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Variable Structure End Point Control of a Flexible Manipulator

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Abstract

We treat the question of control and stabilization of the elastic multibody system developed in the Phillips Laboratory, Edwards Air Force Base, California. The controlled output is judiciously chosen such that the zero dynamics are stable or almost stable. A variable structure control (VSC) law is derived for the end point trajectory control. Although, the VSC law accomplishes precise end point tracking, elastic modes are excited during the maneuver of the arm. A Linear stabilizer is designed for the final capture of the terminal state.

1 INTRODUCTION

Light-weight robotic arms are of considerable importance for space applications. In the Phillips Laboratory, Edwards Air Force Base, CA, a multibody system has been set up for the study of dynamics and control-structure interaction. This project is called the Planar Articulating Controls Experiment (PACE). The system has two elastic links and two revolute joints. The arm rotates in the horizontal plane on a smooth granite table.

Recently some theoretical and experimental studies related to end point control have been done. Inverse control systems and variable structure systems have been designed [1–4]. This research is related to end point trajectory control of the multibody system. For the trajectory control, a control law based on variable structure system (VSS) theory[12,13] is derived. A stabilizer is designed using pole assignment technique for vibration damping. Simulation results are presented to show that in the closed-loop system, precise trajectory tracking and vibration damping are accomplished in spite of the payload uncertainty.

2 MATHEMATICAL MODEL AND PROBLEM STATEMENT

The robotic arm is shown in the Fig.1. The elastic deformations can be expressed as

$$\begin{aligned} \delta_1(l_1,t) &= \sum_{i=1}^{n} \phi_{1i}(l_1) q_{1i}(t) \\ \delta_2(l_2,t) &= \sum_{i=1}^{n} \phi_{2i}(l_2) q_{2i}(t) \end{aligned} \tag{1}$$

Let $z = (\theta^T, q^T)$, $\theta = (\theta_1, \theta_2)^T$, $q = (q_{11}, \cdots, q_{1n}, q_{21}, \cdots, q_{2n})^T$, $u = (u_1, u_2)^T$,is the vector of joint torques, $B_1 = [I_{2\times 2}, O_{2\times 2n}]^T$ The equation of motion is given by

$$M(z)\ddot{z} + h_o(z, \dot{z}) + \partial P(z)/\partial z = B_1 u \tag{2}$$

Defining $x = (z^T, \dot{z}^T)^T$, one obtains a state variable representation of (2) given by

$$\dot{x} = f(x) + B(x)u \tag{3}$$

The controlled output vector is chosen as

$$y = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} \theta_1 + \alpha_1 \frac{\delta_1(L_1,t)}{L_1} \\ \theta_2 + \alpha_2 \frac{\delta_2(L_2,t)}{L_2} \end{bmatrix} \tag{4}$$

Let $y_r(t) = y_{r1}(t), y_{r2}(t)]^T$ be a given reference trajectory. We are interested in designing control system such that in the closed-loop system $y(t)$ tracks $y_r(t)$ and elastic vibration is suppressed.

3 VARIABLE STRUCTURE SYSTEM

Now we proceed to derive the variable structure system. The VSC law is a discontinuous function of state variables. For the design of the variable structure system (VSS), we choose a switching surface in the state space for the control function to have discontinuity, and the control law is chosen such that the trajectories of the system beginning from any initial condition are attracted towards this surface.

The switching surface $S = (S_1, S_2)^T$ is chosen of the form

$$S(\tilde{y}, \dot{\tilde{y}}) = \dot{\tilde{y}} + 2\zeta_e \omega_{ne} \tilde{y} + {\omega_{ne}}^2 w. \tag{5}$$

$$\dot{w} = \tilde{y}, w \in R^2 \tag{6}$$

where $\tilde{y} = (\tilde{y_1}, \tilde{y_2})^T = ((y_1 - y_r1), (y_2 - y_r2))^T$, $y = (y_1, y_2)^T$.

We note that when the trajectory is confined to the surface $S = 0$ during the sliding phase, one has

$$\ddot{\tilde{y}} + 2\zeta_e \omega_{ne} \dot{\tilde{y}} + {\omega_{ne}}^2 \tilde{y} = 0 \tag{7}$$

and $\tilde{y}(t) \rightarrow 0$, as $t \rightarrow \infty$, for $\zeta_e > 0, \omega_{ne} > 0$.

Now we choose the control law such that the trajectories are attracted towards $S = 0$ when $S(\tilde{y}(0), \dot{\tilde{y}}(0)) \neq 0$. Differentiating (5) and using (6) gives

$$\begin{aligned} \dot{S} &= C_1 h(z, \dot{z}) + B^*(z)u - \ddot{y}_r + 2\zeta_e \omega_{ne} \dot{\tilde{y}} + {\omega_{ne}}^2 \tilde{y} \\ &\triangleq a(z, \dot{z}) + B^*(z)u \end{aligned} \tag{8}$$

where $a(z, \dot{z}) = C_1 h(z, \dot{z}) - \ddot{y}_r + 2\zeta_e \omega_{ne} \dot{\tilde{y}} + \omega_{ne}{}^2 \tilde{y}$

We choose control law u such that $\dot{S} < 0$ if $S \neq 0$ and the trajectory reaches the surface $S = 0$ in a finite time. In view of (8), we select u of the form

$$u = B_n^{*-1}(z)[-a_n(x) - ksgn(S) + v] \tag{9}$$

where $v = (v_1, v_2)^T$ is the stabilization signal to be determined later.

4 VIBRATION STABILIZATION

Using the VSC law, one can track given reference trajectories. However, this excites the elastic modes of the arm.

Define the state vector ξ as

$$\xi = \begin{bmatrix} \Delta y^T & S^T & \Delta q^T & \Delta \dot{q}^T & \Delta w^T \end{bmatrix}^T \tag{10}$$

Then the linearized system is given by

$$\dot{\xi} = \tilde{A}\xi + \tilde{B}v \tag{11}$$

The zero dynamics of the system when $y = y^*{}_r$, are given by

$$\tilde{M}_{22}\Delta\ddot{q} + P_{qq}\Delta q = 0 \tag{12}$$

The stability of q-responses depends on the stability of the zero dynamics. For the final capture of the terminal state, a stabilization signal of the form

$$v = -\tilde{F}\xi \tag{13}$$

is chosen such that the closed-loop system matrix $\tilde{A}_c = (\tilde{A} - \tilde{B}\tilde{F})$ is Hurwitz.

5 DIGITAL SIMULATION RESULTS

This section explores the results of the digital simulations carried out for the end effector position. The values of α and n used for the simulation are 0.4 and 2 respectively.

A command trajectory $y_r(t)$ was generated to control $y(0) = 0$ to $y_r{}^*$. It was assumed that the given tip position corresponds to $y_r^* = (90°, 60°)^T$. The command trajectory was generated by a third order filter

$$\dddot{y}_r + P_2\ddot{y}_r + P_1\dot{y}_r + P_0(y_r - y^*) = 0 \tag{14}$$

The matrices P_i of the command generator are taken as $P_i = p_i \ I_{2\times 2}$, i=0,1,2 and are selected such that the poles associated with $y_{ri}(t)$, the i^{th} component of $y_r(t)$, are at

{ -2, -2 ± i2 }. The parameters were selected as $k = 100$, $\zeta_e = 0.707$ $\epsilon = 0.3$ and $\omega_{ne} = 3.5$ yielding poles associated with $\tilde{y}_i$, of values { -333.33, -2.47 ± i2.48 },

For the chosen feedback gains, the sets S_r and S_e of poles of (34) for $\alpha = 0.4$ are

$$S_r = [-333.33, -333.33, -2.47 \pm 2.48i, -2.47 \pm 2.48i] \tag{15}$$

$$S_e = [\pm 19.67i, \pm 71.73i, \pm 449.73i, \pm 459.96i]$$

The closed-loop system defined by (3), (6), (9) and (15) was digitally simulated (with α = 0.4) and the switching logic closes the stabilizer loop when the trajectory enters the neighborhood of the terminal value (3 seconds). The stabilizer dampens out the elastic mode oscillations. The simualtion results are shown in the Figure 2.

6 CONCLUSIONS

The control and stabilization of the multibody system developed in the Phillips Laboratory was considered. Based on the variable structure system theory control laws were derived for the control of the selected output variables. A linear stabilizer was designed for the final capture of the terminal state and vibration suppression.

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Model of the Robotic arm

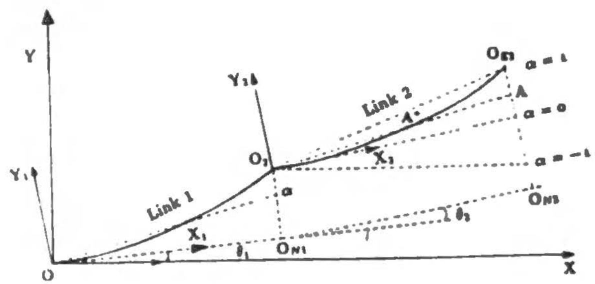

Figure 1

Trajectory tracking: Control with stabilization:nominal payload:
Outputs y_1 and y_2

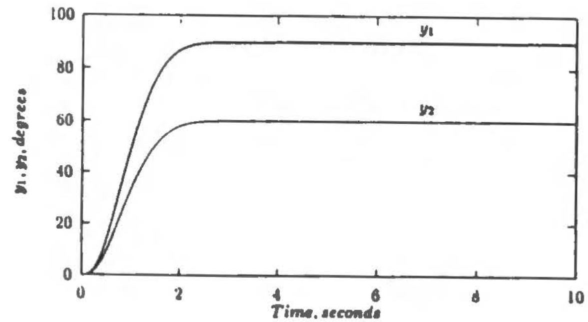

Figure 2

Trajectory tracking: Control with stabilization:nominal payload:
control Torques u_1 and u_2

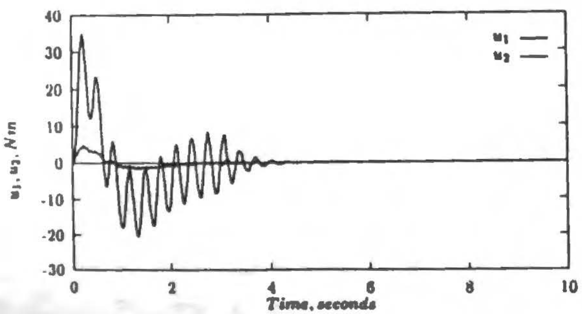

Figure 3

Trajectory tracking: Control with stabilization:nominal payload
Elastic deflections D_{1e} and D_{2e}

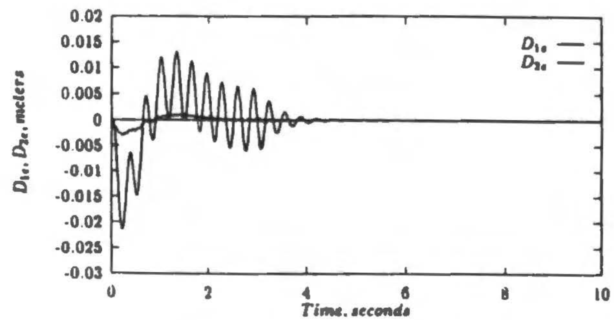

Figure 4