characteristics through PageRank centrality algorithm to identify the most significant nodes in the traffic network. We use these significant nodes as the seeds for a Voronoi diagram based partitioning mechanism of the network. A network wide hierarchical feedback controller is then presented which controls these sub regions individually and the network as a whole. A case study is performed for the Manhattan area in New York city and results are provided through simulations.

2.2 Introduction and Literature Survey

Optimal control of big urban traffic networks remains a big challenge. Challenges include uncertainty in user behavior, accurate estimation of OD trip matrices, and a good estimate of a network state. Traditionally, the developed control algorithms were focused on controlling each link and signalized intersection individually. There are

multiple problems associated with this microscopic network control strategy. Firstly, it is very complex to model traffic dynamics at every link on a huge urban traffic network. Secondly, these models or simulation scenarios are very difficult to calibrate due to high amount of stochasticity involved in traveler behavior and network parameters. Thirdly, even if we manage to develop a realistic model or simulation scenario, a large number of input/output variables associated with the control design makes the implementation infeasible and issues such as controllability and observability of the network come into play ([19], [5], [4]). Hence, a macro approach seems more logical than the micro approach when dealing with large scale dynamical networks. Researchers have started paying attention to the idea of performing perimeter control of a area based on aggregation of traffic conditions in a network. This approach helps in reducing the complexity of the modeling and control design, as well as in the deployment of the traffic control methodologies.

Using aggregation of traffic variables over a region has recently been of active interest to researchers due to the desire to efficiently manage traffic in bigger regions. The concept of MFD is not new, but has only recently started to gain a significant traction and attention from researchers. Godfrey was the first to present the idea of MFD through his research in 1969, where he reported the network-wide relationship among observed speed and density [20]. Paper by Herman and Prigogine [21] presented the idea of aggregation of traffic variables for a city, where they consider moving vehicles as well as the portion of stopped vehicles to estimate aggregated traffic conditions in a traffic network. Recently, the existence of MFD by using experimental data from

downtown Yokohama, Japan was proved in [22] by Daganzo and Geroliminis. The authors further made the case for the existence of the MFD relation between average speed and densities over the network of a certain class, and provided some analytical treatment as well in ([23]). A generalized MFD which uses the inhomogeniety of the traffic has been proposed in [24].

As an example of the importance of MFD and its usage in practical problems, Haddad et. al. ([25]) use the model for the model predictive perimeter control of an area modeled as two aggregated regions. Geroliminis et. al. also developed an optimal perimeter control problem for twin-regions by utilizing MFD concept in [26], and solved it by using model predictive control approach. Another study based on using a model predictive control is [27]. Daganzo developed an adaptive control method to decrease delays and congestion in [28] by observing and controlling aggregate vehicular accumulations in neighborhoods. Ekbatani et. al. also explored the idea of MFD to increase throughput and decrease congestion by applying gating techniques using feedback control [29]. Although the concept of MFD is promising and has a lot of potential, its existence is not guaranteed. A well defined MFD exists only under certain conditions, which were discussed in [17]. Authors in the paper laid down the network properties required for a well structured MFD with negligible spread and without hysteresis effects. Geroliminis and Boyaci further investigated the effect of road lengths on urban network characteristics and generation of an MFD [18].

However sometimes due to large network size and variations within the network

based on dynamics and other conditions, aggregated variables do not truly represent the entire network. Recent research indicates that link density heterogeneity is very crucial in determining the shape and scatter of MFD [16]. Moreover, heterogeneity can also be a cause for hysteresis loops and degradation of network performance [30]. An MFD is expected to be well defined under the condition that is a homogeneous network, i.e. it has similar link properties. However, in reality, large scale urban networks are expected to have congestion varying differently from link to link. Now, to exploit the usefulness of MFD in designing macroscopic control strategies, it becomes very important to subdivide the network into smaller regions for the existence of sub-MFDs in each of these subregions. First attempt in this direction was made by Ji & Geroliminis in [31], focusing on clustering of the network based on spatial congestion distribution for a given time period. The objectives of clustering were as follows (i) link densities inside a cluster have low scatter or variance, and (ii) spatial compactness of each cluster. To achieve this objective a mechanism involving three consecutive algorithm was used. Ji et. al. further extended this methodology and proposed a dynamic partitioning algorithm using probe-data [32]. Authors studied the spatiotemporal relation of congested links and observed congestion propagation from a macroscopic perspective. Maximum connected component of congested links was utilized to capture congestion propagation in a region and to partition the region dynamically.

Using the notion of well defined MFDs in carefully chosen sub-regions, Aboudolas and Geroliminis presented a methodology for perimeter control using feedback in a

multi-reservoir system [33]. Another feedback control based gating mechanism has been developed in [34]. The work was expanded and a novel control strategy for twin cities using MFD and feedback-based gating was introduced in [35]. The gating mechanism was further modified to work well with time-delays in [36]. Haddad proposed modeling and control of uncertain MFD systems for multiple-region networks in [37]. Ramezani et. al. presented two aggregated models for region- and subregion-based MFD and designed a hierarchical perimeter flow control using them [38].

The discussion above, highlights the importance of MFD in designing macroscopic traffic controls for large-scale urban networks and discusses some recent attempts to exploit the idea. However, it also points towards the need for careful study of MFD's existence and for robust methodologies to sub-divide then network into smaller regions. Studies such as [31], [32] and [33] are positive steps in this direction laying the foundation for further research. Our work aims at complementing the existing methodologies and tries to exploit some inherent network characteristics sub-division algorithms. This work shows a specific way, various regions can be identified, where traffic aggregation can be used for network control. We present a novel methodology to divide an urban network into sub-networks so that in each sub-network an MFD can be used to determine the state of that sub-networks. The region division is based on the theory of complex networks [39]. We exploit the inherent network characteristics through PageRank algorithm [40] to identify the most significant nodes in the traffic network. We then use these significant nodes as the seeds for a Voronoi diagram based partitioning mechanism of the network [41] [42]. A network wide hierarchical

feedback controller is then presented which controls these sub regions individually and the network as a whole. A case study is performed for the Manhattan area in New York city and results are provided through simulations.

CHAPTER 3

BACKGROUND

3.1 Complex Networks

Cyber-physical systems are becoming the backbone of the modern society. These systems have a very high level of connectivity and the study area of *complex networks* has been developing to answer many questions in this field. Many introductory survey papers ([39], [43]) and several books exist that provide details of the theory and applications of complex networks ([44], [45], [46]). Different types of networks that come under this area of study include computer networks, social networks, economic networks, biological networks, and also transportation networks. This paper uses the theory of complex networks to perform dynamic aggregation of regions so that MFD based control strategies can be applied to the overall network. Specifically, we use the page rank algorithm to identify the most important nodes in a transportation network. This can be performed based on the structure of a directed graph. It can also be performed on a weighted directed graph where the weights represent the dynamic traffic conditions, for instance using time varying densities, on the links of a graph.

3.2 PageRank Algorithm

PageRank algorithm is used by Google to rank webpages. PageRank algorithm measures the importance of webpages by counting the number and quality of links. We will provide next some basic terminology and discussion on the subject.

Consider a non-weighted network ($\mathcal{N}$, $\mathcal{E}$), where $\mathcal{N}$ is a set representing nodes, and $\mathcal{E}$ is a set representing edges, hence:

$$\mathcal{N} = \{a, b, c, d, \ldots\}, \quad \mathcal{E} = \{(a, b) \mid \text{for each valid pair of connected nodes}\} \tag{3.1}$$

Now the basic node centrality/significance index can be defined in terms of degree of the node. For instance, the degree of node a is representative of the direct number of links it has and is defined as

$$\text{degree } (a) = |\{(a, b) \in \mathcal{E}\}|, \tag{3.2}$$

where $|.|$ is the degree operator giving the total count of valid connections of the node a.

Another idea following immediately after the degree centrality concept is that the node centrality to be proportional to the cumulative degree of its direct neighbors. Mathematically speaking:

$$\text{centrality } (a) = \sum_{(a,b)\in\mathcal{E}} \text{degree } (b), \tag{3.3}$$

Now the underlying argument in the PageRank algorithm is that the weight distribution of each connection is not uniform. Rather, it is inversely proportional to the number of other connections that the neighboring nodes have. PageRank centrality is defined by the following recursive formula:

$$\text{PageRank }(a) = \alpha \sum_{(a,b)\in\mathcal{E}} \frac{\text{PageRank }(b)}{\text{degree }(b)} + \frac{1-\alpha}{n} \tag{3.4}$$

where $0 < \alpha < 1$ is the damping parameter and n represents total nodes.

3.3 Voronoi Diagrams

Voronoi diagrams provide a computational geometric method to divide a region into subregions based on distance from a given set of seed points ([41], [7]). The problem for the Voronoi diagram is as follows.

Problem 3.3.1 (Voronoi Problem)**.** *Given* $\Omega \subset \mathcal{R}^n$*, and* $s_i \in \Omega$*,* $i \in \{1, 2, \cdot, N\}$*, then find* v_i*,* $v_i \subset \Omega$*,* $i \in \{1, 2, \cdot, N\}$*, such that*

$$\bigcup_{i=1}^{N} v_i = \Omega, \quad v_i^0 \cap v_j^0 = \emptyset \quad \textit{and} \quad v_i = \{x \in \Omega \mid d(x, s_i) \le d(x, s_j), \forall j \neq i\} \tag{3.5}$$

Here the subscript on the regions such as v_i^0 means the interior of the set, and $d(x, y)$ is the function representing distance between the points x and y.

CHAPTER 4

CONTROL ARCHITECTURE

4.1 Main Algorithm

The flowchart for the algorithm for the overall subdivision, aggregation, and region feedback control is shown in Figure 4.1.

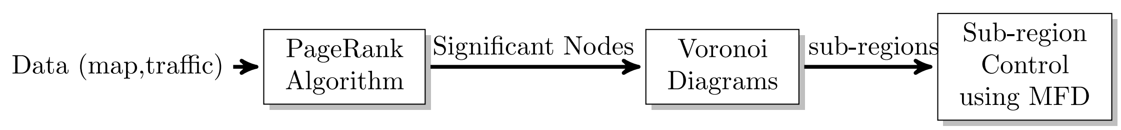


Figure 4.1: Main Algorithm Flowchart

The task of creating subregions based on page rank algorithm can be performed once in a static setting or can be repeatedly applied as the traffic in the system changes which can produce changing boundaries of various regions in a time varying fashion. The static map data can be used as an input in the static setting to perform the page rank algorithm and obtain the most important nodes of the digraph. Alternately, in a dynamic setting we can create weighted digraph, where the weights on the links are obtained via traffic densities, and this would give a changing set of the most important nodes as the traffic conditions change. After the nodes are obtained, we apply the Voronoi algorithm to divide the overall region into subregions. Once the

regions are obtained, we can use MFD for each region in order to develop control law on the simplified network model.

4.2 Traffic Control Design

Once the subregions have been established with their corresponding MFDs, we can write down the conservation flow dynamics for traffic densities for each subregion being its state variable. This provides us with state space control dynamics where control variables are also used to indicate how traffic control has to be performed. For instance if the perimeter control in the region has to be performed using a ramp control ([47]) or a gating mechanism, a corresponding control variable will be present in the dynamics. Once the dynamics are established, we can design a feedback control law to satisfy some design requirements such as some asymptotic performance or optimality ([48], [49]). An example of dynamics using MFD is given in [26].

CHAPTER 5

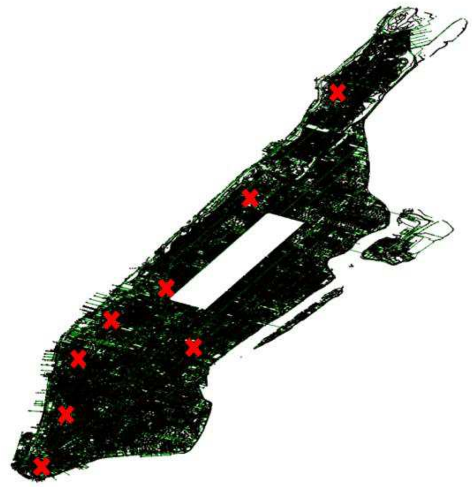
CASE STUDY

We consider Manhattan road network for application of the proposed approach and dividing it into sub-networks based on PageRank algorithm. The Manhattan Open Street Map network was downloaded from *www.openstreetmap.org*. Which was then further parsed and processed to extract the valid nodes (intersections) and their latitude-longitude values. Total number of valid nodes in the data for Manhattan road network were $446,415$.

We next analyze the data using *NetworkX* package in *python* and apply PageRank algorithm to obtain ten most significant nodes. Figure 5.1a shows the top significant nodes marked on the Manhattan road network. Two sets of two nodes which were very close and overlapping were considered as one.

Figure 5.1b shows the markings of nodes on a google map. It is quite interesting to note that most of these node are close to the major entry bridges/tunnels into the Manhattan area. We next apply Voronoi diagram method to divide the network into sub-networks, using these significant nodes as seeds. The divided regions are shown in figure 5.1c.

The subdivided network can now be represented as a weighted digraph so that we can generate a state space representation of finite dimensional traffic dynamics



(a) Top Nodes

(b) Top Nodes

(c) Subnetworks

Figure 5.1: Manhattan, New York Area Divisions

for the entire network. The directed graph is shown in Figure 5.2. In fact, we can process the output of the Voronoi diagram and add more features, such as some extra subregions, depending on the local geography as well as the traffic density distribution. For instance, in this example the water way divides the nodes 2 and nodes 5 further. Each MFD region should have a low variance in its internal traffic density distribution. Hence, the region boundary design can be performed combining the results of the Voronoi algorithm, geographical features and traffic density variance. We can modify the digraph based on these additional changes.

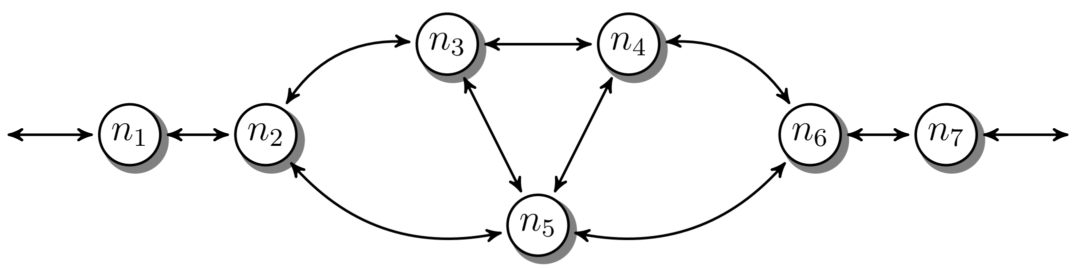


Figure 5.2: Network Digraph

We can now develop the dynamics for the system for the traffic moving on the digraph in Figure 5.2. Control variables can appear at multiple places in the equations depending on what actual mechanism is available in the system. For instance congestion pricing might be in use to control the inflow at a location, or some ramp flows or signalized intersections. Control objectives can be created based on the MFD variables in the system dynamics and control law designed.

5.1 Hierarchical Control

The MFD framework automatically lends itself into a natural hierarchical control structure. Each MFD region should have a low variance traffic density distribution for the MFD to be valid in that region. So, at the higher level MFD can be used to control an inflow, for instance at a bridge to the entire region. Similarly, at a lower level, the traffic signals and ramps inside the region can be used to maintain a smooth flow in the region which would create the uniform traffic density in that region.

5.2 Two Region Simulation Study

An example of dynamics using MFD is given in [26]. Borrowing the dynamics from there we have its labeled digraph and equations in (5.1).

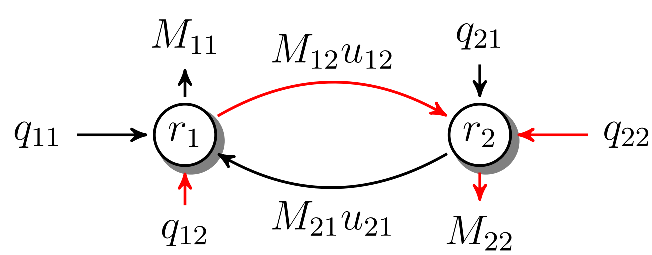


Figure 5.3: Labelled Digraph

$$
\begin{aligned}
\frac{dn_{11}(t)}{dt} &= q_{11}(t) + u_{21}(t)M_{21}(t) - M_{11}(t) \\
\frac{dn_{12}(t)}{dt} &= q_{12}(t) - u_{12}(t)M_{12}(t) \\
\frac{dn_{21}(t)}{dt} &= q_{21}(t) - u_{21}(t)M_{21}(t) \\
\frac{dn_{22}(t)}{dt} &= q_{22}(t) + u_{12}(t)M_{12}(t) - M_{22}(t)
\end{aligned} \tag{5.1}
$$

The region r_1 has n_1 total number of vehicles which is a sum of n_{11}, the number of vehicles in r_1 with destination in r_1, and n_{12}, the number of vehicles in r_1 with destination in r_2. The variables for region r_2 are defined analogously. The variable q_{ij} indicates the trips starting from region i with destination in region j. The variable M_{ij} indicates the trips ending in region j with origin in region i. The control variables $u_{ij} \in [0, 1]$ indicate the fraction of the flow M_{ij} they allow to go through. The flow variables M_{ij} are related to n_i, n_{ij}, and $G_i(n_i)$ where $G_i(n_i)$ is the MFD variable indicating the average flow in a region as a function of the total number of vehicles (the product of the average density in a region and the total lane length in the region) in that region. These relationships are noted in equation (5.2).

$$
n_i(t) = n_{ii}(t) + n_{ij}(t), i \neq j \;, \qquad \text{and} \qquad M_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)} G_i(n_i) \tag{5.2}
$$

In order to design control laws to try to achieve desired flows obtained by maintaining critical densities (or number of vehicles) in each region we combine equation (5.1) with equation (5.2) to obtain

$$\frac{d}{dt}\begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} = \begin{bmatrix} q_{11}(t) + q_{12}(t) - M_{11}(t) \\ q_{21}(t) + q_{22}(t) - M_{22}(t) \end{bmatrix} + \begin{bmatrix} M_{21}(t) & -M_{12}(t) \\ -M_{21}(t) & M_{12}(t) \end{bmatrix} \begin{bmatrix} u_{21}(t) \\ u_{12}(t) \end{bmatrix} \tag{5.3}$$

We rewrite equation (5.3) using matrix notation which is implied by that equation as

$$\frac{dn(t)}{dt} = Q(t) + M(t)u(t) \tag{5.4}$$

Equation (5.3) is decoupled by the control law:

$$u(t) = M^{-1}[-Q + v] \tag{5.5}$$

where the vector v has components v_1 and v_2 that are chosen as

$$v_i(t) = -K_i(n_i(t) - n_{id}) \tag{5.6}$$

In order to make the vector $n(t)$ with components $n_i(t)$ follow a desired vector of number of vehicles in each region given by the vector n_d which has components n_{id}, we choose positive values for k_i the control gains. This control is designed to ensure $n(t) \rightarrow n_d(t)$ as $t \rightarrow \infty$. Figure 5.4 show the simulation results for two regions. Values of K_1 and K_2 are fixed at 1 and 2 respectively. Simulations result show that both the sub-regions achieve desired state (number of vehicles) by using the proposed control law. Error term also goes to zero as $t \rightarrow \infty$.

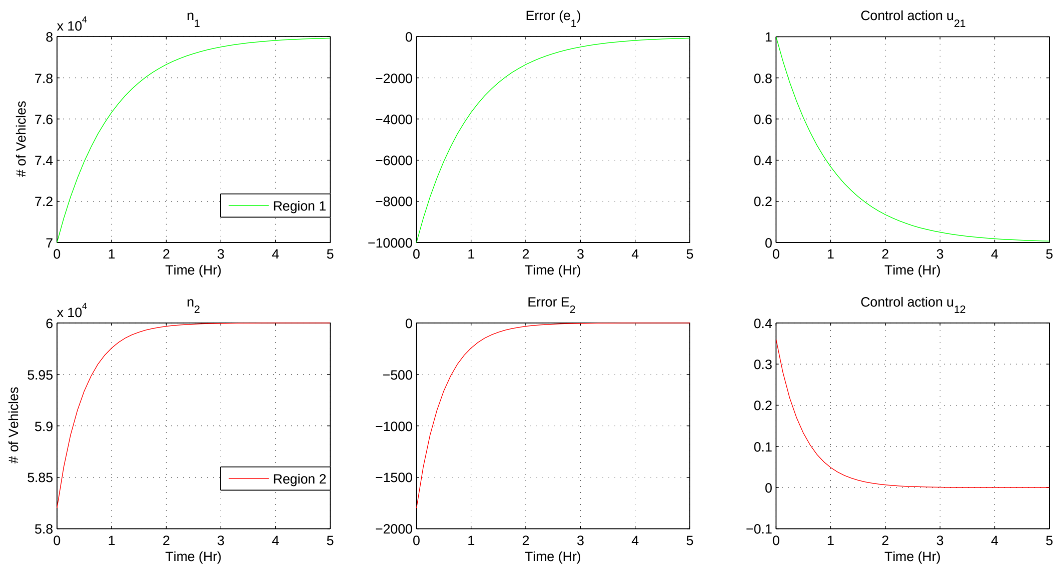


Figure 5.4: Simulation Results

CHAPTER 6

CONCLUSION

This work presented a new algorithm for creating subregions for an area for traffic control based on the application of page rank algorithm from the complex networks theory to identify important nodes followed by the application of the Voronoi diagram algorithm. This algorithm then used MFD for each subregion and used feedback control design on the simplified network model. An example problem was studied showing these steps applied to that problem followed by a simulation performed in a two region network that uses a novel feedback linearization control for perimeter control. In the next part we are designing a model to achieve higher level of traffic control in a network.

Part II

DYNAMIC PRICING

CHAPTER 7

INTRODUCTION

7.1 Summary

In the last part we have performed traffic control on the sub-MFD regions of a road network. Part two of the thesis is about presenting a model to achieve a higher level of traffic control, such as to control an inflow into the network, for instance at a bridge to the entire region. We have designed a model based on dynamic congestion pricing. This part of thesis presents a mathematical framework for dynamic congestion pricing. The objective is to calculate an optimal toll using optimal control theory. The problem consists of tolled lanes or routes and alternate non-tolled lanes or routes. The model is developed using traffic conservation law, queuing theory and fundamental macroscopic relationships. A Logit model is used for establishing the relationship between the price and driver choice behavior. We design a cost function and then use Hamilton-Jacobi-Bellman equation to derive an optimal control law which utilizes real-time traffic parameters to determine an optimal toll price. Simulations are also performed to demonstrate the robustness of this optimal control congestion pricing algorithm.

7.2 Background

Congestion is one of the major area of concern in transportation. Congestion pricing is one of the methods to tackle congestion problem on roads effectively. It is method which charges the vehicles for the using certain roads during certain time of the day. It is aimed at reducing the congestion during peak hours by encouraging travelers to use non-congested alternate routes by giving them benefit of travel time. According to the road pricing theory, a toll is necessary to reach social efficient system.

Congestion pricing can be designed in a static way, where it is formulated based on historical data. This is usually accomplished through hourly, weekly or monthly schedules. However, with the help of technological advancements such as real time sensors etc., dynamic tolling can also be achieved. Dynamic congestion pricing means that the toll rates can be calculated based on the current traffic situation. The dynamic optimal toll mechanism can be helpful to the users as well as to the system as a whole. However, implementing dynamic toll price has its own challenges due to user related and technological challenges. Hence, more often for practical purposes pricing is done in certain step sizes.

As discussed previously, dynamic congestion pricing algorithm is often applied using the step functions that dynamically adjust toll rates. This approach might not lead to optimally desired results. Hence this problem is really important. It requires proper analysis and development of proper mathematical framework [47]. In this work, we will first develop the problem formulation framework for the dynamic tolling problem. We will then design an optimal control methodology using dynamic

tolling. This control aims at achieving the desired travel time at the toll-road while keeping the toll as low as possible.

CHAPTER 8

LITERATURE REVIEW

Congestion pricing is an important research topic in traffic engineering. Various research studies have been conducted aimed at establishing the theoretical and mathematical framework for the pricing models ([50], [51], [52], [53]). Lindsey [54] reviewed road pricing applications in various countries and recommended the best practices. Congestion pricing experiences from other international countries were studied in ([55], [56], [57]). Lessons learned from these implementations were analyzed in these studies.

Congestion pricing is generally implemented by using High-occupancy toll (HOT) lanes lanes. These HOT lanes are accessed by vehicles with required occupancy or vehicles that are ready to pay tolls. There is also another methodology called cordon based pricing. Where drivers are charged upon entering a congested part of a city. This toll price is generally a flat rate rather than distance based or dynamic price. In [58], Q.Meng, Z.Liu and S.Wang, presented a toll scheme using distance calculations that can be efficiently implemented in cordon pricing scheme instead of flat toll charges.

Friesz et al. presented the theory of dynamic congestion pricing using real time optimal control theory in [59]. They analyzed the necessary conditions for optimal

congestion prices to uncover bang-bang, singular and synthesized optimal control decision rules for setting network tolls in a dynamic environment. Yang [60] proposed a system to achieve better system travel time.

Marginal-cost pricing method was implemented on an urban network in [61]. Extending the work further, Zhao and Kockelman studied an online version of the algorithm [62].

B. Hårsman and J.M.Quigley [63] proved that citizens value commute time highly by analyzing a case of road pricing. Residents of city of Stockholm voted to adopt the dynamic toll scheme permanently that reduces congestion on urban motorways. A. de Palma and R.Lindsey in their paper [64] describe different ways and different technologies which can be used for congestion pricing for a single lane or entire road networks. Variation in toll price depends on the multiple factors which are discussed in the paper. Congestion Pricing is the center of traffic implementation projects in many countries. Lot of technologies like digital photography, transponders, satellites and cellular network communications are considered, but the best technology choice depends on the local conditions and systems.

CHAPTER 9

MATHEMATICAL MODELLING

Generally, dynamic toll projects are implemented in such a way that the commuters have a choice between a tolled road and a regular road during peak hours of congestion. Prevailing traffic conditions on both roadways, travel time of both lanes, and dynamic toll rates are displayed to the commuters using signs and boards, so that commuters can make informed decision to pay the toll or take the alternate free road.

Consider the system configuration as given in figure 9.1. Variables used in the mathematical model formulation are summarized below in Table 9.1. Mathematical model and notations are directly borrowed from [47].

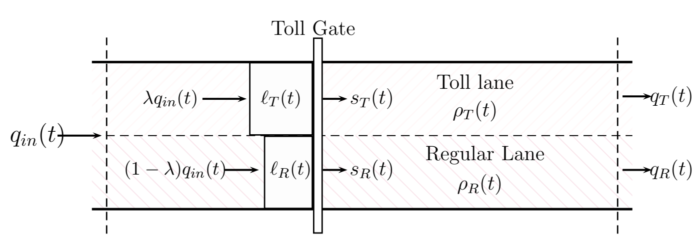


Figure 9.1: System Setup

Dynamics for system shown in figure 9.1 are given by:

Table 9.1: Notation for Variables

Notation	Explanation
q_{in}	Traffic in-flow
λ	Percent flow using toll
ℓ_T	Queue length for toll lane
ℓ_R	Queue length for regular lane
s_T	Service rate for toll lane
s_R	Service rate for regular lane
ρ_T	Traffic density in toll lane
ρ_R	Traffic density in regular lane
ρ_m	Maximm traffic density
v_f	Traffic free flow velocity
q_T	Traffic outflow from toll lane
q_R	Traffic outflow from regular lane
L_T	Length of the toll lane
L_R	Length of the regular lane
T_T	Travel time through toll lane
T_R	Travel time through regular lane

$$
\begin{aligned}
\dot{\ell_T} &= \lambda q_{in}(t) - s_T(t) \\
\dot{\rho_T} &= s_T(t) - q_T(t) \\
\dot{\ell_R} &= (1-\lambda) q_{in}(t) - s_R(t) \\
\dot{\rho_R} &= s_R(t) - q_R(t)
\end{aligned}
\tag{9.1}
$$

We have the system with toll lane and regular lane. There is queuing in both lanes. Equation (9.1) gives the rate of change of the queue at the toll lane and regular lane, $\dot{\ell_T}$ and $\dot{\ell_R}$, and the rate of change of density in toll lane and regular lane, $\dot{\rho_T}$ and $\dot{\rho_R}$, by the application of conservation law.

Now using the Greenshields' relationship between then density and speed, we

have:

$$
\begin{aligned}
q_T(t) &= v_f \rho_T \left(1 - \frac{\rho_T(t)}{\rho_m}\right) \\
q_R(t) &= v_f \rho_R \left(1 - \frac{\rho_R(t)}{\rho_m}\right)
\end{aligned}
\tag{9.2}
$$

If automatic tolling is also introduced using technologies such as RF-tagging or plate scanning, then the system dynamic for such model is represented by equation (9.3), assuming that such setting would allow queuing in every lane. This is the most general model. In this model as shown in Figure 9.2, ℓ_{RF} is the the queue length for tagged vehicles and $s_{RF}(t)$ as the service rate.

$$
\begin{aligned}
\dot{\ell_T} &= \lambda(1-\beta)q_{in}(t) - s_T(t) \\
\dot{\ell_{RF}} &= \lambda\beta q_{in}(t) - s_{RF}(t) \\
\dot{\rho_T} &= s_T(t) + s_{RF}(t) - v_f \rho_T \left(1 - \frac{\rho_T(t)}{\rho_m}\right) \\
\dot{\ell_R} &= (1-\lambda)q_{in}(t) - s_R(t) \\
\dot{\rho_R} &= s_R(t) - v_f \rho_R \left(1 - \frac{\rho_R(t)}{\rho_m}\right)
\end{aligned}
\tag{9.3}
$$

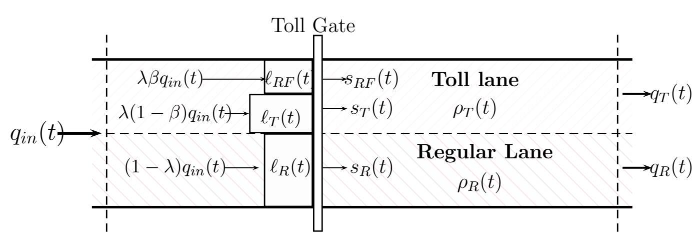


Figure 9.2: General model

The basic model in equation (9.1) can be modified Based on the actual imple-

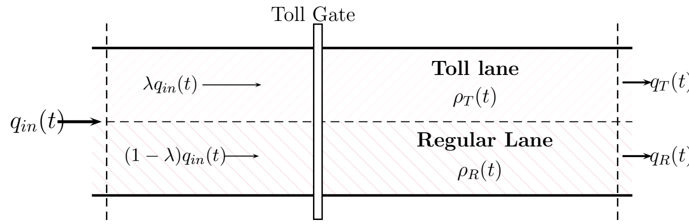


Figure 9.3: Model with noqueue at any Lane

mentation of the tolling system. For example, if there will be no queues at the toll entrance as well at the regular entrance then the system dynamics for this type of implementation are shown in equation (9.4)

$$\begin{aligned} \dot{\rho_T} &= \lambda q_{in}(t) - v_f \rho_T \left(1 - \frac{\rho_T(t)}{\rho_m}\right) \\ \dot{\rho_R} &= (1-\lambda) q_{in}(t) - v_f \rho_R \left(1 - \frac{\rho_R(t)}{\rho_m}\right) \end{aligned} \tag{9.4}$$

CHAPTER 10

OPTIMAL CONTROL LAW FOR CONGESTION PRICING

10.1 Problem Formulation

User-equilibrium means to have equal travel time on lanes with common origin and destination. However, "allowable" user-equilibrium" is defined, when travel time in toll lane and regular lane is compared, to maintain lower value travel time at toll lane than the regular lane. In order to obtain an optimal control design, equation (10.1) is used. A lower travel time must be maintained at the tolling lane otherwise drivers will not be willing to pay for a worse or even equal traffic conditions.

When designing the feedback control system, the error is determined by equation (10.3) where symbol γ is the multiplying factor for toll lane travel time. The γ factor must be larger than 1 so that the tolling lane would be considered worthwhile for the driver. This parameter is taken to be constant in this paper; however, it can vary based on traffic conditions and drivers' behaviour as well as the parameters used in the logit model. In other words, γ could increase or decrease based on the desired λ which is the percent flow using toll.

$$
\begin{aligned}
T_T(t) &= \frac{\ell_T(t)}{s_T(t)} + \frac{L_T}{v_f\left(1 - \dfrac{\rho_T(t)}{\rho_m}\right)} \\
T_R(t) &= \frac{\ell_R(t)}{s_R(t)} + \frac{L_R}{v_f\left(1 - \dfrac{\rho_R(t)}{\rho_m}\right)}
\end{aligned}
\tag{10.1}
$$

For simplification, we are considering the system in figure 9.3, assuming here that the tolling system is fast enough and there are no queues on any lanes. Then the equation (10.1) becomes:

$$
\begin{aligned}
T_T(t) &= \frac{L_T}{v_f\left(1 - \dfrac{\rho_T(t)}{\rho_m}\right)} \\
T_R(t) &= \frac{L_R}{v_f\left(1 - \dfrac{\rho_R(t)}{\rho_m}\right)}
\end{aligned}
\tag{10.2}
$$

The feedback control problem here is to maintain the allowable user-equilibrium, such that the error given by equation (10.3). Error term achieves a closed loop dynamics, that means error goes to zero asymptotically. The desire is that the error in equation (10.3) goes to zero in the feedback system, hence the time in the tolled lane must be $\frac{1}{\gamma}$ when compared to the regular lane.

$$
\begin{aligned}
&e(t) = \gamma T_T(t) - T_R(t) = \\
&\gamma\left(\frac{L_T}{v_f\left(1 - \dfrac{\rho_T(t)}{\rho_m}\right)}\right) - \left(\frac{L_R}{v_f\left(1 - \dfrac{\rho_R(t)}{\rho_m}\right)}\right)
\end{aligned}
\tag{10.3}
$$

Now, the control should be designed such that it achieves the goal to make the error go to zero. In this paper we are using optimal control theory using HJB equations

to derive the control methodology for the above non linear system dynamics. Now lets start by differentiating the error. Differentiating equation (10.3) against time, we obtain the error dynamics as

$$\begin{aligned}
\dot{e}(t) &= \gamma \dot{T}_T(t) - \dot{T}_R(t) \\
&= \gamma \left(\frac{L_T}{v_f \rho_m \left(1 - \frac{\rho_T(t)}{\rho_m}\right)^2} \dot{\rho}_T(t) \right) \\
&- \left(\frac{L_R}{v_f \rho_m \left(1 - \frac{\rho_R(t)}{\rho_m}\right)^2} \dot{\rho}_R(t) \right)
\end{aligned} \tag{10.4}$$

Using system dynamics given by equation (9.4), terms in the above error dynamic are expanded, we get

$$\begin{aligned}
\dot{T}_T(t) &= \frac{L_T}{v_f \rho_m \left(1 - \frac{\rho_T(t)}{\rho_m}\right)^2} \dot{\rho}_T(t) \\
&= \frac{L_T}{v_f \rho_m \left(1 - \frac{\rho_T(t)}{\rho_m}\right)^2} \left[\lambda q_{in}(t) - v_f \rho_T(t) \left(1 - \frac{\rho_T(t)}{\rho_m}\right) \right]
\end{aligned} \tag{10.5}$$

Similarly,

$$\begin{aligned}
\dot{T}_R(t) &= \frac{L_R}{v_f \rho_m \left(1 - \frac{\rho_R(t)}{\rho_m}\right)^2} \dot{\rho}_R(t) \\
&= \frac{L_R}{v_f \rho_m \left(1 - \frac{\rho_R(t)}{\rho_m}\right)^2} \left[(1 - \lambda) q_{in} - v_f \rho_R(t) \left(1 - \frac{\rho_R(t)}{\rho_m}\right) \right]
\end{aligned} \tag{10.6}$$

Substituting (10.5) and (10.6) into (10.4) gives us

$$\dot{e}(t) = f + g\lambda \tag{10.7}$$

where f and g are obtained by combining equations (10.5) and (10.6) with equation (10.4).

$$f = \frac{L_R \rho_R}{\rho_m(1 - \frac{\rho_R}{\rho_m})} - \frac{\gamma L_T \rho_T}{\rho_m(1 - \frac{\rho_T}{\rho_m})} - \frac{L_R q_{in}}{v_f \rho_m (1 - \frac{\rho_R}{\rho_m})^2} \tag{10.8}$$

$$g = \frac{\gamma L_T q_{in}}{v_f \rho_m (1 - \frac{\rho_T}{\rho_m})^2} + \frac{L_R q_{in}}{v_f \rho_m (1 - \frac{\rho_R}{\rho_m})^2} \tag{10.9}$$

10.2 Optimal Control

Now we design the cost function which would be optimized and control law will be found. Let the cost function be

$$J = \frac{1}{2} \int (e^2(t) + \theta^2) dt \tag{10.10}$$

where $\theta = 1 - \lambda$, is our control variable. Now, the Hamiltonian is defined as

$$\mathcal{H} = \frac{1}{2}(e^2(t) + \theta^2) + J_x^*[\dot{e}(t)] \tag{10.11}$$

Differentiating $\mathcal{H}$ partially against control variable θ we obtain

$$\frac{\partial \mathcal{H}}{\partial \theta} = \theta + J_x^* \frac{\partial}{\partial \theta}[f + g(1 - \theta)] \tag{10.12}$$

$$\frac{\partial \mathcal{H}}{\partial \theta} = \theta - gJ_x^* \quad (10.13)$$

For the necessary condition for an extremum we use

$$\frac{\partial \mathcal{H}}{\partial \theta} = 0 \quad (10.14)$$

to get:

$$\theta^* = gJ_x^* \quad (10.15)$$

Now we check,

$$\frac{\partial^2 \mathcal{H}}{\partial \theta^2} = 1 > 0 \quad (10.16)$$

Hence, this is a point of a minima. The Hamilton Jacobi Bellman equation is given by:

$$0 = J_t^*(x(t), t) + \mathcal{H}(x(t), u^*(x(t), J_x^*, t), J_x^*, t) \quad (10.17)$$

In our case: $u = \theta$ and $x(t) = e(t)$. Hence the HJB becomes:

$$0 = J_t^* + \frac{1}{2}(e^2(t) + \theta^{*2}) + J_x^*[\dot{e}(t)] \quad (10.18)$$

Now substituting the value of optimal control (θ^*) and $\dot{e}(t)$ from equation (10.15)

and equation (10.7) to the HJB equation (10.18), we get:

$$0 = J_t^* - g^2/2J_x^{*2} + (f+g)J_x^* + 1/2e^2(t) \tag{10.19}$$

Equation (10.19) is the HJB equation for this problem. It is not trivial to solve this PDE equation analytically. Hence, we will use steady state analysis to obtain a steady state solution after the transients have settled down. The value of J can be plugged back into equation (10.15) to get the the control variable. Section 10.3 provides the steady state analysis.

10.3 Steady State Analysis

Steady state analysis is the analysis after a very long time. hence the time variants have settled down. This means derivatives w.r.t. time are ignored as shown below:

$$J_t^* = 0 \tag{10.20}$$

Hence equation (10.19) converts into a quadratic equation of J_x^*, as described in (10.21)

$$0 = -g^2/2J_x^{*2} + (f+g)J_x^* + 1/2e^2(t) \tag{10.21}$$

For equation (10.21) to have real roots, we must have $b^2 - 4ac \geq 0$.

which implies that

$$(f+g)^2 - 4(g^2/2)(-1/2e^2(t)) \geq 0 \tag{10.22}$$

Further simplification yields,

$$(f+g)^2 + g^2e^2(t) \geq 0 \tag{10.23}$$

The discriminant is positive, therefore the roots of equation (10.21) are real. The value of J_x^* can be written as

$$J_x^* = \frac{S}{g^2} \tag{10.24}$$

where $S = (f+g) \pm \sqrt{(f+g)^2 + g^2e^2(t)}$

10.4 Calculation of Actual Toll

We now have the optimal control law, but we still have to come up with the actual tolling price that must be charged to the commuters. To define relationship between λ and toll price we choose the Logit model to formulate its functional form. The following relationship is used.

$$\lambda = \frac{1}{1 + \exp\left(a_1(T_T(t) - T_R(t)) + a_2p(t) + a_3\right)} \tag{10.25}$$

Now, as we have computed the value of λ, then using relationship from equation (10.25) we will calculate the adjustable toll rate given by equation (10.26).

$$p(t) = \frac{1}{a_2}\left(\ln(\frac{1-\lambda}{\lambda}) - a_1(T_T(t) - T_R(t)) - a_3\right) \tag{10.26}$$

In equation 10.26, variables a_1, a_2 and a_3 are the weights which can be varied by traffic officers to control the toll price depending on the traffic condition. The weight a_1 is given to the travel time difference in toll lane and regular lane. This travel time difference is the deciding factor to driver for choosing between two routes. Deciding the proper toll price depending on the travel time difference is critical as drivers will only be willing to pay the toll price if they receive the benefit of travel time on toll lane by certain amount; a_2 is the weight given to the toll rate, and a_3 represents other factors.

Using equation (10.26), we see that if a_1 increases, which indicates an increased effect of travel time in the tolled lane or if a_2 increases, which indicates an increased effect of the toll rate or if a_3 increases, which indicates an increased effect of other factors in the driver choice, then the toll price $p(t)$ should decrease [47].

CHAPTER 11

SIMULATION RESULTS AND DISCUSSION

We have developed the feedback control law using HJB optimal control technique in steady state. Now, we present the simulation results to check the state variable values ρ_T and ρ_R , error e and control variable λ starting with some initial values of state variables. The simulation is performed for different values of γ.

Using equation (10.15) and equation (10.24), the error dynamics in equation (10.7) takes the form as

$$\dot{e}(t) = \mp\sqrt{(f+g)^2 + g^2e^2(t)} \tag{11.1}$$

Let $M = \sqrt{(f+g)^2 + g^2e^2(t)}$. The value of $\dot{e}(t)$ is chosen based on value of e at that instant. This means if $e > 0$ then $\dot{e}(t) = -M$ and if $e < 0$ then $\dot{e}(t) = M$. The value of control variable λ will be calculated using $\dot{e}(t)$ and ultimately value of ρ_T and ρ_R are calculated at that instant. This is how the simulations are performed.

Figure 11.1, 11.2, and 11.3, show the simulation results for $\rho_T(t)$, $\rho_R(t)$, $e(t)$ and $\lambda(t)$ for values of $\gamma = 1$, $\gamma = 1.5$ and $\gamma = 2$, respectively.

Simulation results come out to be good with error $e(t)$ converging to zero as

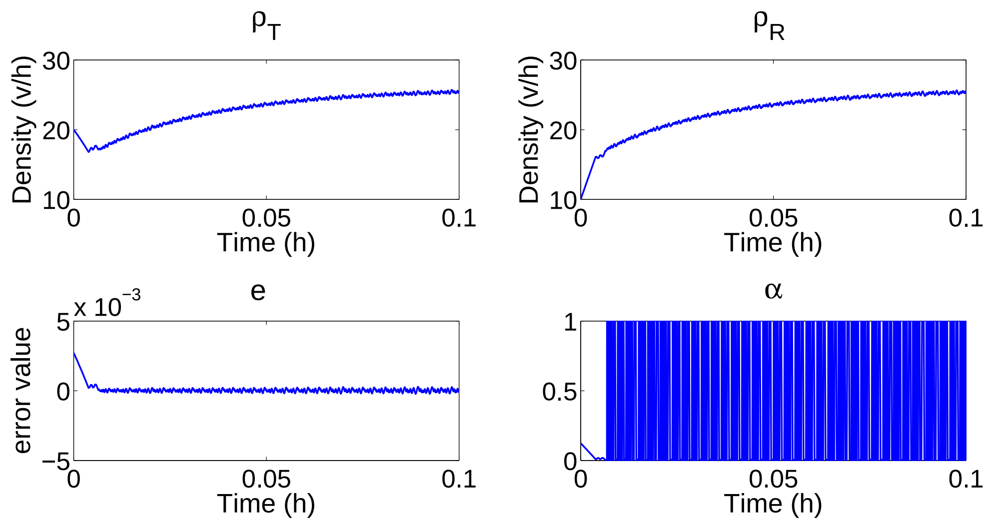


Figure 11.1: Simulation Result (with chattering) for $\gamma = 1$

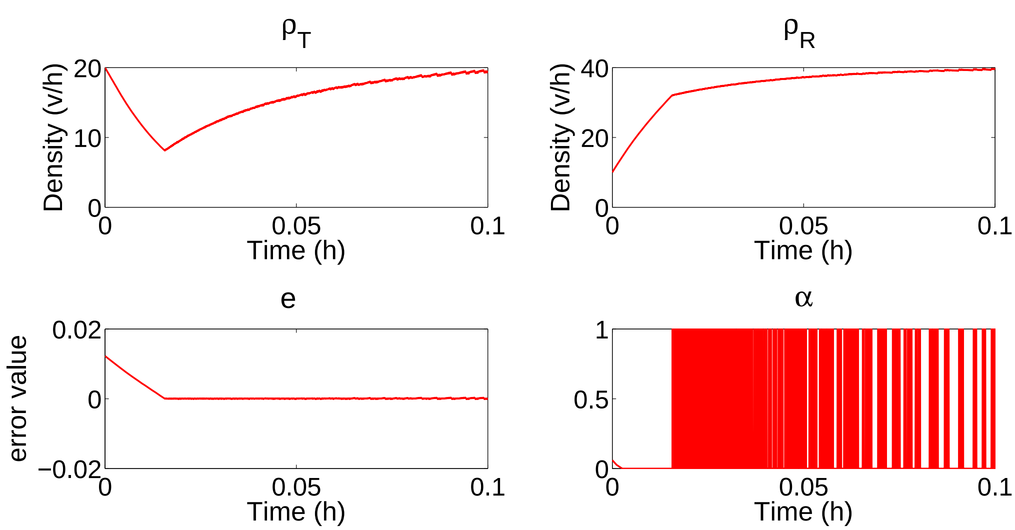


Figure 11.2: Simulation Result (with chattering) for $\gamma = 1.5$

$t \to \infty$, ρ_T and ρ_R attaining steady state value depending on value of γ. However, value of λ oscillates and never achieves steady state value.

This oscillatory behaviour is known as chattering. Chattering is undesirable in practice. There are many methods to reduce chattering. One of the method is to use

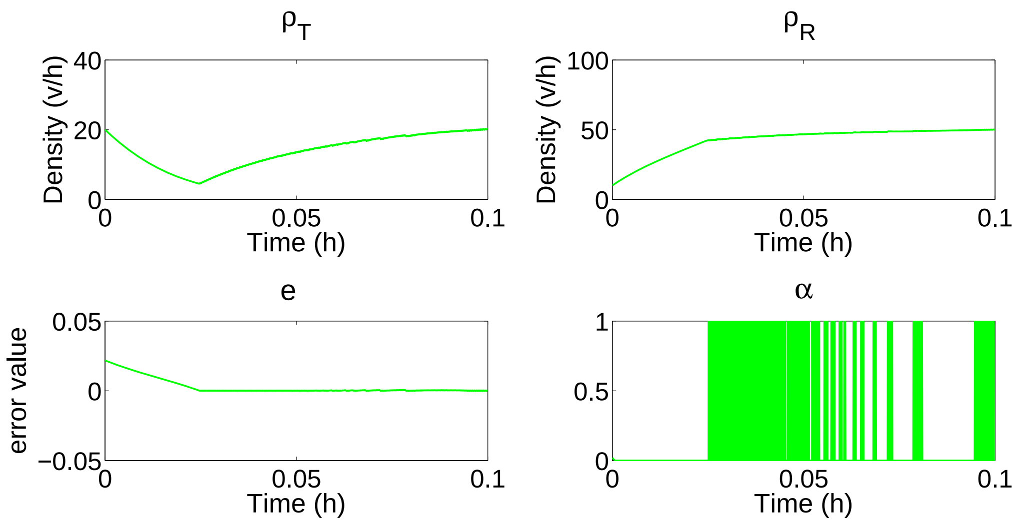


Figure 11.3: Simulation Result (with chattering) for $\gamma = 2$

a saturation function for the controller. Other method is to use low pass first order filter. To remove chattering we introduced a boundary layer about the zero error and a saturation function for that boundary while calculating the λ. The saturation function approximates the sgn term into a continuous function. Basically, this method removes or minimizes chattering by removing the discontinuity of the sgn function and introducing a continuous function in its place. The value of saturation function, here represented as 'sat' is either +1, -1 or e depending on the value of e and then this value is used to calculate the value of $\dot{e}(t) = sat * M$. This can be understood from Figure 11.4. This calculated value of $\dot{e}(t)$ is now use to do simulation which minimizes chattering.

Figures 11.5, 11.6 and 11.7 are the simulation results which show that the chattering has been minimized. The results are now satisfactory with each variable attaining

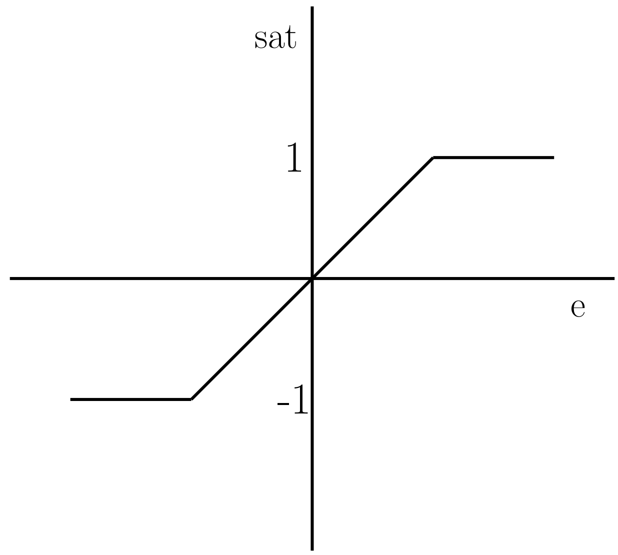


Figure 11.4: Saturation Function

desired results. Control variable λ also attains a steady state value.

Once we have the value of λ, toll price is calculated using equation (10.26). Figure 11.8 shows that the price results are consistent with the other variable values. We have calculated the toll price for values of $\gamma = 1$, $\gamma = 1.5$ and $\gamma = 2$ taking $a_1 = 1000$, $a_2 = 1$ and $a_3 = -4$. Traffic managers can adjust the toll price depending on the traffic scenario by changing a_1, a_2 and a_3 values.

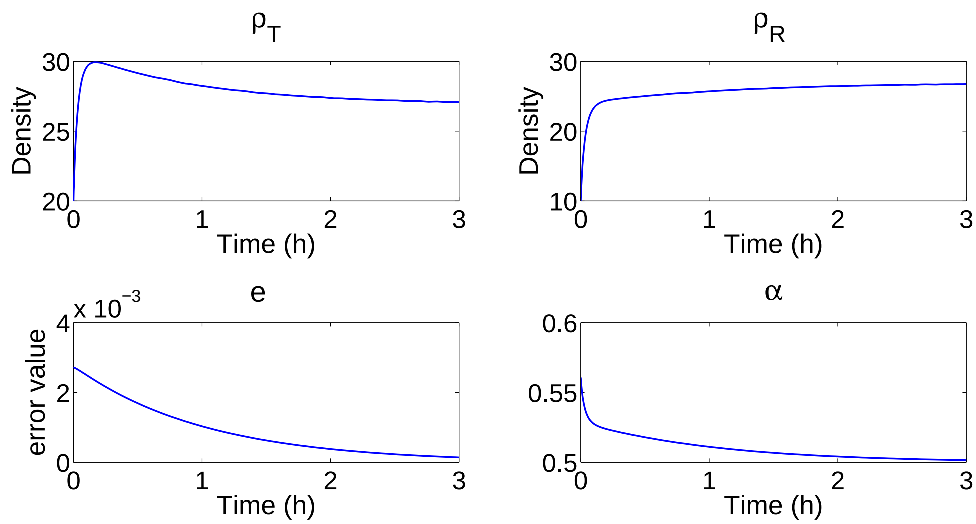


Figure 11.5: Simulation Result for $\gamma = 1$

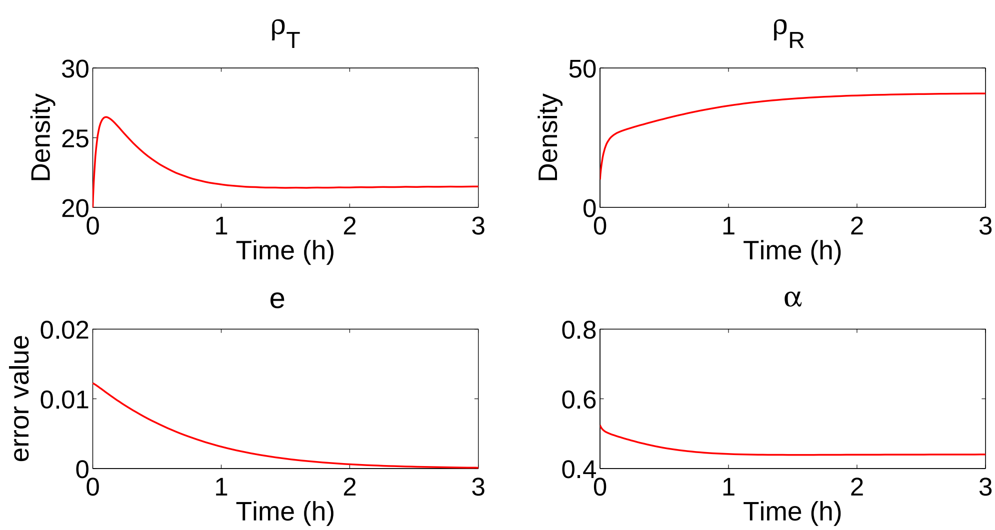


Figure 11.6: Simulation Result for $\gamma = 1.5$

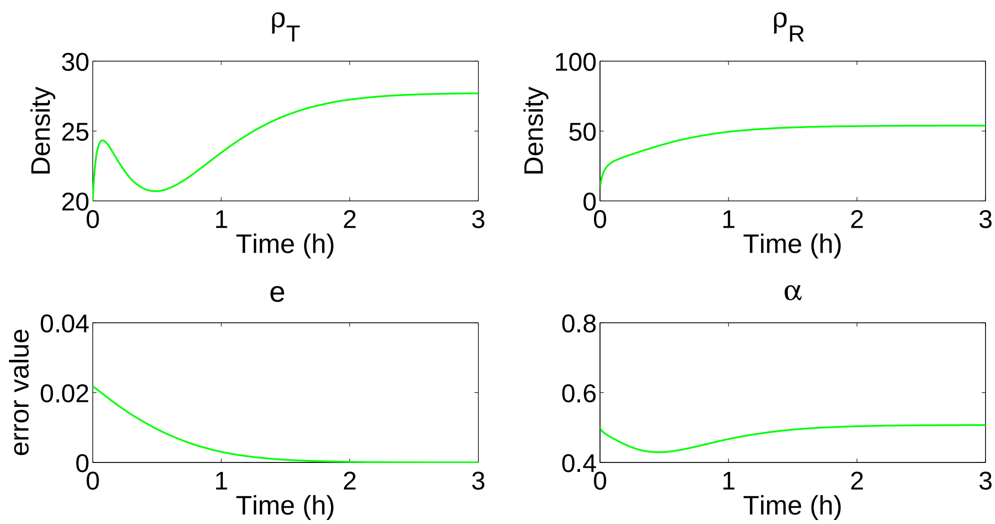
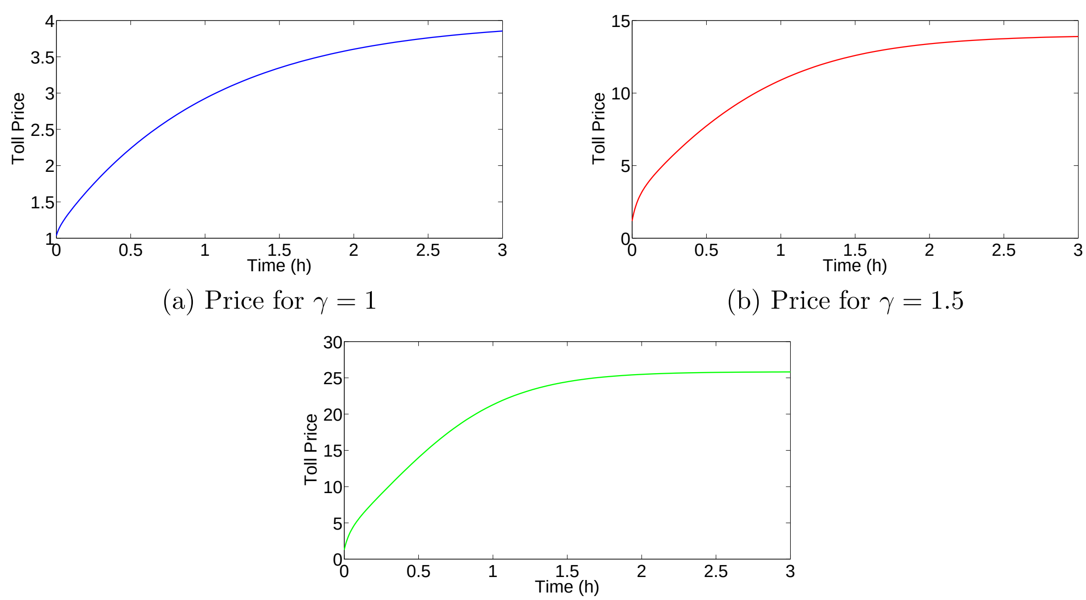


Figure 11.7: Simulation Result for $\gamma = 2$



(a) Price for $\gamma = 1$

(b) Price for $\gamma = 1.5$

(c) Price for $\gamma = 2$

Figure 11.8: Price Calculation

CHAPTER 12

CONCLUSION

This work presented an optimal control technique to approach congestion pricing problem using Hamilton Jacobi bellman equation. We have a non-linear system dynamics which is to be controlled to minimize the cost function. By using HJB we obtain the optimal control law at steady state which minimizes the cost function. Obtained control design utilizes the dynamic traffic conditions to determine the toll rate. Tolling price depends on many factors such as traffic condition, drivers' lane choice behavior. Simulation is also performed to verify the performance of the proposed control methodology. Simulations results validate the proposed control algorithm. However, in this part we have considered the simplest system model, but optimal control technique can be very well applied to general model with RF tags technology lane besides toll lane and regular lane.

BIBLIOGRAPHY

[1] B. Greenshields, W. Channing, H. Miller *et al.*, "A study of traffic capacity," in *Highway research board proceedings*, vol. 1935. National Research Council (USA), Highway Research Board, 1935.

[2] P. Kachroo, S. Agarwal, and S. Sastry, "Inverse problem for non-viscous mean field control: Example from traffic," *Automatic Control, IEEE Transactions on.*

[3] S. Agarwal, P. Kachroo, S. Contreras, and S. Sastry, "Feedback-coordinated ramp control of consecutive on-ramps using distributed modeling and godunov-based satisfiable allocation," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 16, no. 5, pp. 2384–2392, 2015.

[4] S. Agarwal, P. Kachroo, and S. Contreras, "A dynamic network modeling-based approach for traffic observability problem," *Intelligent Transportation Systems, IEEE Transactions on.*

[5] S. Contreras, P. Kachroo, and S. Agarwal, "Observability and sensor placement problem on highway segments: A traffic dynamics-based approach," *Intelligent Transportation Systems, IEEE Transactions on.*

[6] P. Verma, H. Yang, P. Kachroo, and S. Agarwal, "Modeling and estimation of the vehicle-miles traveled tax rate using stochastic differential equations."

[7] S. Agarwal, A. Sancheti, R. Khaddar, and P. Kachroo, "Geospatial framework for integration of transportation data using voronoi diagrams," in *Transportation Research Board 92nd Annual Meeting*, no. 13-5378, 2013.

[8] A. S. Krishen, P. Kachroo, S. Agarwal, S. S. Sastry, and M. Wilson, "Safety culture from an interdisciplinary perspective: Conceptualizing a hierarchical feedback-based transportation framework," *Transportation Journal*, vol. 54, no. 4, pp. 516–534, 2015.

[9] A. S. Krishen, S. Agarwal, P. Kachroo, and R. L. Raschke, "Framing the value and valuing the frame? algorithms for child safety seat use," *Journal of Business Research*, 2015.

[10] P. Maheshwari, A. Paz *et al.*, "Development of a methodology to evaluate projects using dynamic traffic assignment models," *Open Journal of Applied Sciences*, vol. 5, no. 02, p. 50, 2015.

[11] A. Paz, P. Maheshwari, P. Kachroo, and S. Ahmad, "Estimation of performance indices for the planning of sustainable transportation systems," *Advances in Fuzzy Systems*, vol. 2013, p. 2, 2013.

[12] S. Merrill, A. Paz, V. Molano, P. Maheshwari, P. P. Shrestha, R. Conover, and H. Stephen, "A land ferry system to alleviate increasing costs of maintaining the i-80 transportation corridor: An economic assessment," Tech. Rep., 2015.

[13] P. Maheshwari, R. Khaddar, P. Kachroo, and A. Paz, "Dynamic modeling of performance indices for planning of sustainable transportation systems," *Networks and Spatial Economics*, pp. 1–23, 2014.

[14] P. Maheshwari, P. Kachroo, A. Paz, and R. Khaddar, "Development of control models for the planning of sustainable transportation systems," *Transportation Research Part C: Emerging Technologies*, 2015.

[15] P. Maheshwari, "Development of a decision support framework for the planning of sustainable transportation systems," 2015.

[16] C. F. Daganzo, V. V. Gayah, and E. J. Gonzales, "Macroscopic relations of urban traffic variables: Bifurcations, multivaluedness and instability," *Transportation Research Part B: Methodological*, vol. 45, no. 1, pp. 278–288, 2011.

[17] N. Geroliminis and J. Sun, "Properties of a well-defined macroscopic fundamental diagram for urban traffic," *Transportation Research Part B: Methodological*, vol. 45, no. 3, pp. 605–617, 2011.

[18] N. Geroliminis and B. Boyacı, "The effect of variability of urban systems characteristics in the network capacity," *Transportation Research Part B: Methodological*, vol. 46, no. 10, pp. 1607–1623, 2012.

[19] Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabási, "Controllability of complex networks," *Nature*, vol. 473, no. 7346, pp. 167–173, 2011.

[20] J. Godfrey, "The mechanism of a road network," *Traffic Engineering & Control*, vol. 11, no. 7, pp. 323–327, 1969.

[21] R. Herman and I. Prigogine, "A two-fluid approach to town traffic," *Science*, vol. 204, no. 4389, pp. 148–151, 1979.

[22] N. Geroliminis and C. F. Daganzo, "Existence of urban-scale macroscopic fundamental diagrams: Some experimental findings," *Transportation Research Part B: Methodological*, vol. 42, no. 9, pp. 759–770, 2008.

[23] C. F. Daganzo and N. Geroliminis, "An analytical approximation for the macroscopic fundamental diagram of urban traffic," *Transportation Research Part B: Methodological*, vol. 42, no. 9, pp. 771–781, 2008.

[24] V. L. Knoop, H. van Lint, and S. P. Hoogendoorn, "Traffic dynamics: Its impact on the macroscopic fundamental diagram," *Physica A: Statistical Mechanics and its Applications*, vol. 438, pp. 236–250, 2015.

[25] J. Haddad, M. Ramezani, and N. Geroliminis, "Model predictive perimeter control for urban areas with macroscopic fundamental diagrams," in *American Control Conference (ACC), 2012*. IEEE, 2012, pp. 5757–5762.

[26] N. Geroliminis, J. Haddad, and M. Ramezani, "Optimal perimeter control for two urban regions with macroscopic fundamental diagrams: A model predictive approach," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 14, no. 1, pp. 348–359, 2013.

[27] P. Deo, B. De Schutter, and A. Hegyi, "Model predictive control for multi-class traffic flows," in *Control in Transportation Systems*, 2009, pp. 25–30.

[28] C. F. Daganzo, "Urban gridlock: macroscopic modeling and mitigation approaches," *Transportation Research Part B: Methodological*, vol. 41, no. 1, pp. 49–62, 2007.

[29] M. Keyvan-Ekbatani, A. Kouvelas, I. Papamichail, and M. Papageorgiou, "Exploiting the fundamental diagram of urban networks for feedback-based gating," *Transportation Research Part B: Methodological*, vol. 46, no. 10, pp. 1393–1403, 2012.

[30] N. Geroliminis and J. Sun, "Hysteresis phenomena of a macroscopic fundamental diagram in freeway networks," *Transportation Research Part A: Policy and Practice*, vol. 45, no. 9, pp. 966–979, 2011.

[31] Y. Ji and N. Geroliminis, "On the spatial partitioning of urban transportation networks," *Transportation Research Part B: Methodological*, vol. 46, no. 10, pp. 1639–1656, 2012.

[32] Y. Ji, J. Luo, and N. Geroliminis, "Empirical observations of congestion propagation and dynamic partitioning with probe data for large-scale systems," *Transportation Research Record: Journal of the Transportation Research Board*, no. 2422, pp. 1–11, 2014.

[33] K. Aboudolas and N. Geroliminis, "Perimeter and boundary flow control in multi-reservoir heterogeneous networks," *Transportation Research Part B: Methodological*, vol. 55, pp. 265–281, 2013.

[34] M. Keyvan-Ekbatani, M. Yildirimoglu, N. Geroliminis, and M. Papageorgiou, "Traffic signal perimeter control with multiple boundaries for large urban networks," in *Intelligent Transportation Systems-(ITSC), 2013 16th International IEEE Conference on.* IEEE, 2013, pp. 1004–1009.

[35] ——, "Multiple concentric gating traffic control in large-scale urban networks," *Intelligent Transportation Systems, IEEE Transactions on*, vol. 16, no. 4, pp. 2141–2154, 2015.

[36] M. Keyvan-Ekbatani, M. Papageorgiou, and V. L. Knoop, "Controller design for gating traffic control in presence of time-delay in urban road networks," *Transportation Research Part C: Emerging Technologies*, 2015.

[37] J. Haddad, "Robust constrained control of uncertain macroscopic fundamental diagram networks," *Transportation Research Part C: Emerging Technologies*, vol. 59, pp. 323–339, 2015.

[38] M. Ramezani, J. Haddad, and N. Geroliminis, "Dynamics of heterogeneity in urban networks: aggregated traffic modeling and hierarchical control," *Transportation Research Part B: Methodological*, vol. 74, pp. 1–19, 2015.

[39] M. E. Newman, “The structure and function of complex networks,” *SIAM review*, vol. 45, no. 2, pp. 167–256, 2003.

[40] L. Page, S. Brin, R. Motwani, and T. Winograd, “The pagerank citation ranking: Bringing order to the web.” Stanford InfoLab, Technical Report 1999-66, November 1999, previous number = SIDL-WP-1999-0120.

[41] F. Aurenhammer, “Voronoi diagramsa survey of a fundamental geometric data structure,” *ACM Computing Surveys (CSUR)*, vol. 23, no. 3, pp. 345–405, 1991.

[42] A. Okabe, B. Boots, K. Sugihara, and S. N. Chiu, *Spatial tessellations: concepts and applications of Voronoi diagrams.* John Wiley & Sons, 2009, vol. 501.

[43] M. Newman, A.-L. Barabasi, and D. J. Watts, *The structure and dynamics of networks.* Princeton University Press, 2006.

[44] R. Cohen and S. Havlin, *Complex networks: structure, robustness and function.* Cambridge University Press, 2010.

[45] M. Newman, *Networks: an introduction.* Oxford University Press, 2010.

[46] S. N. Dorogovtsev, *Lectures on complex networks.* Oxford University Press Oxford, 2010, vol. 24.

[47] P. Kachroo and K. M. Özbay, *Feedback ramp metering in intelligent transportation systems.* Springer Science & Business Media, 2011.

[48] ——, *Feedback control theory for dynamic traffic assignment.* Springer Science & Business Media, 2012.

[49] S. V. Ukkusuri and K. M. Özbay, *Advances in Dynamic Network Modeling in Complex Transportation Systems.* Springer, 2013.

[50] T. T. and V. S., "Design and Evaluation of Road Pricing: State-of-The-Art and Methodological Advances," *Netnomics*, no. 1, pp. 5–52, 2009.

[51] Y. H. and Z. X., "Determination of Optimal Toll Levels and Toll Locations of Alternative Congestion Pricing Schemes," *Proceedings of the 15th International Symposium on Transportation and Traffic Theory*, pp. 519–540, 2002.

[52] V. E.T, "Second Best Congestion Pricing in General Networks: Heuristic Algorithms for Finding Second Best Optimal Toll Levels and Toll Points," *Transportation Research Part B*, no. 36, pp. 707–729, 2002.

[53] H. D.W. and R. M.V., "Solving congestion Toll Pricing Models," *Equilibrium and Advanced Transportation Modeling. Kluwer Academic Publishers Dordrecht The Netherlands*, pp. 109–124, 1998.

[54] L. R., "Road Pricing Issues and Experiences in the US and Canada," *IMPRINT-EUROPE Fourth Seminar Implementing Pricing Policies in Transport: Phasing and Packaging*, pp. available on July 2009 at http://www.imprint–eu.org/public/Papers/IMPRINT4–lindsey–v2.pdf, 13-14 May 2003.

[55] O. J. and B. S., "Toll Financing in Norway: The Success, the Failures and Perspectives for the Future," *Transport Policy*, no. Vol. 9, pp. 253–260, 2002.

[56] G. M., "Congestion Management and Electronic Road Pricing in Singapore," *Journal of Transport Geography*, no. Vol. 10 Issue 1, pp. 29–38, 2002.

[57] L. T., "London Congestion Pricing Implications for Other Cities," *Victoria Transport Policy Institute (VTPI)*, p. available on July 2009 at http://www.vtpi.org/london.pdf, 2004.

[58] Q. Meng, Z. Liu, and S. Wang, "Optimal distance tolls under congestion pricing and continuously distributed value of time," *Transportation Research Part E: Logistics and Transportation Review*, vol. 48, no. 5, pp. 937–957, 2012.

[59] T. L. Friesz, D. Bernstein, and N. Kydes, "Dynamic congestion pricing in disequilibrium," *Networks and Spatial Economics*, vol. 4, no. 2, pp. 181–202, 2004.

[60] H. Yang, "Evaluating the benefits of a combined route guidance and road pricing system in a traffic network with recurrent congestion," *Transportation*, vol. 26, no. 3, pp. 299–322, 1999.

[61] H. Yang, Q. Meng, and D.-H. Lee, "Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions," *Transportation Research Part B: Methodological*, vol. 38, no. 6, pp. 477–493, 2004.

[62] Y. Zhao and K. M. Kockelman, "On-line marginal-cost pricing across networks: Incorporating heterogeneous users and stochastic equilibria," *Transportation Research Part B: Methodological*, vol. 40, no. 5, pp. 424–435, 2006.

[63] B. Hårsman and J. M. Quigley, "Political and public acceptability of congestion pricing: Ideology and self-interest," *Journal of Policy Analysis and Management*, vol. 29, no. 4, pp. 854–874, 2010.

[64] A. de Palma and R. Lindsey, "Traffic congestion pricing methodologies and technologies," *Transportation Research Part C: Emerging Technologies*, vol. 19, no. 6, pp. 1377–1399, 2011.

CURRICULUM VITAE

Graduate College
University of Nevada, Las Vegas

Saumya Gupta

Home Address:
1304 Rawhide St
Las Vegas, Nevada 89119

Degrees:
Bachelor of Technology, E&I, 2009
Uttar Pradesh Technical University, Lucknow, India

Master of Science, ECG, 2016
University of Nevada Las Vegas, Las Vegas, Nevada

Thesis Title: Complex Traffic Network Modeling & Area-wide Hierarchical Control

Thesis Examination Committee:
Chairperson, Dr. Pushkin Kachroo, Ph.D.
Committee Member, Dr. Ke-Xun Sun, Ph.D.
Committee Member, Dr. Yingtao Jiang, Ph.D.
Graduate Faculty Representative, Dr. Amei Amei, Ph.D.